Determination of Collins-Soper kernel

and TMDPDFs

from global analysis and lattice

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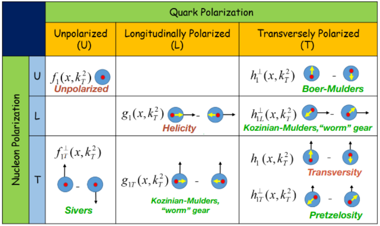
IPARCOS

Comunidad de Madrid

TMDs: Towards a Synergy between Lattice QCD and Global Analysis



+

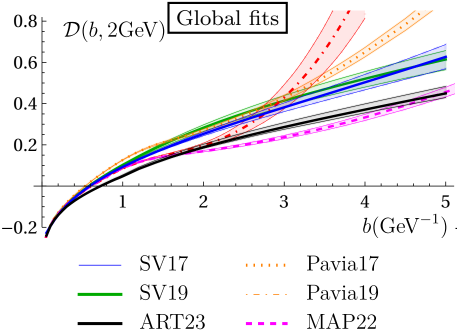
$$\mathcal{D} = -\frac{\tilde{K}}{2}$$

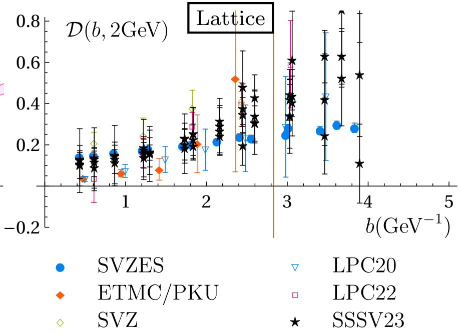
Collins-Soper kernel

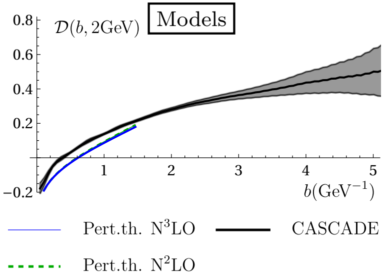
$$\frac{dF(x,b;\mu,\zeta)}{d\ln\zeta} = -\mathcal{D}(b,\mu)$$

Nonperturbative part of the evolution
to be extracted together with TMD distributions

- Fits of collider data
- Fits of lattice simulations
- Models


Global fits
$\mathcal{D}(b, 2\text{GeV})$
0.8
0.6
0.4
0.2
−0.2
1
2
3
4
5
$b(\text{GeV}^{-1})$
SV17
SV19
ART23
Pavia17
Pavia19
MAP22


Lattice
$\mathcal{D}(b, 2\text{GeV})$
0.8
0.6
0.4
0.2
−0.2
1
2
3
4
5
$b(\text{GeV}^{-1})$
SVZES
ETMC/PKU
SVZ
LPC20
LPC22
SSSV23


Models
$\mathcal{D}(b, 2\text{GeV})$
0.8
0.6
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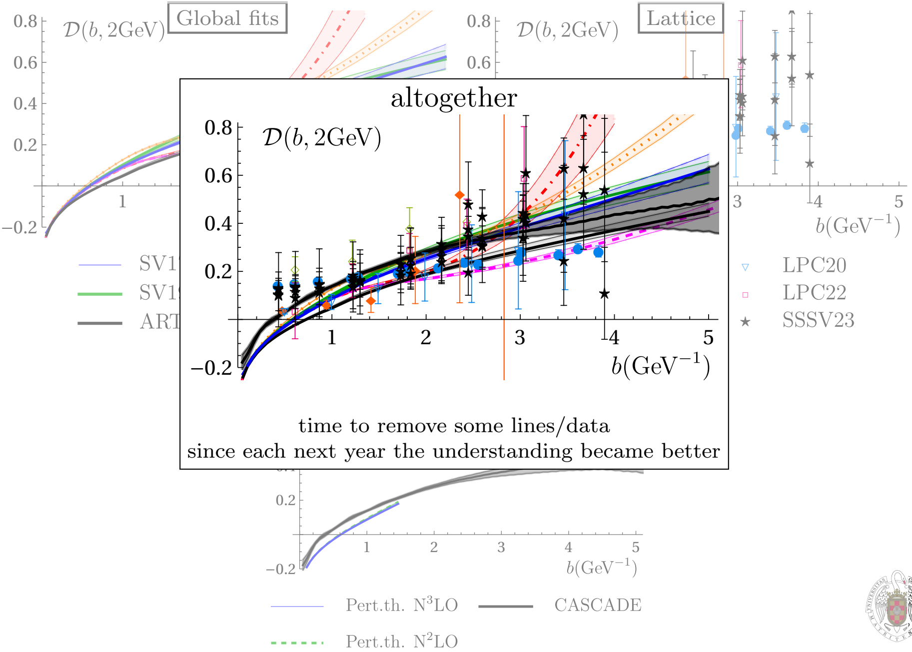
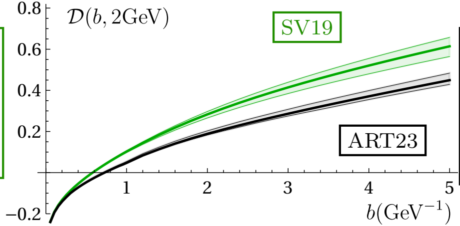
1
2
3
4
5
$b(\text{GeV}^{-1})$
Pert.th. N^{3}LO
CASCADE
Pert.th. N^{2}LO


Global fits
Lattice
$\mathcal{D}(b, 2\text{GeV})$
altogether
0.8
0.6
0.4
0.2
−0.2
1
2
3
4
5
$b(\text{GeV}^{-1})$
SV1
SV1
ART
LPC20
LPC22
SSSV23
Pert.th. N^{3}LO
CASCADE
Pert.th. N^{2}LO
time to remove some lines/data
since each next year the understanding became better

[1912.06532]

- DY (457 pt.)
- SIDIS (582 pt.)
- $2 < Q < 150$GeV
- N^3LL



[2305.07473]

