High-energy dipole scattering amplitude off a proton
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from low-energy light-cone wave function & BK evolution

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for the SURGE collaboration

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BFKL / BK/ JIMWLK evolution eqs are differential eqs, require initial condition:
dipole in Large-Nc approximation

$$\partial_Y S(\vec{x},\vec{y}) = \frac{\alpha_s N_c}{2\pi^2} \int d^2z \, \frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2\,(\vec{z}-\vec{y})^2} \left[S(\vec{x},\vec{z})\, S(\vec{z},\vec{y}) - S(\vec{x},\vec{y})\right]$$

- issues with current implementations of I.C. (next slide)
- body of empirical knowledge of "large-x" structure of proton, can it be incorporated ?
- in the future: proton LCwf from the lattice? (though only x>~ 0.1) If available, how can we use it to construct I.C. for small-x ?

Initial conditions for (rc)BK fits to small-x inclusive HERA data :

$$N(r;x_0) = 1 - \exp\left[-\frac{1}{4}\left(r^2 Q_{s0}^2\right)^\gamma \log\left(\frac{1}{r\Lambda} + e e_c\right)\right]$$

AAMQS: PRD 80 (2009)
EPJ-C 71 (2011)
Lappi, Mäntysaari: PRD 88 (2013)

- fits prefer $\gamma \sim 1.1 - 1.2$; violates pQCD prediction $N(r) \sim r^2$ for small r^2,
  positivity of Fourier transform (UGD)
  issues with generalization to nuclei (Albacete et al, 1209.2001)
- imposed at x_0=0.01, requires re-fit for other x_0
  no prediction for dependence on x_0 as required for NLO (e.g. Ducloue et al, 1902.06637)
- no connection to "large"-x proton structure; parameters fitted to small-x data itself,
  rcBK + LO photon w.f. $\longrightarrow$ $\chi^2/N_{dof} \sim 1$, $\longrightarrow$ "perfect fit" !?
  no statistical evidence for NLO corrections!
  (they'd either make matters worse or they could be absorbed into a retune of the initial condition)

The proton on the light front

The proton on the light front (three quark Fock state; L.C. time $x^+ = 0$)

$$|P\rangle = \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2k_1 d^2k_2 d^2k_3}{(16\pi^3)^3} 16\pi^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$
$$\times \psi(k_1, k_2, k_3) \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1; p_2, i_2; p_3, i_3\rangle$$

+ higher Fock states

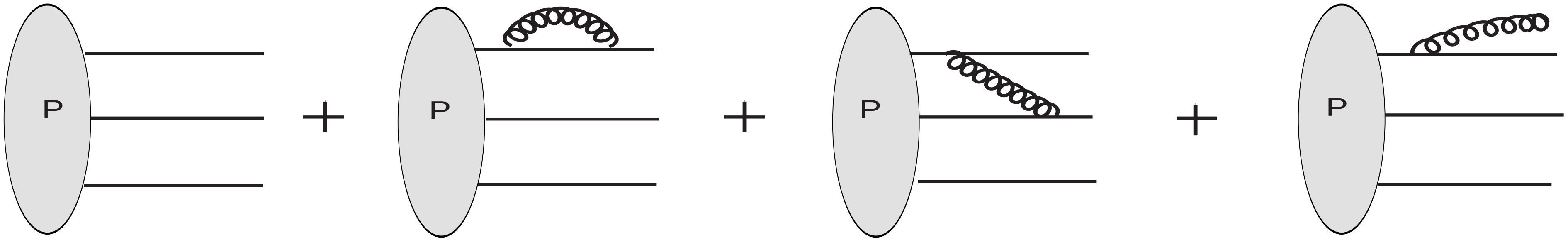
P. Lepage & Brodsky, 1979 - ....
Brodsky, Pauli, Pinsky, PR (1998)

* Fock space amplitude ψ is gauge invariant, universal, and process independent, encodes the non-perturbative structure of hadrons (QCD eigenstates)

* fix 3q wave function Ψqqq from low-energy / "large"-x proton structure, augment by Ψqqqg via LCpt
* evolution adds additional soft gluons into wave function

