Detectors in weakly-coupled field theories arXiv:2209.00008

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small-x physics in scalar ϕ^4 theory

SURGE proposal:

- **Small x evolution:** LO evolution is not sufficient for accuracy. Need the NLO and beyond. How to consistently implement resummation in non-linear evolution and match small with large x, relevant for EIC kinematic regime ?

Talk by SCH (McGill) @ SURGE 1 workshop, Brookhaven, June 29 2023

Motivation

Scalar ϕ^4 theory lacks gluons:
we don't expect saturation-like behavior.

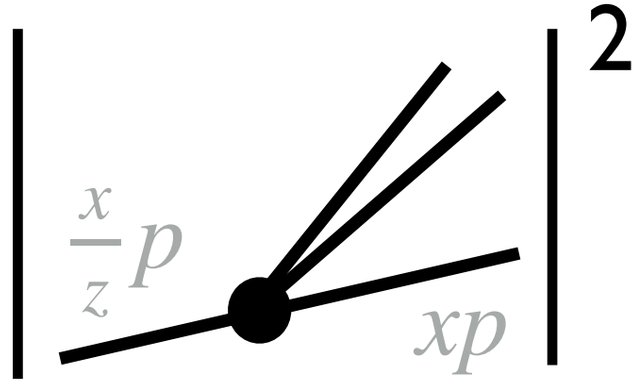
Linear physics is still interesting (DGLAP *and* BFKL-like).

Q: what is the Pomeron intercept in scalar ϕ^4?

DGLAP evolution of ϕ parton distribution:

$$\mu\frac{d}{d\mu}f_\phi(x;\mu) = \int_x^1 \frac{dz}{z} P_{\phi\to\phi}(z) f_\phi(\frac{x}{z};\mu)$$

$$P_{\phi\to\phi}(z) = \frac{\lambda^2}{(4\pi)^4}\left[1-z - \frac{1}{6}\delta(1-z)\right] + O(\lambda^3)$$



DGLAP kernel lacks 1/z singularity from soft emissions.

$\Rightarrow$ expect $\lim_{x\to 0} xf_\phi(x) = 0$: small-x physics stays linear.

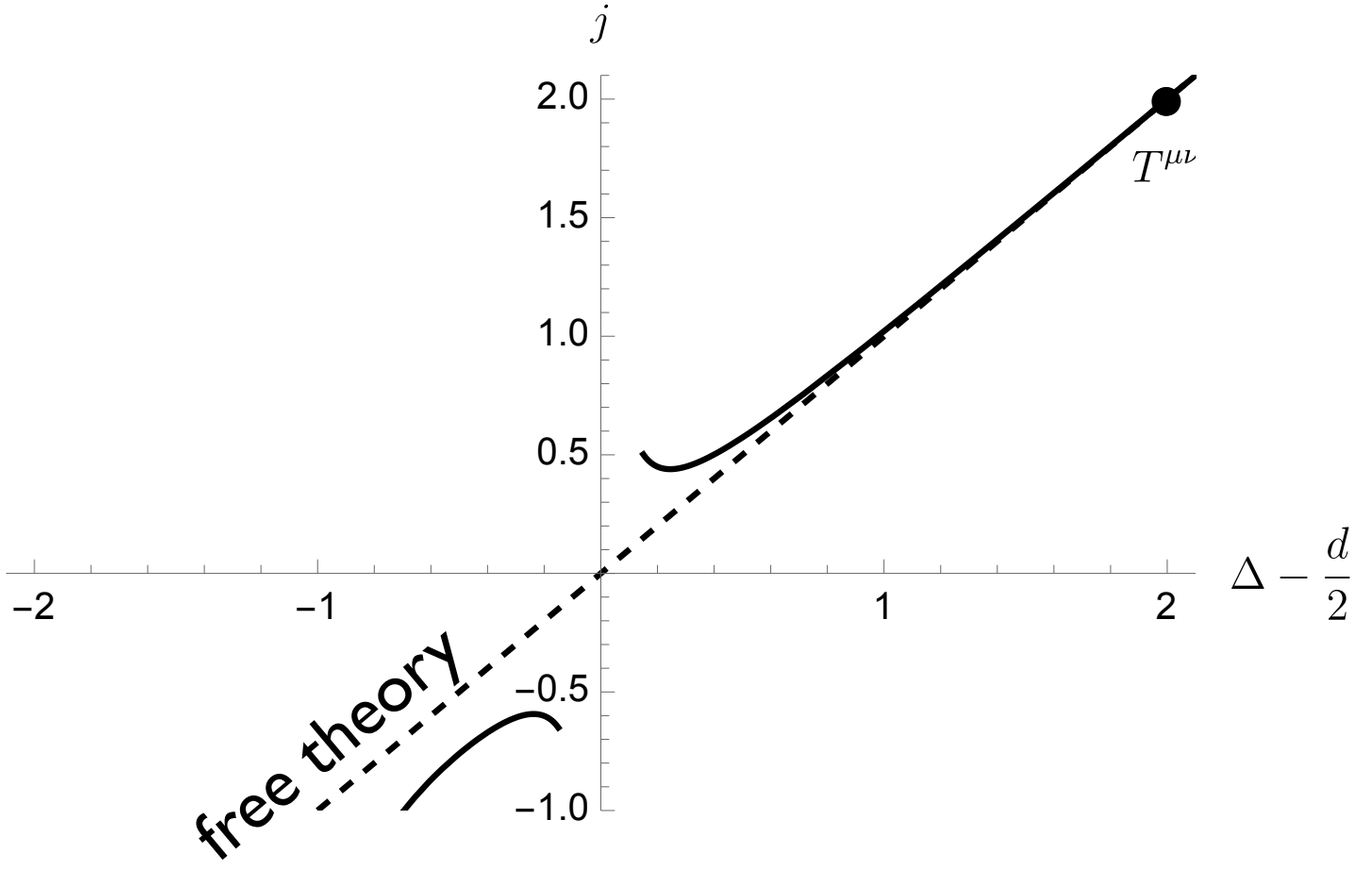
(Contrast with: $xf_g(x) \to \infty$ from QCD DGLAP.)

