Color neutralization in JIMWLK and an improved Gaussian model

Haowu Duan

North Carolina State University
In preparation, with Vladimir Skokov, and Cameron Stephen

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Overview

- Introduction
  - Eikonal Scattering, MV(McLerran-Venugopalan) model
  - What is color neutralization?
  - Color neutralized model as improved initial condition for JIMWLK (The goal of this project)
- Color neutralization and JIMWLK evolution
  - Improved Gaussian model with color neutralization
    (Comparison between MV model and color neutralized model)
  - MV model+JIMWLK generate color neutralization
    (Comparison of MV model and color neutralized model, and result of JIMWLK)
  - Observable which are sensitive to parameters in the model
- Some results from evolution

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Eikonal Scattering through a dense target

- Eikonal amplitude is given by the path-ordered Wilson line

$$V = \mathcal{P}\exp\left\{-ig\int dx^+ A_a^-(x^+,\vec{x}_\perp)t^a\right\}$$

- On the time independent gluon field
  - Classical: $-\partial_\perp^2 A^-_{Cov}(x^+,\vec{x}_\perp) = g\rho_{Cov}(x^+,\vec{x}_\perp)$
- The strong gluon field $A_a^-(x^+,x_\perp) = A_a^-[\rho_a]$, for example, the dipole correlator is

$$d(r_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}\Big(V^\dagger(r_\perp)V(0)\Big)\rangle_T = \frac{1}{N_c}\int D[\rho_a]\,\mathrm{Tr}\Big(V^\dagger(r_\perp)V(0)\Big)W[\rho_a]$$

- MV model

$$W[\rho_a] = \exp\left\{-\int dx^+\int d^2x_\perp \frac{\rho^a(x^+,\vec{x}_\perp)\rho^a(x^+,\vec{x}_\perp)}{2\mu_0^2}\right\}$$

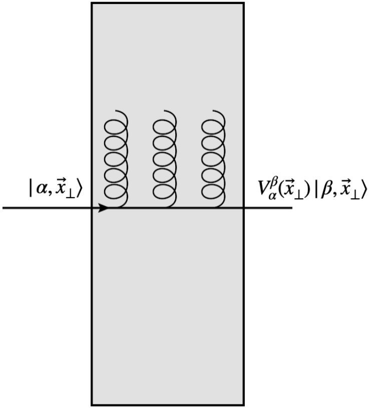

Figure: Eikonal scattering of a quark

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Hadronic structure in MV model

- A hadron at fixed configuration: $|h\rangle \approx |\text{soft gluon}\rangle \otimes |\text{valance charge}(\rho_a(\boldsymbol{x}))\rangle$
- MV model:
  - Classical charge distribution with Gaussian fluctuations.
  - Gluon fields are Coulomb radiation from the classical charge

$$-\partial_\perp^2 A^-_{Cov}(x^+, \vec{x}_\perp) = g\rho_{Cov}(x^+, x_\perp)$$

  - Gluon should be confined, to reflect this fact, we introduce a cut-off to remove long wavelength modes,

$$G_0(k_\perp) = \frac{1}{k_\perp^2} \to \frac{1}{k_\perp^2 + m^2}$$

  - Gaussian noise with "constant" two point function

$$\langle \rho^a(x^+, \vec{x}_\perp)\rho^b(y^+, \vec{y}_\perp)\rangle = \delta^{ab}\delta(x^+ - y^+)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)\mu_0^2(x^+)$$

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What is color neutralization?

- Hadron should be color neutral,

$$\int d^2x_\perp \hat{\rho}^a(\vec{x}_\perp)|h\rangle = 0$$

- Because all hadrons are color neutral, the following generalization should also be true

$$\langle h'| \int d^2x_\perp \hat{\rho}^a(\vec{x}_\perp)|h\rangle = 0$$

- This leads to the constraint for two point function,

$$\int d^2x_\perp d^2y_\perp \langle h|\hat{\rho}^a(\vec{x}_\perp)\hat{\rho}^b(\vec{y}_\perp)|h\rangle = 0$$

- Assuming translational invariant target,

$$\int d^2r_\perp \mu^2(\vec{r}_\perp) = \tilde{\mu}^2(\vec{k}_\perp = 0) = 0$$ Not satisfied by MV model

PRD(2000), C. S. Lam et al

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Modified Gaussian model as initial condition for JIMWLK evolution

- $k_\perp \gg Q_s$
  - $\mu^2(\vec{k}_\perp)$ satisfies BFKL
  - Geometric scaling at $Q_s \leq k_\perp \leq \frac{Q_s^2}{\Lambda_{QCD}}$

    PLB(2001), Iancu et al
- $k_\perp \ll Q_s$
  - $\mu^2(\vec{k}_\perp) \propto \frac{k_\perp^2}{\pi}$

    Nucl.Phys.A (2003), E. Iancu et al
- Interpolation between the two regime happens around Q

  Q is color neutralization scale. In the literature $Q \simeq Q_s$
- Gaussian approximation of JIMWLK evolution

  PLB (2011), A. Dumitru et al.

