

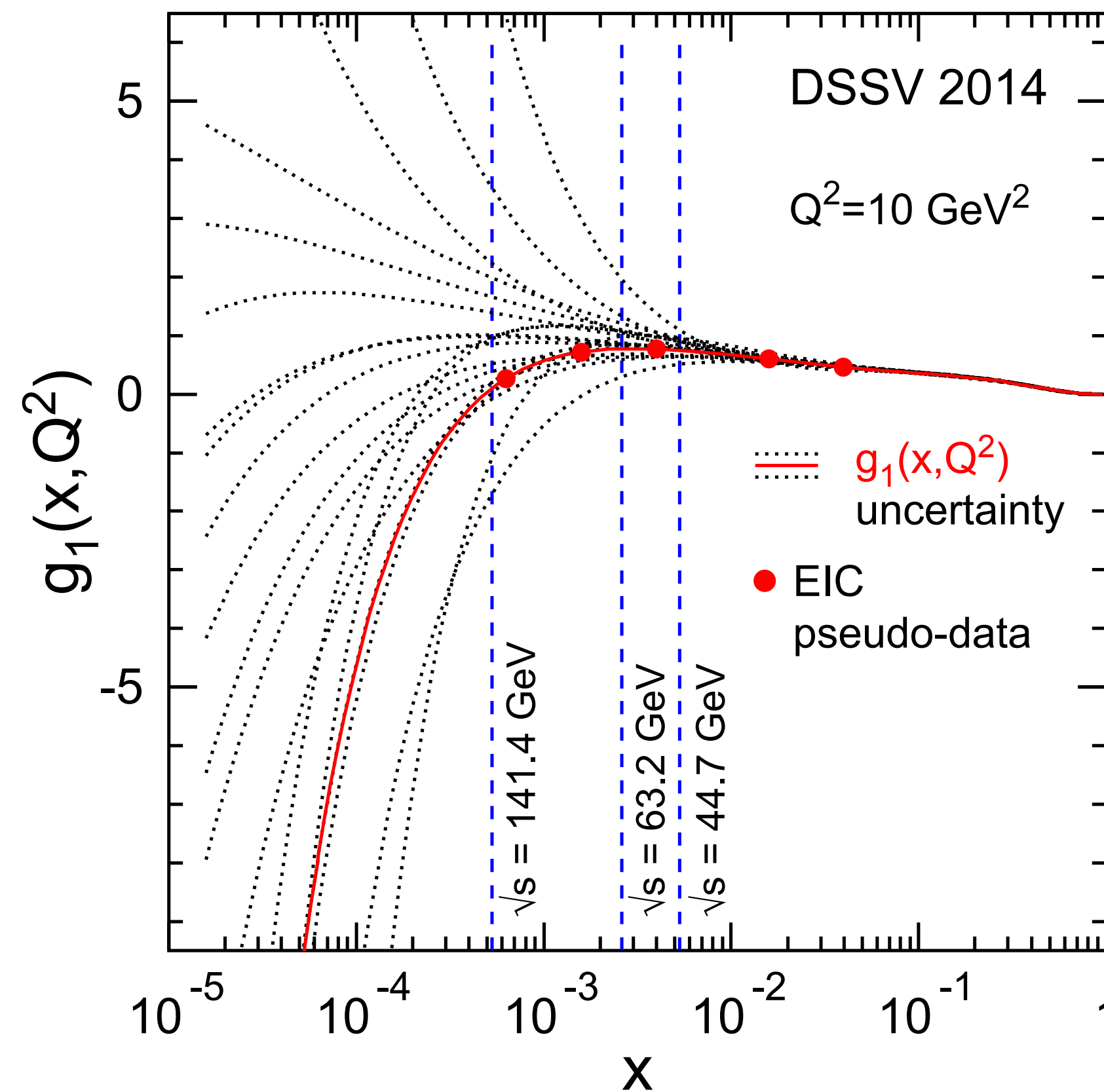
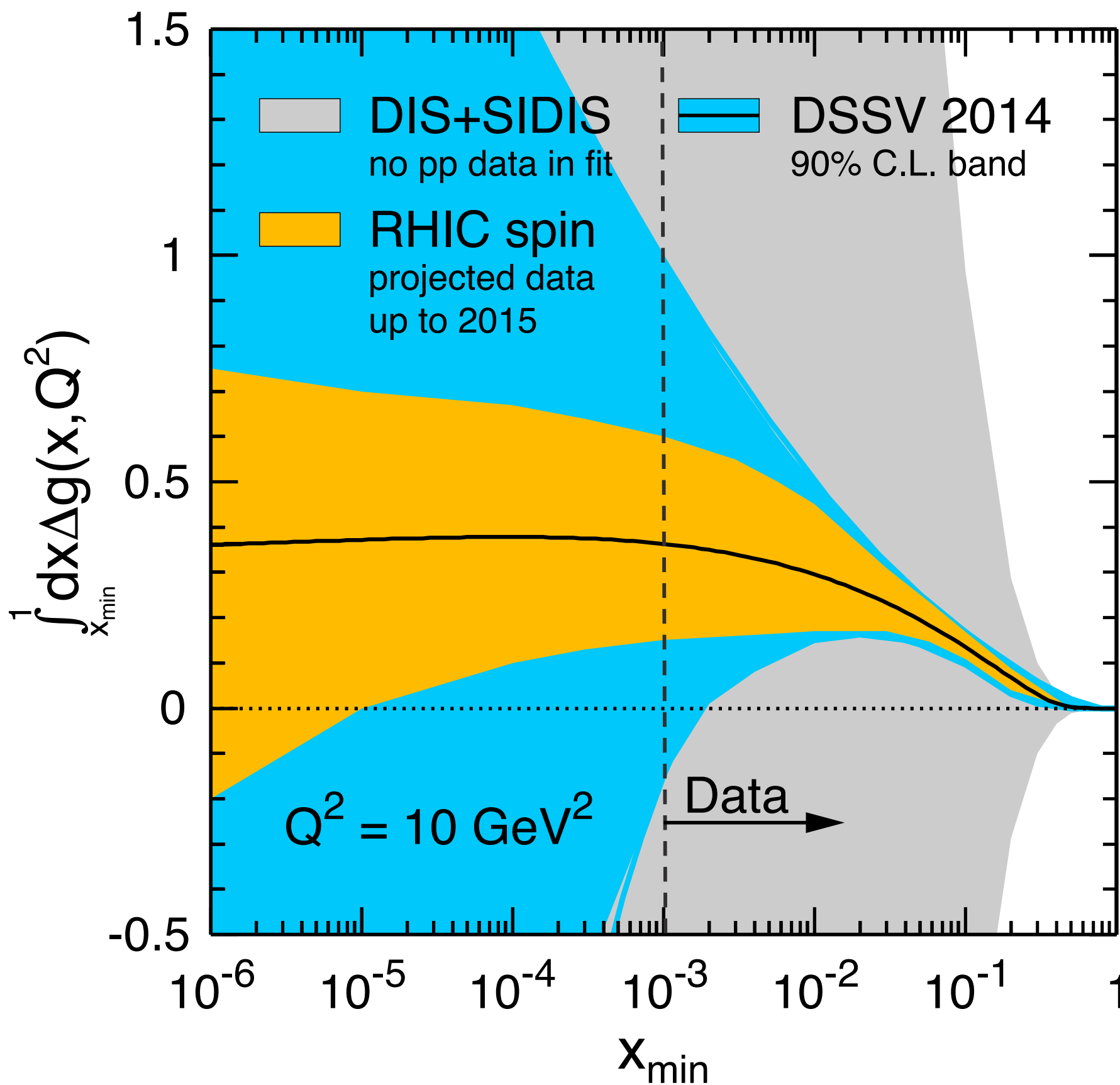
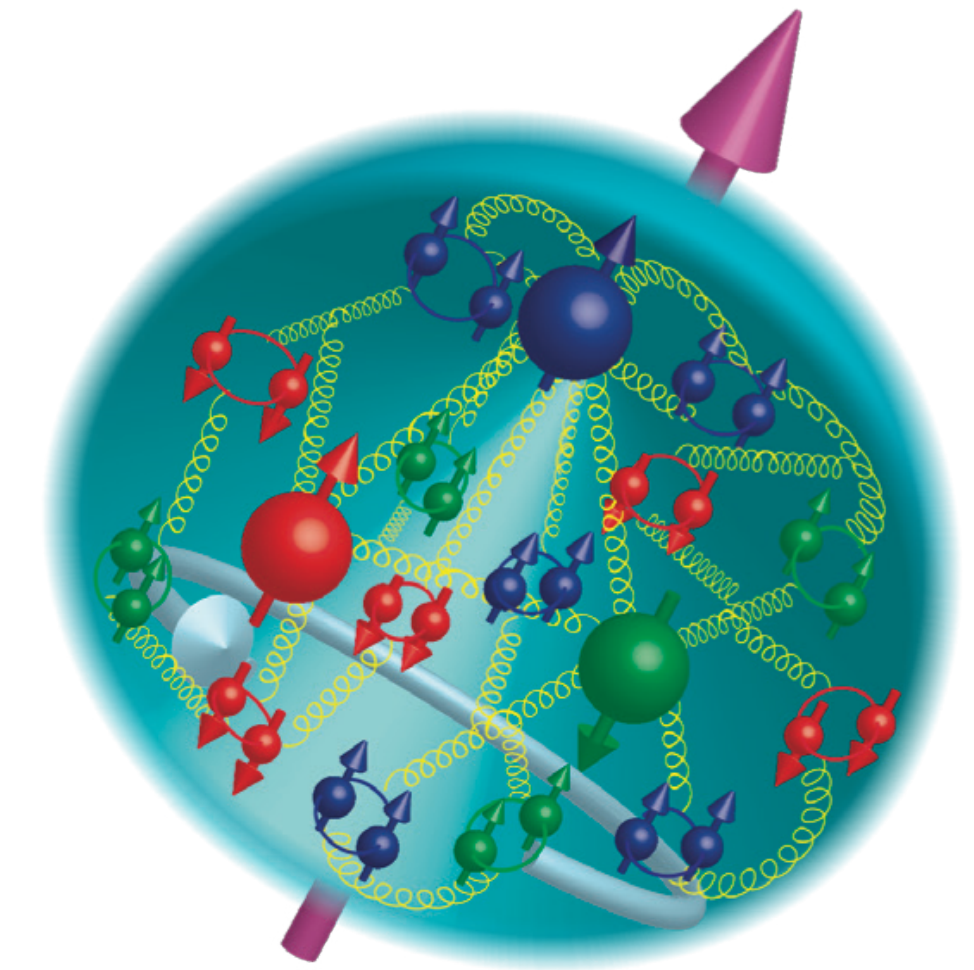
Proton spin at small- x

Andrey Tarasov

TMDs: Towards a Synergy between Lattice QCD and Global Analyses, June 23, 2023

Proton helicity structure

The fundamental properties of hadrons, and in particular its spin, are defined by the complex dynamics of quarks and gluons which form a strongly bonded **many-body parton system**. This dynamics in the context of spin dependent observables is not well understood (spin puzzle, large uncertainties for spin at small-x etc.)



$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g$$

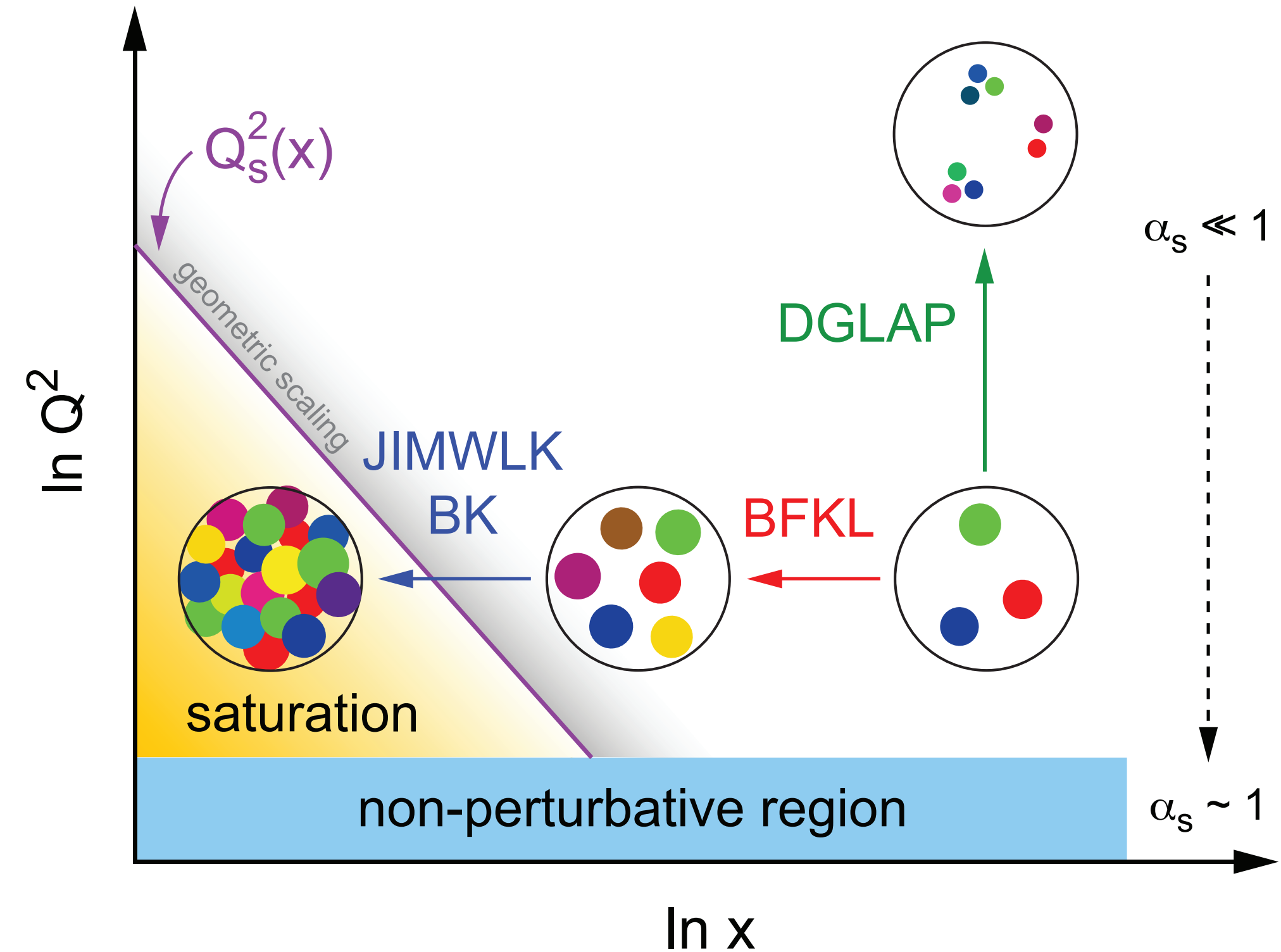
There is a lot of interest in studying proton spin at small-x

D. De Florian, R. Sassot, M. Stratmann,
W. Vogelsang, PRL 113 (2014)

Many-body parton system at small-x

What is a challenge of the small-x proton spin study? What is a difference between Bjorken ($Q^2 \rightarrow \infty$) and Regge ($x_B \rightarrow 0$) limits?

