TMD Factorization and Renormalization at Next-to-Leading Power

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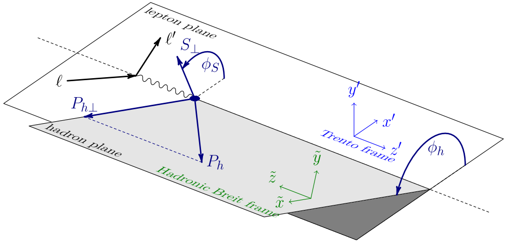
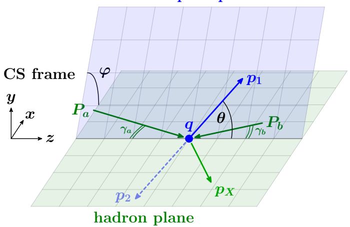
Intro: Azimuthal Asymmetries in SIDIS and Drell-Yan

Semi-Inclusive DIS (SIDIS)



Drell-Yan



- Polarized SIDIS $(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2},\ y = \frac{P_N\cdot q}{P_N\cdot p_\ell})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} \sim W_{UU,T} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h W_{UU}^{\cos\phi_h} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S W_{LT}^{\cos\phi_S} + \cdots$$

- Unpolarized Drell-Yan $pp \to \gamma^*/Z \to \ell^+\ell^-$ [notation from 2006.11382]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\,\mathrm{d}\varphi} \sim L_+\Big[(1+\cos^2\theta)W_{\mathrm{unpol}} + \sin(2\theta)\cos\varphi W_1 + \sin(2\theta)\sin\varphi W_6\Big]$$
$$+ L_-\Big[\sin\theta\cos\varphi W_3 + \sin\theta\sin\varphi W_7\Big] + \cdots$$

- Azimuthal Asymmetries: NLP structure functions in the TMD limit

History and Current Status

- Intrinsic transverse momentum of partons inside hadrons $\Rightarrow$
  $W_{UU}^{\cos\phi_h} \sim \mathcal{F}\Big[\frac{k_{Tx}}{Q} f_1 D_1\Big]$ [Cahn '78, '79]
- A more careful parton model calculation [Mulders, Tangerman '95]

$$\frac{x}{2z} W_{UU}^{\cos\phi_h} = \frac{2M_N}{Q}\mathcal{F}\Bigg\{\frac{-k_{Tx}}{M_N}\left[(f_1 + x\tilde{f}^{\perp})D_1 + \frac{M_h}{M_N} x h_1^{\perp}\frac{\tilde{H}}{z}\right] - \frac{p_{Tx}}{M_h}\left[\left(x\tilde{h} - \frac{k_T^2}{M_N^2}h_1^{\perp}\right)H_1^{\perp} + \frac{M_h}{M_N} f_1 \frac{\tilde{D}^{\perp}}{z}\right]\Bigg\}$$

  - ▷ Mismatch with perturbative results at tree level [Bacchetta et al '08]
  - ▷ Conjecture: Resolved by adding a LP soft function [Bacchetta et al '19]
- Systematic derivation of bare factorization for SIDIS to all order using soft-collinear effective theory (SCET) [our work '21]
- Recent progress from other groups

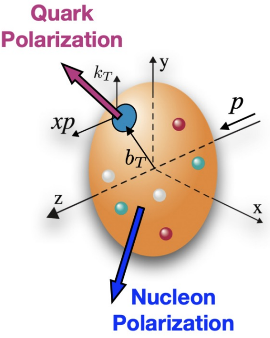
  [Balitsky, Tarasov '17] [Inglis-Whalen, Luke, Roy, Spourdalakis 21']

  [TMD operator expansion by Moos, Rodini, Scimemi, Vladimirov '21 '22 '23]

  [CSS formalism by Gamberg, Kang, Shao, Terry, Zhao 22']

Motivation

- Long-standing/challenging/interesting problem
- Effect can sometimes be large, e.g. Cahn effect
- 3D structure of hadrons
- Spin dependence
- ...



[TMD handbook '23]

More Motivation: Extending to Electroweak Currents

- Leading order for $W_{6,7}$ starts at $\mathcal{O}(\alpha_s^2)$:

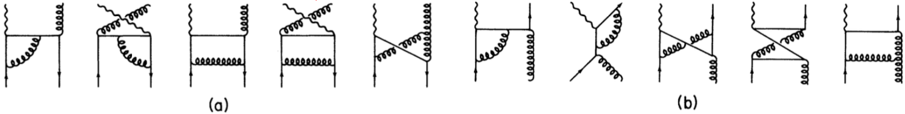
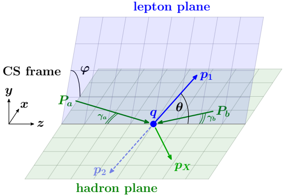
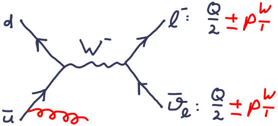


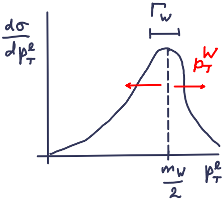
FIG. 1. Feynman diagrams contributing to the absorptive part of the parton scattering amplitude, (a) quark + antiquark → W + gluon and (b) quark + gluon → W + quark, in one-loop order. Wavy lines denote W and curly lines denote gluons. [Phys.Rev.Lett. 52 (1984) 1076]

$\Rightarrow$ Theoretical playground to check nontrivial qgq contribution to $W_{6,7}$

- m_W measurement at the LHC: need theory input since neutrino is lost







$m_W^{\text{LHCb}} = 80354 \pm 23_{\text{stat.}} \pm 10_{\text{exp.syst.}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}}$ MeV [LHCb, 2109.01113]

Most theory uncertainty comes from model for W_3!

Outline

I. Derivation of all-order bare factorization from SCET
(including Z/W production for Drell-Yan)

II. Nontrivial renormalization
of TMD quark-gluon-quark correlators

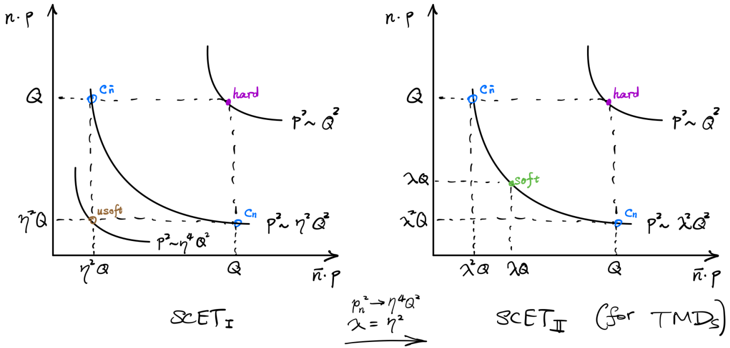
Part I

Factorization

Review of SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

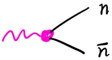
- EFT for collinear/soft d.o.f.s with power counting parameter $\lambda \ll 1$
- Lightcone coordinate $p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2} n\cdot p + p_\perp$
- n_i-collinear particles: $(n_i \cdot p, \bar{n}_i \cdot p, p_{n_i\perp}) \sim Q(\lambda^2, 1, \lambda)$
- Ultrasoft $k^\mu \sim Q\lambda^2$ in $\mathrm{SCET_I}$; Soft $k^\mu \sim Q\lambda$ in $\mathrm{SCET_{II}}$ (for TMD)
- For SIDIS, take $n_1 = n /\!\!/ P_N$ and $n_2 = \bar{n} /\!\!/ P_h$



