Resummation of the photon-gluon impact factor

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in collaboration with **Dimitri Colferai** and **Wanchen Li**

Outline

- Resummation for the BFKL gluon Green's function
- Photon gluon impact factor with exact kinematics
- Resummed impact factor: collinear and small x constraints
- Numerical results for the $\gamma^*\gamma^*$ cross section

Small x BFKL evolution

BFKL evolution equation for high-energy limit

$$\partial_Y \mathrm{G}(Y, \boldsymbol{k}, \boldsymbol{k}_0) = \int \frac{d^2\boldsymbol{k}'}{\pi} \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}')\, G(Y, \boldsymbol{k}', \boldsymbol{k}_0)$$

G is **Gluon Green's function**

$\boldsymbol{k}$ transverse momentum

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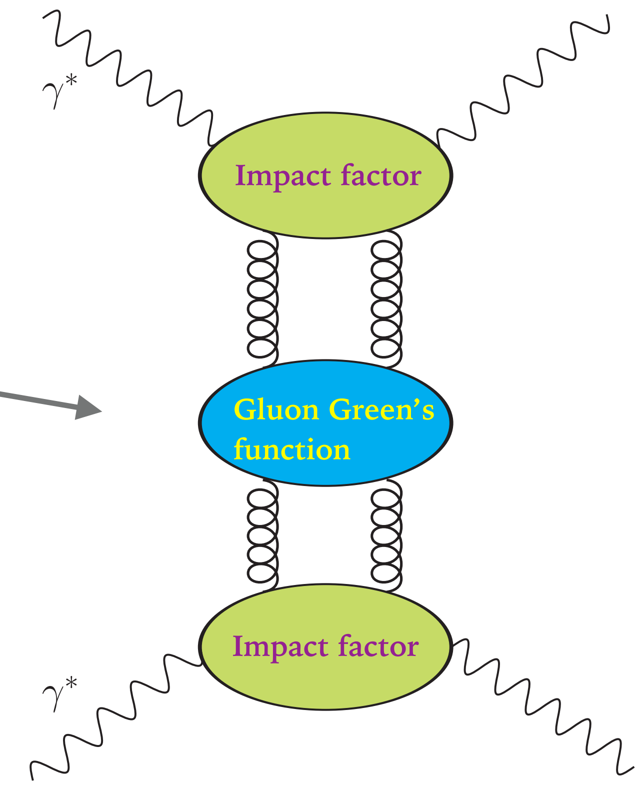

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BFKL **kernel** has expansion $\mathcal{K} = \bar{\alpha}_s \mathcal{K}_0 + \bar{\alpha}_s^2 \mathcal{K}_1 + \ldots$

known up to next-to-leading order in QCD $\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$

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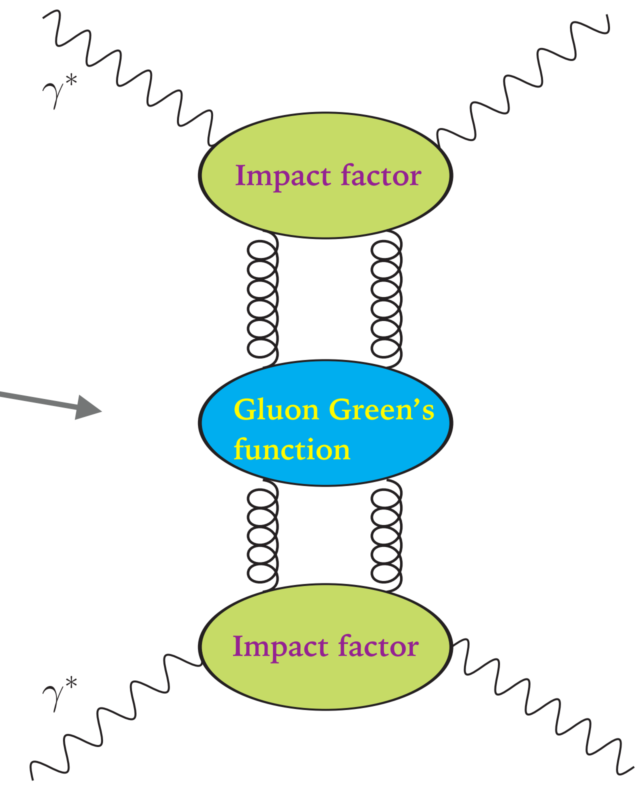

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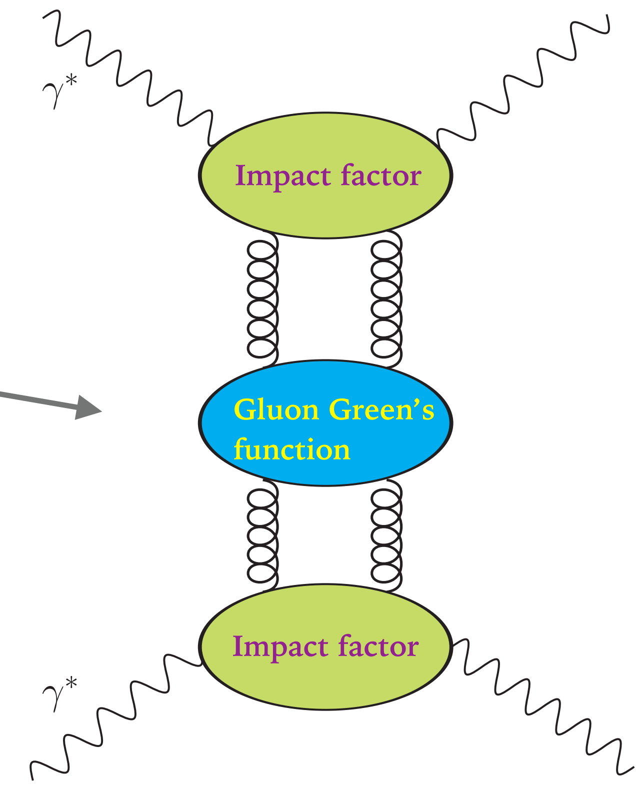

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LL kernel in Mellin space $\gamma \leftrightarrow \ln \boldsymbol{k}^2$

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

$\psi(z) = \Gamma'(z)/\Gamma(z)$

NLL corrections to BFKL

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collinear & anti-collinear poles

Solution to the intercept $\omega_P = \bar{\alpha}_s \chi(\gamma = 1/2) \to 4 \ln 2 \bar{\alpha}_s$

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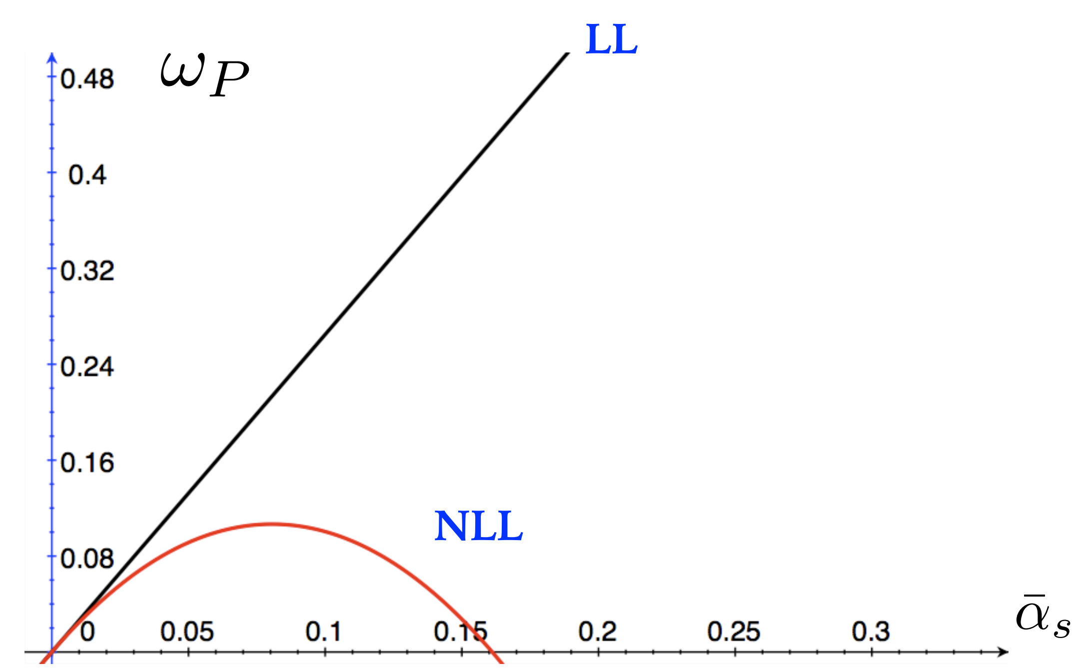
NLL corrections to BFKL equation are **large** and **negative**

Main sources:

- running coupling
- kinematical constraint
- DGLAP anomalous dimension



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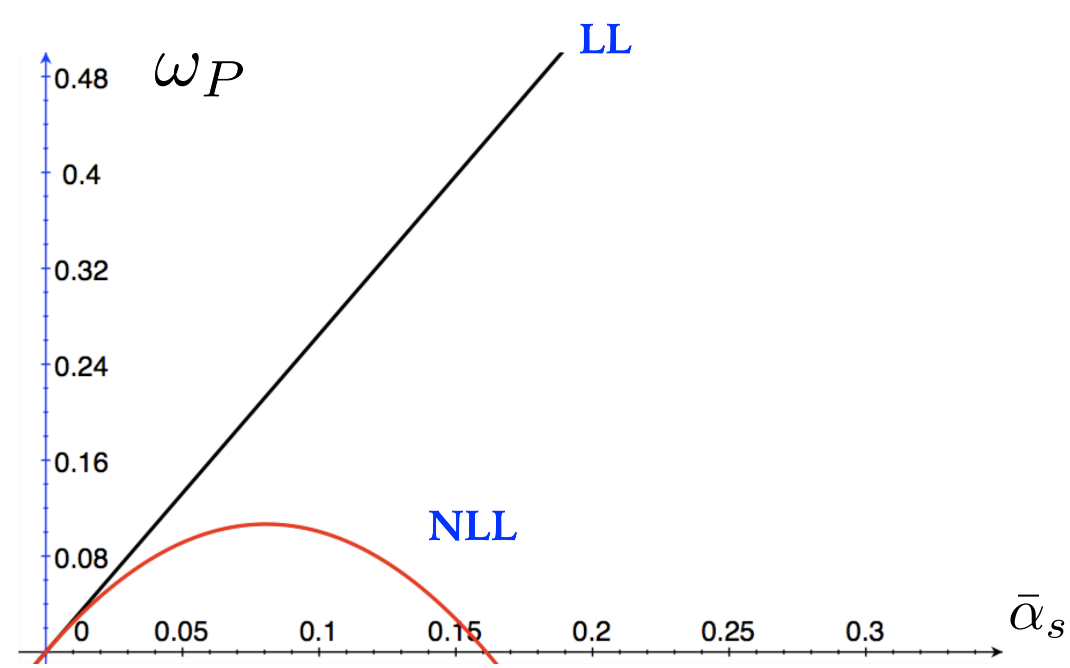

In Mellin space: (negative contributions) double and triple poles

$$\frac{1}{\gamma^2}, \frac{1}{\gamma^3}$$

Small x resummation

Altarelli,Ball,Forte; Thorne, White; Sabio-Vera; Ciafaloni, Colferai, Salam, AS (CCSS)

CCSS resummation (RGI renormalization group improved small x evolution):

Andersson, Gustafson, Kharraziha, Samuelsson; Ciafaloni; Kwiecinski, Martin, Sutton

- Include **kinematical constraint** : leads to **shifts of poles**
- Include DGLAP **splitting function** and **running coupling** in the leading part
- Suitable subtractions to avoid double counting, guarantee momentum sum rule
- Motivation in Mellin space, final equation in the **momentum** space

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$$X(\gamma,\omega) = 2\psi(1) - \psi(\gamma+\frac{\omega}{2}) - \psi(1-\gamma+\frac{\omega}{2}) + \omega A_{gg}(\omega)\left(\frac{1}{\gamma+\frac{\omega}{2}} + \frac{1}{1-\gamma+\frac{\omega}{2}}\right) + \bar{\alpha}_s\tilde{\chi}_1(\gamma,\omega)$$

Mellin variable $\omega \leftrightarrow \ln s$

$A_{gg}(\omega)$ DGLAP anomalous dimension without the $1/\omega$ term

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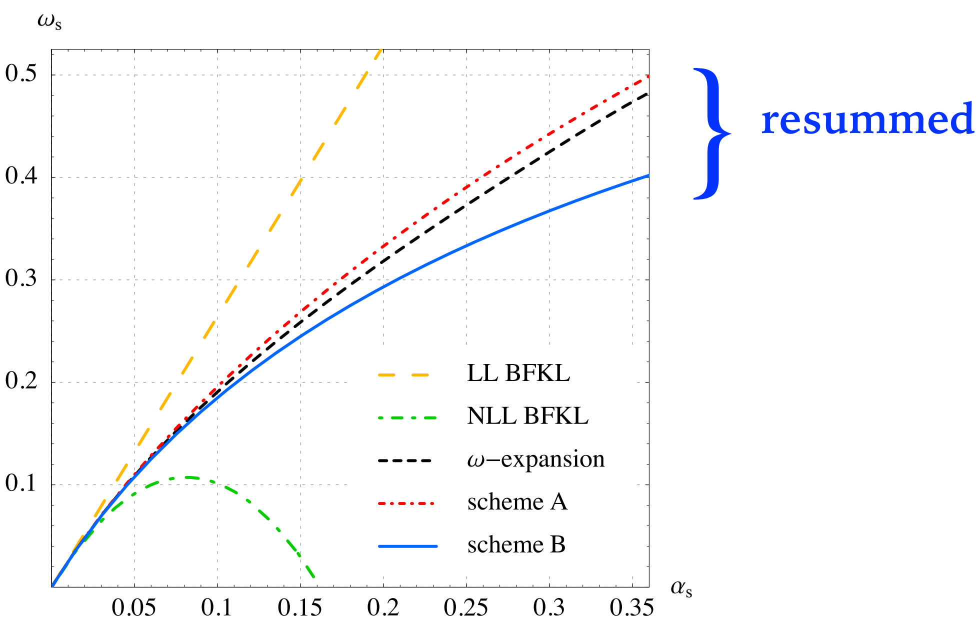
Double and triple poles of NLL recovered when expanding in ω, i.e.

$$-\psi(\gamma+\frac{\omega}{2}) \simeq \frac{1}{\gamma+\frac{\omega}{2}} \simeq \frac{1}{\gamma} - \frac{1}{2}\frac{\omega}{\gamma^2} \simeq \frac{1}{\gamma} - \frac{\bar{\alpha}_s}{2\gamma^3}$$

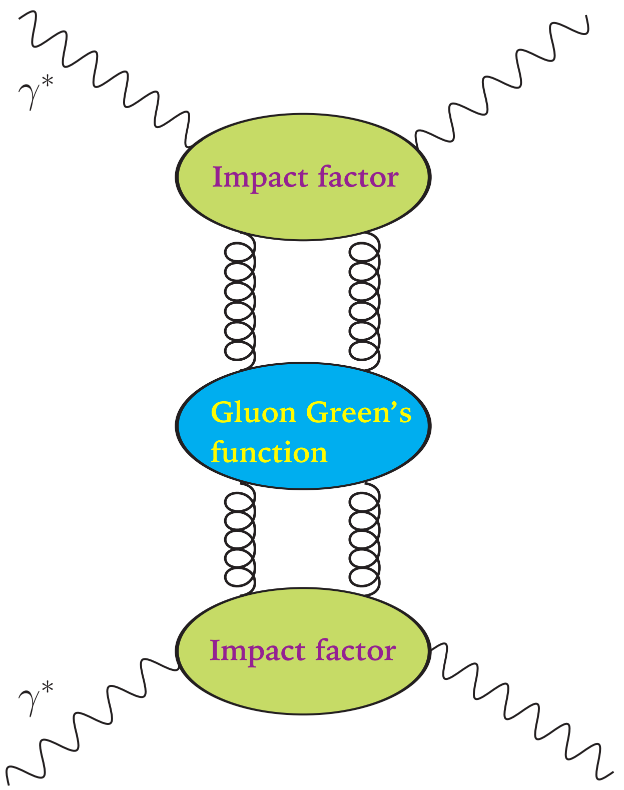


Quintic poles of **NNLL result in N=4 sYM** recovered too *Deak,Kutak,Li,AS*

Gromov,Levkovich-Maslyuk,Sizov;Velizhanin;Caron-Huot,Herranen

Resummation of impact factors: case study of $\gamma^*\gamma^*$ scattering

- The double-tagged process $e^+e^- \longrightarrow e^+e^- +$ hadrons allows to measure the $\gamma^*\gamma^* \longrightarrow$ **hadrons** cross section.
- Excellent process to study BFKL for two **comparable virtualities** of the photons.
- BFKL exchange should be dominant at high energy



