Elucidating the correspondence between CGC and High Twist formalism

SURGE Collaboration Meeting

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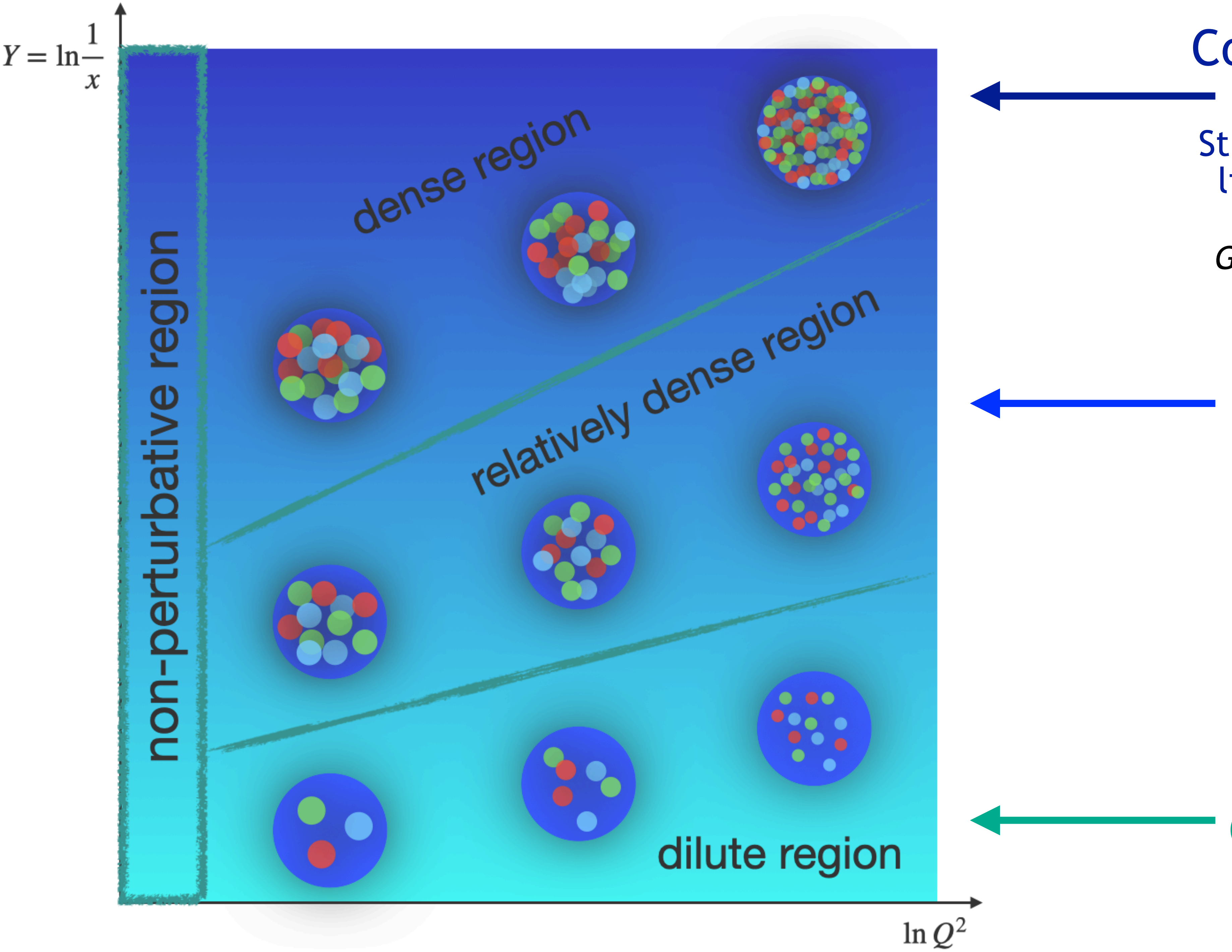
Farid Salazar

work in preparation in collaboration with
Yu Fu, Zhong-Bo Kang, Xin-Nian Wang, and Hongxi Xing

Anatomy of QCD matter

The Physics of dilute and dense regimes of QCD



Color Glass Condensate (CGC)

Strong fields, Wilson lines, non-linear evolution (BK/JIMWLK)

Review:
Gelis, Iancu, Venugopalan (2003)

High-twist formalism

Multiparton correlations, DGLAP type equations

Qiu, Sterman (1991)
Guo, Wang (2000)
Qiu, Vitev (2003)
Kang, Wang, Wang, Xing (2013)

Leading twist Collinear factorization

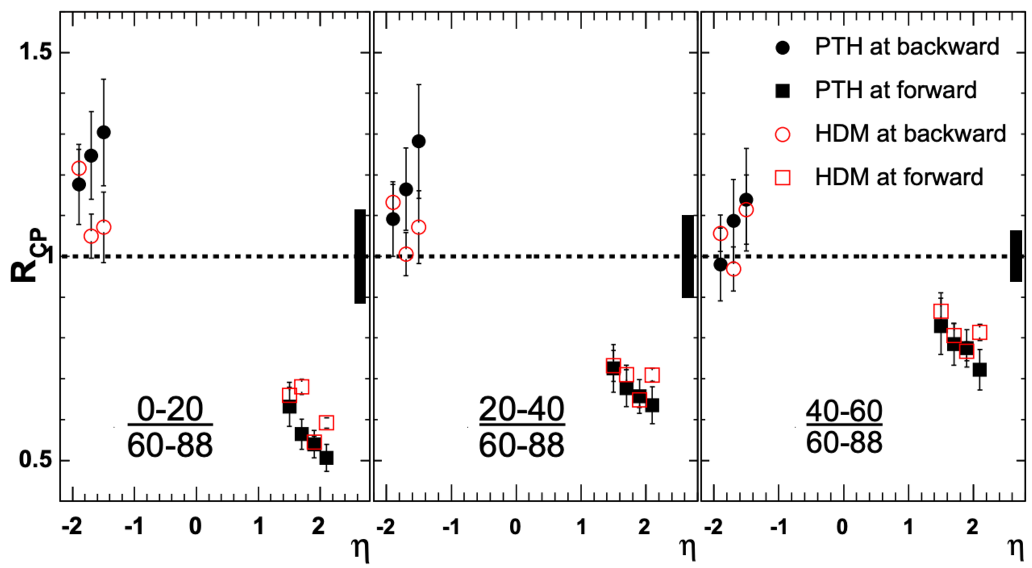
PDFs, DGLAP evolution

Collins, Soper (1981)

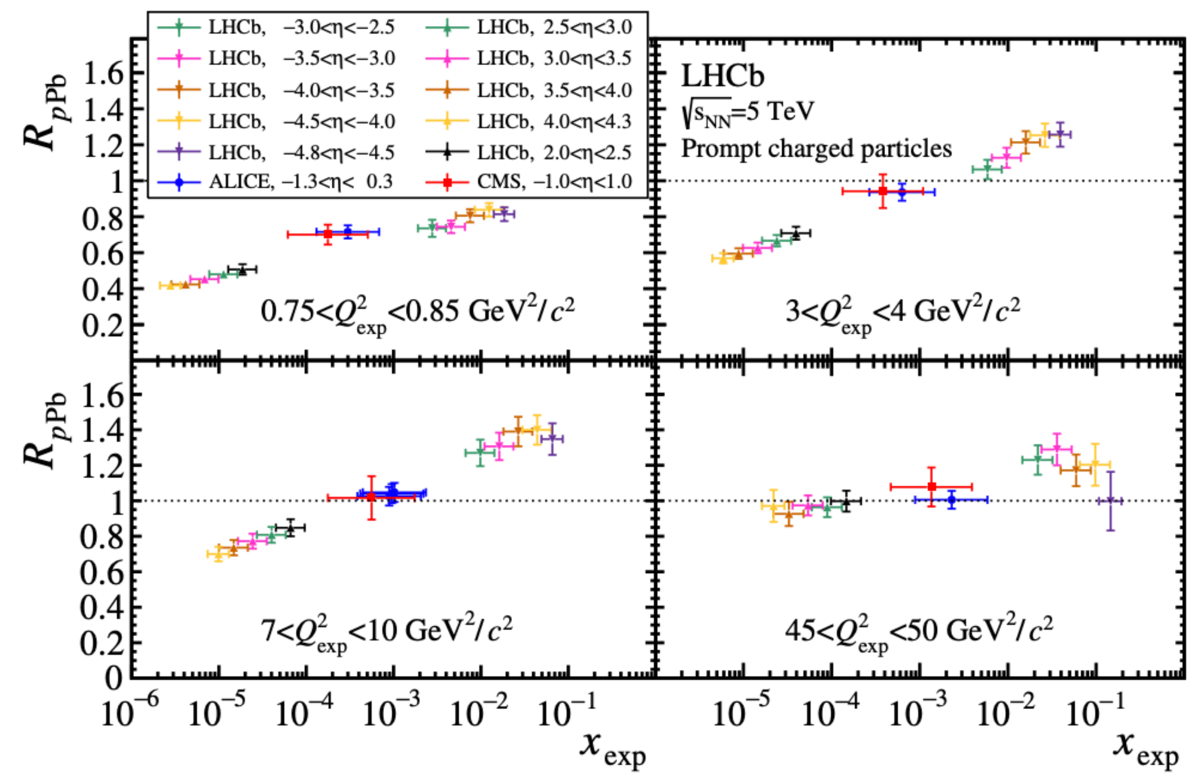
Nuclear modification ratio

Enhancement (backward region) vs suppression (forward region)

