Azimuthal asymmetries in *D*-meson and jet production at the EIC

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In collaboration with

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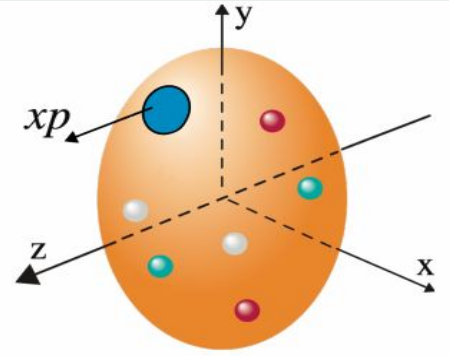
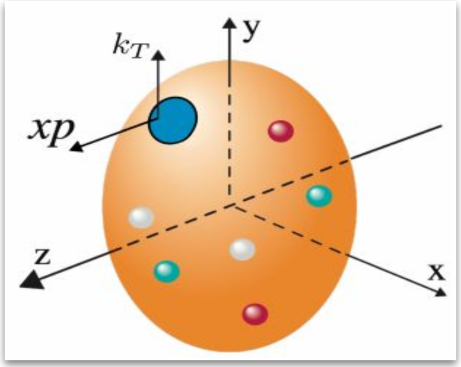
CFNS-POSTDOC MEETING

Parton distribution functions





PDFs - gives the probability to find parton of longitudinal momentum fraction **x** within nucleon.

1D information about the partons $f(x)$

Collinear factorization

Universal PDFs

$l\,p \to l\,X$ Deep inelastic scattering (DIS)

TMD PDFs - gives the probability to find parton of longitudinal momentum fraction **x** and transverse momenta $k_\perp$ within the nucleon.

3D information about the partons $f(x, k_\perp)$

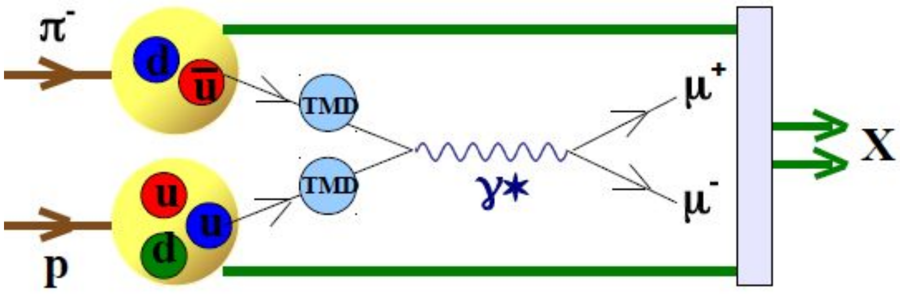
TMD factorization

Non-Universal PDFs

$l\,p \to l\,h\,X$ SIDIS $p\,p \to l^+ l^- X$ Drell Yan

TMDs in Drell Yan and SIDIS process

Drell Yan $p\,\pi \to \mu\,\mu^-\,X$

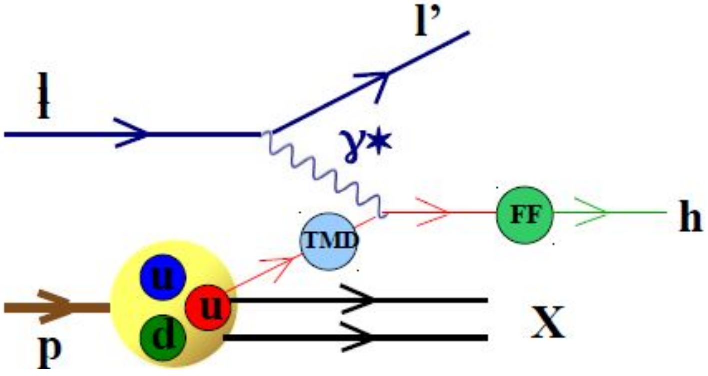


$$\sigma^{DY} \propto f_p(x, k_\perp; Q^2) \otimes f_\pi(y, p_\perp; Q^2) \otimes \hat{\sigma}^{part}$$

TMD PDFs

Hard scattering

SIDIS $l\,p \to l\,h\,X$



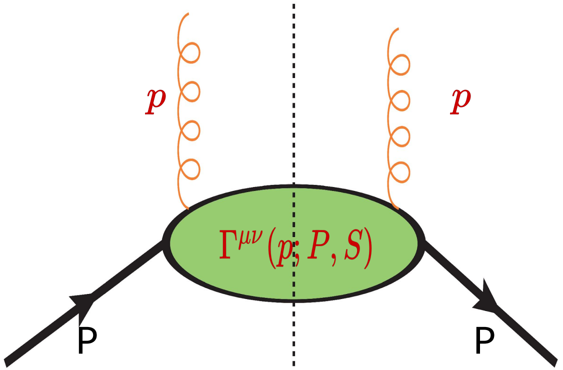
$$\sigma^{SIDIS} \propto f_p(x, k_\perp; Q^2) \otimes \hat{\sigma}^{part} \otimes D_f^h(z, p_\perp; Q^2)$$

TMD PDFs

TMD FFs

Gluon Correlator

$$\Gamma_g^{\mu\nu}(p;P,S) = \frac{n_\rho n_\sigma}{(P.n)^2}\int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi}\langle P,S|\mathrm{Tr}[F^{\mu\rho}(0)U_{[0,\xi]}F^{\nu\sigma}(\xi)U'_{[\xi,0]}]|P,S\rangle_{\xi^+=0}$$



The gauge links connecting the two points ensure that the TMD distribution is gauge invariant.

- Gauge links are path ordered exponential connecting the field strength tensors along a definite path.

$$U^C = \mathcal{P}\exp[ig\int_C dz^\mu A_\mu(z)]$$

- Simplest possible configurations are ++ or - - and + - or - +.

- In the literature related to small-x physics, these are known as Weizsacker-Williams (WW) and Dipole distributions respectively.

