Global analysis of polarized DIS + SIDIS at small x

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Proton Spin Puzzle

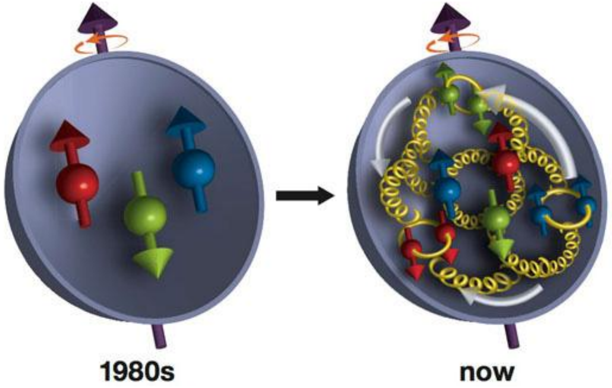
Jaffe-Manohar Spin Sum Rule:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$S_{q,g}$ = Helicity of quarks and gluons

$L_{q,g}$ = Orbital angular momentum

S_q ~ 30% of proton spin!

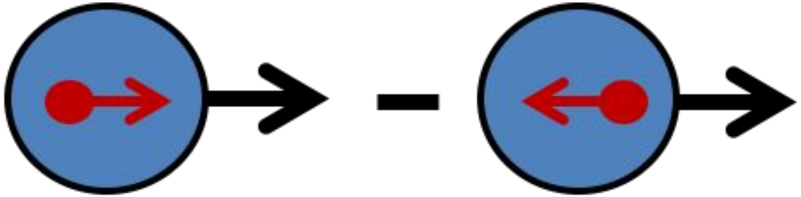


Quark Helicity Parton Distribution Functions

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2))$$

Helicity PDFs:

$$\Delta q =$$



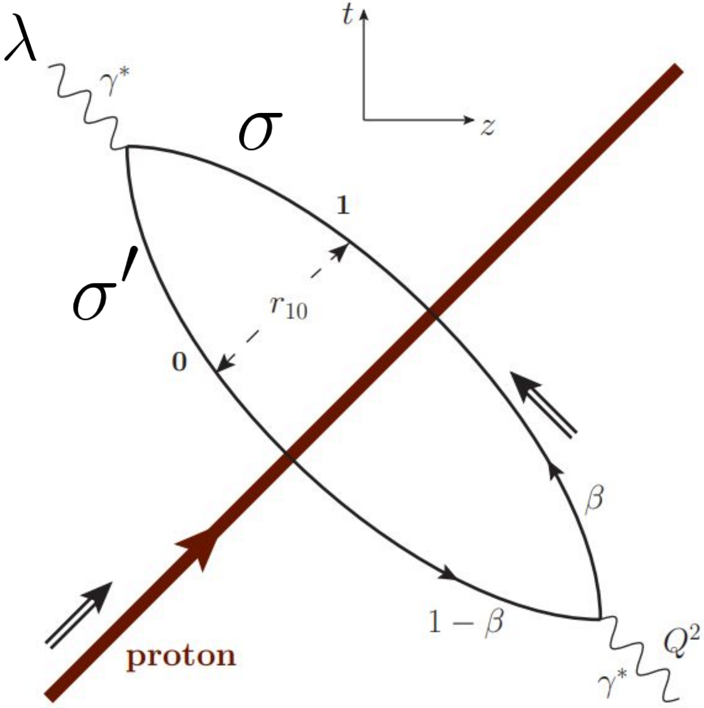
- Q^2 = resolution at which we probe the proton
- Bjorken $x \sim \frac{1}{s}$. We need theory to extrapolate to x=0