Now to $|P> \sim \psi_{qqq}|qqq> + \psi_{qqqg}|qqqg>$

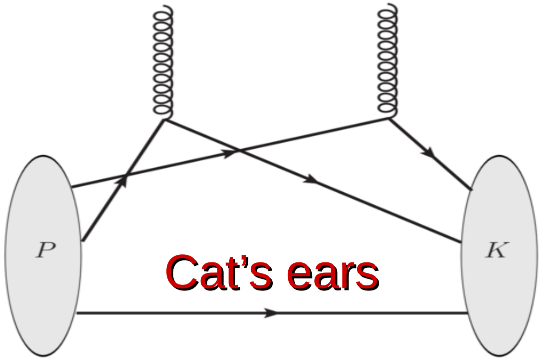
computed in perturbation theory, 1-gluon emission / exchange,
w/o employing $x_g \to 0$ approximation;
built on top of qqq "low-energy structure" of the proton



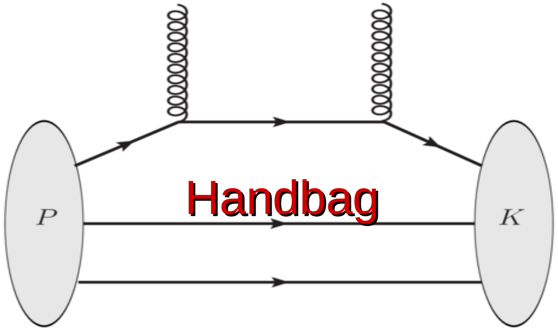
Color charge correlators:

$$\rho^a(\vec{x}) = \int \mathrm{d}x^- \, J^{+a}(x^+ = 0, x^-, \vec{x})$$

$$\langle \rho^a(\vec{q}_1)\, \rho^b(\vec{q}_2) \rangle \sim g^2\, \delta^{ab}\, G_2(\vec{q}_1, \vec{q}_2)$$



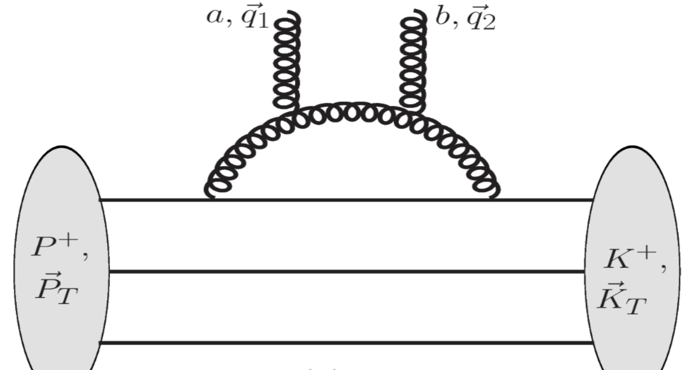
2-body GTMD / Wigner



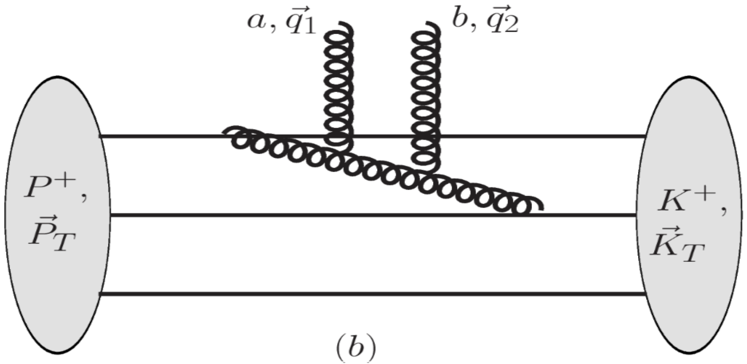
1-body GTMD / Wigner

LO diagrams

NLO color charge correlators

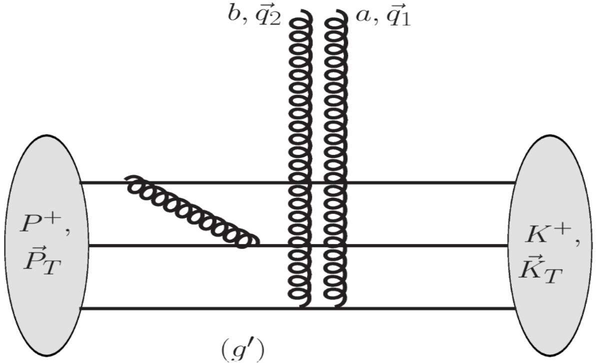


(a)