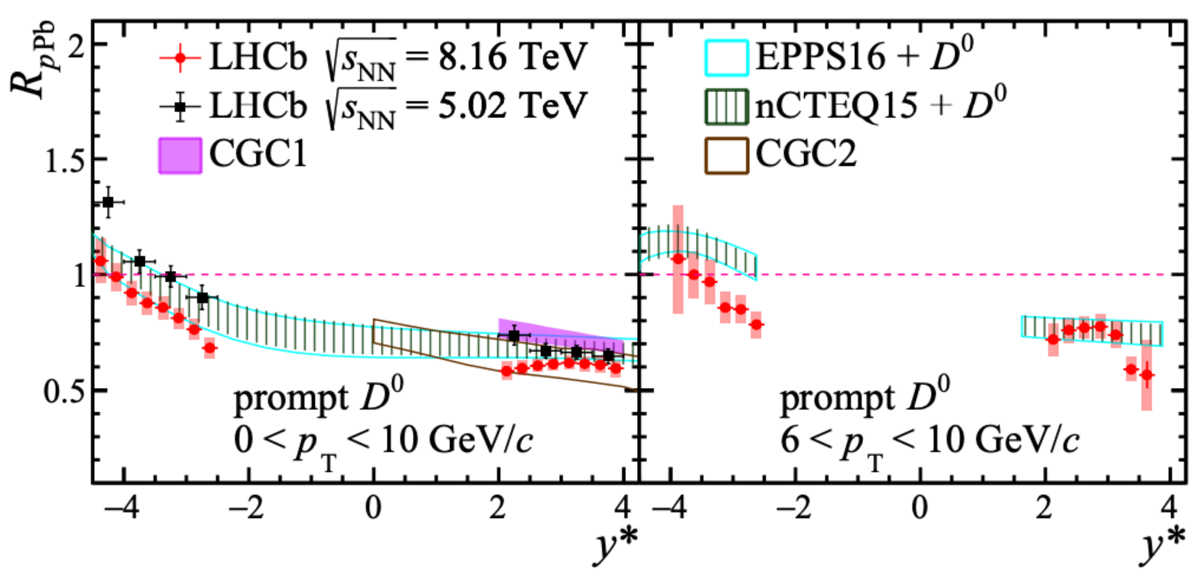
PHENIX (2004)
hadron production in dAu



LHCb (2022)
Charged particle production pPb



LHCb (2022)
prompt D meson in pPb



Nuclear modification ratio

$$R_{pA} = \frac{1}{A^{1/3}} \frac{\sigma_{pA}}{\sigma_{pp}}$$

Values x probed in nuclei

$$x_A \sim \frac{Q}{\sqrt{s}} e^{-y}$$

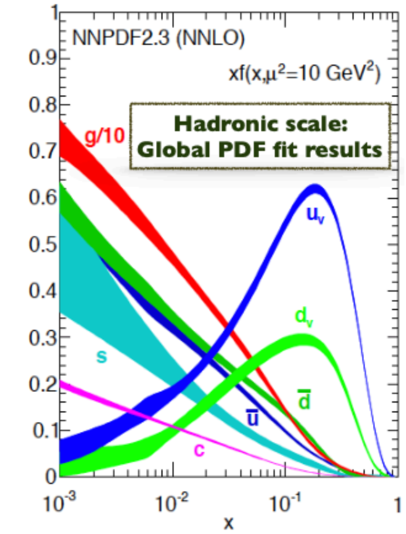
Theoretical framework for multiple scattering

(Generalized) High-Twist

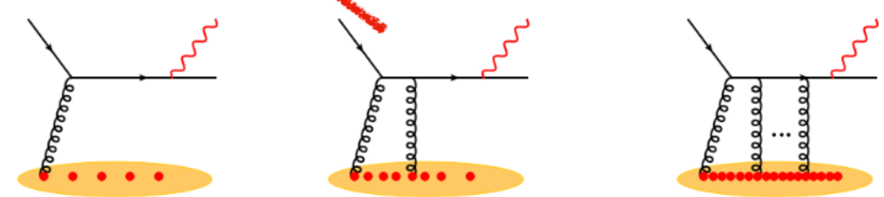
perturbative expansion

$$\sigma^h_{phys} = \left[\alpha_s^0 C_2^{(0)} + \alpha_s^1 C_2^{(1)} + \alpha_s^2 C_2^{(2)} + \ldots\right] \otimes T_2(x)$$
$$+ \frac{1}{Q}\left[\alpha_s^0 C_3^{(0)} + \alpha_s^1 C_3^{(1)} + \alpha_s^2 C_3^{(2)} + \ldots\right] \otimes T_3(x)$$
$$+ \frac{1}{Q^2}\left[\alpha_s^0 C_4^{(0)} + \alpha_s^1 C_4^{(1)} + \alpha_s^2 C_4^{(2)} + \ldots\right] \otimes T_4(x)$$
$$+ \ldots$$