Review of SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

- SCET Lagrangian $\mathcal{L}_{\text{SCET}_{\text{II}}} = \left(\sum_{i\geq 0} \mathcal{L}_{\text{hard}}^{(i)}\right) + \left(\sum_{i\geq 0} \mathcal{L}_{\text{dyn}}^{(i)} + \mathcal{L}_G^{(0)}\right)$
  - ▷ $\mathcal{L}_{\text{hard}}^{(i)} = \sum_k C_k^{(i)} \mathcal{O}_k^{(i)} = \frac{\mathrm{i}e^2}{Q^2} J_{\ell\ell'\mu} \sum_k J_k^{(i)\mu}$, Hard scattering operators



constructed by $\int$out offshell modes above $\lambda^2 Q^2$

  - ▷ $\mathcal{L}_{\text{dyn}}^{(0)} = \mathcal{L}_n^{(0)} + \mathcal{L}_{\bar{n}}^{(0)} + \mathcal{L}_s^{(0)}$, Collinear and soft dynamics factorize

while $\mathcal{L}_{\text{dyn}}^{(i)}$ with $i \geq 1$ can be more involved

  - ▷ $\mathcal{L}_{\text{G}}^{(0)}$, Glauber $(\lambda^2, \lambda^2, \lambda)$: connect different sectors

Factorization = Total Glauber contribution vanishes

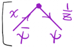
  - Here we assume $\mathcal{L}_G^{(0)}$ doesn't spoil factorization (left for future work)

- Building blocks: Collinear fields $\chi_n = W_n^\dagger \xi_n$, $\mathcal{B}_{n\perp}^\mu = \frac{1}{g}\left[W_n^\dagger \, \mathrm{i}D_{n\perp}^\mu W_n\right] \sim \lambda$

  Soft quark and gluon $\psi_{s(n)} \sim \lambda^{3/2}$, $\mathcal{B}_{s(n)}^\mu \sim \lambda$

  Momentum operators $\mathcal{P}_\perp$, $n \cdot \partial_s$, $\bar{n} \cdot \partial_s \sim \lambda$, ...

Warm Up: Leading Power (LP) TMD Factorization

LP current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S^\dagger_{\bar n} S_n]\, \chi^\beta_{n,\omega_a} \sim C_f^{(0)}(Q) \Big[$ 

(+ Wilson lines) $\Big]$

- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\,\mu} | h, X\rangle \langle h, X | J^{(0)\nu} | N\rangle$
- Collinear fields yield quark correlators

$$\hat B_f^{\beta'\beta}(x, \vec b_T) = \Big\langle N \Big| \bar\chi^\beta_n(b_\perp)\, \delta(\omega_a - \overline{\mathcal{P}}_n)\, \chi_n^{\beta'}(0) \Big| N \Big\rangle$$

$$\hat{\mathcal{G}}_f^{\alpha\alpha'}(z, \vec b_T) = \frac{1}{2z} \sum\!\!\!\!\!\!\int_X \Big\langle 0 \Big| \delta(\omega_b - \overline{\mathcal{P}}_{\bar n})\, \chi^\alpha_{\bar n}(b_\perp) \Big| h, X_{\bar n} \Big\rangle \Big\langle h, X_{\bar n} \Big| \bar\chi^{\alpha'}_{\bar n}(0) \Big| 0 \Big\rangle$$

- Soft Wilson lines $\Rightarrow \mathcal{S}(b_T) = \frac{1}{N_c} \mathrm{tr} \Big\langle 0 \Big| \big[S_n^\dagger S_{\bar n}\big](b_\perp)\, \big[S^\dagger_{\bar n} S_n\big](0) \Big| 0 \Big\rangle$
- Combine into the quark correlators $B_f^{\beta'\beta} = \hat B_f^{\beta'\beta} \sqrt{\mathcal{S}}\,, \quad \mathcal{G}_f^{\alpha'\alpha} = \hat{\mathcal{G}}_f^{\alpha'\alpha} \sqrt{\mathcal{S}}$

$\Rightarrow$ Factorized LP hadronic tensor where $\mathcal{H}_f^{(0)}(Q) = \big|C_f^{(0)}(Q)\big|^2$

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec q_T \cdot \vec b_T}\, \mathcal{H}_f^{(0)}(Q)\, \mathrm{Tr}\Big[B_f(x, \vec b_T)\, \gamma_\perp^\mu\, \mathcal{G}_f(z, \vec b_T)\, \gamma_\perp^\nu\Big]\,.$$

- For Z/W boson, replace $\gamma_\perp^\mu C_f^{(0)}$ by $\gamma_\perp^\mu (v_q + a_q \gamma_5) C_f^{(0)}$ or $\gamma_\perp^\mu (1 - \gamma_5) C_{ff'}^{(0)}$

Warm Up: LP TMD Factorization from SCET

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \, \mathcal{H}_f^{(0)}(Q) \, \mathrm{Tr}\left[B_f(x, \vec{b}_T) \, \gamma_\perp^\mu \, \mathcal{G}_f(z, \vec{b}_T) \, \gamma_\perp^\nu \right] .$$

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$B_f^{\beta'\beta}(x, \vec{p}_T) = \frac{1}{4}\left\{ f_1 \not{n} + \mathrm{i} h_1^\perp \frac{[\not{p}_\perp, \not{n}]}{2M_N} \right\}^{\beta'\beta} + \dots ,$$

$$\mathcal{G}_f^{\alpha'\alpha}(z, \vec{k}_T) = \frac{1}{4}\left\{ D_1 \not{\bar{n}} + \mathrm{i} H_1^\perp \frac{[\not{k}_\perp, \not{\bar{n}}]}{2M_h} \right\}^{\alpha'\alpha}$$

- Contract $W^{(0)\mu\nu}$ with $P_{-1}^{(0)\mu\nu} = x^\mu x^\nu + y^\mu y^\nu$, $P_3^{(0)\mu\nu} = x^\mu x^\nu - y^\mu y^\nu$,

$$W_{UU,T}^{(0)} = \mathcal{F}\left[\mathcal{H}^{(0)} \, f_1 D_1 \right] ,$$

$$W_{UU}^{\cos(2\phi_h)\,(0)} = \mathcal{F}\left[-\frac{2\, p_{Tx} \, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \mathcal{H}^{(0)} \, h_1^\perp H_1^\perp \right] ,$$

[Collins, SCET, ...]

$$\mathcal{F}[\omega \, \mathcal{H} \, g \, D] = 2z \sum_f \int \mathrm{d}^2 p_T \, \mathrm{d}^2 k_T \, \delta^2\Big(\vec{q}_T + \vec{p}_T - \vec{k}_T \Big) \, \omega(\vec{p}_T, \vec{k}_T) \, \mathcal{H}_f(Q) \, g_f(x, p_T) \, D_f(z, k_T)$$

