

Chiral & trace anomalies in Deep **V**irtual **C**ompton **S**cattering

Shohini Bhattacharya

RIKEN BNL/BNL

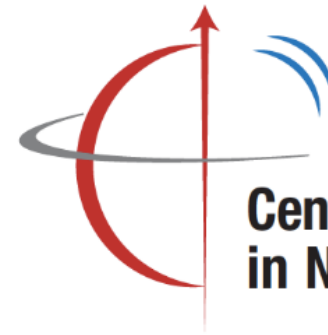
10 March 2023

In Collaboration with:

Yoshitaka Hatta (BNL)

Werner Vogelsang (Tubingen U.)

Based on: [arXiv:2210.13419](https://arxiv.org/abs/2210.13419)



**Center for Frontiers
in Nuclear Science**

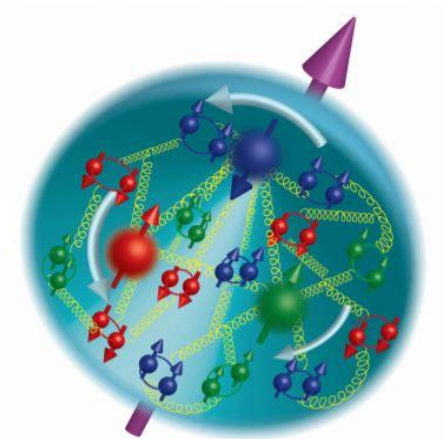


CFNS Monthly Postdoc Meetings



Quantum Chromodynamics (QCD)

- Interested in nucleons (**protons & neutrons**)
- Deep-inelastic electron-proton scattering (Friedman, Kendall, Taylor et al, 1968)
--- Protons are complicated dynamical systems of quarks and gluons (**partons**)
- Pressing question: How to understand nucleons from its constituents?



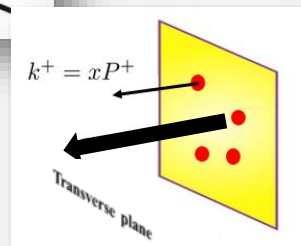
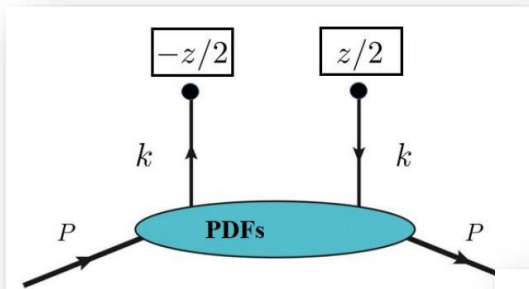
(EIC White Paper)



Quantum Chromodynamics (QCD): Non-perturbative functions

← **Parton motion**
← **Nucleon motion**

Snapshots of the nucleons



Parton **D**istribution **F**unctions

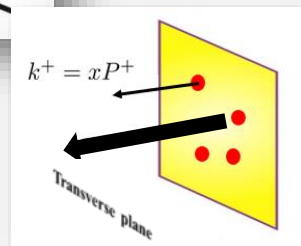
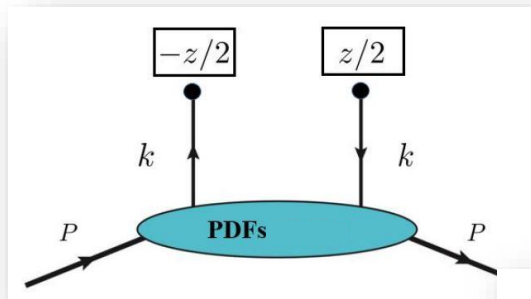
PDFs (x)



Quantum Chromodynamics (QCD): Non-perturbative functions

← **Parton motion**
← **Nucleon motion**

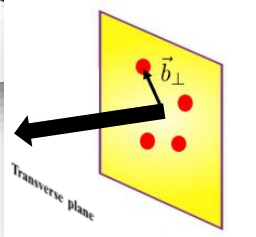
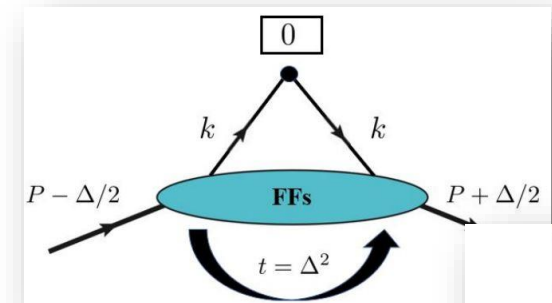
Snapshots of the nucleons



PDFs (x)

Form Factors

FFs (Δ)