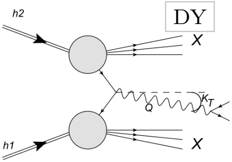
- DY (627 pt.)
- ~~SIDIS~~
- $4 < Q < 1000$GeV
- N^4LL

- Same realisation (ζ-prescription)
- Same code (**artemide**)

- Better model
- More precise data

TMD factorization theorem

$$\frac{d\sigma}{dq_T}=\sigma_0\int\frac{d^2b}{(2\pi)^2}e^{i(bq_T)}C\left(\frac{Q}{\mu}\right)F(x_1,b;\mu,\zeta)F(x_2,b;\mu,\bar\zeta)$$



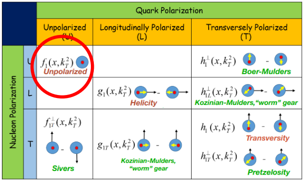
Factorization valid at:

$Q\gg\Lambda \qquad Q\gg q_T \qquad Q\gg k_T$

TMD factorization theorem

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

N^{4}LO



Evolution
N^{4}LO

TMD factorization theorem

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

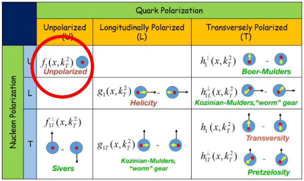
N^{4}LO

N^{3}LO

TMDPDF at small-b

$$\lim_{b\to 0} F_f(x,b) \simeq \int_x^1 \frac{dy}{y} C_{f\leftarrow f;}\left(\frac{x}{y}; \ln(b)\right) q_f(y)$$

(resummation regime)



Evolution
N^{4}LO

TMD factorization theorem

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

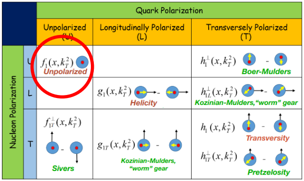
N^{3}LO

N^{4}LO

TMDPDF at small-b

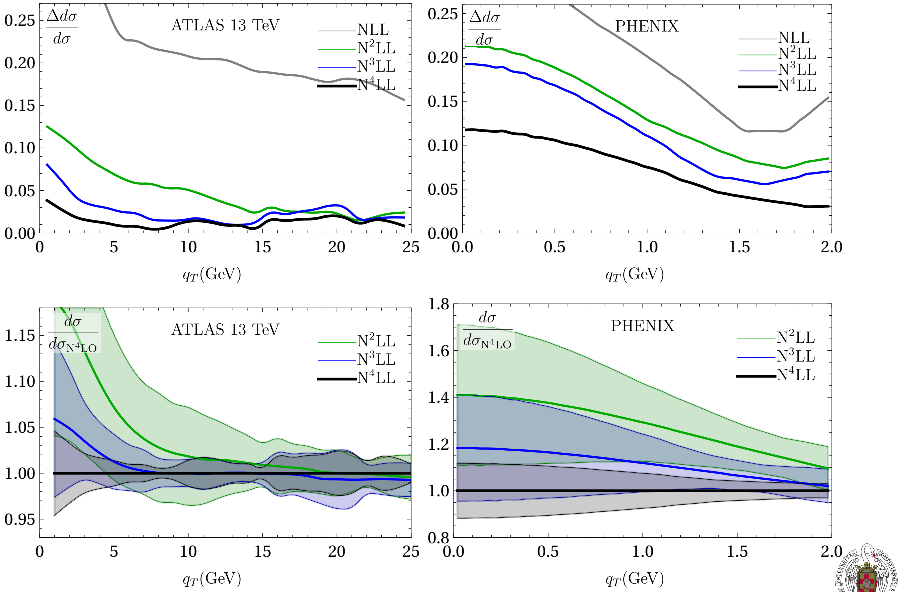
$$\lim_{b\to 0} F_f(x,b) \simeq \int_x^1 \frac{dy}{y} C_{f\leftarrow f'}\left(\frac{x}{y}; \ln(b)\right) q_{f'}(y)$$

(resummation regime)

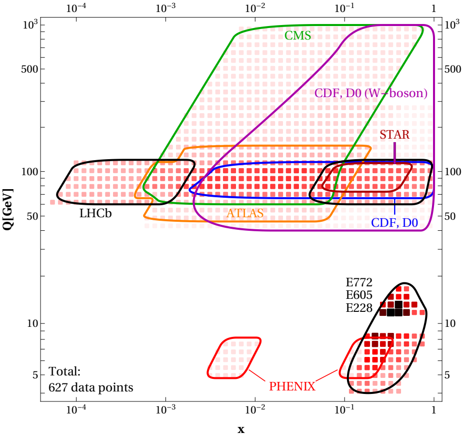


Evolution
N^{4}LO

Altogether N^{4}LL
(in resummation nomenclature)



$\frac{\Delta d\sigma}{d\sigma}$
ATLAS 13 TeV
NLL
N^{2}LL
N^{3}LL
N^{4}LL
q_T(GeV)
PHENIX
$\frac{d\sigma}{d\sigma_{N^4LO}}$



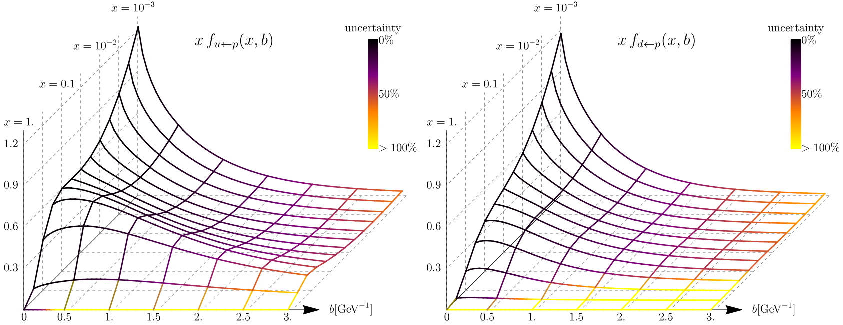
*** data included for the first time**

- ATLAS
  - Z-boson at 8 (y-diff.)
  - **Z-boson at 13 TeV (0.1% prec.!)**
- CMS
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
  - **Z/γ up to $Q = 1000$GeV**
- LHCb
  - Z-boson at 7 and 8 TeV
  - **Z-boson at 13 TeV (y-diff.)**
- **Further more:**
  - Z-boson at Tevatron
  - **W-boson at Tevatron**
  - **Z-boson at RHIC**
  - DY at PHENIX
  - DY at FERMILAB (fix target)