Multiple scattering expansion



twist-4 expansion

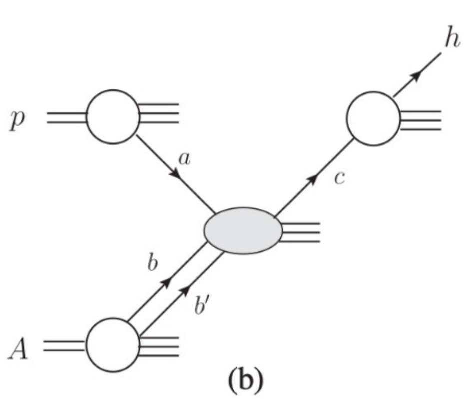


- Nuclear enhanced power correction

$$\frac{1}{Q^2} \rightarrow \frac{A^{1/3}}{Q^2}$$

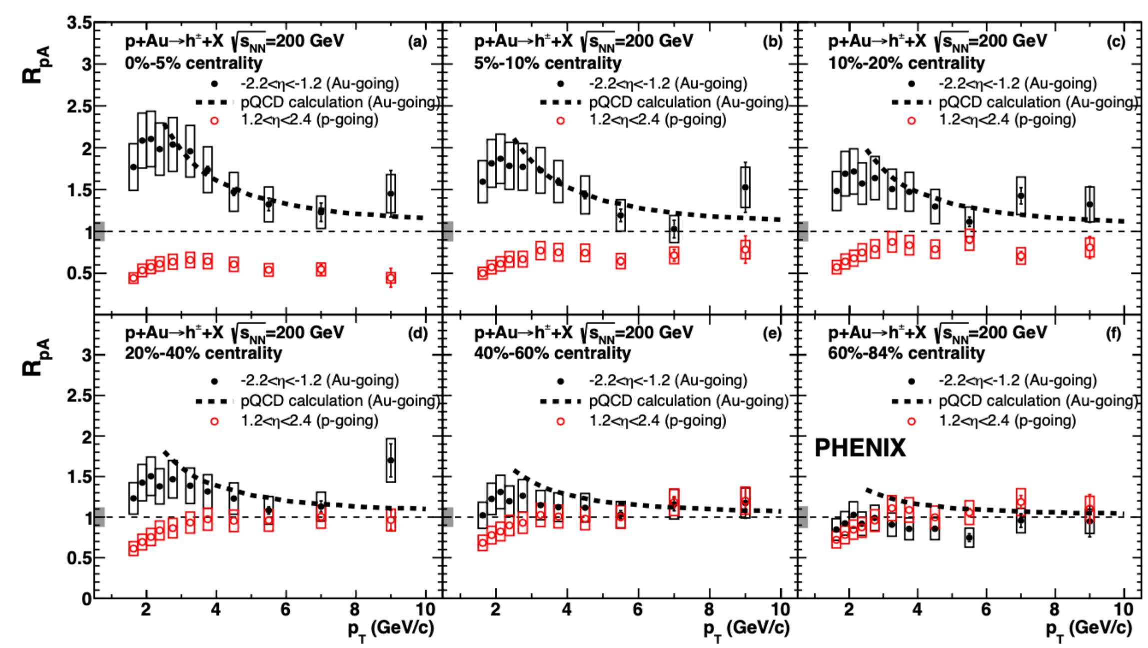
- Enhancement from twist-4 contribution

$$E_h \frac{d\sigma^{(D)}}{d^3 P_h} = \left(\frac{8\pi^2 \alpha_s}{N_c^2 - 1}\right) \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{a/p}(x') \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$
$$\times \sum_{i=I,F} \left[x^2 \frac{\partial^2 T^{(i)}_{b/A}(x)}{\partial x^2} - x \frac{\partial T^{(i)}_{b/A}(x)}{\partial x} + T^{(i)}_{b/A}(x)\right] c^i H^i_{ab\to cd}(\hat{s}, \hat{t}, \hat{u})$$



p+A -> h + X

Kang, Vitev, Xing (2014)



Enhancement in RpA!

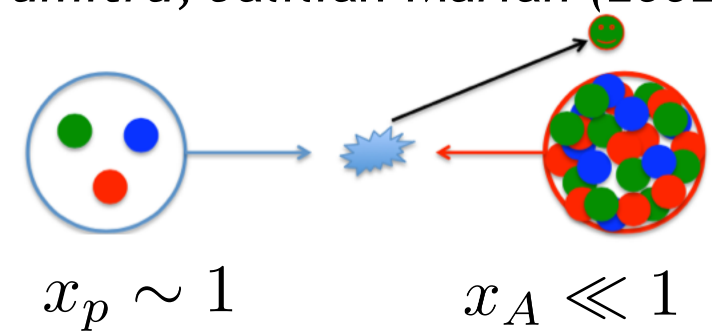
Theoretical framework for multiple scattering

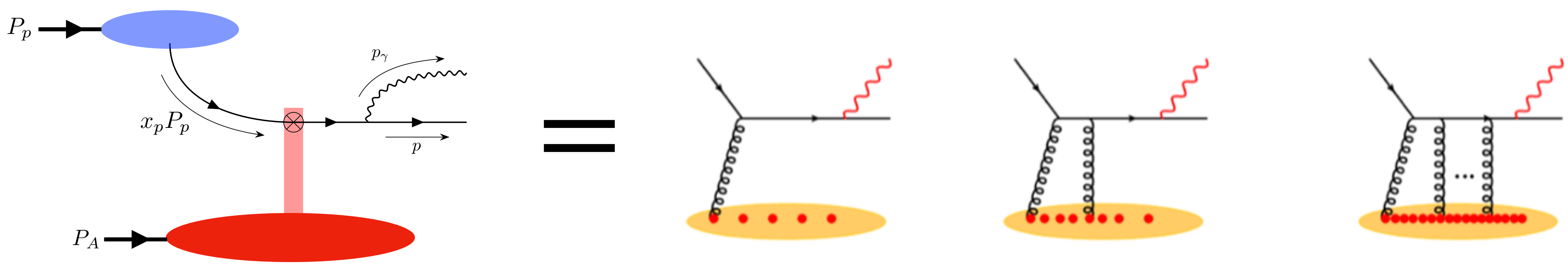
Color Glass Condensate

Dumitru, Jalilian-Marian (2002)

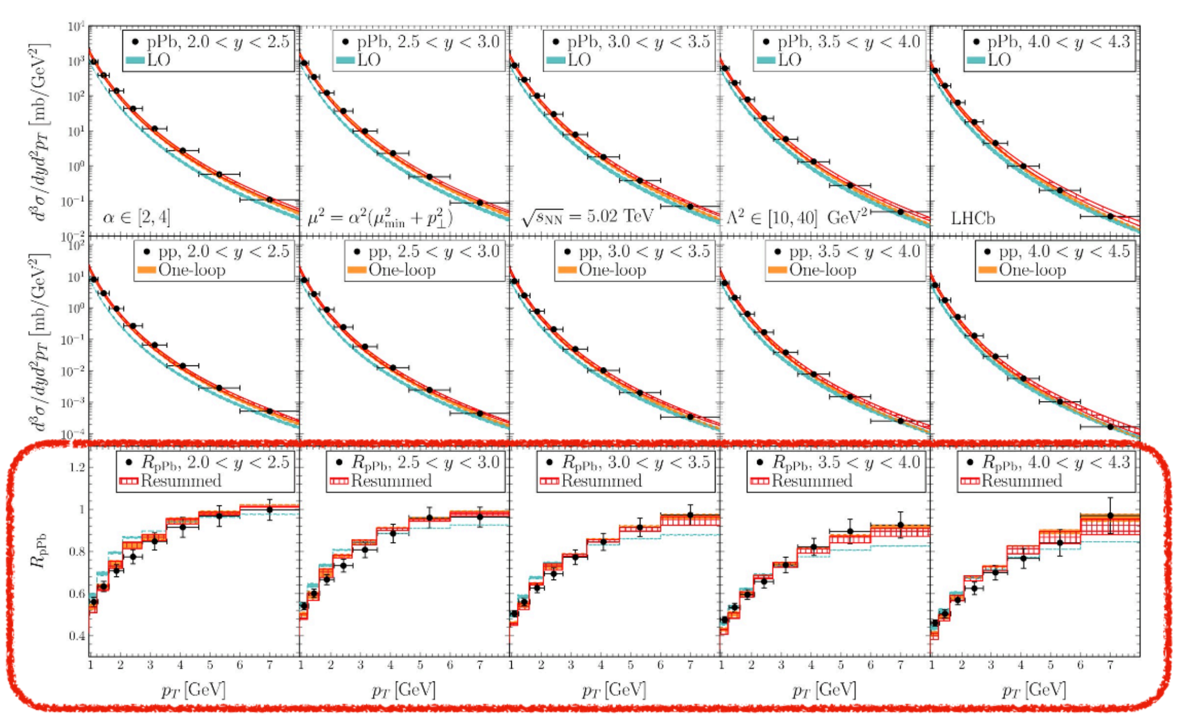
- Hybrid (dilute-dense factorization)

$$\sigma \propto x_p f_{q/p}(x_p) \otimes H \otimes \mathcal{F}(x_A, k_\perp) \otimes D_{h/q}(z)$$





- Multiple (eikonal) scatterings are resummed and are encoded in the unintegrated gluon distribution $\mathscr{F}(x_g, k_\perp)$



p+A -> h + X

Chirilli, Xiao, Yuan (2012)
Stasto, Xiao, Zaslavsky (2014)
Iancu, Mueller, Triantafyllopoulos (2016)
Liu, Kang, Li (2020)
Shi, Wang, Wei, Xiao (2022)

Suppression in RpA!

A unified picture of dilute and dense limits

Several efforts in this direction:

Gluon TMD in particle production from low to moderate x

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Abstract: We study the rapidity evolution of gluon transverse momentum dependent distributions appearing in processes of particle production and show how this evolution changes from small to moderate Bjorken x.

Keywords: Deep Inelastic Scattering (Phenomenology), QCD Phenomenology

ArXiv ePrint: 1603.06548

Next-to-eikonal corrections in the CGC: gluon production and spin asymmetries in pA collisions

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Abstract: We present a new method to systematically include corrections to the eikonal approximation in the background field formalism. Specifically, we calculate the subleading, power-suppressed corrections due to the finite width of the target or the finite energy of the projectile. Such power-suppressed corrections involve Wilson lines decorated by gradients of the background field — thus related to the density - of the target. The method is of generic applicability. As a first example, we study single inclusive gluon production in pA collisions, and various related spin asymmetries, beyond the eikonal accuracy.

