



**U.S. MAGNET
DEVELOPMENT
PROGRAM**

Efforts on HTS Modeling and Code Development: Progress, Challenges, Opportunities

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Outline



- **Introduction**
 - Theory Recap
 - Motivation
 - Project Goals
- **Last year's progress**
 - Solid Modeling (2D)
 - Thin Shell Modeling (2D)
 - Material Database
- **Current work in progress**
 - Thermal Coupling
 - Extension to 3D
- **Summary and Outlook**



Why is HTS modeling so difficult?

- **highly nonlinear material laws**

- power law for resistivity
- critical current and depends on magnetic field and temperature
- so does the thermal conductivity

→ lots of iterations needed for every time step
→ FEM generates ill-conditioned, non positive definite and non-symmetric matrices
(=very difficult to solve)

“if solving nonlinear equations is like downhill skiing, solving nonlinear equations for HTS modeling is like skiing the double black diamond trail without poles”

Ohm's Law

$$e = \rho j$$

$$\rho = \frac{e_c}{j_c} \left(\frac{|j|}{j_c} \right)^{n-1}$$

$$j_c = f(|B|, \angle B, T)$$

Fourier's Law

$$\dot{q} = -k \nabla T$$

$$k = f(T, |B|)$$

$$c_p = f(T)$$





Mixed Formulations: a Quick Recap

- gaining significant interest over the last five years!

- **goal:**

- improve conditioning for system matrix
- reduce degrees of freedom
 - faster computation

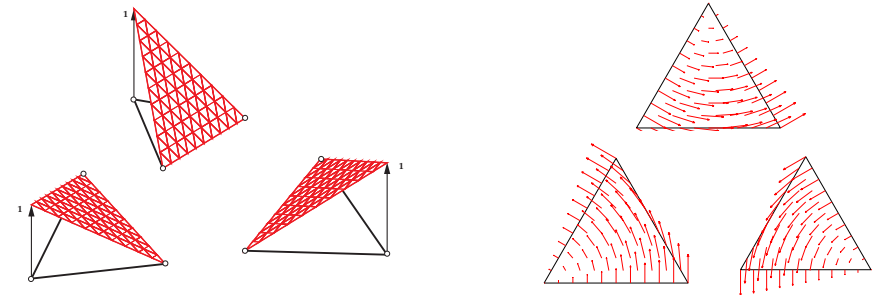
- **basic idea**

- governing equations are domain specific
- domains are connected over interfaces conditions

- **challenges**

- user-friendliness
- mesh generation and memory management
- iterating over non-linear material models
- matrices are difficult to solve
 - (still ill conditioned, non symmetric, non-positive definite)

	Air / Vacuum	Conductor	Ferromagnetic Alloy
Governing Equation	$\nabla \times \mathbf{h} = \mathbf{0}$ Ampère-Maxwell	$\nabla \times \mathbf{h} = \mathbf{j}$ Ampère-Maxwell	$\nabla \times \mathbf{e} = \dot{\mathbf{b}}$ Faraday's Law
Degree of Freedom	$\mathbf{h} = -\nabla \phi$ Magnetic Scalar Potential	\mathbf{h} Magnetic Field	$\mathbf{b} = \nabla \times \mathbf{a}$ Magnetic Vector Potential
Transport Law	none	$\mathbf{e} = \rho \cdot \mathbf{j}$ Ohm's Law	$\mathbf{h} = \mathbf{v} \cdot \mathbf{b}$ Magnetic Law
Comment	minimal number of dofs	need edge elements	simple material law implementation