Phenomenology

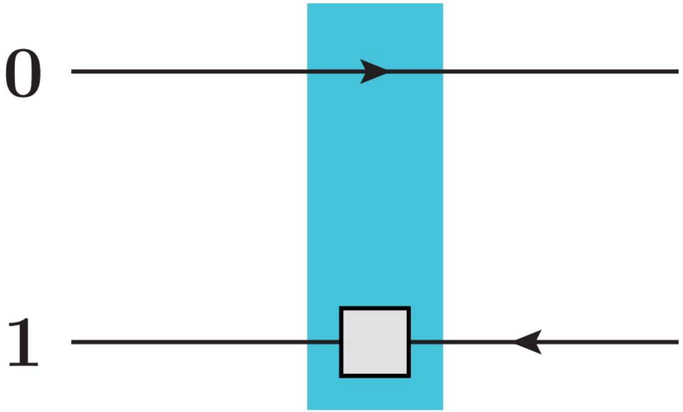
- Describe observables in terms of small-*x* theory (hPDFs and Polarized Dipole Amplitudes)
- Solve the evolution of the Amplitudes
- Identify initial conditions/ undetermined parameters
- Fit parameters to data $(5 \times 10^{-3} < x < 0.1)$
- Extrapolate (to the EIC) $(10^{-4} < x < 5 \times 10^{-3})$

(Polarized) DIS in the (Polarized) Dipole Picture

$$g_1 \propto |\psi|^2 \otimes (Q + 2G_2)$$



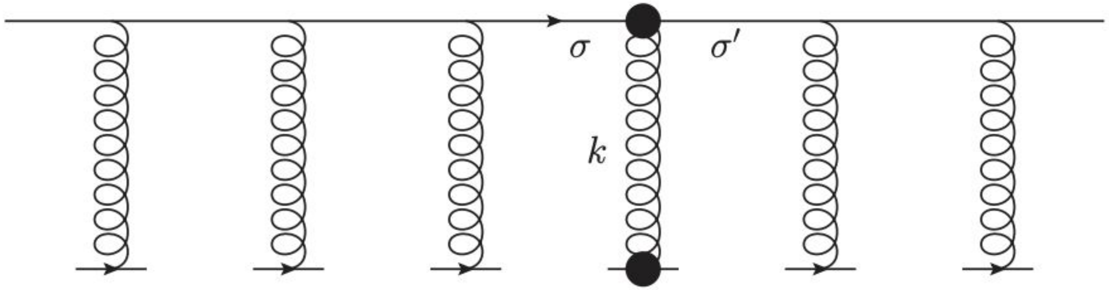
(Polarized) DIS in the (Polarized) Dipole Picture



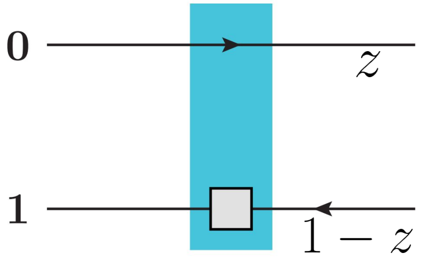
- In pDIS, the electron and proton have their helicity specified
- Cross-section now dependent on **Polarized Dipole Amplitudes**:

$$Q_q, G_2, \tilde{G}$$

- Quark line undergoes one extra helicity exchange, which is **sub-eikonal**



Polarized Dipole Amplitude - Degrees of Freedom



5 Amplitudes:

$$Q_{u,d,s},\ \widetilde{G},\ G_2\ (s_{10},\eta)$$

Polarized Dipole Amplitudes are functions of

- Transverse separation:

$$x_{10}^2 = (\underline{x}_1 - \underline{x}_0)^2$$

- Momentum Fraction times center of mass energy: zs
- Rescaled variables:

$$\eta = \sqrt{\frac{N_c}{2\pi}} \ln \frac{zs}{\Lambda^2} \qquad s_{10} = \sqrt{\frac{N_c}{2\pi}} \ln \frac{1}{x_{10}^2\Lambda^2}$$

Calculating Helicity Distributions

$$\Delta q + \Delta \bar{q} = -\frac{1}{\pi} \int_{0}^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \left[Q_q(s_{10}, \eta) + 2G_2(s_{10}, \eta) \right]$$