Keywords: QCD Phenomenology, Hadronic Colliders

ArXiv ePrint: 1404.2219

Gluon-mediated inclusive Deep Inelastic Scattering from Regge to Bjorken kinematics

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Abstract: We revisit high energy factorization for gluon mediated inclusive Deep Inelastic Scattering (DIS) for which we propose a new semi-classical approach that accounts systematically for the longitudinal extent of the target in contrast with the shockwave limit. In this framework, based on a partial twist expansion, we derive a factorization formula that involves a new gauge invariant unintegrated gluon distribution which depends explicitly on the Feynman x variable. It is shown that both the Regge and Bjorken limits are recovered in this approach. We reproduce in particular the full one loop inclusive DIS cross-section in the leading twist approximation and the all-twist dipole factorization formula in the strict $x = 0$ limit. Although quantum evolution is not discussed explicitly in this work, we argue that the proper treatment of the x dependence of the gluon distribution encompasses the kinematic constraint that must be imposed on the phase-space of gluon fluctuations in the target to ensure stability of small-x evolution.

Keywords: Deep Inelastic Scattering or Small-X Physics, Parton Distributions

ArXiv ePrint: 2112.01412

Quark jets scattering from a gluon field: From saturation to high p_t

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We continue our studies of possible generalization of the color glass condensate effective theory of high energy QCD to include the high p_t (or equivalently large x) QCD dynamics as proposed in [Phys. Rev. D 96, 074020 (2017)]. Here, we consider scattering of a quark from both the small and large x gluon degrees of freedom in a proton or nucleus target and derive the full scattering amplitude by including the interactions between the small and large x gluons of the target. We thus generalize the standard eikonal approximation for parton scattering, which can now be deflected by a large angle (and therefore have large p_t) and also lose a significant fraction of its longitudinal momentum (unlike the eikonal approximation). The corresponding production cross section can thus serve as the starting point toward the derivation of a general evolution equation that would contain the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation at large Q^2 and the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner evolution equation at small x. This amplitude can also be used to construct the quark Feynman propagator, which is the first ingredient needed to generalize the color glass condensate effective theory of high energy QCD to include the high p_t dynamics. We outline how it can be used to compute observables in the large x (high p_t) kinematic region where the standard color glass condensate formalism breaks down.

DOI: 10.1103/PhysRevD.99.014043

Helicity evolution at small x

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Abstract: We construct small-x evolution equations which can be used to calculate quark and anti-quark helicity TMDs and PDFs, along with the g_1 structure function. These evolution equations resum powers of $\alpha_s \ln^2(1/x)$ in the polarization-dependent evolution along with the powers of $\alpha_s \ln(1/x)$ in the unpolarized evolution which includes saturation effects. The equations are written in an operator form in terms of polarization-dependent Wilson line-like operators. While the equations do not close in general, they become closed and self-contained systems of non-linear equations in the large-N_c and large-N_c & N_f limits. As a cross-check, in the ladder approximation, our equations map onto the same ladder limit of the infrared evolution equations for the g_1 structure function derived previously by Bartels, Ermolaev and Ryskin [1].

Keywords: Resummation, Perturbative QCD

ArXiv ePrint: 1511.06737

Quark branching in QCD matter to any order in opacity beyond the soft gluon emission limit

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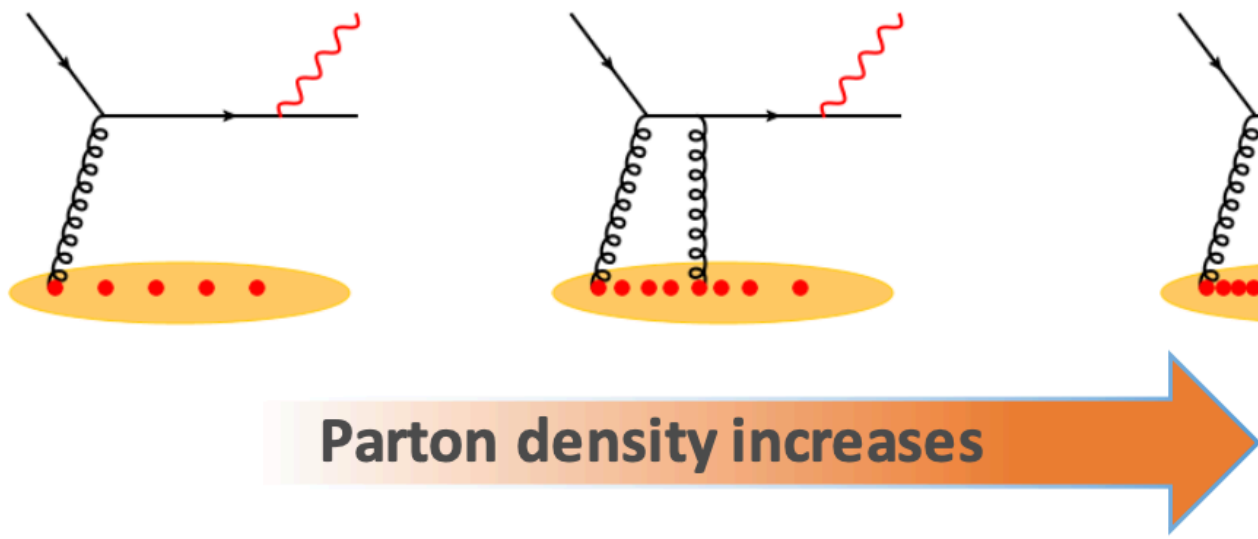
ıld nuclear matter effects in reactions with nuclei at a future electron-ion collider (EIC) lead to a fication of semi-inclusive hadron production, jet cross sections, and jet substructure when compared to acuum. At leading order in the strong coupling, a jet produced at an EIC is initiated as an energetic :, and the process of this quark splitting into a quark-gluon system underlies experimental observables. spectrum of gluons associated with the branching of this quark jet is heavily modified by multiple ering in a medium, allowing jet cross sections and jet substructure to be used as a probe of the um's properties. We present a formalism that allows us to compute the gluon spectrum of a quark jet to bitrary order in opacity, the average number of scatterings in the medium. This calculation goes ıd the simplifying limit in which the gluon radiation is soft and can be interpreted as energy loss of the :, and it significantly extends previous work which computes the full gluon spectrum only to first order acity. The theoretical framework demonstrated here applies equally well to light parton and heavy : branching, and is easily generalizable to all in-medium splitting processes.

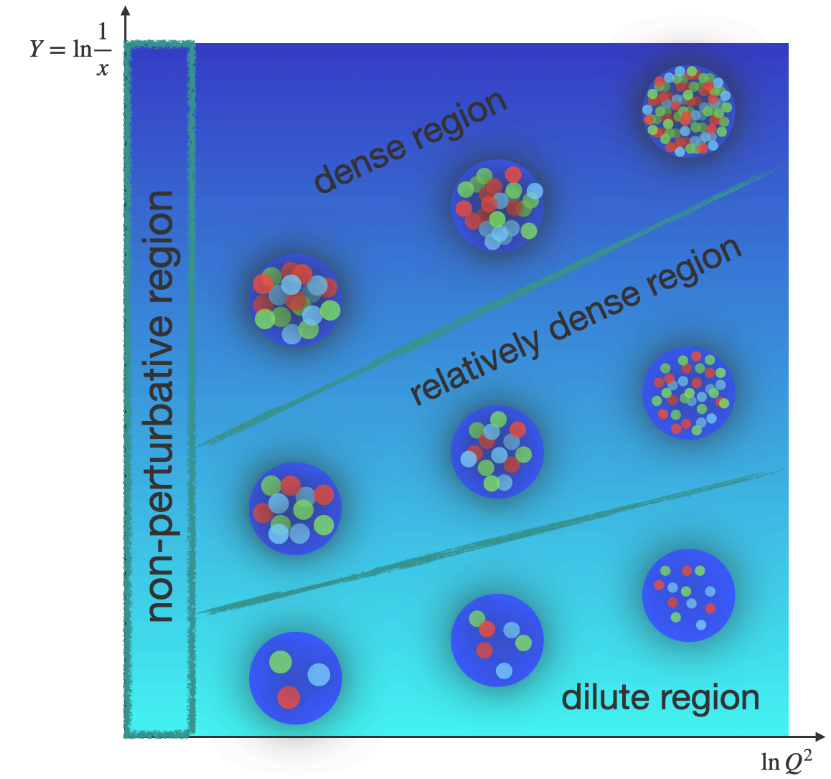
DOI: 10.1103/PhysRevD.98.094010

+ many more!