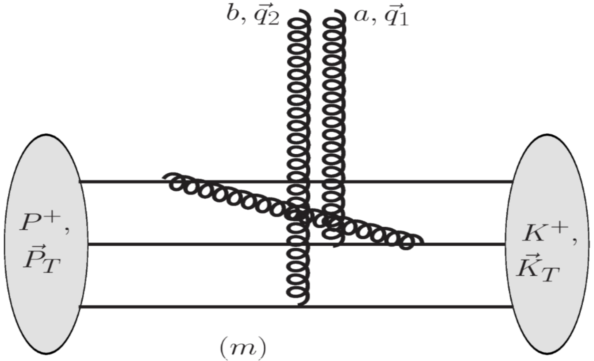


(b)

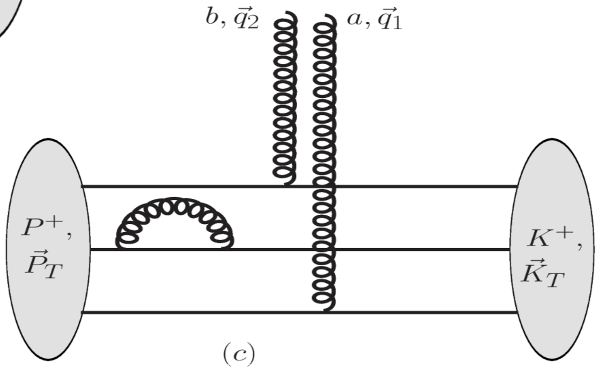
notes:
* soft & collinear div.
* UV divergences cancel
* Ward satisfied



(g')



(m)



(c)

+ many more

Twenty thousand leagues under the seas

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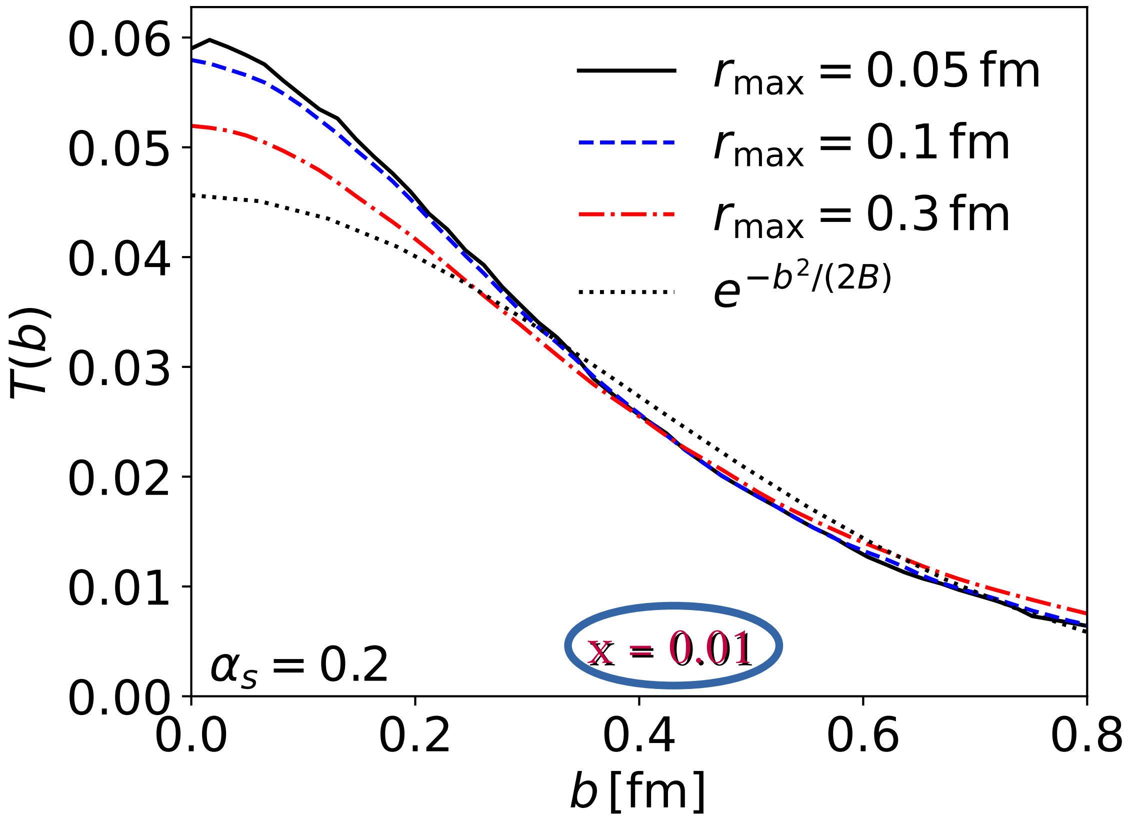
from $x = 0.1 \longrightarrow x = 10^{-4}$ (and below) ?

