NC STATE UNIVERSITY

Center for Frontiers in Nuclear Science

Anomaly zero modes and sub-eikonal corrections at small-x

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Global Analysis

NLO corrections

- next-to-leading order (NLO) computations of process-dependent impact factors
- small-x BK and JIMWLK equations to next-to-leading logarithmic (NLL) accuracy

Initial/Final states

- initial conditions for evolution equations
- fluctuations of the initial state target wave function
- thermalization process

Sub-eikonal corrections

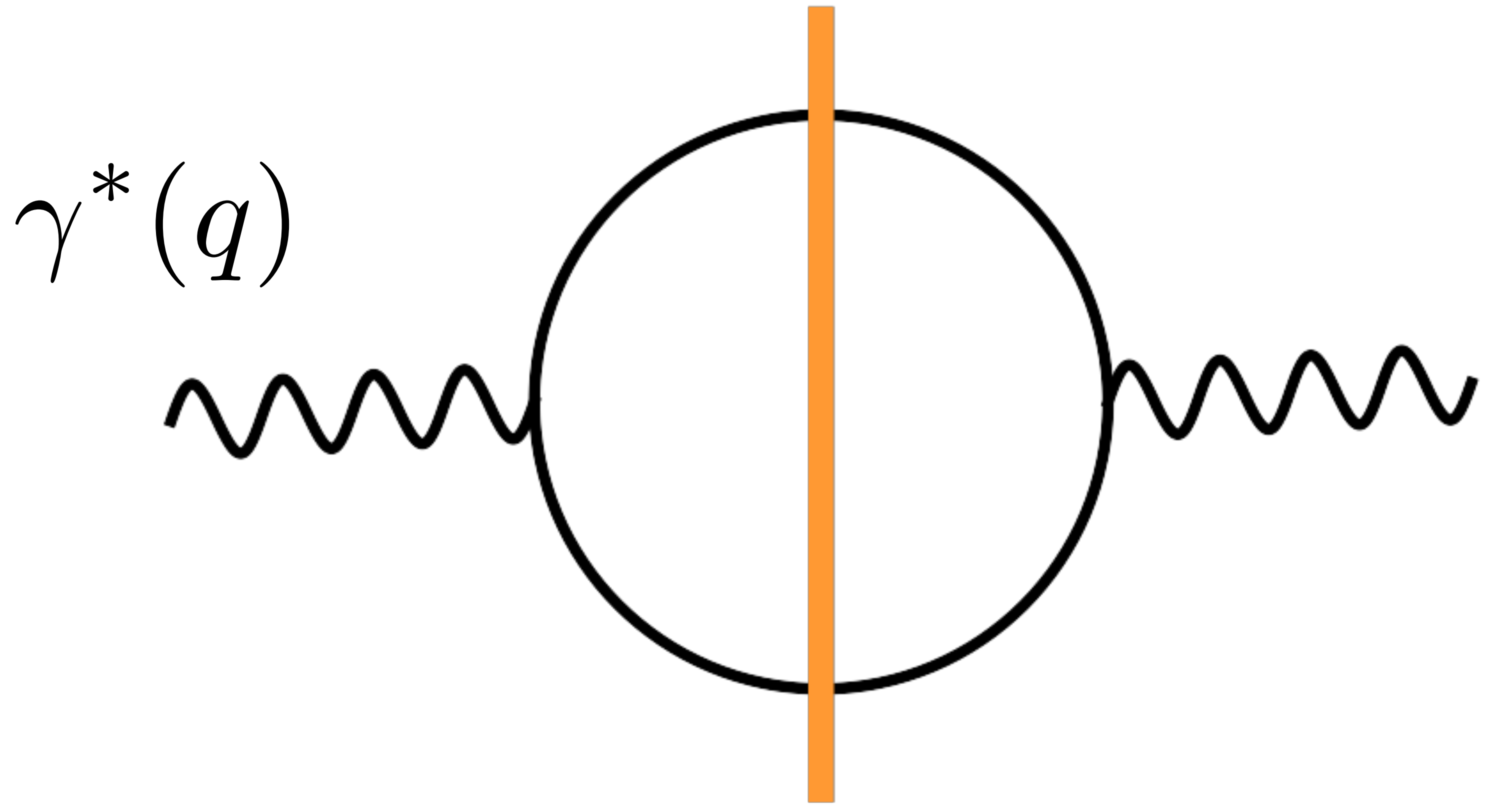
- spin at small-x
- matching between the TMD and the CGC formalisms, BFKL based approach
- proper treatment of the collinear corner of phase space

Beyond the eikonal approximation

The small-x regime of the many-body parton system can be explored in the framework of CGC. In the CGC EFT the target field has an infinitesimally small support (shock-wave) and doesn't have a transverse component:

$$A^+_{\rm cl}(x) = -\frac{1}{\partial^2_\perp}\rho(x_\perp)\delta(x^-); \quad A^-_{\rm cl}(x) = A^i_{\rm cl}(x) = 0$$

To be sensitive to spin effects one has to go beyond the leading eikonal approximation and include two types of corrections:



- Non-zero value of the transverse component of the background field

$$A^i_{\rm cl}(x) \neq 0$$

- Non-zero "size" of the shock-wave

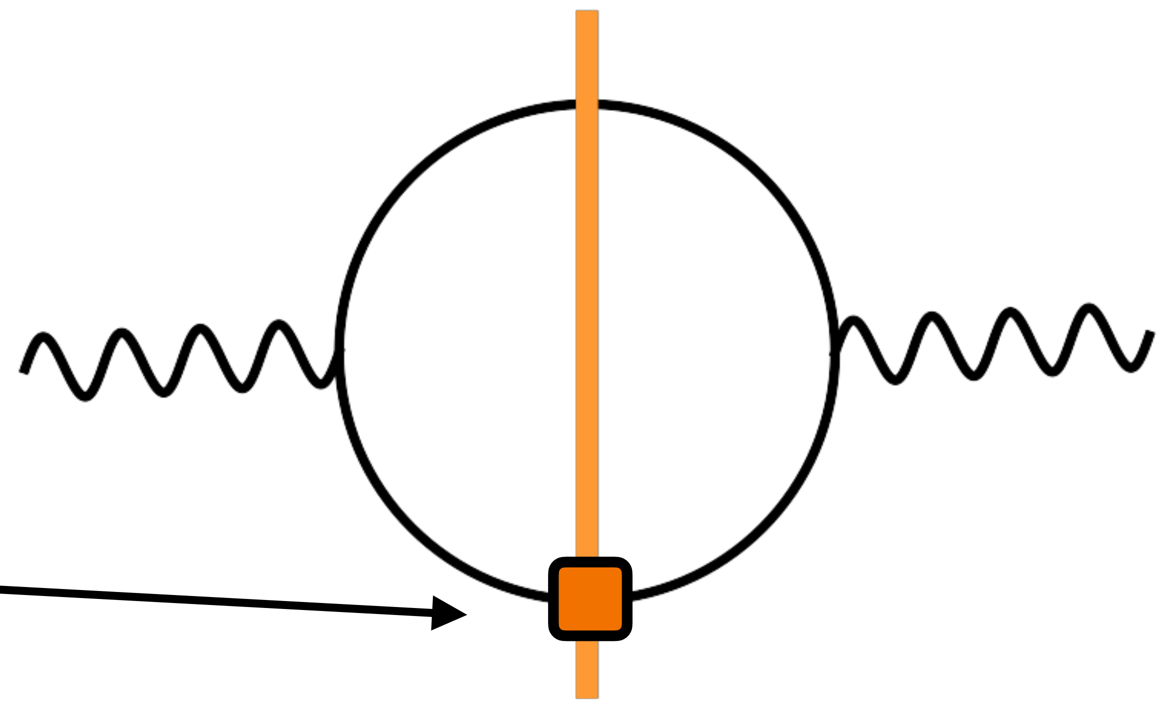
$$A^+_{\rm cl}(x) \not\propto \delta(x^-)$$

Sub-eikonal corrections are important!

- In the case of the small-x helicity evolution, sub-eikonal corrections are the leading order effect. The corrections generate the KPS-CTT evolution (Kovchegov, Pitonyak, Sievert 2016-2019; Cougoulic, Kovchegov, Tarasov, Tawabutr, 2022) which is consistent with the small-x polarized DGLAP

$$ig \int_{-\infty}^{\infty} dz^- \, z^- V_x[\infty, z^-] \, F_{-k} \, V_x[z^-, -\infty]$$