*Lagrange
Elements*

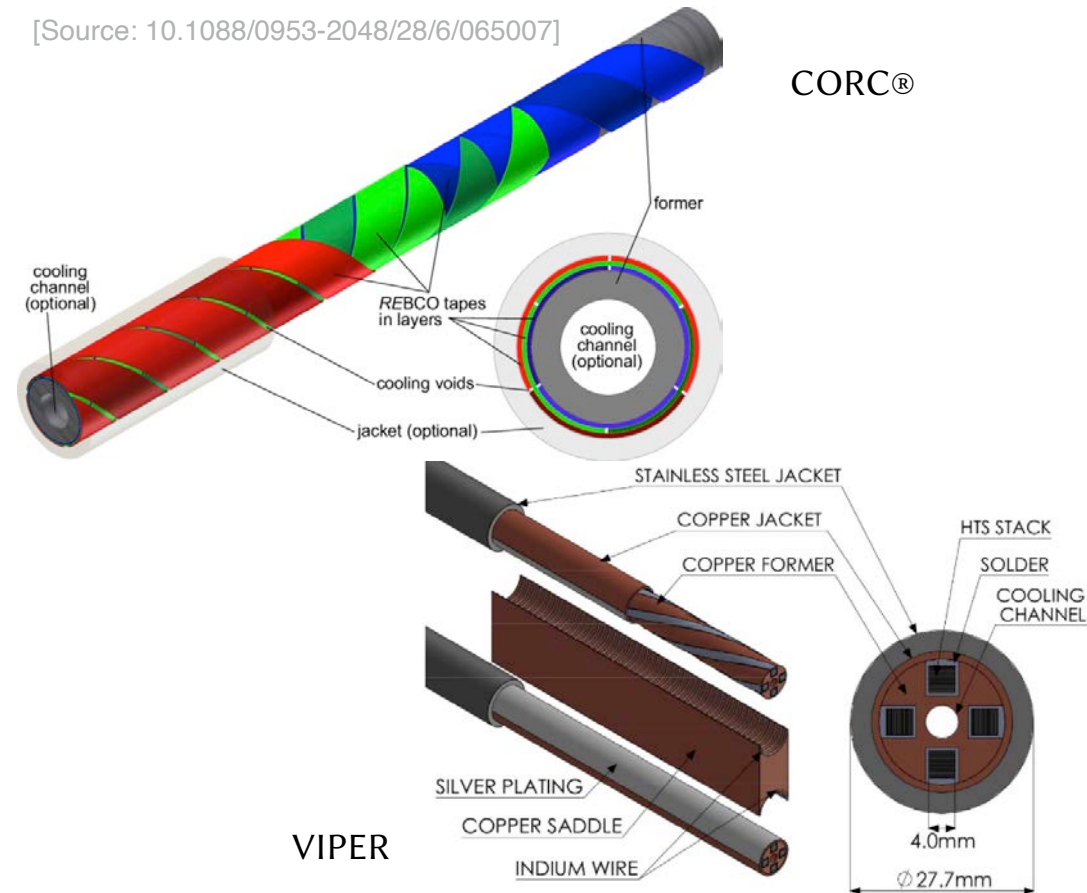
*Nédélec
Elements*



BELFEM: Motivation and Project Goals

- **have a custom C++ codebase that**
 - can predict quench behavior of HTS
 - coupled electromagnetic-thermal simulation
 - is sufficiently fast
 - uses state of the art h - ϕ formulation
 - uses state of the art solver libraries
 - uses state of the art parallelization
 - supports complex geometries and current sharing
 - uses state of the art thin shell models
 - need low level access to data structure
 - can handle highly nonlinear material properties
 - custom database
 - using 3D-B-splines to allow smooth derivatives

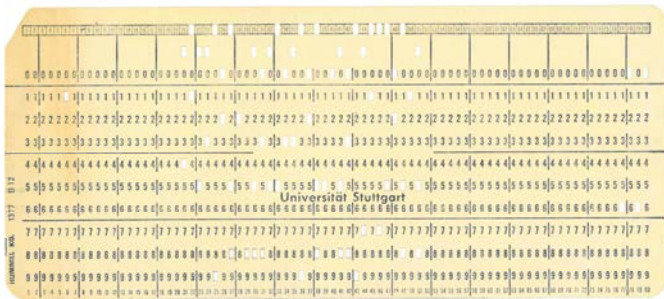
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Codebase Requirements & Philosophy



- **Flexibility & Maintainability**

- consistent naming scheme of functions and classes
- lots of comments in code!
- modular structure

- **simple MATLAB-like dense linear algebra**

- through ARMADILLO or BLAZE

- **text based interface tailored to magnet development**

- ability to write scripts in BASH and Python

- **utilization of community software**

- use open source data formats (GMSH, HDF5, Exodus II)
- link against community libraries: MUMPS, PETSc, STRUMPACK, ...

- **be open source once mature**

- Berkeley Lab specific BSD-3 like license

Last Year's Progress



Solid Formulation Implemented & Validated

1. Element Interpolation

Figure 1.1: Shape function of the order \$k\$ (linear, quadratic, cubic)

Figure 1.2: Cubic Operator

Figure 1.3: 1983 Interface Element for \$h-\Phi\$ coupling

2. Interpolation

Using linear distributions, the mass Matrix reads

The stiffness matrix only exists for the case of \$h\$-coupling

3.2.3 \$h-\Phi\$ Domain Interface

Figure 1.3: 1983 Interface Element for \$h-\Phi\$ coupling

3. Interpolation

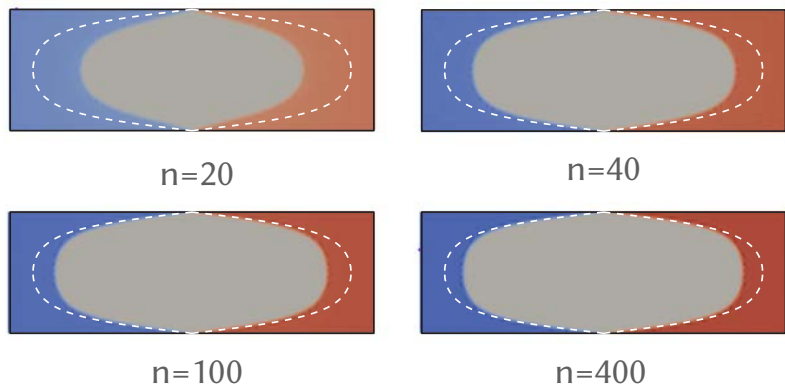
with a pointing node axis from the ferromagnetic domain