  JHEP(2012), E. Iancu et al.

- Interpolating scheme with anomalous dimension

$$\mu^2(\vec{k}_\perp) = \mu_0^2 \frac{k_\perp^2}{Q^2\pi} \frac{(\frac{Q^2}{k_\perp^2})^\gamma}{1 + (\frac{Q^2}{k_\perp^2})^\gamma} \tag{1}$$

This model used for fitting

- $\gamma = 1$ is the special case I will use in the rest of the talk

$$\mu^2(\vec{k}_\perp) = \frac{\mu_0^2}{\pi} \frac{k_\perp^2}{k_\perp^2 + Q^2} \tag{2}$$

Nucl. Phys. A (2016), L. McLerran et al

PRL(2022), A. Kovner et al

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Dipole in MV model

- We parameterize the dipole with the following convention

$$d(r) = \exp\Big\{-2\pi\tilde{Q}_s^2\Gamma(r)\Big\}$$

- In numerical calculations, we extract Q_s $(\neq \tilde{Q}_s)$ using,

$$d(r = \frac{\sqrt{2}}{Q_s}) = e^{-\frac{1}{2}}$$

- Most general form of $\Gamma(r)$

$$\Gamma(r) = 2\int \frac{pdp}{2\pi}\frac{1 - J_0(pr)}{(p^2+m^2)^2}\tilde{f}(p), \quad \tilde{f}(p) \text{ is a dimensionless function, in MV model } \tilde{f}(p) = 1$$

- Generalization of Gaussian noise

$$\langle\rho^a(x^+,\vec{x}_\perp)\rho^b(y^+,\vec{y}_\perp)\rangle = \delta^{ab}\mu^2(x^+)\delta(x^+-y^+)\delta^{(2)}(\vec{x}_\perp-\vec{y}_\perp) \Rightarrow \delta^{ab}\mu^2(x^+)\delta(x^+-y^+)f(\vec{x}_\perp-\vec{y}_\perp)$$

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Definition of the models

- Fundamental dipole

$$d(r) = \exp\left\{-2\pi \tilde{Q}_s^2 \Gamma(r)\right\} \quad (3)$$

- MV model

$$\Gamma_I(r) = 2\int \frac{pdp}{2\pi} \frac{1 - J_0(pr)}{(p^2 + m^2)^2} = \frac{r^2}{\pi}\left[\frac{1 - mrK_1(mr)}{2(mr)^2}\right] \quad (4)$$

- MV model, small m asymptotic expression

$$\tilde{\Gamma}_I(r) \approx \frac{r^2}{8\pi} \ln\left(\frac{1}{r^2 m^2}\right) \quad (5)$$

- Color neutralization with $\gamma = 1$

$$\Gamma_{II}(r) = 2\int \frac{pdp}{2\pi} \frac{1 - J_0(pr)}{(p^2 + m^2)^2} \frac{p^2}{p^2 + Q^2} \quad ;\text{we expect } m \ll m_{\text{MV}} \quad (6)$$

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Comparison between MV small m asymptotic expression and exact expression

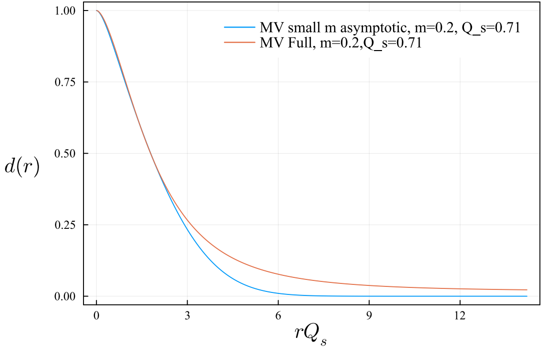

Figure: Start deviate from $rQ_s = 2.5$

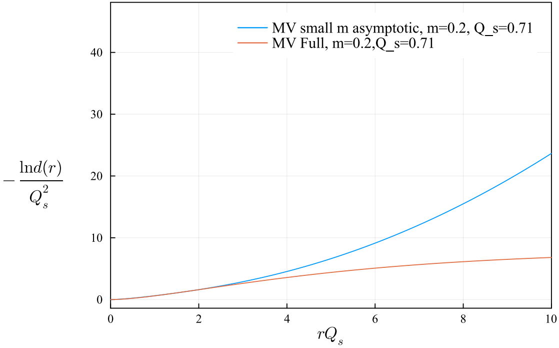

Figure: $\Gamma(r)$ significantly slowed down with color neutralization

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Comparison between MV and Color neutralized model