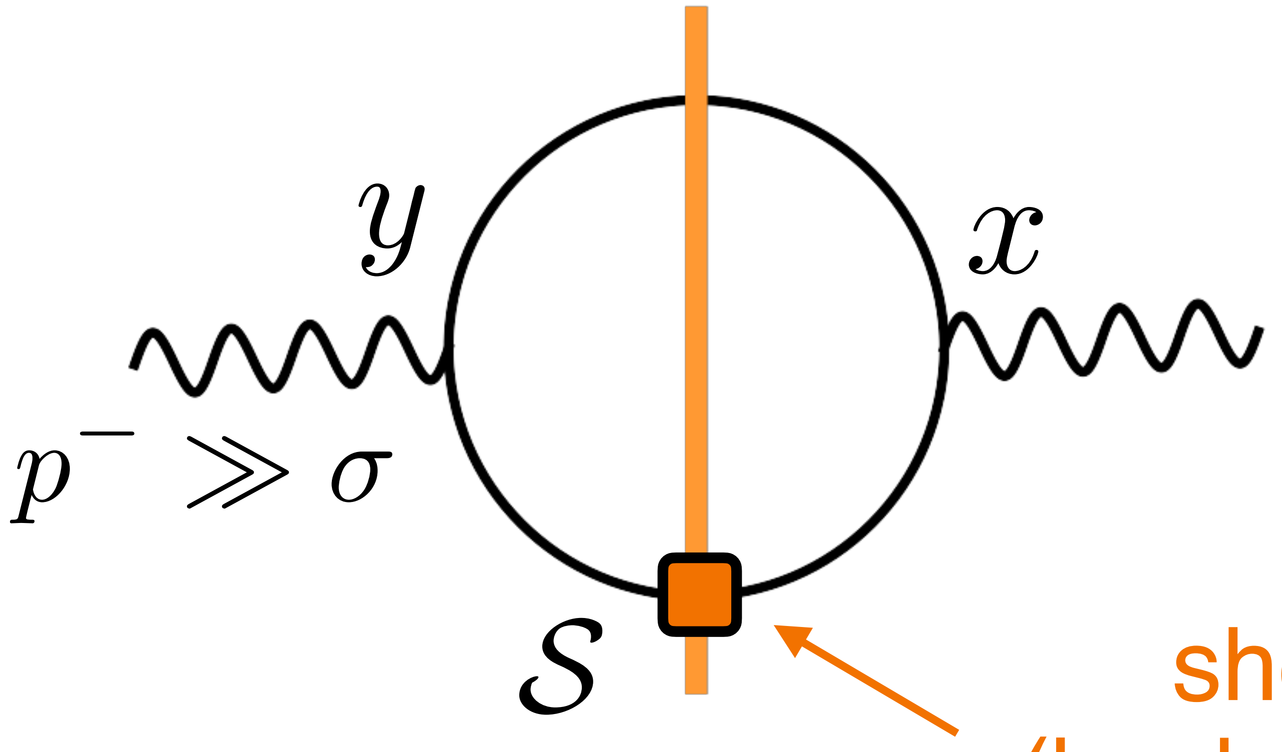
$$= \frac{1}{2} \int_{-\infty}^{\infty} dz^- V_x[\infty, z^-] \left[D_k - \overleftarrow{D}_k \right] V_x[z^-, -\infty]$$



- The sub-eikonal operator which generates the small-x DGLAP evolution
- The operator is related to the Jaffe-Manohar polarized gluon distribution ΔG, which satisfies the DGLAP evolution. It can by obtained by expanding the exponential factor (expansion in x_B) in the definition of the ΔG distribution
- This operator comes from the scalar phase in the propagator when it is expanded onto the light-cone
- This operator describes sub-eikonal corrections due to a non-zero width of the shock-wave

Propagators in the background field: eikonal expansion

- How do we construct the eikonal expansion? Write the most general expression for a propagator



shockwave
(background field)
$p^- \ll \sigma$

$$(x|\frac{1}{P^2 + i\epsilon}|y) = -\frac{i}{2\pi}\theta(x^- - y^-)\int_0^\infty \frac{dp^-}{2p^-}e^{-ip^-(x-y)^+}(x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-}\mathcal{S}(x^-,y^-)e^{i\frac{p_\perp^2}{2p^-}y^-}|y_\perp)$$

operator describing
interaction with a target

- Construct an expansion in powers of $1/p^-$

$$\mathcal{S}(x^-,y^-) = \mathcal{S}_0(x^-,y^-) + \frac{1}{p^-}\mathcal{S}_1(x^-,y^-) + \frac{1}{(p^-)^2}\mathcal{S}_2(x^-,y^-) + \ldots$$

leading eikonal term
(Wilson line)

sub-eikonal correction

Altinoluk, Armesto, Beuf, Martínez, Salgado (2014)
Balitsky, Tarasov (2015-2016)
Chirilli (2019)
Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)

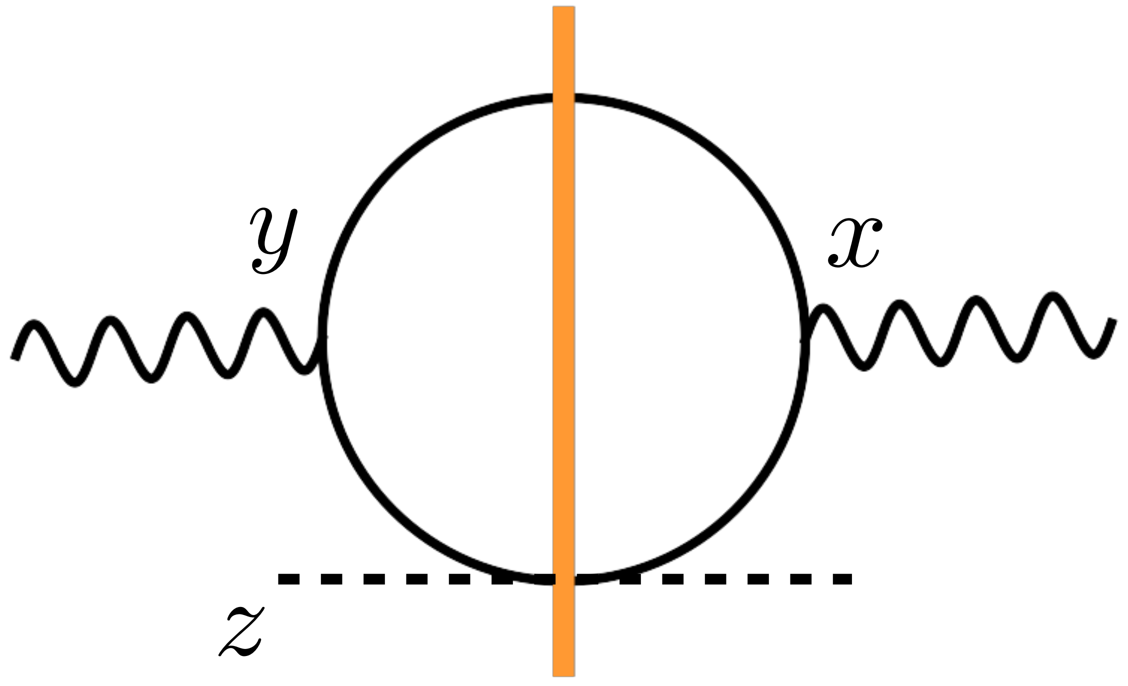
Functional integral (worldline) representation of a propagator

- The expansion is equivalent to an expansion of the background fields onto a given direction

$$(x|\frac{1}{P^2+i\epsilon}|y) = \mathcal{N}^{-1}\int_0^\infty dT \int_{x(0)=y}^{x(T)=x} \mathcal{D}x(\tau)\, e^{-i\int_0^T d\tau \frac{1}{4}\dot{x}^2}\, P\exp\left(ig\int_0^T d\tau\ \dot{x}^\mu A_\mu\right)$$

sum over all trajectories

scalar phase (Wilson line)



- choose an arbitrary direction z (usually the light-cone direction) and expand all fields onto it, gradient expansion

$$A_\mu(x) = A_\mu(z) + (x-z)_\alpha \partial^\alpha A_\mu(z) + \frac{1}{2}(x-z)_\alpha (x-z)_\beta \partial^\alpha \partial^\beta A_\mu(z) + \ldots$$

- write the expansion in terms of strength tensors and Wilson lines. Infinite number of terms!

$$(x|\frac{1}{P^2+i\epsilon}|y) = \Big(-\frac{i}{2\pi}\theta(x^- - y^-)\int_0^\infty \frac{dp^-}{2p^-} + \frac{i}{2\pi}\theta(y^- - x^-)\int_{-\infty}^0 \frac{dp^-}{2p^-}\Big) e^{-ip^-(x-y)^+}(x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-}$$

$$\times\Big\{[x^-,y^-] + \frac{ix^-}{2p^-}(\{p^k,A_k\}+A^kA_k)[x^-,y^-] - \frac{iy^-}{2p^-}[x^-,y^-](\{p^k,A_k\}+A^kA_k)$$

$$-\frac{i}{2p^-}\int_{y^-}^{x^-} dz^- z^-[x^-,z^-]\{P^k,F_{-k}\}[z^-,y^-] + O\Big(\frac{1}{(p^-)^2}\Big)\Big\}^{ab} e^{i\frac{p_\perp^2}{2p^-}y^-}|y_\perp)$$

light-cone Wilson line

sub-eikonal corrections (DGLAP evolution, ΔG distribution)

First moment of the structure function

- Does the eikonal/light-cone expansion always work? Can we write it down for all observables and study their asymptotic in a particular kinematic limit or the expansion breaks down?
- The expansion breaks down for topological quantities. This effects are not local!

The helicity can be extracted from the first moment of the g_1 structure function

$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\boxed{\Sigma(Q^2)}\right)\left(1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N}\sum_f \langle P, S|\bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f|P, S\rangle \equiv \frac{1}{M_N}\langle P, S|J_5^\mu(0)|P, S\rangle$$

Quark contribution to the proton spin is defined by the isosinglet axial vector current J_5^μ