- We incorporate running coupling that runs with size of the dipole

- $\eta_{max} = \sqrt{N_c/2\pi} \ln(Q^2/x\Lambda^2)$
- $s_{10}^{min} = max[0, \eta - \sqrt{N_c/2\pi} \ln(1/x)]$

Calculating Helicity Distributions

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2} G_2\left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x}\right)$$

- Jaffe-Manohar Gluon Helicity Distribution

Large Nc&Nf Helicity Evolution

In the large Nc&Nf, Nc/Nf fixed limit, the evolution equations for the polarized dipole amplitudes close:

$$Q_f(s_{10},\eta) = Q_f^{(0)}(s_{10},\eta) + \int_{s_{10}+y_0}^{\eta} d\eta' \int_{s_{10}}^{\eta'-y_0} ds_{21}\, \alpha_s(s_{21}) \Big[Q_f(s_{21},\eta') + 2\tilde{G}(s_{21},\eta') + 2\tilde{\Gamma}(s_{10},s_{21},\eta') - \bar{\Gamma}_f(s_{10},s_{21},\eta') + 2G_2(s_{21},\eta') + 2\Gamma_2(s_{10},s_{21},\eta') \Big] + \frac{1}{2}\int_{y_0}^{\eta} d\eta' \int_{\max\{0,s_{10}+\eta'-\eta\}}^{\eta'-y_0} ds_{21}\, \alpha_s(s_{21}) \Big[Q_f(s_{21},\eta') + 2G_2(s_{21},\eta') \Big]$$

+ 9 more

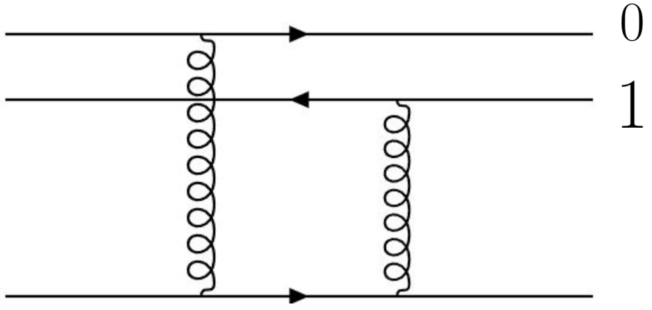
- 5 Polarized dipole amplitudes mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$ - which impose lifetime ordering
- Small-*x* cutoff, $y_0 \propto \ln 1/x_0$

Large Nc&Nf Helicity Evolution

- 5 **Polarized dipole amplitudes** mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$
- For a total of 10 equations that form a **closed system**
- Undetermined initial conditions: $Q^{(0)}_{u,d,s}, \tilde{G}^{(0)}, G_2^{(0)}$

Inhomogeneous term

The inhomogeneous term is given by a Born-inspired ansatz:



$$\propto \int_0^s \frac{dk_\perp^2}{k_\perp^2}\left(1 - e^{-\underline{k}\cdot\underline{x}_{10}}\right) = \pi \ln(s x_{10}^2)$$

$$\propto \eta - s_{10}$$

$$\Gamma_q^{(0)} = Q_q^{(0)} = a\eta + b s_{10} + c$$

- Same form of the other Dipole Amplitudes
- Parameters a,b,c need to be extracted from data
- 15 parameters for singlet hPDFS

Recap:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2))$$

$$S_g(Q^2) = \int_0^1 dx \Delta G(x,Q^2)$$

$$\Delta q + \Delta\bar{q} = -\frac{1}{\pi}\int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10}\,[Q_q(s_{10},\eta) + 2G_2(s_{10},\eta)]$$