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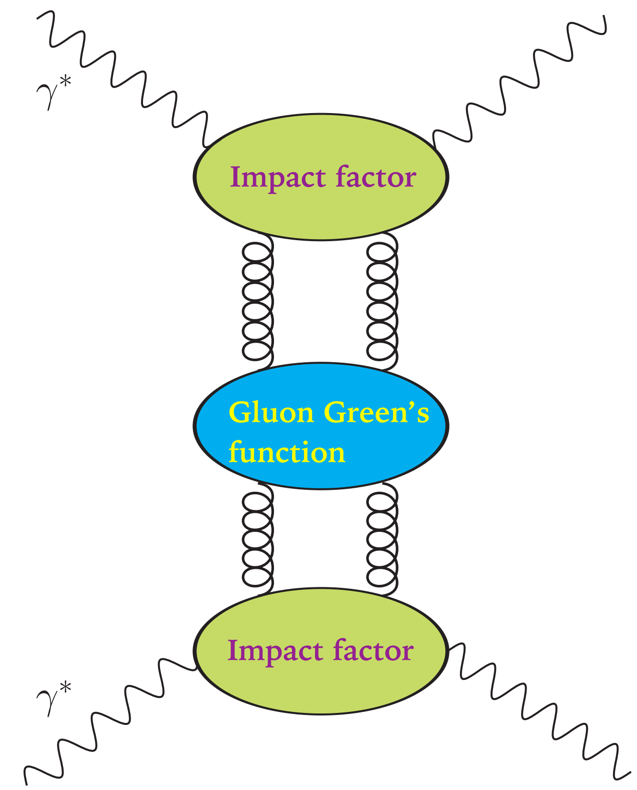


High-energy factorization formula

$$\sigma^{(jk)}(s, Q_1, Q_2) = \frac{1}{2\pi Q_1 Q_2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma - \frac{1}{2}} \phi^{(j)}(\gamma)\, G(\omega, \gamma)\, \phi^{(k)}(1-\gamma)$$

$Q_1^2 = -q_1^2, Q_2^2 = -q_2^2$ are negative photon virtualities

$s = (q_1 + q_2)^2$ for the $\gamma^*\gamma^*$ process

j, k photon polarizations

s_0 energy scale

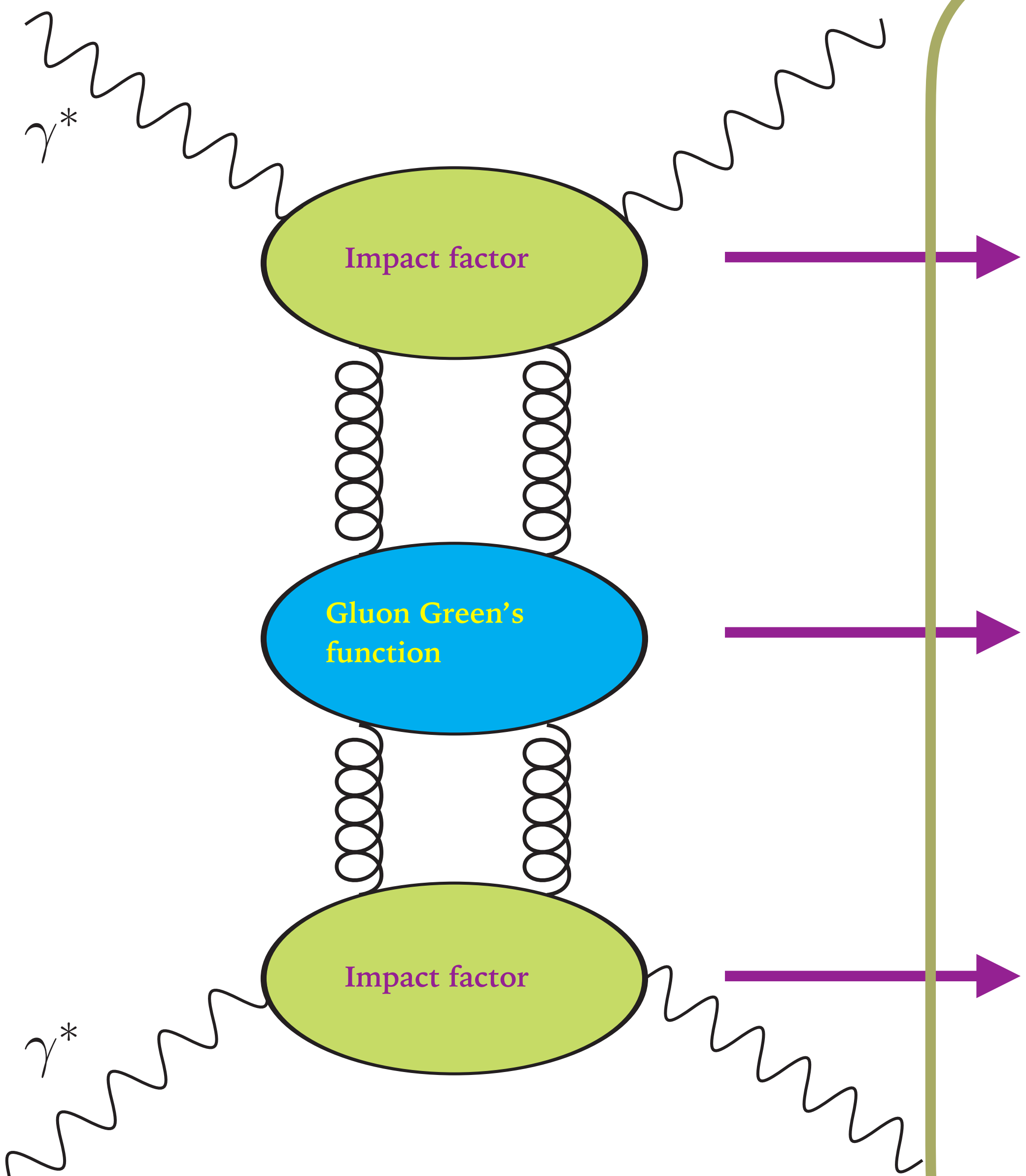
$\phi^{(j,k)}$ impact factors: known up to NLO

Balitsky, Chirilli; Beuf

$G(\omega, \gamma)$ BFKL gluon Green's function

$$G(\omega, \gamma) = \frac{1}{\omega - \bar{\alpha}_s \chi(\gamma)}$$

Resummation with impact factors



Standard high-energy

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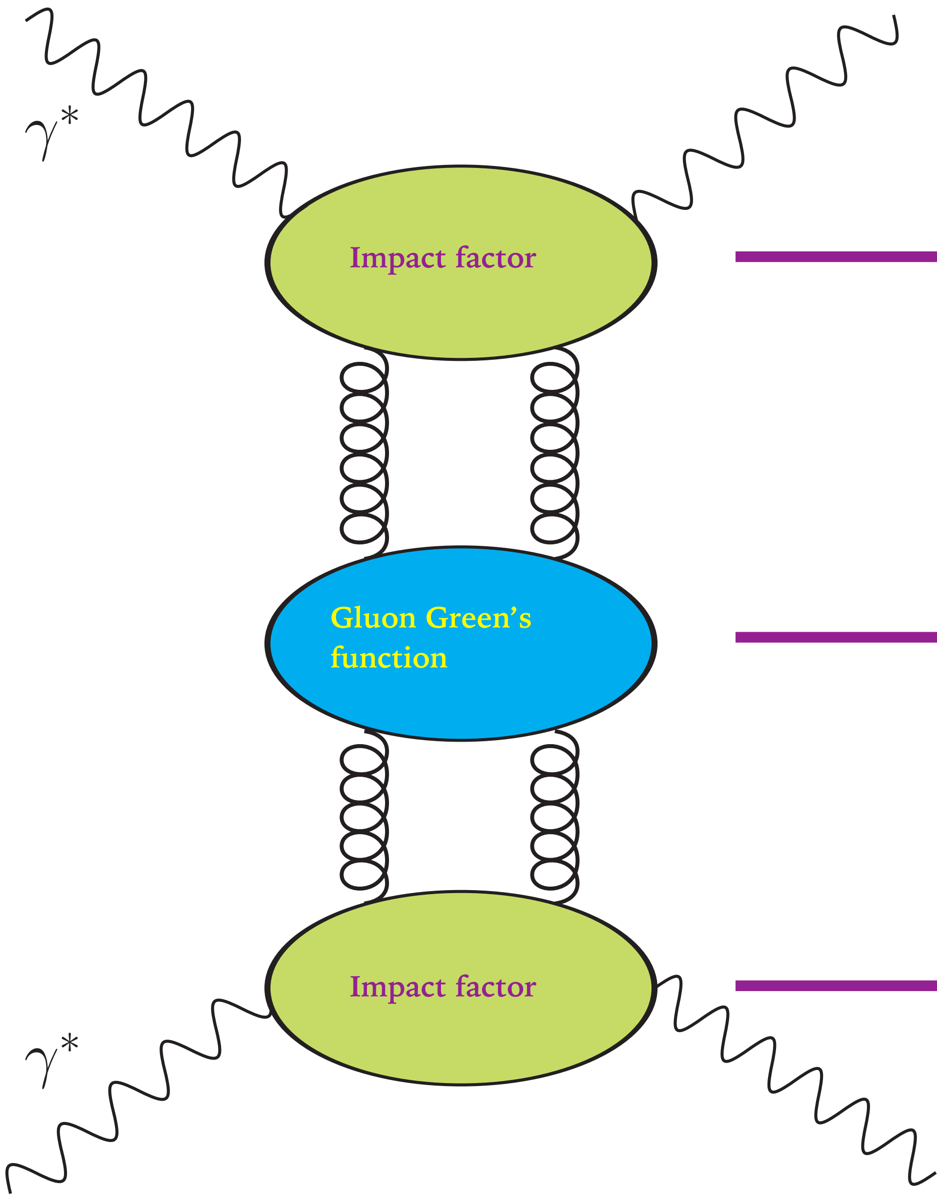

