Overview of WG
Small-x evolution + NLO calculations

Zhongbo Kang
UCLA

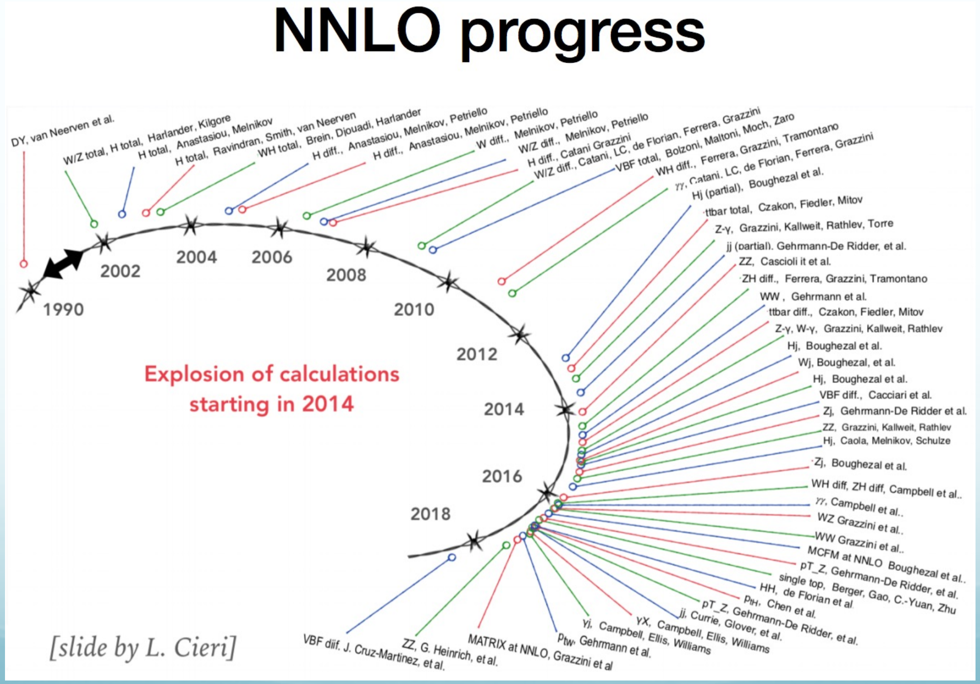
@ZhongboK

Working Group Members

- Faculty
  - Ian Balitsky (ODU & JLab)
  - Simon Caron-Huot (McGill)
  - Yuri Kovchegov (Ohio State)
  - Alex Kovner (Connecticut)
  - Jamal Jalilian-Marian (Baruch College)
  - Vladi Skokov (North Carolina)
  - Anna Stasto (Penn State)
  - Raju Venugopalan (BNL)
  - Yoshitaka Hatta (BNL)
  - Daniel Pitonyak (Lebanon Valley)
  - Matthew Sievert (New Mexico)
- Postdoc and students
  - Farid Salazar (LBL/UCLA)
  - …
  - Please write to me zkang@ucla.edu to include your names

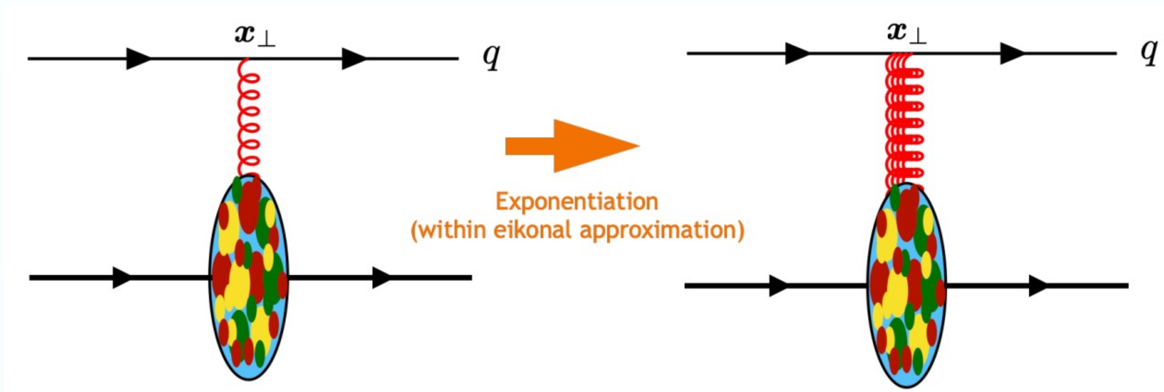
Why?