Q: What is the pomeron intercept: $f_\phi \sim x^{-j_*}$?

Moments: $f(j) \equiv \int_0^1 \frac{dx}{x} x^j f(x) \propto \langle p \,|\, \phi \partial_+^j \phi \,|\, p \rangle$

≡matrix element of $\mathcal{O}_j$: definite boost j & dimension Δ.

DGLAP $\Leftrightarrow$ $\Delta(j) = d - 2 + j + \frac{\lambda^2}{(4\pi)^4}\left[\frac{1}{6} - \frac{1}{j(j+1)}\right]$



j=0 pole suggests

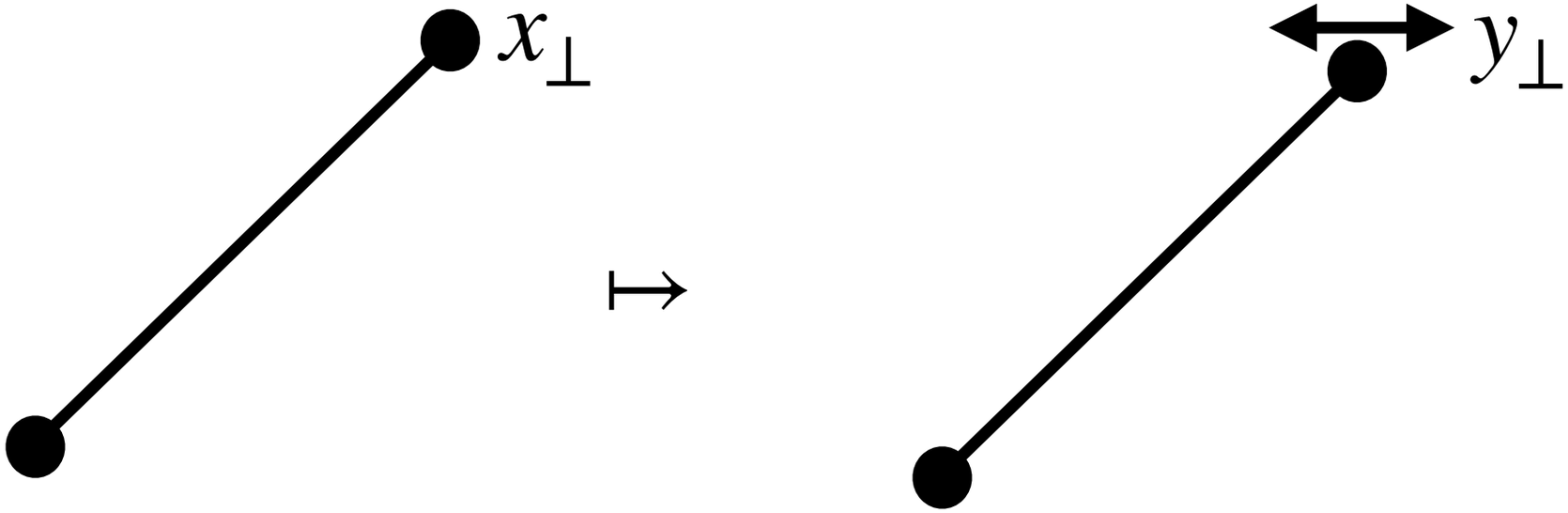
$f_\phi(x) \sim x^{-j_* \approx 0}$

plot axes as: [Brower,Polchinski,Strassler&Tan '06]

Expect: high-energy limit should involve full transverse plane.