selected results from arXiv:2303.16339
(w/ Heikki Mäntysaari & Risto Paatelainen)

Effective transverse “profile”

- $T(b) \sim \int \mathrm{d}^2 r \; N(\vec{r}, \vec{b}; x)$
- r and x-dependent
- close to the Gaussian profile that was fitted to data



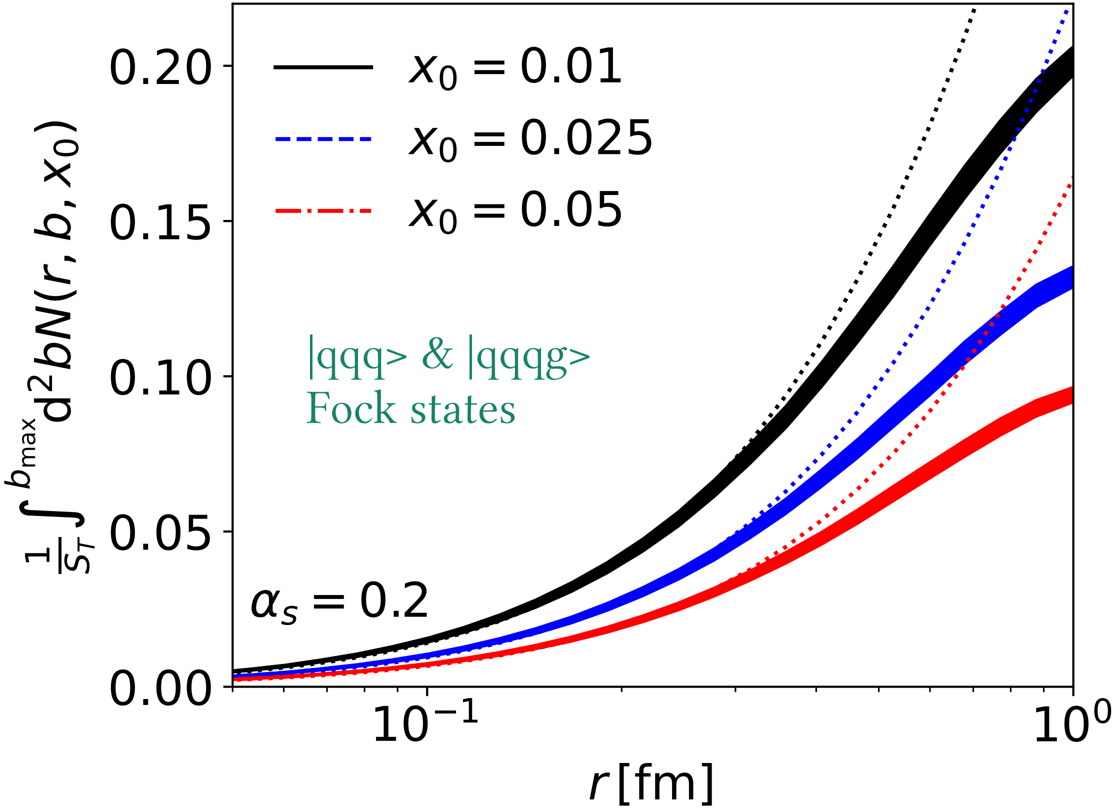
r-dependence of N(r;x)

- ~r^2 in perturbative regime, as expected
- analytic parameterization for r <~ 0.25 fm:

$$N(r) = 1 - \exp\left[-\frac{1}{4}\left(r^2 Q_{s0}^2\right)^\gamma \log\left(\frac{1}{r\Lambda}+1\right)\right]$$