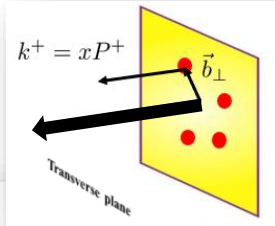


Quantum Chromodynamics (QCD): Non-perturbative functions

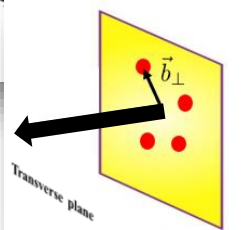
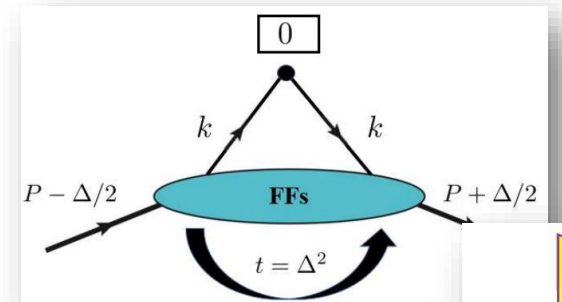
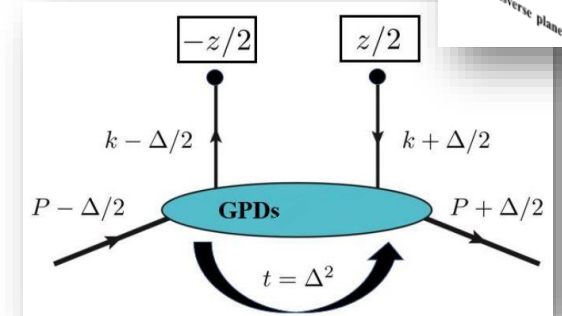
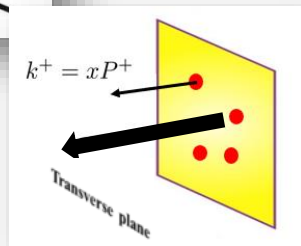
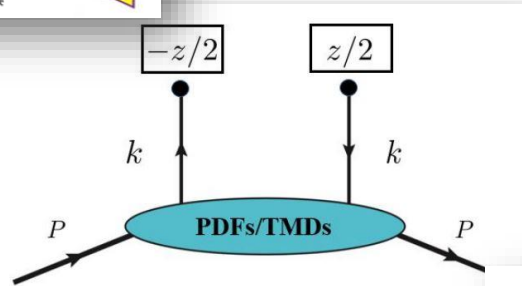
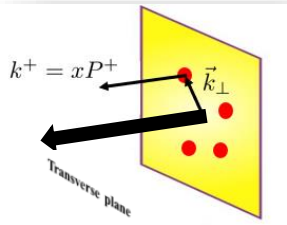
← **Parton motion**
← **Nucleon motion**

Snapshots of the nucleons

Generalized Parton Distributions



Transverse Momentum-dependent Distributions



TMDs (x, \vec{k}_\perp)

GPDs (x, Δ)

$\int d^2 \vec{k}_\perp$

$\Delta = 0$

$\int dx$

PDFs (x)

FFs (Δ)