Focus on ϕ^4 CFT in $d = 4 - \epsilon$ (Wilson-Fisher).
There is a natural *transverse shadow*:

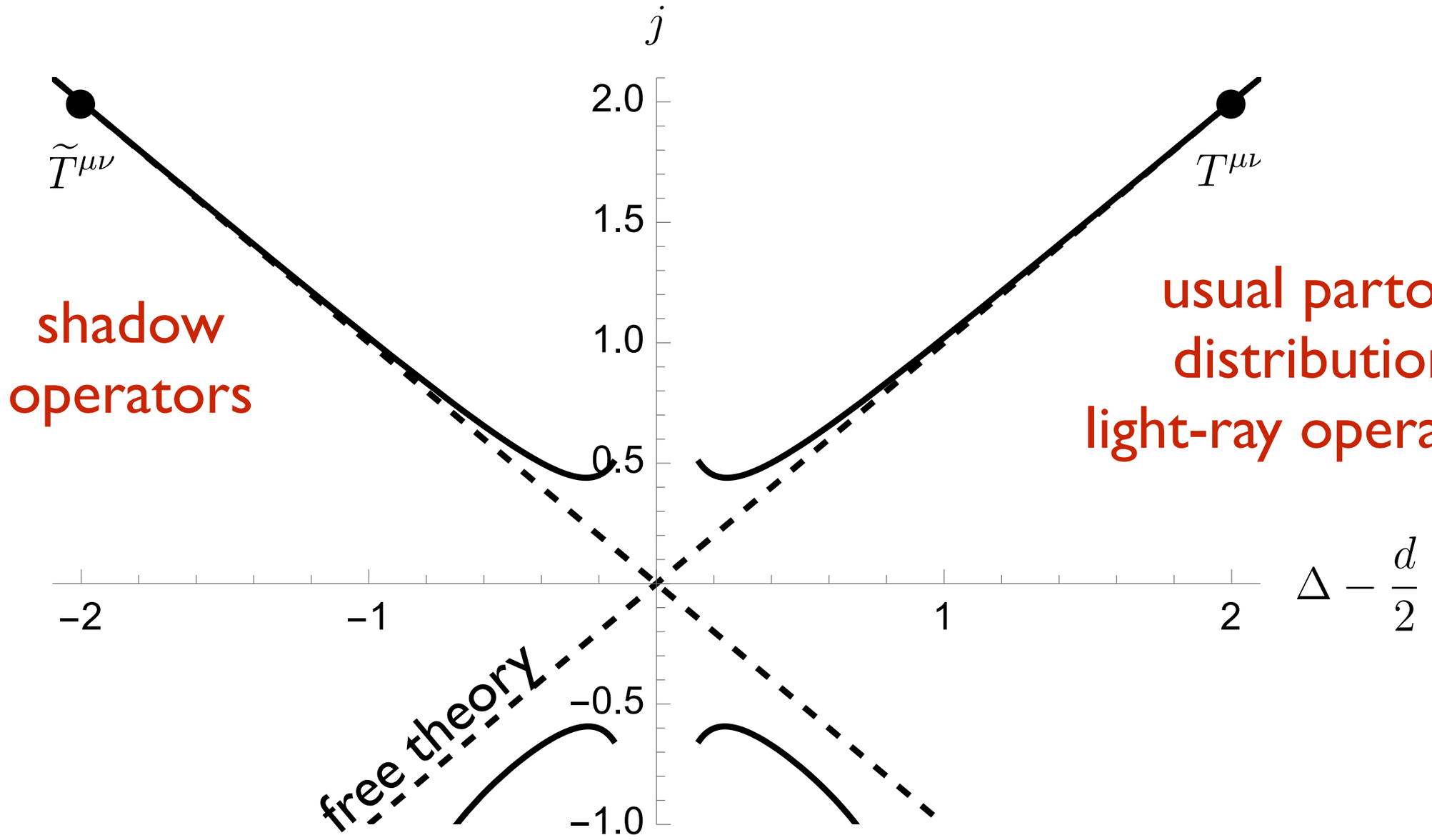
[Simmons-Duffin& Kravchuk '18]



$$\mathcal{O}_j(x_\perp) \quad \mapsto \quad \widetilde{\mathcal{O}}_j(x_\perp) = \int \frac{d^{d-2}y_\perp \, \mathcal{O}_j(y_\perp)}{[(x_\perp - y_\perp)^2]^{d-1-\Delta}}$$

$$\Delta \quad \mapsto \quad d - \Delta$$

Spectrum of lightray operators in any CFT must be invariant. [SCH '17]