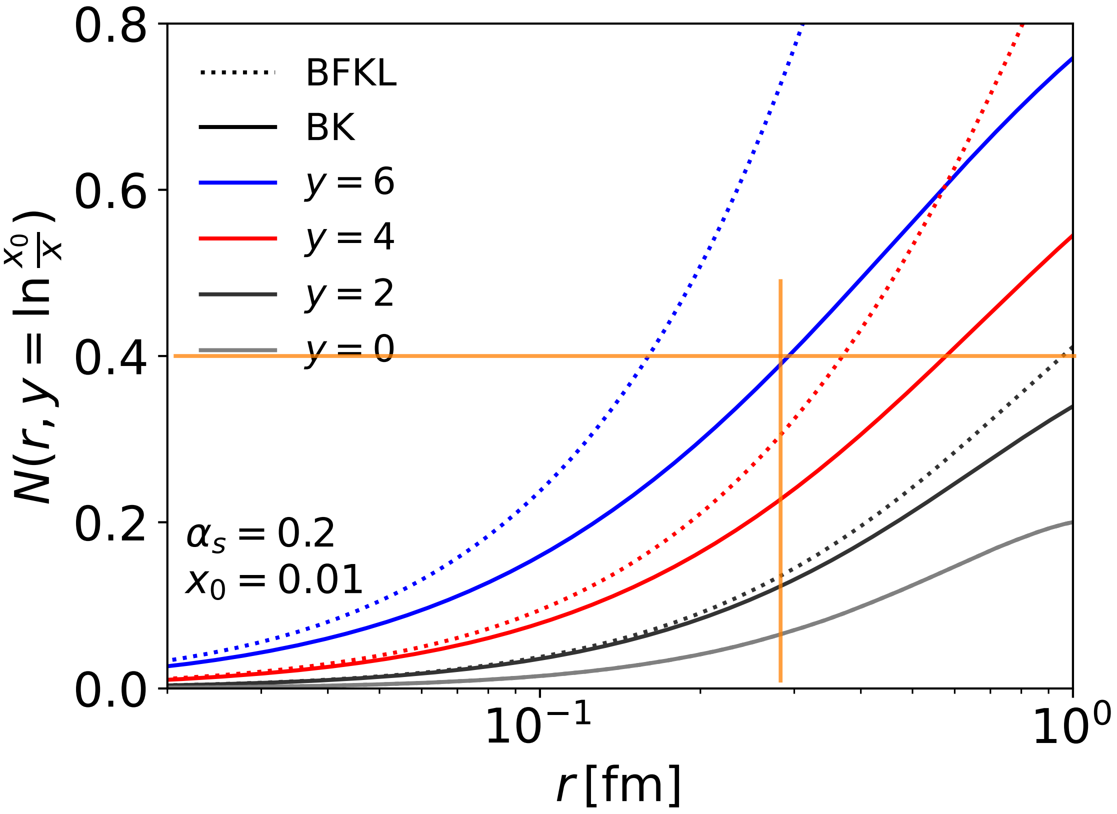
Q_{s0}, γ fitted to numerical data,
Λ = 0.24 GeV was fixed

x	$Q^2_{s,0}$ [GeV^2]	γ
0.01	$0.100^{+0.004}_{-0.004}$	$1.001^{+0.001}_{-0.001}$
0.025	$0.066^{+0.003}_{-0.003}$	$0.998^{+0.001}_{-0.001}$
0.05	$0.047^{+0.002}_{-0.002}$	$0.997^{+0.001}_{-0.001}$