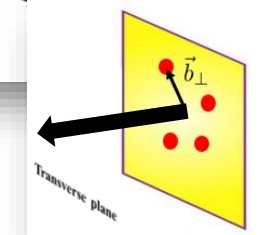
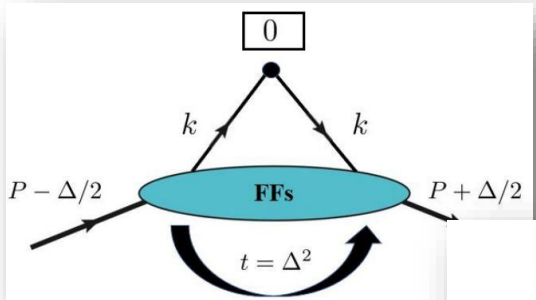
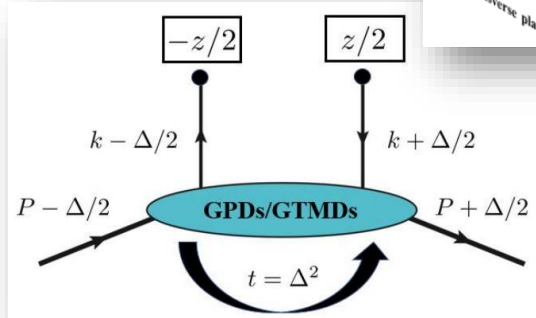
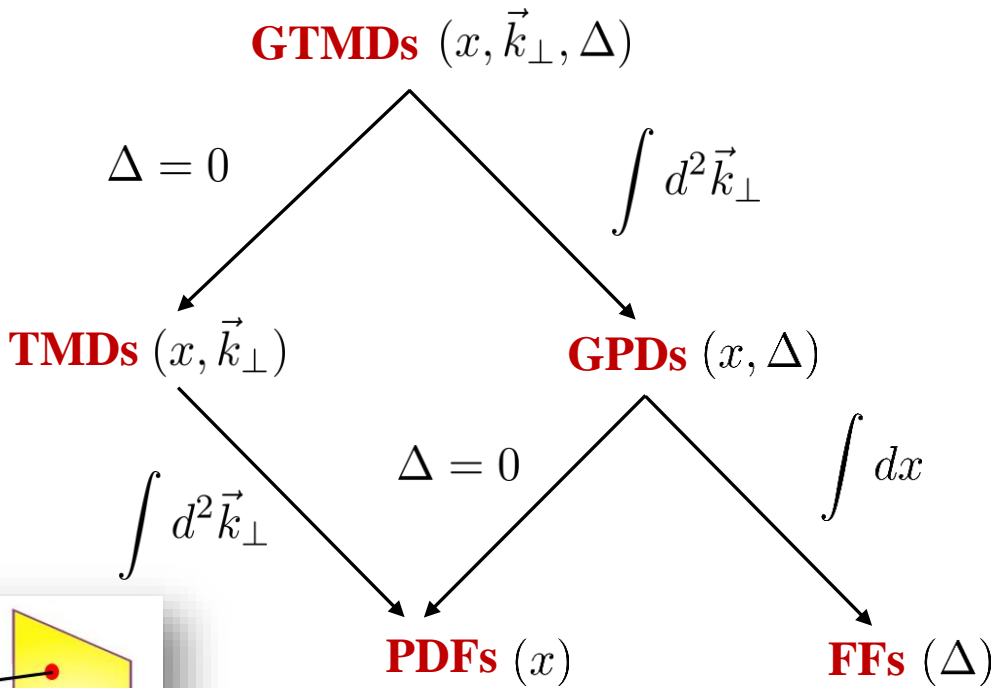
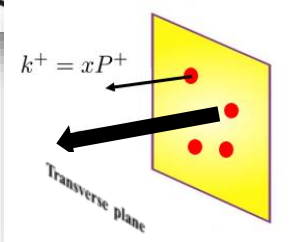
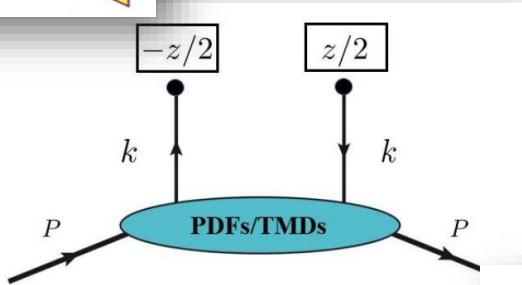
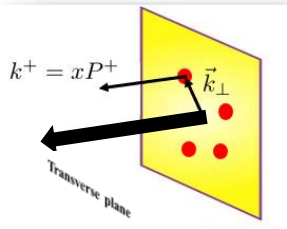


Quantum Chromodynamics (QCD): Non-perturbative functions

 **Parton motion**
 **Nucleon motion**

Snapshots of the nucleons

Generalized **T**ransverse **M**omentum-dependent **D**istributions





Quantum Chromodynamics (QCD): Non-perturbative functions

← **Parton motion**
← **Nucleon motion**

Snapshots of the nucleons

Generalized **T**ransverse **M**omentum-dependent **D**istributions

GTMDs $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$

Focus of this talk:

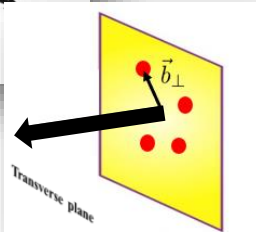
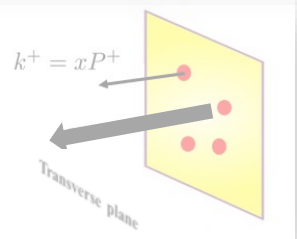
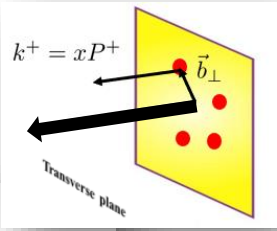
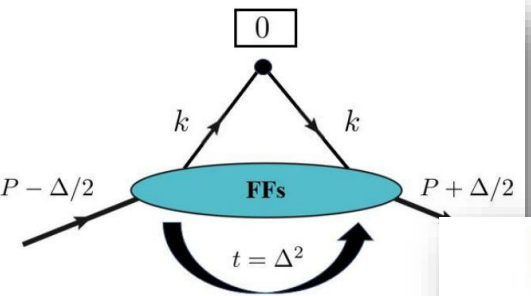
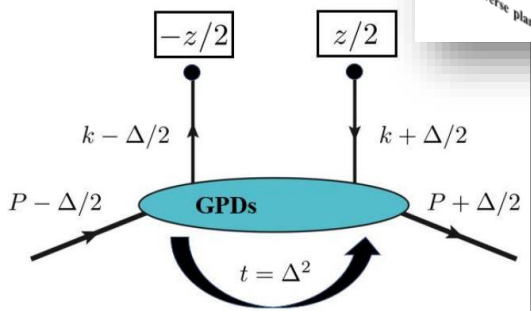
GPDs (x, Δ)