Consistency between CGC and High-Twist formalism

Scanning different regions of hadronic matter





Twist expansion:

$$\mathrm{d}\sigma \sim \frac{1}{p_{\gamma\perp}^4}\left[A + B\frac{\langle k_\perp^2\rangle}{p_{\gamma\perp}^2} + C\frac{\langle k_\perp^2\rangle^2}{p_{\gamma\perp}^4}\ldots\right]$$

leading twist
(twist-2)

Higher twist
(twist-4 and twist-6)

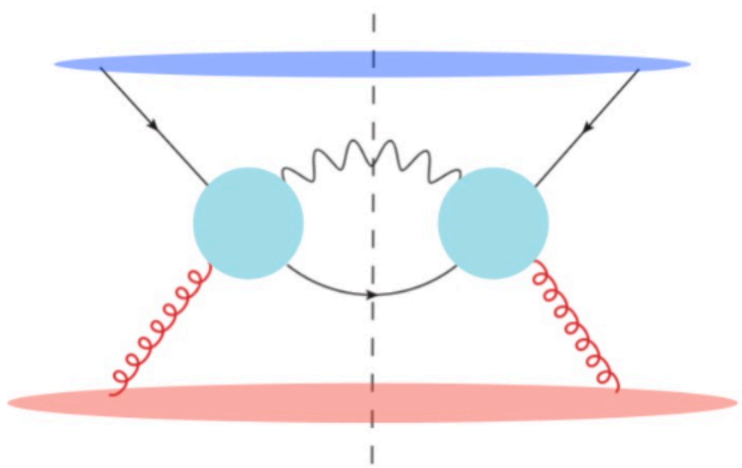
CGC: Saturation scale grows with energy and nuclear number

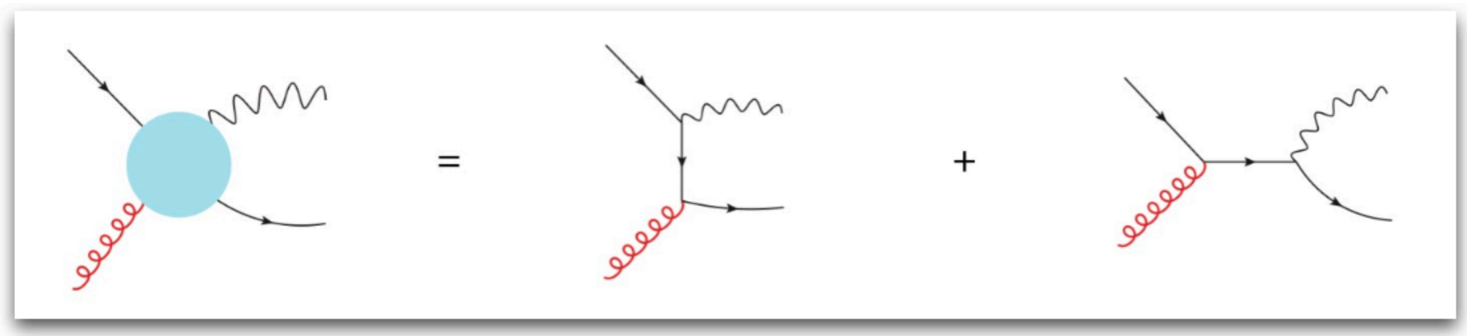
$$\langle k_\perp^2\rangle \propto Q_s^2 \propto A^{1/3}x^{-\lambda}$$

Direct photon production in pA collisions

Single scattering contribution

- Consider quark-gluon initiated channel





- Leading twist collinear factorization

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p_\gamma}_\perp} = \frac{\alpha_{\mathrm{em}}\alpha_s}{N_c}\int \mathrm{d}x_p f(x_p)\frac{\xi^2\left[1+(1-\xi)^2\right]}{\boldsymbol{p_\gamma}_\perp^4} x_A g(x_A)$$

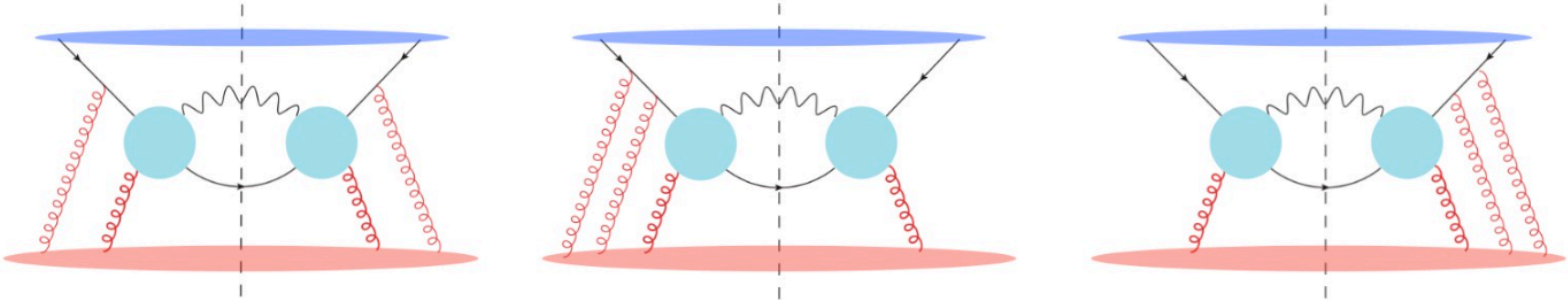
$$\boxed{xg(x) = \frac{1}{P_A^+}\int \frac{\mathrm{d}y^-}{2\pi} e^{ixP_A^+ y^-} \left\langle P_A \middle| F_a^{\alpha+}(y^-) F_a^{\beta+}(0^-) \middle| P_A \right\rangle \delta_{\perp\alpha\beta}}$$

Direct photon production in pA collisions

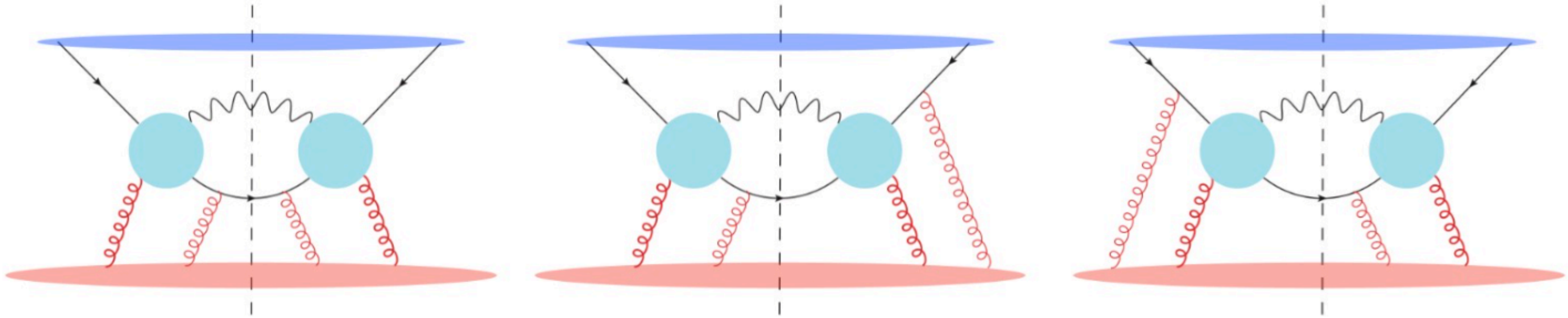
Incoherent multiple scattering from high-twist

Soft gluon scattering insertion

- Initial state: double scattering and single-triple scattering



- Final state double scattering and initial-final state interference



+ other 18 diagrams

Direct photon production in pA collisions

Incoherent multiple scattering from high-twist

- Twist-4 contribution to the differential cross-section

$$E_\gamma \frac{d\sigma^D_{qA\to\gamma}}{d^3\boldsymbol{p}_\gamma} = \int dx_p f_q(x_p) x_b \frac{4\pi^2\alpha_s^2\alpha_e}{N_c^2} \frac{\xi^2 - 2\xi + 2}{\boldsymbol{p}^6_{\gamma\perp}} \left[\cdots\right]_{x_1=x_b, x_2=x_3=0}$$

$$T(x_1,x_2,x_3) = \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi} e^{ix_1P^+y^-} e^{ix_2P^+(y_1^- - y_2^-)} e^{ix_3P^+y_2^-} \frac{1}{xP^+}\left\langle P_A | F^{+\omega}(0^-) F^{+\kappa}(y_2^-) F^+{}_\kappa(y_1^-) F^+{}_\omega(y^-) | P_A \right\rangle$$

result from initial state rescattering

$[\cdots]$ Cuts / Derivatives	Central	Asymmetric
2nd	$\xi^4[x_b^2 \frac{\partial^2 T^{C,I}}{\partial x_1^2}]$	0
1st	$-3\xi^4[x_b \frac{\partial T^{C,I}}{\partial x_1}] + (1-\xi)\xi^3[x_b \frac{\partial T^{C,I}}{\partial x_2}]$	$(1-\xi)\xi^3[x_b \frac{\partial T^{A,I}}{\partial x_2}]$
0th	$4\xi^4 T^{C,I}$	0