TMD Factorization at Next-to-Leading Power (NLP)

- Kinematic corrections: trivial, proportional to LP $W_{UU,T}$, $W_{UU}^{\cos 2\phi_h}$
- NLP dynamic Lagrangian contributions vanish
- NLP current contributions
  - Soft currents: vanish, but play an important role in ren.
    e.g. Currents involving $\mathcal{B}_{s\perp}^{(n_i)\mu}$ yield
    $$\frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger\, S_{\bar n}\right](b_\perp)\left[S_{\bar n}^\dagger\, S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar n)\rho} S_{\bar n}^\dagger\, S_n\right](0)\middle|0\right\rangle = 0$$
  - Collinear currents

$$J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a}\bar\chi_{\bar n,\omega_b}\left[S_{\bar n}^\dagger S_n\right]\gamma^\mu\, \mathcal{P}\!\!\!/_\perp\, \bar n\!\!\!/\chi_{n,\omega_a} + \text{h.c.}$$

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar n^\mu)\int \mathrm{d}\omega_a \mathrm{d}\omega_b \mathrm{d}\omega_c\, C_f^{(1)}(Q, \xi = \omega_c/Q)$$

$$\times\Big[\delta(\omega_a+\omega_c-Q)\,\delta(\omega_b-Q)\,\bar\chi_{\bar n,\omega_b}\left[S_{\bar n}^\dagger S_n\right]\mathcal{B}\!\!\!/_{\perp n,-\omega_c}\chi_{n,\omega_a}$$

$$+\,\delta(\omega_a-Q)\,\delta(\omega_b+\omega_c-Q)\,\bar\chi_{\bar n,\omega_b}\mathcal{B}\!\!\!/_{\perp \bar n,\omega_c}\left[S_{\bar n}^\dagger S_n\right]\chi_{n,\omega_a}\Big]$$

$\sim\; \mathcal{C}_f^{(1)}(Q,\xi)\Big[$ 

$\Big]\frac{1}{z} + x\Big[$ 

$\frac{1}{z}$ (+ Wilson lines)$\Big]$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

- $\mathcal{P}_\perp$ current $J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a} \bar{\chi}_{\bar{n},\omega_b} [S_{\bar{n}}^\dagger S_n] \gamma^\mu \not{\mathcal{P}}_\perp \not{\bar{n}} \chi_{n,\omega_a}$ + h.c.
- $\left\langle J^{(0)\mu} J_{\mathcal{P}}^{(1)\nu} \right\rangle$ gives

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c} \sum_f \int \mathrm{d}^2\vec{b}_T \, \mathcal{H}_f^{(0)}(Q)$$
$$\times \left\{ \mathrm{Tr}\left[B_{\mathcal{P}f}(x,\vec{b}_T)\, \gamma^\mu \, \mathcal{G}_f(z,\vec{b}_T)\, \gamma^\nu \right] + \mathrm{Tr}\left[B_f(x,\vec{b}_T)\, \gamma^\mu \, \mathcal{G}_{\mathcal{P}f}(z,\vec{b}_T)\, \gamma^\nu \right] \right\} .$$

- $\mathcal{P}_\perp \sim \mathrm{i}\partial_\perp \Rightarrow$ Same leading power functions appear

$$B_{\mathcal{P}f}(x,\vec{b}_T) = \frac{-\mathrm{i}}{2Q} \frac{\partial}{\partial b_\perp^\rho} \left[\gamma_\perp^\rho \not{\bar{n}} \,,\, B_f(x,\vec{b}_T) \right]$$
$$= \frac{M_N}{2Q} \left\{ -\mathrm{i} M_N f_1^{(1)} \not{b}_\perp - \frac{\mathrm{i}}{4} h_1^{\perp\,(0')} [\not{n}, \not{\bar{n}}] \right\} + \cdots$$

$$\mathcal{G}_{\mathcal{P}\,f}(z,\vec{b}_T) = \frac{\mathrm{i}}{2Q} \frac{\partial}{\partial b_\perp^\rho} \left[\gamma_\perp^\rho \not{n} \,,\, \mathcal{G}_{h/f}(x,\vec{b}_T) \right]$$
$$= \frac{M_h}{2Q} \left\{ \mathrm{i} D_1^{(1)} M_h \not{b}_\perp + \frac{\mathrm{i}}{4} H_1^{\perp\,(0')} [\not{n}, \not{\bar{n}}] \right\} .$$

Subleading Current: with $\mathcal{B}_{n_i\perp}$ Insertion

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar{n}^\mu)\int \mathrm{d}\omega_a \mathrm{d}\omega_b \mathrm{d}\omega_c\, C_f^{(1)}(Q,\xi)\Big[\delta(\omega_a+\omega_c-Q)\,\delta(\omega_b-Q)\,\bar{\chi}_{\bar{n},\omega_b}[S_{\bar{n}}^\dagger S_n]\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$
$$+\,\delta(\omega_a-Q)\,\delta(\omega_b+\omega_c-Q)\,\bar{\chi}_{\bar{n},\omega_b}\not{\mathcal{B}}_{\perp\bar{n},\omega_c}[S_{\bar{n}}^\dagger S_n]\chi_{n,\omega_a}\Big]$$

- $\left\langle J^{(0)\dagger\mu} J_{\mathcal{B}}^{(1)\nu}\right\rangle$ gives ($\xi = \omega_c/Q$, energy fraction of the collinear gluon)

$$W_{\mathcal{B}}^{(1)\mu\nu} = \frac{2z}{Q}\sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\int \mathrm{d}\xi\, \mathcal{H}^{(1)}(Q,\xi)(n^\mu+\bar{n}^\mu)$$
$$\times\,\mathrm{Tr}\Big[\tilde{B}_{\mathcal{B}\,f}^{\rho}(x,\xi,\vec{b}_T)\,\gamma_\rho\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma_\perp^\nu + B_f(x,\vec{b}_T)\,\gamma_\perp^\nu\,\tilde{\mathcal{G}}_{\mathcal{B}\,f}^{\rho}(z,\xi,\vec{b}_T)\,\gamma_\rho\Big] + \text{h.c.}\,.$$