Manifestation of anomalies in
high-energy exclusive processes

PDFs (x)

FFs (Δ)

$\int dx$



Chiral anomaly



Recap on U(1) chiral anomaly in QCD:

- **Lagrangian invariant under global chiral rotation** $\psi \rightarrow e^{i\alpha\gamma_5}\psi$
- **Conserved charge** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

Chiral anomaly



Anomaly equation:

$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

A fundamental property of axial-vector current is the anomaly equation



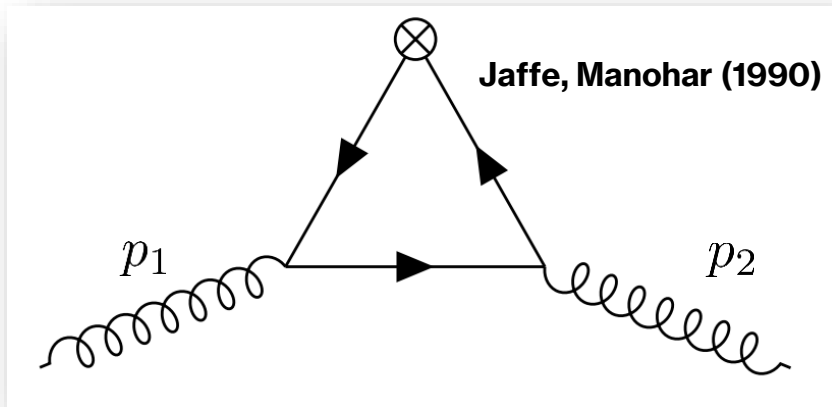
Chiral anomaly

Anomaly equation:

$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

A fundamental property of axial-vector current is the anomaly equation

A perturbative solution to anomaly equation:



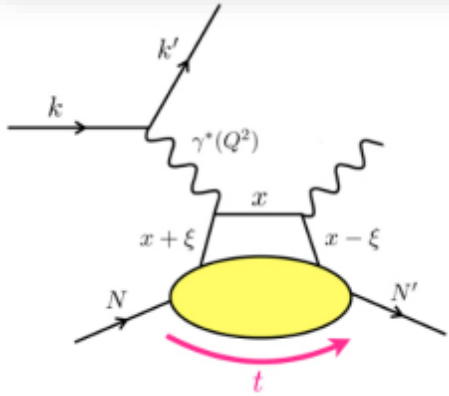
Calculation in off-forward kinematics ($l = p_2 - p_1$):

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}_{\alpha\beta}^a | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Imprint of Anomalies in QCD Compton scattering

Compton scattering:

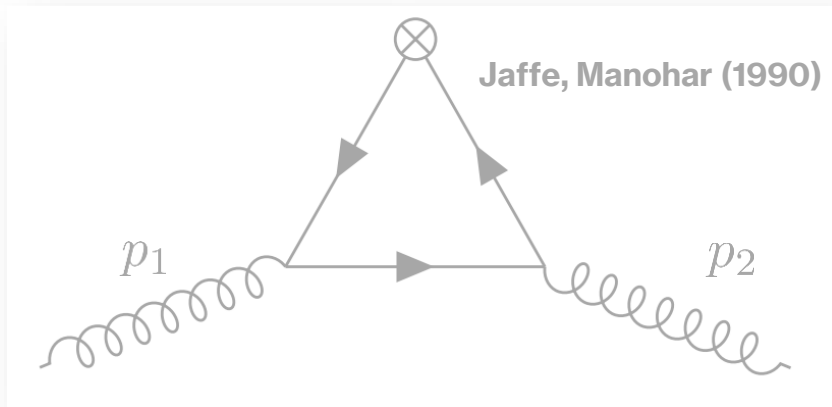


$$L = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

In QCD Compton scattering, box diagrams appear in perturbation theory at one-loop