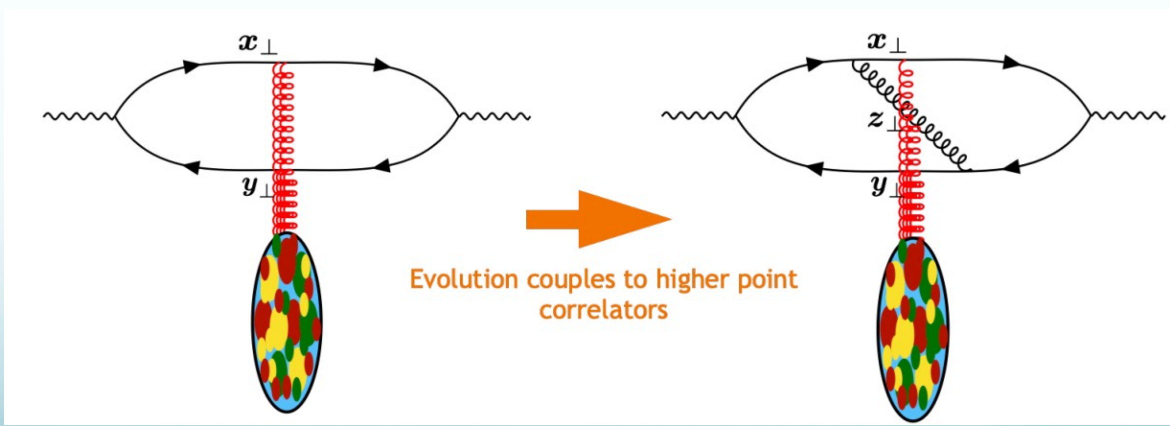
- In comparison with conventional collinear pQCD computations
  - NNLO and beyond
- Higher-order computations in the small-x regime is much more challenging
  - NLO becomes state of the art only in the last couple of years
- This also means a lot of new opportunities



Complexity

- In the small-x regime, one has to simultaneously include
  - Coherent multiple scattering to all order (*not existing in the usual collinear factorization nor in the more advanced TMD factorization*)
  - Radiative corrections order by order





Courtesy of Farid Salazar

Challenges

- NLO computations in the small-x regime is extremely complex
- Numerical implementation is also quite complicated
- One would like to have interpolation between small x and moderate x to describe the data in the full kinematic region

Main goals of WG

- Perform NLO calculations at small x
- Include corrections that allow computations at moderate x
- Develop NLO+NLL numerical packages for phenomenology

Milestones of WG

- Year 1: Skokov, Stasto, Venugopalan
  - Assess and formulate numerical implementation of dipole and quadrupole correlators NLLx BK and JIMWLK
- Year 2-3: Kang, Venugopalan, Jallian-Marian, Kovner
  - Perform NLO computations for dihadron production in e+A and assess its status in p+A and formulate strategy for small-x evolution and resummation at NLLx and beyond
  - Calculate corrections to small x observables such as single inclusive particle production in DIS to extend saturation studies to high Q^2 and mid rapidity
- Year 3: Stasto, Venugopalan, Skokov, Kovchegov, Pitonyak, Sievert
  - Numerical implementation of NLLx BK

Milestones of WG

- Year 4: Jalilian-Marian, Kovner
  - Based on the Born Oppenheimer approximation and other perturbative techniques, develop a consistent unified evolution NLO framework applicable to small as well as moderate values of x.
  - Study saturation in single inclusive particle production in DIS on nuclei from very forward (dominated by small x gluons) to mid rapidity at EIC (dominated by moderate x gluons)
- Year 4-5: Kang, Venugopalan
  - Study and implement dijet and dihadron production at NLO in p+A collisions
- Year 4-5: Hatta
  - Calculate single and double spin asymmetries in dijet production at small-x to NLO including the quark and gluon OAM contributions and make predictions for the EIC

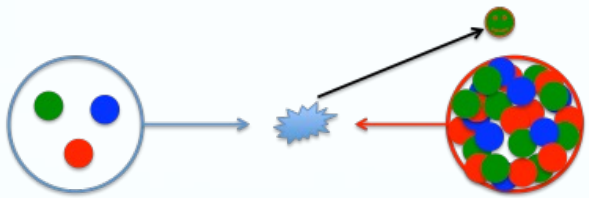
Current status

- Arguably, NLO computations in the color-glass condensate / gluon saturation regime is one of the most active research directions in the field and a lot of progress has been made

- "Single inclusive hadron production in p+A collisions at the NLO" by Liu, Kang, Liu [3] and by Shi, Wang, Wei, Xiao [4]
- "Single inclusive jet production in p+A collisions at the NLO" by Liu, Xie, Kang, Liu [5] and by Wang, Chen, Gao, Shi, Wei, Xiao [6]
- "Dijet photoproduction in γ+A collisions at the NLO" by Taels, Altinoluk, Beuf, Marquet [7]
- "Dijet impact factor in e+A collisions at the NLO" by Caucal, Salazar, Venugopalan [8–10]
- "Single inclusive hadron production in e+A collisions at the NLO" by Bergabo, Jalilian-Marian [11]
- "Dihadron production in e+A collisions at the NLO" by Bergabo, Jalilian-Marian [12, 13] and by Iancu, Mulian [14]
- "Diffractive dihadron production in γ^*+A collisions at the NLO" by Fucilla, Grabovsky, Li, Szymanowski, Wallon [15]
- ...