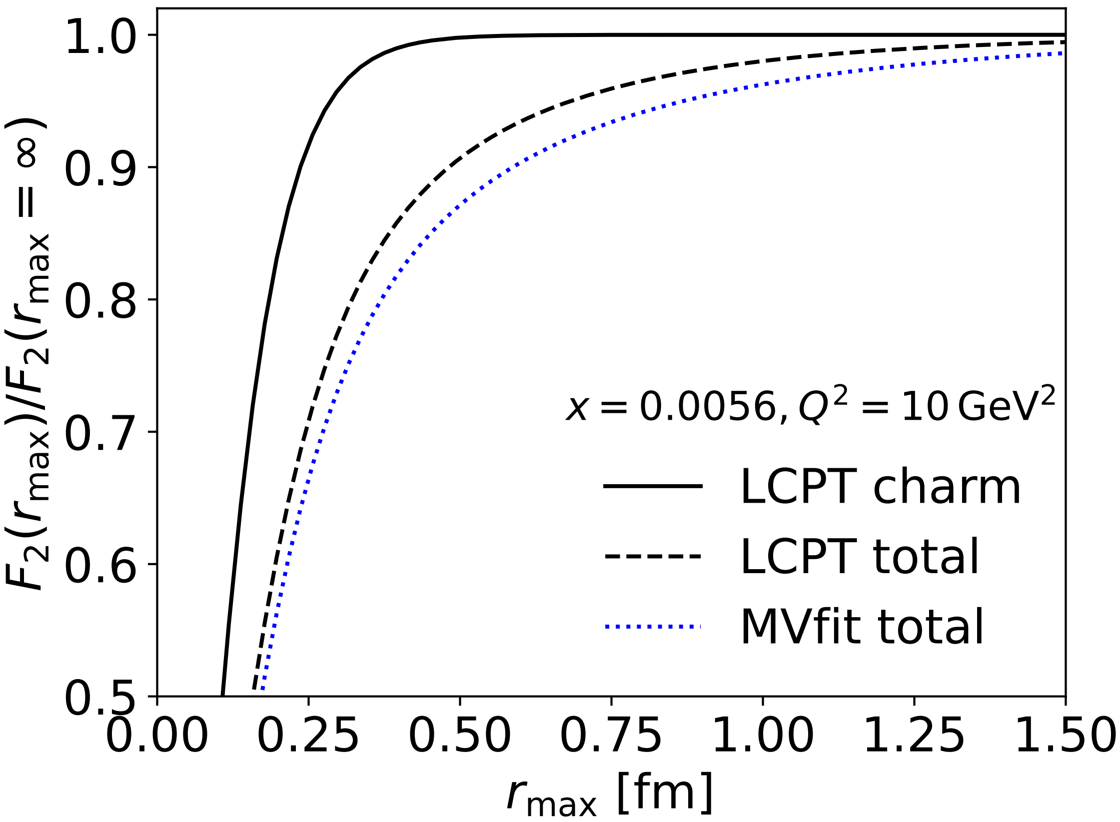
LL evolution of N(r;x) from x_0 = 0.01

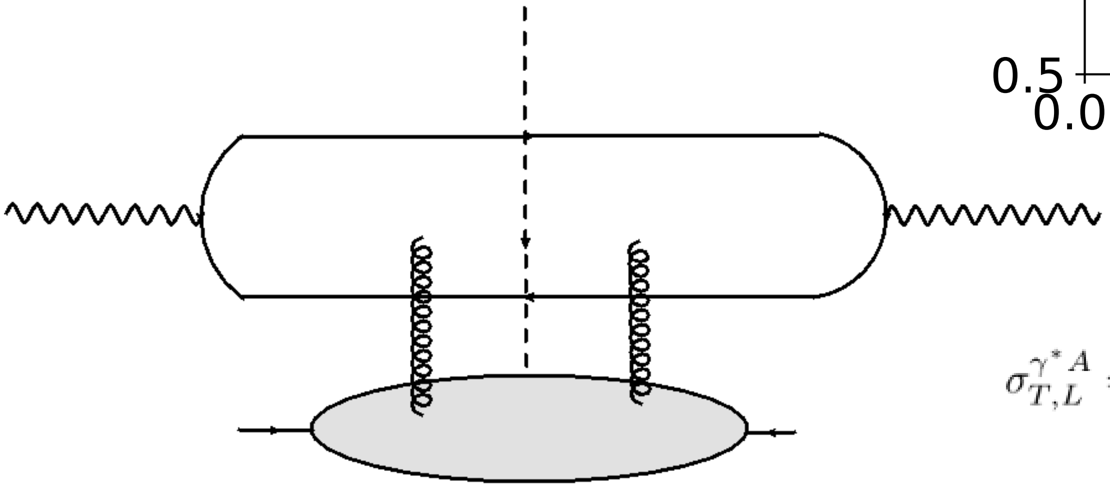
- adds Fock states with addl soft gluons
- BFKL only for y <~ 2
- saturation requires y~6 ↔ x ~ 2E-5: r_s=0.3fm, Qs=1GeV