P. J. Mulders and J. Rodrigues PRD 63 094021(2001)

Gluon TMDs

$$\Phi^{\mu\nu}(x, q_T) = \int \frac{d\xi^- d^2\xi_T}{M_p (2\pi)^3} e^{iq\cdot\xi} \langle P, S | \mathrm{Tr}[F^{+\mu}(0) U^{[C]} F^{+\nu}(\xi) U'^{[C]}] | P, S \rangle_{\xi^+ = 0}$$

Target polarization		Gluon polarization		
		Unpolarised	Circularly	Linearly
	Unpolarised	f_1^g		$h_1^{\perp g}$
	Longitudinal		g_{1L}^g	$h_{1L}^{\perp g}$
	Transverse	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Linearly polarized ($h_1^{\perp g}$)

Helicity (g_{1L}^g)

Kotzinin-Mulders ($h_{1L}^{\perp g}$)

Sivers function ($f_{1T}^{\perp g}$)

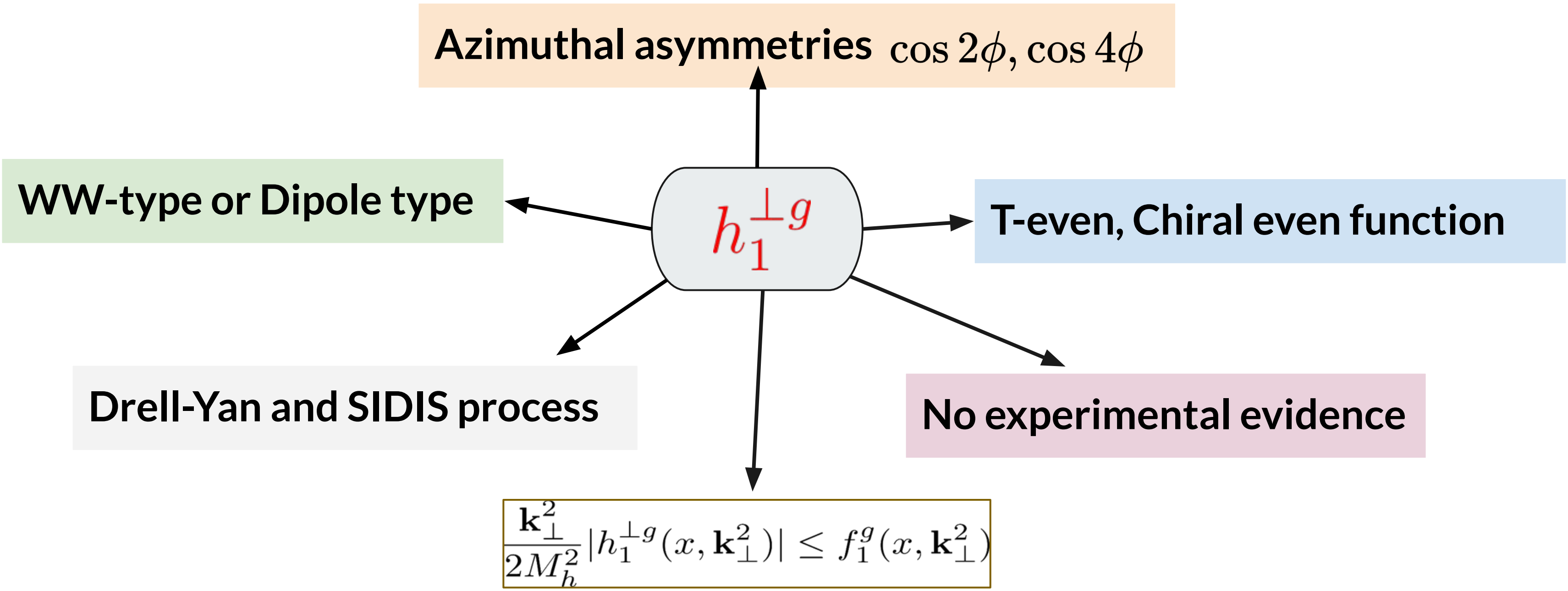
worm-gear (g_{1T}^g)

transversity (h_{1T}^g)

Pretzelosity ($h_{1T}^{\perp g}$)

D. Boer *et al* arXiv 1507.05267

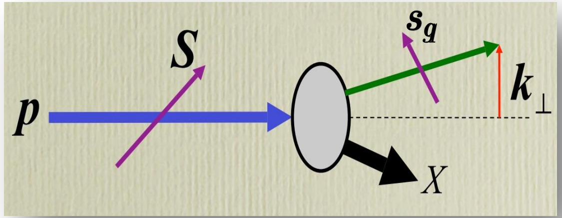
Linearly polarized gluon distribution function



D. Boer et al. PRL 106 132001 (2011)

Mulders and Rodrigues, PRD 63, 094021 (2001)

Gluon Sivers function



$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$: **Sivers effect**

- In 1990 Sivers proposed that SSA can be explained by allowing the correlation between transverse momentum of parton and polarization direction of its parent hadron. D. Sivers, PRD 41, 83(1990)

- Sivers function is Time-reversal odd function.

- Sivers function in DY is equal in magnitude but opposite in sign compared to Sivers function in SIDIS.

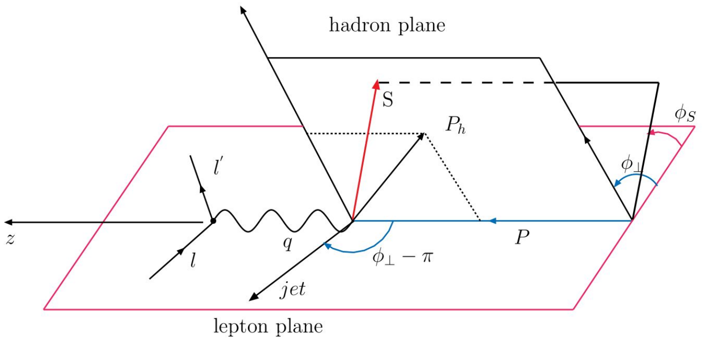
$$\Delta^N f_{g/p^\uparrow}(x, \mathbf{k}_\perp)|_{\mathrm{DY}} = -\Delta^N f^\perp_{g/p^\uparrow}(x, \mathbf{k}_\perp)|_{\mathrm{SIDIS}}$$