627 data points

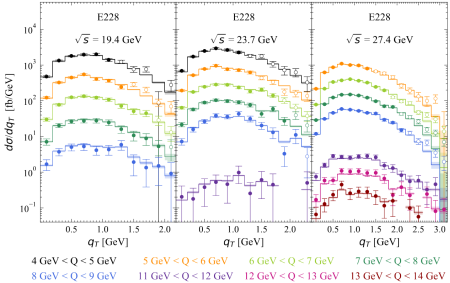
vs. 457 in SV19
vs. 484 in MAP22

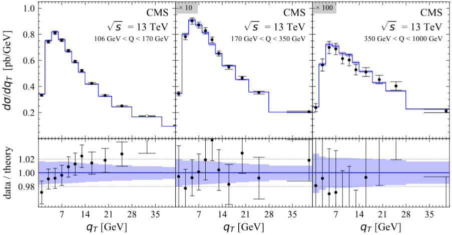
ART23



Extra features of analyses:

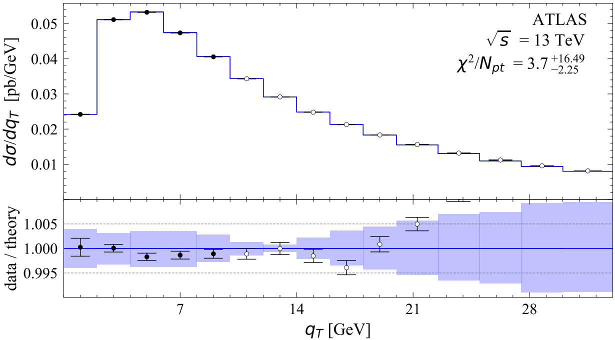
- Flavor dependent NP-ansatz (first time!)
  - 2 parameters per flavor
  - u, d, $\bar{u}$, $\bar{d}$, rest
- New parametrization for Collins-Soper kernel (3 parameters)
- Consistent inclusion of the PDF uncertainty (first time!)
- *artemide*





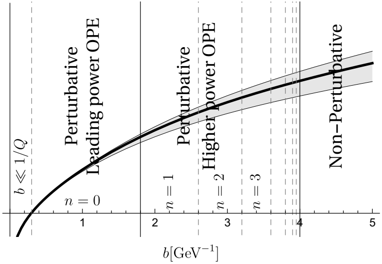
4GeV → 1000GeV

Very presice test of TMD evolution



$$\text{TOTAL } (N_{\text{pt}} = 627): \quad \chi^2/N_{\text{pt}} = 0.96^{+0.09}_{-0.01}$$

Bless and curse of small-b matching



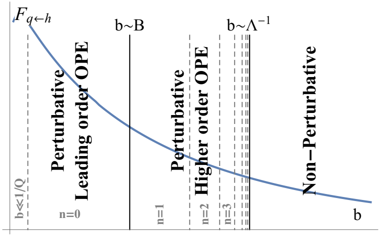
$$\lim_{b\to 0} \mathcal{D}(b) \sim a_s(\mu) 2C_F \mathbf{L}_\mu + a_s^2 \ldots$$

+ power corrections

Usual model:

$$\mathcal{D}(b,\mu) = \mathcal{D}_{\text{small-b}}(b^*,\mu) + g_{\text{NP}}(b)$$

$$g_{\text{NP}}(b) = \mathcal{O}(b^2)$$



$$\lim_{b\to 0} F(x,b) \sim f(x,\mu) + a_s \ldots$$

+ power corrections

Usual model:

$$F(x,b) = f_{\text{small-b}}(x,b^*) f_{\text{NP}}(x,b)$$

$$f_{\text{NP}}(x,b) = 1 + \mathcal{O}(b^2)$$

Bless and curse of small-b matching

Why is it good:

- use the power of perturbation theory in the important region
- re-use/agreement with collinear fits
- *conceptually* the model is still very general

Why is it bad:

- Extremely restrict the freedom (if one uses a "small" number of parameters)

??-bias

$$\lim_{b\to 0}\mathcal{D}(b)\sim a_s(\mu)2C_F\mathbf{L}_\mu+a_s^2...$$

+ power corrections

Usual model:

$$\mathcal{D}(b,\mu)=\mathcal{D}_{\text{small-b}}(b^*,\mu)+g_{\text{NP}}(b)$$

$$g_{\text{NP}}(b)=\mathcal{O}(b^2)$$

PDF-bias

$$\lim_{b\to 0}F(x,b)\sim f(x,\mu)+a_s...$$

+ power corrections

Usual model:

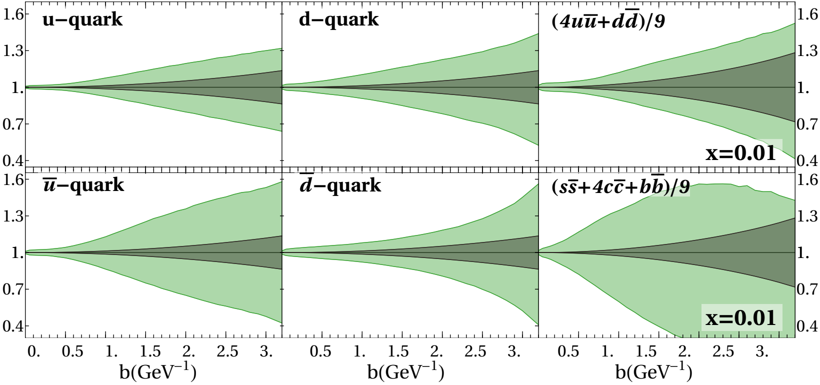
$$F(x,b)=f_{\text{small-b}}(x,b^*)f_{\text{NP}}(x,b)$$

$$f_{\text{NP}}(x,b)=1+\mathcal{O}(b^2)$$

How to fight PDF-bias?