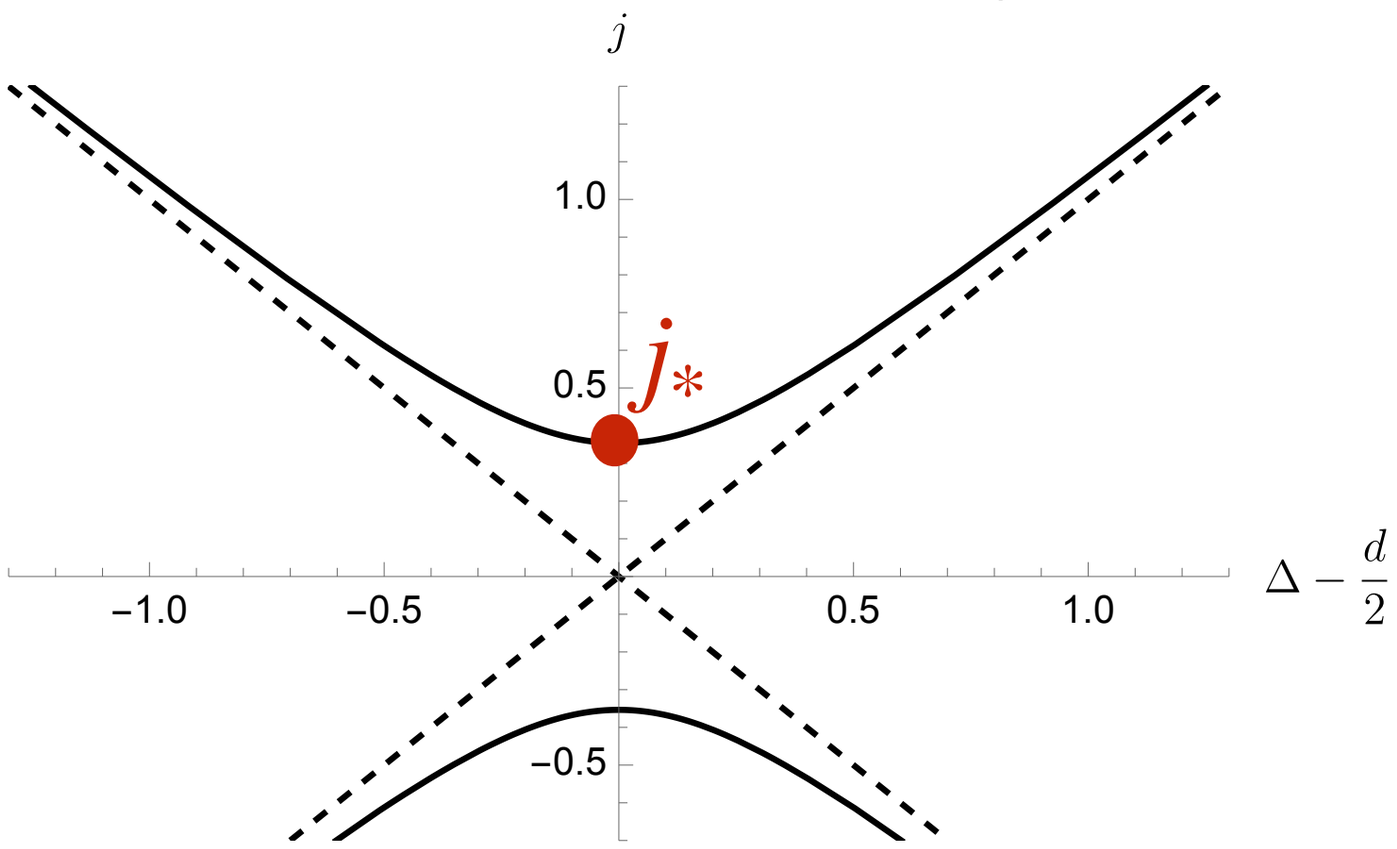
Idea: singularities in perturbation theory caused by free theory level crossing.