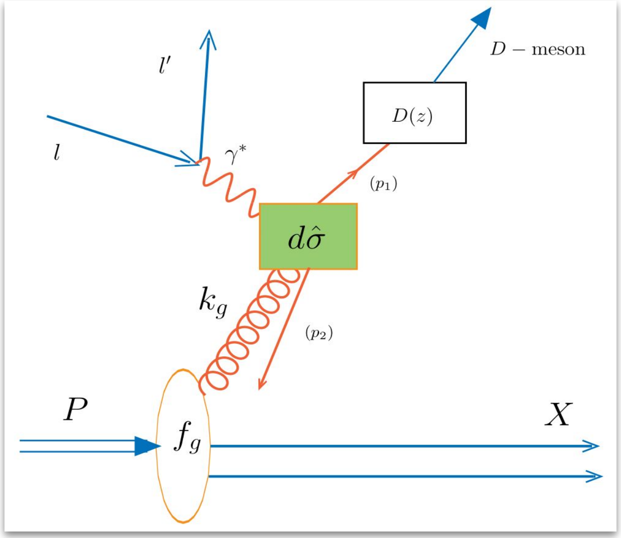
- In some models, it is related to the orbital angular momentum. A, Bacchetta and M, Radici, PRL **107**, 212001 (2011)

- lepton-pair production, back-to-back jet production.

D-meson and jet production at EIC

$$e(l) + p^{\uparrow}(P) \rightarrow e(l') + D(P_h) + \text{jet} + X$$





$$\gamma^*(q) + g(k) \rightarrow c(p_1) + \bar{c}(p_2)$$

Kinematic variables:

$$Q^2 = -q^2$$ Virtuality of photon

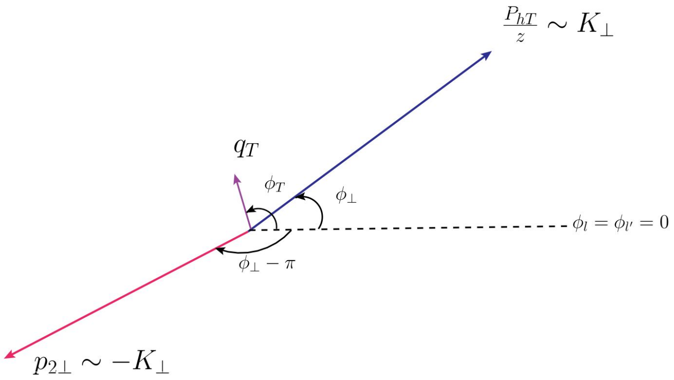
$$x = \frac{Q^2}{2P\cdot q}$$ Bjorken variable

$$y = \frac{P\cdot q}{P\cdot k}$$ inelasticity

$$s = (l+P)^2$$ centre of mass energy

$$z_h = \frac{P.P_h}{P.q}$$

Differential scattering cross-section



$$\mathbf{q}_T = \frac{\mathbf{P}_{hT}}{z} + \mathbf{p}_{2\perp} \qquad \mathbf{K}_\perp = \frac{\frac{\mathbf{P}_{hT}}{z} - \mathbf{p}_{2\perp}}{2}$$

- In the case where $|\mathbf{q}_T| \ll |\mathbf{K}_\perp|$, the *D*-meson and jet are almost back to back in the transverse plane.

Assuming the TMD factorization holds, the total differential scattering cross-section can be written as

$$d\sigma^{ep\to e+D+\bar{c}+X} = \frac{1}{2s}\frac{d^3\mathbf{l}'}{(2\pi)^3 2E_{l'}}\frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2}\int dx_g\, d^2\mathbf{k}_{\perp g}\, dz\,(2\pi)^4\,\delta^4(q+k-p_1-p_2)$$
$$\times \frac{1}{Q^4}L^{\mu\nu}(l,q)\,\Phi_g^{\rho\sigma}(x_g,\mathbf{k}_{\perp g})\,H_{\mu\rho}^{\gamma^* g\to c\bar{c}}\,H_{\nu\sigma}^{*;\gamma^* g\to c\bar{c}}\,D(z)\,J(z)$$

Differential scattering cross-section

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$$\times\frac{1}{Q^4}L^{\mu\nu}(l,q)\,\Phi_g^{\rho\sigma}(x_g,\mathbf{k}_{\perp g})\,H_{\mu\rho}^{\gamma^* g\to c\bar{c}}\,H_{\nu\sigma}^{*;\gamma^* g\to c\bar{c}}\,D(z)\,J(z)$$

The gluon correlator (non-perturbative) for unpolarized proton is given as

$$\Phi_U^{\rho\sigma}(x_g,\mathbf{k}_{\perp g}) = \frac{1}{2x_g}\left[-g_T^{\rho\sigma} f_1^g(x,\mathbf{k}_{\perp g}^2) + \left(\frac{k_{\perp g}^{\rho}k_{\perp g}^{\sigma}}{M_P^2} + g_T^{\rho\sigma}\frac{\mathbf{k}_{\perp g}^2}{2M_P^2}\right) h_1^{\perp g}(x,\mathbf{k}_{\perp g}^2)\right]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

The gluon correlator for transversely polarized proton is given as

$$\Phi_T^{\mu\nu}(x_g,\mathbf{k}_{\perp g}) = \frac{1}{2x_g}\Bigg\{-g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}k_{\perp g\rho}S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x_g,\mathbf{k}_{\perp g}^2) + i\epsilon_T^{\mu\nu}\frac{k_{\perp g}\cdot S_T}{M_p} g_{1T}^g(x_g,\mathbf{k}_{\perp g}^2)$$
$$+\frac{k_{\perp g\rho}\epsilon_T^{\rho\{\mu}k_{\perp g}^{\nu\}}}{2M_p^2}\frac{k_{\perp g}\cdot S_T}{M_p} h_{1T}^{\perp g}(x_g,\mathbf{k}_{\perp g}^2) - \frac{k_{\perp g\rho}\epsilon_T^{\rho\{\mu}S_T^{\nu\}} + S_{T\rho}\epsilon_T^{\rho\{\mu}k_{\perp g}^{\nu\}}}{4M_p} h_{1T}^g(x_g,\mathbf{k}_{\perp g}^2)\Bigg\}$$