- Hard function $\mathcal{H}^{(1)}(Q,\xi) = C_f^{(1)}(Q,\xi)\,C_f^{(0)}(Q)$
- The TMD qgq correlators are defined as

$$\tilde{B}_{\mathcal{B}\,f}^{\rho\,\beta'\beta}(x,\xi,\vec{b}_T) \equiv Q\,\langle N|\,\big[\bar{\chi}_{n,\omega_a}^{\beta}\,\mathcal{B}_{\perp n,-\omega_c}^{\rho}\big](b_\perp^\mu)\,\chi_n^{\beta'}(0)\,|N\rangle\,\sqrt{S(b_T)}\,,$$
$$\tilde{\mathcal{G}}_{\mathcal{B}\,\bar{f}}^{\rho\,\beta\beta'}(z,\xi,\vec{b}_T) \equiv \frac{Q}{2z}\sum\!\!\!\!\!\!\!\int_{X_{\bar{n}}}\langle 0|\,\big[\bar{\chi}_{\bar{n},\omega_b}^{\beta}\,\mathcal{B}_{\perp\bar{n},\omega_c}^{\rho}\big](b_\perp^\mu)\,|h,X_{\bar{n}}\rangle\,\langle h,X_{\bar{n}}|\,\chi_{\bar{n}}^{\beta'}(0)\,|0\rangle\,\sqrt{S(b_T)}$$

can be decomposed as [Boer, Mulders, Pijlman '03; Bacchetta, Mulders, Pijlman '04]

$$\tilde{B}_{\mathcal{B}\,f/N}^{\rho}(x,\xi,\vec{b}_T) = \frac{M_N}{4P_N^-}\Big\{-\mathrm{i}M_N\,\underline{\tilde{f}}^{\perp(1)}\,b_{\perp\sigma}\,(g_\perp^{\rho\sigma}-\mathrm{i}\epsilon_\perp^{\rho\sigma}\gamma_5) + \underline{\tilde{h}}\,\mathrm{i}\gamma_\perp^\rho\Big\} + \dots$$
$$= \frac{M_N}{4P_N^-}\Big\{-\mathrm{i}M_N\Big(\tilde{f}^{\perp(1)}-\mathrm{i}\tilde{g}^{\perp(1)}\Big)\,b_{\perp\sigma}\,(g_\perp^{\rho\sigma}-\mathrm{i}\epsilon_\perp^{\rho\sigma}\gamma_5) + \Big(\tilde{h}+\mathrm{i}\tilde{e}\Big)\mathrm{i}\gamma_\perp^\rho\Big\} + \dots\,.$$

Results: Examples of NLP Factorized Asymmetries

SIDIS (including kinematic corrections, $\mathcal{P}_\perp$, $\mathcal{B}_{n_i\perp}$ operator contributions)

$$
\begin{aligned}
W_{UU}^{\cos\phi_h} = \mathcal{F}\Big\{ & \frac{q_T}{Q}\,\mathcal{H}^{(0)}\left[-f_1 D_1 + h_1^{\perp(1)} H_1^{\perp(1)}\right] \\
& + \mathcal{H}^{(0)}\left[-\frac{M_N}{Q} f_1^{(1)} D_1 - \frac{M_h}{Q} f_1 D_1^{(1)} + \frac{M_N}{Q} h_1^{\perp(0')} H_1^{\perp(1)} + \frac{M_h}{Q} h_1^{\perp(1)} H_1^{\perp(0')}\right]\Big\} \\
& - \Re\left[\mathcal{H}^{(1)}\left[\frac{2xM_N}{Q}\left(\underline{\tilde{f}}^{\perp(1)} D_1 + \underline{\tilde{h}}\, H_1^{\perp(1)}\right) + \frac{2M_h}{zQ}\left(f_1 \underline{\tilde{D}}^{\perp(1)} + h_1^{\perp(1)} \underline{\tilde{H}}\right)\right]\right]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}[\mathcal{H}\, g^{(n)} D^{(m)}] = 2z \sum_f \int \mathrm{d}\xi\, \mathcal{H}_f(q^+ q^-, \xi) \int_0^\infty \frac{\mathrm{d}b_T\, b_T}{2\pi} (M_N b_T)^n (-M_h b_T)^m J_{n+m}(b_T q_T) \\
\times\; g_f^{(n)}(x, (\xi), b_T)\, D_f^{(m)}(z, (\xi), b_T) + (f \to \bar{f})
\end{aligned}
$$

Examples of NLP Factorized Asymmetries

Drell-Yan (in the CS frame)

$$W_3 = \mathcal{F}\Big\{-\frac{M_N}{Q}\Big[\mathcal{H}_4^{(0)} f_1^{(1)} f_1 - \mathcal{H}_4^{(0)} f_1 f_1^{(1)} - \mathcal{H}_5^{(0)} h_1^{\perp(1)} h_1^{\perp(0')} + \mathcal{H}_5^{(0)} h_1^{\perp(0')} h_1^{\perp(1)}\Big]$$
$$-\frac{2M_N}{Q}\Re\Big[x_a \mathcal{H}_\times^{(1)} \underline{\tilde{f}}^{\perp(1)} f_1 - x_b \mathcal{H}_\times^{(1)} f_1 \underline{\tilde{f}}^{\perp(1)} + x_b \mathcal{H}_\otimes^{(1)} h_1^{\perp(1)} \underline{\tilde{h}} - x_a \mathcal{H}_\otimes^{(1)} \underline{\tilde{h}}\, h_1^{\perp(1)}\Big]\Big\},$$

$$W_6 = \mathcal{F}\Big\{-\frac{2M_N}{Q}\Im\Big[x_a \mathcal{H}_\times^{(1)} \underline{\tilde{f}}^{\perp(1)} f_1 - x_b \mathcal{H}_\times^{(1)} f_1 \underline{\tilde{f}}^{\perp(1)} + x_b \mathcal{H}_\otimes^{(1)} h_1^{\perp(1)} \underline{\tilde{h}} - x_a \mathcal{H}_\otimes^{(1)} \underline{\tilde{h}}\, h_1^{\perp(1)}\Big]\Big\}$$

- For photon, $W_3 = W_6 = 0$ (P-odd hard functions vanish)
- For $W^\pm$, $\mathcal{H}_5^{(0)} = \mathcal{H}_\otimes^{(1)} = 0$ (Boer-Mulders effects & NLP analog vanish)

$$\mathcal{H}_{4\,W^+W^+\,f\bar{f}'} = \frac{4\pi\alpha_{\rm em}}{N_c}\frac{|V_{ff'}|^2}{\sin^2\theta_w}|C_q|^2\,, \quad \mathcal{H}^{(1)}_{\times\,W^+W^+\,f\bar{f}'} = \frac{4\pi\alpha_{\rm em}}{N_c}\frac{|V_{ff'}|^2}{\sin^2\theta_w}C_q^* C_q^{(1)}$$

- W_6 starts at $\mathcal{O}(\alpha_s^2)$: probe the most non-trivial part of qgq, Interesting to check with fixed-order calculation

Part II

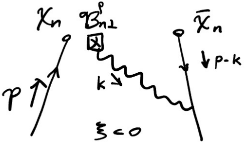
Renormalization

Novel Rapidity Divergence

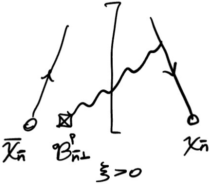
$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{SIDIS}} \sim \hat{t}^{\nu} \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\overbrace{\tilde{B}^{\rho}_{\mathcal{B}\ f}(x,\xi,\vec{b}_T)\, \gamma^{\mu}_{\perp}\, \mathcal{G}(z,\vec{b}_T)\, \gamma_{\perp\rho}}^{+\frac{1}{\eta}} + \overbrace{B(x,\vec{b}_T)\, \gamma^{\mu}_{\perp}\, \tilde{\mathcal{G}}^{\rho}_{\mathcal{B}}(z,\xi,\vec{b}_T)\, \gamma_{\perp\rho}}^{-\frac{1}{\eta}} \Big]$$