Resolve mixing using 2x2 matrix acting on $(\mathcal{O}_\Delta, \widetilde{\mathcal{O}}_\Delta)$:

$$j(\Delta) - 1 = \begin{pmatrix} \Delta - 2 & 0 \\ 1 & 2 - \Delta \end{pmatrix} + \frac{\lambda^2}{(4\pi)^4} \begin{pmatrix} \cdots & 2 \\ \cdots & \cdots \end{pmatrix} + \ldots$$

[SCH,Kologlu,Kravchuk,Meltzer&Simmons-Duffin '22]

eigenvalues: $j_\pm(\Delta) = \pm\sqrt{(\Delta-2)^2 + 2\lambda^2/(4\pi)^4} + \text{regular}$



- matches poles:

$$j_+ \approx \Delta - 2 + \frac{\lambda^2/(4\pi)^4}{\Delta - 2} + \ldots$$

- j^{free} not diagonalizable

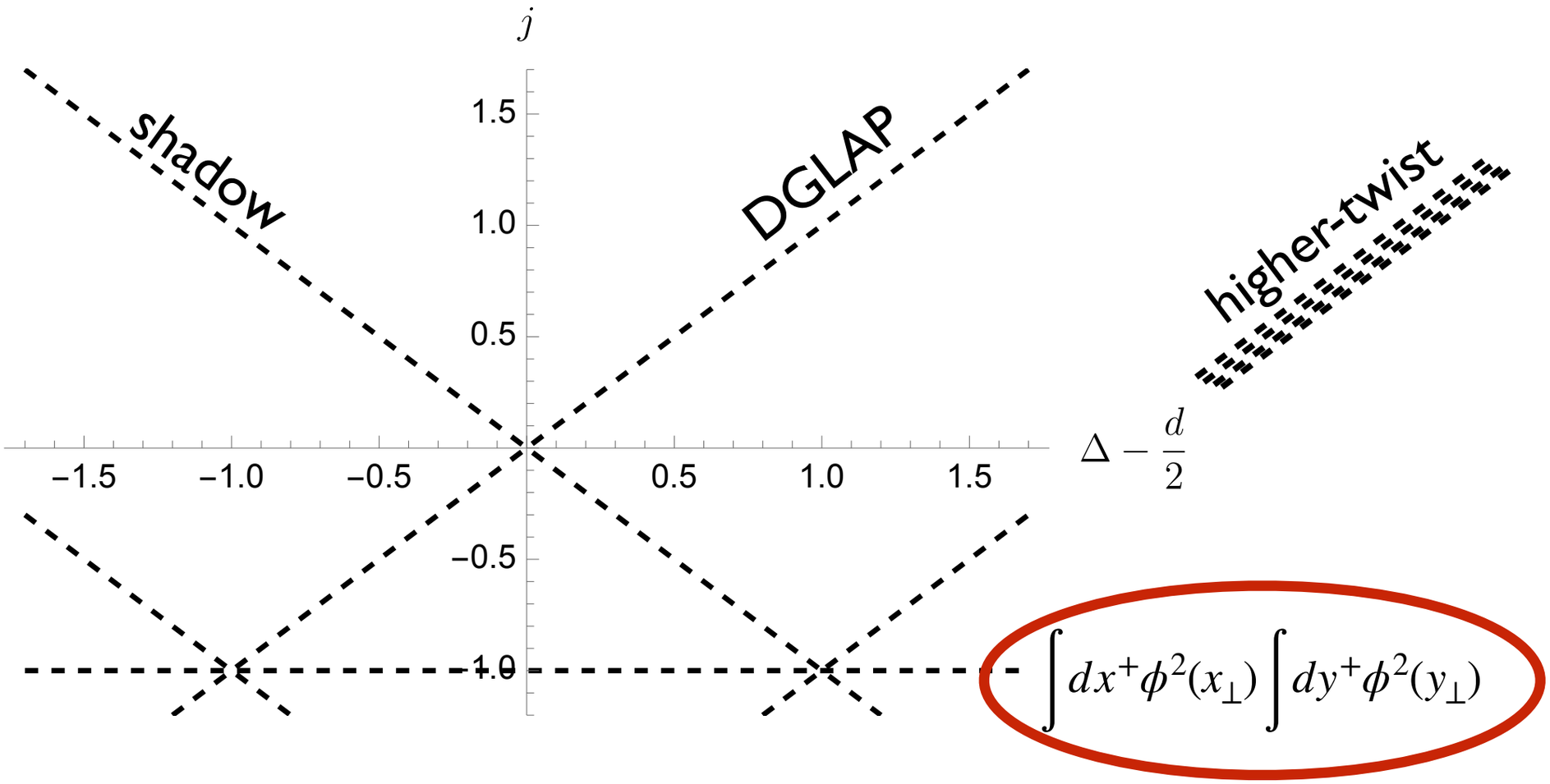
Summary of results

from arXiv:2209.00008

- Method to resolve intersections (double-logs)
- Exponent $f_\phi(x) \sim x^{-j_*}$ in ϕ^4 in $d = 4 - \epsilon$.
  Supports $j_* \approx 0.8 < 1$ in critical 3D Ising.
  [SCH+Zaharee '20]