Gluon Sivers function

Differential scattering cross-section

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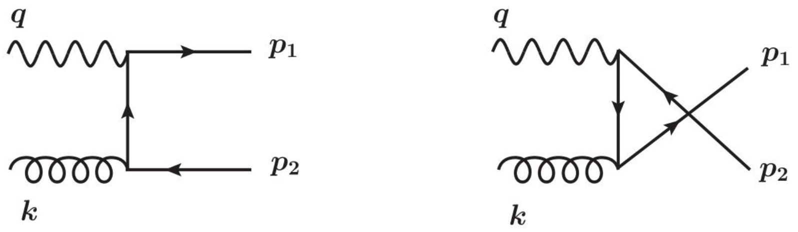
$$\times\frac{1}{Q^4}L^{\mu\nu}(l,q)\,\Phi_g^{\rho\sigma}(x_g,\mathbf{k}_{\perp g})\,H_{\mu\rho}^{\gamma^* g\to c\bar{c}}\,H_{\nu\sigma}^{*;\gamma^* g\to c\bar{c}}\,D(z)\,J(z)$$

Leptonic tensor

$$L^{\mu\nu} = e^2\frac{Q^2}{y^2}\Big[-(1+(1-y)^2)g_T^{\mu\nu} + 4(1-y)\epsilon_L^\mu\epsilon_L^\nu + 4(1-y)\left(\hat{l}_\perp^\mu\hat{l}_\perp^\nu + \tfrac{1}{2}g_T^{\mu\nu}\right)$$
$$+ 2(2-y)\sqrt{1-y}\left(\epsilon_L^\mu\hat{l}_\perp^\nu + \epsilon_L^\nu\hat{l}_\perp^\mu\right)\Big]$$

Scattering amplitude (perturbative part)

$$\gamma^*(q) + g(k) \to c(p_1) + \bar{c}(p_2)$$



Feynman diagram for D-meson production in SIDIS process

Differential scattering cross-section

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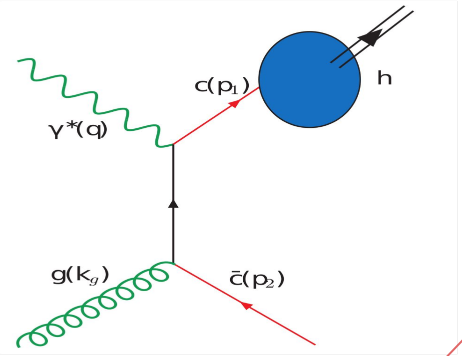
Jacobian

$$D_{D/c}(z,\mu) = \frac{Nz(1-z)^2}{\left[(1-z)^2+\epsilon z\right]^2}$$

Fragmentation function (non-perturbative part)

$$\mu = m_c = 1.5\ \text{GeV}$$
$$N = 0.694$$
$$\epsilon = 0.101$$



$$z_1 = \frac{P\cdot p_1}{P\cdot q}$$
$$z_2 = \frac{P\cdot p_2}{P\cdot q}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$z = \frac{P\cdot P_h}{P\cdot p_1} = \frac{z_h}{z_1}$$

energy fraction of the virtual photon taken by the observed *D*-meson in protons rest frame

Kniehl and Kramer Phys. Rev. D **74**, 037502

$\cos 2\phi_T$ asymmetry

The cross-section as the sum of unpolarized and transversely polarized cross-sections,

$$\frac{d\sigma}{dQ^2\,dy\,dz_h\,d^2\mathbf{q}_T\,d^2\mathbf{K}_\perp} \equiv d\sigma(\phi_S,\phi_T) = d\sigma^U(\phi_T,\phi_\perp) + d\sigma^T(\phi_S,\phi_T)$$

The cross-section for the unpolarized proton is written as the linear sum of $\cos\phi_\perp$ and $\cos\phi_T$ harmonics convoluted with the fragmentation function,

$$d\sigma^U = \mathcal{N}\int dz\Big[\big(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp\big)f_1^g(x,\mathbf{q}_T^2) + \big(\mathcal{B}_0\cos 2\phi_T + \mathcal{B}_1\cos(2\phi_T-\phi_\perp) + \mathcal{B}_2\cos 2(\phi_T-\phi_\perp)$$
$$+\mathcal{B}_3\cos(2\phi_T-3\phi_\perp) + \mathcal{B}_4\cos(2\phi_T-4\phi_\perp)\big)\frac{\mathbf{q}_T^2}{M_p^2}h_1^{\perp g}(x,\mathbf{q}_T^2)\Big]D(z)$$

The weighted azimuthal asymmetry, gives the ratio of specific gluon TMD over unpolarized f_1^g and is defined as

$$A^{W(\phi_S,\phi_T)} \equiv 2\frac{\int d\phi_S\,d\phi_T\,d\phi_\perp\,W(\phi_S,\phi_T)\,d\sigma(\phi_S,\phi_T,\phi_\perp)}{\int d\phi_S\,d\phi_T\,d\phi_\perp\,d\sigma(\phi_S,\phi_T,\phi_\perp)}$$

The $h_1^{\perp g}$ gluon TMD could be extracted by studying the following two azimuthal asymmetries

$$A^{\cos 2\phi_T} = \frac{\mathbf{q}_T^2}{M_p^2}\frac{\int dz\,\mathcal{B}_0\,D(z)\,h_1^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\mathbf{q}_T^2)}$$

$$A^{\cos 2(\phi_T-\phi_\perp)} = \frac{\mathbf{q}_T^2}{M_p^2}\frac{\int dz\,\mathcal{B}_2\,D(z)\,h_1^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\mathbf{q}_T^2)}$$