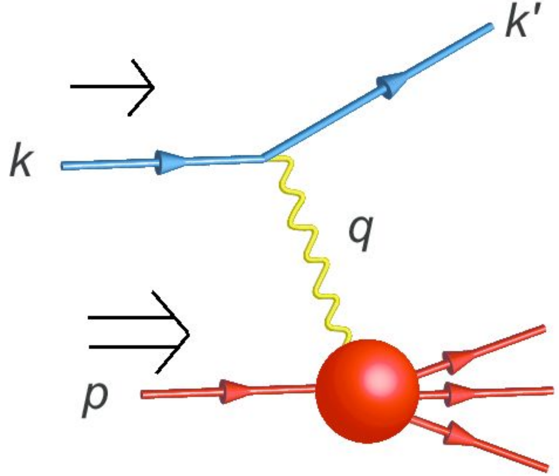
$$\Delta G(x,Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2}\, G_2\left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x}\right)$$

Large $N_c \& N_f$ Helicity Evolution

$$Q_q^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}$$

Observables - Double Spin Asymmetries in DIS

$$A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto A_1 \propto g_1^{p,n}$$



- $\uparrow$ $(\downarrow)$ is positive (negative) helicity electron
- $\Uparrow$ $(\Downarrow)$ is positive (negative) helicity proton
- A_1 is virtual photoproduction asymmetry

Describing Observables - pDIS

What enters into observables are linear combinations of hPDFs

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$
$$\Delta q^- = \Delta q - \Delta \bar{q}$$

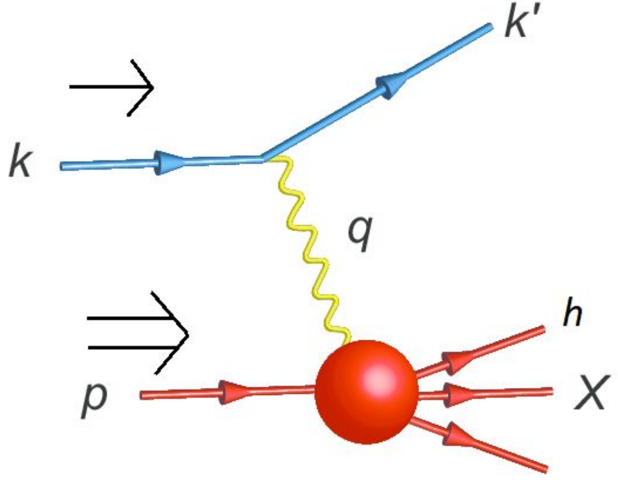
- Three relevant hPDFs in DIS: $\Delta u^+, \Delta d^+, \Delta s^+$
- Data exist for two observables that contain these hPDFs in linearly independent combinations: g_1^p and g_1^n

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q Z_q^2 \Delta q^+(x, Q^2)$$

- Z_q is the quark charge fraction

Observables - Double Spin Asymmetries in SIDIS

$$A_{||}(z) = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto g_1^h(z)$$



- *h* is the tagged hadron
- *z* is the momentum fraction of the virtual photon carried by the tagged hadron

g_1^h Structure Functions

$$g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q Z_q^2 \Delta q(x,z,Q^2) D_q^h(z,Q^2)$$

- D_q^h are fragmentation functions - giving the probability quark *q* fragments into hadron *h*
- z Is the fraction of the virtual photons momentum carried by the hadron
- The flavour hPDF is obtained via $\Delta q = \frac{1}{2}(\Delta q^+ + \Delta q^-)$
- In pSIDIS, we are able to scatter on 2 targets (proton, neutron), tag 2 outgoing hadrons (pion, kaon) that each have 2 charges - 2x2x2=8 new observables

Describing Observables - pSIDIS

- Expand our horizons to Semi-Inclusive DIS - all hPDFs are relevant here, both singlet, Δq^{+} and non-singlet, Δq^{-}
- **Non-singlet distributions obey their own small-*x* evolution that has been solved**

$$\Delta q^{-} = \frac{N_c}{2\pi^3} \int\limits_{\frac{\Lambda^2}{s}}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} G_f^{\mathrm{NS}}(x_{10}^2, zs)$$