- Contribution responsible for nuclear enhancement at large-x

$$E_\gamma \frac{d\sigma^D_{pA\to\gamma}}{d^3\boldsymbol{p}_\gamma} = \frac{4\pi^2\alpha_s^2\alpha_e}{N_c}\frac{1}{s}\int \frac{dx_p}{x_p} f(x_p) \int \frac{dx}{x} c^I H^U_{qg\to q\gamma}(\hat{s},\hat{t},\hat{u})\,\delta(\hat{s}+\hat{t}+\hat{u}) \boxed{\left[x^2\frac{\partial^2 T^I(x)}{\partial x^2} - x\frac{\partial T^I(x)}{\partial x} + xT^I(x)\right]}$$

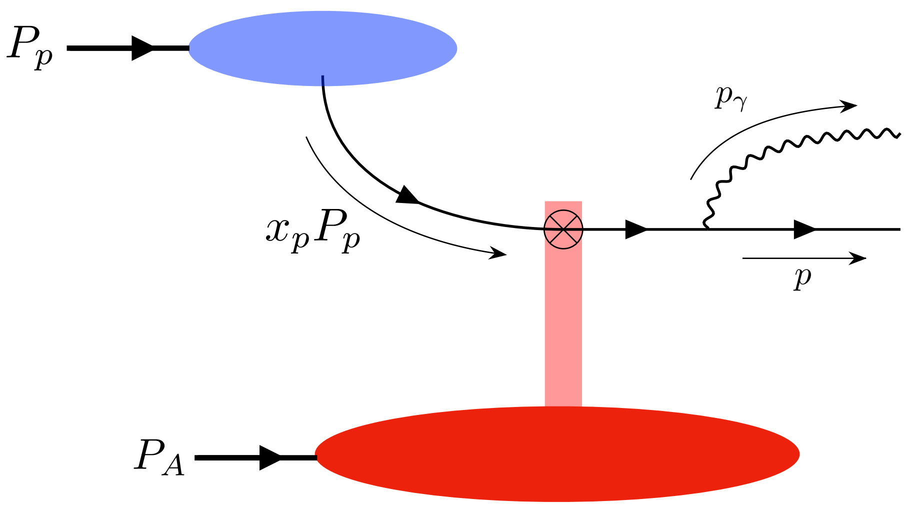
$$c^I = -\frac{1}{\hat{s}} - \frac{1}{\hat{t}}$$

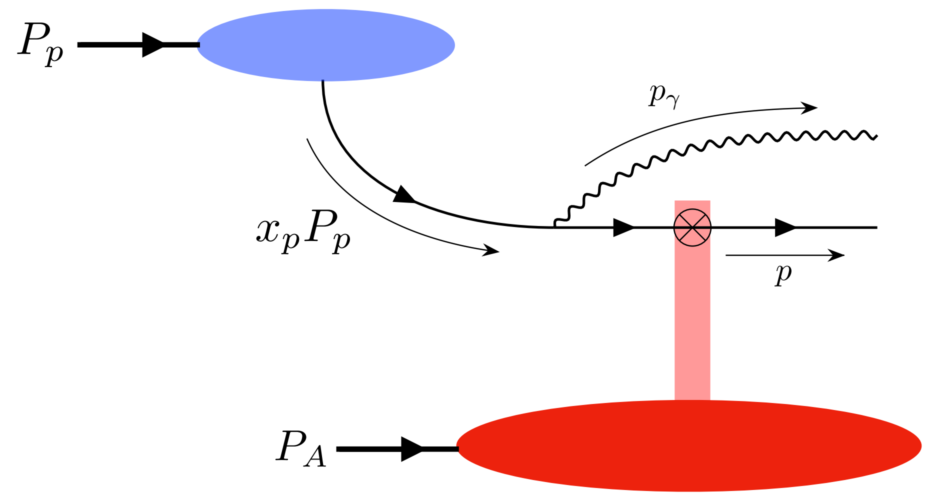
Only initial state rescattering contributes positive -> nuclear enhancement

Direct photon production in pA collisions

Coherent multiple scattering from CGC

- Amplitudes





- Differential cross-section CGC

Gelis, Jalilian-Marian (2002)

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\boldsymbol{\gamma}\perp}} = \frac{\alpha_{\mathrm{em}}}{2\pi^2}\int \mathrm{d}x_p f(x_p)\xi^2\left[1+(1-\xi)^2\right]$$
$$\times \int \mathrm{d}^2\boldsymbol{l}_\perp \int \frac{\mathrm{d}^2\boldsymbol{y}_\perp}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{y}'_\perp}{2\pi} e^{-i\boldsymbol{l}_\perp\cdot(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}\frac{\boldsymbol{l}_\perp^2 D(x_A;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{\left(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\boldsymbol{\gamma}\perp}\right)^2\boldsymbol{p}_{\boldsymbol{\gamma}\perp}^2}$$

- Dipole correlator

$$\boxed{D(x_A,\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\left[V(\boldsymbol{y}_\perp)V^\dagger(\boldsymbol{y}'_\perp)\right]\right\rangle_{x_A}}$$

From CGC to leading twist collinear factorization

Correspondence between CGC and single scattering

$$\frac{D(x_A;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp})^2\,\boldsymbol{p}_{\gamma\perp}^2}=\underbrace{\frac{D(x;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{\boldsymbol{p}_{\gamma\perp}^4}}_{\text{Twist-2}}+\frac{1}{2}\boldsymbol{l}_\perp^\alpha\boldsymbol{l}_\perp^\beta\frac{\partial}{\partial\boldsymbol{l}_\perp^\alpha}\frac{\partial}{\partial\boldsymbol{l}_\perp^\beta}\underbrace{\left[\frac{D(x_A;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp})^2}\right]}_{\text{Twist-4}}\Bigg|_{\boldsymbol{l}_\perp=\boldsymbol{0}_\perp}+\dots$$

- Twist-2

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma\mathrm{d}^2\boldsymbol{p}_{\gamma\perp}}=\frac{\alpha_{\mathrm{em}}\alpha_s}{N_c}\int\mathrm{d}x_p f(x_p)\frac{\xi^2\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\gamma\perp}^4}x_A g(x_A)\Big|_{x\to0}$$

Twist-2 gluon PDF = **second moment** dipole correlator *Baier, Mueller, Schiff (2004)*

$$xg(x)\overset{x\to0}{\overbrace{=}}\frac{N_c}{2\pi^2\alpha_s}\int\boldsymbol{l}_\perp^2\mathrm{d}^2\boldsymbol{l}_\perp C(x,\boldsymbol{l}_\perp)$$

Dipole correlator in momentum space

Phase $e^{ixP^+y^-}$ dropped out ("sub-eikonal")

$$\boxed{xg(x)\Big|_{x\to0}=\frac{1}{P_A^+}\int\frac{\mathrm{d}y^-}{2\pi}\langle P_A|\mathrm{Tr}\left[F_a^{\alpha+}(y^-)F_a^{\beta+}(0^-)\right]|P_A\rangle\,\delta_{\perp\alpha\beta}}$$

From CGC to twist-4 collinear factorization

Correspondence between CGC and double scattering

- Twist-4

$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\boldsymbol{\gamma}\perp}} = \frac{(2\pi)^2\alpha_{\mathrm{em}}\alpha_s^2}{N_c^2}\int \mathrm{d}x_p f(x_p)\frac{\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\boldsymbol{\gamma}\perp}^6}\xi^4 T_{\mathrm{HT}}(x_A)$$

