

Tomography of pions and protons via transverse momentum dependent distribution

Leonard Gamberg



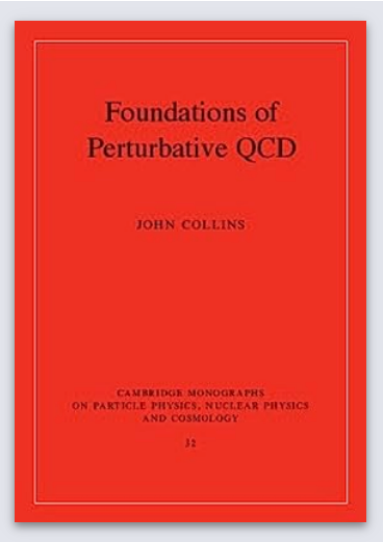
w/ Patrick Barry, Eric Moffat, Alexei Prokudin
Wally Melnitchouk, Nobuo Sato, Daniel Pitonyak
"JAM 3-D"

TMDs: Towards a Synergy between Lattice QCD and Global Analyses
BNL-EIC Theory Institute & CFNS
Stony Brook 06/21-23/2023



Here we focus on TMDs and novel properties of the transverse separation of quark fields as a function of their longitudinal momenta for the proton and pion, giving deeper insights into color confined systems that emerge from QCD.

Aybat Rogers 2011 PRD
 Collins 2011 red book
 Collins Rogers 2015 PRD
 TMD Handbook arXiv <https://arxiv.org/abs/2304.03302>



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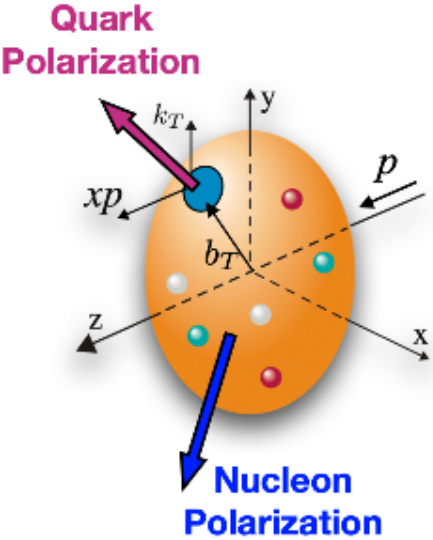


Figure 1.1: Illustration of the momentum and spin variables probed by TMD parton distributions.

Outline of talk

- **Intro** Pion π structure
- **Analysis framework**: provides motivation to perform simultaneous fit of
 - pion & proton TMD pdfs
 - w/ pion collinear pdfs

from DY & LN data within JAM QCD analysis

- **Pheno**
- **Results**

...with an eye toward lattice & TMDs

Motivation of my talk

- We explore the impact on [JAM 21](#) pion pdfs extracted from a simultaneous fit of low energy fixed target p_T dependent DY & collinear π -nuclear cross section data

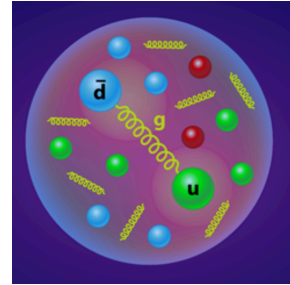
In particular:

- **First** we carry out a fit of the non-perturbative parameters of the pion TMD from the available data.
- As a **second step** we open up the fit of both collinear pion pdf parameters along with non-perturbative parameters.
- As a **final step**, we perform a fit of the p_T integrated and p_T dependent data to carry out a simultaneous fit of the pion collinear pdfs and pion TMDs. This constitutes a first such study.
- We also compare the impact of various scenarios for describing non-perturbative content of the TMD contribution.

<https://arxiv.org/abs/2302.01192>

Intro

The Pion as bound state QCD



- Pion plays a central/“outsized” role in hadron physics
- @ low energy, as nearly massless $\bar{q}q$ bound state Goldstone boson is a critical ingredient for understanding dynamical χ symmetry breaking from small current quark masses

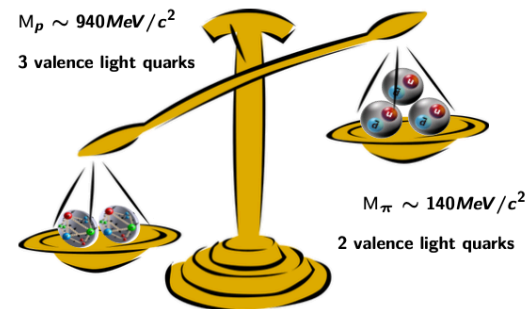
Pion pole condition from Bethe-Salpeter

$$m_\pi^2 = \frac{m}{2G_\pi M I(m_\pi^2)}$$

Gell Mann Reiner Oaks Relation

$$f_\pi^2 m_\pi^2 = \frac{1}{2} (m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle$$

- ★ **Mass without mass**” bulk of pion mass due to QCD quantum fluctuations of $\bar{q}q$ pairs, gluons, & energy associated with quarks moving at close to speed of light



The Pion recent progress

@ high energies pion's partonic structure unfolded/revealed from DY process as predicted from Collinear Factorization \longrightarrow momentum distributions, $f_{i/\pi}(x, \mu)$

$$\frac{d\sigma^{\text{DY}}}{dQ^2 dy} = \sum_{a,b} \int dx_a dx_b H_{a,b}^{\text{DY}}(x_a, x_b, y, Q^2, \mu^2) f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2)$$

$$\frac{d^3\sigma^{\text{LN}}}{dx_B dQ^2 dx_L} = \frac{4\pi\alpha^2}{x_B Q^4} \left(1 - y_e + \frac{y_e^2}{2}\right) F_2^{\text{LN}}(x_B, Q^2, x_L)$$

Jefferson Lab Angular Momentum (JAM)

Barry, Sato, Melnitchouk, C.-R. Ji PRL 2018

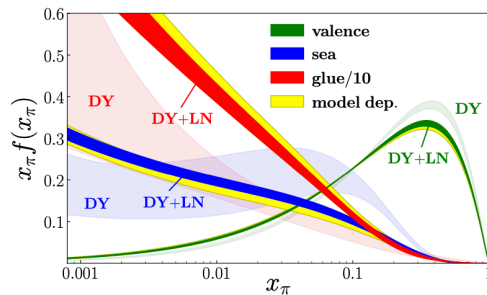
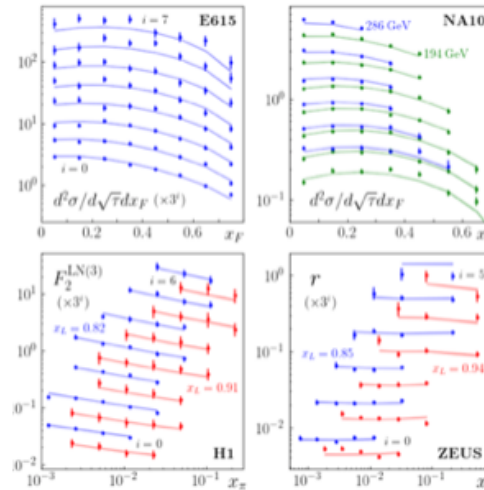
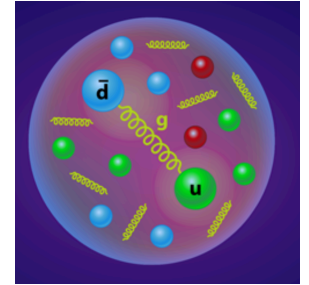
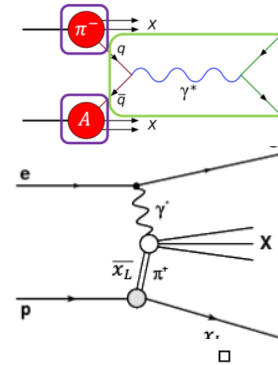


FIG. 2. Pion valence (green), sea quark (blue) and gluon (red, scaled by 1/10) PDFs versus x_π at $Q^2 = 10 \text{ GeV}^2$, for the full DY + LN (dark bands) and DY only (light bands) fits. The bands represent 1σ uncertainties, as defined in the standard Monte Carlo determination of the uncertainties [42] from the experimental errors. The model dependence of the fit is represented by the outer yellow bands.