Box diagram can be viewed as a non-local generalization of triangle diagram

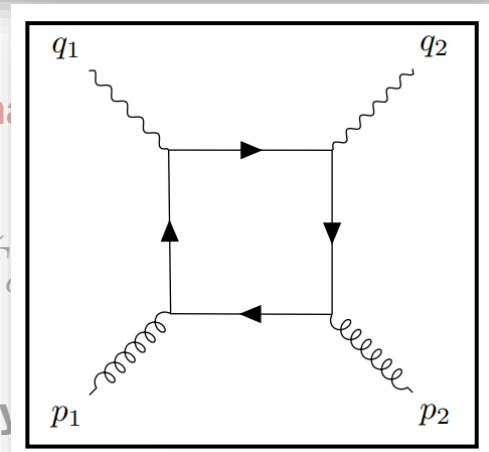
If triangle is dominated by anomaly pole, trace of that should be visible in box diagram



Calculation in off-forward kinematics

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}_a^{\gamma\delta} | p_1 \rangle$$

Triangle diagram is dominated by



Box diagram

Imprint of Anomalies in QCD Compton scattering

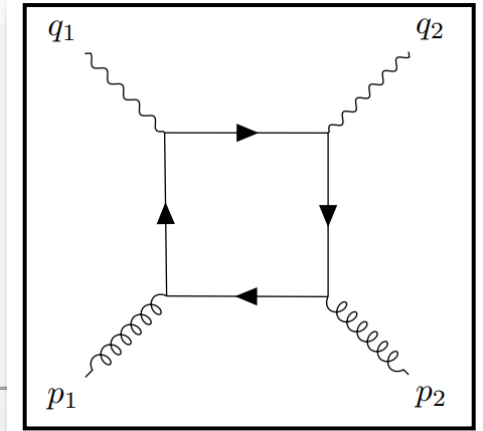
The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov^{1,2} and Raju Venugopalan³

$$O_\mu J_5^\mu = -\frac{1}{4\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

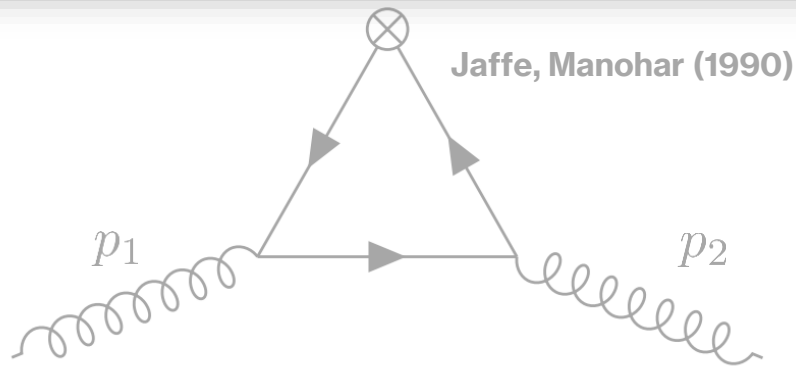
The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov^{1,2} and Raju Venugopalan³



Box diagram

Andrey and Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics



Calculation in off-forward kinematics ($l = p_2 - p_1$):

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}_{\alpha\beta}^a | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole



Imprint of Anomalies in QCD Compton scattering

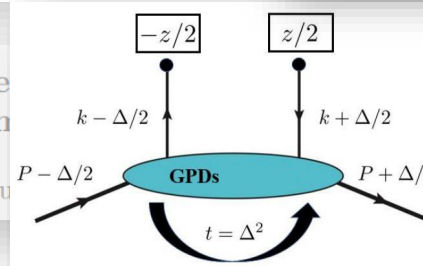
The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov^{1,2} and Raju Venugopalan³

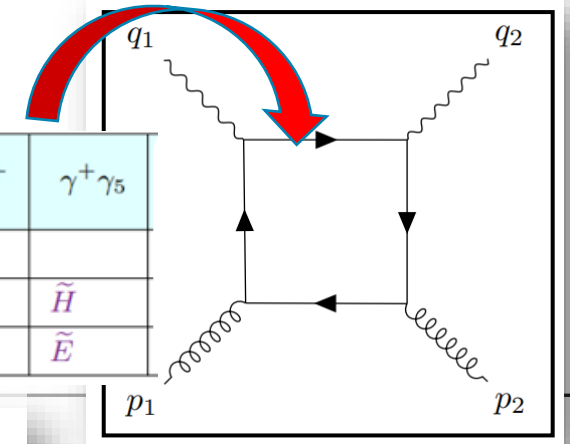
The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from Bjorken to Regge asymptotics

Andrey Tarasov^{1,2} and Raju Venugopalan³

Twist-2 GPDs



Γ	γ^+	$\gamma^+\gamma_5$
Pol.		
U	H	
L		\tilde{H}
T	E	\tilde{E}



Box diagram

Andrey and Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics



Jaffe, Manohar (1990)