Upper bound

- ➔ Linearly polarized gluon distribution satisfies the positivity bound
- ➔ Upper limit of asymmetry obtained when this bound is saturated

$$\frac{\mathbf{q}_T^2}{2M_p^2}\,|h_1^{\perp\,g}(x,\mathbf{q}_T^2)| \leq f_1^g(x,\mathbf{q}_T^2)$$

$$\frac{|q_T|}{M_p}\,|f_{1T}^{\perp\,g}(x,q_T^2)| \leq f_1^g(x,q_T^2)$$

$$\frac{\mathrm{q}_t^2}{2M_p^2}|h_1^{\perp g}(x,\mathrm{q}_t^2)| = f_1^g(x,\mathrm{q}_t^2)$$

$$A^{\cos 2\phi_T} = \frac{\mathbf{q}_T^2}{M_p^2}\frac{\int dz\,\mathcal{B}_0\;D(z)\;h_1^{\perp\,g}(x,\mathbf{q}_T^2)}{\int dz\,\mathcal{A}_0\;D(z)\;f_1^g(x,\mathbf{q}_T^2)}$$

$$A^{\cos 2\phi_T} \to U = \frac{2*|\mathbb{B}_0|}{\mathbb{A}_0}$$

$$A^{\cos 2(\phi_T-\phi_\perp)} = \frac{q_T^2}{M_p^2}\frac{\int dz\,\mathcal{B}_2\;D(z)\;h_1^{\perp\,g}(x,q_T^2)}{\int dz\,\mathcal{A}_0\;D(z)\;f_1^g(x,q_T^2)}$$

$$A^{\cos 2(\phi_T-\phi_\perp)} \to U = \frac{2*|\mathbb{B}_2|}{\mathbb{A}_0}$$

Parametrization of TMDs

Gaussian Parametrization of TMDs

$$f_1^g(x, \mathbf{q}_T^2) = f_1^g(x, \mu) \frac{e^{-\mathbf{q}_T^2/\langle q_T^2 \rangle}}{\pi \langle q_T^2 \rangle}$$

$$h_1^{\perp g}(x, \mathbf{q}_T^2) = \frac{M_p^2 f_1^g(x,\mu)}{\pi \langle q_T^2 \rangle^2} \frac{2(1-r)}{r} e^{1-\frac{\mathbf{q}_T^2}{r\langle q_T^2 \rangle}}$$

$f_1^g(x, \mu)$ is the collinear gluon PDF

$r (0 < r < 1)$ and $\langle \mathrm{q}_T^2 \rangle$ are parameters

$r = 1/3$ $\langle \mathrm{q}_T^2 \rangle = 1 \text{ GeV}^2$

D. Boer, C. Pisano, PRD 86, 094007 (2012)

QCD Scale : $\mu = \sqrt{m_D^2 + Q^2}$

Mass of D-meson : $m_D = 1.8 \text{ GeV}$

MSTW2008 PDF

The European Physical Journal C 63, 189 (2009)

Sivers asymmetry

- The cross-section for the transversely polarized proton is written as

$$\int d\phi_\perp d\sigma^T = 2\pi|\boldsymbol{S}_T|\frac{|\boldsymbol{q}_T|}{M_p}\int dz\left[\mathcal{A}_0\sin(\phi_S-\phi_T)f_{1T}^{\perp g}(x,\boldsymbol{q}_T^2)-\frac{1}{2}\mathcal{B}_0\sin(\phi_S-3\phi_T)\frac{|\boldsymbol{q}_T|^2}{M_p^2}h_{1T}^{\perp g}(x,\boldsymbol{q}_T^2)\right.$$
$$\left.+\mathcal{B}_0\sin(\phi_S+\phi_T)h_1^g(x,\boldsymbol{q}_T^2)\right]D(z)\,,$$

- Sivers asymmetry can be extracted through the azimuthal asymmetry

$$A^{\sin(\phi_S-\phi_T)}=\frac{|\mathbf{q}_T|}{M_p}\frac{\int dz\,\mathcal{A}_0\,D(z)\,f_{1T}^{\perp g}(x,\mathbf{q}_T^2)}{\int dz\,\mathcal{A}_0\,D(z)\,f_1^g(x,\mathbf{q}_T^2)}$$

Gaussian Parametrization of Sivers function

$$f_{1T}^{\perp g}(x,q_T)=\frac{\sqrt{2e}}{\pi}\mathcal{N}_g(x)\,f_{g/p}(x)\sqrt{\frac{1-\rho}{\rho}}\frac{e^{-q_T^2/\rho\langle q_T^2\rangle}}{\langle q_T^2\rangle^{3/2}}$$

$$\mathcal{N}_g(x)=N_g x^\alpha(1-x)^\beta\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha\beta^\beta}$$

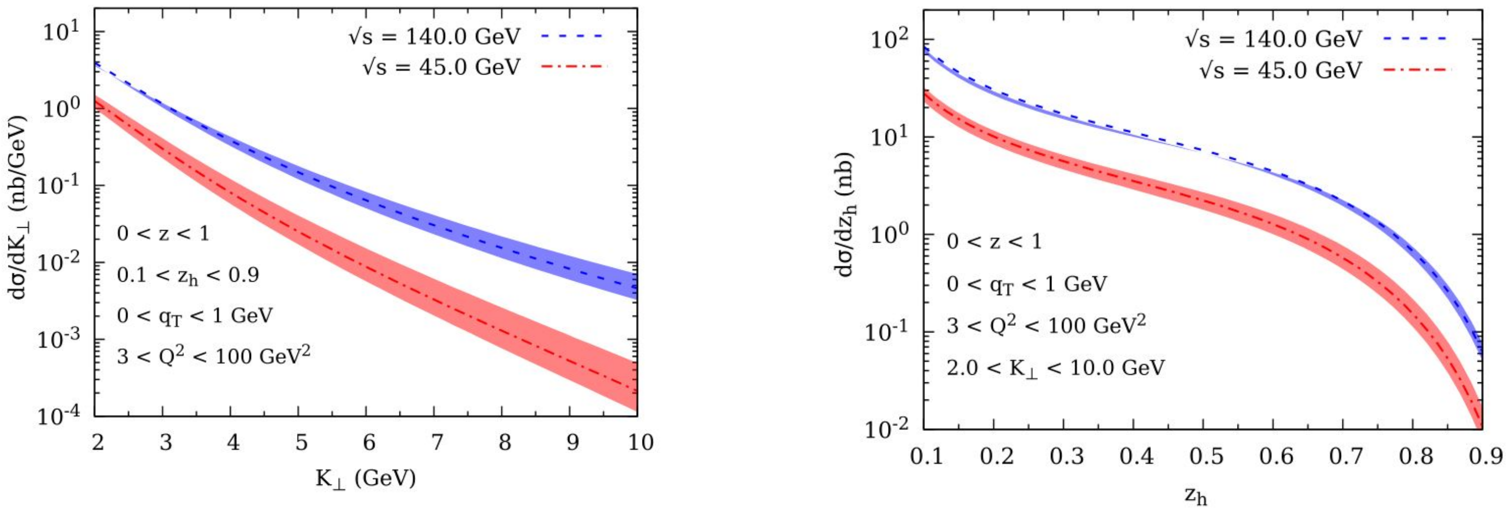
the extracted best fit parameters are $N_g=0.25\,,\quad \alpha=0.6\,,\quad \beta=0.6\,,\quad \rho=0.1$