- While at large-x the proton can be viewed as a collection of weakly interacting partons, at small-x the proton is characterized by a rapid rise of the number of partons leading to a constant recombination of partons. At small-x we aim to sum **multiple interactions between partons**. These interactions are described by so-called **dipole amplitudes** (similar to TMDs).
- The large logarithms to be resummed are different at large- and small-x. While at large-x we resum logarithms $\alpha_s \log Q^2$ originating in transverse integrals, at small-x we resum **longitudinal logarithms** $\alpha_s \ln 1/x$ (rapidity divergence).



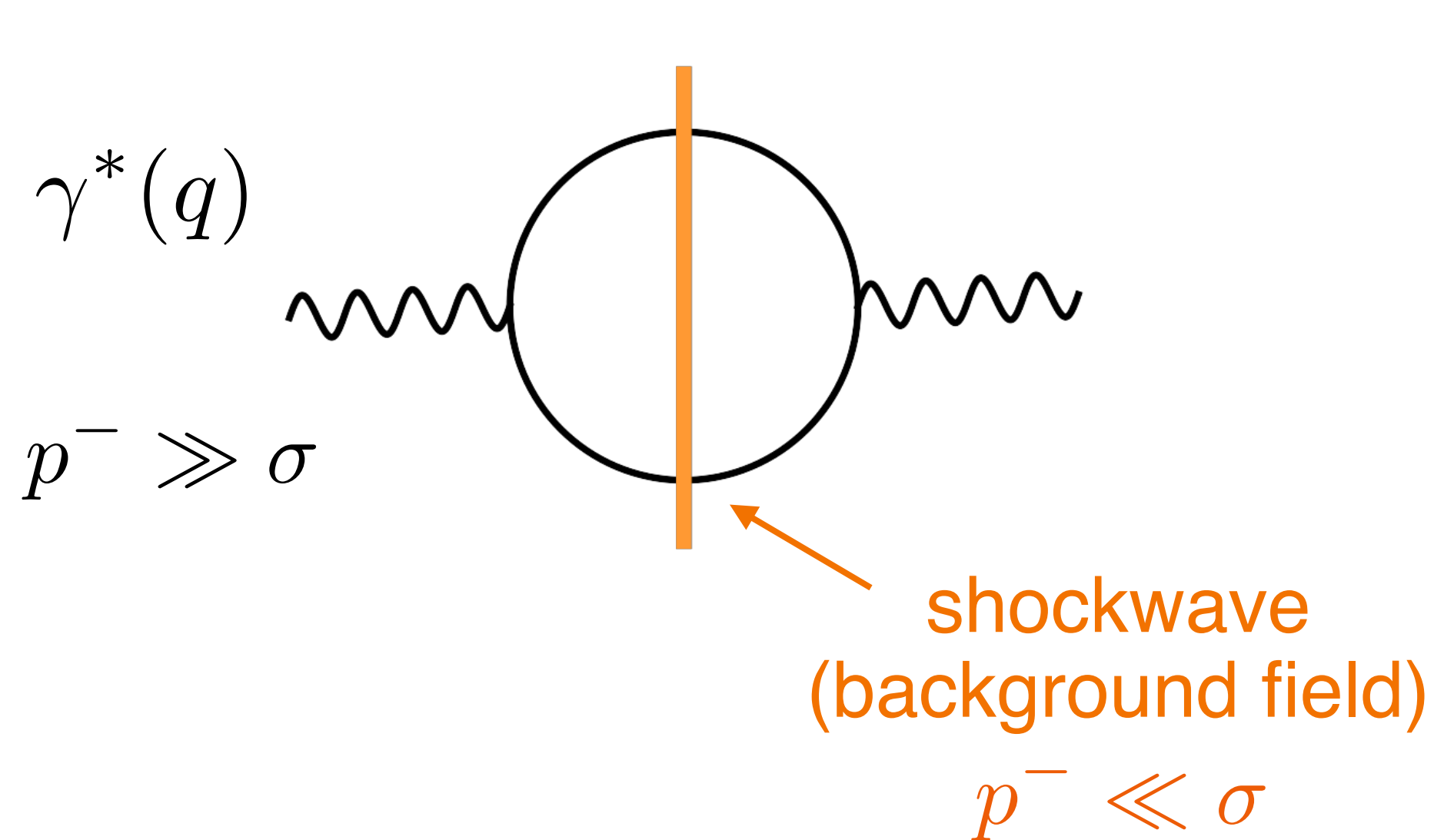
Color Glass Condensate

The small- x regime of the many-body parton system can be explored in the framework of CGC. In the CGC EFT the target field has an infinitesimally small support (shock-wave) and doesn't have a transverse component:

$$A_{\text{cl}}^+(x) = -\frac{1}{\partial_{\perp}^2} \rho(x_{\perp}) \delta(x^{-}); \quad A_{\text{cl}}^-(x) = A_{\text{cl}}^i(x) = 0$$

McLerran, Venugopalan (1994)

Deep inelastic scattering as a scattering on a shock-wave:



$$\sigma \propto \int \frac{d^2 p_{\perp}}{4\pi^2} I(p_{\perp}, q_{\perp}) \langle P | \text{tr} V(p_{\perp}) V^{\dagger}(q_{\perp} - p_{\perp}) | P \rangle$$

impact factor

↑
operator describing
interaction with a target

Balitsky (1996)

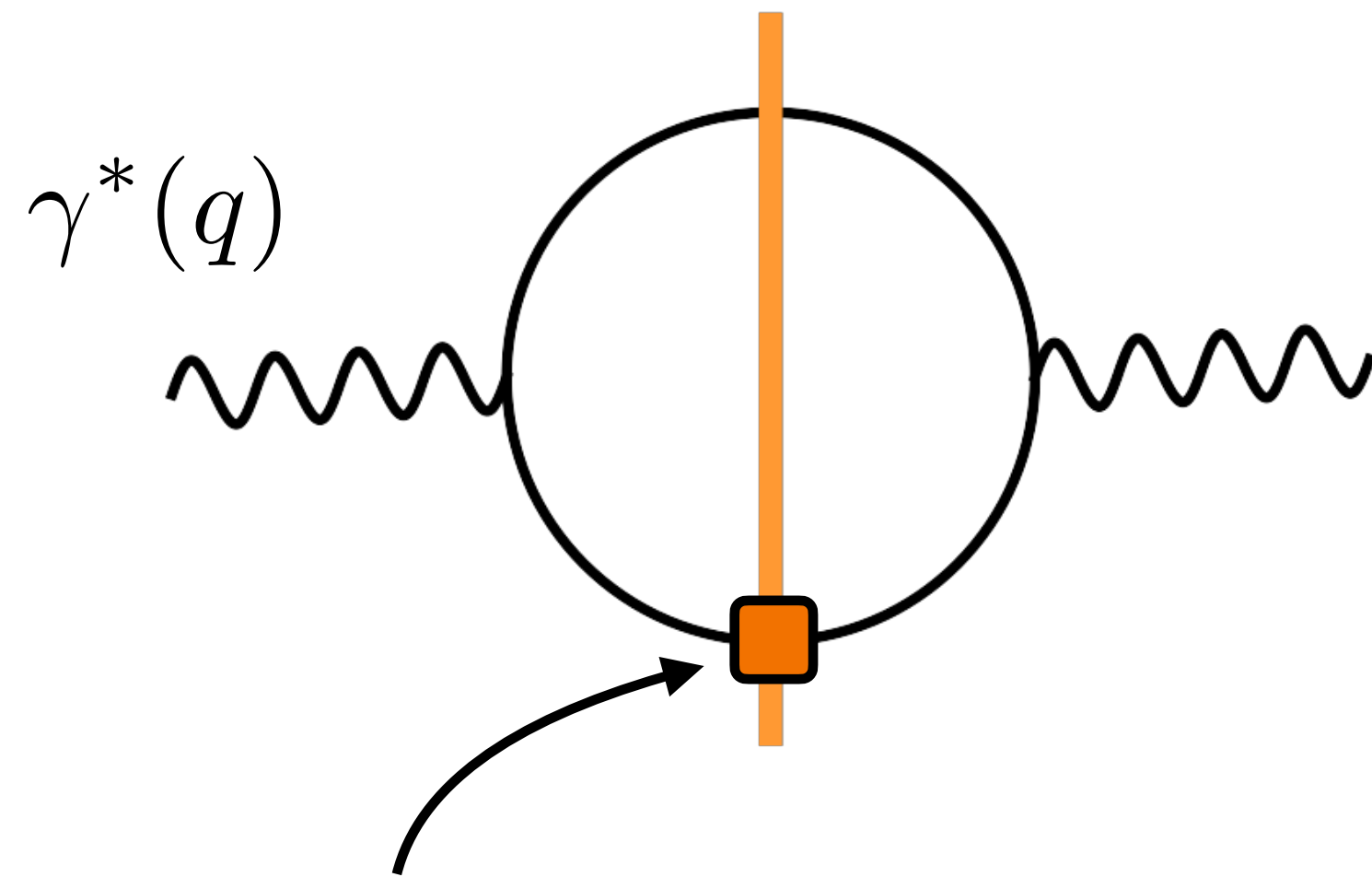
In the leading (eikonal) approximation the operator is constructed from light-cone Wilson lines (color dipole) which makes it insensitive to spin effects

Beyond the eikonal approximation

The small- x regime of the many-body parton system can be explored in the framework of CGC. In the CGC EFT the target field has an infinitesimally small support (shock-wave) and doesn't have a transverse component:

$$A_{\text{cl}}^+(x) = -\frac{1}{\partial_{\perp}^2} \rho(x_{\perp}) \delta(x^{-}); \quad A_{\text{cl}}^-(x) = A_{\text{cl}}^i(x) = 0$$

To be sensitive to spin effects one has to go beyond the leading eikonal approximation and include two types of corrections:



Helicity dependent interaction
via sub-eikonal operators

- Non-zero value of the transverse component of the background field

$$A_{\text{cl}}^i(x) \neq 0$$

- Non-zero “size” of the shock-wave

$$A_{\text{cl}}^+(x) \approx \delta(x^{-})$$

Non-zero value of the transverse component

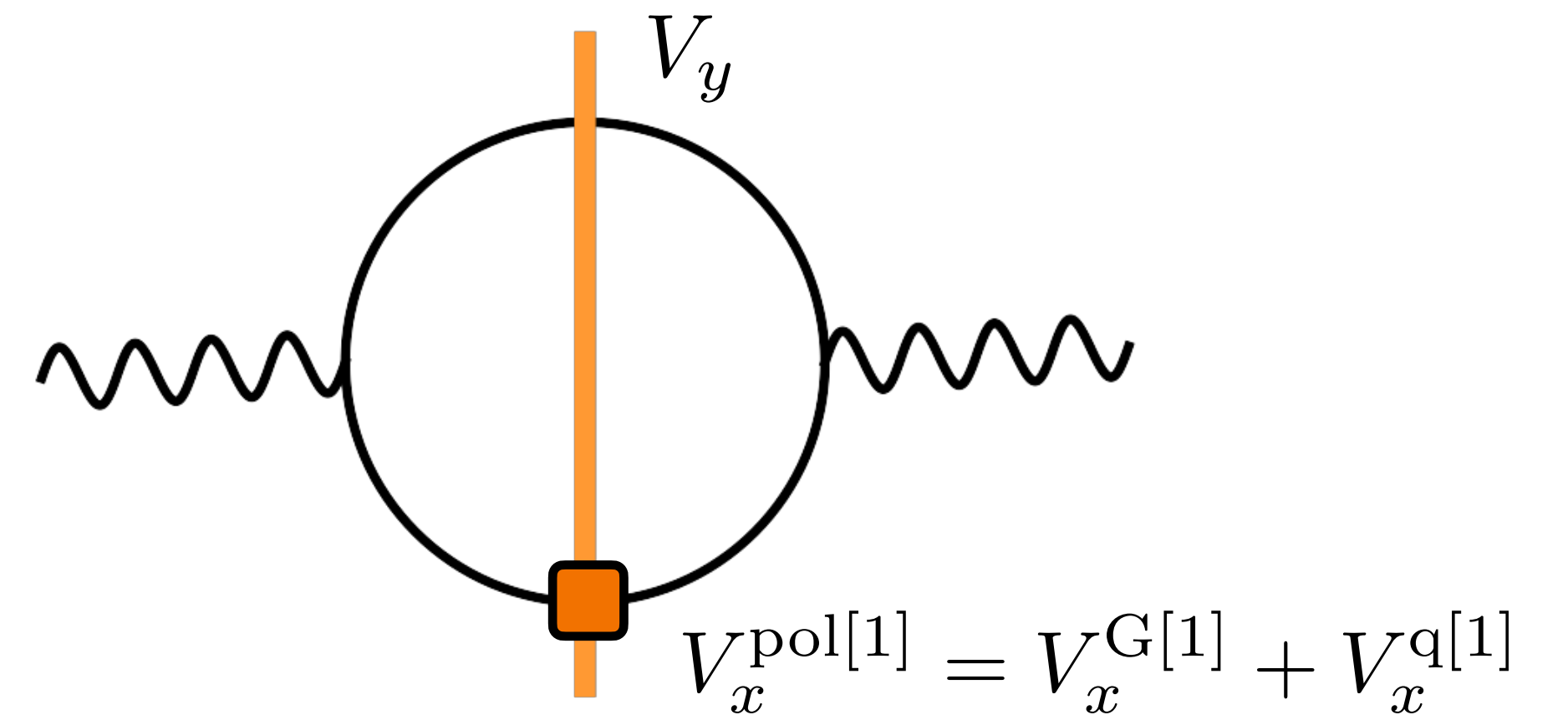
Originally, the first type of corrections was considered by [Kovchegov, Pitonyak, Sievert \(2016-2019\)](#). There are two helicity dependent operators at the sub-eikonal level.

$$V_x^{\text{G}[1]} = \frac{igP^+}{s} \int_{-\infty}^{\infty} dx^- V_x[\infty, x^-] F^{12}(x^-, x_\perp) V_x[x^-, -\infty],$$

$$V_x^{\text{q}[1]} = \frac{g^2 P^+}{2s} \int_{-\infty}^{\infty} dx_1^- \int_{x_1^-}^{\infty} dx_2^- V_x[\infty, x_2^-] t^b \psi_\beta(x_2^-, x_\perp) U_x^{ba}[x_2^-, x_1^-] [\gamma^+ \gamma^5]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, x_\perp) t^a V_x[x_1^-, -\infty]$$

These operators generate a polarized dipole amplitude

$$Q_{xy}(\sigma) \equiv \frac{1}{2N_c} \left\langle\left\langle \text{T tr} \left[V_y V_x^{\text{pol}[1] \dagger} \right] + \text{T tr} \left[V_x^{\text{pol}[1]} V_y^\dagger \right] \right\rangle\right\rangle(\sigma)$$



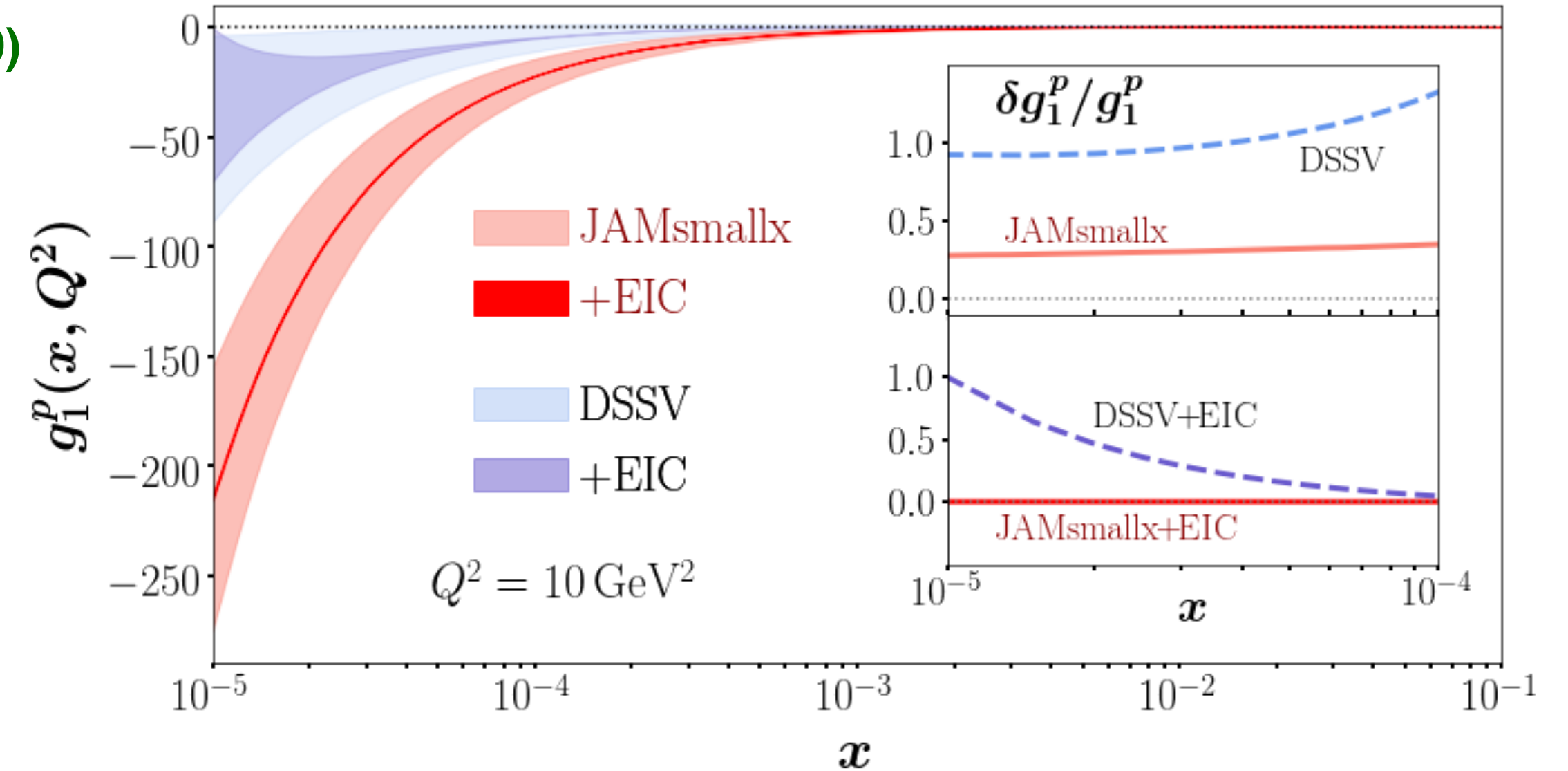
Inclusion of these operators leads to the KPS evolution, which has been extensively studied.

KPS evolution

Kovchegov, Pitonyak, Sievert (2016-2019)

- Sums up powers of $\alpha_s \ln 1/x$ and $\alpha_s \ln^2 1/x$.
- The equations are closed in the large- N_c and large- N_c & N_f limits.
- The equations were obtained in both flavor singlet and non-singlet channels.
- The flavour singlet helicity evolution equations were solved numerically and analytically in large- N_c limit.
- Flavor non-singlet equations at large- N_c were solved analytically.
- A numerical solution of the large- N_c & N_f equations was obtained in [Y. V. Kovchegov and Y. Tawabutr \(2020\)](#)

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{2.31} \sqrt{\frac{\alpha_s N_c}{2\pi}}$$



[D. Adamiak, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato and M. D. Sievert, PRD 104 \(2021\) L031501](#)

- Contradicts the asymptotic obtained in the infrared evolution equations (IREE) formalism

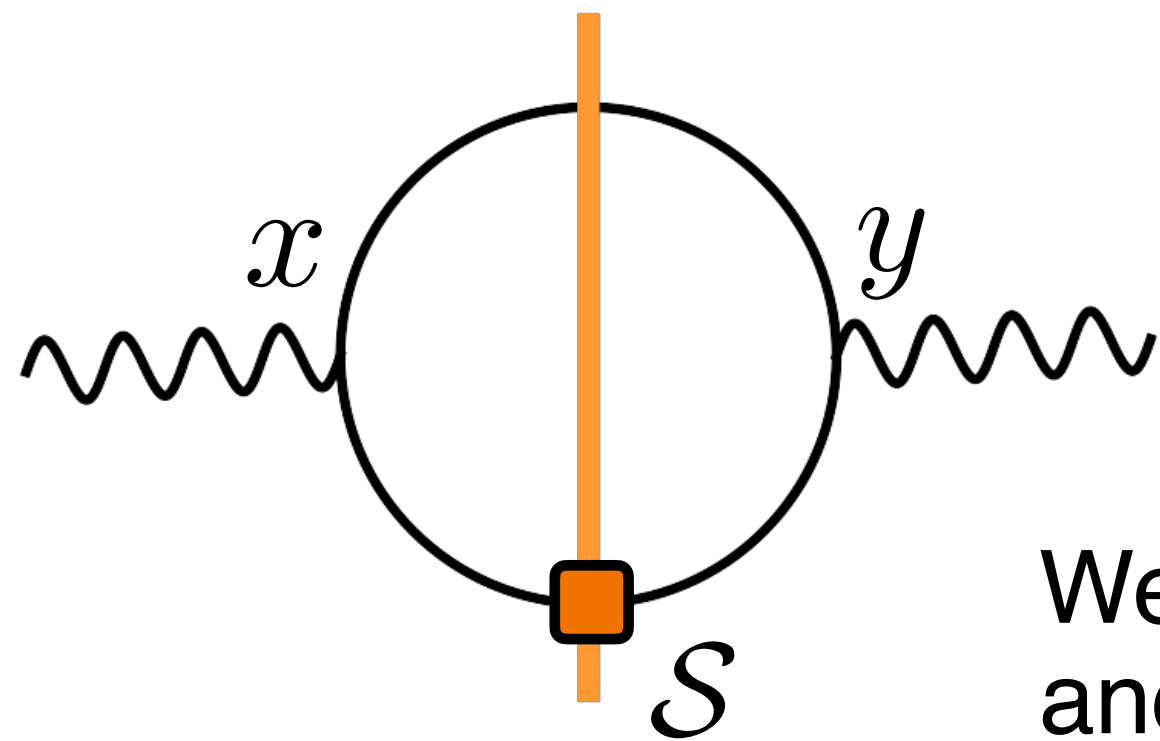
$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66} \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

[Bartels, Ermolaev, Ryskin 1996](#)
[R. Boussarie, Y. Hatta and F. Yuan 2019](#)

Sub-eikonal corrections

Recently a systematic treatment of the sub-eikonal corrections was performed in the background field method

Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)



$$(x | \frac{1}{P^2 + i\epsilon} | y) = -\frac{i}{2\pi} \theta(x^- - y^-) \int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+} (x_\perp | e^{-i\frac{p_\perp^2}{2p^-} x^-} \mathcal{S}(x^-, y^-) e^{i\frac{p_\perp^2}{2p^-} y^-} | y_\perp)$$

We construct an expansion in eikinality for the operator and find the structure of the sub-eikonal correction:

operator describing interaction with a target

$$\mathcal{S}(x^-, y^-) = \mathcal{S}_0(x^-, y^-) + \frac{1}{p^-} \mathcal{S}_1(x^-, y^-) + \frac{1}{(p^-)^2} \mathcal{S}_2(x^-, y^-) + \dots$$

↑
leading eikonal term
(Wilson line)

↑
sub-eikonal correction

Altinoluk, Armesto, Beuf, Martínez, Salgado (2014)
Balitsky, Tarasov (2015-2016)
Chirilli (2019)

Eikonal expansion of the gluon propagator (axial gauge)

The general form of the propagators has to be simplified. We construct an eikonal expansion in the shock-wave approximation of the propagators which is suited to the rapidity factorization

$$\begin{aligned} \mathbb{T} [C_\mu^a(x) C_\nu^b(y)] &= -\frac{1}{2\pi} \int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+} \\ &\times (x_\perp | (g_{\mu i} - \frac{n_\mu}{p^-} p_i)^{ac} e^{-i\frac{p_\perp^2}{2p^-} x^-} \mathcal{G}^{ij}(\infty, -\infty) e^{i\frac{p_\perp^2}{2p^-} y^-} (g_{j\nu} - p_j \frac{n_\nu}{p^-})^{db} | y_\perp) + \dots \end{aligned}$$

describes interaction with the background field

eikonal contribution

$$\mathcal{G}^{ij}(\infty, -\infty) = \boxed{g^{ij} U} + \boxed{\frac{g^{ij} s}{2P^+ p^-} U^{\text{q}[2]} + \frac{i\epsilon^{ij} s}{2P^+ p^-} U^{\text{pol}[1]}}$$

F_{12} terms

$$\begin{aligned} & -\frac{ig g^{ij}}{2p^-} p^k \int_{-\infty}^\infty dz^- z^- U[\infty, z^-] \mathcal{F}_{-k} U[z^-, -\infty] - \frac{ig g^{ij}}{2p^-} \int_{-\infty}^\infty dz^- z^- U[\infty, z^-] \mathcal{F}_{-k} U[z^-, -\infty] p^k \\ & + \frac{ig^2 g^{ij}}{2p^-} \int_{-\infty}^\infty dz_1^- \int_{-\infty}^{z_1^-} dz_2^- (z_1^- - z_2^-) U[\infty, z_1^-] \mathcal{F}_{-k} U[z_1^-, z_2^-] \mathcal{F}_{-k} U[z_2^-, -\infty] + O\left(\frac{1}{(p^-)^2}\right). \end{aligned}$$