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\ f/N}(x,\xi,\vec{b}_T) \equiv \langle N|\, \bar{\chi}^{\beta}_{n}(b^{\mu}_{\perp})\, \big[\mathcal{B}^{\rho}_{\perp n,-\xi Q}\, \chi^{\beta'}_{n,(1-\xi)Q}\big](0)\, |N\rangle\, \sqrt{S(b_T)}\,,$$

At leading order (LO), matching onto quark PDF $f_{q/N}(\frac{x}{z})$, FF $d_{h/q}(\frac{z_h}{z})$



$$= -\alpha_s 2C_F \underbrace{\frac{\theta(-\xi)}{(-\xi)^{1+\eta}}}_{= -\frac{\delta(\xi)}{\eta} + \mathcal{L}_0(-\xi)} \left(\frac{Q}{\nu}\right)^{-\eta} \delta\left(\frac{1-z}{z}+\xi\right) \underbrace{\mu^{2\epsilon}\int d^{2-2\epsilon}k_\perp (i k^{\rho}_{\perp})}_{\equiv \mathcal{I}^{\rho}_{NLP}(b_\perp) = \frac{b^{\rho}_{\perp}}{b^{2}_{\perp}}\frac{1}{2\pi} + \mathcal{O}(\epsilon)} + \mathcal{O}(\xi^0)$$



$$= \alpha_s 2C_F \underbrace{\frac{\theta(\xi)}{\xi^{1+\eta}}}_{= -\frac{\delta(\xi)}{\eta} + \mathcal{L}_0(\xi)} \left(\frac{Q}{\nu}\right)^{-\eta} \delta(1-\xi-z)\, \mathcal{I}^{\rho}_{NLP}(b_\perp) + \mathcal{O}(\xi^0)$$

$$\mathcal{L}_0(x) \equiv \left[\frac{\theta(x)}{x}\right]_+$$

- The two terms in $[\ldots]$ are individually rapidity divergent, although divergence cancels in $W^{(1)\mu\nu}_{\mathcal{B}}$ [first observed at LO in Rodini, Vladimirov '22]

Novel Rapidity Divergence

$+\frac{1}{\eta}$ $\qquad$ $-\frac{1}{\eta}$

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{SIDIS}} \sim \hat{t}^\nu \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\,\gamma^\mu_\perp\,\mathcal{G}(z,\vec{b}_T)\,\gamma_{\perp\rho} + B(x,\vec{b}_T)\,\gamma^\mu_\perp\,\tilde{\mathcal{G}}^\rho_{\mathcal{B}}(z,\xi,\vec{b}_T)\,\gamma_{\perp\rho}\Big]$$

where the quark-gluon-quark (qgq) correlators are defined as

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f/N}(x,\xi,\vec{b}_T) \equiv \langle N|\,\bar{\chi}^\beta_n(b^\mu_\perp)\,\big[\mathcal{B}^\rho_{\perp n,-\xi Q}\,\chi^{\beta'}_{n,(1-\xi)Q}\big](0)\,|N\rangle\,\sqrt{S(b_T)}\,,$$

- Rapidity poles start at LO, which is $\mathcal{O}(\alpha_s)$!

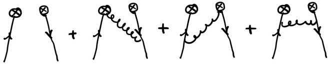
$\Rightarrow$ Can never be canceled by a multiplicative counterterm $Z = 1 + \mathcal{O}(\alpha_s)$

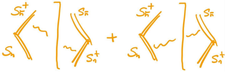
$\Rightarrow$ Need additive counterterm to get separately finite matrix elements
[LO counterterm involving $\partial^\rho\gamma_\zeta^{1-\mathrm{loop}}$ proposed in Rodini, Vladimirov '22]

- Additive counterterm also in subleading $\mathrm{SCET_I}$ applications: dijet [Moult et al '18], threshold [Beneke et al '18], ...
- Here $\mathrm{SCET_{II}}$ has $1/\eta$'s at $\mathcal{O}(\alpha_s\lambda)$
  See also $\mathrm{SCET_{II}}$ @ $\mathcal{O}(\alpha_s\lambda^2)$: q_T [Ebert et al '18], EEC [Moult et al '19], $h\to\gamma\gamma$ [Liu et al '19], ...

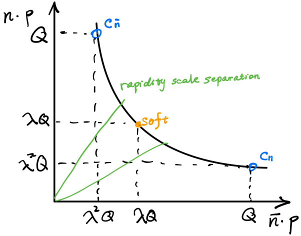
Rap. Div. Cancellation: Use Soft Matrix Elements (ME)

- At LP, rap. div. cancels multiplicatively (eg. η reg.)

$\hat{B}_{q/q} =$  $+\mathcal{O}(\alpha_s^2) = 1 - \alpha_s 4C_F \frac{1}{\eta}\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0) + \mathcal{O}(\alpha_s^2)$

$S = 1+$  $+\mathcal{O}(\alpha_s^2) = 1 + \alpha_s 8C_F \frac{1}{\eta}\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0) + \mathcal{O}(\alpha_s^2)$

$\Rightarrow\ \hat{B}_{q/q}\sqrt{S} \sim \mathcal{O}(\eta^0)$



- Idea: exploit individual pieces of vanishing NLP soft contribution

Construction of the "Counterterm"

$$W_{\mathcal{B}}^{(1)\mu\nu} \sim \hat{t}^{\nu} \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\tilde{B}_{\mathcal{B}}^{\rho}(x,\xi,\vec{b}_T)\, \gamma_{\perp}^{\mu}\, \mathcal{G}(z,\vec{b}_T)\, \gamma_{\perp\rho} + B(x,\vec{b}_T)\, \gamma_{\perp}^{\mu}\, \tilde{\mathcal{G}}_{\mathcal{B}}^{\rho}(z,\xi,\vec{b}_T)\, \gamma_{\perp\rho}\Big]\,.$$

- Use the fact that rap. div. cancels between the two terms in $W_{\mathcal{B}}^{(1)}$
- Can take $\mathcal{B}^{\rho}_{\bar{n}\perp}$ to soft $\mathcal{B}^{(n)\rho}_{s\perp}$ in SCET without changing the rap. div.

$$B(x,\vec{b}_T)\tilde{\mathcal{G}}_{\mathcal{B}}^{\rho}(z,\xi,\vec{b}_T) \to \delta(\xi) B\mathcal{G}\, \frac{S_{\mathcal{B}}^{(n)\rho}}{S}$$

$$S_{\mathcal{B}}^{(n)\rho}(b_\perp) \equiv \frac{1}{N_c}\, \mathrm{tr}\, \Big\langle 0 \Big| \big[S_n^{\dagger}(b_\perp)\, S_{\bar{n}}(b_\perp)\big] \big[S_{\bar{n}}^{\dagger}(0)\, S_n(0) g\mathcal{B}_{s\perp}^{(n)\rho}(0)\big] \Big| 0 \Big\rangle\,,$$