[Bury, et al: 2201.07114]

- **(Ultimately)** Fit PDF and TMDPDF together
- **(Poor man solution)**Include PDF uncertainty into the TMD fit
- Increase flexibility of ansatz (flavor-dependence)



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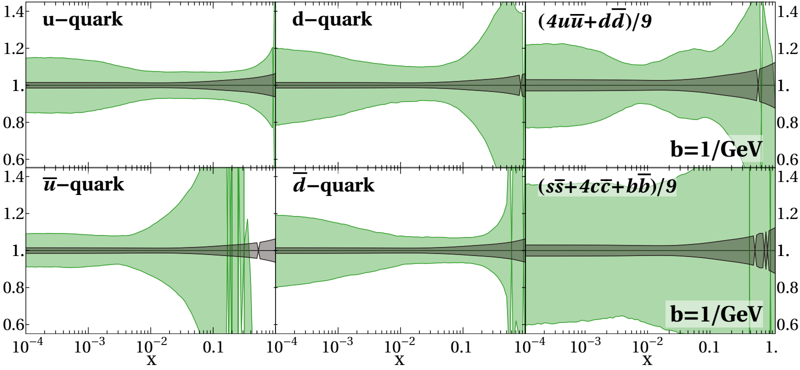

How to fight PDF-bias?

[Bury, et al: 2201.07114]

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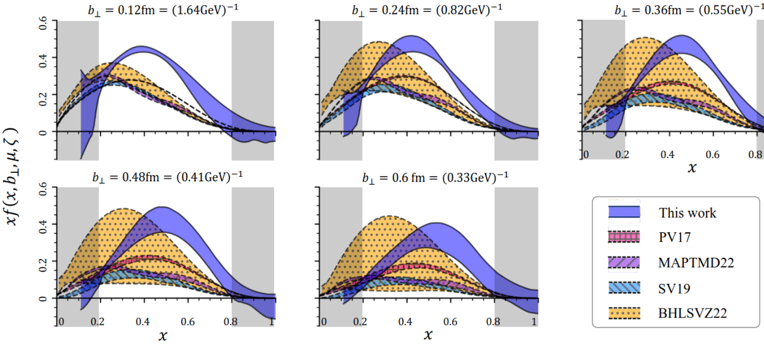
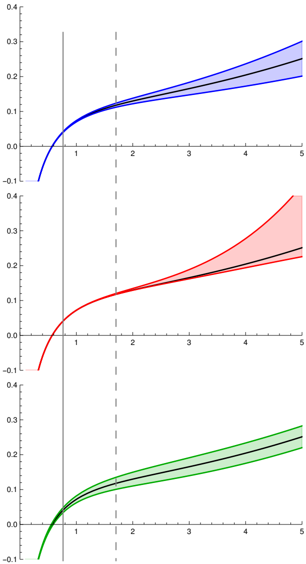
[LPC:2211.02340]



How to fight ??-bias?

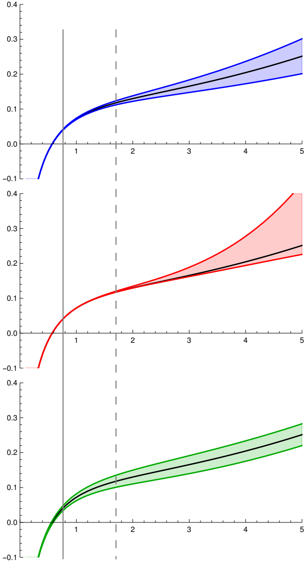


$\sim b^2$

$\sim b^4$

$\sim b^2 \ln(b^*)$

How to fight ??-bias?



$$\mathcal{D}(b,\mu) = \mathcal{D}_{\text{small-b}} + bb^*(c_0 + c_1 \ln(b^*/B_{\text{NP}}))$$

$$B_{\text{NP}} = 1.56^{+0.13}_{-0.09}\text{GeV}$$

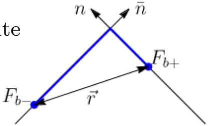
$$c_0 = 3.69^{+0.65}_{-0.61} \cdot 10^{-2}$$

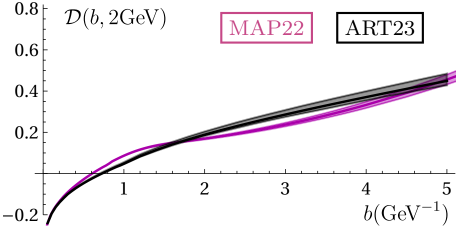
$$c_1 = 5.82^{+0.64}_{-0.88} \cdot 10^{-2}$$

$$\mathcal{D} = \mathcal{D}_0 + \frac{\mathbf{b}^2}{2}\left(\int d\mathbf{r}^2 \frac{\varphi_1(\mathbf{r}^2,0,0)}{\mathbf{r}^2} + \mathcal{O}(a_s)\right) + \mathbf{b}^4 \ldots$$

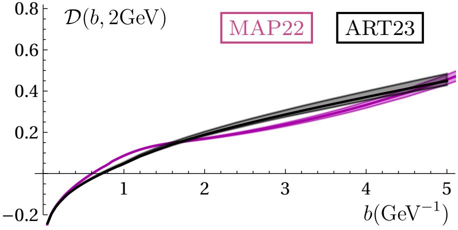
[AV:2003.02288]

? Time to compute log correction ?