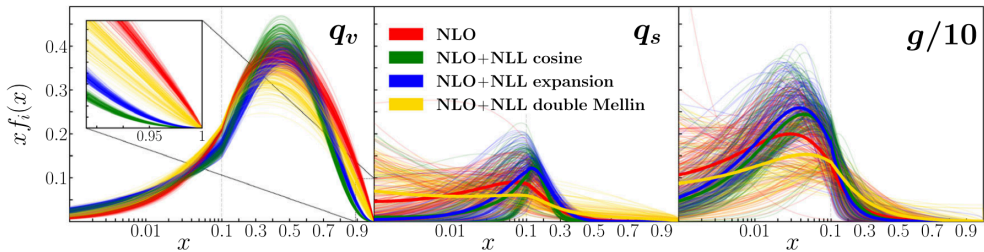


- Top row – Drell-Yan
- Bottom row – Leading neutron
- Good agreement with data
- $\chi^2_{\text{npts}} = 0.979$

□

The Pion PDFs NLL resummation & Lattice progress

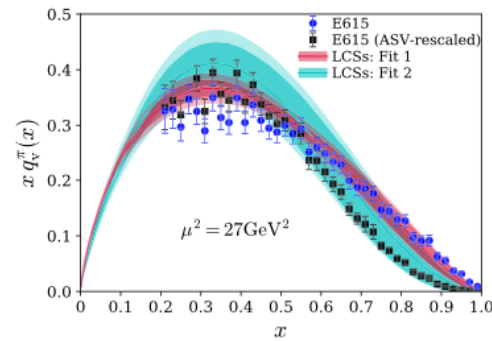
Barry, C.R. Ji, Sato, Melnitchouk, PRL 2021
 global analysis w/ **threshold resummation**



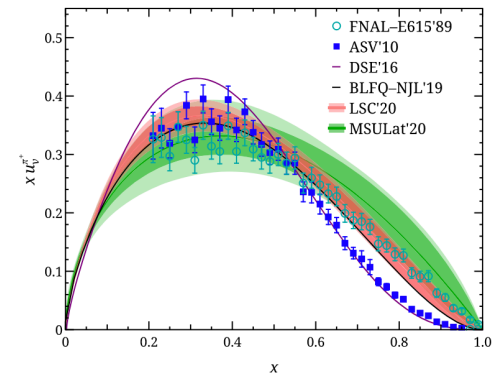
NLO + NLL cosine (green), expansion (blue), and double Mellin (gold) analyses. Inset in the left panel magnifies the very large- x region. The central values of the sea quark and gluon posterior samples are indicated by solid lines.

Lattice

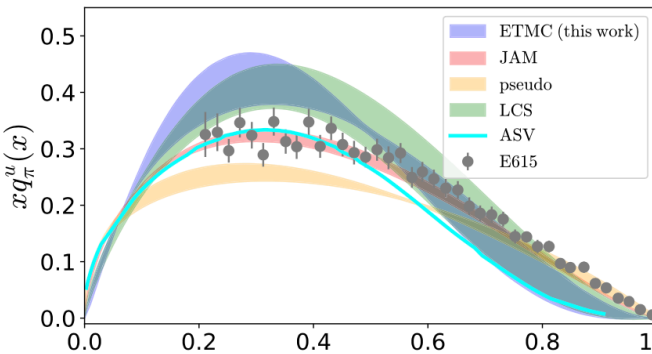
Sufian et al. PRD 102 (2020)
 Had-structure/JLab



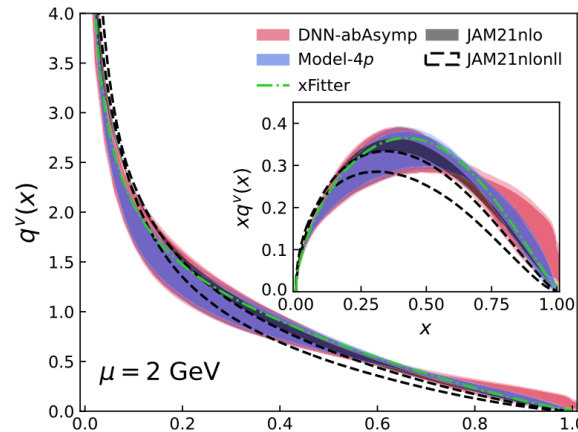
Huey-Wen Lin, et al. PRD 103 (2021)



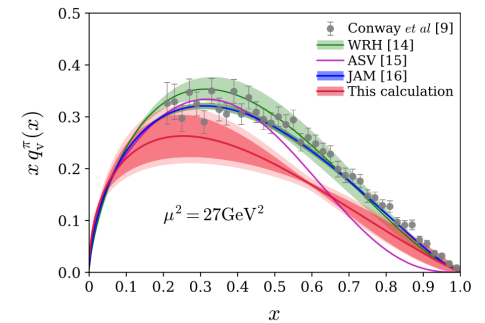
PRD 104 (2021) Alexandrou et al. EMT



PRD 106 (2022) Gao et al. BNL



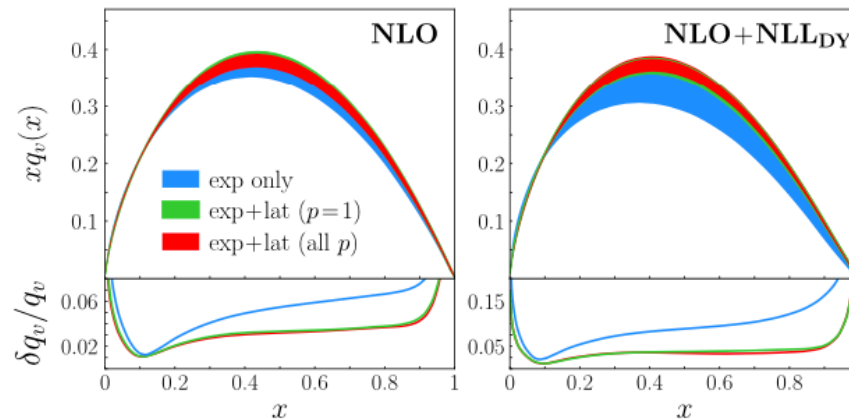
Bálint Joó, et al. PRD 100 (2019)
 Had-struc /JLab



The Pion PDFs progress

First analysis of kind, where experimental data on pion-nucleus Drell-Yan and LN electro-production reactions supplemented by [lattice QCD data](#); reduced Ioffe time pseudo-distributions to constrain the PDFs of the pion.

PRD 105 (2022) Barry et al. JAM/Had-struct.