Missing terms with derivatives of twist-4 distribution

Twist-4 gluon distribution = **fourth moment** dipole correlator

$$T_{\mathrm{HT}}(x) = \frac{2N_c^2}{(2\pi)^4\alpha_s^2}\int \boldsymbol{l}_\perp^4 \mathrm{d}^2\boldsymbol{l}_\perp F(x,\boldsymbol{l}_\perp)$$

$$\begin{aligned} T_{\mathrm{HT}}(x_1,x_2,x_3) =& \frac{1}{P_A^+}\int\frac{\mathrm{d}y^-}{2\pi}\left\langle P_A\right|F_a^{\alpha+}(y^-)F_b^{\rho+}(z^-)F_b^{\delta+}(z'^-)F_a^{\beta+}(0^-)\left|P_A\right\rangle\delta_{\perp\alpha\beta}\delta_{\perp\rho\delta} \\ &\times\left[\Theta(y^--z^-)\Theta(-z'^-)+\Theta(z^--y^-)\Theta(-z'^-)+\Theta(y^--z^-)\Theta(z'^-)+\Theta(z^--y^-)\Theta(z'^-)\right]\end{aligned}$$

Contains all orderings (central cut)

At small-x all twist-4 distributions collapse into a single distribution – no distinction between soft and hard gluons!

$$\lim_{x_1,x_2,x_3\to 0} T_{\mathrm{C,I}}(x_1,x_2,x_3) = \lim_{x_1,x_2,x_3\to 0} T_{\mathrm{C,F}}(x_1,x_2,x_3) = \lim_{x_1,x_2,x_3\to 0} T_{\mathrm{C,FI}}(x_1,x_2,x_3)$$

Initial state | final state | Interference

From CGC to twist-4 collinear factorization

Bringing back the longitudinal “sub-eikonal” phases

$$\mathrm{d}\sigma \propto \int \mathrm{d}x_p f(x_p)\ \mathcal{H} \otimes \mathcal{T}$$

- Expand the Wilson line and include sub-eikonal phase

$$(2\pi)\delta(l^- - l'^-)\gamma^- \int \mathrm{d}^2\boldsymbol{y}_\perp e^{-i(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp)\cdot\boldsymbol{y}_\perp} \int \mathrm{d}y^- \boxed{e^{i(l^+ - l'^+)y^-}} igA_a^+(y^-, \boldsymbol{y}_\perp)(t^a)_{ij}$$

- Collinear expansion (in powers $1/p_{\gamma,\perp}^2$)

$$\mathcal{H}_2^{coll}(p_\gamma; y, y') = 8H(\xi, \boldsymbol{p}_{\gamma\perp}) \boxed{e^{i\bar{x}_A P_A^+(y^- - y'^-)}} \frac{\partial^2 \delta^{(2)}(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)}{\partial \boldsymbol{y}_\perp \cdot \partial \boldsymbol{y}'_\perp}$$

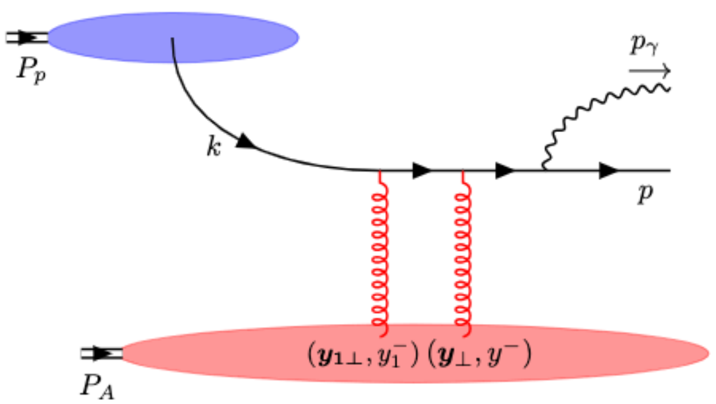
$$\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} = \frac{\alpha_{\mathrm{em}} e_f^2 \alpha_s}{N_c} \int_{x_{p,min}}^1 \mathrm{d}x_p f(x_p) H(\xi, \boldsymbol{p}_{\gamma\perp}) \bar{x}_A f_{g/A}^{(0)}(\bar{x}_A)$$

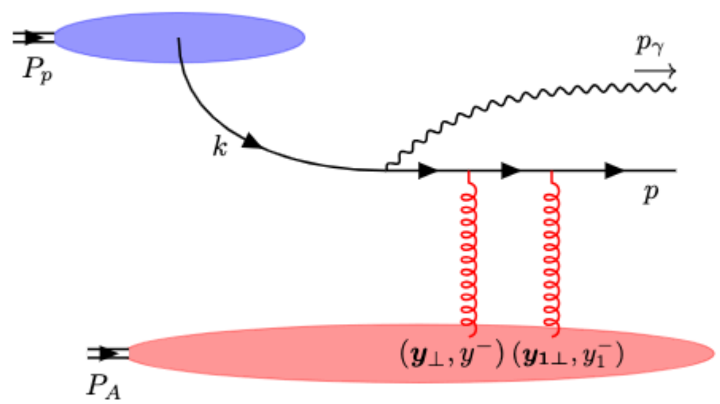
$$H(\xi, \boldsymbol{p}_{\gamma\perp}) = \frac{\xi^2\left[1 + (1-\xi)^2\right]}{\boldsymbol{p}_{\gamma\perp}^4} \qquad f_{g/A}^{(0)}(x) = \frac{1}{xP_A^+} \int \frac{\mathrm{d}y^-}{2\pi} \boxed{e^{ixP_A^+ y^-}} \langle P_A | F_\alpha^+(0^-) F^{+\alpha}(y^-) | P_A \rangle$$

Matches exactly to leading-twist result beyond small-x limit

From CGC to twist-4 collinear factorization

LPM effect: incoherent vs coherent regimes





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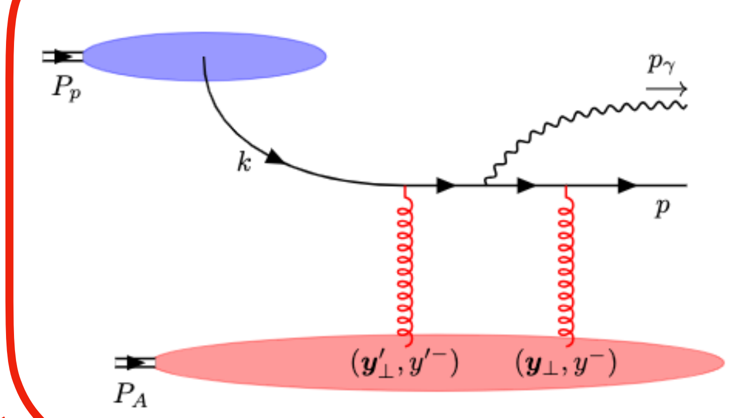


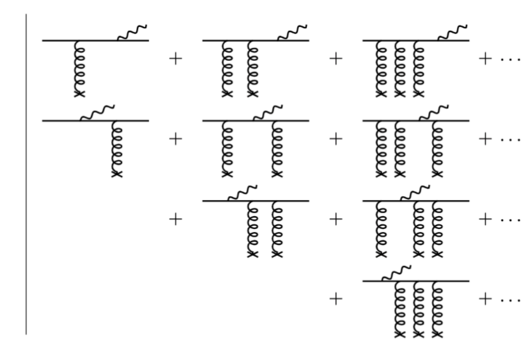
$$\text{phases} = e^{\frac{i}{x_p s}\left[\frac{\boldsymbol{p}_\perp^2}{(1-\xi)} + \frac{\boldsymbol{p}_{\gamma\perp}^2}{\xi} - \boldsymbol{l}_\perp^2\right] P_A^+ y'^-} e^{\frac{i}{x_p s}\boldsymbol{l}_\perp^2 P_A^+ y^-} \left\{ 1 - e^{i\frac{(y^- - y'^-)}{\tau_{\gamma,\text{form}}}} \right\}$$

- (Inverse) formation time for photon production: $$\tau_{\gamma,\text{form}}^{-1} = \frac{1}{x_p s}\frac{\left[\boldsymbol{p}_{\gamma\perp} - \xi \boldsymbol{l}_\perp\right]^2}{\xi(1-\xi)} P_A^+ = \frac{\left[\boldsymbol{p}_{\gamma\perp} - \xi \boldsymbol{l}_\perp\right]^2}{2k^- \xi(1-\xi)}$$

- Landau Pomeranchuk Migdal (LPM) effect
  $\tau_{\gamma,\text{form}} \gg y^- - y'^-$ (coherent) -> contribution vanishes
  $\tau_{\gamma,\text{form}} \ll y^- - y'^-$ (incoherent) -> contribution survives