Very small uncertanties
(despite huge in TMDPDFs)



Very small uncertanties
(despite huge in TMDPDFs)

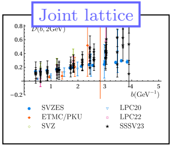
Can lattice compete with it?

PRO

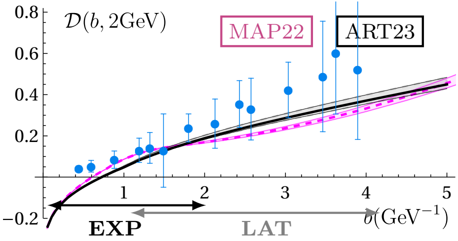
- Can access large-b
- Can study "exotic" sources
- Directly in b-space

CONTRA

- Large power corrections
- Lattice artifacts
- Unknown scheme factor



w.average
(please, do not hit me)



Very small uncertanties
(despite huge in TMDPDFs)

~~Can lattice compete with it?~~
What can lattice add to it?

PRO

- Can access large-b
- Can study "exotic" sources
- Directly in b-space

CONTRA

- Large power corrections
- Lattice artifacts
- Unknown scheme factor

Measuring evolution in experiment and lattice

$$d\sigma(Q,q_T)=\int d^2b e^{i(qb)} H_{\text{DY}}\ F(x_1,b)\ F(x_2,b)$$

$$\Omega(\ell,b;(vP))=\int dx e^{ix\ell p} H_{\text{qTMD}}\ F(x,b)\ \Psi(b)$$

N^2LO
[O.Rio, AV:2304.14440]

"reduced SF"
"instant-jet" TMD

Measuring evolution in experiment and lattice

$$d\sigma(Q,q_T)=\int d^2b e^{i(qb)}H_{\text{DY}}\;F(x_1,b)\;F(x_2,b)\qquad\longrightarrow\quad\frac{\mathcal{F}^{-1}d\sigma(Q_1)}{\mathcal{F}^{-1}d\sigma(Q_2)}=\frac{H_{\text{DY}}(Q_1)}{H_{\text{DY}}(Q_2)}R(Q_1\to Q_2)[\mathcal{D}(b)]$$

$$\Omega(\ell,b;(vP))=\int dx e^{ix\ell p}H_{\text{qTMD}}\;F(x,b)\;\Psi(b)\qquad\longrightarrow\quad\frac{\mathcal{F}^{-1}\Omega((vp_1))}{\mathcal{F}^{-1}\Omega((vp_2))}=\frac{H_{\text{q}}(vp_1)}{H_q(vp_2)}R((vp_1)\to(vp_2))[\mathcal{D}(b)$$

N^2LO
[O.Rio, AV:2304.14440]

"reduced SF"
"instant-jet" TMD

Measuring evolution in experiment and lattice

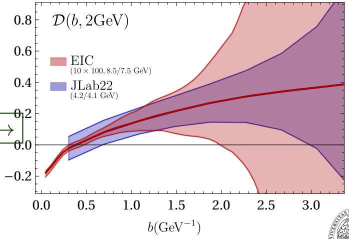
$$d\sigma(Q,q_T)=\int d^2b e^{i(qb)}H_{\rm DY}\ F(x_1,b)\ F(x_2,b) \qquad \longrightarrow \qquad \frac{\mathcal{F}^{-1}d\sigma(Q_1)}{\mathcal{F}^{-1}d\sigma(Q_2)}=\frac{H_{\rm DY}(Q_1)}{H_{\rm DY}(Q_2)}R(Q_1\to Q_2)[\mathcal{D}(b)]$$

$$\Omega(\ell,b;(vP))=\int dx e^{ix\ell p}H_{\rm qTMD}\ F(x,b)\ \Psi(b) \qquad \longrightarrow \qquad \frac{\mathcal{F}^{-1}\Omega((vp_1))}{\mathcal{F}^{-1}\Omega((vp_2))}=\frac{H_{\rm q}(vp_1)}{H_q(vp_2)}R((vp_1)\to(vp_2))[\mathcal{D}(b)$$

N^2LO
[O.Rio, AV:2304.14440]

"reduced SF"
"instant-jet" TMD

Direct measurement of CS kernel from collider data →



In future lattice will be preciser, but experiment will be also preciser.

The true power of lattice simulations is access to “difficult” or impossible for experiment channels

- x-moments of TMDs
- Gluon CS-kernel
- Gluon TMDs
- Meson TMDs
- Higher-twist TMDs
- .....

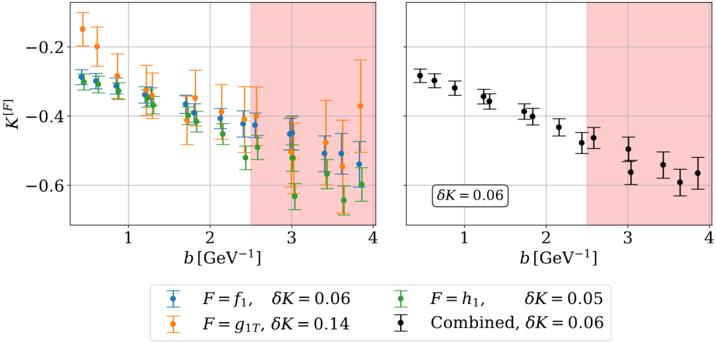
Latest example:
test of of universality of CS kernel
[Hai-Tao Shu, M.Schlemmer, T.Sizmann, et al: 2302.06502]

Collins-Soper kernel is the evolution kernel for TMDs
and it universal for

- All TMDPDFs/TMDFFs of twist-2 (all types and hadrons)
- All TMDPDFs/TMDFFs of twist-3 (all types and hadrons) [AV, et al, 2008.01744],[Ebert,at al, 2112.09771]
- All quasi-partonic TMDPDFs/TMDFFs of twist-3 (all types and hadrons) [AV, et al, 2008.01744]

Check of universality for $\{f_1, g_{1T}, h_1\}$

[M.Schlemmer, et al,2103.16991]



$$K = -2\mathcal{D}$$

NLP TMD factorization is very complicated!