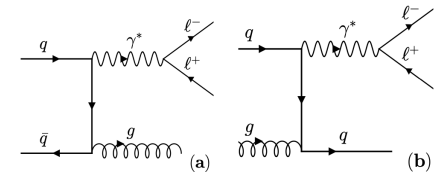


Valence quark distributions (top) extracted from experimental data alone (blue), combined with the $p=1$ lattice data (green), and combined with all the lattice data (red) for the NLO (left) and NLO & NLL_{DY} (right) cases, along with the relative uncertainties (bottom). The bands represent a 1σ uncertainty level.

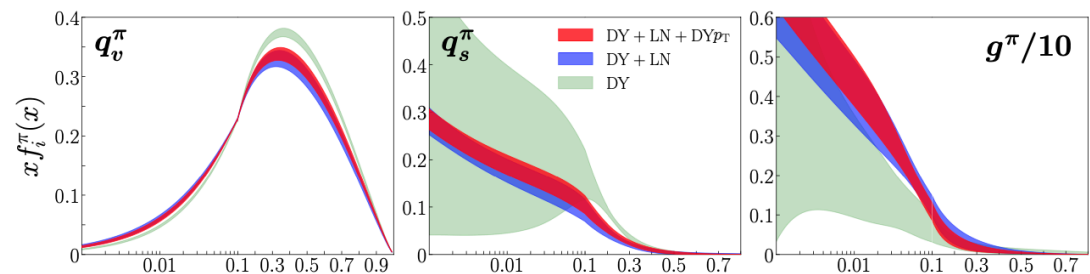
Extended fit to include “large” p_T Drell Yan data

Cao et al *Phys.Rev.D* 103 (2021), added transverse momentum dependent DY data in a globalQCD analysis of large transverse momentum $p_T \sim Q$ dominated hard QCD radiation

$$\frac{d\sigma^{\text{DY}}}{dQ^2 dy dp_T} = \sum_{a,b} \int dx_a dx_b H_{a,b}^{\text{FO}}(x_a, x_b, y, p_T, Q^2, \mu^2) f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2)$$



The inclusion of p_T -dependent data only slightly reduce uncertainties of the gluon distribution at large x & impacts on other distributions negligible



Understanding how these contrasting manifestations of the same $\bar{q}q$ bound state arise dynamically at different energy scales from first principles remains a major challenge in QCD

Extend fit to include “low” p_T Drell Yan data

- We consider impact on collinear pion pdfs from $p_T (\equiv q_T) \sim k_T \ll Q$ “TMD” region (*n.b.* smaller statistical uncertainties on the data)
- Pion induced DY scattering processes provide possibility to extract TMDs of the pion and nucleon when the cross section is kept differential in the transverse momentum of the produced lepton pair
- Factorized according to the framework of Collins-Soper-Sterman (CSS)

Factorization for low Drell-Yan $q_T \sim k_T \ll Q$

W-term, valid for low q_T

$$\frac{d^3\sigma}{d\tau dY dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int d^2b_T e^{ib_T \cdot q_T} \\ \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2)$$

- Factorization commonly carried out in $b = (b^-, b^+, \mathbf{b}_T)$ space
- Bare Fourier transform (FT) TMD commonly carried out in $b = (b^-, b^+, \mathbf{b}_T)$ space

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{db^-}{4\pi} e^{-ixP^+b^-} \text{Tr}[\langle \mathcal{N} | \bar{\psi}_q(b) \gamma^+ \mathcal{W}(b, 0) \psi_q(0) | \mathcal{N} \rangle]$$

- \mathbf{b}_T is the Fourier conjugate to the intrinsic transverse momentum of quarks k_T

The Pion TMDs: factorization, renormalization, and evolution

“Bare” TMD Factorization Parton Model

◆ Mulders Tangerman NPB 1995, Boer Mulders PRD 1997

$$q_T \sim k_T \ll Q$$

Leading Quark TMDPDFs



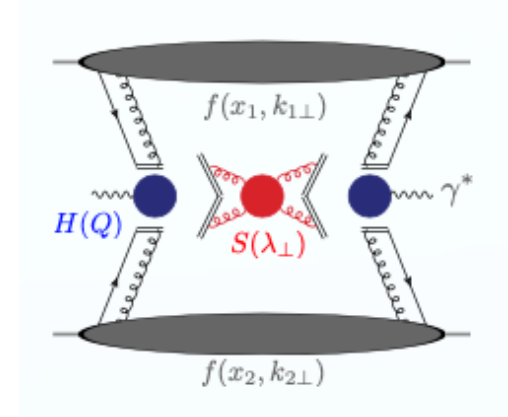
	Quark Polarization		
	Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
u	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$

$$f(x, k_T, s_T) = \frac{1}{2} \left[f_1^\pi(x, k_T^2) + \frac{s_T^i \epsilon^{ij} k_T^j}{m_\pi} h_1^{\pi \perp}(x, k_T^2) \right]$$

- Factorization carried out Fourier b_T space FT TMDs $\tilde{f}(x, b_T)$
- Real QCD need QFT definitions of TMDs LC & UV divergences reflected in the CS & RG Eqs. $\tilde{f}(x, b_T; \mu, \zeta)$
- TMD Evolution depends on rapidity ζ and RGE scales μ

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{db^-}{4\pi} e^{-ixP^+b^-} \text{Tr}[\langle \mathcal{N} | \bar{\psi}_q(b) \gamma^+ \mathcal{W}(b, 0) \psi_q(0) | \mathcal{N} \rangle]$$

$$\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \rightarrow 0} Z_{\text{uv}}(\mu, \zeta, \epsilon) \lim_{y_B \rightarrow -\infty} \frac{\tilde{f}_{i/p}^{(u)}(x, \mathbf{b}_T, \epsilon, y_B, xP^+)}{\sqrt{\tilde{S}_{n_A(2y_n)n_B(2y_B)}(b_T, \epsilon, 2y_n - 2y_B)}}$$



TMD Factorization

- ◆ Collins Soper Serman NPB 1985
- ◆ Ji Ma Yuan PRD PLB ...2004, 2005
- ◆ Aybat Rogers PRD 2011
- ◆ Collins 2011 Cambridge Press
- ◆ Echevarria, Idilbi, Scimemi JHEP 2012, ...
- ◆ SCET Becher & Neubert, 2011 EJPC

Aybat Rogers 2011 PRD
Collins 2011 red book
Collins Rogers 2015 PRD
TMD Handbook <https://arxiv.org/abs/2304.03302>

Renormalization and TMD Evolution- $\{\zeta, \mu\}$

* Collins Soper Eq.
$$\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

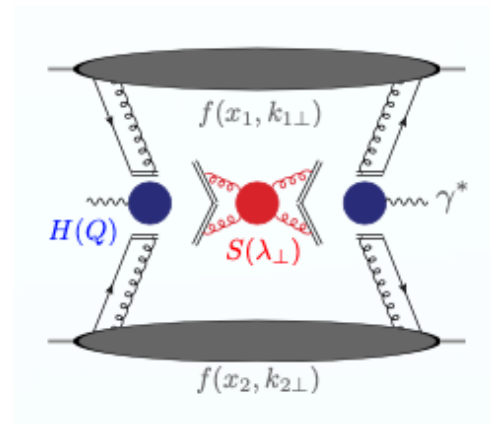
$$\tilde{K}(b_T, \mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, \infty)}$$

* RGE for C.S. kernel

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_k(\alpha_s(\mu))$$

* RGE for TMD

$$\frac{d \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(\alpha_s(\mu), \zeta/\mu)$$