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RGI formulation

$$\Phi^{(j)}(\omega,\gamma)$$

$$\mathcal{G}(\omega,\gamma) = \frac{1}{\omega - \bar{\alpha}_s X(\omega,\gamma)}$$

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Solve the nonlinear equation in ω . Gives the leading energy behavior

$$\omega = \bar{\alpha}_s X(\omega, \gamma) \equiv \omega^{\text{eff}}(\gamma, \bar{\alpha}_s) \equiv \bar{\alpha}_s \chi^{\text{eff}}(\gamma, \bar{\alpha}_s)$$

ω integral singles out the residue

$$\text{Res}_{\omega=\omega^{\text{eff}}} [\omega - \bar{\alpha}_s X(\omega, \gamma)]^{-1} = [1 - \bar{\alpha}_s \partial_\omega X(\omega^{\text{eff}}, \gamma)]^{-1}$$

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Resummed impact factor
with ω dependence

$$\Phi^{(j)}(\omega,\gamma)$$

Renormalization Group Improved formulation

Relation between two formulations:

standard high-energy

resummed

$$\chi(\gamma) = X(\omega^{\mathrm{eff}}, \gamma)$$

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<u>leading order</u>

$$\chi_0(\gamma) = X_0(0, \gamma)$$

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next-to-leading order

$$\chi_1(\gamma) = X_1(0, \gamma) + \chi_0(\gamma)\partial_\omega X_0(0, \gamma)$$

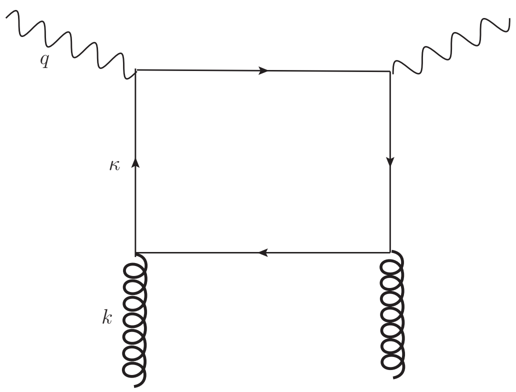
$$\begin{aligned}\phi_0^{(j)}(\gamma)\phi_1^{(k)}(1-\gamma) + \phi_1^{(j)}(\gamma)\phi_0^{(k)}(1-\gamma) = {} & \Phi_0^{(j)}(0, \gamma)\left[\Phi_1^{(k)}(0, 1-\gamma) + \chi_0(1-\gamma)\partial_\omega\Phi_0^{(k)}(0, 1-\gamma)\right] \\ & + \left[\Phi_1^{(j)}(0, \gamma) + \chi_0(\gamma)\partial_\omega\Phi_0^{(j)}(0, \gamma)\right]\Phi_0^{(k)}(0, 1-\gamma) \\ & + \Phi_0^{(j)}(0, \gamma)\Phi_0^{(k)}(0, 1-\gamma)\partial_\omega X_0(0, \gamma)\end{aligned}$$

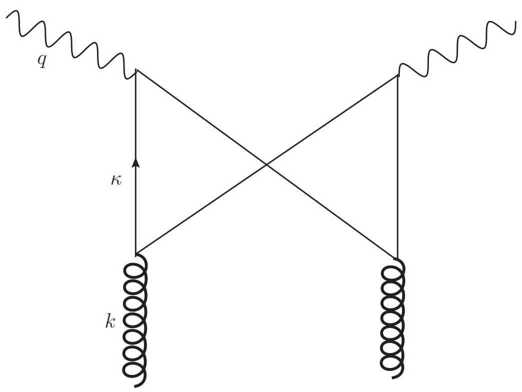
HE formula with exact kinematics: argument of the gluon density

How to get correct ω dependence of the impact factors ?

$$\phi^{(k)}(\gamma) \longrightarrow \Phi^{(k)}(\omega, \gamma)$$

Graphs at LO





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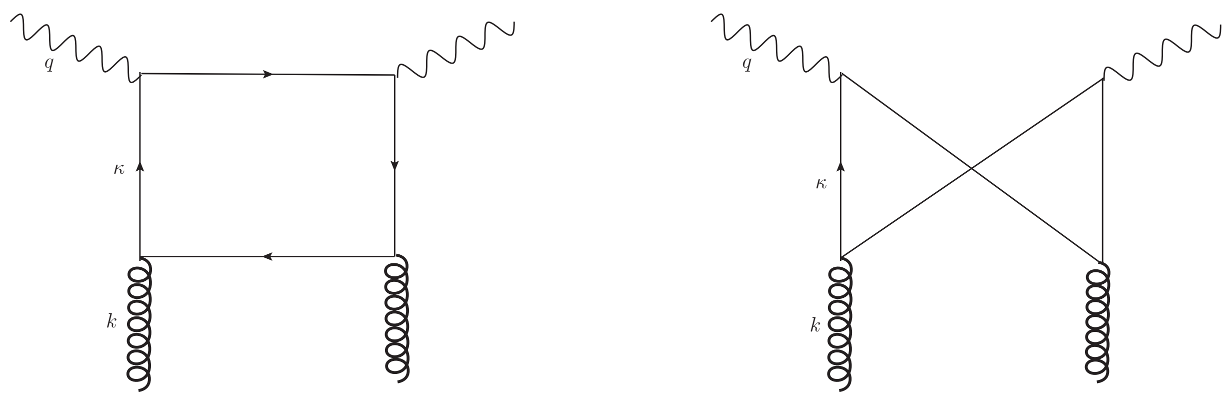
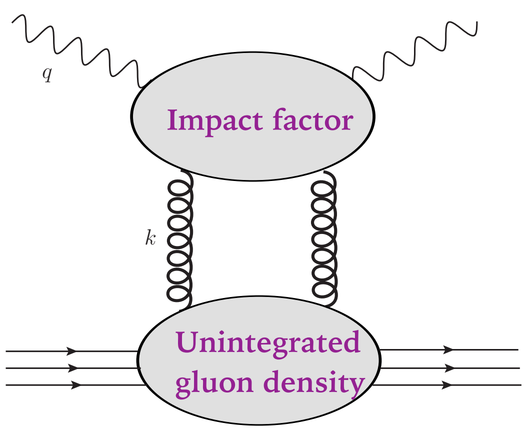


Consider structure function in DIS in high-energy factorization (momentum space)



Impact factor(in mom.space)

$$F_{2,L}(x, Q^2) = \hat{F}^0_{2,L}(Q^2, \boldsymbol{k}, \boldsymbol{\kappa}, z) \otimes f(x_g, \boldsymbol{k}^2)$$

unintegrated gluon density

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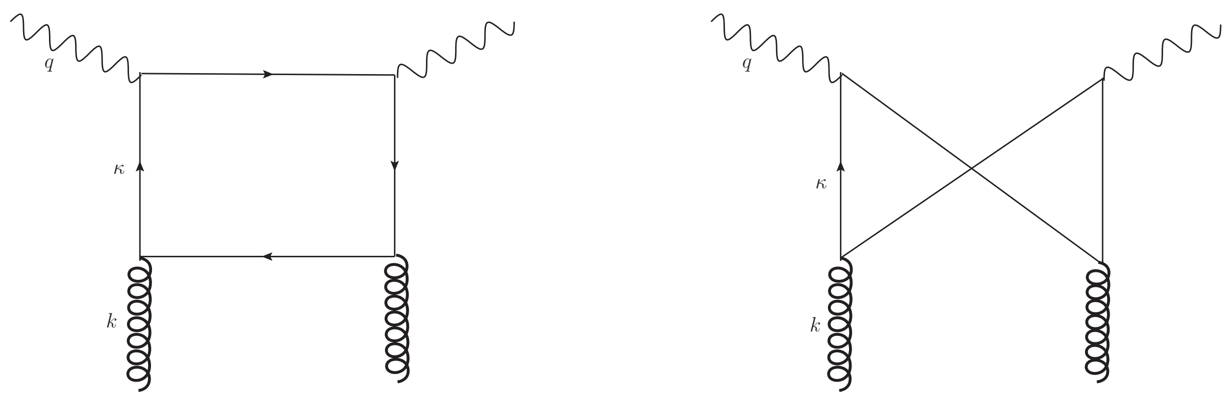