- Discussed subleading powers, but could not resum.
- (spacelike-timelike map: j^{th} moment of PDF $\Leftrightarrow$ generalized calorimeter that weights E^{j-1} of each particle='detector')

more on subleading powers (operators contribute $\sim x^{-j}$):

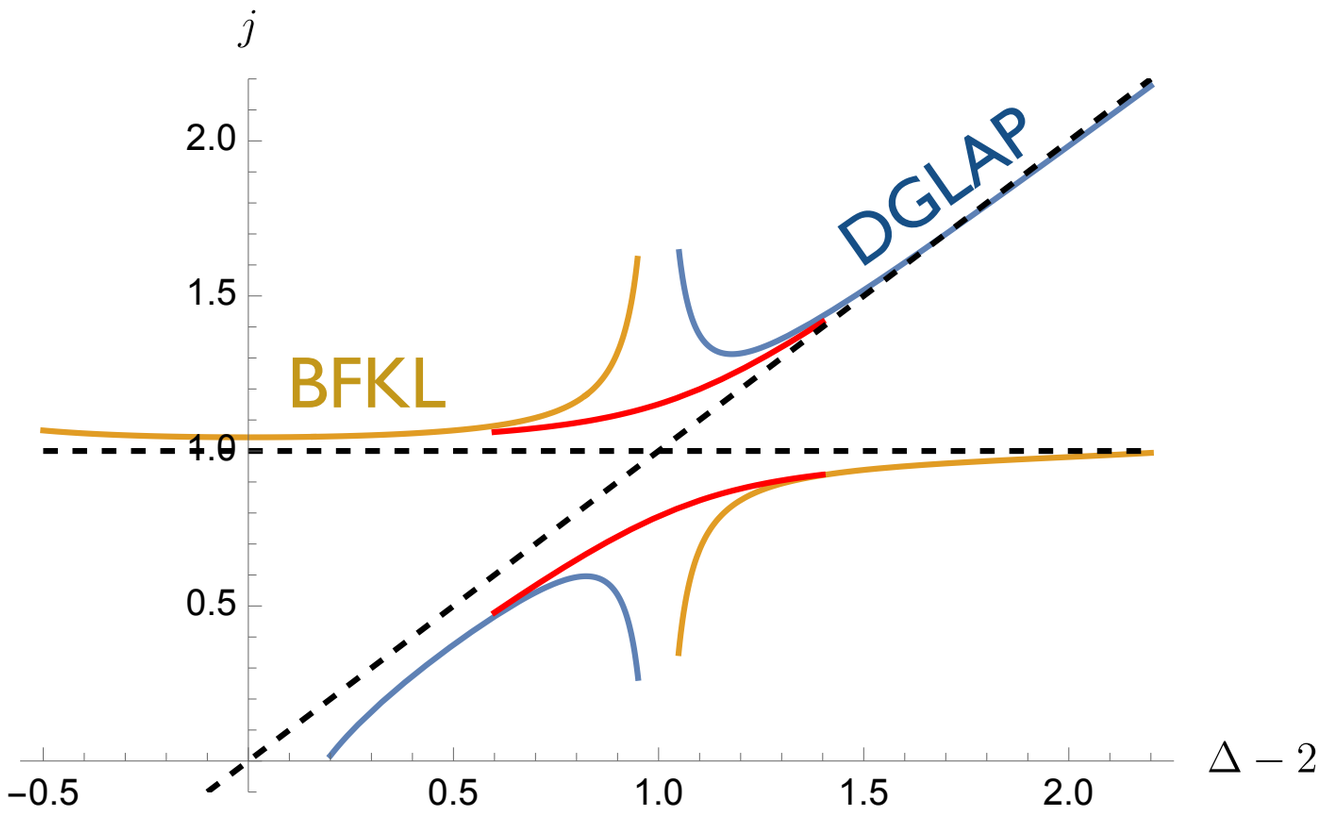


We could renormalize individual diagrams,
but it is still an open problem how to exponentiate them.