Anomaly equation

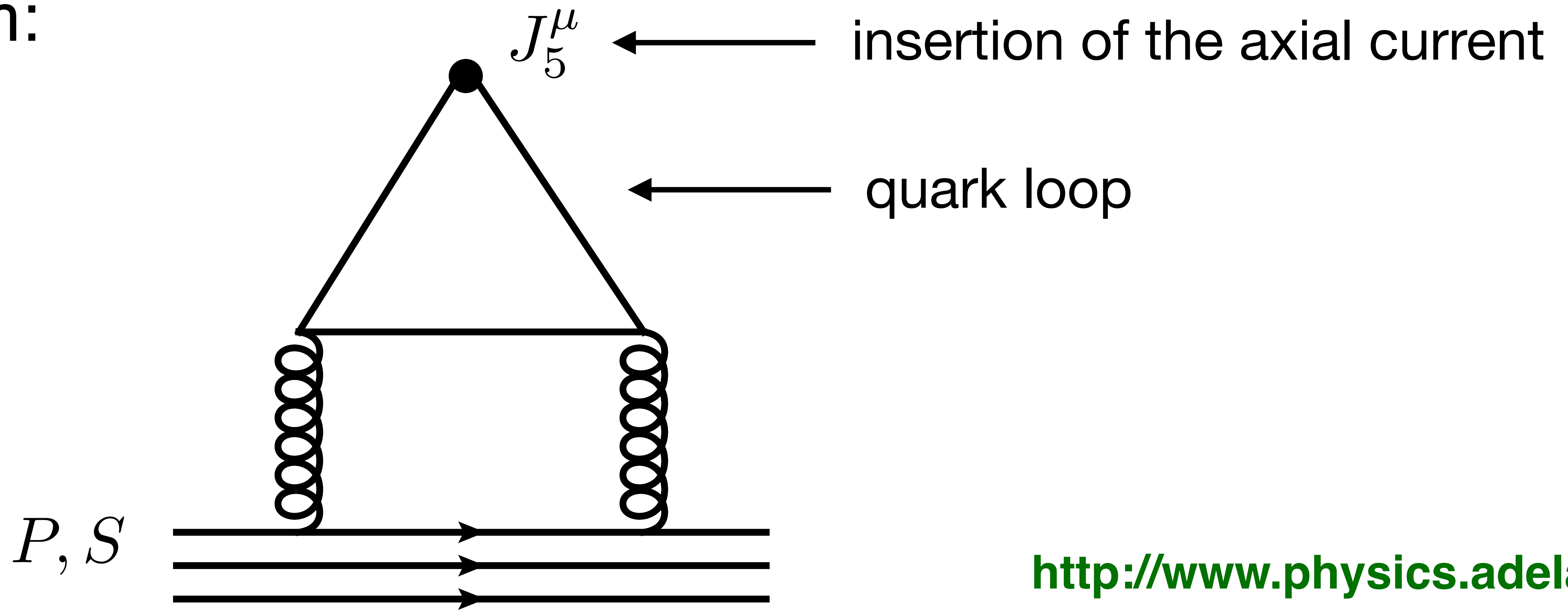
The fundamental property of the J_5^μ current is the anomaly equation:

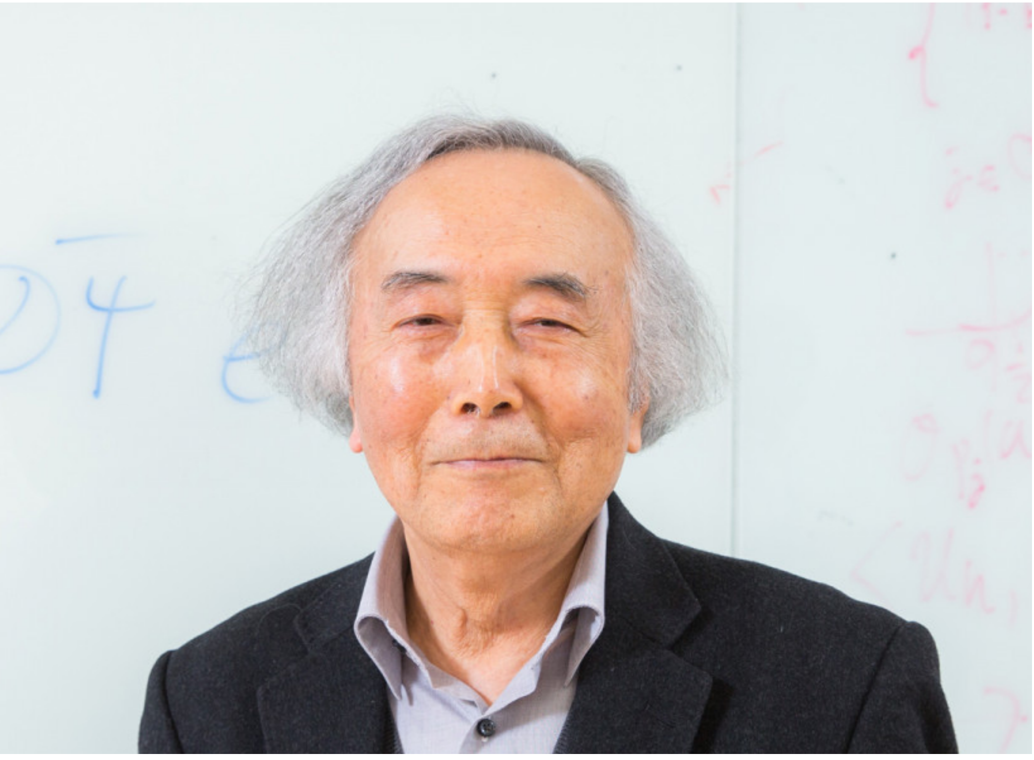
$$\partial^\mu J_\mu^5(x) = \frac{n_f \alpha_s}{2\pi}\,\mathrm{Tr}\left(F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)\right) = 2\,n_f\,\partial_\mu K^\mu$$

The isosinglet current couples to the topological charge density in the polarized proton!

The anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations. Topological properties of the QCD vacuum! **K. Fujikawa, PRL. 42, 1195 (1979)**

In the leading order the coupling is generated by the triangle diagram:

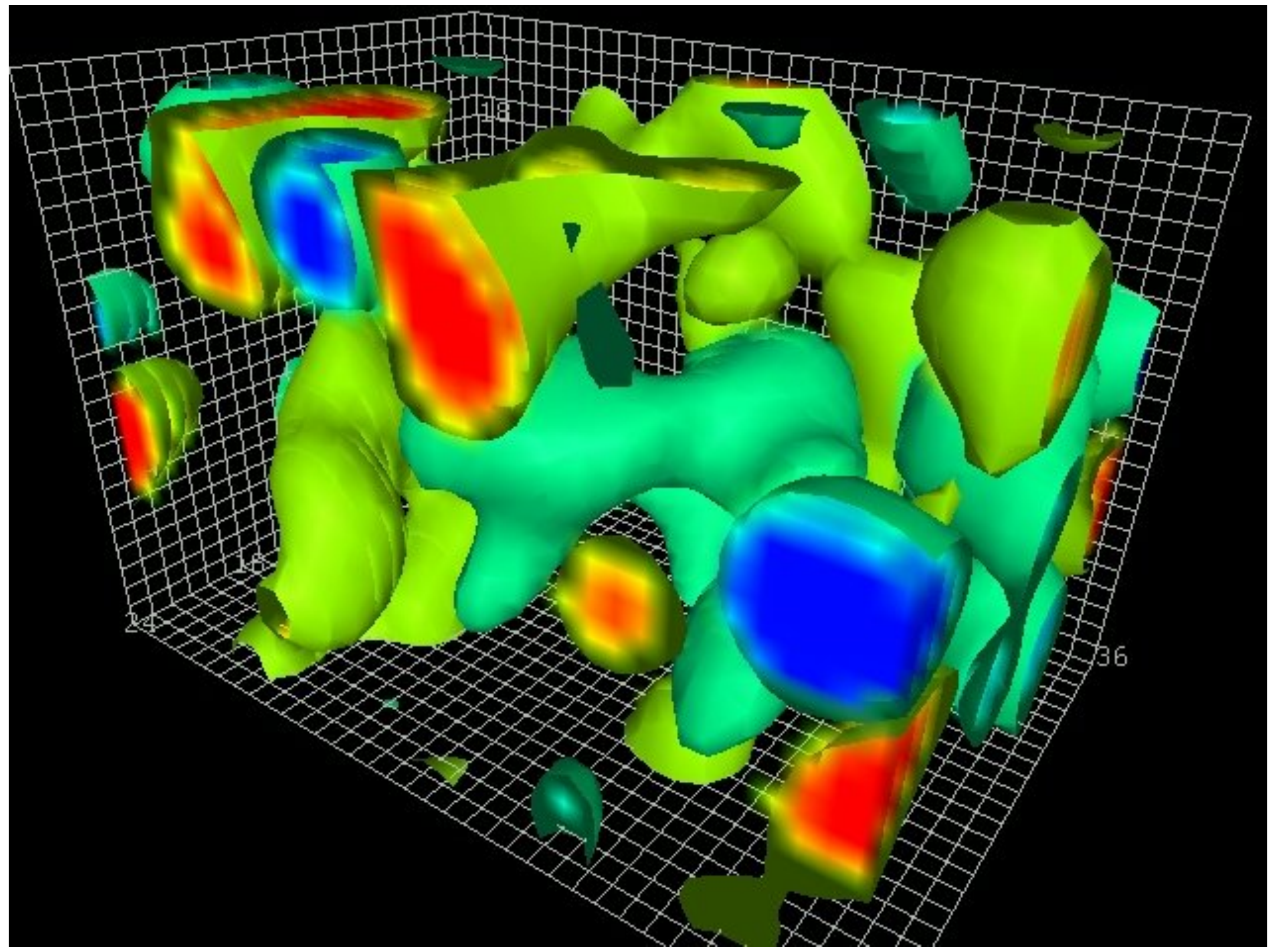




Kazuo Fujikawa

Chern-Simons current:

$$K_\mu = \frac{\alpha_S}{4\pi}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma\right)\right]$$