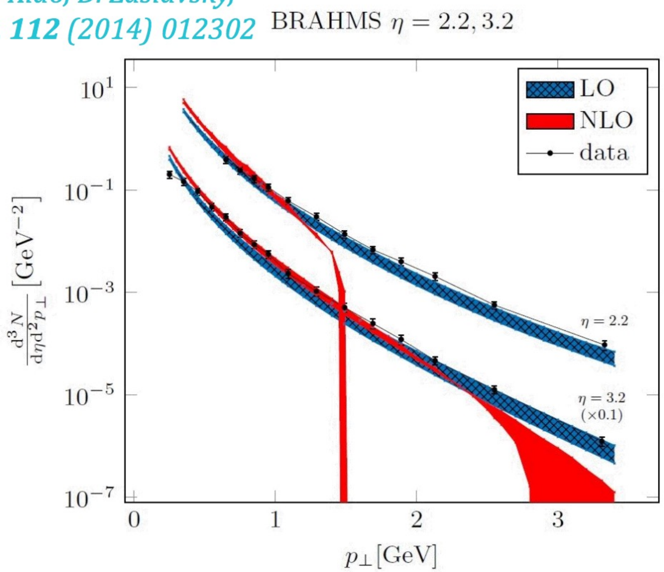
Phenomenology example: single hadron

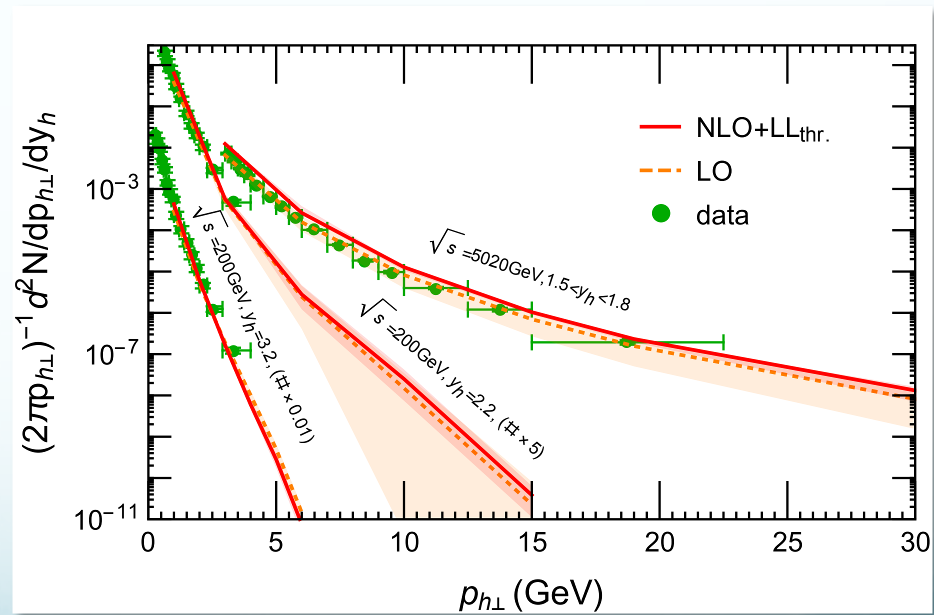
- Forward single hadron production in p+A collisions
  - Lesson: threshold resummation $x_1 \sim 1$ and small-x resummation $x_2 << 1$



$$x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim \mathcal{O}(1)$$
$$x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$$