[Rodini,AV:2211.04494]

$$
\begin{aligned}
F(x,b;\mu) = \frac{1}{x}\left(\frac{(2|x|(vP))^2}{\zeta}\right)^{-\mathcal{D}(b,\mu)} \Big\{ & \mathbb{C}_{11}(\mathbf{L}_p,\mu) A(x,b;\mu,\zeta) \\
& + \mathbb{C}_{11}(\mathbf{L}_p,\mu)\left(\mathbf{\Psi}_2(b) + \mathring{\mathcal{D}}(b)\ln\left(\frac{\mu(2|x|(vP))}{\zeta}\right)\right) B(x,b;\mu,\zeta) \\
& + \int_{-1}^{1} dx_2 \left(\mathbb{C}_R(\mathbf{L}_p,x,x_2) C(\tilde{x},b;\mu,\zeta) + s\pi\,\mathbb{C}_I(\mathbf{L}_p,x,x_2) D(\tilde{x},b;\mu,\zeta)\right) \Big\}
\end{aligned}
$$

Twist-3 qTMD

Derivative twist-2 TMD

twist-2 TMD

twist-3 "reduced SF"

Twist-3 TMDs $\langle \bar{q} G q \rangle$

Derivative CS-kernel

NLP TMD factorization is very complicated!

[Rodini,AV:2211.04494]

Derivative twist-2 TMD

Twist-3 qTMD

$$F(x,b;\mu) = \frac{1}{x}\left(\frac{(2|x|(vP))^2}{\zeta}\right)^{-\mathcal{D}(b,\mu)}\Big\{\mathbb{C}_{11}(\mathbf{L}_p,\mu)A(x,b;\mu,\zeta)$$
$$+\mathbb{C}_{11}(\mathbf{L}_p,\mu)\left(\boldsymbol{\Psi}_2(b)+\mathring{\mathcal{D}}(b)\ln\left(\frac{\mu(2|x|(vP))}{\zeta}\right)\right)B(x,b;\mu,\zeta)$$
$$+\int_{-1}^{1}dx_2\,\big(\mathbb{C}_R(\mathbf{L}_p,x,x_2)C(\tilde{x},b;\mu,\zeta)+s\pi\mathbb{C}_I(\mathbf{L}_p,x,x_2)D(\tilde{x},b;\mu,\zeta)\big)\Big\}$$

twist-2 TMD

twist-3 "reduced SF"

Twist-3 TMDs $\langle\bar{q}Gq\rangle$

Derivative CS-kernel

Γ	qTMD	A	B	C	D
$\mathbb{1}$	E			$2\mathbf{h}_{\oplus}$	$2\mathbf{h}_{\ominus}$
	$E_T^{\perp}$			$2\mathbf{h}_{\oplus T}^{A\perp}$	$2\mathbf{h}_{\ominus T}^{A\perp}$
$i\gamma^5$	E_L			$2\mathbf{h}_{\oplus L}$	$2\mathbf{h}_{\ominus L}$
	E_T			$2\mathbf{h}_{\oplus T}^{D\perp}$	$2\mathbf{h}_{\ominus T}^{D\perp}$
γ^α	F_T	$-f_{1T}^{\perp}-\frac{b^2M^2}{2}\mathring{f}_{1T}^{\perp}$	$\frac{b^2M^2}{2}f_{1T}^{\perp}$	$\mathbf{f}_{\ominus T}-\mathbf{g}_{\oplus T}$	$-\mathbf{f}_{\oplus T}-\mathbf{g}_{\ominus T}$
	$F_L^{\perp}$			$-\mathbf{f}_{\ominus L}^{\perp}+\mathbf{g}_{\oplus L}^{\perp}$	$\mathbf{f}_{\oplus L}^{\perp}+\mathbf{g}_{\ominus L}^{\perp}$
	$F^{\perp}$	$\mathring{f}_1$	$-f_1$	$\mathbf{f}_{\ominus}^{\perp}-\mathbf{g}_{\oplus}^{\perp}$	$-\mathbf{f}_{\oplus}^{\perp}-\mathbf{g}_{\ominus}^{\perp}$
	$F_T^{\perp}$	$\mathring{f}_{1T}^{\perp}$	$-f_{1T}^{\perp}$	$-\mathbf{f}_{\ominus T}^{\perp}+\mathbf{g}_{\oplus T}^{\perp}$	$\mathbf{f}_{\oplus T}^{\perp}+\mathbf{g}_{\ominus T}^{\perp}$
$\gamma^\alpha\gamma^5$	G_T	$g_{1T}+\frac{b^2M^2}{2}\mathring{g}_{1T}$	$-\frac{b^2M^2}{2}g_{1T}$	$-\mathbf{f}_{\oplus T}-\mathbf{g}_{\ominus T}$	$-\mathbf{f}_{\ominus T}+\mathbf{g}_{\oplus T}$
	$G_L^{\perp}$	$\mathring{g}_1$	$-g_1$	$\mathbf{f}_{\oplus L}^{\perp}+\mathbf{g}_{\ominus L}^{\perp}$	$\mathbf{f}_{\ominus L}^{\perp}-\mathbf{g}_{\oplus L}^{\perp}$
	$G^{\perp}$			$\mathbf{f}_{\oplus}^{\perp}+\mathbf{g}_{\ominus}^{\perp}$	$\mathbf{f}_{\ominus}^{\perp}-\mathbf{g}_{\oplus}^{\perp}$
	$G_T^{\perp}$	$\mathring{g}_{1T}$	$-g_{1T}$	$\mathbf{f}_{\oplus T}^{\perp}+\mathbf{g}_{\ominus T}^{\perp}$	$\mathbf{f}_{\ominus T}^{\perp}-\mathbf{g}_{\oplus T}^{\perp}$
$i\sigma^{\alpha\beta}\gamma^5$	$H_T^{\perp}$	$-h_{1T}^{\perp}+\mathring{h}_1-\frac{b^2M^2}{4}\mathring{h}_{1T}^{\perp}$	$-h_1+\frac{b^2M^2}{4}h_{1T}^{\perp}$	$2\mathbf{h}_{\ominus T}^{A\perp}$	$-2\mathbf{h}_{\oplus T}^{A\perp}$
	H	$-2h_1^{\perp}$		$-2\mathbf{h}_{\ominus}$	$2\mathbf{h}_{\oplus}$
$i\sigma^{+-}\gamma^5$	$H_L^{\perp}$	$-2h_{1L}^{\perp}-b^2M^2\mathring{h}_{1L}^{\perp}$	$b^2M^2h_{1L}^{\perp}$	$-2\mathbf{h}_{\ominus L}$	$2\mathbf{h}_{\oplus L}$
	H_T	$-h_{1T}^{\perp}-\mathring{h}_1-\frac{b^2M^2}{4}\mathring{h}_{1T}^{\perp}$	$h_1+\frac{b^2M^2}{4}h_{1T}^{\perp}$	$-2\mathbf{h}_{\ominus T}^{D\perp}$	$2\mathbf{h}_{\oplus T}^{D\perp}$