Topological charge density

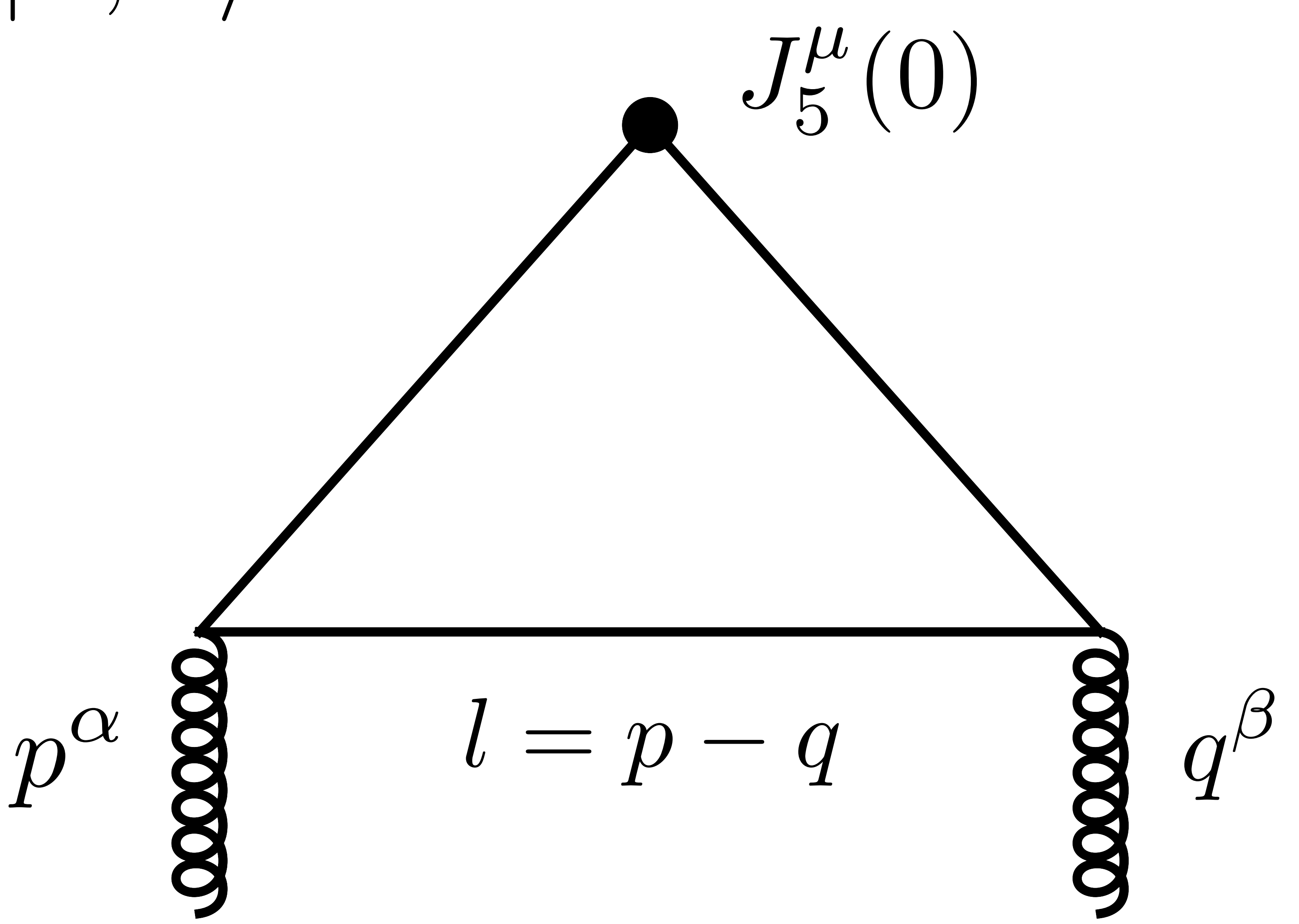
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

Quark helicity and the triangle anomaly

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S|\bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f|P, S\rangle \equiv \frac{1}{M_N} \langle P, S|J_5^\mu(0)|P, S\rangle$$

The key role of the anomaly can be seen from the structure of the triangle graph in the off-forward limit. An exact calculation in the worldline approach gives

$$\langle P', S|J_5^\mu(0)|P, S\rangle = -i \boxed{\frac{l^\mu}{l^2}} \frac{\alpha_s n_f}{2\pi} \langle P', S|\boxed{\mathrm{Tr}\left(F\tilde{F}\right)}|P, S\rangle$$



Tarasov, Venugopalan (2021)

infrared (anomaly) pole.
The anomaly is not local!

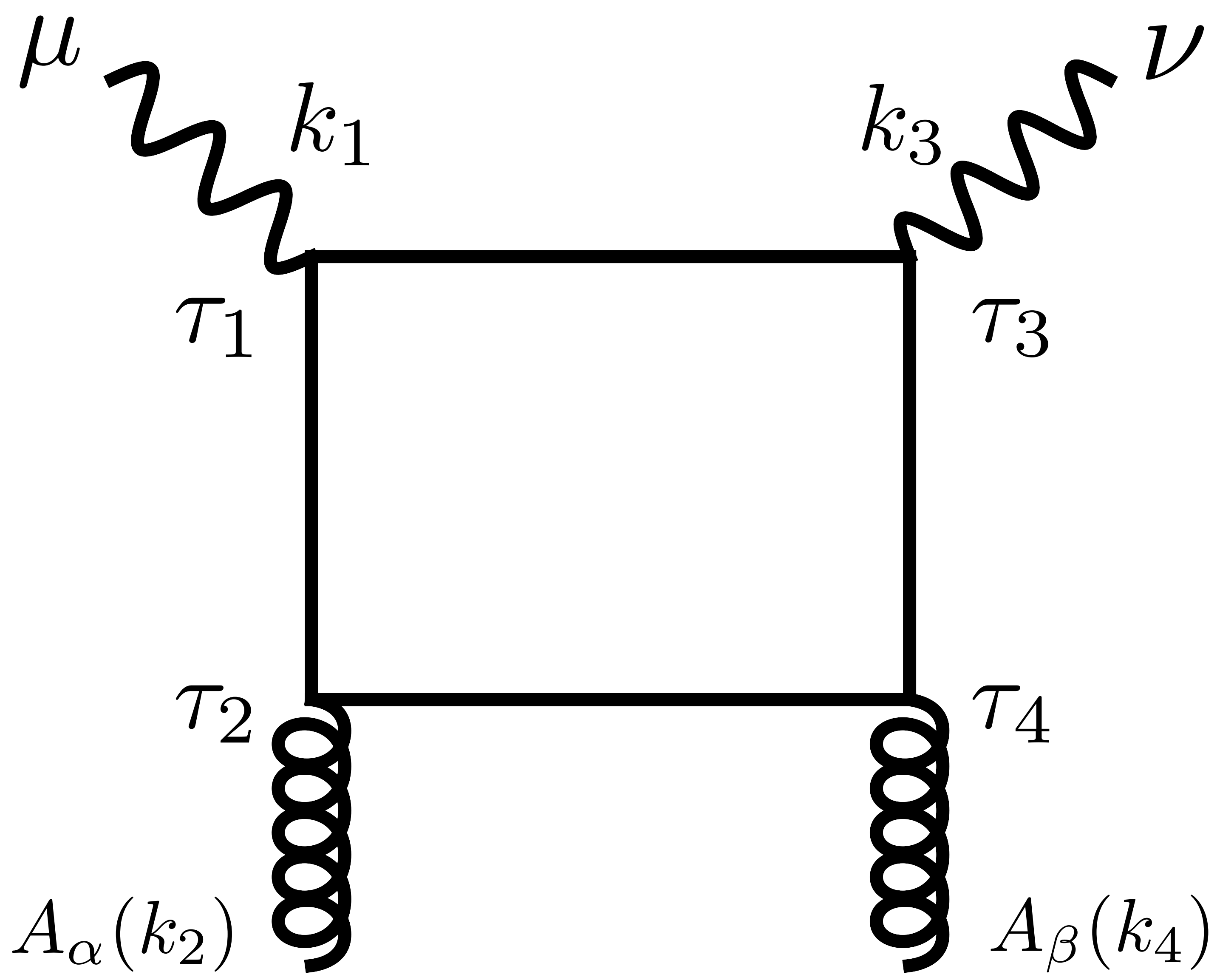
topological charge density

R. L. Jaffe, A. Manohar (1990)
Shore, Veneziano (1990)
K.-F. Liu (1992)

Adler-Bell-Jackiw anomaly

Exact result, which is obtained without performing any light-cone expansion!

Infrared pole and the structure function g_1



We use powerful worldline QFT formalism to compute box diagram contribution to g_1 in exact kinematics of internal variables

$$g_1(x_B, Q^2) \propto -\frac{g^2 e^2 e_f^2}{2} \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \int \mathcal{D}\psi \ \exp\Big\{ - S_{w.l.}(x,\psi)\Big\}$$

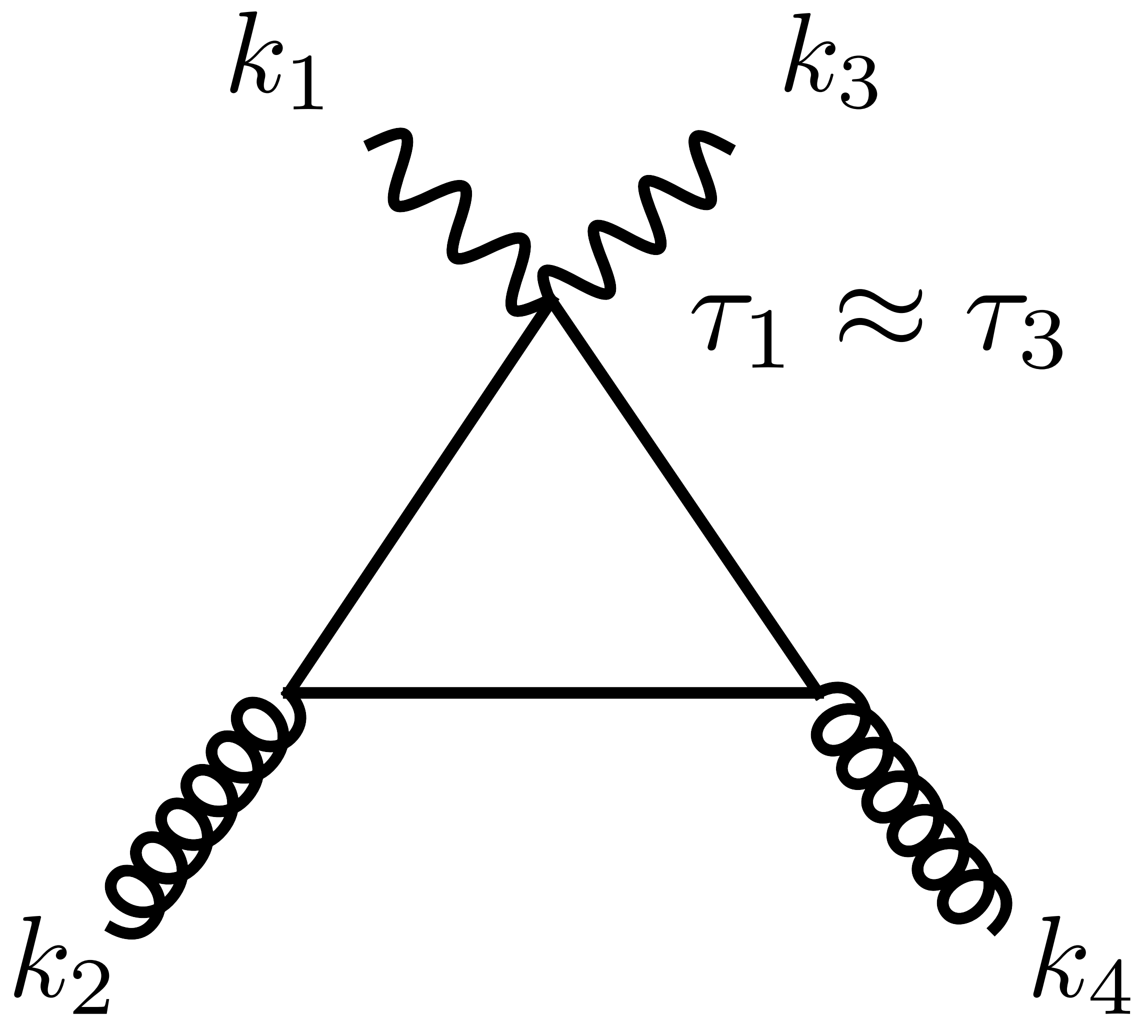
$$\times\Big[V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4) - (\mu \leftrightarrow \nu)\Big]$$

where the vertex corresponds to the interaction of a worldline with the external current