*A. Stasto, B.-W. Xiao, D. Zaslavsky, Phys. Rev. Lett. **112** (2014) 012302*





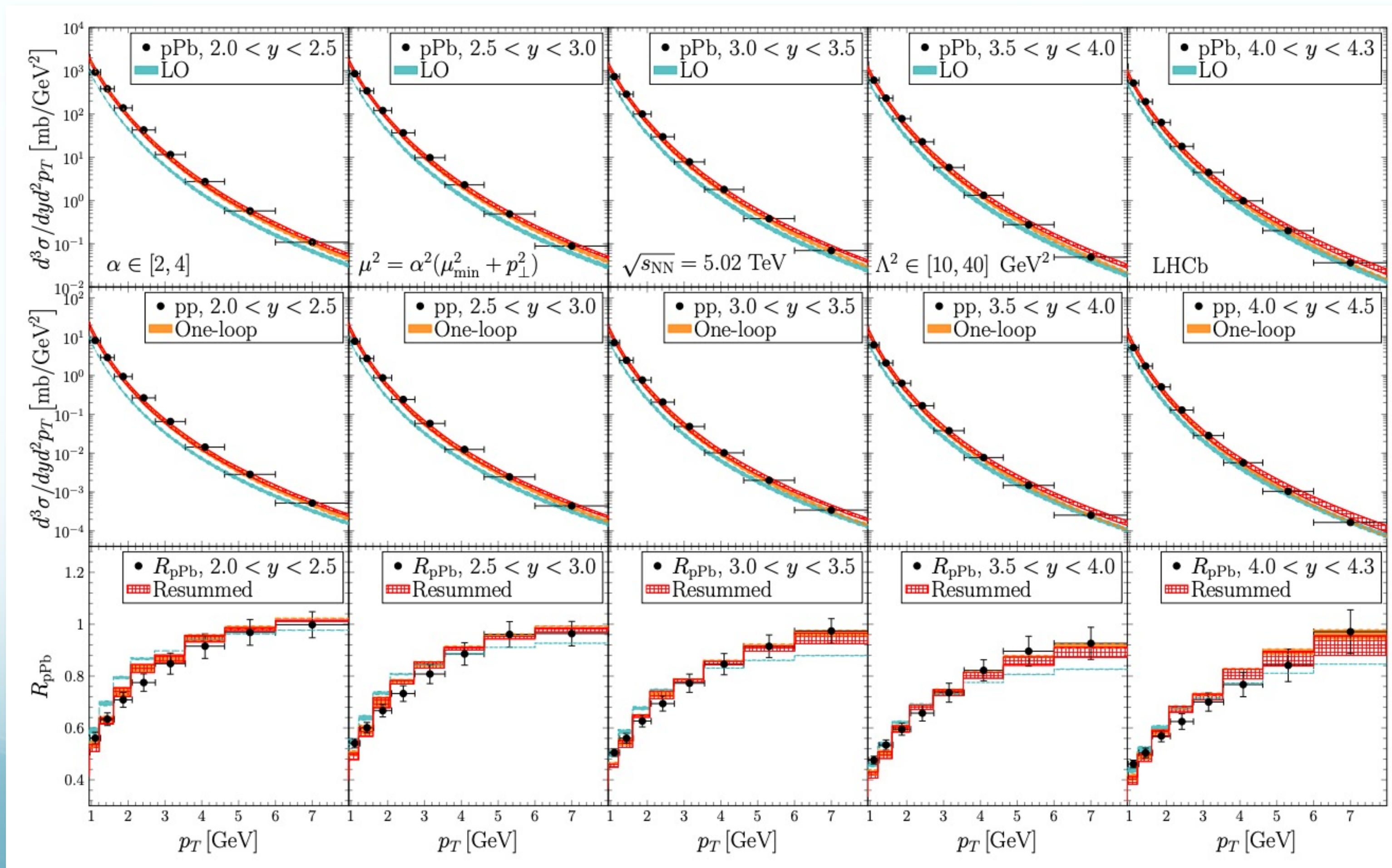
Kang, Liu, Liu, 2004.11990, PRD 20

Full phenomenology

PHYSICAL REVIEW LETTERS **128**, 202302 (2022)

Pursuing the Precision Study for Color Glass Condensate in Forward Hadron Productions

Yu Shi,[1,2,*] Lei Wang,[2,†] Shu-Yi Wei,[1,3,‡] and Bo-Wen Xiao[4,§]



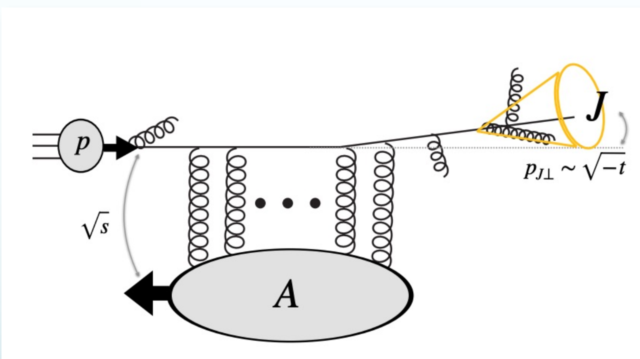
Phenomenology example: single jet

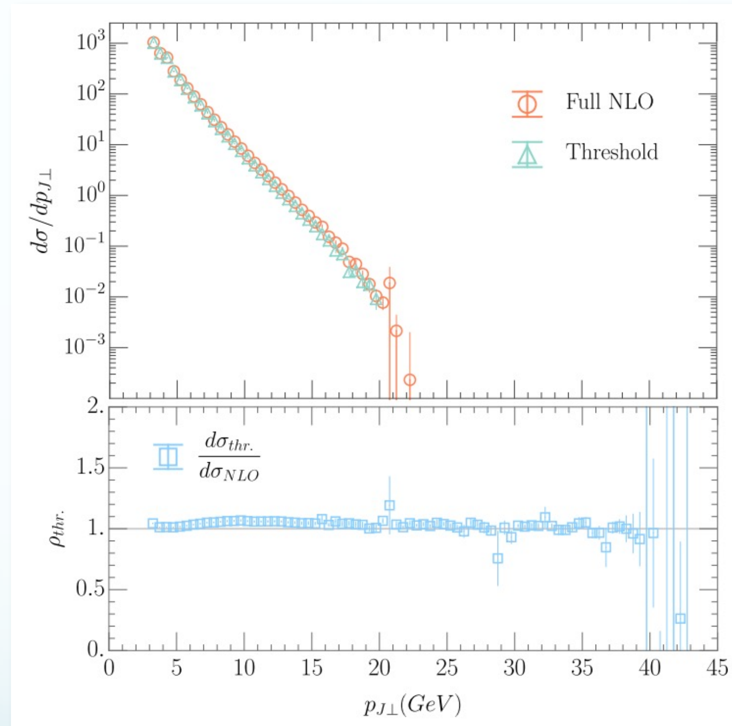
- Forward single jet production in p+A collisions
  - Complete NLO with full jet algorithm implementation
  - Analytical computations for jets with small radius R

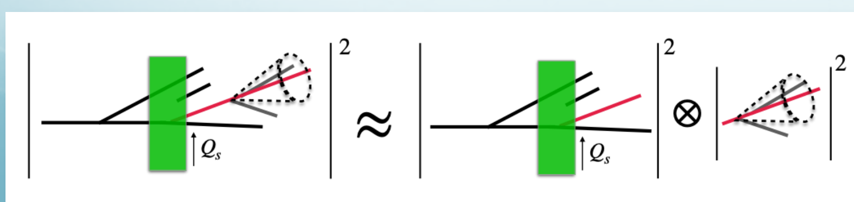
JHEP
PUBLISHED FOR SISSA BY SPRINGER
RECEIVED: April 20, 2022
REVISED: May 27, 2022
ACCEPTED: May 31, 2022
PUBLISHED: July 7, 2022

Single inclusive jet production in pA collisions at NLO in the small-x regime

Hao-yu Liu,[a,b] Kexin Xie,[c] Zhong-Bo Kang[d,e,f] and Xiaohui Liu[a,b,g]