- By construction, $\boxed{\tilde{B}_{\mathcal{B}}^{\prime\rho[\Gamma]}(\xi) \equiv \tilde{B}_{\mathcal{B}}^{\rho[\Gamma]}(\xi) + \delta(\xi)\, \frac{S_{\mathcal{B}}^{(n)\rho}}{S}\, B^{[\Gamma]}} \sim \mathcal{O}(\eta^0)$,

- Similarly, $S_{\mathcal{B}}^{(\bar{n})\rho} \equiv \frac{1}{N_c}\, \mathrm{tr}\, \Big\langle 0 \Big| \big[S_n^{\dagger}(b_\perp)\, S_{\bar{n}}(b_\perp)\big] \big[g\mathcal{B}_{s\perp}^{(\bar{n})\rho}(0) S_{\bar{n}}^{\dagger}(0)\, S_n(0)\big] \Big| 0 \Big\rangle$ absorbs rap. div. of the second term: $\tilde{\mathcal{G}}_{\mathcal{B}}^{\prime\rho[\Gamma]}(\xi) \equiv \tilde{\mathcal{G}}_{\mathcal{B}}^{\rho[\Gamma]}(\xi) + \delta(\xi)\, \frac{S_{\mathcal{B}}^{(\bar{n})\rho}}{S}\, \mathcal{G}^{[\Gamma]} \sim \mathcal{O}(\eta^0)$

Construction of the "Counterterm"

- Note that $S_{\mathcal{B}}^{(n)\rho} + S_{\mathcal{B}}^{(\bar{n})\rho}$

$$= \frac{1}{N_c} \operatorname{tr} \Big\langle 0 \Big| \big[S_n^\dagger S_{\bar{n}}\big](b_\perp) \Big[S_{\bar{n}}^\dagger S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar{n})\rho} S_{\bar{n}}^\dagger S_n\Big](0) \Big| 0 \Big\rangle = 0$$

due to C & P & Poincaré invariance of the vacuum!

[Ebert, AG, Stewart 21']

- $$\tilde{B}_{\mathcal{B}}^{\rho}\mathcal{G} + B\tilde{\mathcal{G}}_{\mathcal{B}}^{\rho} = \underbrace{\left[\tilde{B}_{\mathcal{B}}^{\rho} + \delta(\xi)\, B\,\frac{S_{\mathcal{B}}^{(n)\rho}}{S}\right]}\mathcal{G} + B\underbrace{\left[\delta(\xi)\mathcal{G}\,\frac{S_{\mathcal{B}}^{(\bar{n})\rho}}{S} + \tilde{\mathcal{G}}_{\mathcal{B}}^{\rho}\right]}$$

free of $\frac{1}{\eta}$ divergences to all orders (additive + LP multiplicative div.)

$$= \tilde{B}_{\mathcal{B}}^{\prime\rho}\mathcal{G} + B\tilde{\mathcal{G}}_{\mathcal{B}}^{\prime\rho}$$

see backup for $\mathcal{O}(\alpha_s^2)$ checks

- So we can simply replace the old by the new qgq correlators

A Neat Property of the "Counterterm"

$$S_{\mathcal{B}}^{(n)\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger(b_\perp)\,S_{\bar n}(b_\perp)\right]\left[S_{\bar n}^\dagger(0)\,S_n(0)g\mathcal{B}_{s\perp}^{(n)\rho}(0)\right]\middle|0\right\rangle$$
$$S_{\mathcal{B}}^{(\bar n)\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger(b_\perp)\,S_{\bar n}(b_\perp)\right]\left[g\mathcal{B}_{s\perp}^{(\bar n)\rho}(0)S_{\bar n}^\dagger(0)\,S_n(0)\right]\middle|0\right\rangle$$

- Manipulation of Wilson lines gives

$$\begin{aligned}\left[\mathcal{P}_\perp^\rho S_{\bar n}^\dagger S_n\right] &= \left[S_{\bar n}^\dagger \mathrm{i}D_{s\perp}^\rho S_n\right] + \left[S_{\bar n}^\dagger \mathrm{i}\overleftarrow{D}_{s\perp}^\rho S_n\right]\\ &= S_{\bar n}^\dagger S_n\left[S_n^\dagger \mathrm{i}D_{s\perp}^\rho S_n\right] + \left[S_{\bar n}^\dagger \mathrm{i}\overleftarrow{D}_{s\perp}^\rho S_{\bar n}\right] S_{\bar n}^\dagger S_n\\ &= S_{\bar n}^\dagger S_n\, g\mathcal{B}_{s\perp}^{(n)\rho} - g\mathcal{B}_{s\perp}^{(\bar n)\rho}\, S_{\bar n}^\dagger S_n\end{aligned}$$

- Since $S_{\mathcal{B}}^{(n)\rho} + S_{\mathcal{B}}^{(\bar n)\rho} = 0$, we have $S_{\mathcal{B}}^{(n)} = -S_{\mathcal{B}}^{(\bar n)} = \frac{\mathrm{i}}{2}\partial_\perp^\rho S$ so that

$$S_{\mathcal{B}}^{(n)\rho}/S = \frac{\mathrm{i}}{2}\partial_\perp^\rho \ln S$$

$\Rightarrow$ Due to non-Abelian exponentiation for S, $S_{\mathcal{B}}^{(n)\rho}/S \propto 1/\eta + \mathcal{O}(\eta^0)$ (no ϵ poles, no double poles in η)

Renormalization and Evolution

- Renormalization

$$W_{\mathcal{B}\ \mathrm{DY}}^{(1)\mu\nu} \sim \int \mathrm{d}\xi' \, \mathcal{H}^{(1)\mathrm{ren}}(\xi') \, Z^{(0)} Z^{(1)}(\xi', \xi) \otimes \Big[\tilde{B}_{\mathcal{B}}^{\prime\rho}(\xi) \ \bar{B} + B \ \tilde{\bar{B}}_{\mathcal{B}}^{\prime\rho}(\xi) \Big]$$

$Z^{(1)}(\xi', \xi)$: counterterm for $C^{(1)}$, calculated in [Freedman, Goerke '14, Goerke, Inglis-Whalen '17; Beneke et al '17; Vladimirov, Moos, Scimemi '21]

- Due to charge conjugation $\tilde{B}_{\mathcal{B}}^{\prime\rho}(\xi) \leftrightarrow \tilde{\bar{B}}_{\mathcal{B}}^{\prime\rho}(\xi)$, we know that $Z(\xi', \xi)$ renormalizes $\tilde{B}_{\mathcal{B}}^{\prime\rho}(\xi) = \tilde{B}_{\mathcal{B}}^{\rho}(\xi) + \delta(\xi) B \frac{\mathrm{i}}{2} \partial_\perp^\rho \ln S$,

$$\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho}(\xi) \equiv Z^{(1)}(\xi', \xi) \otimes \tilde{B}_{\mathcal{B}}^{\prime\rho}(\xi)$$

- This implies μ anomalous dimension

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho[\Gamma]}(\xi) = \gamma_\mu(\xi, \xi') \otimes \tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho[\Gamma]}(\xi')$$

- Also get all order ζ RGE (equivalently: ν RGE) for renormalized objects

$$2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho[\Gamma]} = -\delta(\xi) \big(\mathrm{i}\partial_\perp^\rho \gamma_\zeta \big) B^{\mathrm{ren}[\Gamma]} + \gamma_\zeta \tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho[\Gamma]}$$

Are we done?