(PHENIX Collaboration at RHIC)

U. D'Alesio, C. Flore, F. Murgia, C. Pisano, and P. Taels, Phys. Rev. D99(2019)036013

Numerical Results

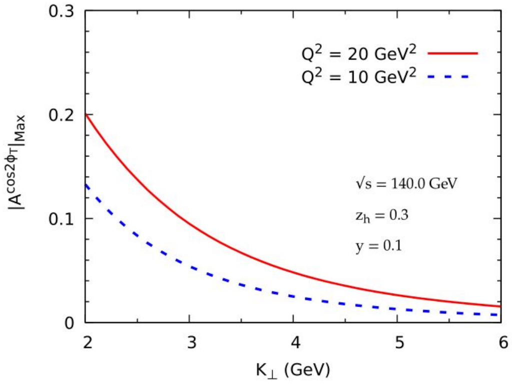
Unpolarized differential scattering cross-section

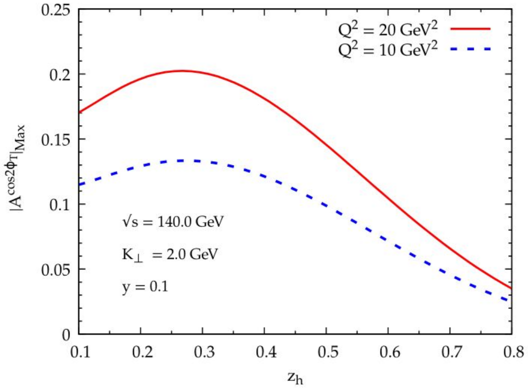


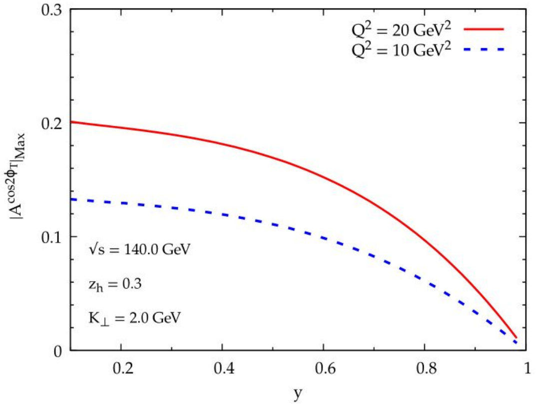
The bands are obtained by varying the factorization scale in the range $\frac{1}{2}\mu < \mu < 2\mu$

Numerical Results – $\cos 2\phi_T$ Azimuthal Asymmetry

Upper bound

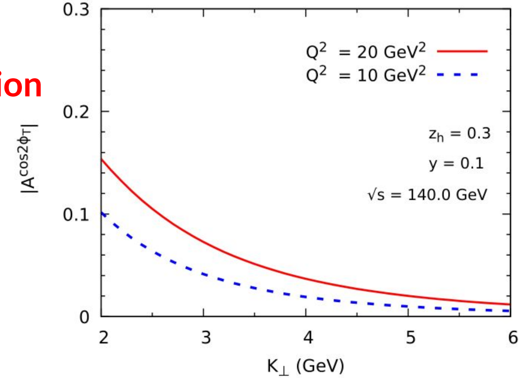


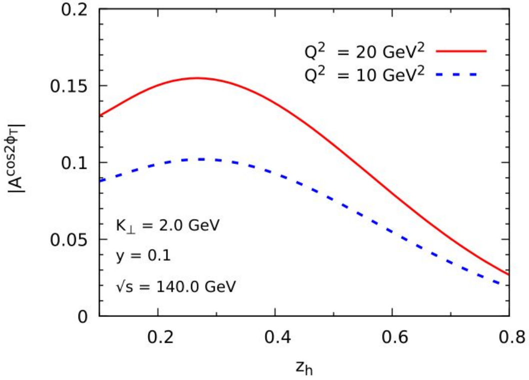


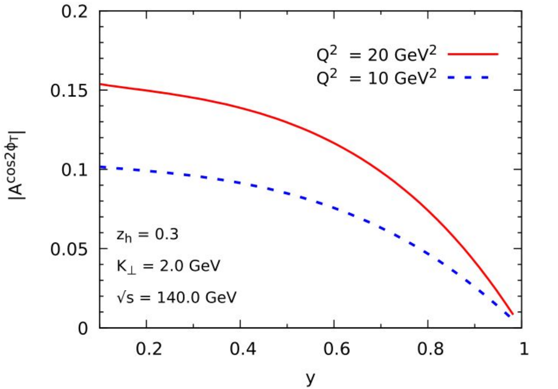


The z is integrated over 0 < z < 1 and q_T is integrated over 0 < q_T< 1 GeV.