Calculation in off-forward kinematics ($l = p_2 - p_1$):

arXiv: 2210.13419 (2022)

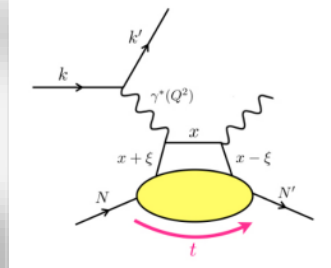
Chiral and trace anomalies in Deeply Virtual Compton Scattering

Shohini Bhattacharya,^{1,*} Yoshitaka Hatta,^{1,2,†} and Werner Vogelsang^{3,‡}

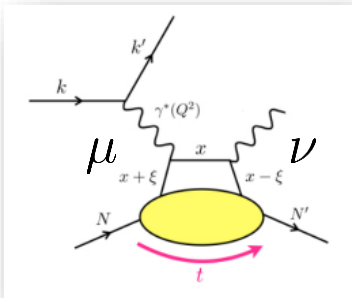
Perturbative Feynman-diagram approach:

Interpretation of results in terms of GPDs

Implication of anomaly poles on factorization of Compton scattering processes



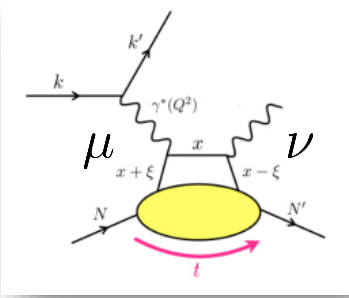
Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude ($\xi = 0$)

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}}$$

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude ($\xi = 0$)

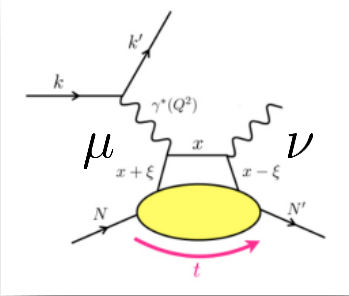
$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \bar{u}(P_2) \left[\left(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Expected terms:

Splitting function $\Delta P_{qg}(\hat{x}) = 2T_R(2\hat{x} - 1)$

Coefficient function $\delta C_g^{\text{off}}(\hat{x}) = 2T_R(2\hat{x} - 1) \left(\ln \frac{1}{\hat{x}(1 - \hat{x})} - 1 \right)$

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude ($\xi = 0$)

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \bar{u}(P_2) \left[\left(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Pole term

In agreement with Tarasov, Venugopalan

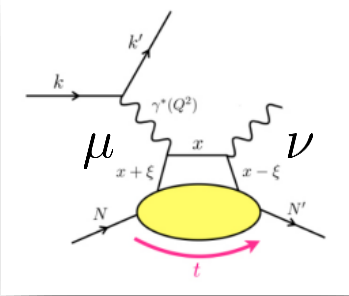
Coefficient function $\delta C_g^{\text{anom}}(\hat{x}) = 4T_R(1 - \hat{x})$

Twist-4 GPD:

$$\tilde{\mathcal{F}}(x, l^2) \equiv \frac{iP^+}{\bar{u}(P_2) \gamma_5 u(P_1)} \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \langle P_2 | F_a^{\mu\nu}(-z^-/2) \tilde{F}_{\mu\nu}^a(z^-/2) | P_1 \rangle$$

Chiral anomaly manifests itself in high energy amplitude

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \bar{u}(P_2) \left[\left(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Twist-4 GPD

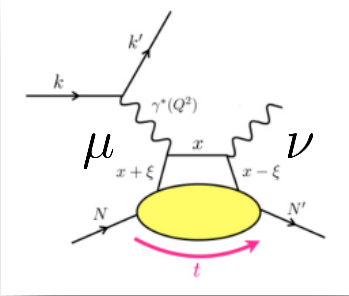
The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} = \frac{1}{2} \sum_f e_f^2 \bar{u}(P_2) \left[\gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2)) \right] u(P_1) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$$

↓
Twist-2 GPDs to all orders

Chiral anomaly manifests itself in high energy amplitude & possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \bar{u}(P_2) \left[\left(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 - \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Anomalous contribution to GPD \tilde{E} at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

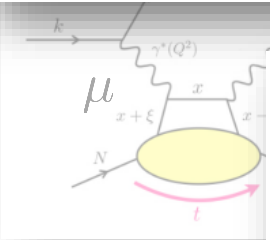
$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} = \frac{1}{2} \sum_f e_f^2 \bar{u}(P_2) \left[\gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2)) \right] u(P_1) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$$