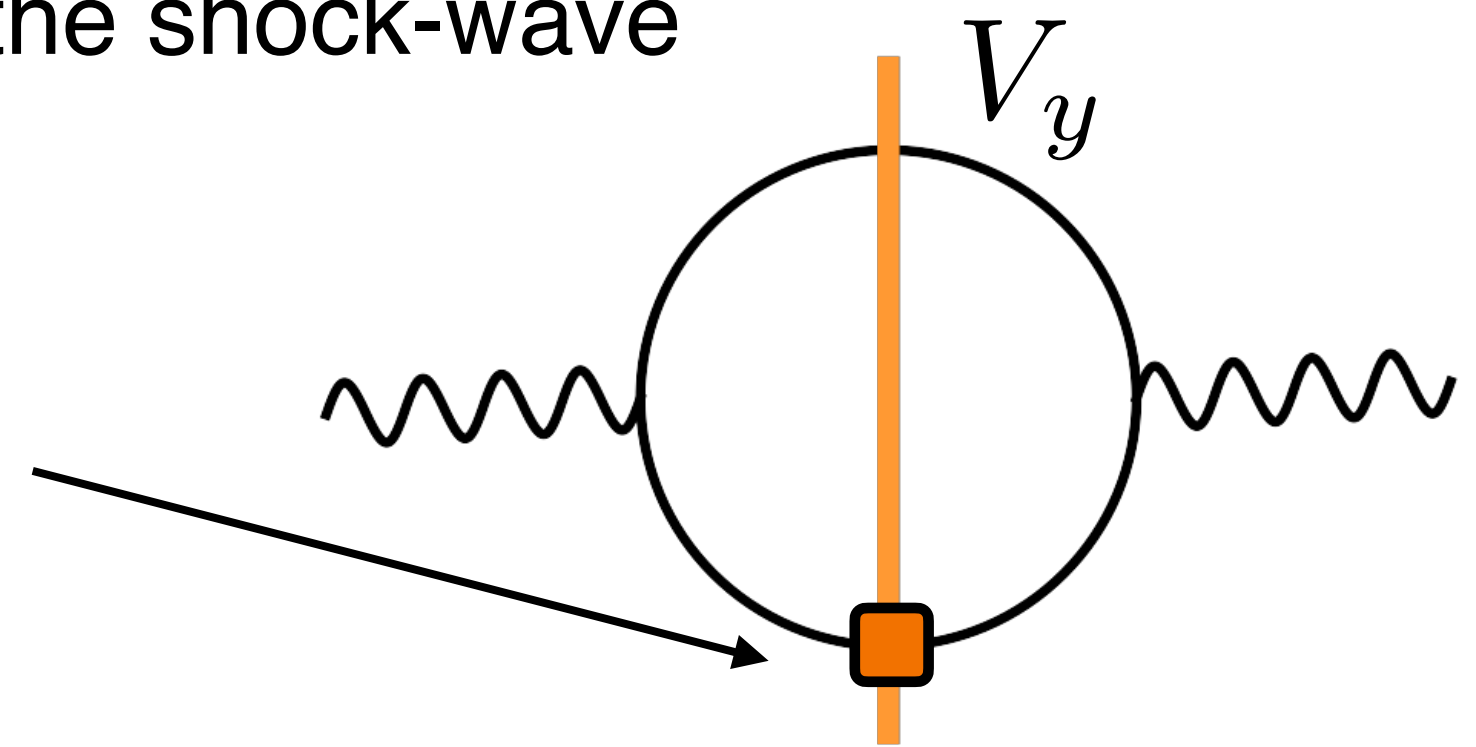
Sub-eikonal corrections are suppressed by $1/p^-$

new terms

A new operator related to a non-zero “size” of the shock-wave

- We find another operator at the sub-eikonal level which generates the **small-x DGLAP evolution**
- This operator is related to the Jaffe-Manohar polarized gluon distribution ΔG which satisfies the DGLAP evolution. It can be obtained by expanding the exponential factor (expansion in x_B) in the definition of the JM distribution
- This operator comes from the **scalar phase** in the propagator when it is expanded onto the light-cone
- This operator describes sub-eikonal corrections due to a non-zero width of the shock-wave

$$ig \int_{-\infty}^{\infty} dz^- z^- V_x[\infty, z^-] F_{-k} V_x[z^-, -\infty]$$



This operator in turn generates a new type of the polarized dipole amplitude:

$$G_{xy}^i(\sigma) \equiv \frac{igP^+}{2sN_c} \left\langle\left\langle \text{T tr} \left[V_y^\dagger \int_{-\infty}^{\infty} dz^- z^- V_x[\infty, z^-] F^{+i} V_x[z^-, -\infty] \right] + \text{c.c.} \right\rangle\right\rangle(\sigma)$$

KPS-CTT evolution

- Sums up powers of $\alpha_s \ln 1/x$ and $\alpha_s \ln^2 1/x$
- **Contains mixing between different types of operators**
- Consistent with small-x DGLAP evolution
- The equations are closed in the large- N_c and large- N_c & N_f limits.
- A system of equations in the large- N_c limit in the DLA approximation:

$$G(x_{10}^2, z s) = G^{(0)}(x_{10}^2, z s) + \frac{\alpha_s N_c}{2\pi} \int_{\frac{1}{s x_{10}^2}}^z \frac{dz'}{z'} \int_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z' s) + 3 G(x_{21}^2, z' s) + 2 G_2(x_{21}^2, z' s) + 2 \Gamma_2(x_{10}^2, x_{21}^2, z' s) \right]$$

+ three similar equations, where amplitudes G , Γ , G_2 and Γ_2 parametrize dipole amplitudes G_{10}^i and Q_{10}

- Large- N_c equations have been solved numerically (**CKTT 2022**) and analytically (**J. Borden and Y. V. Kovchegov, 2023**). The result is in agreement with the BER result:

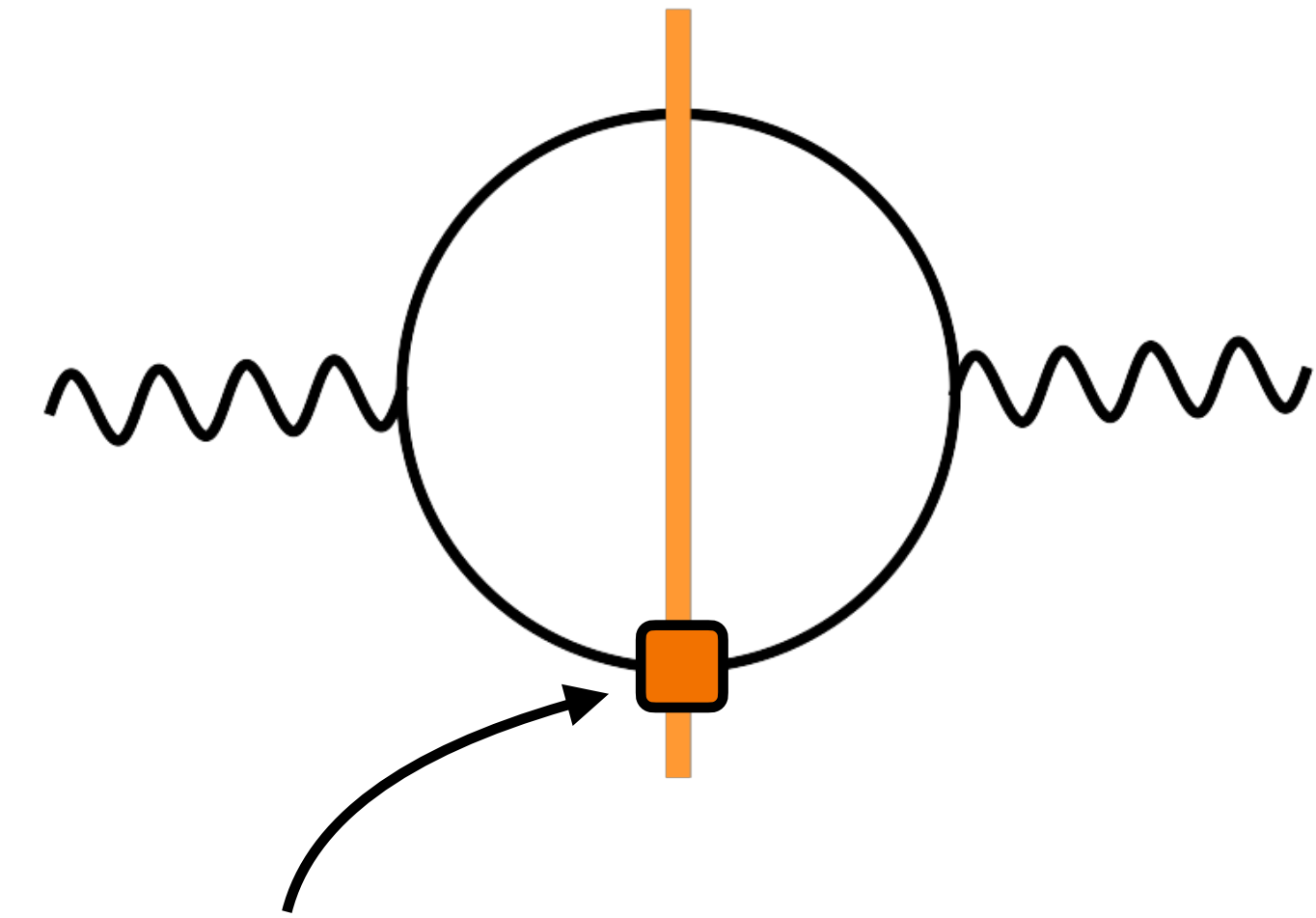
$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

- Large- N_c & N_f has been recently solved numerically (**D. Adamiak, Y. V. Kovchegov and Y. Tawabutr, 2023**) and showed disagreement with IREE of the order of 2-3%

Calculation of observables

$$\sigma^{\gamma^* p} \propto - \sum_f \frac{N_c Z_f^2}{4\pi^4} \int d^2 x_{10} \int_{\Lambda^2/s}^1 \frac{dz}{z} \left\{ \begin{aligned} & 2 [z^2 + (1-z)^2] a_f^2 [K_1(x_{10} a_f)]^2 G_2(x_{10}^2, zs) \\ & + \left[(1-2z) a_f^2 [K_1(x_{10} a_f)]^2 - m_f^2 [K_0(x_{10} a_f)]^2 \right] Q(x_{10}^2, zs) \end{aligned} \right\}$$

impact factor
↓



Helicity dependent interaction via sub-eikonal operators

where dipole amplitudes are integrated over impact parameter:

$$\int d^2 \left(\frac{x_1 + x_0}{2} \right) G_{10}^i(zs) = (x_{10})_{\perp}^i G_1(x_{10}^2, zs) + \epsilon^{ij} (x_{10})_{\perp}^j G_2(x_{10}^2, zs)$$

$$\int d^2 \left(\frac{x_0 + x_1}{2} \right) Q_{10}(zs) = Q(x_{10}^2, zs)$$

Q: Is the eikonal expansion valid for all polarization dependent observables? Is there any contribution for which it breaks down?

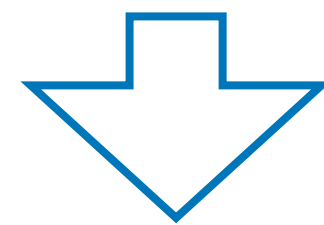
First moment of the structure function

The helicity can be extracted from the first moment of the g_1 structure function

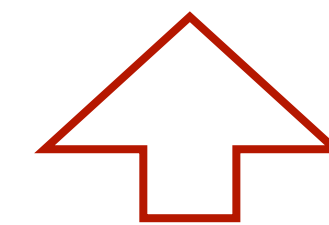
$$\int_0^1 dx_B g_1(x_B, Q^2) = \frac{1}{18} (3F + D + 2\Sigma(Q^2)) \left(1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

In terms of quark PDFs the helicity can be defined as

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B (\Delta q_f(x_B, Q^2) + \Delta \bar{q}_f(x_B, Q^2)) \quad \Delta q_f(x_B) = \text{red circle with right arrow} - \text{red circle with left arrow}$$



$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S | \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f | P, S \rangle \equiv \frac{1}{M_N} \langle P, S | J_5^\mu(0) | P, S \rangle$$



Quark contribution to the proton spin is defined by the isosinglet axial vector current J_5^μ