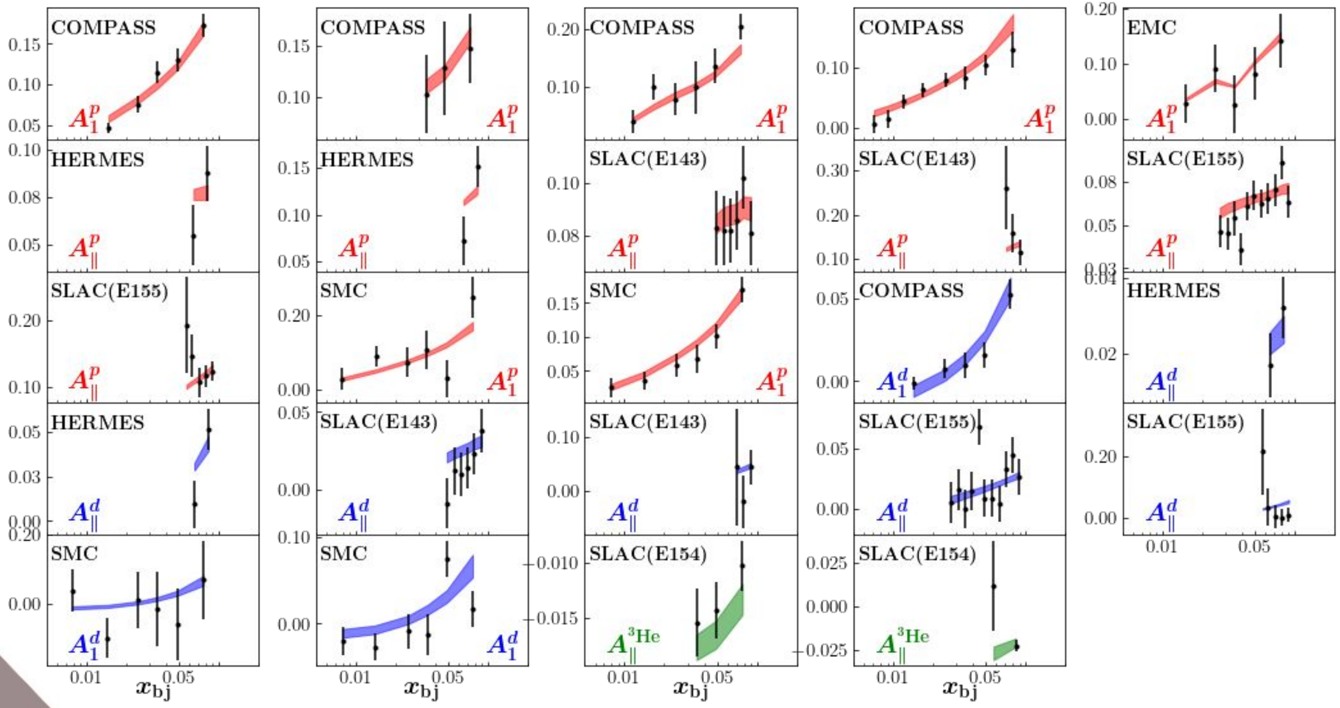
- Q_q^{NS} is the non-singlet Polarized Dipole Amplitude - obeys its own evolution equation
- pSIDIS grants us access to the semi-inclusive, spin dependent structure functions g_1^h

Actually doing phenomenology - JAM framework

The Jefferson Angular Momentum (JAM) framework is a pipeline that enables the statistical comparison of theory to data. Thanks to Nobuo Sato and Wally Melnitchouk for granting us access. The JAM framework is