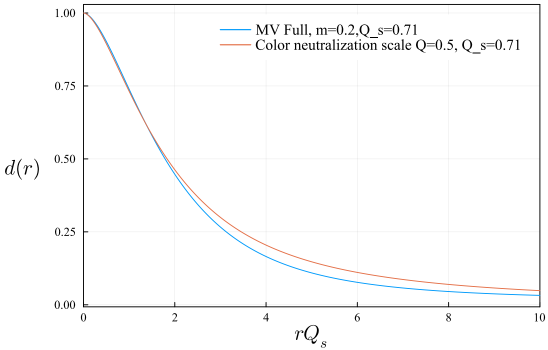

Figure: Dipole not sensitive to different models

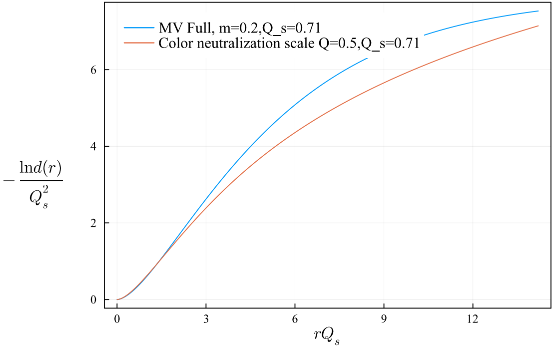

Figure: Difference in $\Gamma(r)$ is more obvious.
$m_{cn} = 0$ was chosen

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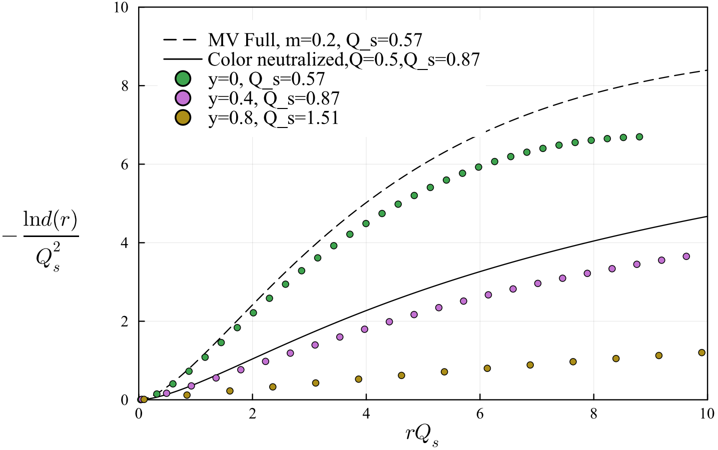
Comparison with JIMWLK evolution (500 configurations)

- Requires large distance data to capture the full behavior of $\Gamma(r)$, requires more configurations.
- Since only two out of 4 parameters: $m, \tilde{Q}_s, Q, \gamma$ were used, the room left to constrain γ and m is limited.



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WW Gluon distribution function

$$xG^{ij}(\boldsymbol{x},\boldsymbol{y}) = 8\pi\langle \mathrm{Tr}\, A_i(\boldsymbol{x}) A_j(\boldsymbol{y})\rangle$$

with the gauge field in the lightcone gauge,

$$A^i = \frac{1}{ig} V^\dagger(\boldsymbol{x})\partial^i V(\boldsymbol{x})$$

$$xG^{(1)}_{ij}(\boldsymbol{r}) = \frac{1}{2}\delta_{ij} xG^{(1)}(\boldsymbol{r}) - \frac{1}{2}\left(\delta_{ij} - 2\frac{r_i r_j}{r^2}\right) xh^{(1)}(\boldsymbol{r})$$

In a Gaussian model, with asymptotically small m

$$xG^{(1)}(\boldsymbol{r}) = \frac{4N_c S_\perp}{\alpha_s}\left(1 - D(\boldsymbol{r})\right)/r^2,$$

$$xh^{(1)}(\boldsymbol{r}) = \frac{4N_c S_\perp}{\alpha_s}\left(1 - D(\boldsymbol{r})\right)/\Gamma(r^2) \;\; ; \Gamma(r^2) \approx \frac{r^2}{8\pi}\ln\left(\frac{1}{r^2 m^2}\right)$$