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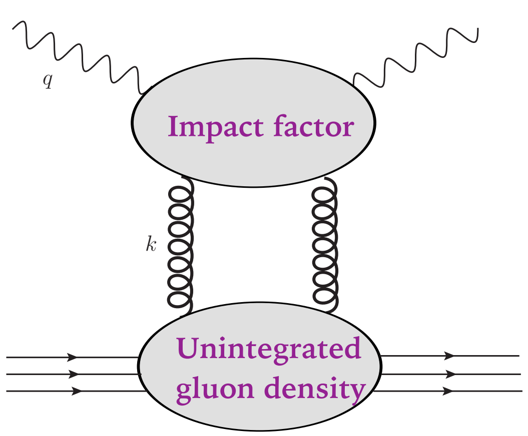

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In the high-energy limit at LO (or in dipole model) : $x_g = x_{Bj}$

Exact kinematics: $$x_g = x_{Bj}\left(1+\frac{\boldsymbol{k}^2}{Q^2}+\frac{\boldsymbol{\kappa}^2+m^2}{z(1-z)Q^2}\right)$$

Askew, Kwiecinski, Martin, Sutton; Kwiecinski, Martin, AS

LO impact factor with exact kinematics: shift of poles

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Bialas, Navelet, Peschanski

↓ Mellin space

$\Phi(\omega, \gamma)$ ω dependent impact factor

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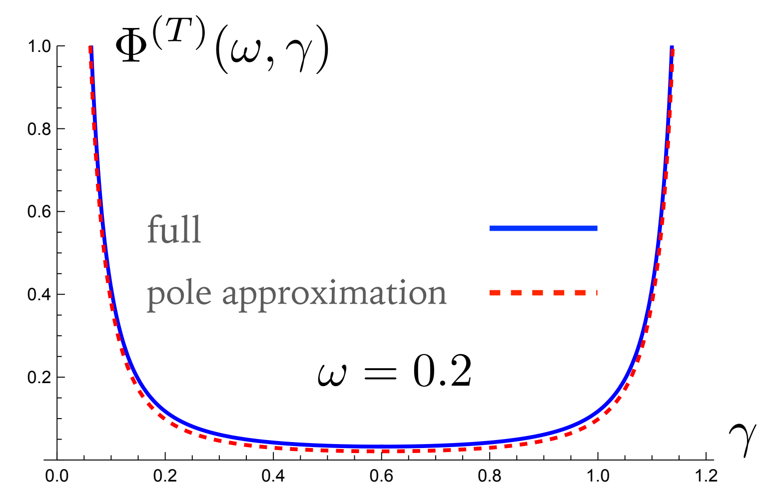
$$\Phi(\omega,\gamma)$$

ω dependent impact factor

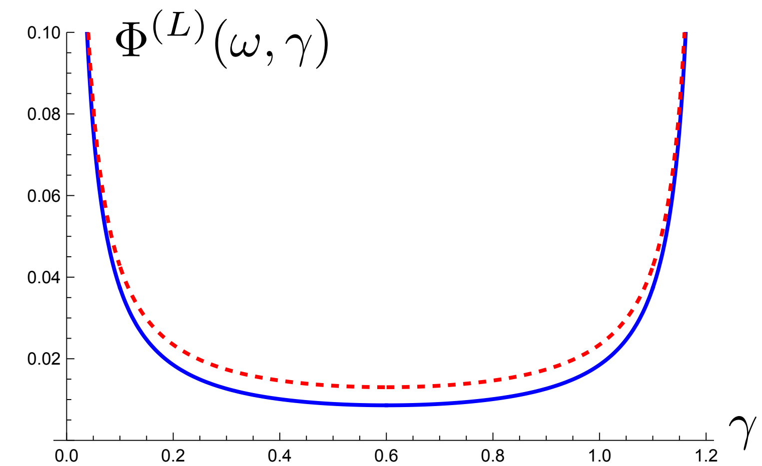
Transverse

$$\Phi^{(T)}(\omega,\gamma) \sim \frac{1}{\gamma^2} + \frac{1}{(1-\gamma+\omega)^2}$$



Longitudinal

$$\Phi^{(L)}(\omega,\gamma) \sim \frac{1}{\gamma} + \frac{1}{1-\gamma+\omega}$$



Reduce to $\phi_0^{(T,L)}(\gamma)$ when $\omega \to 0$

Similar ω shifts of poles as in GGF

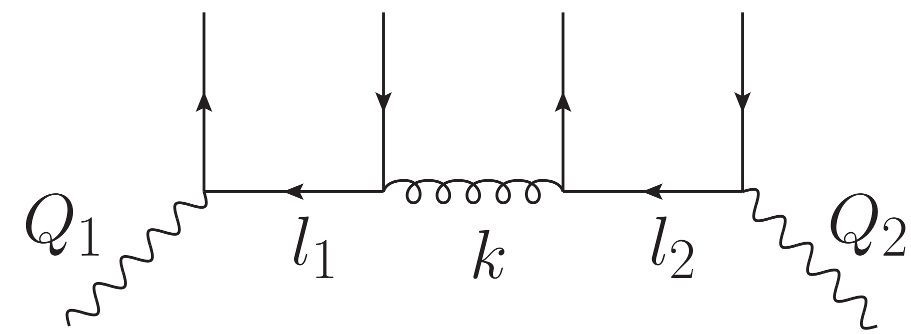
Collinear analysis

Back to $\gamma^*\gamma^*$ cross section…

More information by collinear limit: photons with unequal virtualities $Q_1^2 \gg Q_2^2$

Strong ordering of transverse momenta:

$$Q_1^2 \gg l_1^2 \gg k^2 \gg l_2^2 \gg Q_2^2$$



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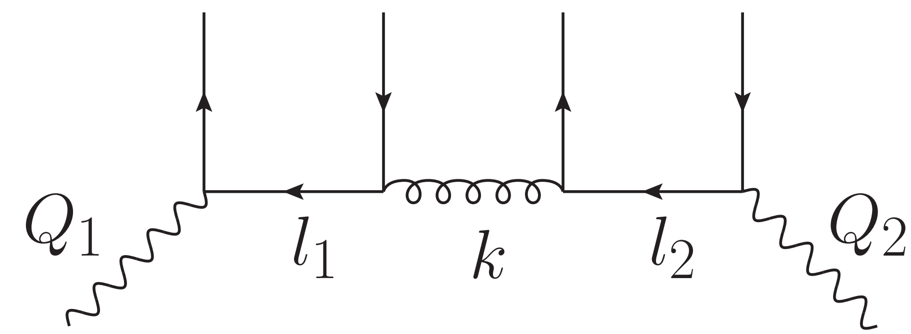

$$\tilde{\sigma}^{(TT)}(\omega, Q_1, Q_2) = (2\pi)^3 \alpha \Big(\sum_{q \in A} e_q^2 \Big) \times$$

$$\int_{Q_2^2}^{Q_1^2} \frac{\mathrm{d}l_1^2}{l_1^2} \frac{\alpha_\mathrm{s}(l_1^2)}{2\pi} P_{qg}(\omega) \int_{Q_2^2}^{l_1^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_\mathrm{s}(k^2)}{2\pi} P_{gq}(\omega) \int_{Q_2^2}^{k^2} \frac{\mathrm{d}l_2^2}{l_2^2} \frac{\alpha}{2\pi} \Big(\sum_{q \in B} e_q^2 \Big) P_{q\gamma}(\omega)$$

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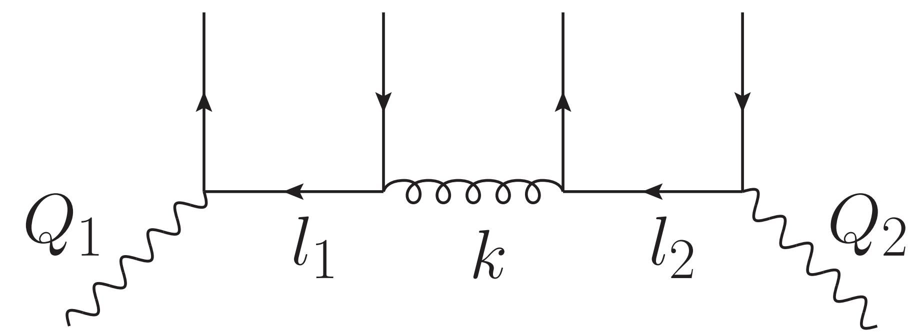

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$$\int_{Q_2^2}^{Q_1^2} \frac{\mathrm{d}l_1^2}{l_1^2} \frac{\alpha_\mathrm{s}(l_1^2)}{2\pi} P_{qg}(\omega) \int_{Q_2^2}^{l_1^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_\mathrm{s}(k^2)}{2\pi} P_{gq}(\omega) \int_{Q_2^2}^{k^2} \frac{\mathrm{d}l_2^2}{l_2^2} \frac{\alpha}{2\pi} \Big(\sum_{q\in B} e_q^2\Big) P_{q\gamma}(\omega)$$