Evolution Renormalization and TMD $\{\zeta, \mu\}$

$$\tilde{f}_{i/P}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_{i/P}(x, \mathbf{b}_T, \mu_0, \zeta_0) \exp \left\{ \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_q[\alpha_s(\mu'); \zeta_0/\mu'^2] \right\} \exp \left\{ \tilde{K}(b_T; \mu) \ln \sqrt{\frac{\zeta}{\zeta_0}} \right\}$$

Integral extends from $b = 0$ to ∞ one cannot avoid using parton densities and \tilde{K} in the non-perturbative large- b_T region

Evolution Renormalization and TMD $\{\zeta, \mu\}$

$$\tilde{f}_{i/P}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_{i/P}(x, \mathbf{b}_T, \mu_0, \zeta_0) \exp \left\{ \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_q[\alpha_s(\mu'); \zeta_0/\mu'^2] \right\} \exp \left\{ \tilde{K}(b_T; \mu) \ln \sqrt{\frac{\zeta}{\zeta_0}} \right\}$$

Integral extends from $b = 0$ to ∞ one cannot avoid using parton densities and \tilde{K} in the non-perturbative large- b_T region

- At small b_T , the TMD PDF can be described in terms of its OPE:

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{q/j}(x/\xi, b_T; \mu, \zeta_F) f_{q/\mathcal{N}}(\xi; \mu) + \mathcal{O}((\Lambda_{\text{QCD}} b_T)^a)$$

- where \tilde{C} are the Wilson coefficients, and $f_{q/\mathcal{N}}$ is the collinear PDF
- Breaks down when b_T gets large

Evolution Renormalization and TMD $\{\zeta, \mu\}$

$$\tilde{f}_{i/P}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_{i/P}(x, \mathbf{b}_T, \mu_0, \zeta_0) \exp \left\{ \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_q[\alpha_s(\mu'); \zeta_0/\mu'^2] \right\} \exp \left\{ \tilde{K}(b_T; \mu) \ln \sqrt{\frac{\zeta}{\zeta_0}} \right\}$$

Integral extends from $b = 0$ to ∞ one cannot avoid using parton densities and \tilde{K} in the non-perturbative large- b_T region

- Introduction of non-perturbative functions @large b_T freeze to b_{max} chosen to transition from perturbative to nonperturbative physics

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{max}^2}}$$

$$\begin{aligned} \frac{\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta)}{\tilde{f}_{i/p}(x, \mathbf{b}_*, \mu, \zeta)} &= \frac{\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu_0, \zeta'_0)}{\tilde{f}_{i/p}(x, \mathbf{b}_*, \mu_0, \zeta'_0)} \exp \left[\ln \sqrt{\frac{\zeta}{\zeta'_0}} \left(\tilde{K}(b_T, \mu) - \tilde{K}(b_*, \mu) \right) \right] \\ &= \exp \left[-g_{i/p}(x, b_T) \right] \exp \left[-\ln \sqrt{\frac{\zeta}{\zeta'_0}} g_k(b_T; b_{max}) \right], \end{aligned}$$

$$g_k(b_T; b_{max}) = \tilde{K}(b_*, \mu_0) - \tilde{K}(b_T, \mu_0)$$

CSS evolution $F.T.$ -TMD B.C. OPE & b_* prescript.

$$\{\zeta, \mu\} \rightarrow \zeta = Q^2, \quad \mu = \mu_Q \sim Q$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + \mathbf{b}_T^2 / \mathbf{b}_{\max}^2}}$$

Low- b_T : perturbative
high- b_T : non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*) \times \exp \left\{ -g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\}$$

Relates the TMD at small- b_T to the **collinear** PDF
 \Rightarrow TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
 g_K : universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

Collins, Soper, Sterman, NPB **250**, 199 (1985).

W – term

Aybat Rogers 2011 PRD
 Collins 2011 red book
 Collins Rogers 2015 PRD
 TMD Handbook arXiv <https://arxiv.org/abs/2304.03302>

TMD factorization in Drell-Yan

- In small- q_T region, use the Collins-Soper-Sterman (CSS) formalism and b_* prescription

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H_{j\bar{j}}^{\text{DY}}(Q, \mu_Q, a_s(\mu_Q)) \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{iq_T \cdot \mathbf{b}_T}$$

Can these data constrain the
 pion collinear PDF?

$$\begin{aligned} & \times e^{-g_{j/A}(x_A, b_T; b_{\max})} \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A; \mu_{b_*}) \tilde{C}_{j/j_A}^{\text{PDF}}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \\ & \times e^{-g_{j/B}(x_B, b_T; b_{\max})} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B; \mu_{b_*}) \tilde{C}_{j/j_B}^{\text{PDF}}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \\ & \times \exp\left\{ -g_K(b_T; b_{\max}) \ln \frac{Q^2}{Q_0^2} + \tilde{K}(b_*; \mu_{b_*}) \ln \frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma_j(a_s(\mu')) - \ln \frac{Q^2}{(\mu')^2} \gamma_K(a_s(\mu')) \right] \right\} \end{aligned}$$

Non-perturbative pieces

Perturbative pieces

Non-perturbative piece of the CS kernel

Summary of details of analysis framework

- Nuclear TMD model linear combination of bound protons and neutrons
 - Include an additional A -dependent nuclear parameter
- We use the MAP collaboration's parametrization for non-perturbative TMDs
 - Only tested parametrization flexible enough to capture features of Q bins
- Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs
 - Include both q_T -dependent and collinear pion data and fixed-target pA data

Summary of details of analysis framework

Perturbative accuracy used in our fits

$\gamma_K(\alpha_s(\mu))$	$\beta[\alpha_s(\mu)]$	$\gamma_q(\alpha_s(\mu); 1)$	$\tilde{K}(\bar{b}_T; 1/\bar{b}_T)$	$\tilde{C}_{j/\gamma}$	accuracy	accuracy (SCET)
—	—	—	—	0	QPM	
1	1	—	—	0	LO-LL	LL
2	2	1	1	0	LO-NLL	NLL
3	3	2	2	0	LO-NNLL	
2	2	1	1	1	NLO-NLL	NLL'
3	3	2	2	1	NLO-NNLL	NNLL
3	3	2	2	2	NNLO-NNLL	NNLL'
4	4	3	3	2	NNLO-N ³ LL	N ³ LL
4	4	3	3	3	N ³ LO-N ³ LL	N ³ LL'



The collinear observables we use at NLO

Nuclear TMD PDFs

- The TMD factorization allows for the description of a quark inside a nucleus to be $\tilde{f}_{q/A}$
- However, the intrinsic non-perturbative structure will in-principle change from nucleus-to-nucleus
- Want to model these in terms of protons and neutrons as we don't have enough observables to separately parametrize different nuclei

Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A} \tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A} \tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
 - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x, b_T)} \rightarrow \frac{Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x, b_T)} \\ + \frac{A-Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x, b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x, b_T)} \rightarrow \frac{Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x, b_T)} \\ + \frac{A-Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x, b_T)}.$$

Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1 \right) \right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

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MAP parametrization [See Chiara's talk](#)

- A recent work from the MAP collaboration ([arXiv:2206.07598](#)) used a complicated form for the non-perturbative function

$$f_{1NP}(x, \mathbf{b}_T^2; \zeta, Q_0) = \frac{g_1(x) e^{-g_1(x) \frac{\mathbf{b}_T^2}{4}} + \lambda^2 g_{1B}^2(x) \left[1 - g_{1B}(x) \frac{\mathbf{b}_T^2}{4} \right] e^{-g_{1B}(x) \frac{\mathbf{b}_T^2}{4}} + \lambda_2^2 g_{1C}(x) e^{-g_{1C}(x) \frac{\mathbf{b}_T^2}{4}}}{g_1(x) + \lambda^2 g_{1B}^2(x) + \lambda_2^2 g_{1C}(x)} \left[\frac{\zeta}{Q_0^2} \right]^{\frac{g_K(\mathbf{b}_T^2)/2}{2}}, \quad (38)$$