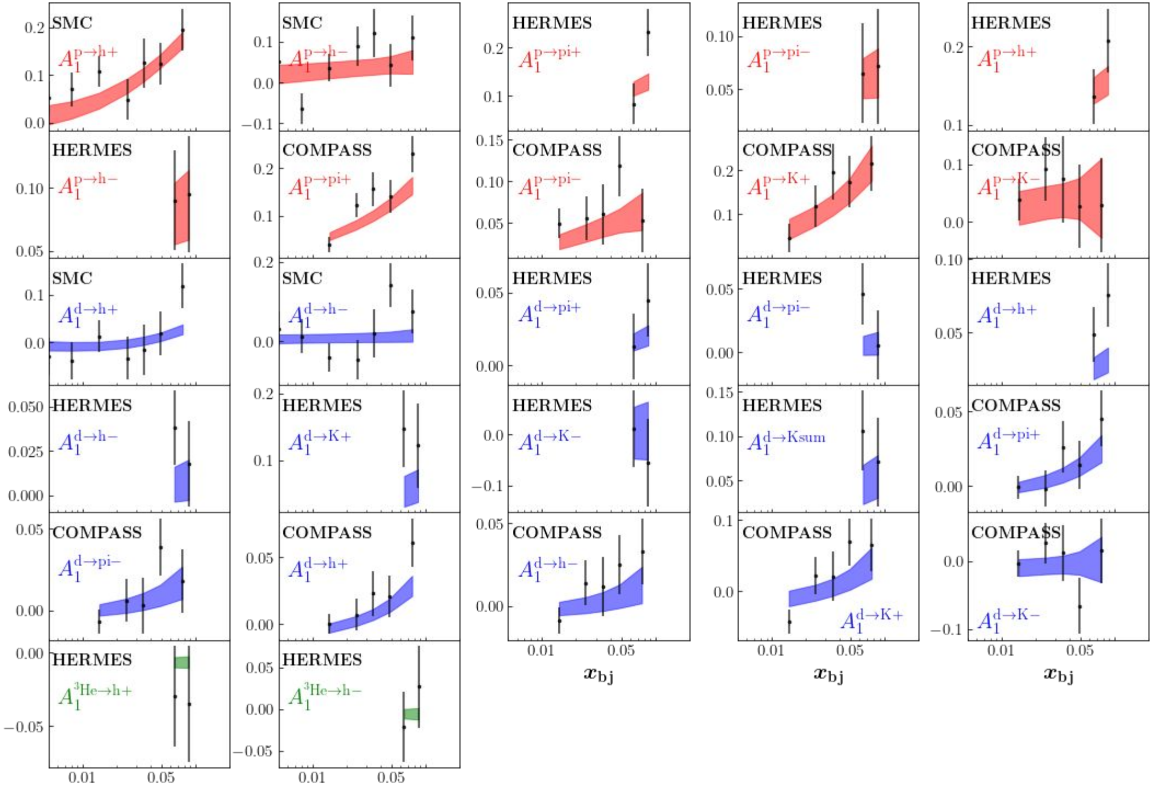
- Bayesian, Monte-Carlo fitting process
- That samples parameters efficiently
- To minimise χ^2
- While incorporating priors
- And returns statistical uncertainties

Global fit of DIS - Data vs Theory



- Red curves - our theory
- Black dots - data
  - COMPASS
  - EMC
  - SMC
  - SLAC
  - HERMES
- Preliminary results