In Mellin space up to order α_s^2

$$\tilde{\tilde{\sigma}}^{(TT,0)}(\omega,\gamma)\big|_{p=1}^{\mathrm{coll}} = \Phi_0^{(T)}\, \mathcal{G}_0\, \Phi_0^{(T)}\big|_{p=1}^{\mathrm{coll}}$$

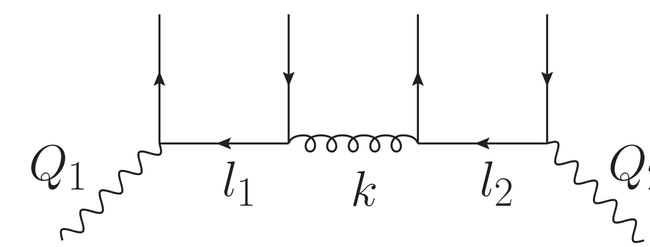
$$= (2\pi)^3 \alpha \Big(2\sum_{q\in A} e_q^2\Big)\frac{1}{\gamma} \cdot \frac{\alpha_\mathrm{s}}{2\pi}\frac{P_{qg}(\omega)}{\gamma} \cdot \frac{\alpha_\mathrm{s}}{2\pi}\frac{P_{gq}(\omega)}{\gamma} \cdot \frac{\alpha}{2\pi}\Big(2\sum_{q\in B} e_q^2\Big)\frac{P_{q\gamma}(\omega)}{\gamma}$$

Additional term $\mathcal{O}(\alpha_s^3/\gamma^5)$ from the running coupling contributing at NLO

Collinear analysis

$$\tilde{\tilde{\sigma}}^{(TT,0)}(\omega,\gamma)\big|^{\text{coll}}_{p=1} = \Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\big|^{\text{coll}}_{p=1}$$
$$= (2\pi)^3\alpha\Big(2\sum_{q\in A} e_q^2\Big)\frac{1}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{qg}(\omega)}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{gq}(\omega)}{\gamma}\cdot\frac{\alpha}{2\pi}\Big(2\sum_{q\in B} e_q^2\Big)\frac{P_{q\gamma}(\omega)}{\gamma}$$

$Q_1^2 \gg Q_2^2$



- Collinear analysis singles out leading logarithmic behavior in ratio Q_1^2/Q_2^2, thus obtained leading poles at $\gamma \sim 0$.
- This corresponds to energy scale $s_0 \sim Q_1^2$. Changing to symmetric energy scale $s_0 \sim Q_1 Q_2$ the pole at $\gamma \sim 0$ gets shifted by $-\omega/2$.
- In anticollinear limit , $Q_2^2 \gg Q_1^2$, the same result is obtained with pole at $\gamma \sim 1$ when scale $s_0 \sim Q_2^2$ is chosen. For scale $s_0 \sim Q_1^2$ the pole at $\gamma \sim 1$ gets shifted to $\gamma \sim 1 + \omega$.

Collinear analysis

$$\tilde{\tilde{\sigma}}^{(TT,0)}(\omega,\gamma)\Big|_{p=1}^{\rm coll} = \Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\Big|_{p=1}^{\rm coll}$$
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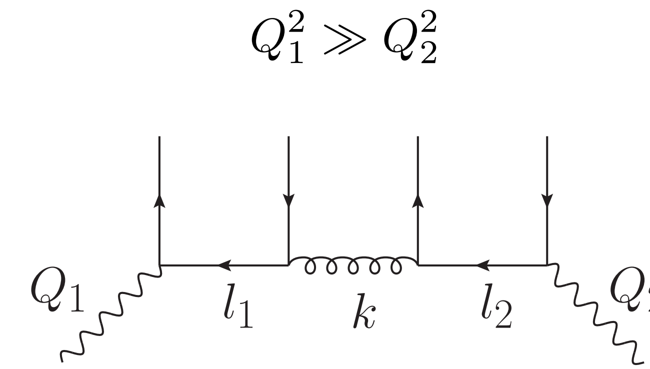

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Taking both collinear and anticollinear limit for scale choice $s_0 \sim Q_1^2$ we get

$$\Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\Big|_{p=1}^{2\times{\rm coll}} = (2\pi)^3\alpha\Big(2\sum_{q} e_q^2\Big)\frac{1}{\gamma}\cdot\frac{\alpha_{\rm s}}{2\pi}\frac{P_{qg}(\omega)}{\gamma}\cdot\frac{\alpha_{\rm s}}{2\pi}\frac{P_{gq}(\omega)}{\gamma}\cdot\frac{\alpha}{2\pi}\Big(2\sum_{q} e_q^2\Big)\frac{P_{q\gamma}(\omega)}{\gamma}$$
$$+\Big(\gamma\to 1+\omega-\gamma\Big)\,.$$

Resummed LO impact factor

Using exact expressions of in Mellin space

$$P_{qq}(\omega) = C_F\left(\frac{5}{4} - \frac{\pi^2}{3}\right)\omega + \mathcal{O}(\omega^2)$$

$$P_{gq}(\omega) = \frac{2C_F}{\omega}\left[1 + \omega A_{gq}(\omega)\right] \qquad A_{gq}(0) = -\frac{3}{4}$$

$$P_{qg}(\omega) = \frac{2}{3}T_R\left[1 + \omega A_{qg}(\omega)\right] \qquad A_{qg}(0) = -\frac{13}{12}$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega}\left[1 + \omega A_{gg}(\omega)\right] \qquad A_{gg}(0) = -\frac{11}{6} + \bar{b}\,, \quad \bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R}P_{qg}(\omega)\,.$$

Leading structure of impact factor with ω dependent coefficient by *Bialas, Navelet, Peschanski*

$$\Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\Big|_{p=1}^{2\times\text{coll}} = \left[\alpha\alpha_s\Big(\sum_q e_q^2\Big)2P_{qg}(\omega)\sqrt{2(N_c^2-1)}\left(\frac{1}{\gamma^2} + \frac{1}{(1+\omega-\gamma)^2}\right)\right]^2 \times \frac{1}{\omega}\Big(1 + \omega A_{gq}(\omega)\Big)$$

Additional term from collinear analysis. Can be included by modification of impact factor:

Term from the gluon exchange (Born level GGF)

Resummed LO impact factor

Using exact expressions of in Mellin space

$$P_{qq}(\omega) = C_F\left(\frac{5}{4} - \frac{\pi^2}{3}\right)\omega + \mathcal{O}(\omega^2)$$

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Term from the gluon exchange (Born level GGF)

Additional term from collinear analysis. Can be included by modification of impact factor:

One possibility

RGI LO transverse impact factor

$$\Phi_0^{(T)}(\omega,\gamma;1) = \Phi_0^{(T,\text{BNP})}(\omega,\gamma)\left[1 + \frac{\omega}{2}A_{gq}(\omega)\right]$$

RGI NLO impact factor

Need two constraints for RGI NLO impact factor

1) RGI cross section agrees with the BFKL at NLO

General relation between two formulations:

$$\chi(\gamma) = X(\omega^{\text{eff}}, \gamma) \qquad \phi^{(j)}(\gamma)\phi^{(k)}(1-\gamma) = \frac{\Phi^{(j)}(\omega^{\text{eff}}, \gamma)\Phi^{(k)}(\omega^{\text{eff}}, 1-\gamma)}{1 - \bar{\alpha}_s \partial_\omega X(\omega^{\text{eff}}, \gamma)}$$

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At NLO this leads to the relation for the impact factors:

$$\phi_1(\gamma) + \phi_1(1-\gamma) = \Phi_1(0,\gamma) + \Phi_1(0,1-\gamma) + \chi_0(\gamma)[\partial_\omega \Phi_0(0,\gamma) + \partial_\omega \Phi_0(0,1-\gamma)] + \phi_0(\gamma)\partial_\omega X_0(0,\gamma)$$

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At NLO this leads to the relation for the impact factors:

$$\phi_1(\gamma)+\phi_1(1-\gamma) = \Phi_1(0,\gamma)+\Phi_1(0,1-\gamma) + \chi_0(\gamma)[\partial_\omega\Phi_0(0,\gamma)+\partial_\omega\Phi_0(0,1-\gamma)] + \phi_0(\gamma)\partial_\omega X_0(0,\gamma)$$

For the highest poles (quartic in transverse case) this gives:

$$\chi_0(\gamma)[\partial_\omega\Phi_0(0,\gamma)+\partial_\omega\Phi_0(0,1-\gamma)] + \phi_0(\gamma)\partial_\omega X_0(0,\gamma) \xrightarrow{\gamma\to 0} \phi_0(\gamma)\left(-\frac{5}{2}\frac{1}{\gamma^2}\right)$$