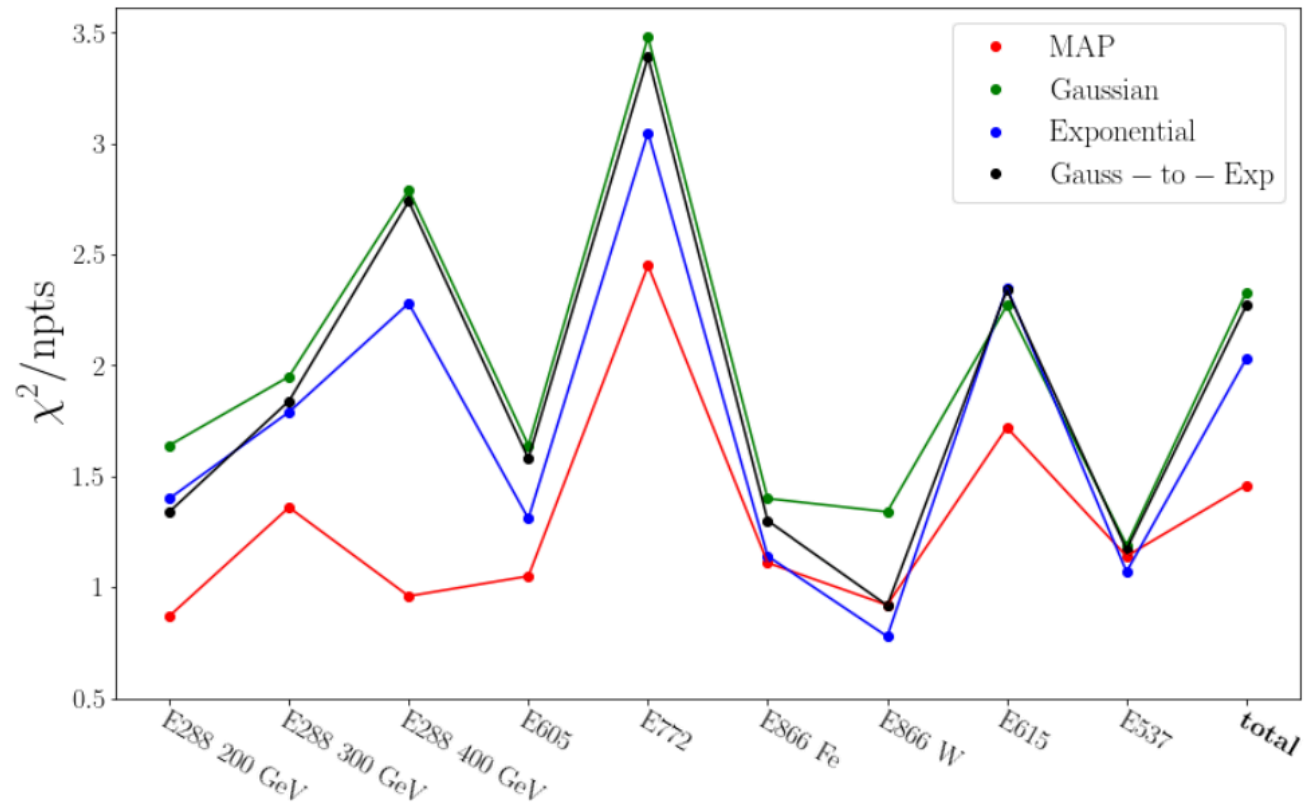
$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha_{\{1,2,3\}}^2}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha_{\{1,2,3\}}^2}},$$

$$g_K(\mathbf{b}_T^2) = -g_2^2 \frac{\mathbf{b}_T^2}{2}$$

Universal CS kernel

Resulting χ^2 for each parametrization

- MAP gives best overall
- How significant?



Perform the Monte Carlo

- We use the MAP parametrization
- Now, we can include the **pion collinear PDF** and its collinear datasets
- Include an additional 225 collinear data points
- Simultaneously extract
 1. Pion TMD PDFs
 2. Pion collinear PDFs
 3. Proton TMD PDFs
 4. Nuclear dependence
 5. Non-perturbative CS kernel

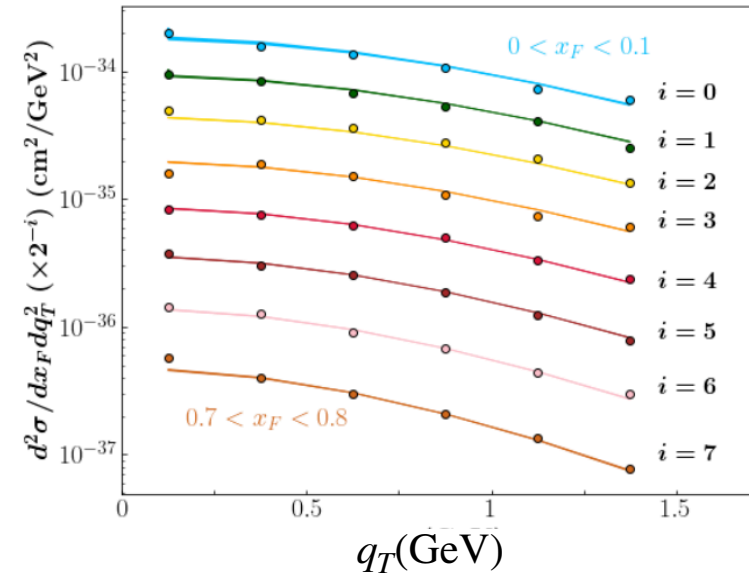
Pheno-Aspects of the fit

This analysis both q_T dependent and collinear data, and are consequently able to, first time simultaneously extract the pion's TMD and collinear PDFs