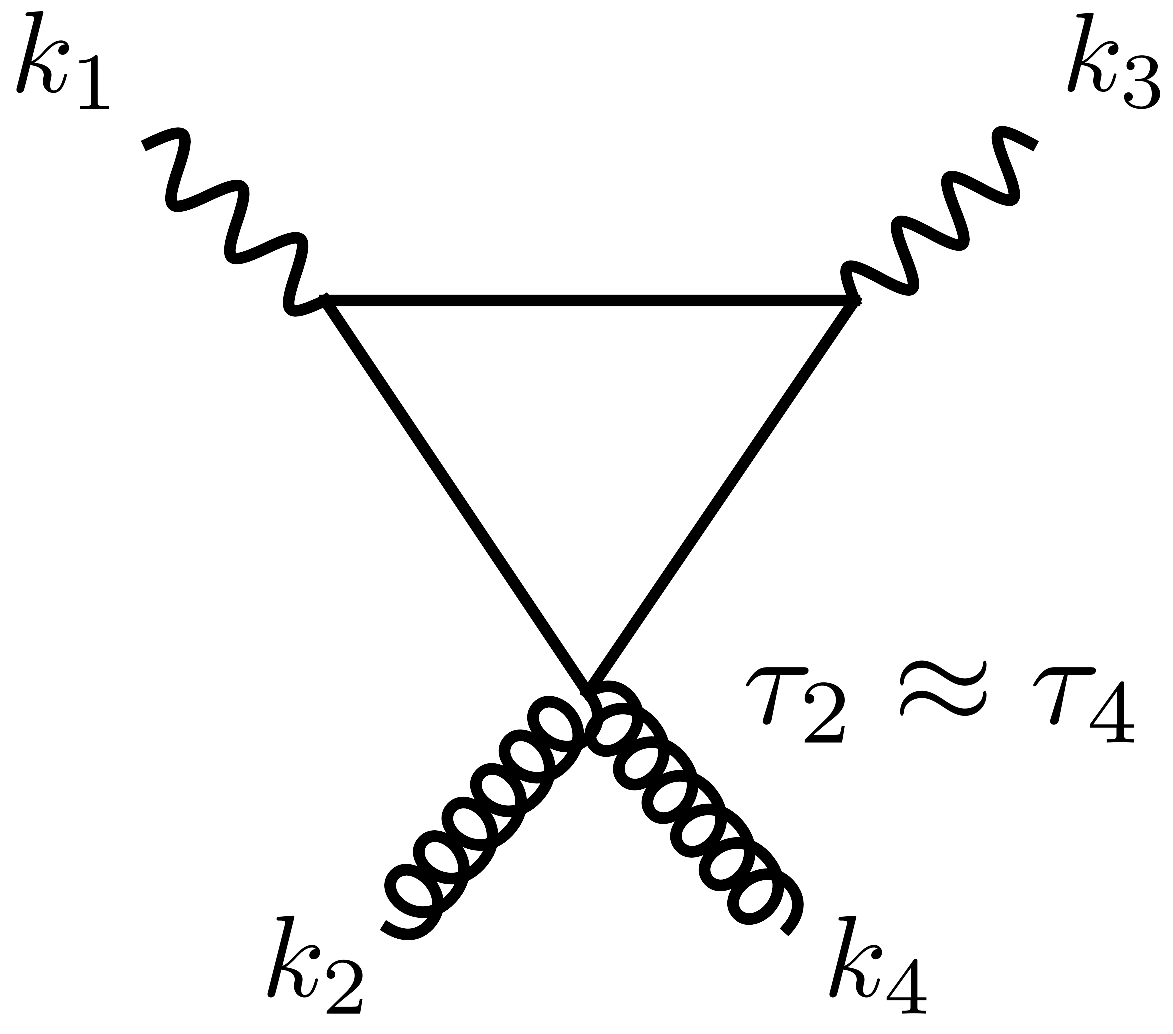
$$V_i^\mu(k_i) \equiv \int_0^T d\tau_i(\dot{x}_i^\mu + 2i\psi_i^\mu k_j \cdot \psi_j)e^{ik_i\cdot x_i}$$

This is the most general expression for the box diagram. We identify the anomalous contribution in Bjorken (large Q^2) and Regge (small x_B) asymptotic limits.

Tarasov, Venugopalan (2021)



point-like interaction with a virtual photon at large Q^2



Lorentz contraction of background fields at small x_B

Infrared pole and the structure function g_1

We find that g_1 is dominated by the triangle anomaly - g_1 is topological in both asymptotic limits of QCD.
First moment of g_1 matches calculation of the triangle diagram

infrared pole

$$S^\mu g_1(x_B, Q^2)\Big|^{anom.}_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \boxed{\frac{l^\mu}{l^2}} \langle P', S| \boxed{\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)} |P, S\rangle$$

$$S^\mu g_1(x_B, Q^2)\Big|^{anom.}_{x_{\mathrm{Bj}}\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x}\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \boxed{\frac{l^\mu}{l^2}} \langle P', S| \boxed{\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)} |P, S\rangle$$

How is the pole cancelled? Interplay between perturbative and non-perturbative physics. The mechanism of the cancelation is deeply related to the $U_A(1)$ problem in QCD - topological mass generation of the $m^2_{\eta'}$.

$\hookrightarrow$ Implications for DIS and DVCS

Anomaly pole and the $U_A(1)$ problem

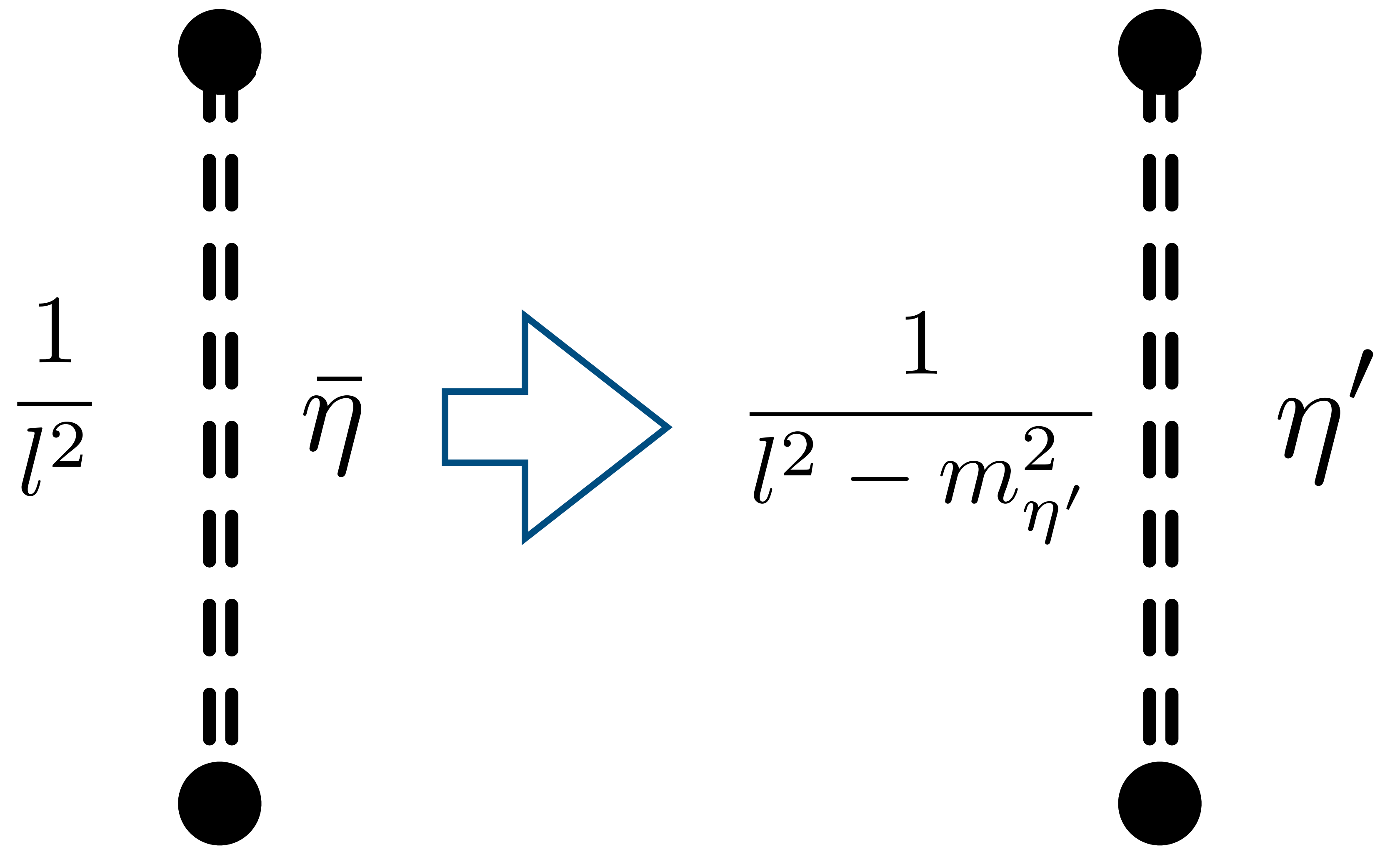
To resolve the pole one has to take into account exchanges of the $\bar{\eta}$ massless “primordial” ninth Goldstone boson arising from the spontaneous symmetry breaking of the flavor group.