From CGC to twist-4 collinear factorization

Consistency between CGC and High-Twist formalism



$$\mathrm{d}\sigma \propto \int \mathrm{d}x_p f(x_p)\ \mathcal{H} \otimes \mathcal{T}$$

$$\mathcal{T}(z_1, z_2, z_3, z_4) = \frac{1}{N_c} \left\langle \mathrm{Tr}\left[A^+(z_1^-, \boldsymbol{z_{1\perp}}) A^+(z_2^-, \boldsymbol{z_{2\perp}}) A^+(z_3-, \boldsymbol{z_{3\perp}}) A^+(z_4^-, \boldsymbol{z_{4\perp}})\right]\right\rangle$$

$$\begin{aligned}
&\mathcal{H}^{coll}_{\mathrm{C,I}}(p_\gamma; y, y', y_1, y_2) \\
&= 8H(\xi, \boldsymbol{p}_{\gamma\perp}) e^{i\bar{x}_A P_A^+(y^- - y'^-)} \frac{\partial \delta^{(2)}(\boldsymbol{y}_\perp - \boldsymbol{y}_{1\perp})}{\partial \boldsymbol{y}_\perp} \cdot \frac{\partial \delta^{(2)}(\boldsymbol{y}'_\perp - \boldsymbol{y}_{2\perp})}{\partial \boldsymbol{y}'_\perp} \times \Bigg[\boxed{\delta^{(2)}(\boldsymbol{y}_{1\perp} - \boldsymbol{y}_{2\perp})} \\
&+ \frac{1}{\boldsymbol{p}^2_{\gamma\perp}} \boxed{\frac{\partial^2 \delta^{(2)}(\boldsymbol{y}_{1\perp} - \boldsymbol{y}_{2\perp})}{\partial \boldsymbol{y}_{1\perp} \cdot \partial \boldsymbol{y}_{2\perp}}} \left[4\xi^2 + \xi(1-\xi)(i\bar{x}_A P_A^+ \Delta y_{12}^-) - 3\xi^2 (i\bar{x}_A P_A^+ \Delta y^-) + \xi^2 (i\bar{x}_A P_A^+ \Delta y^-)^2\right]\Bigg]
\end{aligned}$$

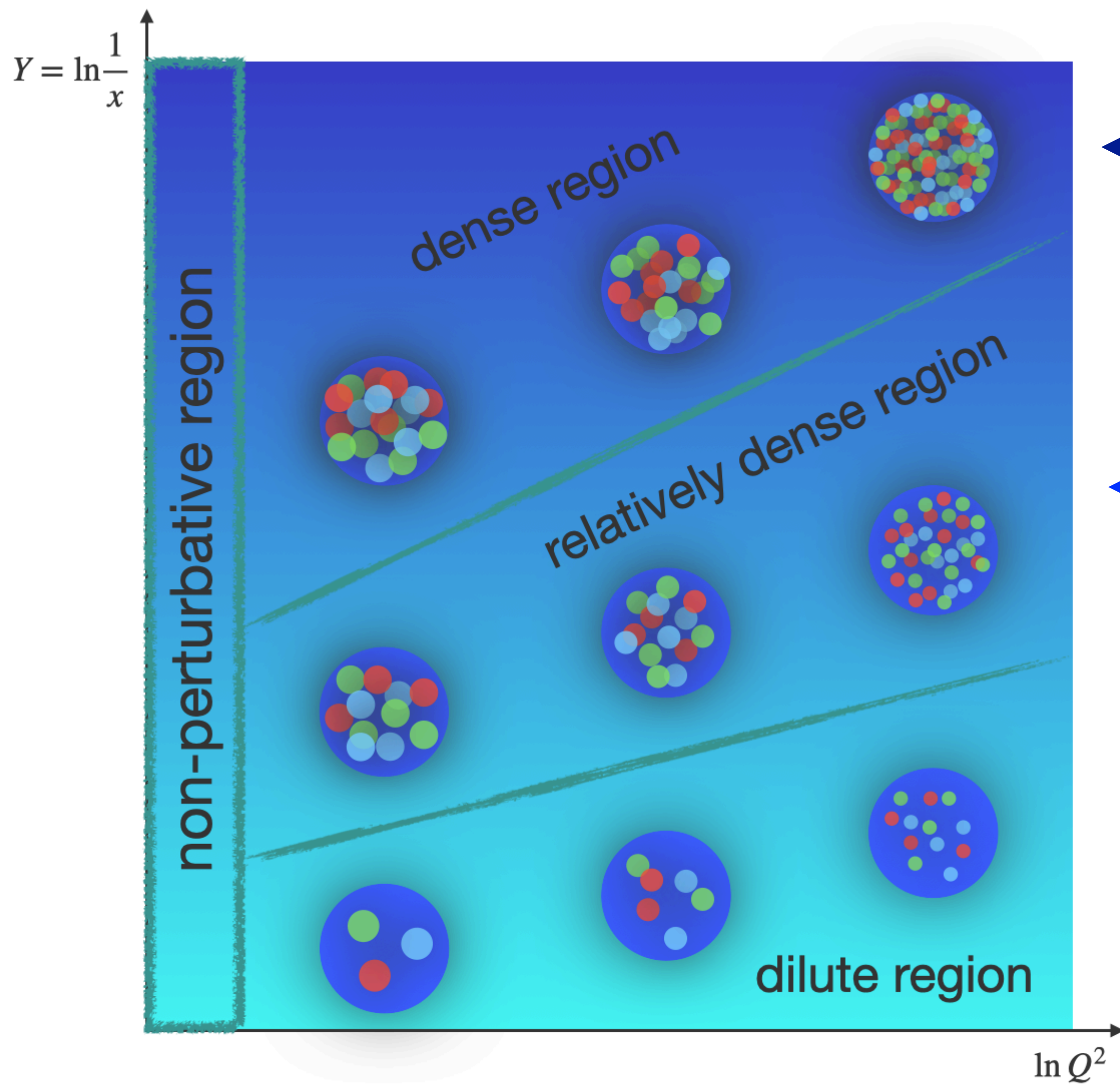
$$\begin{aligned}
\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}_{\mathrm{C,I}}}{\mathrm{d}\eta_\gamma \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} &= \frac{\alpha_{\mathrm{em}} e_f^2 \alpha_{\mathrm{s}}}{N_c} \int_{x_{\min}}^1 \mathrm{d}x_p f(x_p) H(\xi, \boldsymbol{p}_{\gamma\perp}) \boxed{\bar{x}_A f_{g/A}^{(\text{gauge link})}(\bar{x}_A)} \\
&+ \frac{(2\pi)^2 \alpha_{\mathrm{em}} e_f^2 \alpha_{\mathrm{s}}^2}{N_c^2 \boldsymbol{p}^2_{\gamma\perp}} \int_{x_{\min}}^1 \mathrm{d}x_p f(x_p) H(\xi, \boldsymbol{p}_{\gamma\perp}) \mathcal{D}_{\mathrm{C,I}}(\xi, \bar{x}_A, x_1, x_2, x_3) \boxed{T_{\mathrm{C,I}}(x_1, x_2, x_3)}\Big|_{\substack{x_1 = \bar{x}_A \\ x_2 = x_3 = 0}}
\end{aligned}$$

$$\mathcal{D}_{\mathrm{C,I}}(\xi, \bar{x}_A, x_1, x_2, x_3) = \left[4\xi^2 + \xi(1-\xi)\bar{x}_A \frac{\partial}{\partial x_2} - 3\xi^2 \bar{x}_A \frac{\partial}{\partial x_1} + \xi^2 \bar{x}_A \frac{\partial^2}{\partial x_1^2}\right]$$

Only showing initial state central cut contribution, analogous expansion for the others

Matches exactly to twist-4 result and the gauge link in the twist-2

Summary



Color Glass Condensate (CGC)

Strong fields, Wilson lines, non-linear evolution (BK/JIMWLK)

High-twist formalism

Multiparton correlations, DGLAP type equations

Leading twist Collinear factorization

PDFs, DGLAP evolution

Taking direct photon production in pA collisions as an example we show the consistency between CGC and High-Twist formalism in their common domain of validity

Outlook

- Does the consistency between CGC and generalized high-twist formalism persist at NLO?

- Matching between CGC and twist-4 TMDs

- Establish a framework that allows to resum all twists (modify Wilson lines to keep track of phases?)

- Can this correspondence help us implement small-x physics into current event generators such eHIJING?