Twist-2 GPDs to all orders

Chiral anomaly manifests itself in high energy amplitude & possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

Can we still justify factorization? part of Compton amplitude



Redefine

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \underset{\text{Tree level}}{\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2)} + \underset{\text{One loop}}{\frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)}$$

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Postulate that the perturbative pole cancels the pre-existing pole in “bare” GPD:

$$\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2) \approx -\frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2 = 0)$$

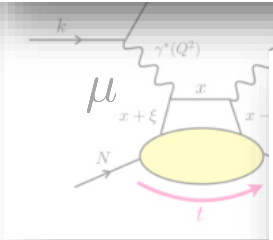
Postulate that the “renormalized” GPD integrates to $g_P(l^2)$:

$$g_P(l^2) = \sum_f \int_{-1}^1 dx \tilde{E}_f(x, \xi, l^2) = \sum_f \int_0^1 dx (\tilde{E}_f(x, \xi, l^2) + \tilde{E}_f(-x, \xi, l^2))$$



Imprint of Anomalies in QCD Compton scattering

Can we still justify factorization? part of Compton amplitude



Redefine

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \underset{\substack{\uparrow \\ \text{Tree level}}}{\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2)} + \underset{\substack{\uparrow \\ \text{One loop}}}{\frac{\alpha_s}{2\pi} \frac{2M}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)}$$

$$\gamma_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \Big] u(P_1)$$

\tilde{E} at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

Consistency:

$$T_{\mu\nu}^{\text{asym}} = \frac{1}{s} \sum_f e_f^2 \bar{u}(P_2) \left[\gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{s} (\tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2)) \right] u(P_1)$$

We find:

$$\frac{g_P(l^2)}{2M} = -\frac{i}{l^2} \left(\frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \Big|_{l^2=0} - \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \right)$$

Exactly the scenario mentioned by **Jaffe-Manohar & Tarasov-Venugopalan**

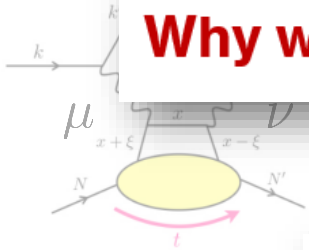
Conversion of massless pole to massive pole:

$$\frac{g_P(l^2)}{2M} \approx \frac{-2M \Delta \Sigma}{l^2 - m_{\eta'}^2}$$



Imprint of Anomalies in QCD Compton scattering

Why was the pole unnoticed in literature?



$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \text{Im} T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \bar{u}(P_2) \left[\left(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\text{off}} \right) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Pole was unnoticed in the GPD literature because one typically assumes

$$l^\mu = -2\xi p^\mu \rightarrow t = l^2 = 0$$

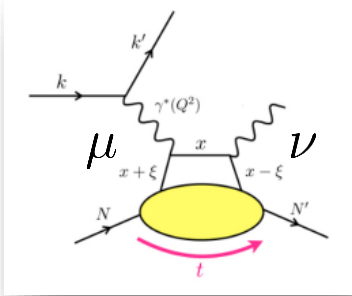
before loop integration

Usual rationale: Corrections supposedly higher twist $\frac{t}{Q^2}$

Twist-2 GPDs to all orders

Chiral anomaly manifests itself in high energy amplitude & possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

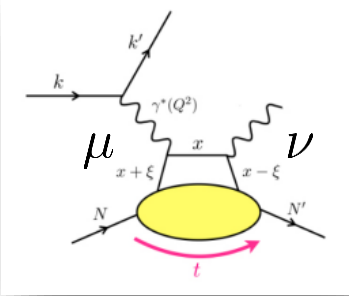


Symmetric part of Compton amplitude ($\xi \neq 0$)

Example:

$$\bar{F}_1^{\text{off}}(x_B, l)$$

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude ($\xi \neq 0$)

Example:

$$\bar{F}_1^{\text{off}}(x_B, l) \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \left[\left(P_{qg} \ln \frac{Q^2}{-l^2} + C_{1g}^{\text{off}} \right) \otimes g(x_B) + \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \frac{\bar{u}(P_2)u(P_1)}{2M} \right]$$

Pole! (New result)

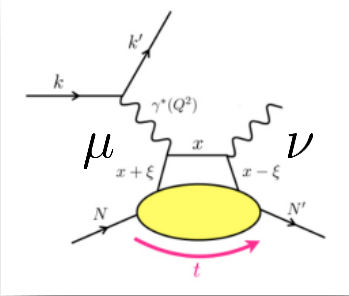
Twist-4 GPD:

$$\mathcal{F}(x, \xi, l^2) = -4xP^+M \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) F_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2)u(P_1)}$$