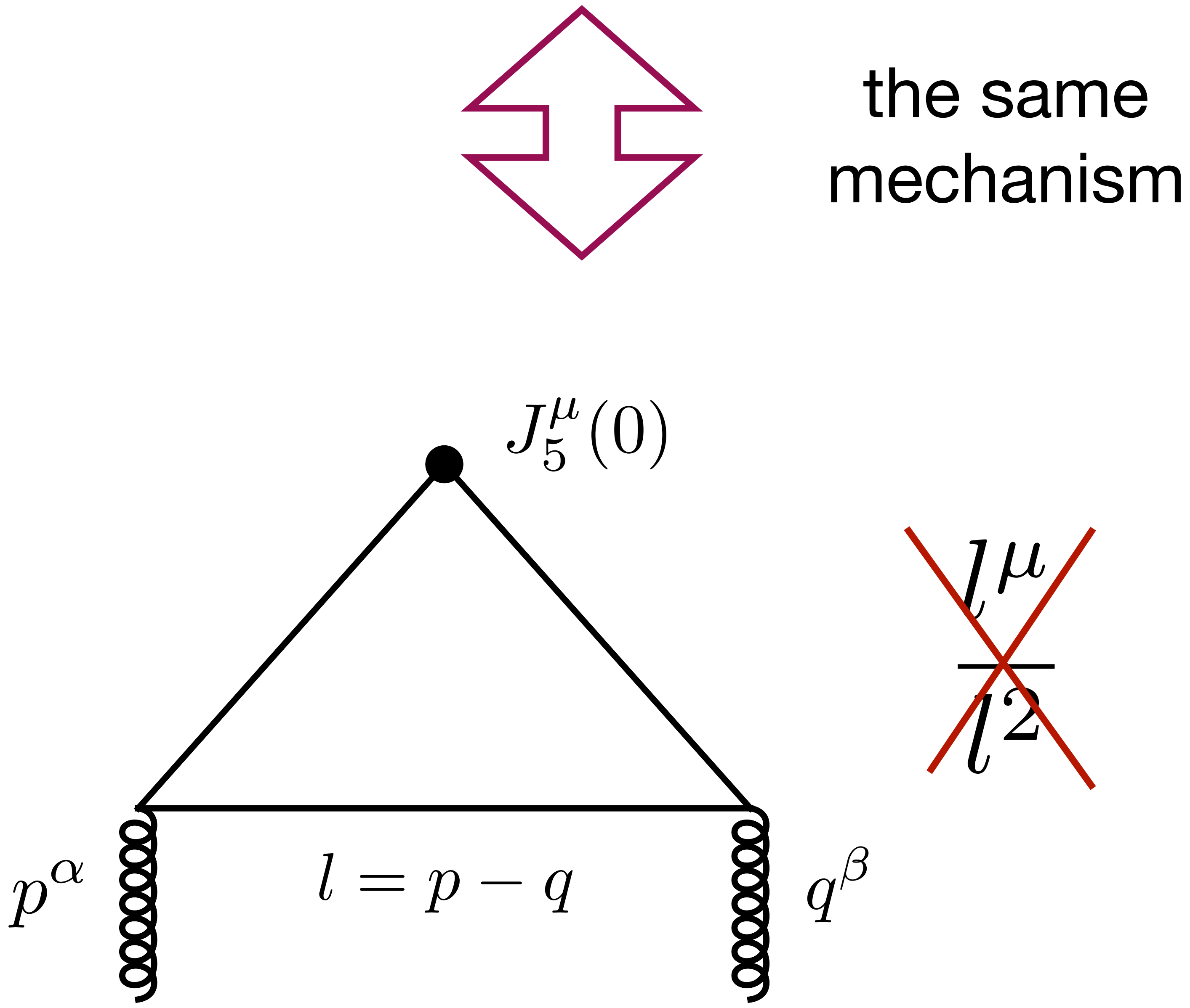
↳ $\boxed{\text{Factorization, eikonal/twist expansion breaking}}$

However $\bar{\eta}$ is not observed. Instead there is a heavy η' ($m_{\eta'} \approx 957 MeV$) - the famous $U_A(1)$ problem.

There is no Goldstone pole just as there is no anomaly pole in the QCD spectrum

We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole. The fundamental role in the pole cancellation is played by the Wess-Zumino-Witten (WZW) coupling. Tarasov, Venugopalan (2022)

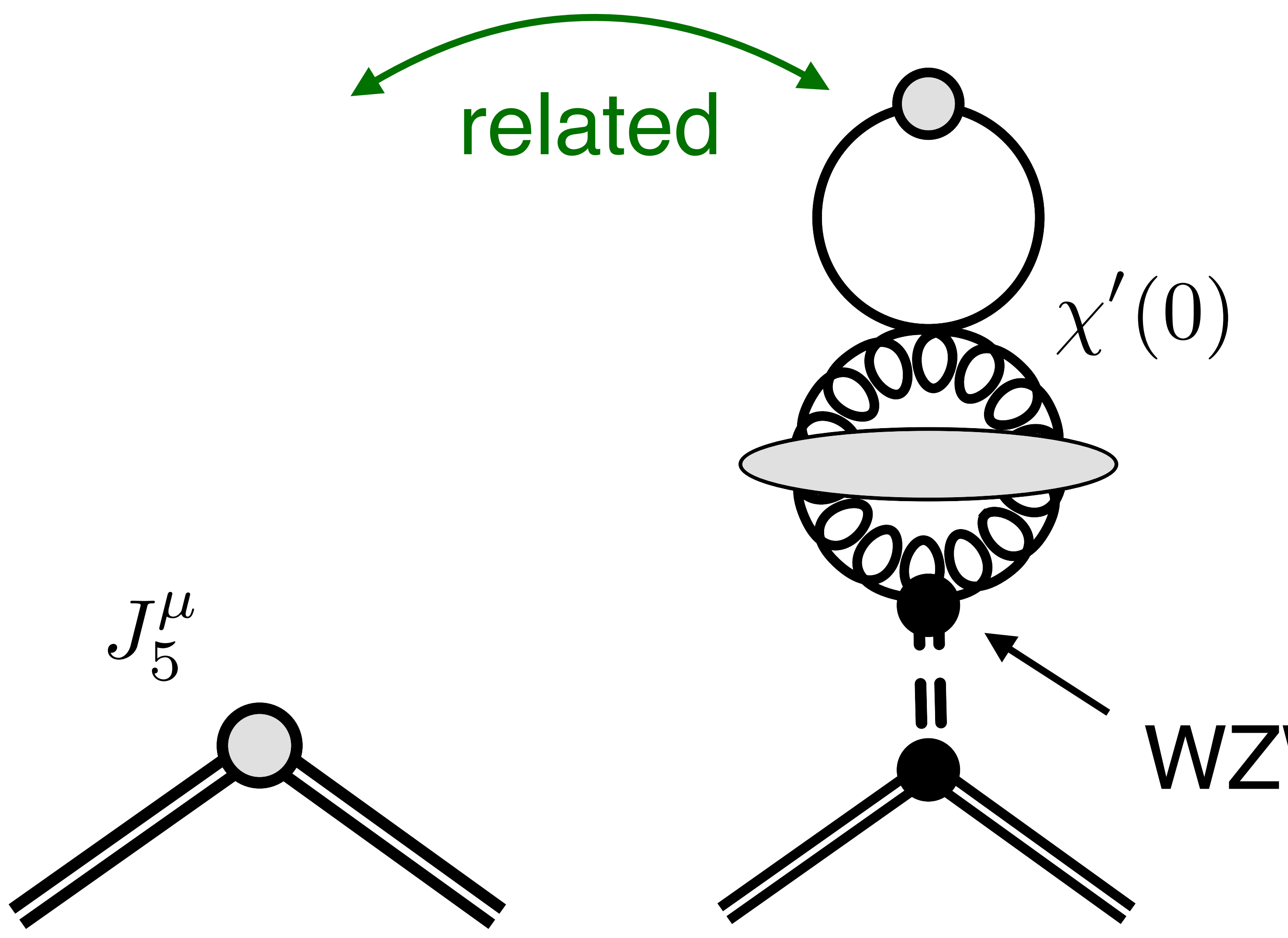




This mechanism relates the helicity structure of the proton to the topology of the QCD vacuum

↳ possibility to detect sphaleron-like topological transitions at EIC

Infrared pole cancelation



From the cancellation of the anomaly pole, using Goldberger-Treiman relation one can relate helicity and the QCD topological susceptibility - topological screening

$$\boxed{\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\sqrt{\chi'(0)}}$$

Shore, Veneziano (1992)

- The first moment of g_1 is determined by the non-perturbative effects
- Can this relation be checked on the lattice?