Anomaly equation

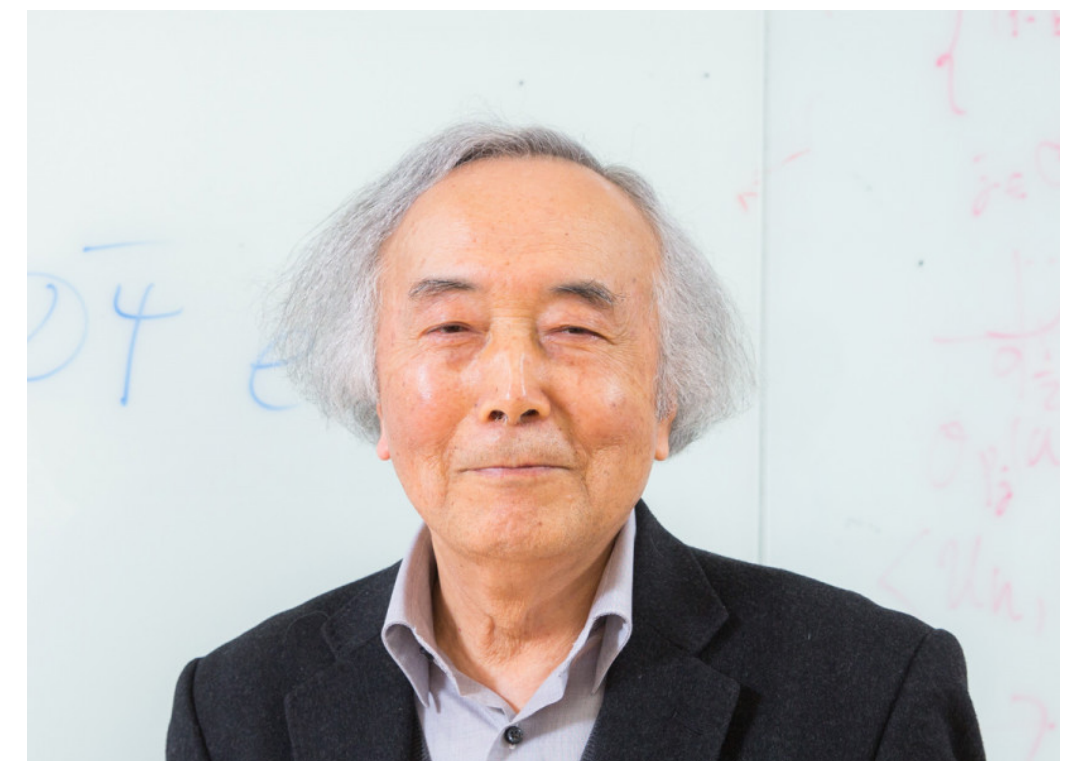
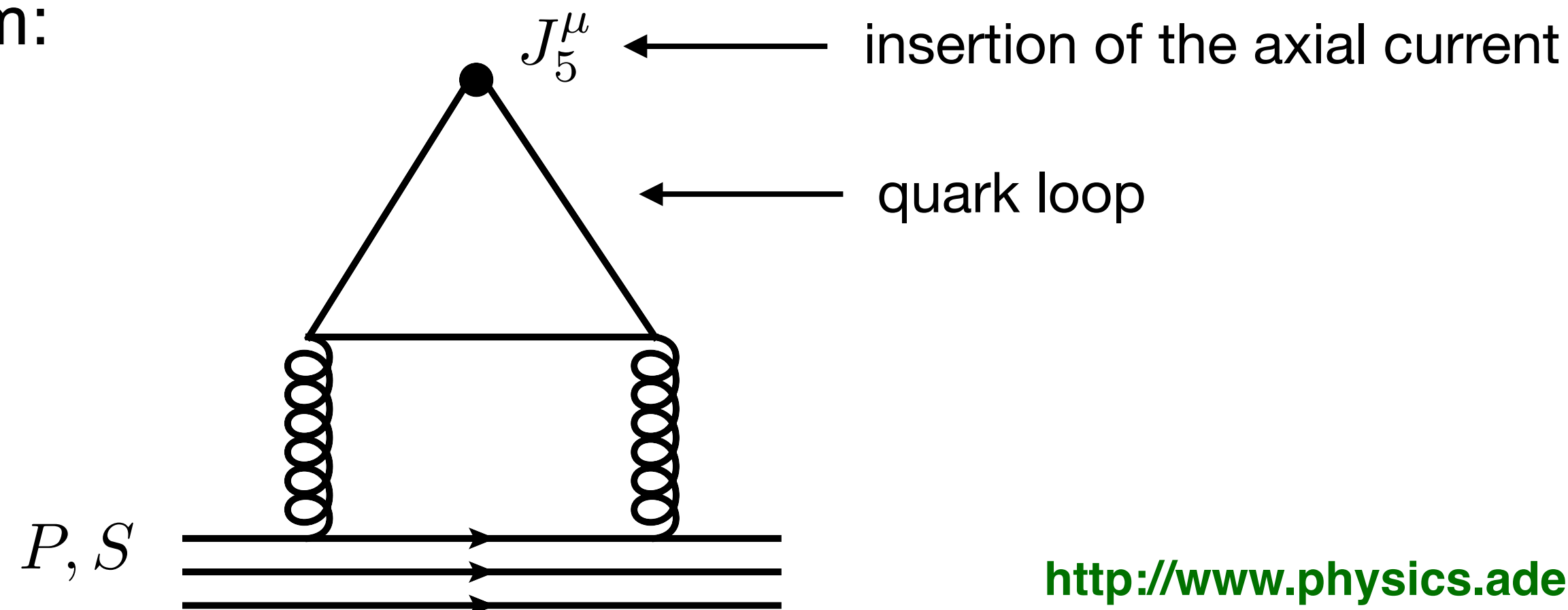
The fundamental property of the J_5^μ current is the anomaly equation:

$$\partial^\mu J_\mu^5(x) = \frac{n_f \alpha_s}{2\pi} \text{Tr} \left(F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right) = 2 n_f \partial_\mu K^\mu$$

The isosinglet current couples to the **topological charge density** in the polarized proton!

The anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations. Topological properties of the QCD vacuum! **K. Fujikawa, PRL. 42, 1195 (1979)**

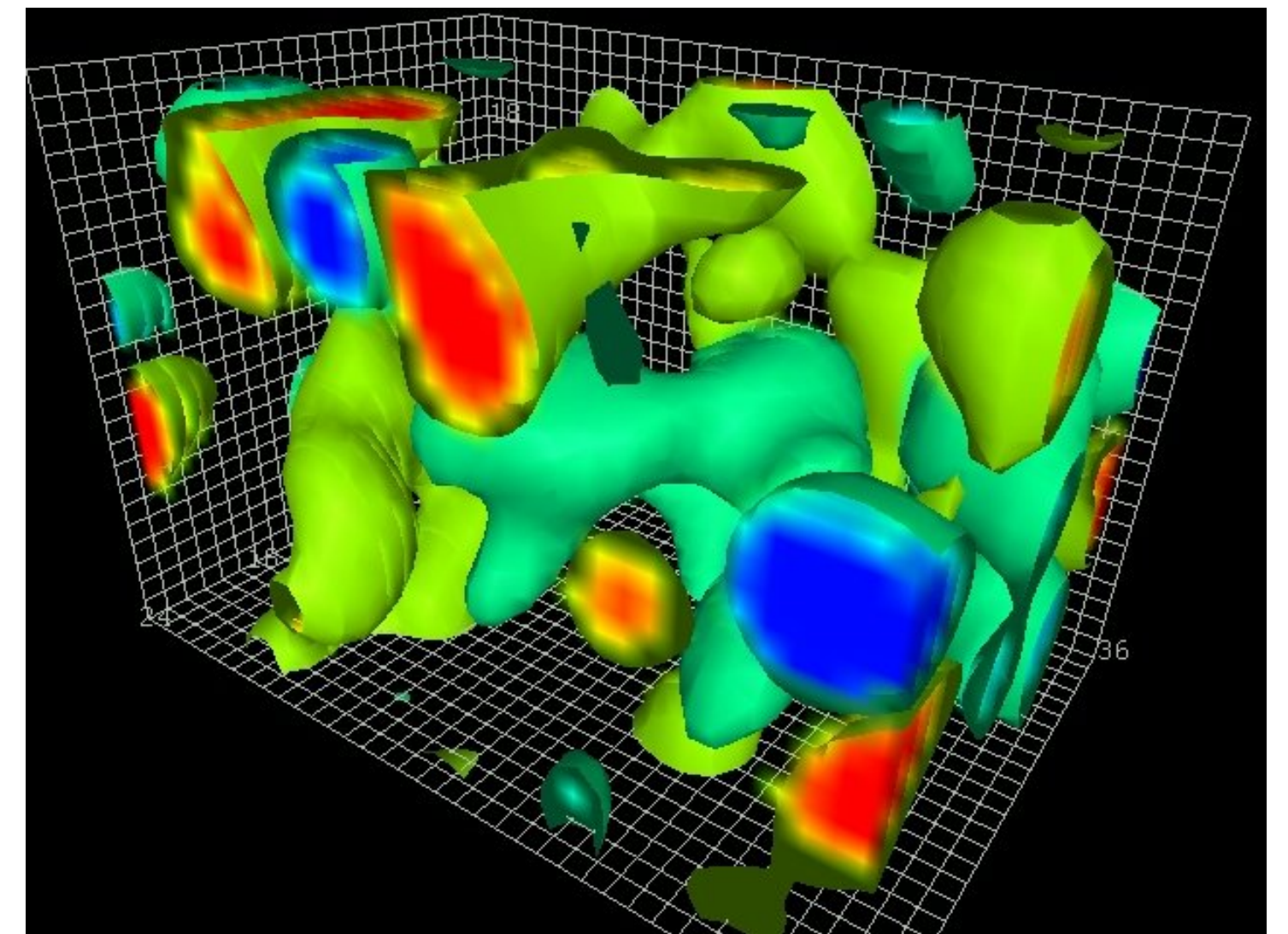
In the leading order the coupling is generated by the triangle diagram:



Kazuo Fujikawa

Chern-Simons current:

$$K_\mu = \frac{\alpha_S}{4\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$$



Topological charge density

Quark helicity and the triangle anomaly

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S | \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f | P, S \rangle \equiv \frac{1}{M_N} \langle P, S | J_5^\mu(0) | P, S \rangle$$

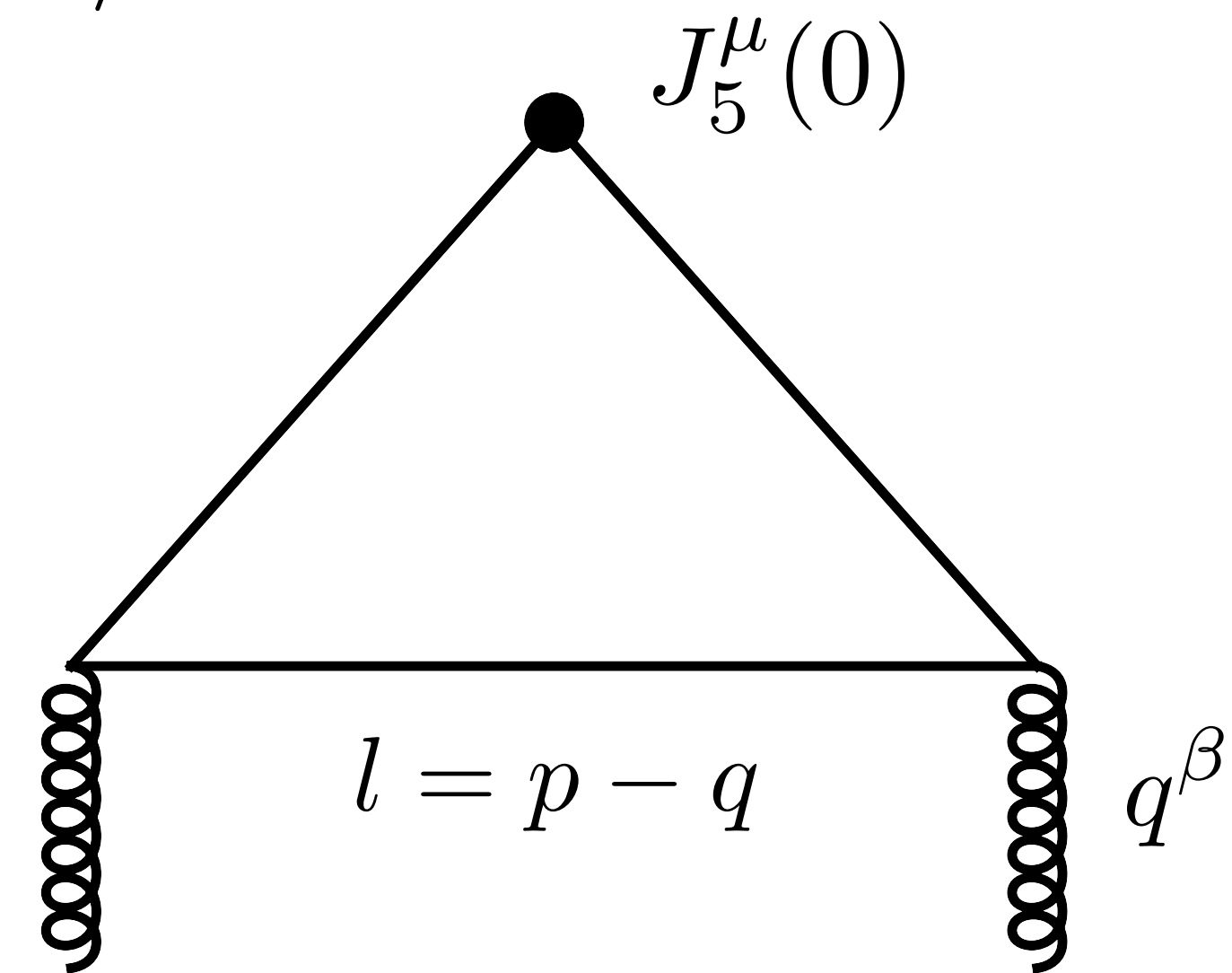
The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit. An exact calculation in the worldline approach gives

$$\langle P', S | J_5^\mu(0) | P, S \rangle = -i \frac{l^\mu}{l^2} \frac{\alpha_s n_f}{2\pi} \langle P', S | \text{Tr}(F \tilde{F}) | P, S \rangle \quad p^\alpha$$

Tarasov, Venugopalan (2021)

infrared
(anomaly) pole

Exact result!
topological charge density
not F_{12} or JM operator!