NLP TMD factorization is very complicated!

[Rodini,AV:2211.04494]

$$F(x,b;\mu) = \frac{1}{x}\left(\frac{(2|x|(vP))^2}{\zeta}\right)^{-\mathcal{D}(b,\mu)}\Big\{\mathbb{C}_{11}(\mathbf{L}_p,\mu)A(x,b;\mu,\zeta)$$
$$+\mathbb{C}_{11}(\mathbf{L}_p,\mu)\left(\mathbf{\Psi}_2(b)+\mathring{\mathcal{D}}(b)\ln\left(\frac{\mu(2|x|(vP))}{\zeta}\right)\right)B(x,b;\mu,\zeta)$$
$$+\int_{-1}^{1}dx_2\left(\mathbb{C}_R(\mathbf{L}_p,x,x_2)C(\tilde{x},b;\mu,\zeta)+s\pi\mathbb{C}_I(\mathbf{L}_p,x,x_2)D(\tilde{x},b;\mu,\zeta)\right)\Big\}$$

Twist-3 qTMD

Derivative twist-2 TMD

twist-2 TMD

twist-3 "reduced SF"

Twist-3 TMDs $\langle\bar{q}Gq\rangle$

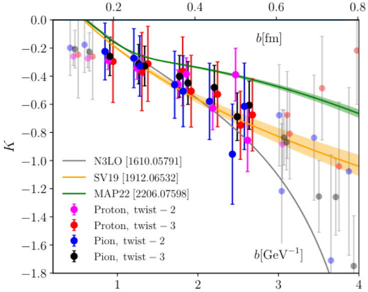
Derivative CS-kernel

Γ	qTMD	A	B	C	D
$\mathbb{1}$	E			$2\mathbf{h}_{\oplus}$	$2\mathbf{h}_{\ominus}$
	$E_T^{\perp}$			$2\mathbf{h}_{\oplus T}^{A\perp}$	$2\mathbf{h}_{\ominus T}^{A\perp}$
$i\gamma^5$	E_L			$2\mathbf{h}_{\oplus L}$	$2\mathbf{h}_{\ominus L}$
	E_T			$2\mathbf{h}_{\oplus T}^{D\perp}$	$2\mathbf{h}_{\ominus T}^{D\perp}$
γ^{α}	F_T	$-f_{1T}^{\perp}-\frac{b^2M^2}{2}\mathring{f}_{1T}^{\perp}$	$\frac{b^2M^2}{2}f_{1T}^{\perp}$	$\mathbf{f}_{\ominus T}-\mathbf{g}_{\oplus T}$	$-\mathbf{f}_{\oplus T}-\mathbf{g}_{\ominus T}$
	$F_L^{\perp}$			$-\mathbf{f}_{\ominus L}^{\perp}+\mathbf{g}_{\oplus L}^{\perp}$	$\mathbf{f}_{\oplus L}^{\perp}+\mathbf{g}_{\ominus L}^{\perp}$
	$F^{\perp}$	$\mathring{f}_1$	$-f_1$	$\mathbf{f}_{\ominus}^{\perp}-\mathbf{g}_{\oplus}^{\perp}$	$-\mathbf{f}_{\oplus}^{\perp}-\mathbf{g}_{\ominus}^{\perp}$
	$F_T^{\perp}$	$\mathring{f}_{1T}^{\perp}$	$-f_{1T}^{\perp}$	$-\mathbf{f}_{\ominus T}^{\perp}+\mathbf{g}_{\oplus T}^{\perp}$	$\mathbf{f}_{\oplus T}^{\perp}+\mathbf{g}_{\ominus T}^{\perp}$
$\gamma^{\alpha}\gamma^5$	G_T	$g_{1T}+\frac{b^2M^2}{2}\mathring{g}_{1T}$	$-\frac{b^2M^2}{2}g_{1T}$	$-\mathbf{f}_{\oplus T}-\mathbf{g}_{\ominus T}$	$-\mathbf{f}_{\ominus T}+\mathbf{g}_{\oplus T}$
	$G_L^{\perp}$	$\mathring{g}_1$	$-g_1$	$\mathbf{f}_{\oplus L}^{\perp}+\mathbf{g}_{\ominus L}^{\perp}$	$\mathbf{f}_{\ominus L}^{\perp}-\mathbf{g}_{\oplus L}^{\perp}$
	$G^{\perp}$			$\mathbf{f}_{\oplus}^{\perp}+\mathbf{g}_{\ominus}^{\perp}$	$\mathbf{f}_{\ominus}^{\perp}-\mathbf{g}_{\oplus}^{\perp}$
	$G_T^{\perp}$	$\mathring{g}_{1T}$	$-g_{1T}$	$\mathbf{f}_{\oplus T}^{\perp}+\mathbf{g}_{\ominus T}^{\perp}$	$\mathbf{f}_{\ominus T}^{\perp}-\mathbf{g}_{\oplus T}^{\perp}$
$i\sigma^{\alpha\beta}\gamma^5$	$H_T^{\perp}$	$-h_{1T}^{\perp}+\mathring{h}_1-\frac{b^2M^2}{4}\mathring{h}_{1T}^{\perp}$	$-h_1+\frac{b^2M^2}{4}h_{1T}^{\perp}$	$2\mathbf{h}_{\ominus T}^{A\perp}$	$-2\mathbf{h}_{\oplus T}^{A\perp}$
	H	$-2h_1^{\perp}$		$-2\mathbf{h}_{\ominus}$	$2\mathbf{h}_{\oplus}$
$i\sigma^{+-}\gamma^5$	$H_L^{\perp}$	$-2h_{1L}^{\perp}-b^2M^2\mathring{h}_{1L}^{\perp}$	$b^2M^2h_{1L}^{\perp}$	$-2\mathbf{h}_{\ominus L}$	$2\mathbf{h}_{\oplus L}$
	H_T	$-h_{1T}^{\perp}-\mathring{h}_1-\frac{b^2M^2}{4}\mathring{h}_{1T}^{\perp}$	$h_1+\frac{b^2M^2}{4}h_{1T}^{\perp}$	$-2\mathbf{h}_{\ominus T}^{D\perp}$	$2\mathbf{h}_{\oplus T}^{D\perp}$