$$\langle P,S|J_5^\mu|P,S\rangle = M_N S^\mu \Sigma(Q^2) = 2M_N S^\mu a_0$$

$$a^0|_{Q^2=10GeV^2} = 0.33 \pm 0.05$$

Gives a natural resolution of the spin crisis

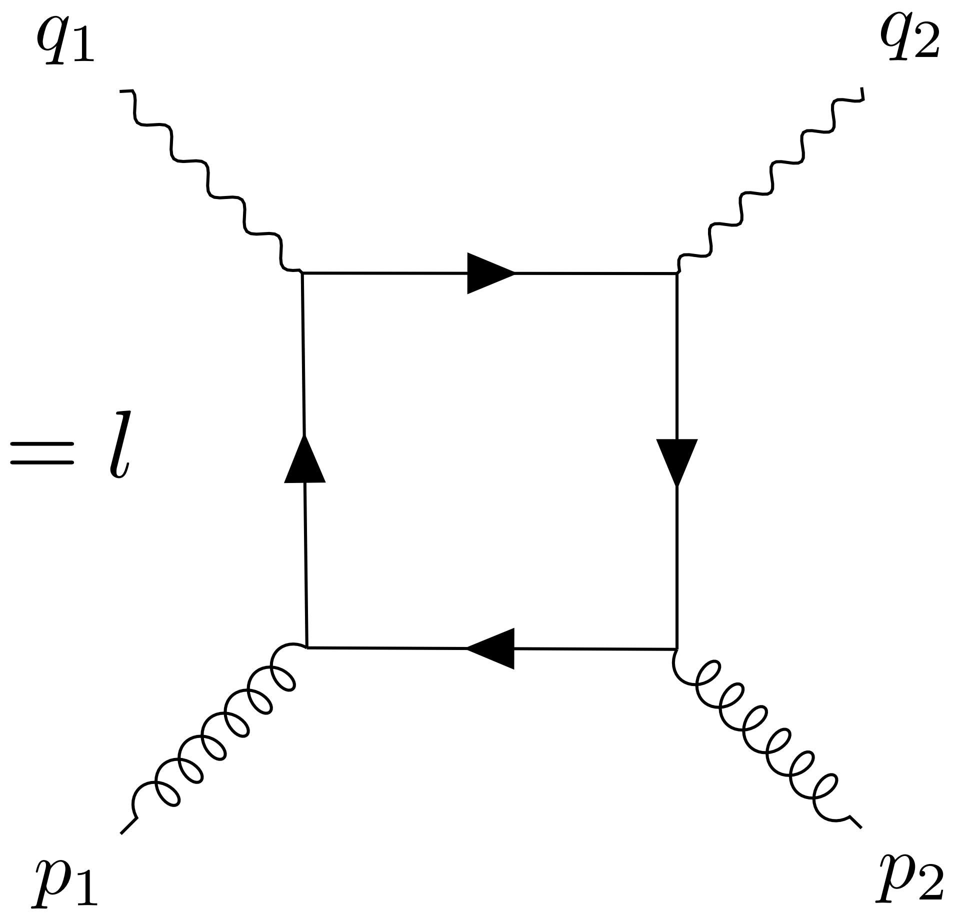
In agreement with COMPASS ($a^0|_{Q^2=3GeV^2} = 0.35 \pm 0.08$) and HERMES data ($a^0|_{Q^2=5GeV^2} = 0.330 \pm 0.064$)

Shore (2007), Narison (2021)


Shore


Veneziano

Anomaly and the DGLAP evolution



Calculation of the box diagram in the Feynman diagram approach at $l^2 \neq 0$

$$P_2 - P_1 = q_1 - q_2 = l$$

S. Bhattacharya, Y. Hatta, W. Vogelsang (2022-2023)

$$\mathcal{J}^\alpha \equiv -\epsilon^{\alpha\beta\mu\nu} P_\beta \mathrm{Im} T^{\mathrm{asym}}_{\mu\nu}$$

$$\mathcal{J}^\alpha|_{\mathrm{box}} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\Delta P_{qg}\ln\frac{Q^2}{-l^2} + \delta C_g^{\mathrm{off}}\right)\otimes \Delta G(x_B)\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5\right] u(P_1)$$

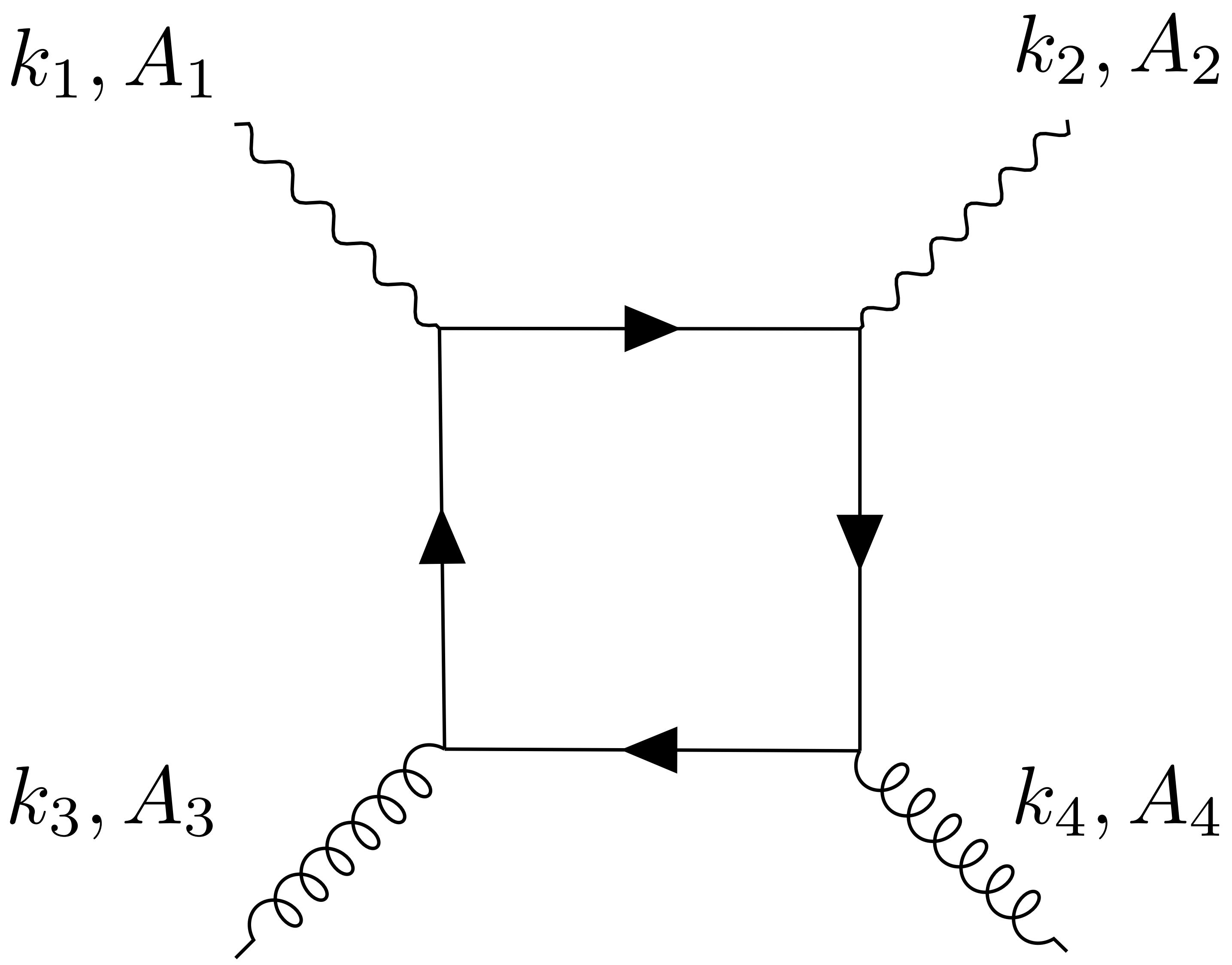
DGLAP evolution, ΔG distribution

Anomaly pole previously observed in the worldline approach, $F\tilde{F}$ operator

According to this result, there are two independent contributions associated with $\Delta G \propto \langle P, S\,|\,F^{+i}F^{+j}\,|\,P, S\rangle$ and $F\tilde{F}$. But the relation between two quantities might be much deeper:

- There is a l^2 scale in the DGLAP log, but this scale defines the anomaly effects
- First moment of the JM operator coincides with the Chern-Simons current in the axial gauge: $$\int_0^1 dx \Delta G(x) = -\frac{1}{2}\langle P, S|K^+|P, S\rangle$$ at the same time $$\partial_\mu K^\mu = \frac{\alpha_s}{4\pi}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$
- If we add higher twist/sub-sub-eikonal corrections, can we relate ΔG to the operator $F\tilde{F}$?

Worldline calculation of the box diagram



- To resolve an ambiguity introduced by the light-cone expansion, we need to perform a complete calculation of the box diagram in the most general kinematics, which can be efficiently done in the worldline approach
- In the worldline approach all loop momenta can be integrated out without making any eikonal/light-cone expansion

$$\Gamma_{\rm scal}(k_1;\ldots;k_4) = g^4 \int_0^\infty \frac{dT}{T}(4\pi T)^{-\frac{D}{2}} e^{-m^2 T} \int_0^T \prod_{i=1}^{4} d\tau_i Q_4(\dot{G}_{ij}) \exp\left[\sum_{i,j=1}^{4} \frac{1}{2} G_{ij} k_i \cdot k_j\right]$$

Ahmadiniaza, Schuberta, Villanueva (2013)

- The result is written in terms of integrals over proper-time variables τ, related to Feynman parameters, and worldline propagators. Integrals over τ define the kinematics of the diagram

$$G^{ij} = |\tau_i - \tau_j| - \frac{1}{T}(\tau_i - \tau_j)^2$$

- Dependence on background fields A is contained in

$$Q_4 = Q_4^4 + Q_4^3 + Q_4^2 + Q_4^{22}$$

What is the correct operator?