Data and theory agreement

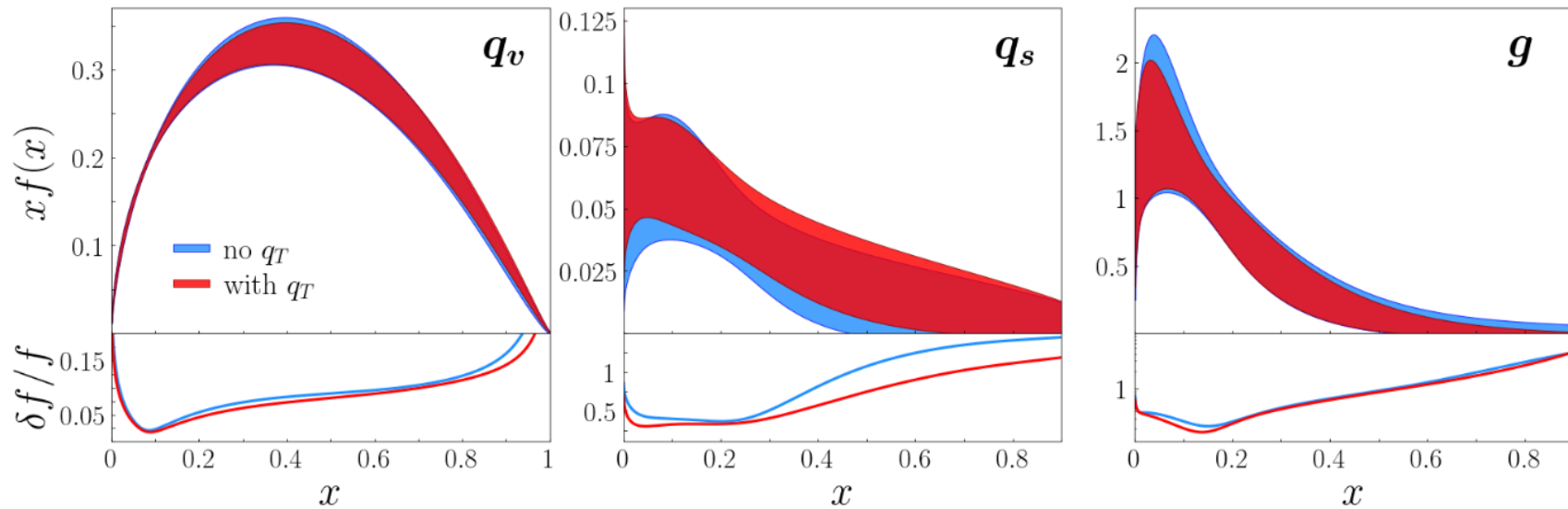
- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	\sqrt{s} GeV	χ^2/np	Z-score
q_T -integr. DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [37]	21.8	0.86	0.76
	NA10 [38]	19.1	0.54	2.27
	NA10 [38]	23.2	0.91	0.18
Leading neutron $ep \rightarrow e' n X$	H1 [73]	318.7	0.36	4.61
	ZEUS [74]	300.3	1.48	2.16
q_T -dep. pA DY $pA \rightarrow \mu^+ \mu^- X$	E288 [67]	19.4	0.93	0.25
	E288 [67]	23.8	1.33	1.54
	E288 [67]	24.7	0.95	0.23
	E605 [68]	38.8	1.07	0.39
	E772 [69]	38.8	2.41	5.74
	E866 (Fe/Be) [70]	38.8	1.07	0.29
	E866 (W/Be) [70]	38.8	0.89	0.11
q_T -dep. πA DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [37]	21.8	1.61	2.58
	E537 [71]	15.3	1.11	0.57
Total			1.15	2.55



Pheno-Aspects of the fit

Extracted pion PDFs



- The small- q_T data do not constrain much the PDFs

By def., the TMDPDF is a $2 - D$ number density dependent on x & b_T “joint” “probability distribution”

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$$

Here we study the probability distribution in b_T for a given x : this is a quantity in which describes the ratio of the $2 - D$ density to the integrated or b_T —independent density; that is dependent on “ b_T given x ”
→ “conditional density”

$$\equiv \tilde{f}_{q/\mathcal{N}}(b_T|x; \mu, \zeta)$$

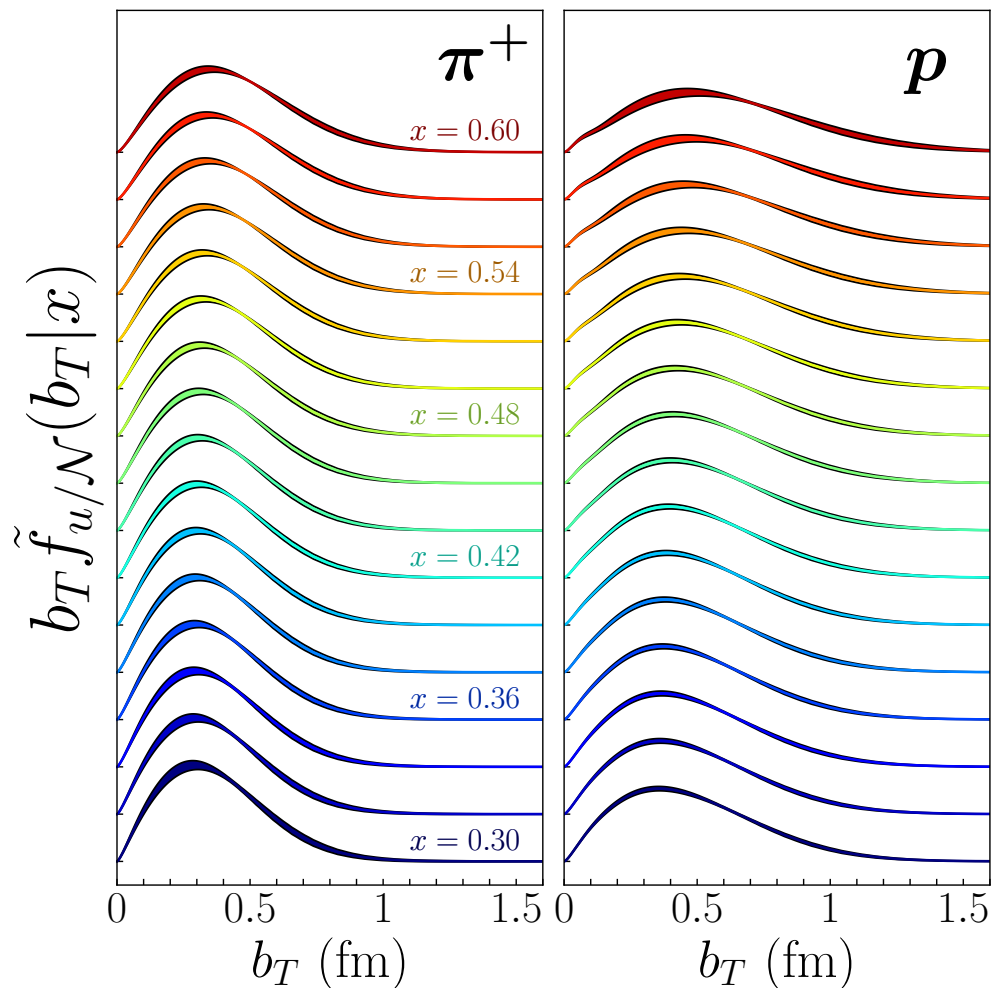
Naturally follow from Bayes’ theorem define a conditional density on b_T — given x

$$\tilde{f}(x, b_T; \mu, \zeta) = \tilde{f}(b_T|x; \mu, \zeta)f(x, \mu)$$

Operationally :

$$\tilde{f}_{q/\mathcal{N}}(b_T|x; \mu, \zeta) = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\int d^2\mathbf{b}_T \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}$$

Resulting TMD PDFs of proton & pion



$$\tilde{f}_{q/\mathcal{N}}(b_T|x; \mu, \zeta) = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\int d^2\mathbf{b}_T f_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}$$

- Shown in x range $x \in [0.3, 0.6]$ where π and p are both constrained: each TMDpdf show w/ 1σ uncertainty band from analysis
- u -quark in π is narrower than u -quark in p & “both” become wider w/increasing x

To make quantitative comparison btwn hadron distributions we consider the average b_T as a function of x

defined as

$$\langle b_T|x \rangle_{q/\mathcal{N}} = \int d^2\mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T|x; Q, Q^2)$$

“Average b_T ”