Gaussian Parametrization

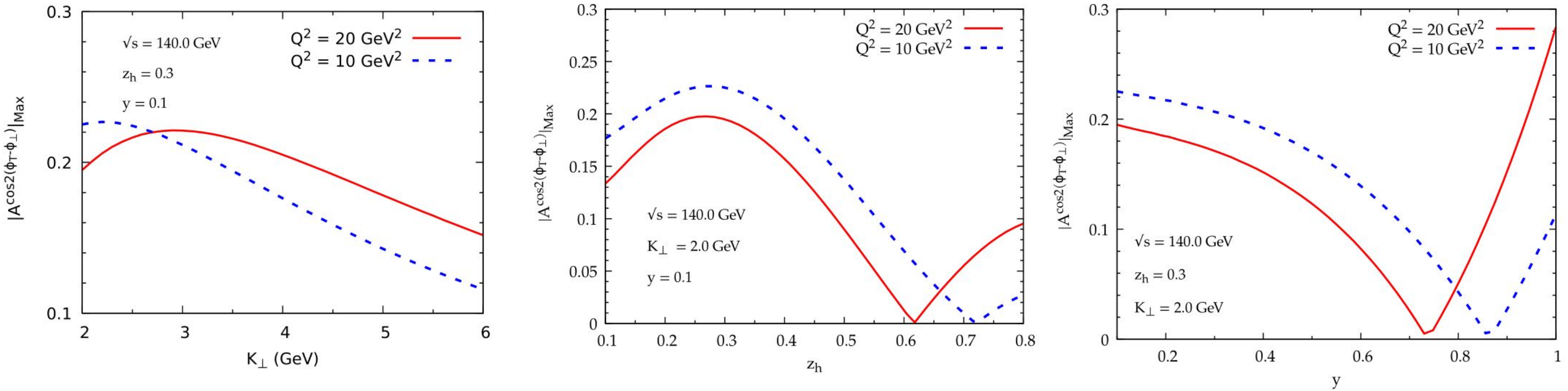






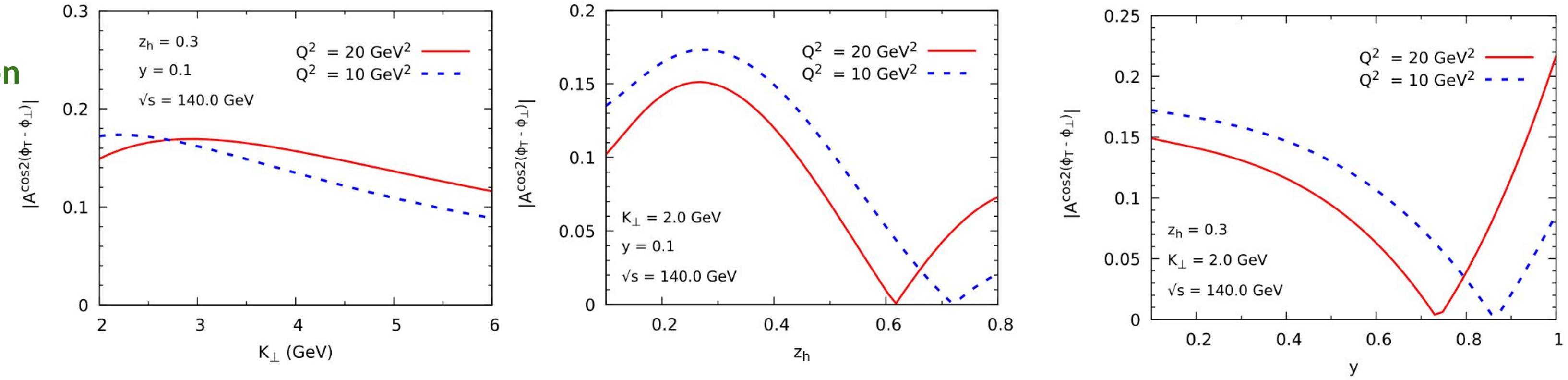
Numerical Results - $\cos 2(\phi_T - \phi_\perp)$ Azimuthal Asymmetry

Upper bound

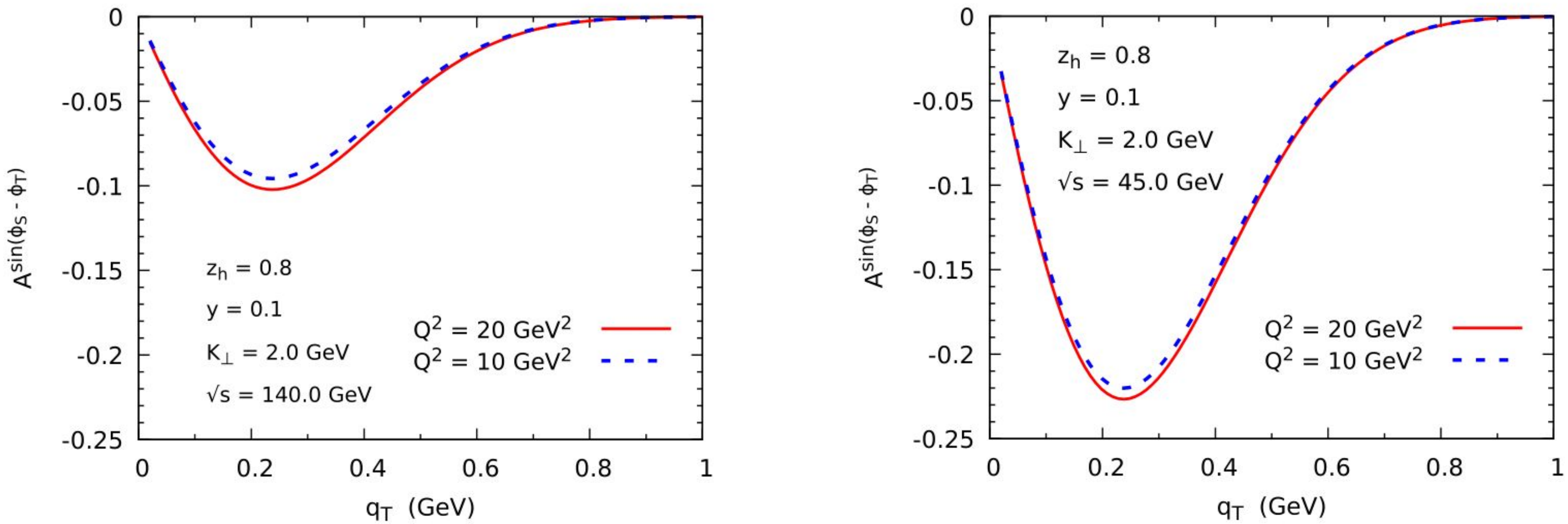


The z is integrated over 0 < z < 1 and q_T is integrated over 0 < q_T< 1 GeV.

Gaussian Parametrization



Numerical Results - Sivers Asymmetry



Sivers Asymmetry

The z is integrated over 0 < z < 1.