R. L. Jaffe, A. Manohar (1990)
Shore, Veneziano (1990)
K.-F. Liu (1992)

Adler-Bell-Jackiw anomaly

The triangle diagram is not local! **The anomaly manifests itself as an infrared pole.** Taking a divergence we obtain the anomaly equation

$$\partial^\mu J_\mu^5 = i l^\mu J_\mu^5 = \frac{n_f \alpha_s}{2\pi} \text{Tr} \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

Infrared pole and the structure function g_1

We find that g_1 is dominated by the triangle anomaly - g_1 is topological in both asymptotic limits of QCD.
 First moment of g_1 matches calculation of the triangle diagram

$$S^\mu g_1(x_B, Q^2) \Big|_{Q^2 \rightarrow \infty}^{anom.} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \left(1 - \frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \rightarrow 0} \frac{l^\mu}{l^2} \langle P', S | \text{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) | P, S \rangle$$

infrared pole

Tarasov, Venugopalan (2021)

$$S^\mu g_1(x_B, Q^2) \Big|_{x_{Bj} \rightarrow 0}^{anom.} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \rightarrow 0} \frac{l^\mu}{l^2} \langle P', S | \text{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) | P, S \rangle$$

How is the pole cancelled? Interplay between perturbative and non-perturbative physics. The mechanism of the cancellation is deeply related to the $U_A(1)$ problem in QCD - topological mass generation of the $m_{\eta'}^2$.

Anomaly pole and the $U_A(1)$ problem

To resolve the pole one has to take into account exchanges of the $\bar{\eta}$ massless “primordial” ninth Goldstone boson arising from the spontaneous symmetry breaking of the flavor group.

↳ Factorization, eikonal/twist expansion breaking

However $\bar{\eta}$ is not observed. Instead there is a heavy η' ($m_{\eta'} \approx 957 MeV$) - the famous $U_A(1)$ problem.

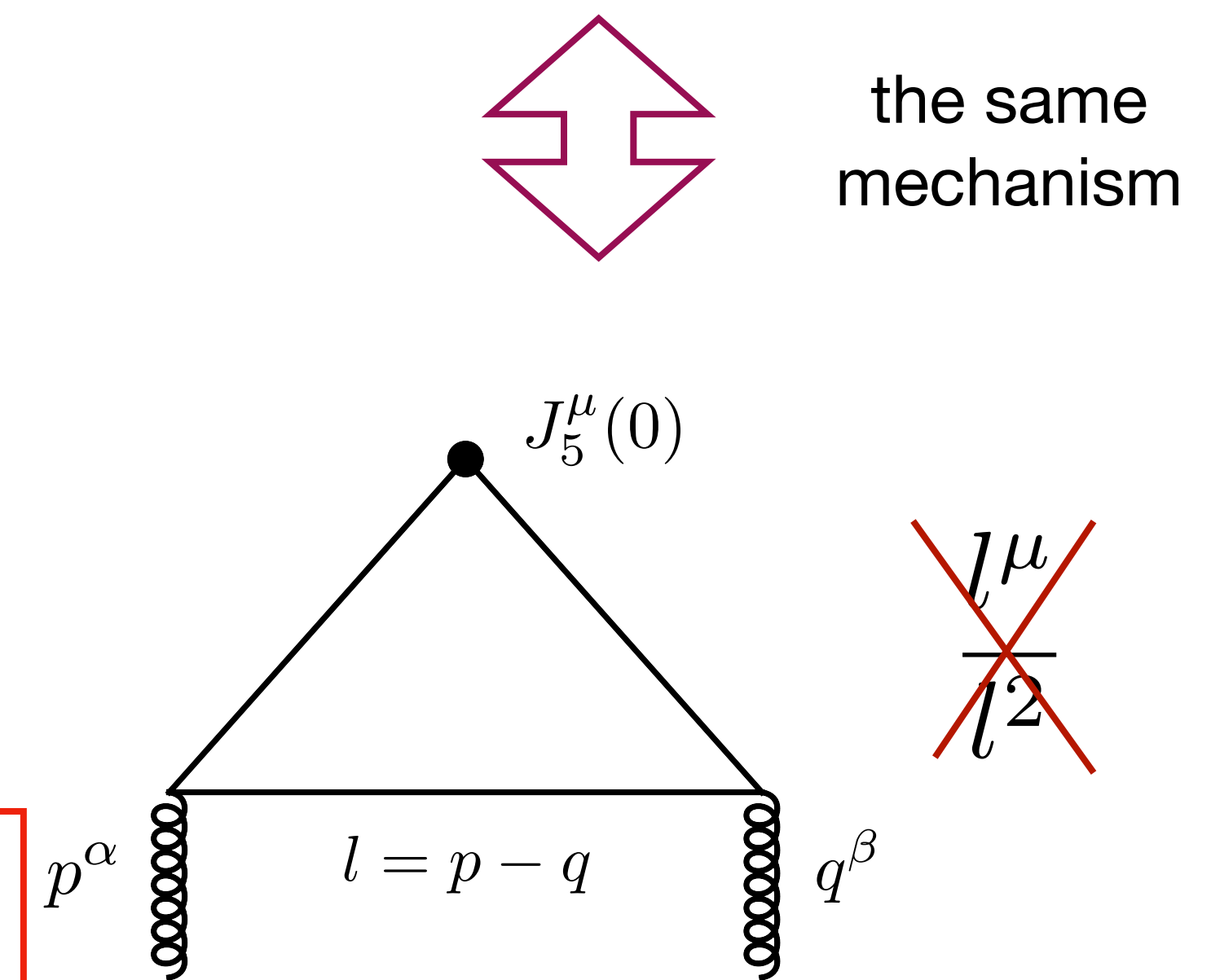
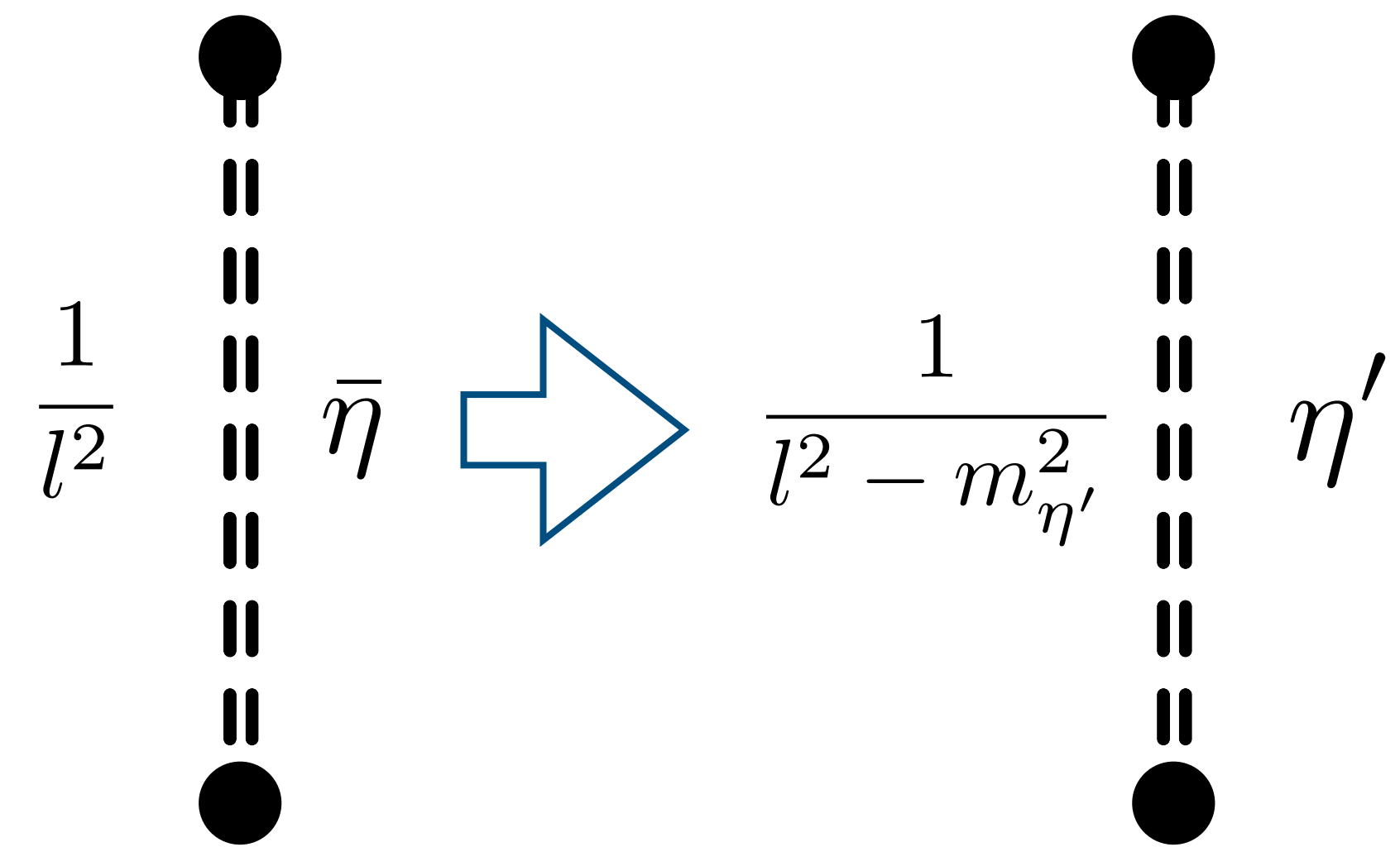
There is no **Goldstone pole** just as there is no **anomaly pole** in the QCD spectrum

We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole

Tarasov, Venugopalan (2022)

This mechanism relates the helicity structure of the proton to the topology of the QCD vacuum

↳ possibility to detect sphaleron-like topological transitions at EIC



Wess-Zumino-Witten coupling

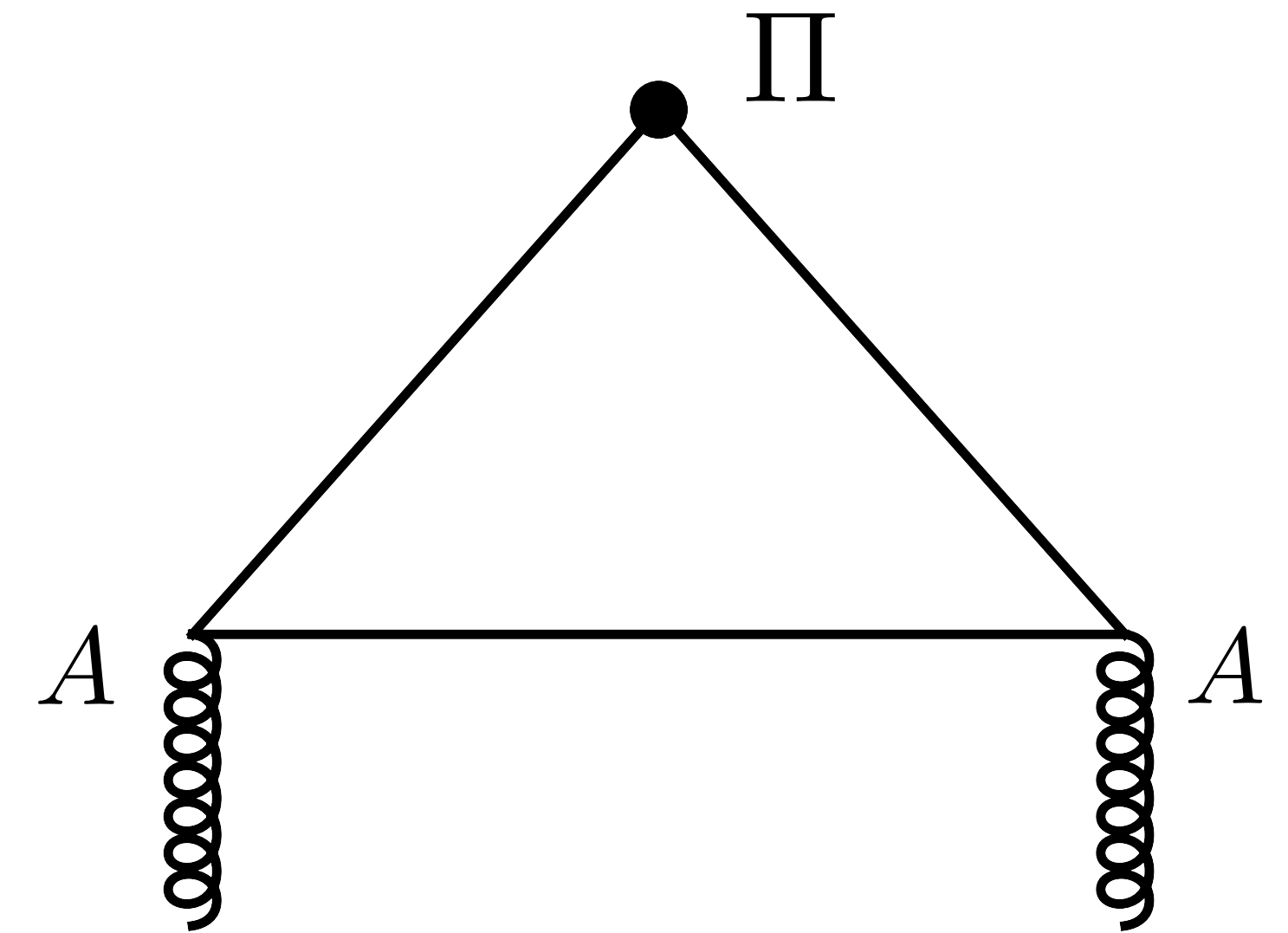
We calculate the imaginary part of the effective action which generates the isosinglet Wess-Zumino-Witten coupling $\propto \bar{\eta} F \tilde{F}$

$$\mathcal{W}_{\mathcal{I}}[\Pi A^2] = \frac{ig^2 2n_f}{16\pi^2} \frac{1}{\Phi} \text{tr}_c \int d^4x \Pi(x) F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x)$$