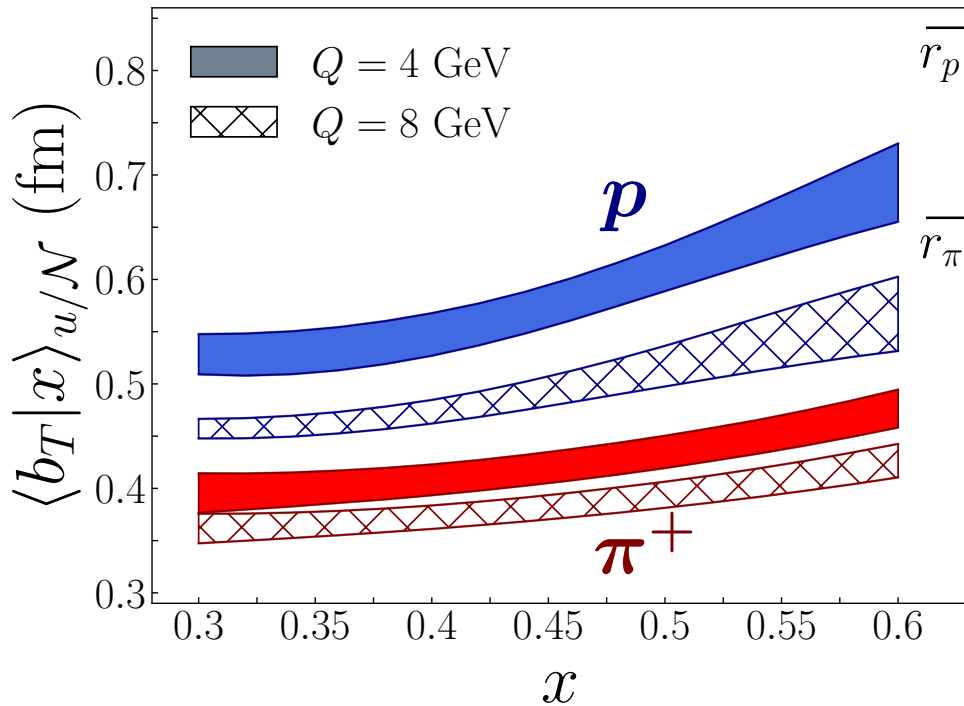
The conditional expectation value of b_T for a given x

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2 \mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

Gives a measure of the transverse correlation in coordinate space of the quark in a hadron for a given x

The conditional expectation value of b_T for a given x

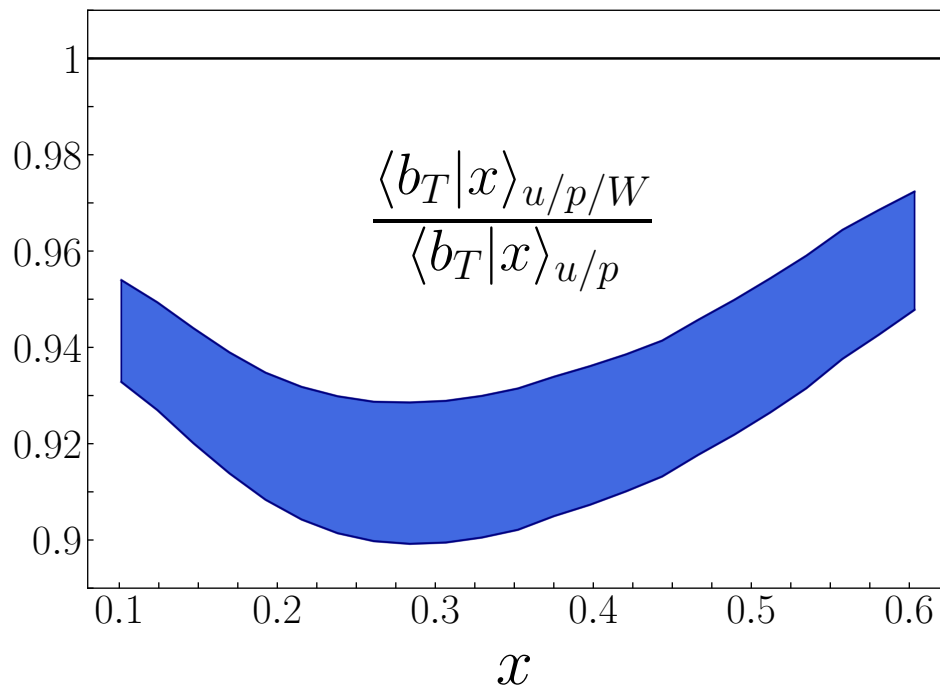
$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2 \mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$



- On av. $\sim 20\%$ reduction of u -quark transverse correlations in pion relative to proton within a $(5.3 - 7.5) \sigma$ confidence level
- Decreases as x decreases
- Interestingly: charge radius of the pion is also about 20% smaller than that of the proton, using the nominal PDG values, $r_p = 0.8409(4)$ fm, $r_\pi = 0.659(4)$ fm) [PDG]
- Also, within each hadron, the average spatial separation of quark fields in the transverse direction does not exceed its charge radius, as shown on the right edge of fig.
- As $x \rightarrow 1$ phase space for the transverse motion k_T of partons becomes smaller, since most of the momentum is along the light-cone direction, thus one expects an increase in the transverse correlations in b_T space
- & as Q increases more glue is radiated, which makes TMDpdf wider in k_T & therefore narrow in b_T space

both quantitatively confirmed in figure

The effect of nuclear environment on quark correlations inside nucleon EMC effect for conditional $\langle b_T | x \rangle$ av



- taking the ratio of $\langle b_T | x \rangle$ for a bound proton in a nucleus to that of a free proton
- Find analogous suppression at $x \sim 0.3$ similar to that found for collinear distributions “transverse EMC” effect
- Results are consistent with Alrasheed et al. PRL 129 (2022) extend beyond by looking at x dependence of non-perturbative transverse structure within a simultaneous collinear & TMD QCD global analysis

Summary

- We have presented a comprehensive analysis of π and P TMD PDFs at N2LL perturbative precision using fixed-target DY data.
- For the first time used both q_T -integrated and q_T - differential DY data, as well as LN measurements, to simultaneously extract π collinear and TMD PDFs and P TMD PDFs.
- The combined analysis, including an exploration of the nuclear dependence of TMDs, allows us to perform a detailed comparison of π and P TMDs and to study the similarities and differences of their transverse spatial & momentum dependence
- We have determined conclusively that the transverse correlations of quarks in a pion are $\approx 20\%$ smaller than those in a proton
- The observed characteristic decrease of the average separation of quark fields for decreasing x may indicate the influence of dynamical chiral symmetry breaking [102].
- We also found evidence for a transverse EMC effect, as discussed earlier by Alrashed et al. [73].