Summary

- *We estimate the* $\cos 2\phi_T$ *and Sivers asymmetry in almost back to back D-meson and jet electroproduction at the future EIC.*
- *We have used fragmentation function to describe the production of D-meson.*
- *We estimate the asymmetry using Gaussian parametrization of TMDs.*
- *We observed that* $\cos 2\phi_T$ *asymmetry and Sivers asymmetry is maximum for large value of*
- *Back to back production of D-meson and jet can be a promising channel to access the ratio of* q^2 *linearly polarized gluon TMD and the gluon Sivers TMD to unpolarized gluon TMD at EIC.*

Thank you for attention

Back up slides

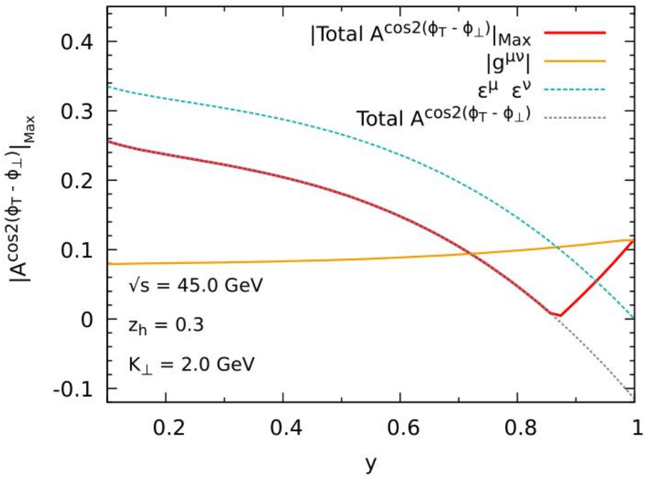
The reason for discontinuities

$$L^{\mu\nu} = e^2 Q^2 \left(-g^{\mu\nu} + \frac{2}{Q^2}(2l^\mu l^\nu - l^\mu q^\nu - l^\nu q^\mu)\right)$$

Virtual photon polarizations

Unpolarized — Longitudinally polarized — Linearly polarized — Interference

$$L^{\mu\nu} = e^2 \frac{Q^2}{y^2}\Big[-(1+(1-y)^2)g_T^{\mu\nu} + 4(1-y)\epsilon_L^\mu \epsilon_L^\nu + 4(1-y)\left(\hat{l}_\perp^\mu \hat{l}_\perp^\nu + \tfrac{1}{2}g_T^{\mu\nu}\right) + 2(2-y)\sqrt{1-y}\left(\epsilon_L^\mu \hat{l}_\perp^\nu + \epsilon_L^\nu \hat{l}_\perp^\mu\right)\Big]$$



$$\mathrm{U} + \mathrm{L} \rightarrow \mathcal{B}_2$$

$$\text{Linearly polarized} \rightarrow \mathcal{B}_0, \mathcal{B}_4$$

$$\mathrm{I} \rightarrow \mathcal{B}_1, \mathcal{B}_3$$

Positivity bound

Gluon Helicity

$|+\rangle$ $|+\rangle$ $|-\rangle$ $|-\rangle$

$\Phi^{\mu\nu}$

ϵ^{ν}

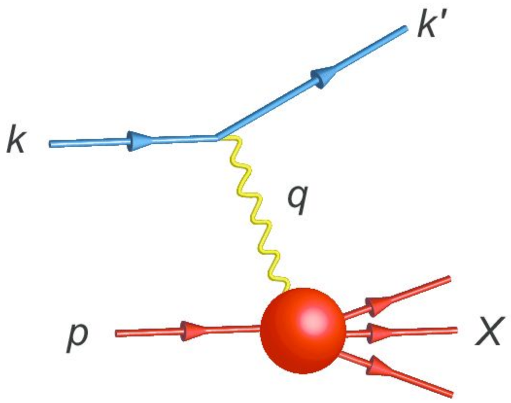
$$
\begin{array}{c}
\langle +| \\ \langle +| \\ \langle -| \\ \langle -|
\end{array}
\left\{
\begin{array}{cccc}
f_1^g + g_{1L}^g & \frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g - i f_{1T}^{\perp g}) & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} + i h_{1L}^{\perp g}) & -i\frac{q_T e^{-3i\phi}}{M_p} h_{1T}^{\perp g} \\
\frac{q_T e^{i\phi}}{M_p}(g_{1T}^g + i f_{1T}^{\perp g}) & f_1^g - g_{1L}^g & -i\frac{q_T e^{-i\phi}}{M_p} h_1^g & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} - i h_{1L}^{\perp g}) \\
-\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} - i h_{1L}^{\perp g}) & i\frac{q_T e^{i\phi}}{M_p} h_1^g & f_1^g - g_{1L}^g & -\frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g + i f_{1T}^{\perp g}) \\
i\frac{q_T e^{3i\phi}}{M_p} h_{1T}^{\perp g} & -\frac{q_T^2 e^{-2i\phi}}{M_p^2}(h_1^{\perp g} + i h_{1L}^{\perp g}) & -\frac{q_T e^{-i\phi}}{M_p}(g_{1T}^g - i f_{1T}^{\perp g}) & f_1^g + g_{1L}^g
\end{array}
\right\}
$$

$\epsilon^{*\mu}$

$$M_p^2\ \Phi^{ii} = 2(P^+)^2 \int_0^1 dx_g x_g f_1^g(x_g, Q^2) = \langle P, S|T^{++}|P, S\rangle$$

Mulders and Rodrigues, PRD 63, 094021 (2001)

Deep inelastic e-p scattering



DIS-proton breaks up, and we end up with many final particle states.

$$Q^2 = -q^2$$ Virtuality of photon

$$x = \frac{Q^2}{2P\cdot q}$$ Bjorken variable

$$y = \frac{P\cdot q}{P\cdot k}$$ inelasticity

$$q = \frac{\hbar}{\lambda}$$

$$\frac{d\sigma^{lp\to eX}}{dx\, dQ^2} = \sum q(x, Q^2)\frac{d\hat{\sigma}^{lq\to lq}}{dQ^2}$$

Parton distribution functions

Partonic cross-section