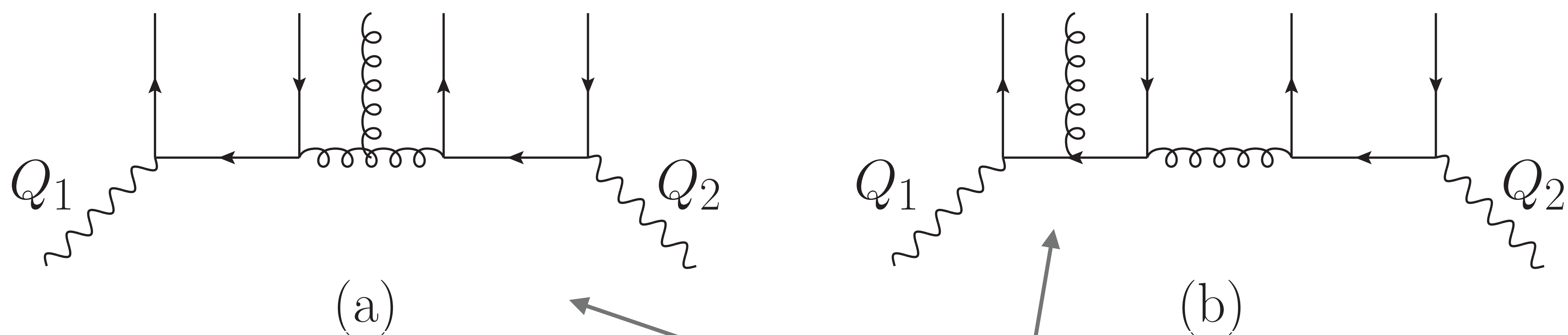
This coincides with NLO result (by *Chirilli,Kovchegov* in the form written *by Ivanov, Murdaca, Papa*)

$$\phi_1(\gamma)+\phi_1(1-\gamma) \xrightarrow{\gamma\to 0} \phi_0(\gamma)\left(-\frac{5}{2}\frac{1}{\gamma^2}+\ldots\right)$$

Collinear analysis at NLO

2) RGI cross section agrees with the DGLAP collinear limits $Q_1 \gg Q_2$ and $Q_1 \ll Q_2$ at $\mathcal{O}(\alpha_s^3)$

This determines the structure of the impact factors at the poles at $\gamma \sim 0$ and at $\gamma \simeq 1 + \omega$



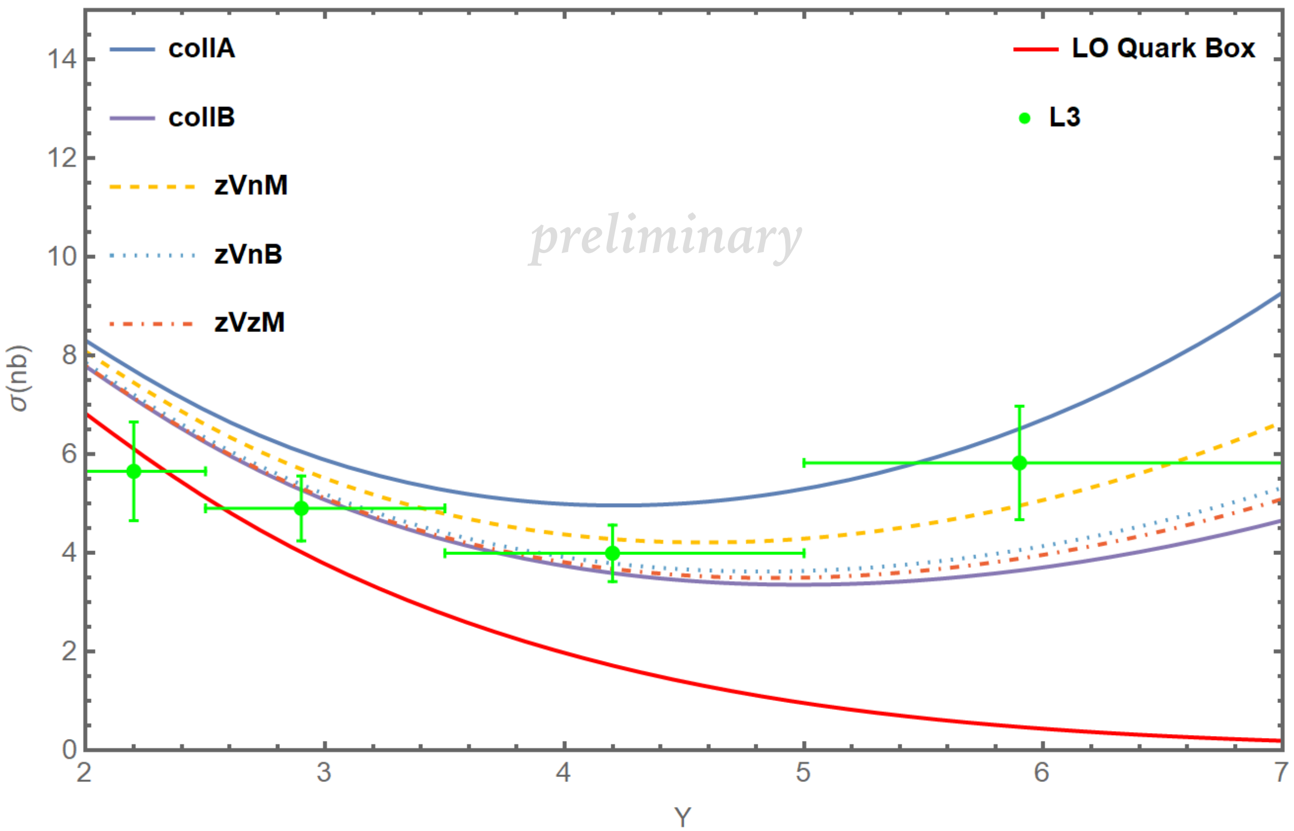
$$\tilde{\tilde{\sigma}}_1^{(TT)}(\omega, \gamma; 1) = \tilde{\tilde{\sigma}}_0^{(TT)}(\omega, \gamma; 1) \left[\frac{\alpha_s}{2\pi} \frac{P_{gg}}{\gamma} + 2 \frac{\alpha_s}{2\pi} \frac{P_{qq}}{\gamma} - \frac{\alpha_s b_0}{\gamma} + \mathcal{O}(\gamma^0) \right]$$

LO

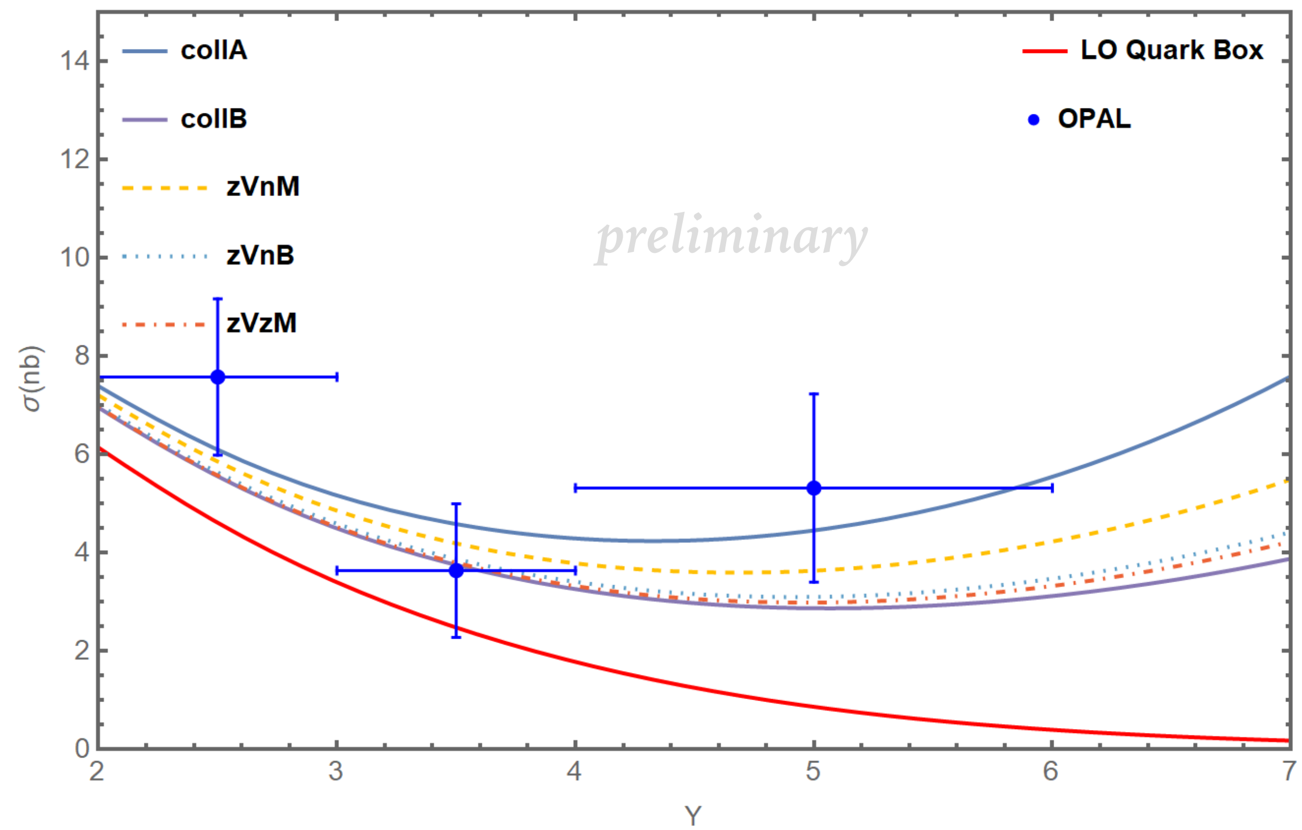
- Can attribute it to the GGF
- Can attribute it to the impact factor
- Running coupling can attribute to either or both