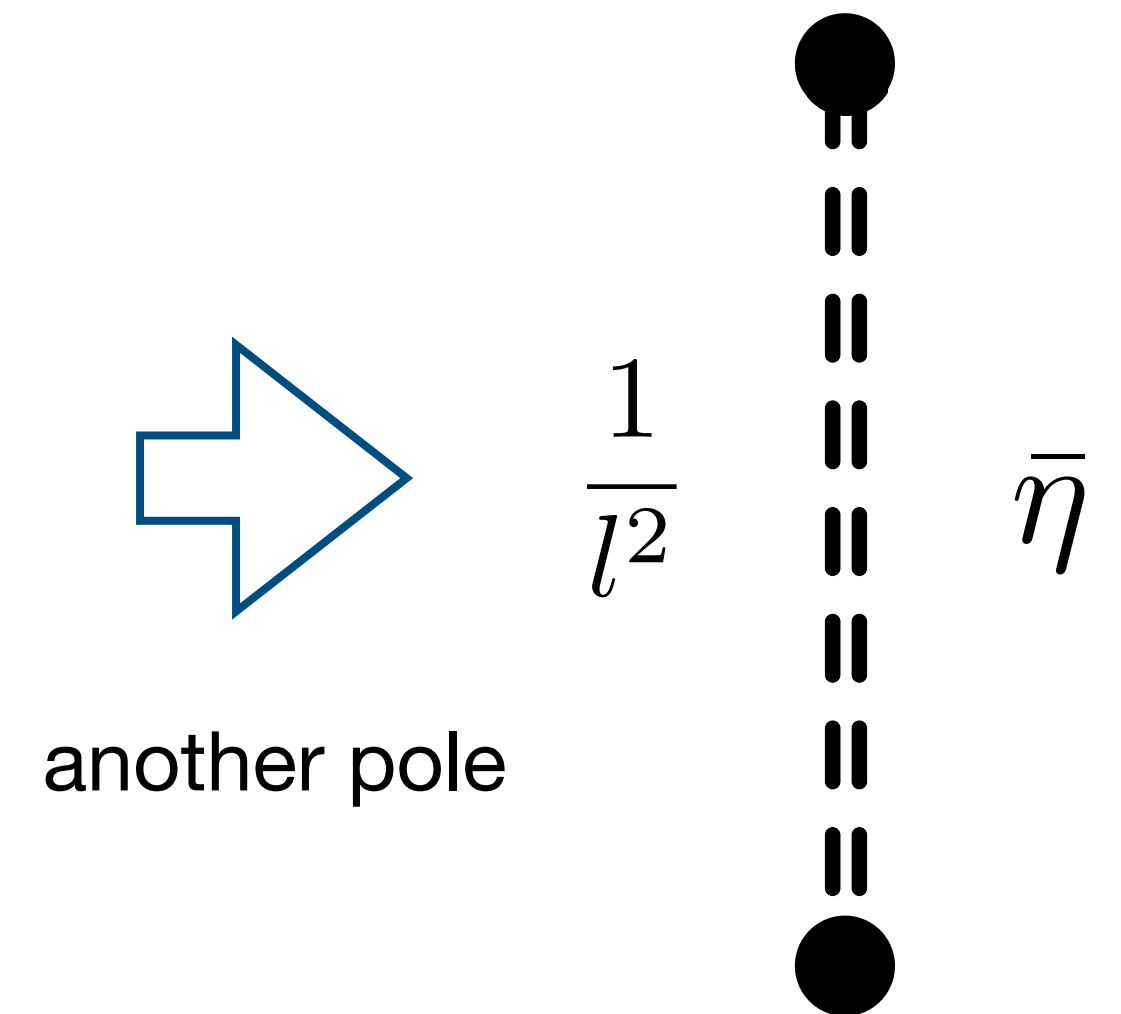
in agreement with the corresponding term in \mathcal{L}_{WZW} which was derived from chiral perturbation theory [Leutwyler \(1996\)](#); [Herrara-Sikody et al \(1997\)](#); [Leutwyler-Kaiser \(2000\)](#)

$$S_{\text{WZW}}^{\bar{\eta}} = -i \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \int d^4x \bar{\eta} \Omega \quad \Omega = \frac{\alpha_s}{4\pi} \text{Tr} \left(F \tilde{F} \right)$$

where $\bar{\eta}$ is a massless “primordial” ninth Goldstone boson arising from the spontaneous symmetry breaking of the flavor group $U_L(3) \times U_R(3)$



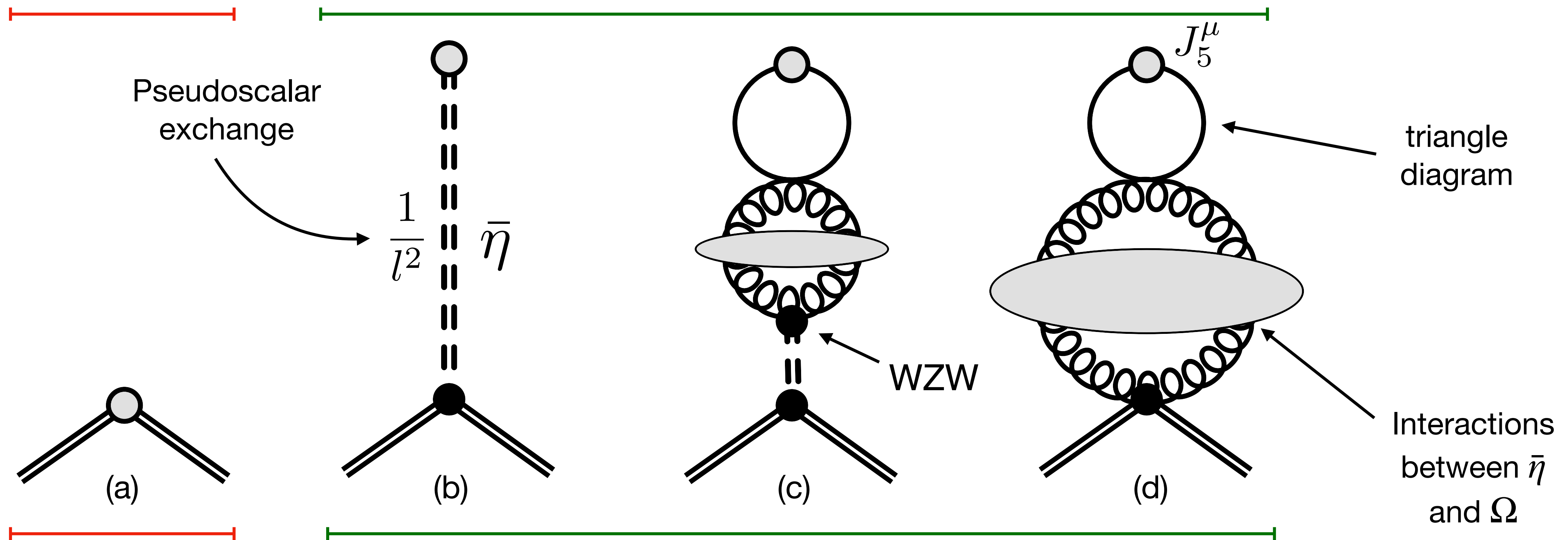
[Tarasov, Venugopalan \(2022\)](#)



Pseudovector vs. pseudoscalar coupling

The fundamental role in the pole cancellation is played by the Wess-Zumino-Witten (WZW) coupling between the topological charge density Ω to a primordial massless isosinglet $\bar{\eta}$.

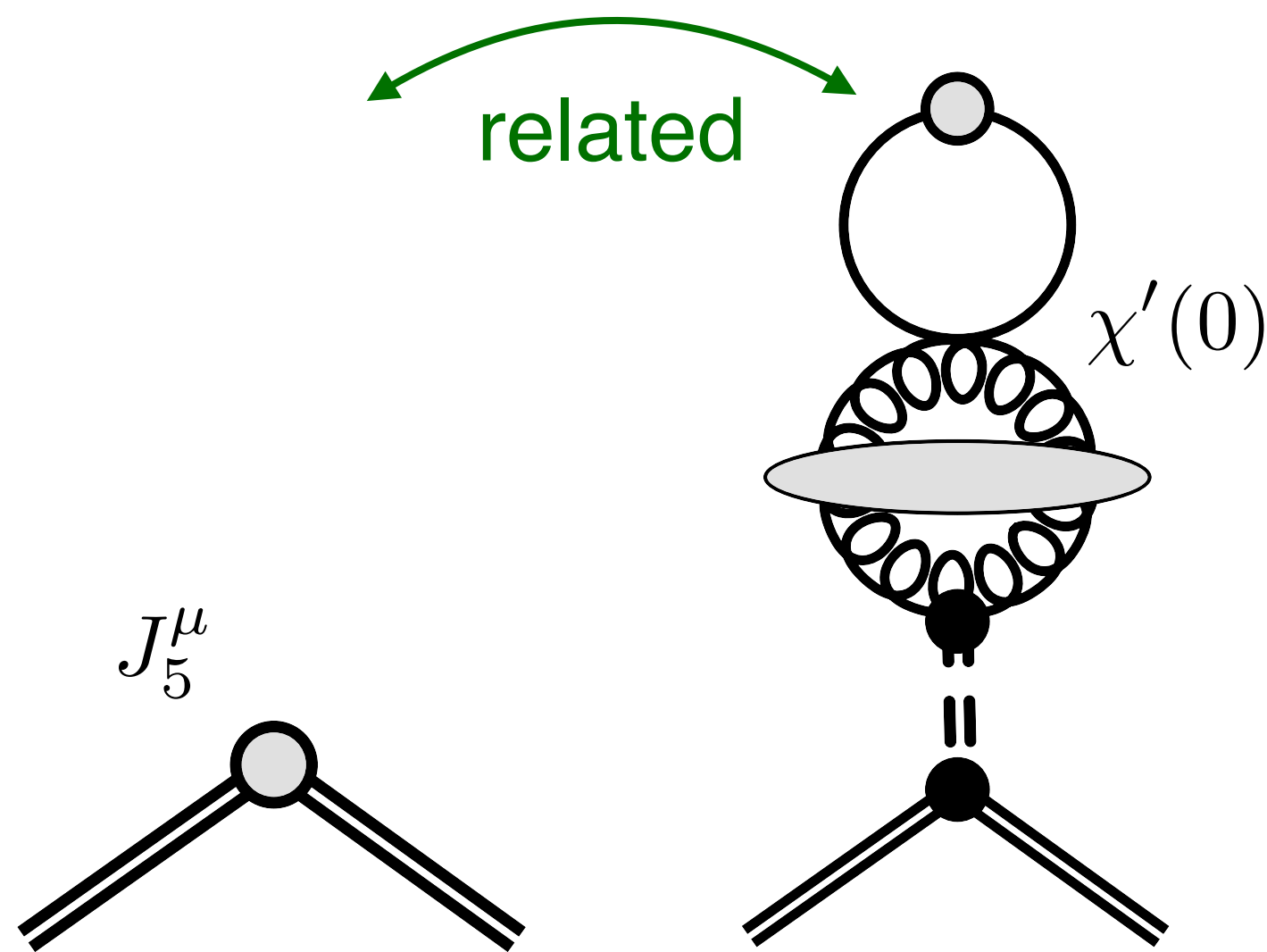
$$\langle P', S | J_5^\mu | P, S \rangle = \bar{u}(P', S) \left[\gamma^\mu \gamma_5 G_A(l^2) + l^\mu \gamma_5 G_P(l^2) \right] u(P, S)$$



Direct axial-vector coupling

Pseudoscalar coupling to the polarized proton

Infrared pole cancellation



From the cancellation of the anomaly pole, using Goldberger-Treiman relation one can relate helicity and the QCD topological susceptibility - **topological screening**

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}} \frac{2n_f}{M_N} g_{\eta_0 NN} \sqrt{\chi'(0)}$$

Shore, Veneziano (1992)

- The first moment of g_1 is determined by the non-perturbative effects
- Can this relation be checked on the lattice?

$$\langle P, S | J_5^\mu | P, S \rangle = M_N S^\mu \Sigma(Q^2) = 2M_N S^\mu a_0$$

$$a^0 |_{Q^2=10 GeV^2} = 0.33 \pm 0.05$$

Gives a natural resolution of the spin crisis



Shore



Veneziano

In agreement with COMPASS ($a^0 |_{Q^2=3 GeV^2} = 0.35 \pm 0.08$) and HERMES data ($a^0 |_{Q^2=5 GeV^2} = 0.330 \pm 0.064$)

Shore (2007), Narison (2021)

Anomaly and the DGLAP evolution

Calculation of the box diagram in the Feynman diagram approach at $l^2 \neq 0$

$$\mathcal{J}^\alpha \equiv -\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}}$$

S. Bhattacharya, Y. Hatta, W. Vogelsang (2022-2023)

$$\mathcal{J}^\alpha|_{\text{box}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \bar{u}(P_2) \left[\left(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