3.3.3 \$h-a\$ Domain Interface

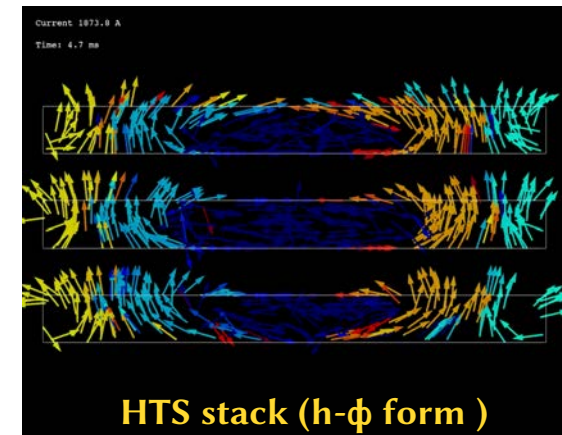
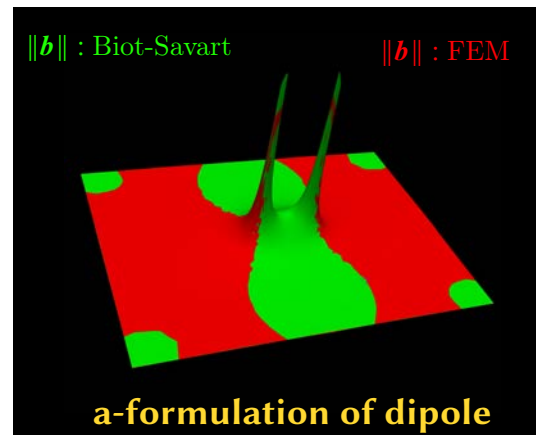
3.3.3.1 Cuts

Figure 1.5: Making the domain simply connected, after Thaler

- **Mixed Formulation (solids)**
 - detailed derivation of h-Φ and h-a forms and element matrices
 - description and improvement of domain interfaces
 - proof of concept implementation in 2D
 - validation against Biot-Savart + Brandt
- **Invited Talk presented at HTS 2022 in Nancy, France**



current distribution validation against Brandt



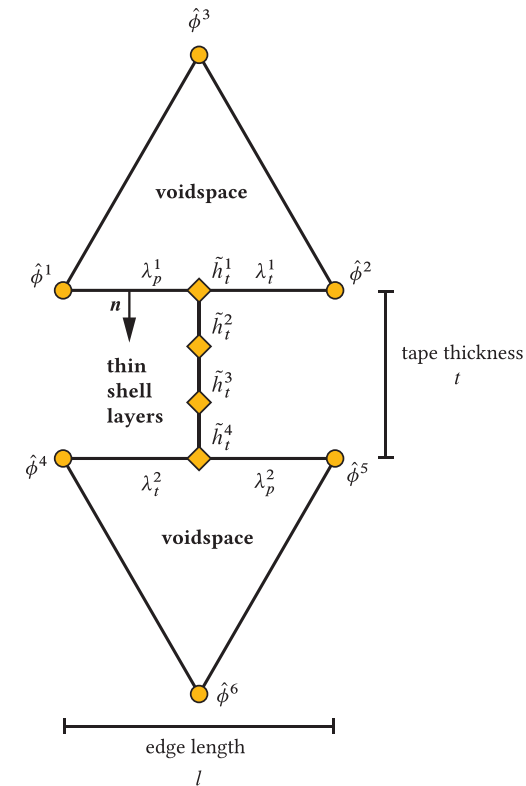
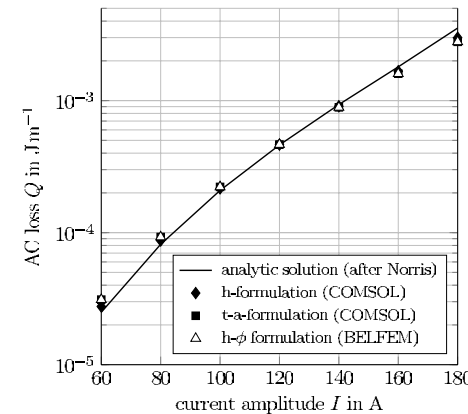
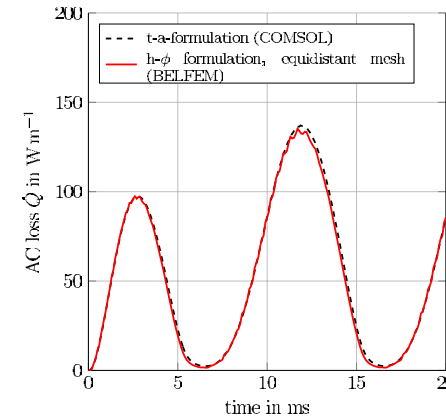
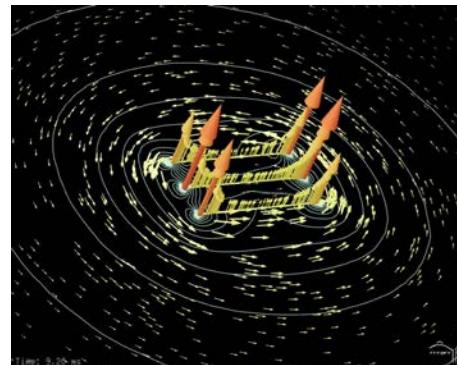
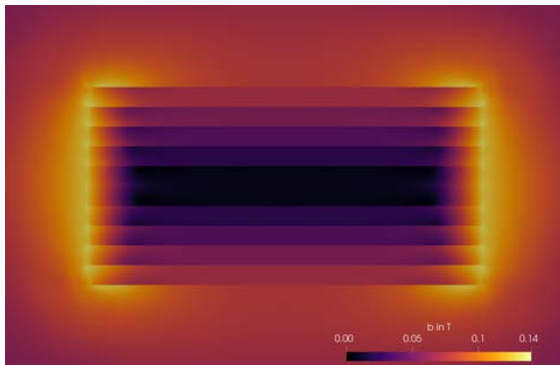
Thin Shell Formulation Implemented & Validated