6 qTMDs (out of 16)
can be used to determine
CS-kernel
(alike in twist-2 case)

Check of universality for $\{f_1(proton), f_1(pion), e(proton), e(pion)\}$

[Hai-Tao Shu, et al,2302.06502]



$$K = -2\mathcal{D}$$

Conclusion

TMDs: Towards a Synergy between Lattice QCD and Global Analysis

The synergy in the phenomenology of lattice and collider data
is in their complementarity

b-space $\longleftrightarrow$ k_T-space

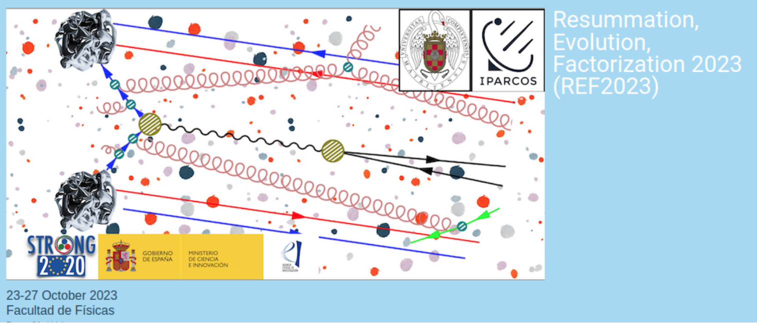
low-energy $\longleftrightarrow$ high-energy

low-statistic many channels $\longleftrightarrow$ high-statistic few channels

... $\longleftrightarrow$ ...

Outline of talk:

- ART23 extraction
  - N^4LL
  - Larger data set (mainly due to LHC data)
  - (more) Accurate determination of uncertanties
  - *artemide:* https://github.com/VladimirovAlexey/artemide-public
- Universality of CS kernel
  - Evolution for different polarizations is the same
  - Evolution for twist-2 and twist-3 TMDs is the same
  - Evolution for pion and proton TMDs is the same



Universidad Complutense de Madrid

23-27 October 2023

https://indico.fis.ucm.es/event/19/

(registration is open)

Backup slides

data set	N_{pt}	χ^2_D/N_{pt}	χ^2_λ/N_{pt}	χ^2/N_{pt}
CDF (run1)	33	0.51	0.16	$0.67^{+0.05}_{-0.03}$
CDF (run2)	45	1.58	0.11	$1.59^{+0.26}_{-0.14}$
CDF (W-boson)	6	0.33	0.00	$0.33^{+0.01}_{-0.01}$
D0 (run1)	16	0.69	0.00	$0.69^{+0.08}_{-0.03}$
D0 (run2)	13	2.16	0.16	$2.32^{+0.40}_{-0.32}$
D0 (W-boson)	7	2.39	0.00	$2.39^{+0.20}_{-0.18}$
ATLAS (8TeV, $Q \sim M_Z$)	30	1.60	0.49	$2.09^{+1.09}_{-0.35}$
ATLAS (8TeV)	14	1.11	0.11	$1.22^{+0.47}_{-0.21}$
ATLAS (13 TeV)	5	1.94	1.75	$3.70^{+16.5}_{-2.24}$
CMS (7TeV)	8	1.30	0.00	$1.30^{+0.03}_{-0.01}$
CMS (8TeV)	8	0.79	0.00	$0.78^{+0.02}_{-0.01}$
CMS (13 TeV, $Q \sim M_Z$)	64	0.63	0.24	$0.86^{+0.23}_{-0.11}$
CMS (13 TeV, $Q > M_Z$)	33	0.73	0.12	$0.92^{+0.40}_{-0.15}$
LHCb (7 TeV)	10	1.21	0.56	$1.77^{+0.53}_{-0.31}$
LHCb (8 TeV)	9	0.77	0.78	$1.55^{+0.94}_{-0.50}$
LHCb (13 TeV)	49	1.07	0.10	$1.18^{+0.25}_{-0.01}$
PHENIX	3	0.29	0.12	$0.42^{+0.15}_{-0.10}$
STAR	11	1.91	0.28	$2.19^{+0.51}_{-0.31}$
E288 (200)	43	0.31	0.07	$0.38^{+0.12}_{-0.05}$
E288 (300)	53	0.36	0.07	$0.43^{+0.08}_{-0.04}$
E288 (400)	79	0.37	0.05	$0.48^{+0.11}_{-0.03}$
E772	35	0.87	0.21	$1.08^{+0.08}_{-0.05}$
E605	53	0.18	0.21	$0.39^{+0.03}_{-0.00}$
Total	627	0.79	0.17	$0.96^{+0.09}_{-0.01}$