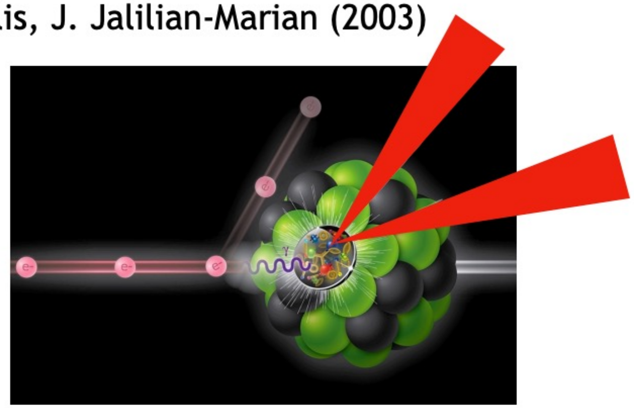


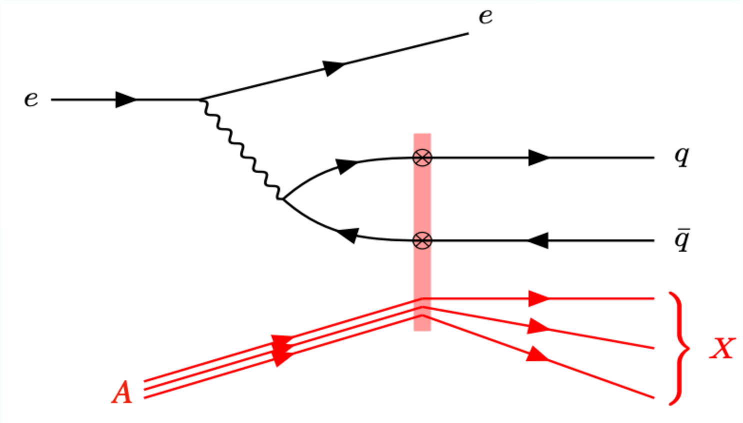
Consistent with Wang, Chen, Gao, Shi, Wei, Xiao, PRD 23

Phenomenology example: dijet in e+A

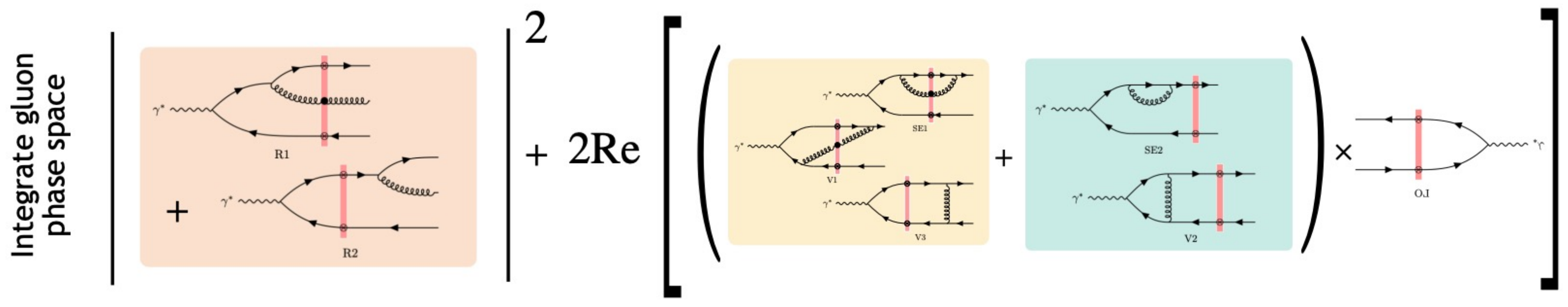
- Dijet production in e+A collisions at NLO

F. Gelis, J. Jalilian-Marian (2003)





P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*

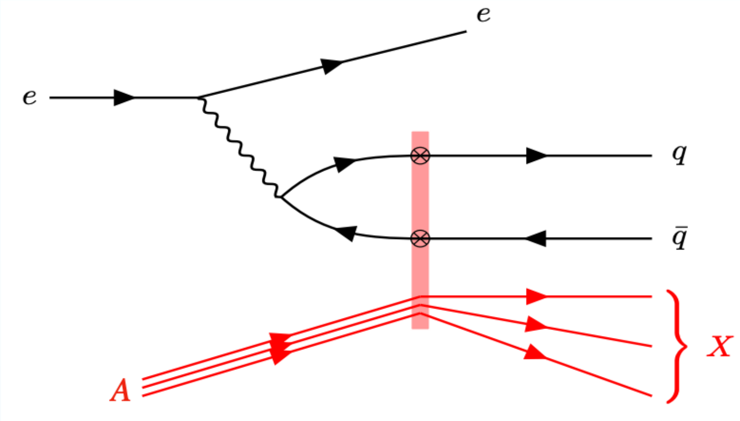


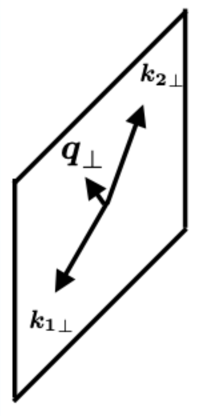
Caucal, Salazar, Schenke, Stebel, Venugopalan, 21, 22, 23

Courtesy of Farid Salazar

One very interesting question

- What happens when the dijet are produced back-to-back in the transverse plane?
  - Forwards jets but back-to-back in transverse plane