Chiral anomaly	Trace anomaly
Anomaly equation: $\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$	Anomaly equation: $\Theta_\mu^\mu = \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta} \quad \Theta^{\mu\nu} : \text{EMT}$
Form Factors: (g_A, g_P)	Gravitational Form Factors: (A, B, \bar{C}, D)

Trace anomaly manifests itself in high energy amplitude & possibly breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude ($\xi \neq 0$)

Example:

$$\bar{F}_1^{\text{off}}(x_B, l) \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left(\sum_f e_f^2 \right) \left[\left(P_{qg} \ln \frac{Q^2}{-l^2} + C_{1g}^{\text{off}} \right) \otimes g(x_B) + \frac{1}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B, \xi, l^2) \frac{\bar{u}(P_2)u(P_1)}{2M} \right]$$

Pole! (New result)

Anomalous contribution to GPDs H, E at one loop

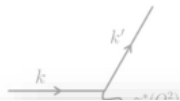
Twist-4 GPD:

$$\mathcal{F}(x, \xi, l^2) = -4xP^+M \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) F_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2)u(P_1)}$$

Trace anomaly manifests itself in high energy amplitude & possibly breaks QCD factorization



Imprint of Anomalies in QCD Compton scattering



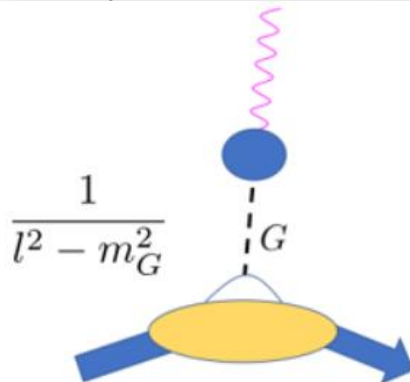
Symmetric part of Compton amplitude ($\xi \neq 0$)

Pole! (New result)

We proposed a possible scenario of pole cancellation in an attempt to rescue QCD factorization

Twist-4

$$\mathcal{F}(x, \xi, l^2) = -4xP^+M \int \frac{dz^-}{2\pi} e^{ixF} \dots$$



anomalous contribution to GPDs H, E at one loop

Trace anomaly manifests itself in high energy amplitude & glueball mass generations
possibly breaks QCD factorization



Summary

- **QCD factorization for Compton scattering: Backbone of the GPD programs at ongoing & future experiments including EIC**
- **Unexpected contributions originating from chiral & trace anomalies**

$$T^{\mu\nu} \sim \frac{\langle F^{\alpha\beta} \tilde{F}_{\alpha\beta} \rangle}{l^2}, \quad \frac{\langle F^{\alpha\beta} F_{\alpha\beta} \rangle}{l^2}$$

Unnoticed in literature, possible violation of factorization



Summary

Perturbative calculations suggest that massless poles are induced in GPDs \tilde{E} , H , E

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

- Unexpected contributions originating from chiral & trace anomalies

$$T^{\mu\nu} \sim \frac{\langle F^{\alpha\beta} \tilde{F}_{\alpha\beta} \rangle}{l^2}, \quad \frac{\langle F^{\alpha\beta} F_{\alpha\beta} \rangle}{l^2}$$

Unnoticed in literature, possible violation of factorization



Summary

Perturbative calculations suggest that massless poles are induced in GPDs \tilde{E} , H , E

However, we know there are no massless poles in axial and gravitational form factors (moments of GPDs)

We proposed a possible scenario of **pole cancellation**

This has to do with eta-meson & glueball mass generations

Antisymmetric case:

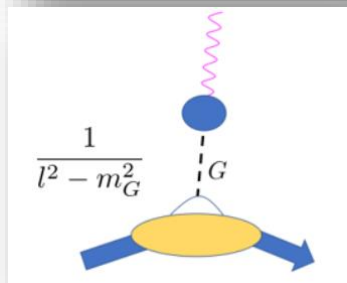
Unnoticed

cf, the η' mass problem

$$\frac{1}{l^2 - m_{\eta'}^2} \eta'$$

$$g_P \sim \int dx \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

Symmetric case:





Summary & outlook

Perturbative calculations suggest that massless poles are induced in GPDs \tilde{E} , H , E

However, we know

Novel connections between DVCS & chiral/trace anomalies:
This could be a new & potentially rich avenue for GPD research

We propose

Higher-loop calculations

pole cancellation

trace anomalies

This has to do with eta-meson & glueball mass generations

Imprint of anomaly on other physical processes:

(Example: Deeply-virtual meson production)

Antisymmetric

Unnoticed

$$g_P \sim \int dx \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

Symmetric case:

$$\frac{1}{l^2 - m_{\eta}^2}$$