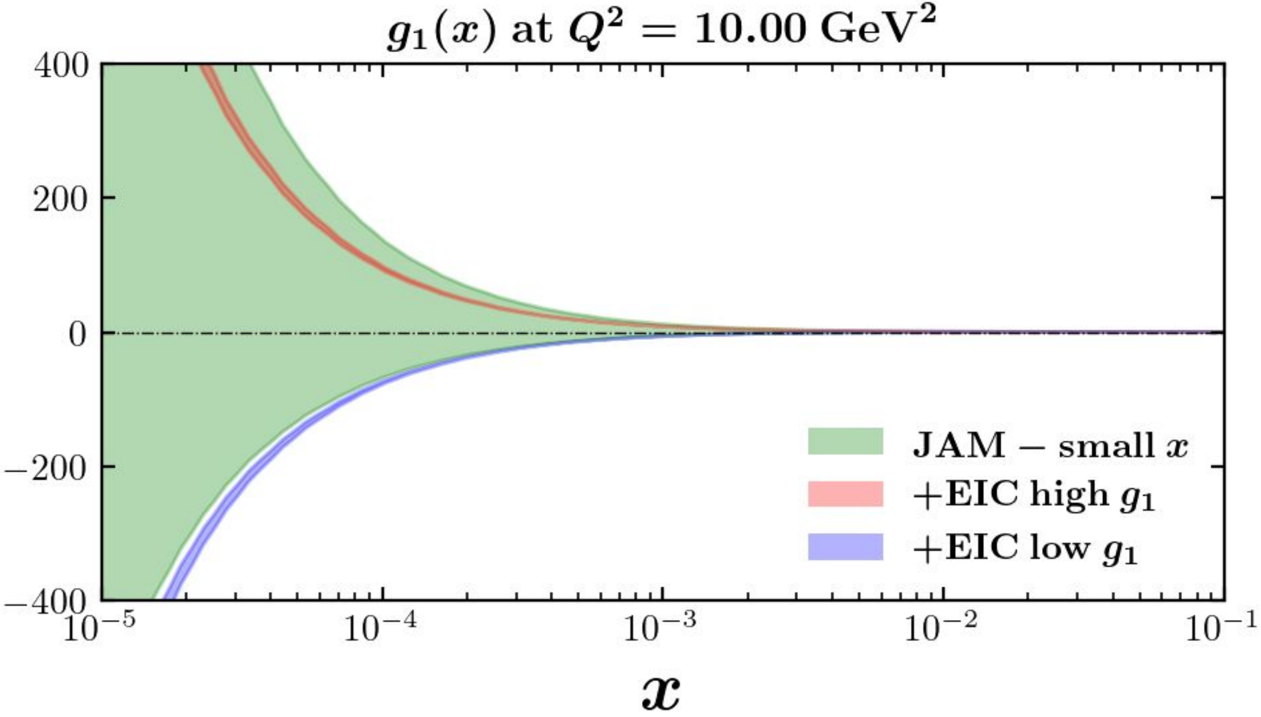
Global fit of SIDIS - Data vs Theory



χ^2 and Data Cuts

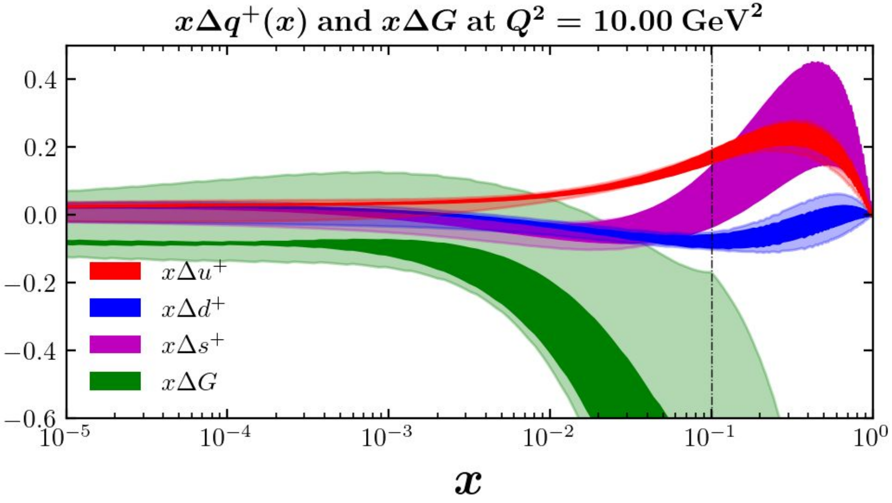
- First simultaneous fit of small-*x* theory to polarized DIS & SIDIS data
- Cut of $0.005 < x < 0.1$
- Cut of $1.69\, GeV^2 < Q^2 < 10.5\, GeV^2$
- Cut of $0.2 < z < 1.0$
- Describing 234 data points
- With a $\chi^2/npts = 1.03$