- The worldline approach relates to gauge invariance, since it leads to the absorption of background fields into field strength tensors

Strassler (1992)
Ahmadiniaza, Lopez-Arcos, Lopez-Lopez, Christian Schubert (2020)

$$Q_4^4 = \dot{G}_{12}\dot{G}_{23}\dot{G}_{34}\dot{G}_{41}F_{\mu\nu}(k_1)F^{\nu\rho}(k_2)F_{\rho\sigma}(k_3)F^{\sigma\mu}(k_4)$$
$$+\dot{G}_{23}\dot{G}_{31}\dot{G}_{14}\dot{G}_{42}F_{\mu\nu}(k_2)F^{\nu\rho}(k_3)F_{\rho\sigma}(k_1)F^{\sigma\mu}(k_4)$$
$$+\dot{G}_{31}\dot{G}_{12}\dot{G}_{24}\dot{G}_{43}F_{\mu\nu}(k_3)F^{\nu\rho}(k_1)F_{\rho\sigma}(k_2)F^{\sigma\mu}(k_4)$$

- In the off-forward kinematics with $l^2 = 0$, a standard result for the handbag diagram with its characteristic $\log(Q^2)$ appears in both the DGLAP and ERBL regions

Tarasov, Venugopalan, in progress

Ji, Osborne (1998)
Friot, Pire, Szymanowski (2007)

- In the low-energy limit operator $F\tilde{F}$ appears

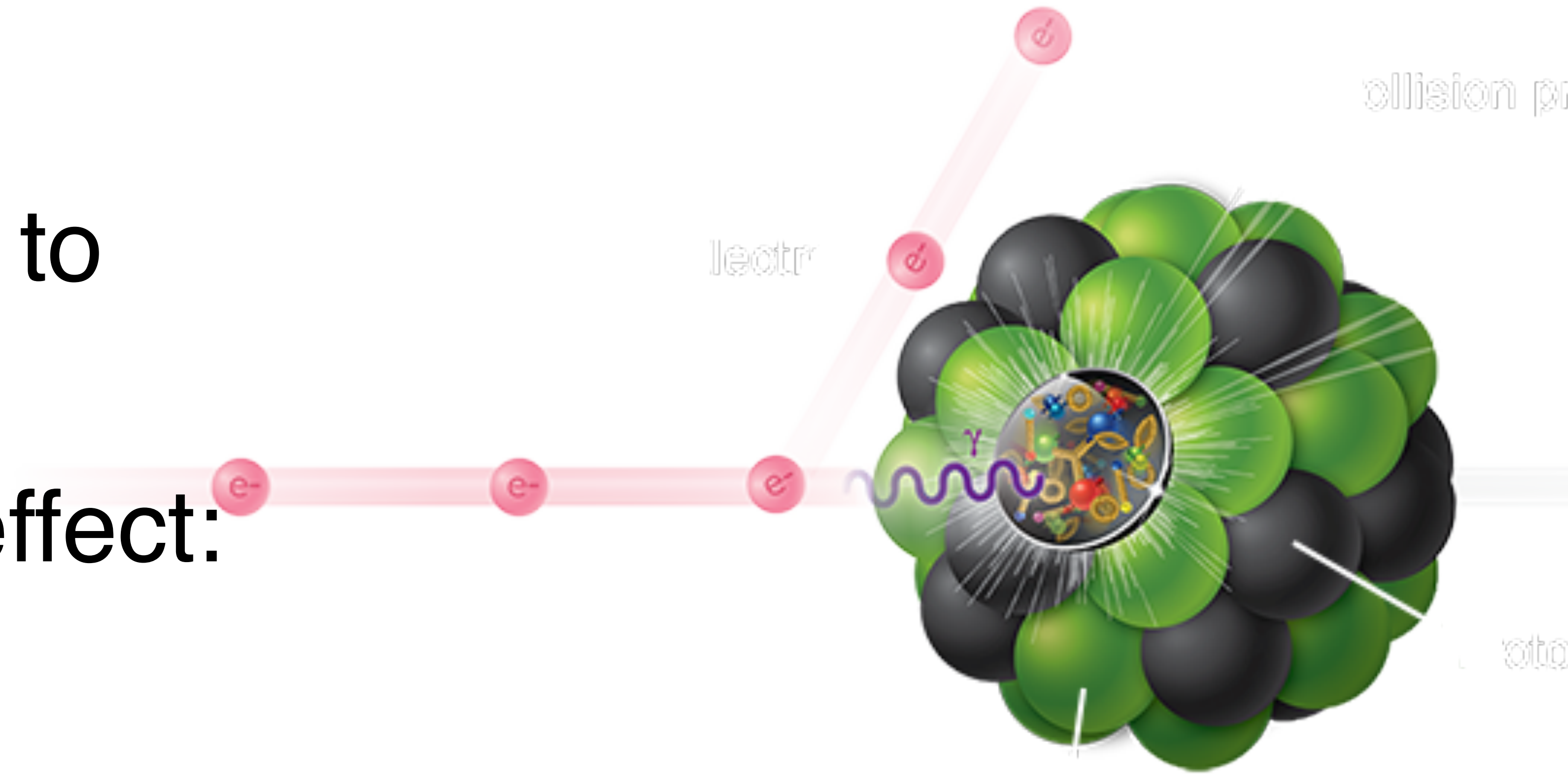
$$\Gamma_4 = \frac{2g^4}{(4\pi)^2 m^4}\left\{\frac{1}{2\cdot 4^2\cdot 3^2}(F^{\mu\nu}F_{\nu\mu})^2 + \frac{1}{8\cdot 45}F^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu}\right\}$$
$$= \frac{g^4}{(4\pi)^2 m^4}\left\{\frac{7}{180}(E^2 - B^2)^2 + \frac{1}{45}(E\cdot B)^2\right\}$$

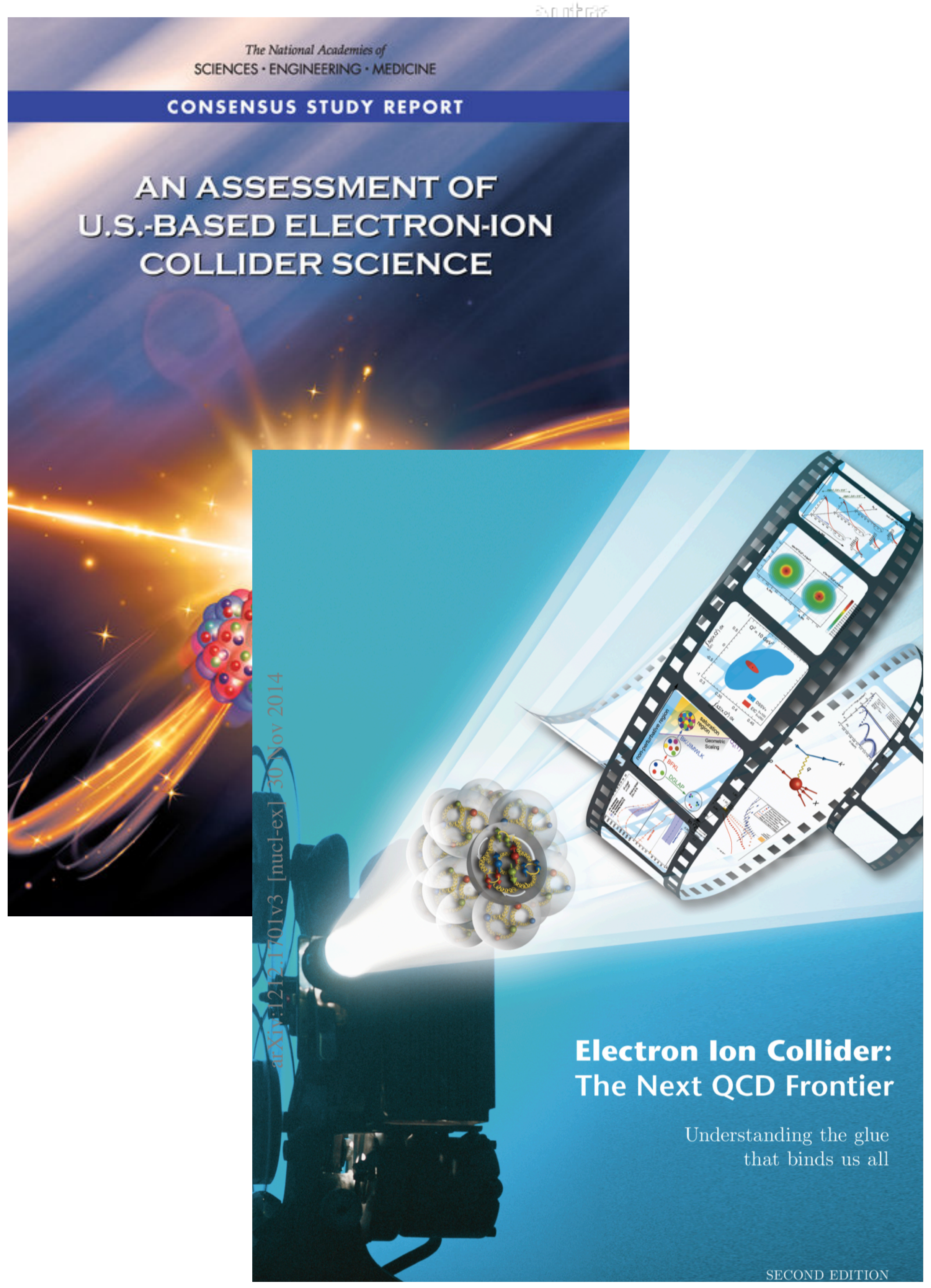
Strassler, (1992)

- Using this approach one can understand the relation between $\Delta G \propto \langle P, S\,|\,F^{+i}F^{+j}\,|\,P, S\rangle$ and $F\tilde{F}$

Summary

- Calculation of sub-eikonal corrections in the eikonal expansion is necessary to achieve high accuracy in phenomenological estimates of observables
- For some observables sub-eikonal corrections constitute the leading order effect: helicity evolution at small-x
- However, for some observables the eikonal/light-cone expansion breaks down
- Example: contribution of the triangle anomaly in DIS and DVCS. The anomaly manifests itself as an infrared pole and is defined by a matrix element of an operator $F\tilde{F}$
- The cancellation of the pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD. This relates corresponding observables to the properties of the QCD vacuum
- Fundamental role of the WZW term both in topological mass generation of the η' and in the cancellation of the infrared pole
- Relation between eikonal expansion (ΔG distribution) and the anomaly (operator $F\tilde{F}$) and the role of the higher twist/sub-sub-eikonal corrections is not fully understood, but can be efficiently explored in the worldline approach





Thank you for your attention!