DIS cross section

- total (light q) X-sec sensitive to large dipoles
- charm X-sec saturated by perturbative dipoles $\longrightarrow$ focus on charm





$$\sigma_{T,L}^{\gamma^* A} = 2\sum_f \int \mathrm{d}^2\mathbf{b}\,\mathrm{d}^2\mathbf{r}\,\mathrm{d}z\,\left|\Psi^{\gamma^*\to q\bar{q}}(\mathbf{r},z,Q^2)\right|^2 N(\mathbf{r},\mathbf{b},\bar{x})$$

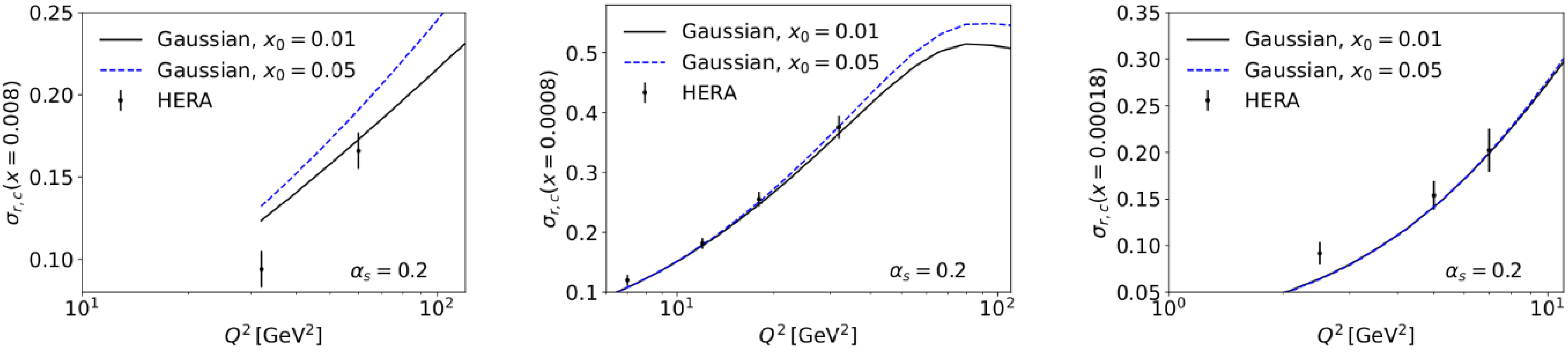
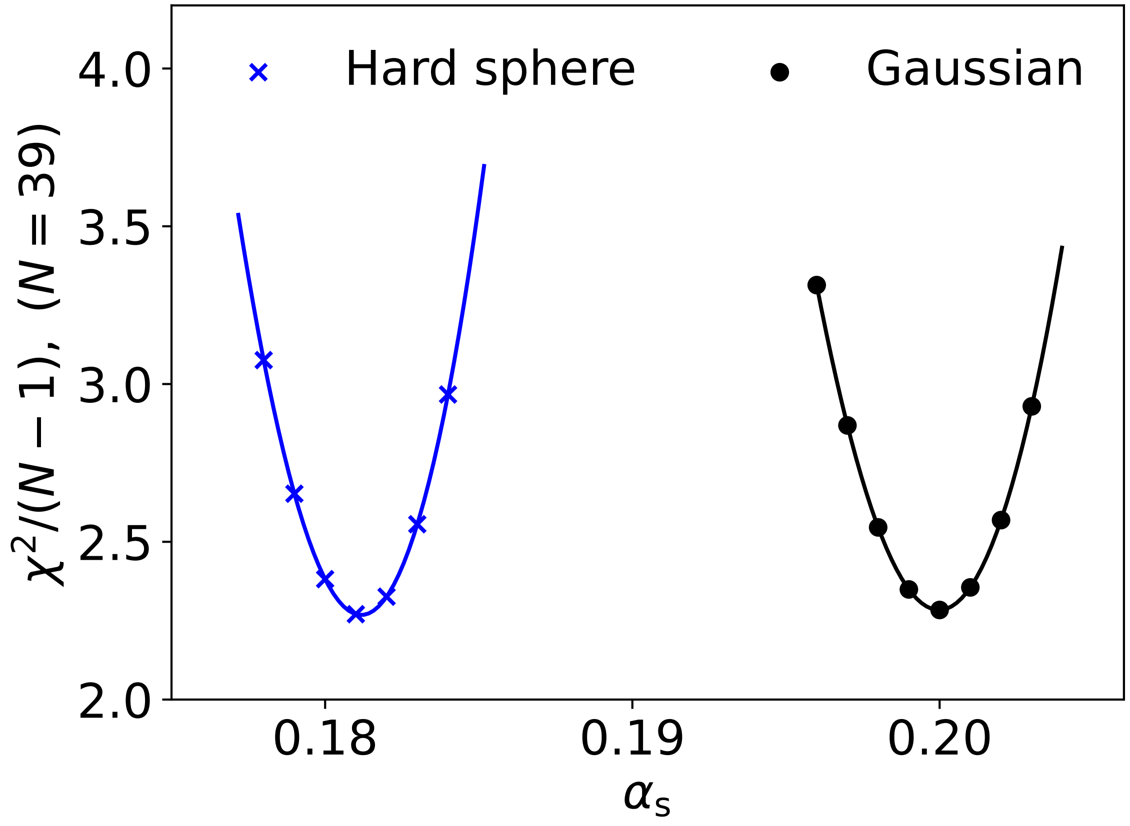