- **Mixed Formulation (thin shells)**

- implementation of Alves-Element in 2D
- validation against analytical methods + COMSOL / GetDP

- **Invited Paper for SuST 2023 (submitted)**

- Christian Messe, Berkeley Lab
- Nico Riva, MIT
- Sofia Viarengo, Politecnico di Torino

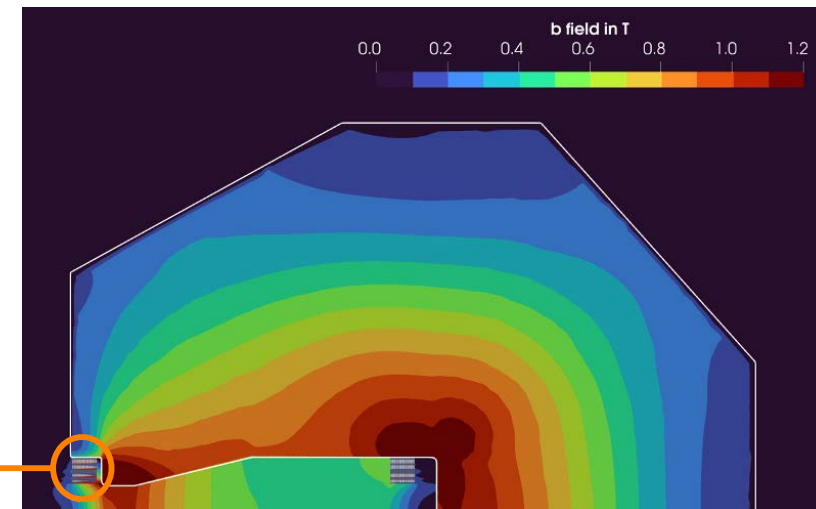
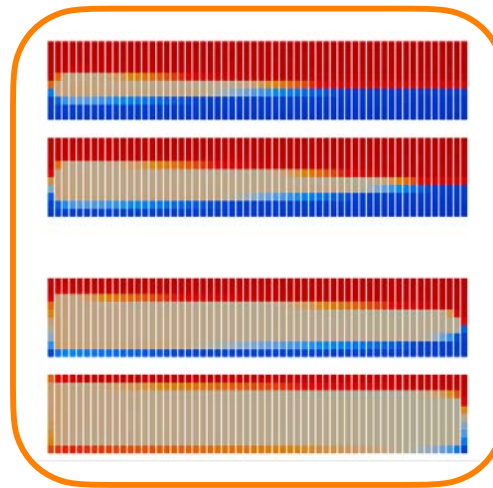
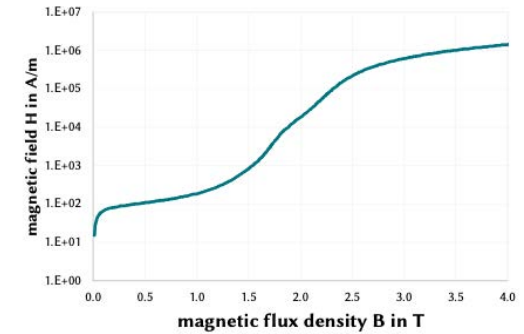
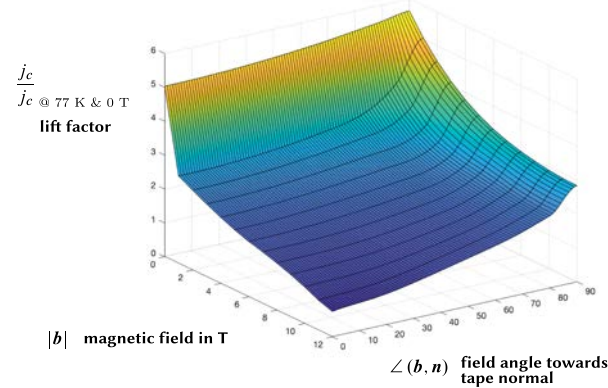




Last Year's progress: Application to Gantry Magnet

• Gantry Magnet Field Quantity Simulation

- proof of concept for complicated structure (254 individual tapes of BSCCO 2223)
- highly nonlinear material properties
- poster presentation at ASC 2022, HI