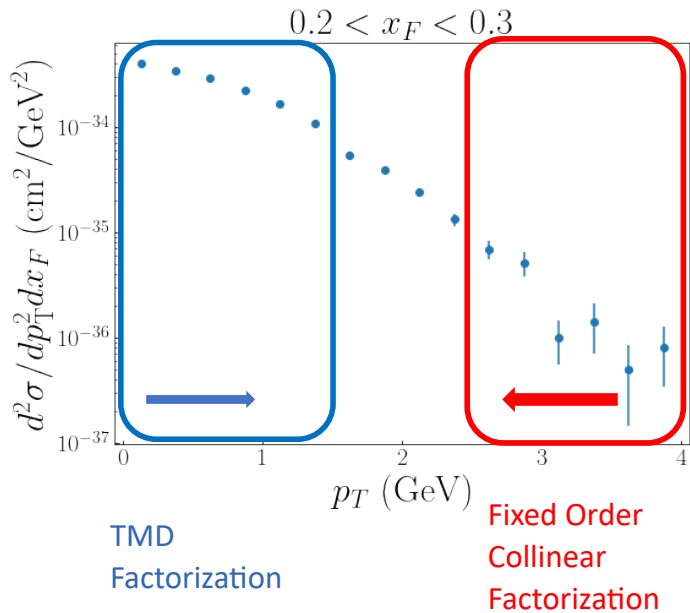
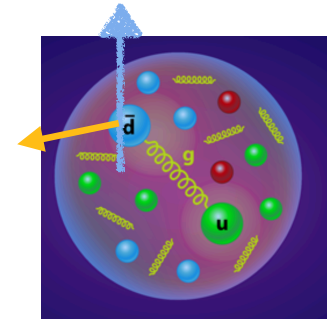
Outlook

- Exploration of the quark transverse correlations in pions and protons can be extended to other hadrons, such as kaons and neutrons, in the near future, when the tagged SIDIS programs at Jefferson Lab and the EIC become available.
- [Include lattice data from collinear \$\pi\$ pdfs](#)
- Also, studying in various NP schemes NP schemes, flavor dependence, ...
 - ★ Future entails the study of matching low and hi p_T data

Extras

“More granular” $p_T \sim k_T \ll Q$ access to the Pion TMDs

To describe the transverse momentum “region” $p_T \sim k_T \ll Q$ is the regime of TMDs of the pion
 Requires fitting “region” $p_T \sim k_T \ll Q$ differential pion-induced Drell-Yan cross section



TMD Factorization

$$\frac{d\sigma^W}{dQ^2 dx_F dp_T^2} = \int \frac{d^2 b_T}{(2\pi)^2} e^{i p_T \cdot b_T} \tilde{W}(x_F, b_T, Q)$$

$$\tilde{W}(x_F, b_T, Q) = \sum_j H_{j\bar{j}}^{\text{DY}}(Q, \mu, a_s(\mu)) \tilde{f}_{j/A}(x_A, b_T; \zeta_A, \mu) \tilde{f}_{\bar{j}/B}(x_B, b_T; \zeta_B, \mu)$$

Considering Non-perturbative Universal content schemes

$$e^{-g_k(b_T; b_{max}) \log\left(\frac{Q}{Q_0}\right)}$$

- Perturbative content calculated from first principles of QFT
- Non-perturbative content (as for collinear case) to be fit

$$\ast \quad g_K(b_T; b_{max}) = \frac{g_2 b_{NP}^2}{2} \ln\left(1 + \frac{b_T^2}{b_{NP}^2}\right) = g_2 b_{NP}^2 \ln \frac{b_T}{b_*}$$

$$\lim_{b_T \rightarrow 0} g_K(b_T; b_{max}) \approx \frac{g_2}{2} b_T^2$$

Aidala, Field, Gamberg, Rogers PRD 2014 “Large b_T “
Sun, Yuan 2015, used in many pheno analyses

$$\ast \quad g_k(b_T; b_{max}) = \frac{g_2}{2} b_T b_*$$

Vladimirov JHEP 2019 “Large b_T “

$$\ast \quad g_k(b_T; b_{max}) = g_0 \left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2}\right]\right)$$

Collins Rogers PRD 2015 “Large b_T “

Considering Non-perturbative “intrinsic” content schemes

$$\tilde{f}_{NP}^j(b_T, x, Q) = e^{-g_j(x, b_T; b_{max})}$$

- Non-perturbative content (as for collinear case) to be fit

* $\tilde{f}_{NP}^j(b_T, x, Q) = \left(\frac{b_*}{b_T}\right)^{2-\nu} \frac{K_{1-\nu}(Mb_T x^{g_1}(1-x)^{g_4}) \times (1 + C_A b_T + C_B/b_T)}{K_{1-\nu}(Mb_* x^{g_1}(1-x)^{g_4}) \times (1 + C_A b_* + C_B/b_*)}$ Bessel parametrization Aidala Gamberg Rogers 2014
large exponential behavior $b_T \sim$ advocated Collins Rogers 2014

* $e^{-g_j(x, b_T; b_{max})} \quad g_j(x, b_T) = \frac{(\lambda_1(1-x) + \lambda_2 x + \lambda_3 x(1-x)) b_T^2}{\sqrt{1 + \lambda_4 x^{\lambda_5} b_T^2}}$ Vladimirov JHEP 2019 has large exponential behavior in b_T

* $e^{-g_j(x, b_T; b_{max})} \quad g_j(x, b_T) = b_T^2 \left(\frac{g_1}{2} + g_1 g_3 \log \left(\frac{10 x x_0}{(x + x_0)} \right) \right)$ Aybat Rogers 2011
generalization of Landry et al.
2001...

* $F_{QZ}^{NP}(b, Q, x_A, x_B; b_{max}) = \exp \left\{ -\ln \left(\frac{Q^2 b_{max}^2}{c^2} \right) \left\{ g_1 [(b^2)^{\alpha} - (b_{max}^2)^{\alpha}] + g_2 (b^2 - b_{max}^2) \right\} - \bar{g}_2 (b^2 - b_{max}^2) \right\}$ Qiu & Zhang PRL 2001 effectively g_k and g_j contained

* $f_{1NP}(x, b_T^2; \zeta, Q_0) = \frac{g_1(x) e^{-g_1(x) \frac{b_T^2}{4}} + \lambda^2 g_{1B}^2(x) \left[1 - g_{1B}(x) \frac{b_T^2}{4} \right] e^{-g_{1B}(x) \frac{b_T^2}{4}} + \lambda^2 g_{1C}(x) e^{-g_{1C}(x) \frac{b_T^2}{4}}}{g_1(x) + \lambda^2 g_{1B}^2(x) + \lambda^2 g_{1C}(x)} \left[\frac{\zeta}{Q_0^2} \right]^{g_K(b_T^2)/2}$ Bacchetta et al. JHEP 10 (2022)

Explored nonperturbative parametrization of TMDs

- First perform single fits of these data to explore various aspects
- Many types of parametrizations have been used in the past
- For the “intrinsic” non-perturbative TMD, we perform fits with each of the following

Gaussian

$$\exp(-g_{q/N}(x, b_T)) = \exp(-g_q(x, A) b_T^2),$$

Exponential

$$\exp(-g_{q/N}(x, b_T)) = \exp(-g_q(x, A) b_T),$$

Gaussian-to-Exponential

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A) \frac{b_T^2}{\sqrt{1 + B_{NP}(x) b_T^2}}\right),$$

Parametrizations

- We can test whether or not the x -dependence is important for these functions (it is!)
- For these g_q functions, we have the following

$$g_q(x, A) = |g^q + g_2^q x + g_3^q (1 - x)^2| (1 + g_1(A^{1/3} - 1)) ,$$
$$B_{NP}(x) = b_{NP} x^2 ,$$

- 4 free parameters for each scheme (5 for Gaussian-to-Exponential)
- We may also open up these for each flavor in the proton (u , d , and *sea*) and for the pion (*val*, *sea*)

Problem describing data

- The E288 400 GeV data are difficult to describe the same above and below the Υ resonance
- Theory overpredicts data when $Q > 11\text{GeV}$
- Could treat as separate datasets – separate normalizations:

E228-200	39
E228-300 $Q < 9\text{GeV}$	43
E228-300 $Q > 11\text{GeV}$	10
E228-400 $Q < 9\text{GeV}$	34
E228-400 $Q > 11\text{GeV}$	42
E772	24
E605 $Q < 9\text{GeV}$	21
E605 $Q > 11\text{GeV}$	32