QCD

DGLAP: $$j = \Delta - 2 + 2\frac{\alpha_s C_A}{\pi}\left(\frac{1}{(\Delta-3)(\Delta-2)} + \frac{2}{\Delta-1} - H_\Delta + b_0\right) + O(\alpha_s^2)$$

BFKL: $$j = 1 + \frac{\alpha_s C_A}{\pi}\left(2\psi(1) - \psi(\tfrac{\Delta-1}{2}) - \psi(\tfrac{3-\Delta}{2})\right) + O(\alpha_s^2)$$



DGLAP-BFKL duality = curves meet analytically.

[Jaroscewicz '82; Salam '98; Brower,Polchinski,Strassler&Tan '06; Ball&Forte '07; Iancu,Madrigal,Mueller,Soyez&Triantafyllopoulos '15,…]

In N=4 super Yang-Mills: [Kotikov,Lipatov,Rej,Staudacher&Velizhanin '07; Basso,SCH&Sever '14; Alfimov,Gromov&Kazakov'14; SCH&Herranen '16,…]

QCD

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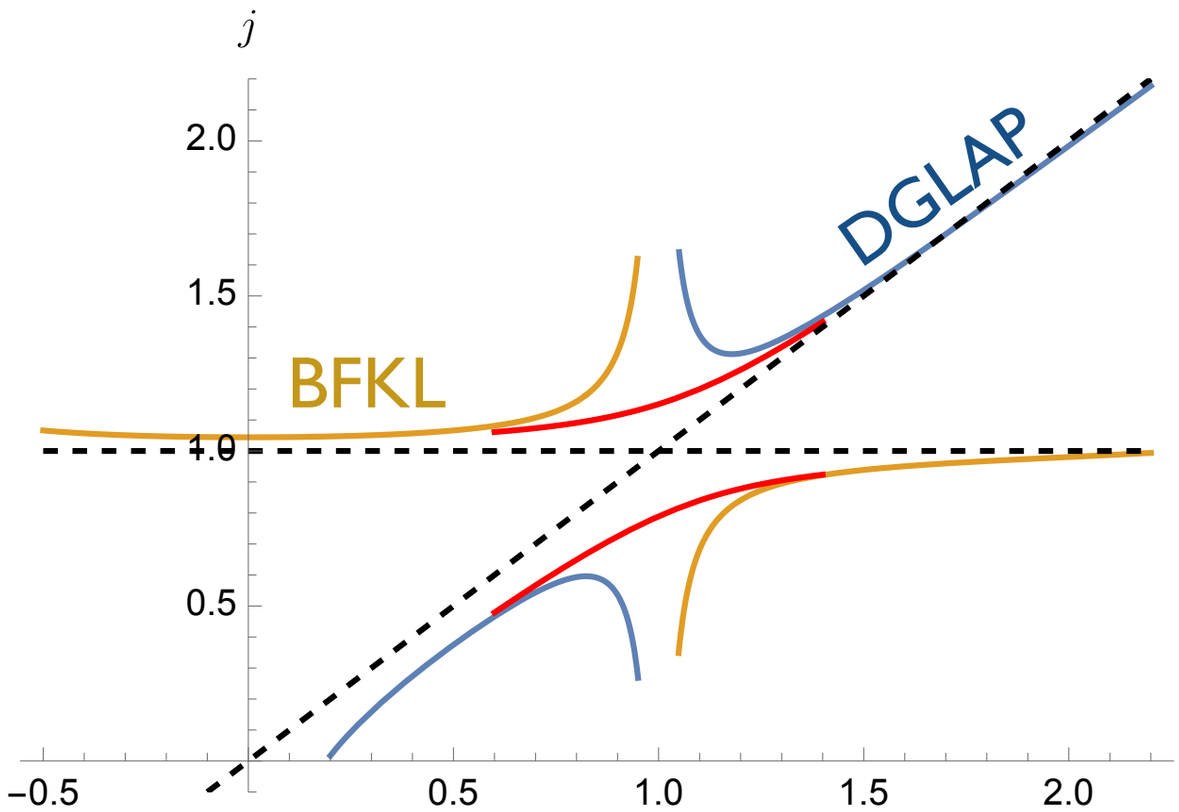