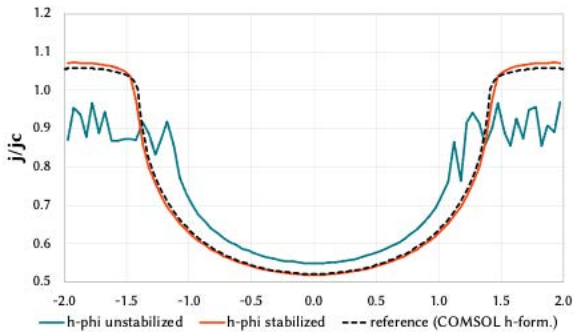




Misadventures: Application to Gantry Magnet

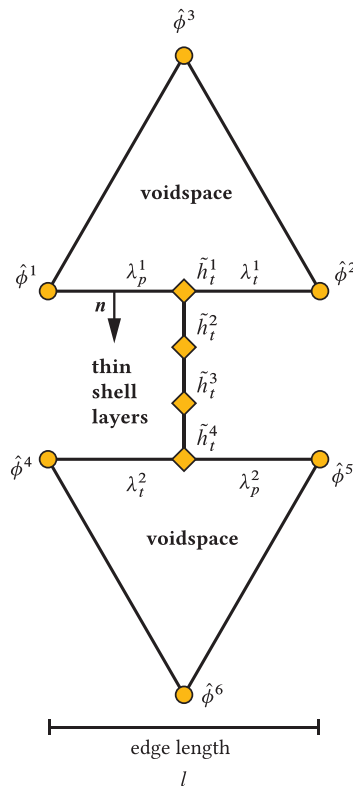
Challenges

- a- ϕ interface unstable for highly nonlinear ferromagnet under high magnetic fields
→ current subject of research
- second order implementation oscillates
→ considering research project in cooperation with CU Boulder

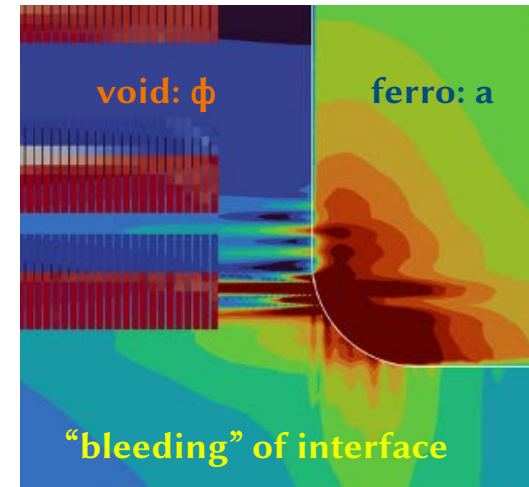
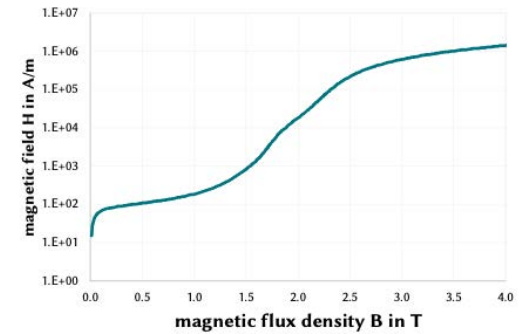
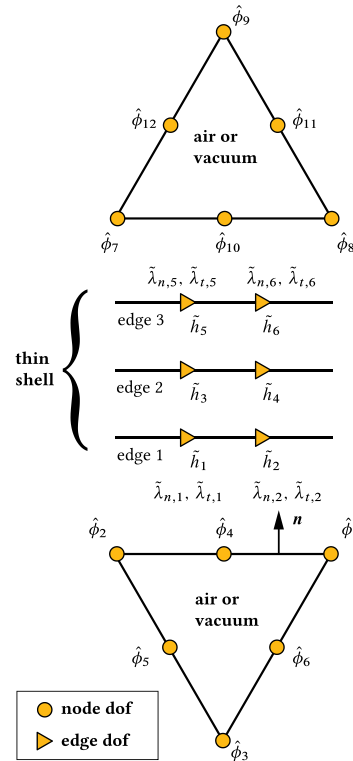


oscillation with 2nd order model

1st order model



2nd order model



Last Year's progress: Material Database

• Material Database

- can provide properties depending on up to three parameters
e.g. ρ , j_c , c_p , k , E , G , α ...
currently free: Copper, Silver, Hastelloy, ...
- values precomputed in lookup tables
→ very fast evaluation
- utilizes B-Splines with up to third order (smooth derivatives!)
→ very suitable for Newton-Raphson and Picard Maneuver
- stand alone database for use in other codes
→ uses HDF5 as data format

• Coupled against SparseLizard

- results presented at ASC 2022, HI
- results published in SuST 2022, Halbach et al.
DOI: 10.1109/TASC.2023.3240389

• Consider publishing under open source license

- interest in the community?

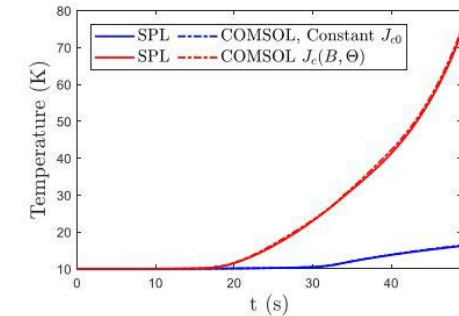
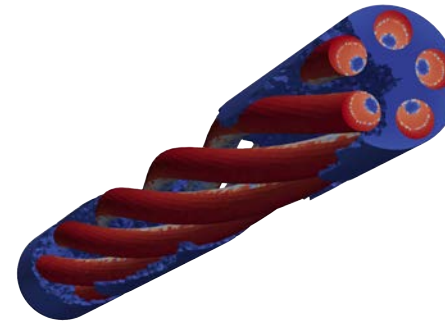
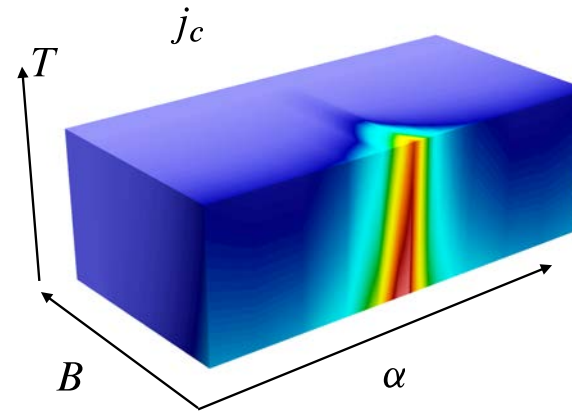