Are we done? NO!

Endpoint Divergence

$C^{(1)}$ calculated in [J. Strohm's master thesis '20, Vladimirov, Moos, Scimemi '21]
where $L_Q = \ln \frac{-q^2}{\mu^2}$

$$C^{(1)}(q^2,\xi) = 1 + \frac{\alpha_s}{4\pi}\bigg[C_F\Big(-L_Q^2 + L_Q - 3 + \frac{\pi^2}{6}\Big) - C_A\frac{\ln\xi}{1-\xi} - \Big(C_F - \frac{C_A}{2}\Big)\frac{\ln(1-\xi)}{\xi}\Big(2L_Q + \ln(1-\xi) - 4\Big)\bigg] + \mathcal{O}(\alpha_s^2),$$

- SCET$_\text{I}$ ME $\sim \mathcal{O}(\alpha_s \xi^{-1/2}) \Rightarrow$ convergent,
  SCET$_\text{II}$ ME $\sim \mathcal{O}(\alpha_s \xi^{-1}) \Rightarrow$ endpoint divergence

$\Rightarrow$ Divergent integral as $\xi \to 0$ at $\mathcal{O}(\alpha_s^2)$

$$\alpha_s^2 \int \mathrm{d}\xi \ln\xi\, \mathcal{L}_0(\pm\xi) = ???$$

- Special: hard coefficient and rapidity divergence conspire to give an endpoint divergence in a soft gluon limit with back-to-back $n\,\&\,\bar{n}$
- Need refactorization for endpoint divergences

  [Z. L. Liu et al, M. Beneke et al, ...]

Solution: Hard-Collinear Matching to $\mathcal{B}_{s\perp}$

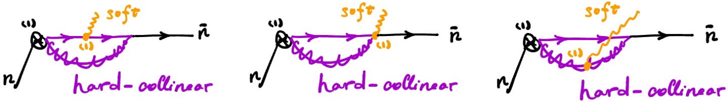
Q^2 λQ^2

- The full $\mathcal{B}_s^\perp$ current receives hard and hard-collinear contributions ($\hat{p}_s = p_s^\pm$) [Ebert, AG, Stewart 21']

$$J^{(1)\mu}_{\mathcal{B}_s^\perp}(0) = J^{(1)\mu}_{\mathrm{h}\,\mathcal{B}_s^\perp}(0) + J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0)$$

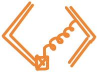
$$J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0) \sim \int \mathrm{d}\hat{p}_s \int \mathrm{d}\tilde{\xi}\, C^{(1)}(q^2,\tilde{\xi})\, \tilde{J}_{\mathcal{B}_s^\perp}(\hat{p}_s,\tilde{\xi})$$
$$\times\, \bar{\chi}_{\bar{n},-\omega_2}\Big\{\big[S_{\bar{n}}^\dagger S_n g\mathcal{B}^{(n)}_{s\perp}\big](p_s^+) + \big[g\mathcal{B}^{(\bar{n})}_{s\perp} S_{\bar{n}}^\dagger S_n\big](p_s^-)\Big\}\chi_{n,-\omega_1}\,,$$

- $T\big[J_{\mathrm{I}}^{(1)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in $\mathrm{SCET_I}$ $\to$ hard scattering operators in $\mathrm{SCET_{II}}$



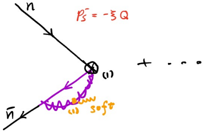
- These $\tilde{J}_{\mathcal{B}_s^\perp}$ graphs reproduce $\xi^{-\epsilon}$ behavior of $C^{(1)}$ $(p_s^- = -\xi Q)$!

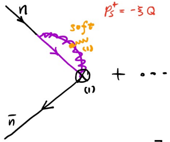
$$\int \mathrm{d}\tilde{\xi}\, \tilde{J}_{\mathcal{B}_s^\perp}(p_s^-,\tilde{\xi}) \sim \sum \text{graphs} = \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left(\frac{p_s^- Q}{\mu^2}\right)^{-\epsilon} + \mathcal{O}(\epsilon^0) = \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}(1-\epsilon\ln(-\xi)) + \mathcal{O}(\epsilon^0\xi^0$$

Soft ME $\equiv \tilde{\mathcal{S}}^{(\bar{n})}_{\mathcal{B}}(p_s^-) \sim$  $(p_s^-) \sim \frac{\alpha_s}{\pi} C_F \frac{\theta(p_s^-)}{p_s^-} = \frac{\alpha_s}{\pi} C_F \frac{\theta(-\xi)}{-\xi Q} \longleftrightarrow \tilde{B}^{\mathrm{ren}}_{\mathcal{B}}(\xi \to 0)$

Solution

- Exploit the vanishing hc soft contribution for Drell-Yan





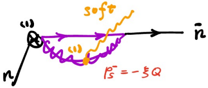
$$
\begin{aligned}
0 &= B\bar{B}\left[\int_0^\infty \mathrm{d}p_s^- \,\frac{1}{\epsilon}\left(\frac{-p_s^- Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^-} - \int_0^\infty \mathrm{d}p_s^+ \,\frac{1}{\epsilon}\left(\frac{-p_s^+ Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^+}\right] \\
&\propto B\bar{B}\int \mathrm{d}\xi \frac{1}{\epsilon}\left(\frac{-\xi q^2}{\mu^2}\right)^{-\epsilon}\left[\theta(|\xi| - \xi_{\mathrm{cut}}) + \theta(\xi_{\mathrm{cut}} - |\xi|)\right]\left[\frac{\theta(-\xi)}{-\xi} - \frac{\theta(-\xi)}{-\xi}\right] \\
&= B\bar{B}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\int \mathrm{d}\xi\, \frac{\xi^{-\epsilon}}{\epsilon}\theta(\xi_{\mathrm{cut}} - |\xi|)\Big(\mathcal{L}_0(-\xi) - \mathcal{L}_0(-\xi)\Big)
\end{aligned}
$$

- $\theta(\xi_{\mathrm{cut}} - |\xi|) \ln\xi\, \mathcal{L}_0(-\xi)$ cancels the divergence in qgq as $\xi \to 0$
- The following convolution is finite at $\xi \to 0$!