FIG. 9: Charm production reduced cross section at $\sqrt{s} = 318\,\text{GeV}$ compared to the HERA data. Results are shown in the region where $\bar{x} \leq 0.01$.

- decent agreement, only fit parameter is (fixed) α_s, no re-tune of Brodsky & Schlumpf non-perturbative qqq wave function was performed !
- we covered 3 decades in energy from x ~ 0.1 ⟶ x ~ 10^{-4}

Statistical evidence for systematic corrections :

- HERA data is very accurate!
- χ^2/N_{dof} = 2.27 for N_{dof}=38: p-value (integral over χ^2 distribution) ~ 10^{-5} !
- evidence for moderate but *systematic* corrections: photon w.f. (here: LO) evolution eqn (here: LL-BK) initial condition



Done:

- tables & analytical parameterizations for b-averaged $N_0(r;x)$ using Brodsky & Schlumpf non-pert. qqq LCwf + LCpt one gluon correction
- tables & interpolation routines for $N_0(\vec{r},\vec{b};x)$, incl. angular dependence !

Later on (if requested):

- option to try other non-pert. qqq wave functions
- imaginary part due to perturbative C-odd ggg exchange
- etc...

Backup

Model LFwf for the proton (Brodsky & Schlumpf, PLB 329, 1994)

$$\psi_{\mathrm{H.O.}}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) = N_{\mathrm{H.O.}} \exp(-\mathcal{M}^2/2\beta^2) \ ,$$

$$\psi_{\mathrm{Power}}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) = N_{\mathrm{Power}}(1+\mathcal{M}^2/\beta^2)^{-p} \ .$$

$$\mathcal{M}^2 = \sum_{i=1}^{3} \frac{\vec{k}^2_{\perp i} + m^2}{x_i}$$

m = 0.26 GeV, β = 0.55 for H.O. wf

m = 0.263, β = 0.607, p = 3.5 for PWR wf

With these parameters they fit:

- proton radius $R^2 = -6 \left. \frac{dF_1(Q^2)}{dQ^2} \right|_{Q^2=0} = (0.76 \ \mathrm{fm})^2$

- proton / neutron magnetic moments $1 + F_2(Q^2 \to 0) = 2.81 \ / \ -1.66$

- axial vector coupling $g_A = 1.25$

The definition of χ^2

If ν independent variables x_i are each normally distributed with mean μ_i and variance σ_i^2, then the quantity known as *chi-square*[2] is defined by

$$\chi^2 \equiv \frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} + \cdots + \frac{(x_\nu-\mu_\nu)^2}{\sigma_\nu^2} = \sum_{i=1}^{\nu} \frac{(x_i-\mu_i)^2}{\sigma_i^2} \qquad (1)$$

Note that ideally, given the random fluctuations of the values of x_i about their mean values μ_i, each term in the sum will be of order unity. Hence, if we have chosen the μ_i and the σ_i correctly, we may expect that a calculated value of χ^2 will be approximately equal to ν. If it is, then we may conclude that the data are well described by the values we have chosen for the μ_i, that is, by the hypothesized function.

The χ^2 distribution

The quantity χ^2 defined in Eq. 1 has the probability distribution given by

$$f(\chi^2) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} e^{-\chi^2/2} (\chi^2)^{(\nu/2)-1}$$

This is known as the χ^2-*distribution with* ν *degrees of freedom.* ν is a positive integer.