Fig. 5

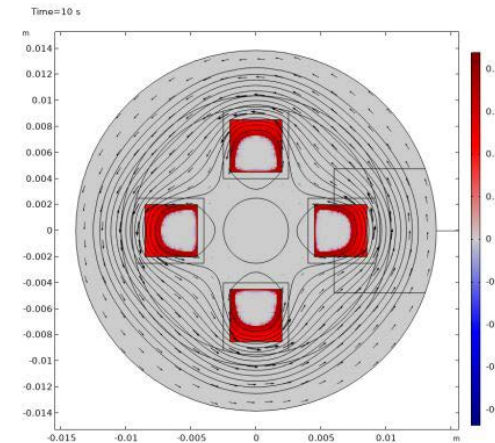


Fig. 6



Last Year's progress: Theory Refinement

• Visiting Professor Frédéric Sirois, Polytechnique Montreal

- review of current state of the art
- discussion of domain interfaces, quench simulation and current sharing
- frequent meetings with former PhD Students Bruno Alves and Alexandre Arsenault
- lots and lots of hours in front of the whiteboard!
- organized a five months visit of PhD student Gregory Giard



$\nabla \cdot \vec{E} = 4\pi\rho$
 DIVERGENCE OF \vec{E} CHARGE DENSITY
 \vec{E} DIVERGES OUT FROM POSITIVE CHARGES AND IN TOWARD NEGATIVE CHARGES. THE TOTAL FLUX OF \vec{E} THROUGH ANY CLOSED SURFACE IS PROPORTIONAL TO THE CHARGE INSIDE.

$\nabla \times \vec{E} = -\frac{1}{c} \frac{d\vec{B}}{dt}$
 CURL OF \vec{E} SPEED OF LIGHT RATE \vec{B} IS CHANGING
 \vec{E} CURLS AROUND CHANGING \vec{B} FIELDS (FARADAY'S LAW) IN A DIRECTION THAT WOULD MAKE A CURRENT THAT WOULD PRODUCE A \vec{B} FIELD TO OPPOSE THE CHANGE IN \vec{E} FLUX (LENZ'S LAW).

$\nabla \cdot \vec{B} = 0$
 DIVERGENCE OF \vec{B}
 \vec{B} NEVER DIVERGES, IT JUST LOOPS AROUND ON ITSELF.

$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{d\vec{E}}{dt}$
 CURL OF \vec{B} SPEED OF LIGHT CURRENT DENSITY RATE \vec{E} IS CHANGING
 \vec{B} CURLS AROUND CURRENTS AND CHANGES IN \vec{E} FIELDS

Current Work in Progress



Work in Progress: Thermal Coupling

- **Extension of Alves-Element with Thermal Model**

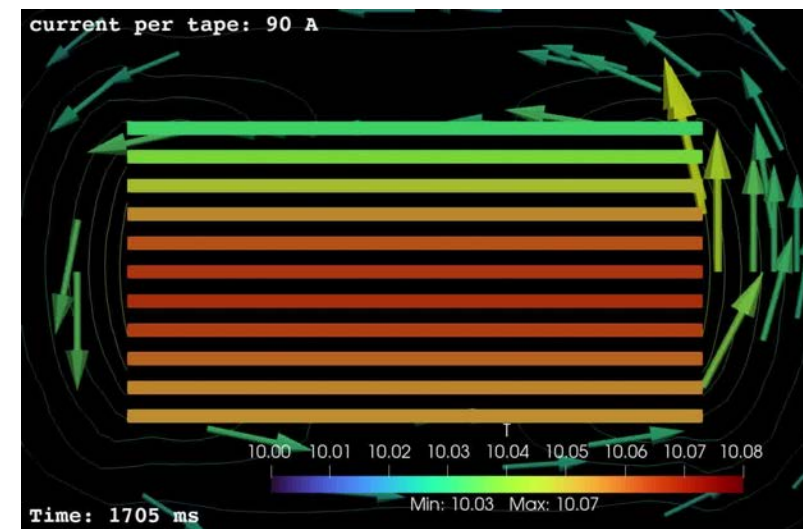
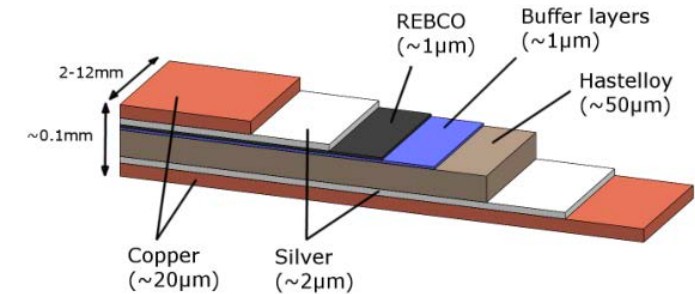
- each layer is modeled individually using lumped masses
 - no mixture rule required
 - very stable convergence
- implicit coupling of physical fields
 - using STRUMPACK to solve quasi-magneto-static problem
 - using PETSc (GMRES) to solve thermal problem