Natural speculation: singularities resolved by 2x2 matrix

$$j - 1 \approx \begin{pmatrix} \Delta - 3 & 2\alpha_s C_A/\pi \\ 1 & 0 \end{pmatrix}$$

$$= \frac{\Delta - 3 \pm \sqrt{(\Delta-3)^2 + 8\alpha_s C_A/\pi}}{2}$$

$$\to \frac{\pm 2\alpha_s C_A}{\pi|\Delta - 3|} + \ldots$$ matches leading poles

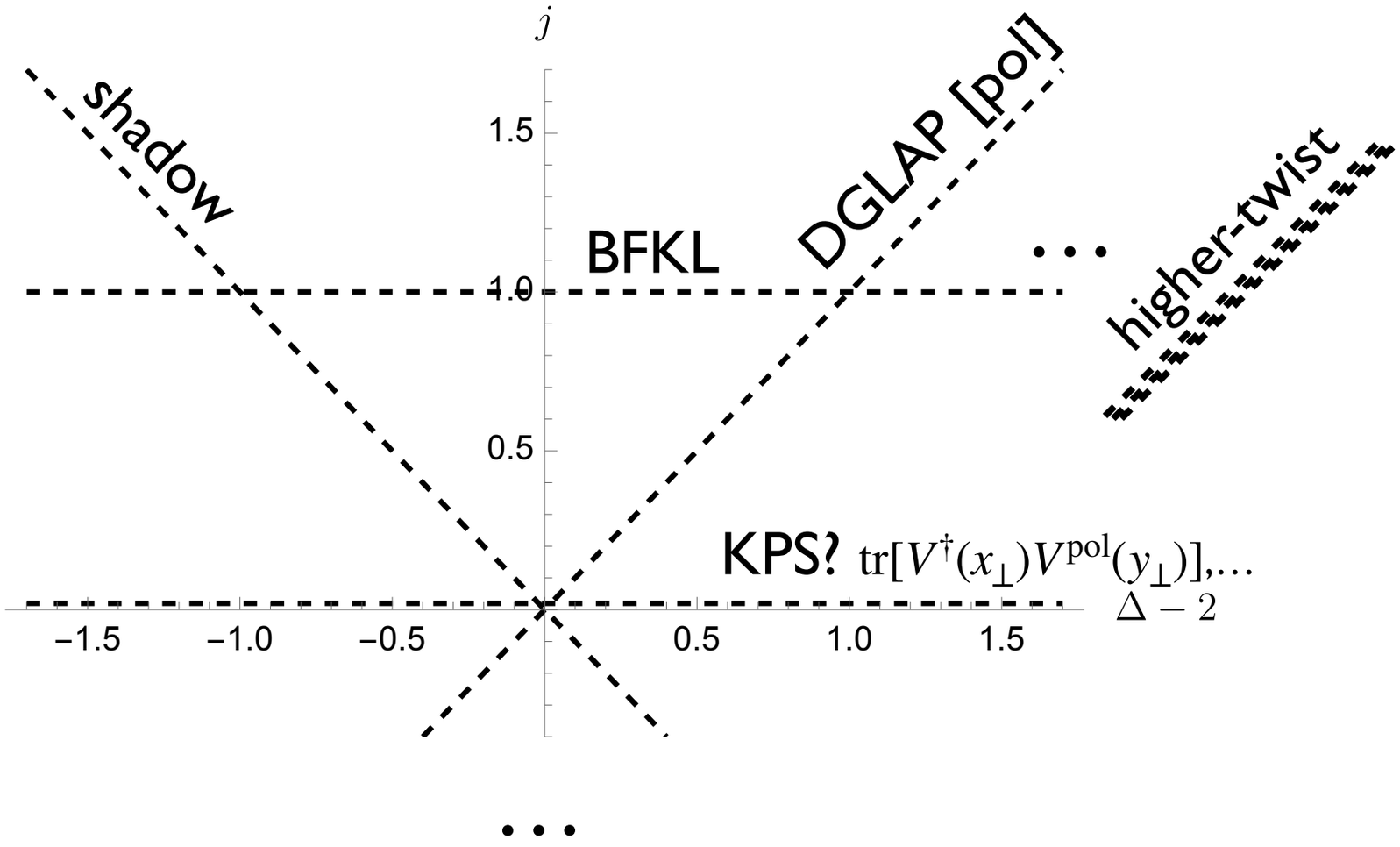
cf [Iancu,Madrigal,Mueller,Soyez&Triantafyllopoulos '15]

Q: Is ignoring lower branch a $\sim x^{-\text{gap}}$ error?

Some considered projects:

- Combine DGLAP with *nonlinear* BK into 2x2 matrix?
- Better understand evolution of subeikonal terms?

Free QCD:



More potential projects:

Mathieu Giroux

- O(d-4) corrections to 2-loop BK or B-JIMWLK?

  (=piece of 3-loop BK in spacelike-timelike correspondence, namely $H^{(3)}_{\mathrm{BK}} - H^{(3)}_{\mathrm{NGL}}$)

- Impact factors, dijet cross-section…?