Results for $\gamma^*\gamma^*$ cross section

L3: $Q^2 = 16\ \mathrm{GeV}^2$



OPAL: $Q^2 = 17.9\ \mathrm{GeV}^2$



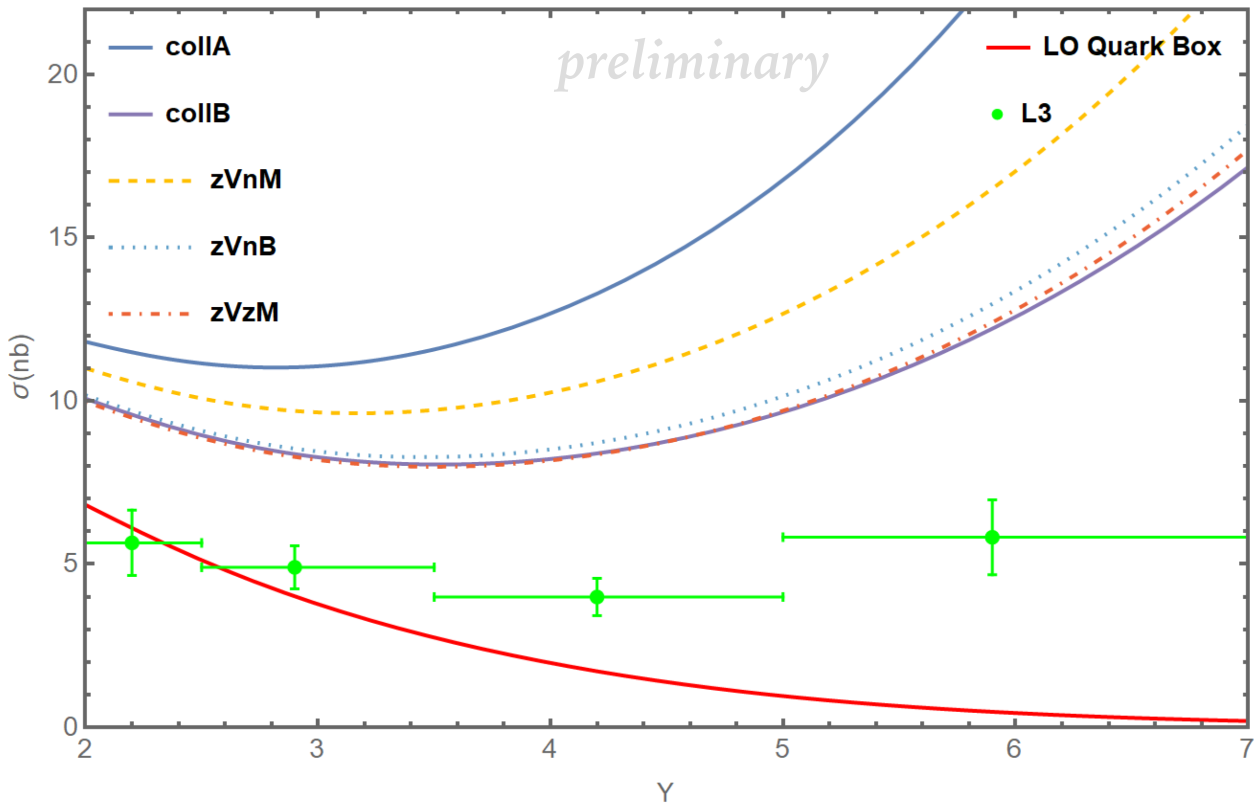
Spread in resummation models

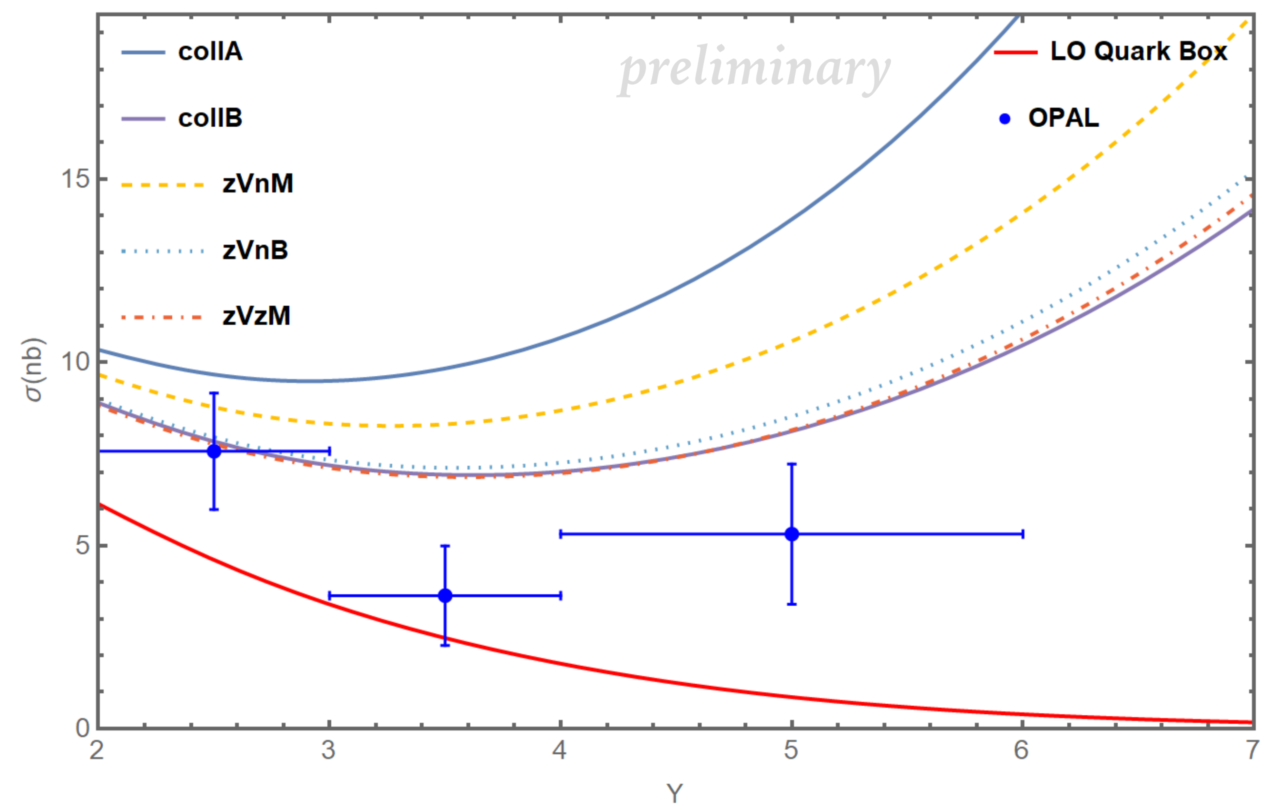
Overall consistent with the data

But this resummed calculation is for $n_f = 3$ light (massless) flavors…

Results for $\gamma^*\gamma^*$ cross section

…for $n_f = 4$ massless, cross section effectively multiplied by 2.78.





Calculation overshoots the data

However, charm is treated as **massless**, so not realistic

Mass effects are large in NLO massive impact factor

Need resummation with masses

NLO corrections to quark box+resummation of double logs, change little bit the result

Conclusions

- **Resummation** needed to for both gluon Green's function and impact factors
- Impact factors get **shift of collinear poles**, similar to GGF
- **Collinear limit** imposed to constraint the RGI impact factors
- Resummed result matches to **BFKL and DGLAP.** (consistency between BFKL and DGLAP for $1/\gamma^4, 1/\gamma^3$ poles in transverse photon case; $1/\gamma^3$, longitudinal case, partially $1/\gamma^2$)
- Resummation gives result **consistent with LEP data**, lower than from BFKL LL and higher than BFKL NLL
- NLO and double log resummation in the **quark box** will slightly modify the result, though not large energy behavior
- **Mass effects** (charm) need to be treated separately in resummation