- **Gregory Giard (visiting PhD student, Politechnique Montreal)**

- improvement of domain interfaces
- adaptive relaxation method
- extension of thermal model to 3D

- **Erik Schnaubelt (PhD student at CERN / TU Darmstadt)**

- defining benchmark problems + validation against COMSOL

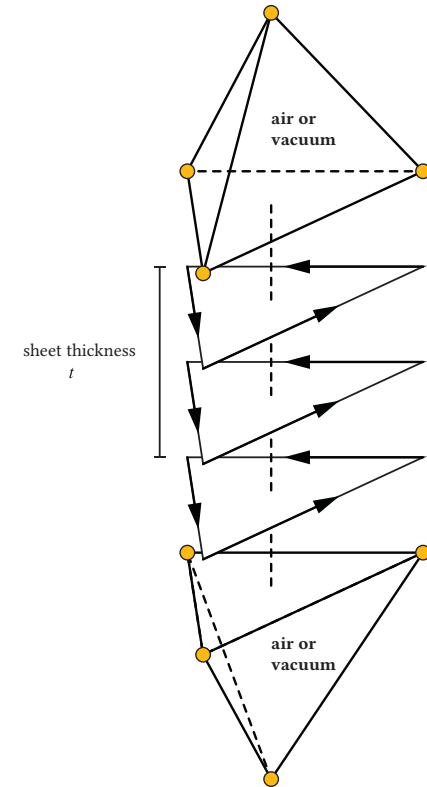
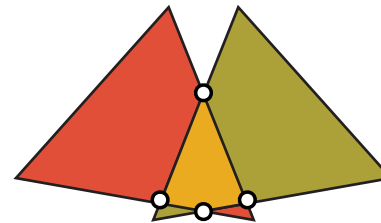
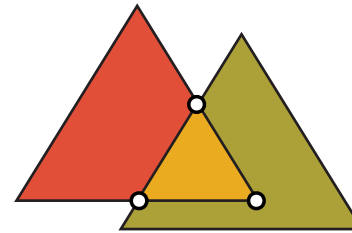
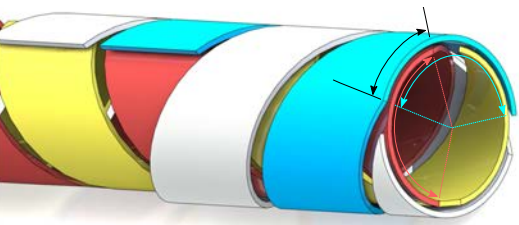