  - In this region, Sudakov logarithms become important
  - Connections with TMD factorization

- Important progress and insights have been made

Caucal, Salazar, Schenke, Stebel, Venugopalan, 23

Observation and question

- One first perform NLO computations
  - Extremely challenging, so it requires truly remarkable efforts
- One then takes the back-to-back limit to achieve both Sudakov + small-x logarithm resummation
  - Also not easy

JHEP

Published for SISSA by Springer
Received: *September 6, 2021*
Accepted: *November 6, 2021*
Published: *November 29, 2021*

108 pages

Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate

Paul Caucal,[a] Farid Salazar[a,b,c] and Raju Venugopalan[a]

JHEP

Published for SISSA by Springer
Received: *September 12, 2022*
Accepted: *November 14, 2022*
Published: *November 30, 2022*

77 pages + two more papers

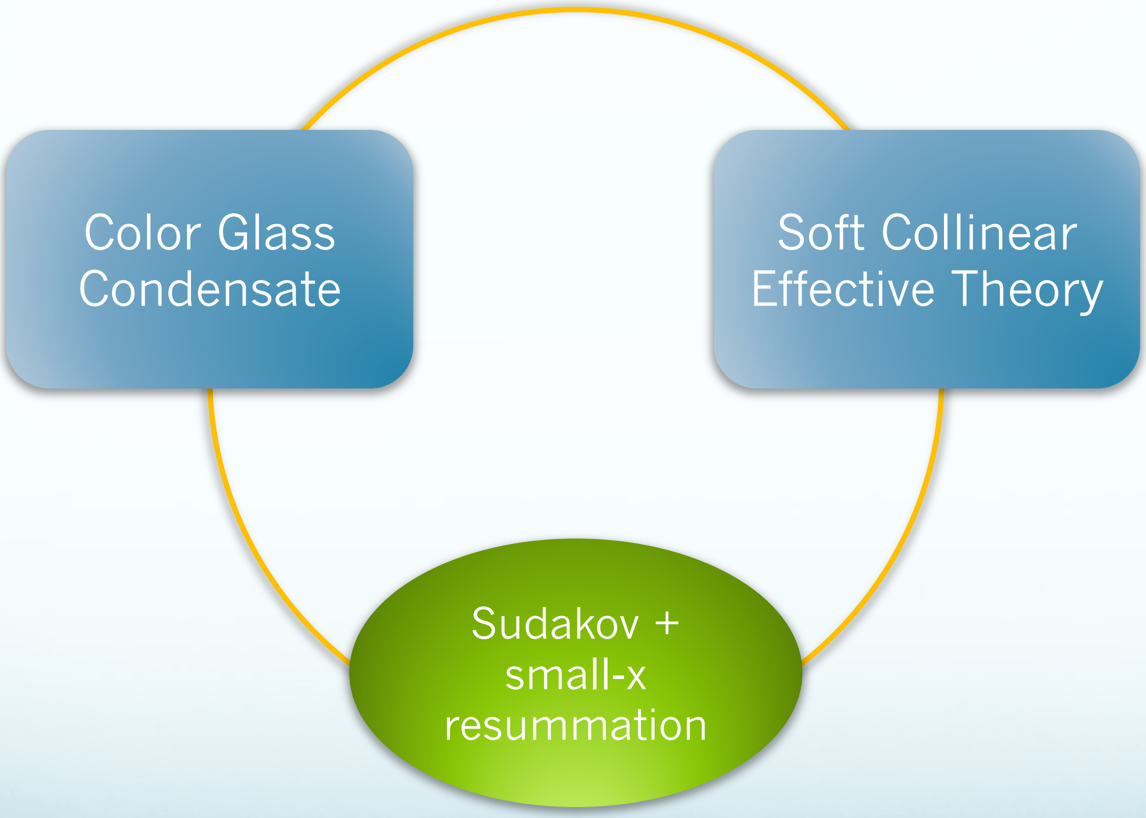
Back-to-back inclusive dijets in DIS at small x: Sudakov suppression and gluon saturation at NLO

Paul Caucal,[a] Farid Salazar,[b,c,d,e] Björn Schenke[a] and Raju Venugopalan[a]

- Is it possible to deal with the back-to-back limit from the very beginning?