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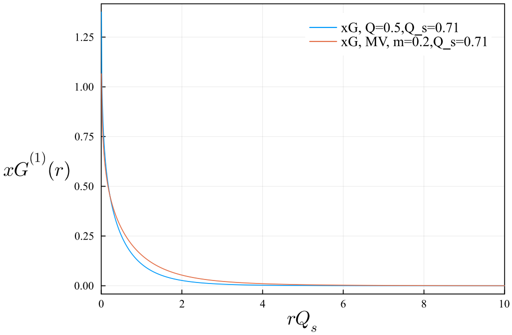
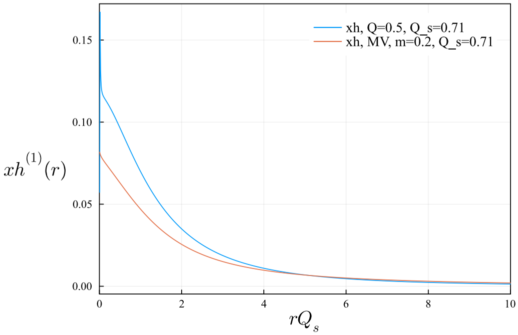
$xG^{(1)}(x_\perp)$ and $xh^{(1)}(x_\perp)$



(a) Not very sensitive



(b) Difference shifted to small distance

Figure: Color neutralized model with $\gamma = 1$ and MV model with same Q_s

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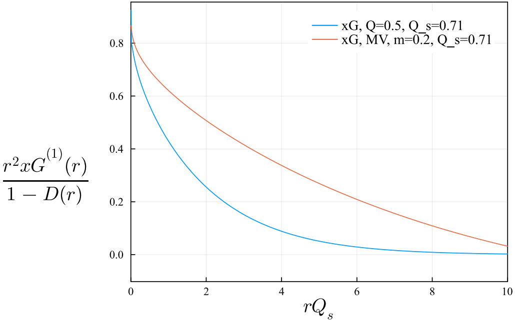
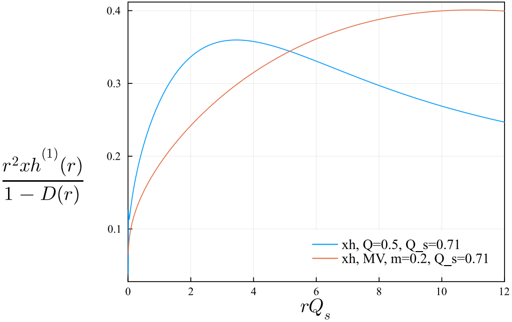
$\frac{r^2 xG^{(1)}(x_\perp)}{1-D(r)}$ and $\frac{r^2 xh^{(1)}(x_\perp)}{1-D(r)}$



(a) Can be used to cross check



(b) Sensitive overall

Figure: Color neutralized model with $\gamma = 1$ and MV model with same Q_s

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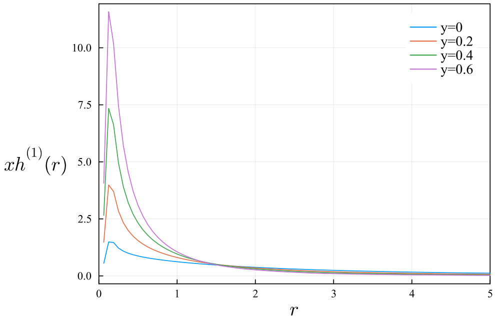
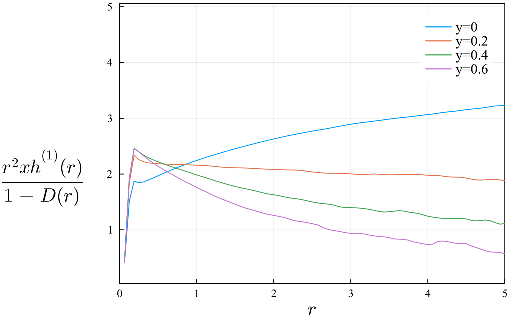
$xh^{(1)}$ behavior from JIMWLK



(a) Can be used to cross check



(b) requires more statistics

Figure: Data from 500 configuration, with L=32 and N=512

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Summary

- Gaussian model with color neutralization
  - At large momentum, $\mu^2(k_\perp)$ satisfys BFKL behavior
  - Small momentum modes will be suppressed, $\mu^2(k_\perp) \to 0$

- We identify $\frac{r_\perp^2 xh^{(1)}(r_\perp)}{1-D(r_\perp)}$ as good observable for fitting $(A, m, Q, \gamma$, 4 parameters in total)

$$\frac{r_\perp^2 xh^{(1)}(r_\perp)}{1 - D(r_\perp)} = Ar^2 \frac{\Gamma^{(2)}(r_\perp^2)}{\Gamma(r_\perp^2)} \quad ; \Gamma^{(2)}(r_\perp^2) = \frac{\partial^2 \Gamma(r_\perp^2)}{\partial(r_\perp^2)^2}$$

- We need to completely understand the rapidity dependence of parameters.

- Simulation of JIMWLK evolution with improved initial condition

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