Work in Progress: Extension to 3D

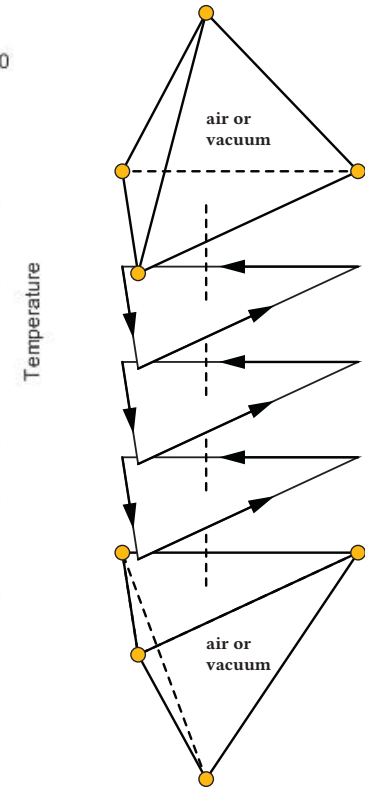
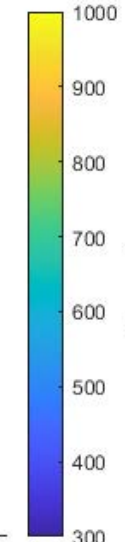
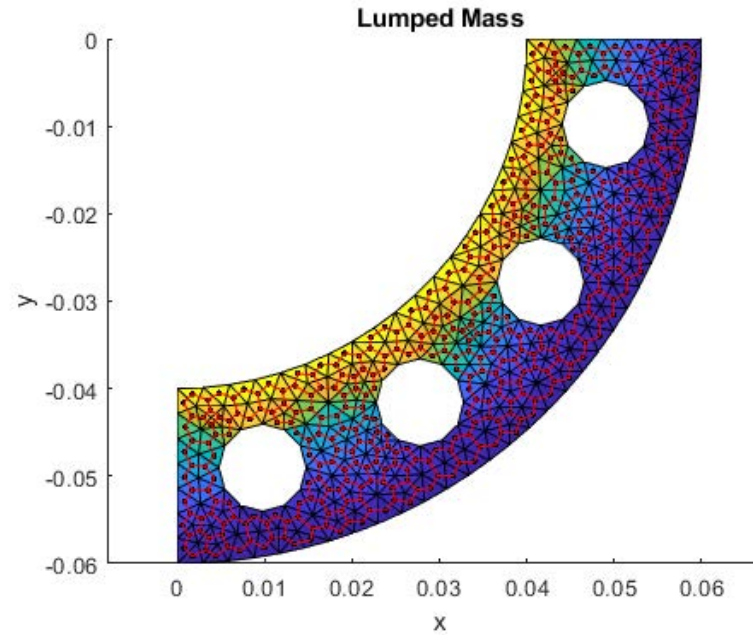
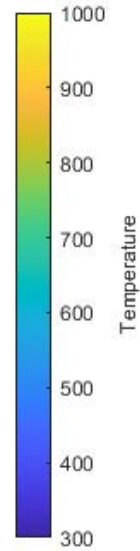
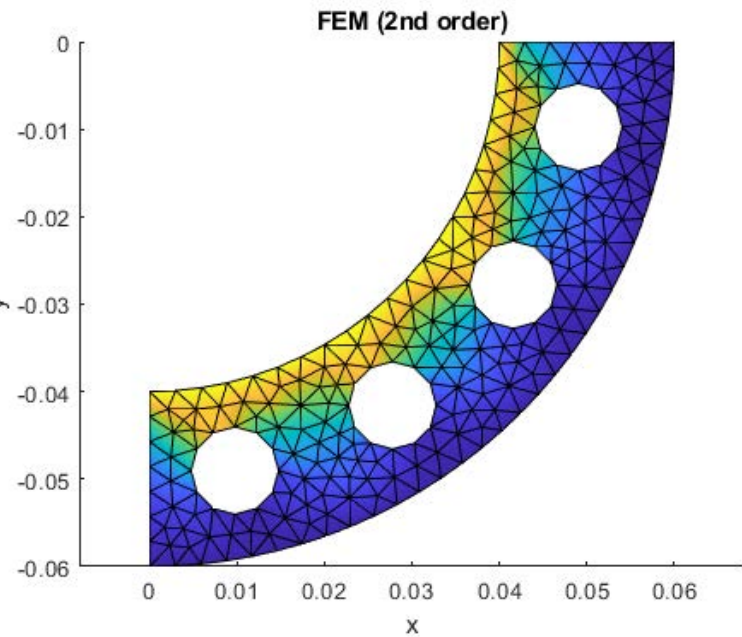
Goal: Thermal-Magnetostatic Modeling of CORC cables

- proof of concept already demonstrated by Bruno Alves
- implementation of magnetic model in BELFEM
- implementation of lumped mass model for 3D
- improvement of geometry engine
- first thoughts about current sharing
- definition of benchmarks





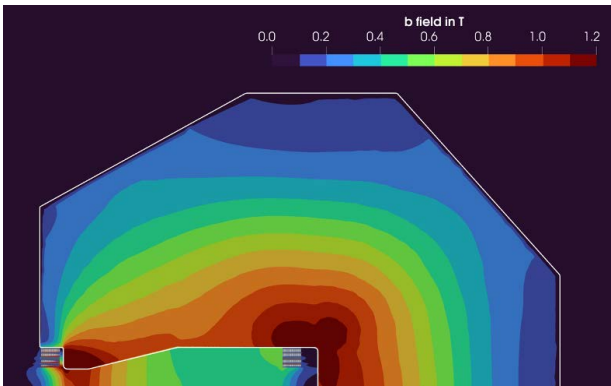
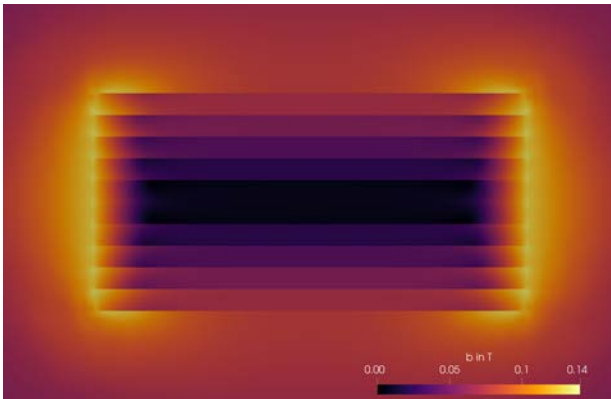
Benchmark: Lumped Mass Model for Triangles



• benchmark

- arbitrary geometry with strong gradients
- goal: prove that lumped mass model works for triangles
- result: very promising!

Summary and Outlook



- developing a finite-element framework tailored to HTS cable & magnet development needs
- support modern mixed formulations such as h-a and h- ϕ
- support thin shells & multi-physics (work in progress)
- code designed to run in parallel on HPC node
- writing textbook-like theory manual
- working on a stand alone material database



during the next months:

- improve domain interfaces
- complete validation for thin shell implementation for and $h-\phi$
- complete work on TS-quenching

during this year:

- finalize implementation of 3D model (together with Gregory Girard)
- work on geometry-preprocessor for 3D current sharing
- benchmark against simple experiments
- further improvements and code hardening
- streamline workflow for real world applications

future plans:

- work on cryogenic fluid database for cooling
- second order thin shells (PhD thesis?)
- publish code under BSD-3-like license