DGLAP evolution, JM operator

Anomaly pole previously observed in the worldline approach, $F\tilde{F}$ operator

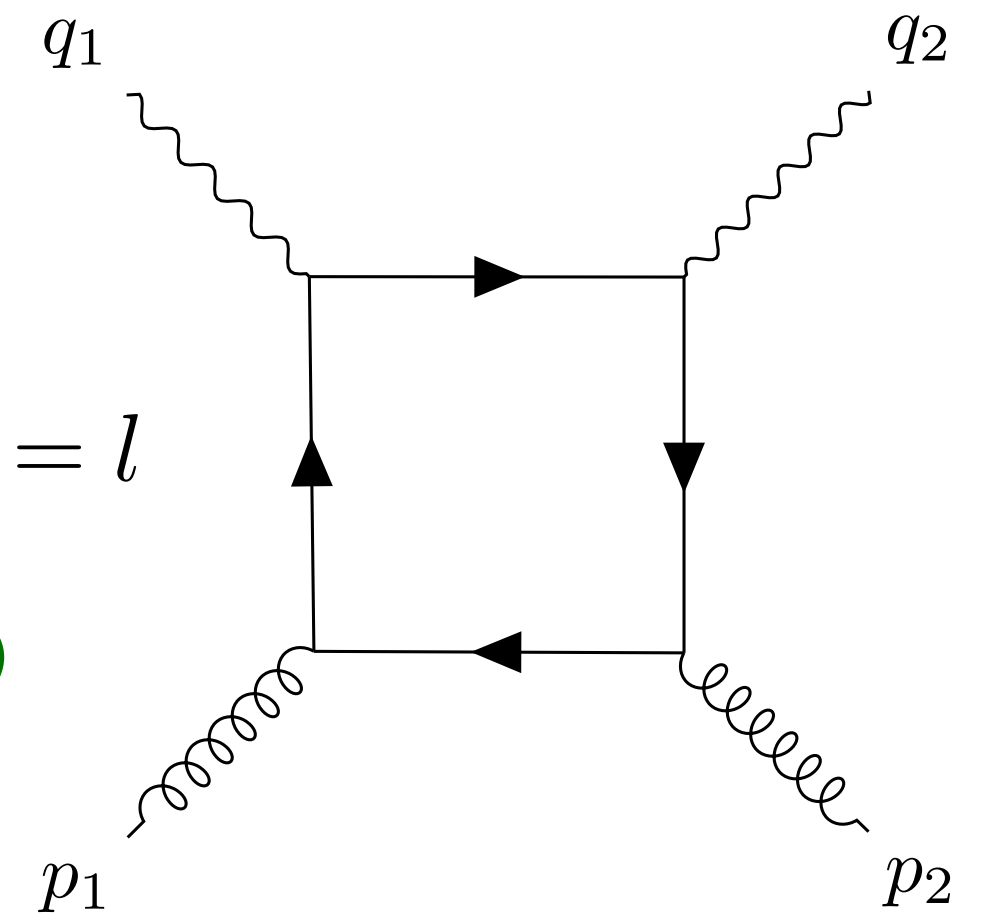
According to this result, there are two independent contributions associated with JM and $F\tilde{F}$. But we believe that the relation might be much deeper:

- There is a l^2 scale in the DGLAP log, but this scale defines the anomaly effects
- First moment of the JM operator coincides with the Chern-Simons current in the axial gauge, but

$$\partial_\mu K^\mu = \frac{\alpha_s}{4\pi} \text{Tr} \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

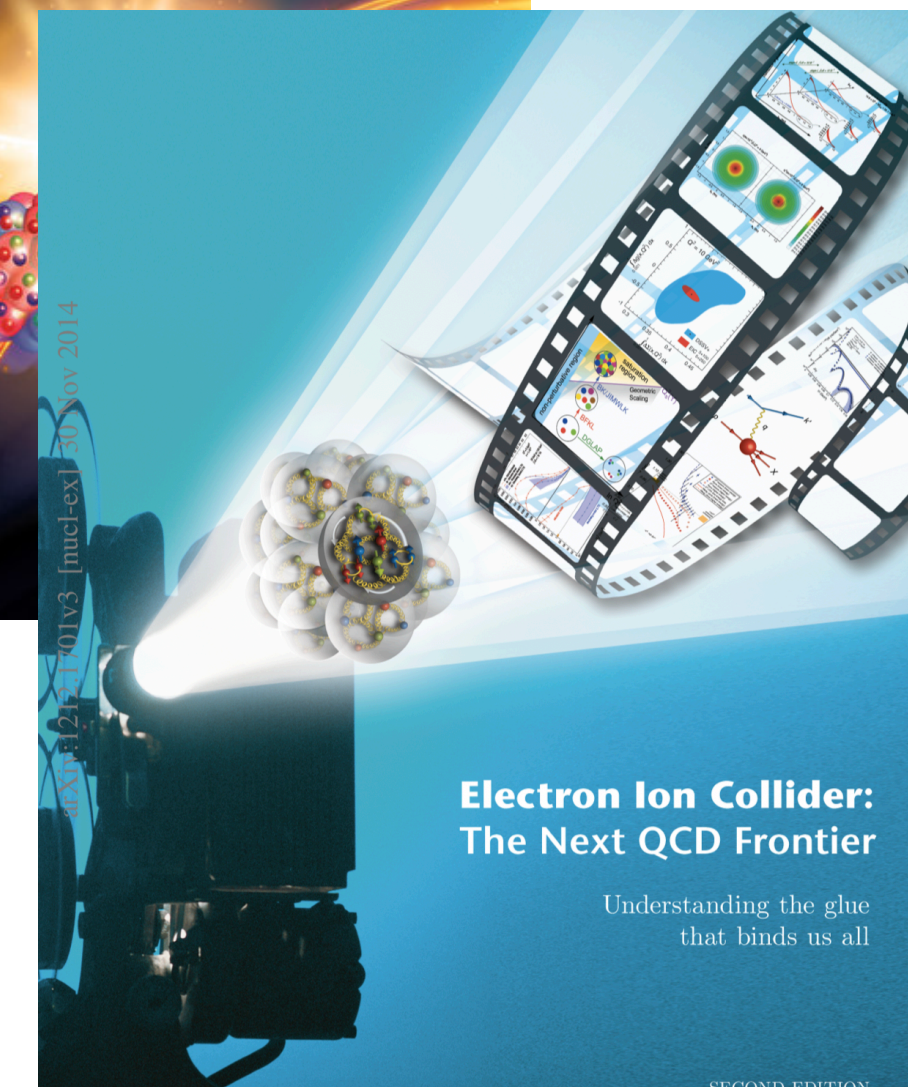
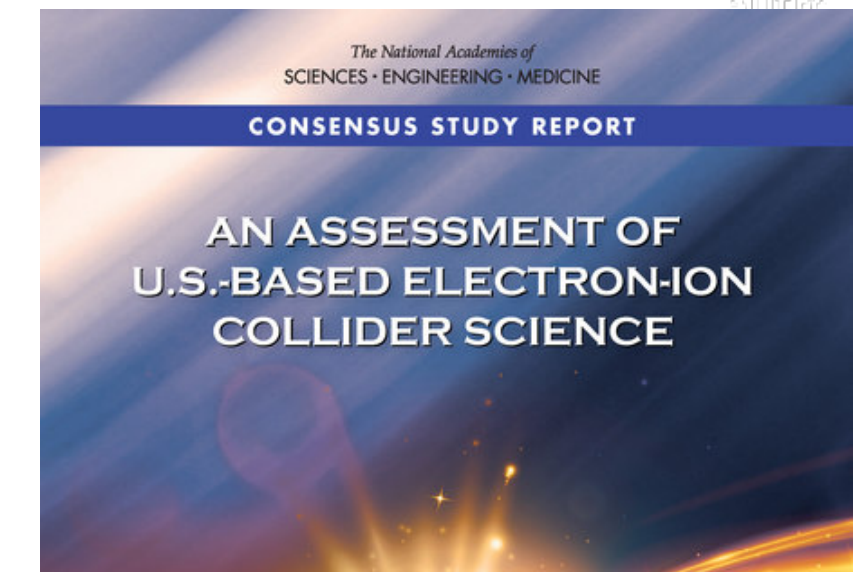
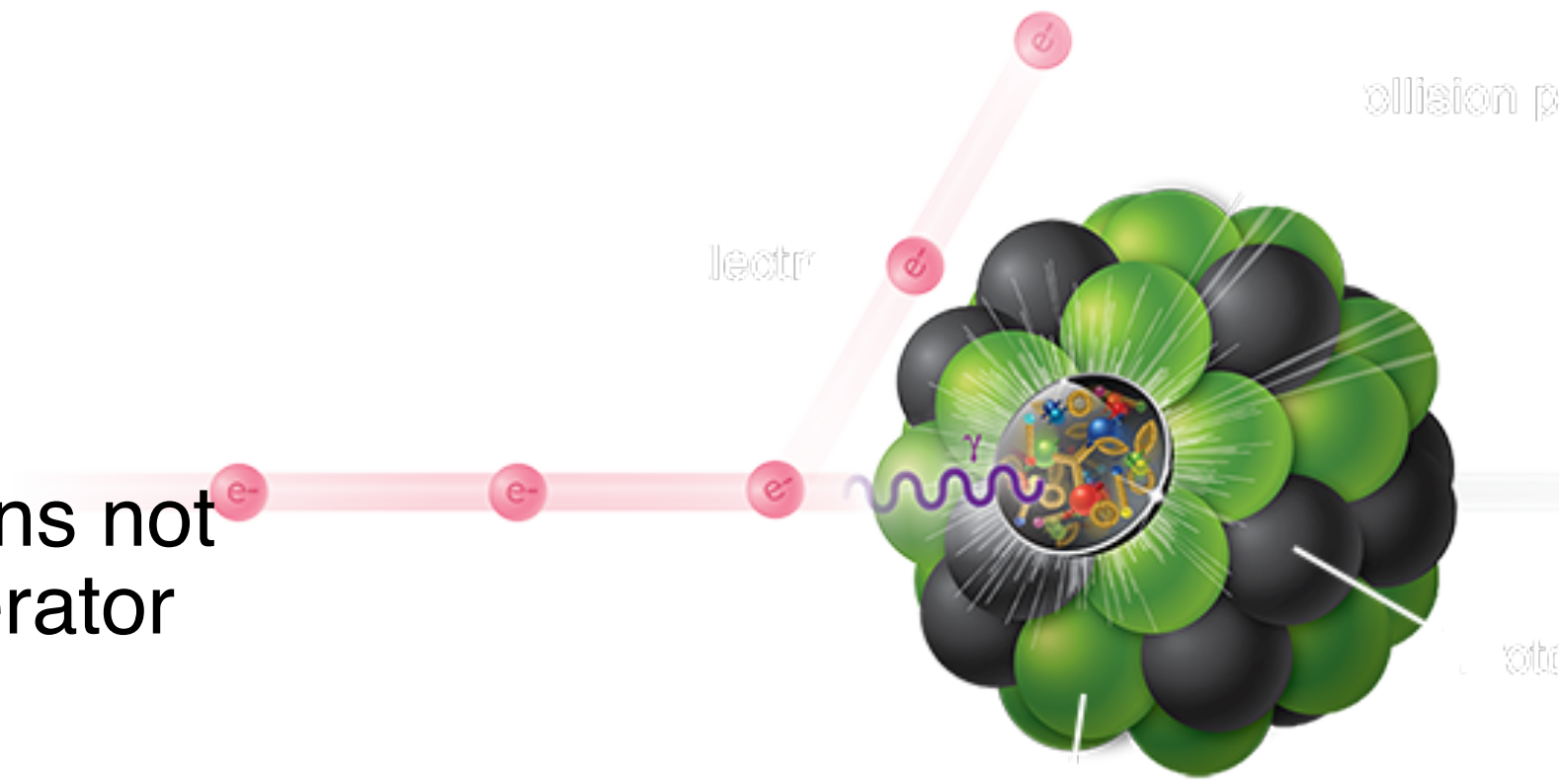
- If we add higher twist/sub-sub-eikonal corrections, can we relate ΔG to the operator $F\tilde{F}$?

Tarasov, Venugopalan, in preparation



Summary

- “Missing” spin of the proton may be found at small values of Bjorken-x.
- To study asymptotic of observables at small-x, one has to calculate sub-eikonal corrections
- A complete set of the sub-eikonal corrections relevant to the small-x helicity evolution contains not only fields strength operator F_{12} and quark axial current $\bar{\psi}\gamma^+\gamma_5\psi$, but also a sub-eikonal operator $D^i - \overleftarrow{D}^i$
- The operator $D^i - \overleftarrow{D}^i$ is related to the Jaffe-Manohar polarized gluon distribution ΔG and has a meaning of the sub-eikonal (covariant) phase
- The KPS-CTT helicity evolution equations contains mixing between these operators
- The helicity evolution doesn't provide a complete picture of the problem since it lacks the anomaly contribution
- There is a class of spin-dependent observables which are dominated by the triangle anomaly in both Bjorken ($Q^2 \rightarrow \infty$) and Regge ($x_B \rightarrow 0$) asymptotic. The anomaly manifests itself as an infrared pole which appears in both limits.
- The cancellation of the pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD. This relates g_1 to the properties of the QCD vacuum
- We demonstrate the fundamental role of the WZW term both in topological mass generation of the η' and in the cancellation of the infrared pole
- Relation between ΔG and $F\tilde{F}$ and the role of the higher twist/sub-sub-eikonal corrections is not fully understood.



Thank you for your attention!