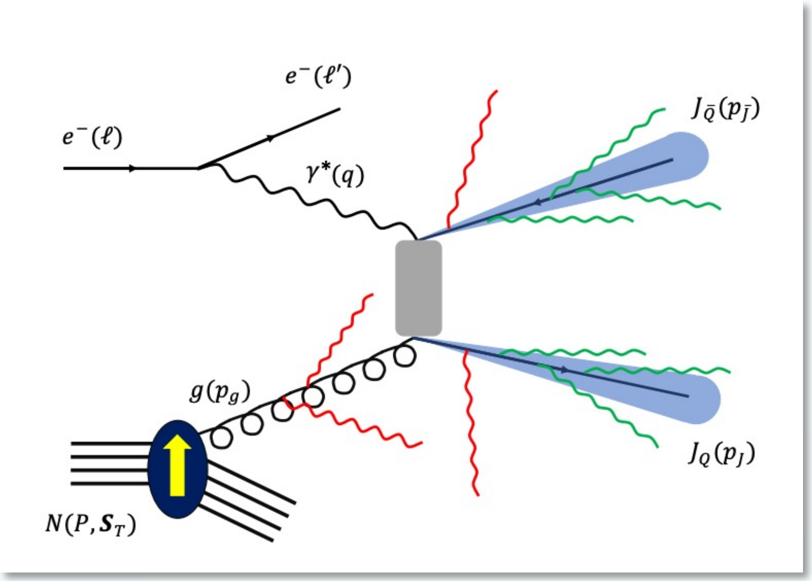
Maybe: CGC + SCET

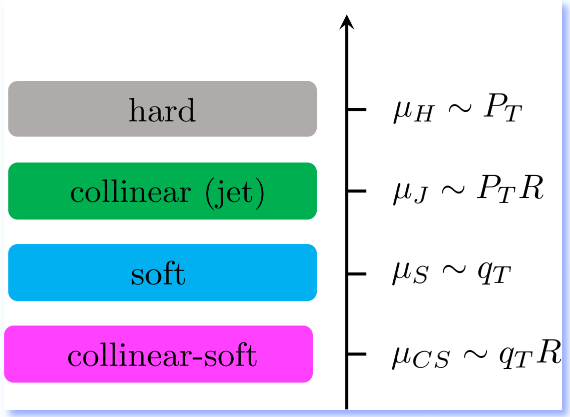
- SCET/TMD factorization is suitable for Sudakov resummation
- CGC effective theory is suitable for small-x resummation



TMD factorization for dijet

- TMD factorization derived using SCET: back-to-back dijet in e+p





$$\frac{d\sigma}{dq_T dP_T} \propto \boldsymbol{H}_{\gamma * g \to q\bar{q}}(P_T) f_{g/A}(x_a, k_{aT})$$
$$\otimes \boldsymbol{S}^{\text{global}}(q_T) \otimes S_c^{\text{cs}}(q_T R) S_d^{\text{cs}}(q_T R) J_c(P_T R) J_d(P_T R)$$

See Kang, Reiten, Shao, Terry, 2012.01756,
Castillo, Echevarria, Makris, Scimemi, 2008.07531

Conjecture at the small x region

- Sudakov logs can be obtained via RG equations for gluon TMD, hard function, soft function, collinear soft function, and jet function

$$\frac{d\sigma^{UU}}{dQ^2 dy d^2\boldsymbol{q}_T dy_J d^2\boldsymbol{p}_T} = H(Q, y, p_T, y_J, \mu_h) \int_0^\infty \frac{bdb}{2\pi} J_0(b\, q_T) f_{g/N}(x, \mu_{b*})$$
$$\times \exp\left[-\int_{\mu_{b*}}^{\mu_h} \frac{d\mu}{\mu}\Gamma^h(\alpha_s) - 2\int_{\mu_{b*}}^{\mu_j} \frac{d\mu}{\mu}\Gamma^{j_Q}(\alpha_s) - \int_{\mu_{b*}}^{\mu_{cs}} \frac{d\mu}{\mu}\left(\bar{\Gamma}^{cs_Q}(\alpha_s) + \bar{\Gamma}^{cs_{\bar{Q}}}(\alpha_s)\right)\right]$$
$$\times \exp\left[-S_{\rm NP}(b, Q_0, n\cdot p_g)\right], \qquad (2.75)$$

See for example, Kang, Reiten, Shao, Terry, 2012.01756

- Then you could expand gluon TMD in terms of WW gluon distribution in the small-x limit

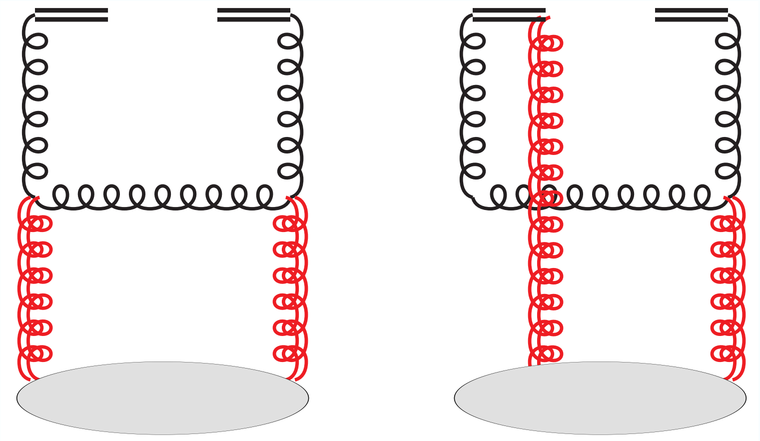
Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
CrossMark
Nuclear Physics B 921 (2017) 104–126
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Transverse momentum dependent parton distributions at small-*x*

Bo-Wen Xiao[a], Feng Yuan[b,*], Jian Zhou[c]