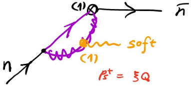
$$
\int \mathrm{d}\xi \left[C_{\mathrm{ren}}^{(1)}(\xi)\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\rho}(\xi) + \theta(\xi_{\mathrm{cut}} - |\xi|)\, C_{\mathrm{ren}}^{(1)} \otimes \tilde{J}_{\mathcal{B}_s^\perp}^{\mathrm{ren}}(-\xi Q)\frac{\tilde{S}_{\mathcal{B}}^{(n)\mathrm{ren}\,\rho}(-\xi Q)}{S^{\mathrm{ren}}} B^{\mathrm{ren}}\right]
$$

Solution

- Exploit the vanishing hc soft contribution for SIDIS



$+\cdots$



$+\cdots$

$$0 = B\mathcal{G}\left[\int_0^\infty \mathrm{d}p_s^- \,\frac{1}{\epsilon}\left(\frac{p_s^- Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^-} - \int_0^\infty \mathrm{d}p_s^+\left(\frac{-p_s^+ Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^+}\right]$$

$$\propto B\mathcal{G}\int \mathrm{d}\xi\,\frac{1}{\epsilon}\left(\frac{-\xi q^2}{\mu^2}\right)^{-\epsilon}\left[\theta(|\xi|-\xi_{\rm cut})+\theta(\xi_{\rm cut}-|\xi|)\right]\left[\frac{\theta(-\xi)}{-\xi}-\frac{\theta(\xi)}{\xi}\right]$$

$$= B\mathcal{G}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\left[\cancel{-\mathrm{i}\pi\left(\frac{1}{\epsilon}-\ln\xi_{\rm cut}\right)} - \frac{1}{2}\pi^2 + \int \mathrm{d}\xi\,\frac{\xi^{-\epsilon}}{\epsilon}\theta(\xi_{\rm cut}-|\xi|)\Big(\mathcal{L}_0(-\xi)-\mathcal{L}_0(\xi)\Big)\right]$$

after including h.c.

- $\theta(\xi_{\rm cut}-|\xi|)\ln\xi\,\mathcal{L}_0(\mp\xi)$ cancels the divergence in $\int \mathrm{d}\xi C_{\rm ren}^{(1)}\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho}$, $\tilde{\mathcal{G}}_{\mathcal{B}}^{\mathrm{ren}\,\rho}$

$$\int \mathrm{d}\xi\left[C_{\rm ren}^{(1)}(\xi)\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho}(\xi) + \theta(\xi_{\rm cut}-|\xi|)\,C_{\rm ren}^{(1)}\otimes\tilde{J}_{\mathcal{B}_s^\perp}^{\rm ren}(-\xi Q)\tilde{\mathcal{S}}_{\mathcal{B}}^{(n)\mathrm{ren}\,\rho}(-\xi Q)/S^{\rm ren}\,B^{\rm ren}\right]$$

$$\int \mathrm{d}\xi\left[C_{\rm ren}^{(1)}(\xi)\tilde{\mathcal{G}}_{\mathcal{B}}^{\mathrm{ren}\,\rho}(\xi) + \theta(\xi_{\rm cut}-|\xi|)\,C_{\rm ren}^{(1)}\otimes\tilde{J}_{\mathcal{B}_s^\perp}^{\rm ren}(\xi Q)\tilde{\mathcal{S}}_{\mathcal{B}}^{(\bar{n})\mathrm{ren}\,\rho}(\xi Q)/S^{\rm ren}\,\mathcal{G}^{\rm ren}\right]$$

- We are left with a remainder term $\boxed{R_{\rm SIDIS}\propto \alpha_s C_A \alpha_s\pi^2 C_F B\mathcal{G} + \mathcal{O}(\alpha_s^3)}$
- For $q_T\sim\Lambda_{\rm QCD}$, $\alpha_s\pi^2 C_F \longrightarrow$ NP function defined by $\tilde{\mathcal{S}}_{\mathcal{B}}^{\rho}(\hat{p}_s, b_\perp)$

Summary

- Motivated studying azimuthal asymmetries in Drell-Yan and SIDIS
- Derived all-order bare factorization
- Also included Z/W production in Drell-Yan
- Constructed all-order definitions of the renormalized TMD qgq correlators which involves both additive and multiplicative soft factors
  - ▷ soft subtraction for $\frac{\delta(\xi)}{\eta}$ divergences
  - ▷ soft subtraction for removal of endpoint $\ln\xi/\xi$ divergences
- Obtained an all order ζ evolution equation for renormalized TMD qgq correlators
- Renormalization procedure for SIDIS leaves behind remainder R_{SIDIS}
- Yields renormalized factorization theorems for $\mathcal{O}(\lambda)$ SIDIS and Drell-Yan

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Are we done?

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Are we done? For now!

Summary

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  - ▷ soft subtraction for $\frac{\delta(\xi)}{\eta}$ divergences
  - ▷ soft subtraction for removal of endpoint $\ln \xi/\xi$ divergences
- Perturbative cross checks at $\mathcal{O}(\alpha_s)$ and C_F^2/η^2, $C_F C_A/\eta^2$ for $\mathcal{O}(\alpha_s^2)$
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Thanks for your attention!

Backup: Checking Cancellation of Rapidity Divergence

Check deepest $(1/\eta^2)$ pole in $\alpha_s^2 C_F^2$ channel:

$$\Big[\hat{\tilde{B}}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi)\underbrace{\frac{S^{(n)\rho}_{\mathcal{B}}}{S}}_{\mathcal{O}(C_F/\eta)}\overbrace{\hat{B}\Big]\sqrt{S}}^{\mathcal{O}(C_F\eta^0)} = \mathcal{O}(\eta^0)$$

$\hat{\tilde{B}}^{(2)}_{\mathcal{B}}$ 

$$= -\,\alpha_s^2\, 8C_F^2\,\frac{1}{\eta^2}\,\delta(1-z)\,\delta(\xi)\,\mathcal{I}^{\rho}_{\mathrm{NLP}}\mathcal{I}_{\mathrm{LP}} + \mathcal{O}(\frac{1}{\eta})$$

$\hat{\tilde{B}}^{(1)}_{\mathcal{B}}$ 

$$= +\,\alpha_s\, 2C_F\,\frac{1}{\eta}\,\delta(1-z)\,\delta(\xi)\,\mathcal{I}^{\rho}_{\mathrm{NLP}} + \mathcal{O}(\eta^0)$$

$\sqrt{S}$ $\frac{1}{2}$ 

$$= +\,\alpha_s\, 4C_F\,\frac{1}{\eta}\,\delta(1-z)\,\delta(\xi)\,\mathcal{I}_{\mathrm{LP}} + \mathcal{O}(\eta^0)$$

Backup: Checking Cancellation of Rapidity Divergence

Check deepest $(1/\eta^2)$ pole in $\alpha_s^2 C_F C_A$ channel:

$$\Big[\underbrace{\hat{\tilde{B}}^{\rho}_{\mathcal{B}}(\xi)}_{\text{have to cancel within}} + \delta(\xi)\underbrace{\frac{S^{(n)\rho}_{\mathcal{B}}}{S}}_{\mathcal{O}(C_F/\eta)}\,\overbrace{\hat{B}\Big]\underbrace{\sqrt{S}}_{1+\alpha_s C_F/\eta}}^{\mathcal{O}(\eta^0)} = \mathcal{O}(\eta^0)$$

$A =$  +  $= \mathcal{O}(1/\eta^2)$

$B =$  +  +  +  + ... $= \mathcal{O}(1/\eta^2)$

$A + B = \mathcal{O}(1/\eta)$ ✓