Gluon TMDs at small x

- Work by Xiao, Yuan, Zhou 2017 + Zhou 2019



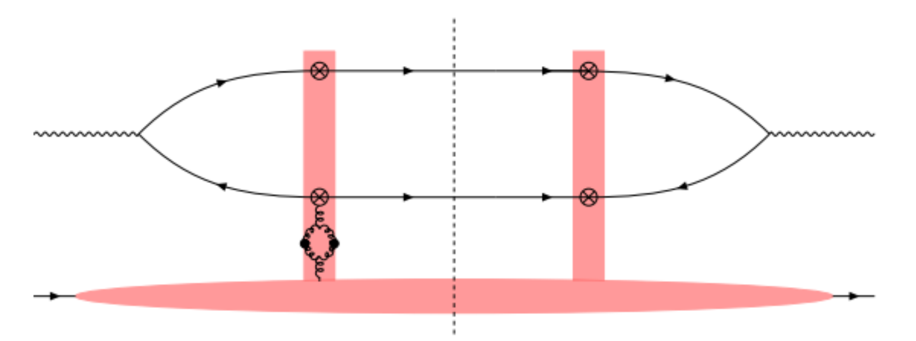
$$xG(x, k_\perp, \zeta_c^2 = \mu_F^2 = Q^2) = -\frac{2}{\alpha_s}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{ik_\perp\cdot b_\perp}$$
$$\times \mathcal{H}(\alpha_s(Q))\mathrm{Exp}\left\{-\int_{\mu_b}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}$$
$$\times \mathcal{F}(x_\perp, y_\perp),$$

$$\mathcal{F}(x_\perp, y_\perp) = \langle \mathrm{Tr}[\partial_\perp^i U(x_\perp)U^\dagger(y_\perp)\partial_\perp^i U(y_\perp)U^\dagger(x_\perp)]\rangle$$

Recent insight

- Quantum correction to Shockwave formalism

Caucal, Salazar, Schenke, Stebel, Venugopalan, 2304.03304



$$\hat{G}_Y^{(1)}(\boldsymbol{r}_{bb'}) = -\alpha_s \beta_0 \left[\frac{1}{\varepsilon} + \text{finite}\right] \hat{G}_Y^{(0)}(\boldsymbol{r}_{bb'})$$

$$G_Y^{bare}(r) = Z\, G_Y \quad (\leftarrow \text{renormalized}) \qquad Z = +\frac{\alpha_s}{4\pi}\beta_0\frac{1}{\varepsilon}$$

$$\Rightarrow \mu\frac{d}{d\mu} G_Y = \gamma_G G_Y$$

Where

$$\gamma_G = -Z^{-1}\mu\frac{d}{d\mu}Z$$

$$= -\mu\frac{d}{d\mu}\left[\frac{\alpha_s}{4\pi}\beta_0\frac{1}{\varepsilon}\right]$$

$$\Downarrow \quad \mu\frac{d}{d\mu}\alpha_s = (-2\epsilon)\alpha_s + \cdots$$

$$= -(-2\epsilon)\frac{\alpha_s}{4\pi}\beta_0\frac{1}{\varepsilon} = 2\frac{\alpha_s}{4\pi}\beta_0$$

$$= -\frac{\alpha_s}{\pi}\left[-\frac{\beta_0}{2}\right]$$

This exactly gives us

$$G_Y(r,\mu) = \exp\left[-\int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\frac{\alpha_s}{\pi}\left(-\frac{\beta_0}{2}\right)\right] G_Y(r,\mu_b)$$

$$\mu_b = \frac{c_0}{r}$$

$$c_0 = 2e^{-\gamma_E}$$

- Gluon TMD in small-x

$$f_{g/A}^{TMD}(x_A, r_\perp, \mu, \zeta_2) = \exp\left[-\int_{\mu_{r_\perp}}^{\mu}\frac{d\mu'}{\mu'}\frac{\alpha_s}{\pi}\left(C_A \ln\frac{\zeta_2}{\mu^2} - \frac{\beta_0}{2}\right)\right] * F_Y^{WW}(r_\perp, R_\perp)$$

Checking with Higgs prodction

- gg Higgs at small transverse momentum
  - Start with a TMD factorization
  - Replace gluon TMD in the nuclear target by the small-x expansion
  - One obtains the correct formula now has both Sudakov + small-x logs

Mueller, Xiao, Yuan, 2013

For pA → Higgs + X production, we'll then have

$$d\sigma \propto f^{TMD}_{g/p}(x_p, r_\perp, \mu, \zeta_1)\, f^{TMD}_{g/A}(x_A, r_\perp, \mu, \zeta_2)\, H(Q)$$

↓ replace this TMD in the target by its expansion in terms of WW gluon distribution

at NLL

$$f^{TMD}_{g/p}(x_p, r_\perp, \mu, \zeta_1) = f_{g/p}(x_p, \mu_{r_\perp}) * \exp\left[-\int_{\mu_{r_\perp}}^{\mu} \frac{d\mu'}{\mu'} \frac{\alpha_s}{\pi}\left(C_A \ln\frac{\zeta_1}{\mu^2} - \frac{\beta_0}{2}\right)\right]$$

for gluon TMD on the nuclear target

$$f^{TMD}_{g/A}(x_A, r_\perp, \mu, \zeta_2) = \exp\left[-\int_{\mu_{r_\perp}}^{\mu} \frac{d\mu'}{\mu'} \frac{\alpha_s}{\pi}\left(C_A \ln\frac{\zeta_2}{\mu^2} - \frac{\beta_0}{2}\right)\right] * F^{WW}_{Y}(r_\perp, R_\perp)$$

Summary

- WG on small-x evolution + NLO calculations should be extremely fun
- We expect to have a lot of exciting developments in the coming years
- We will try our best to work as a team to accomplish milestones we have promised to DOE
- We welcome interested postdocs and students to join us
- Check talks by our WG members:
  - Wednesday: Jamal, Ian, Tomasz
  - Thursday: Simon, Vladimir, Anna, Farid

Thank you!