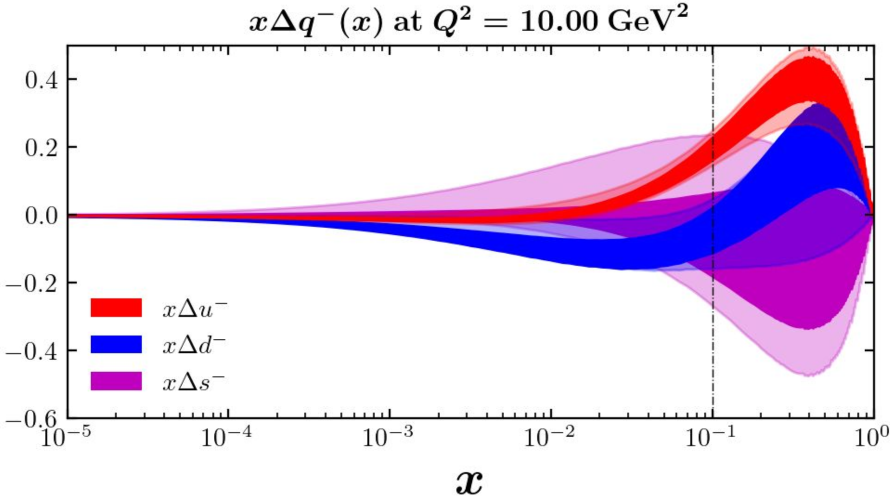
Extraction of g1 structure function (preliminary)



- **Narrow bands are EIC projections**

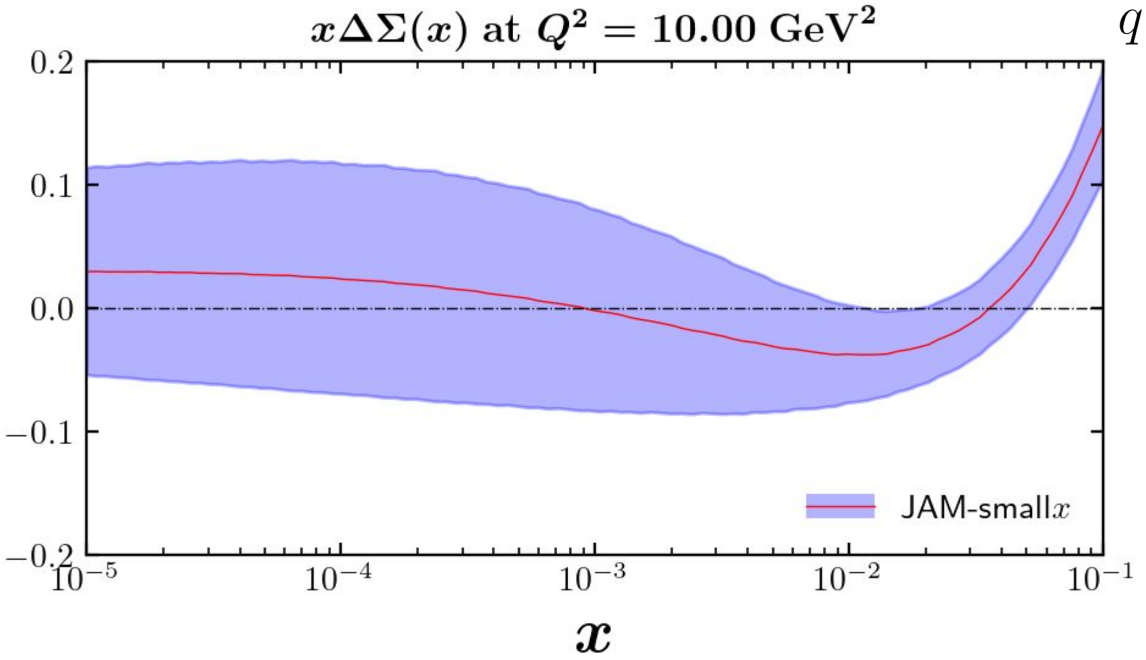
hPDFs - Preliminary





Contribution from Quark Spin

$$\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$$



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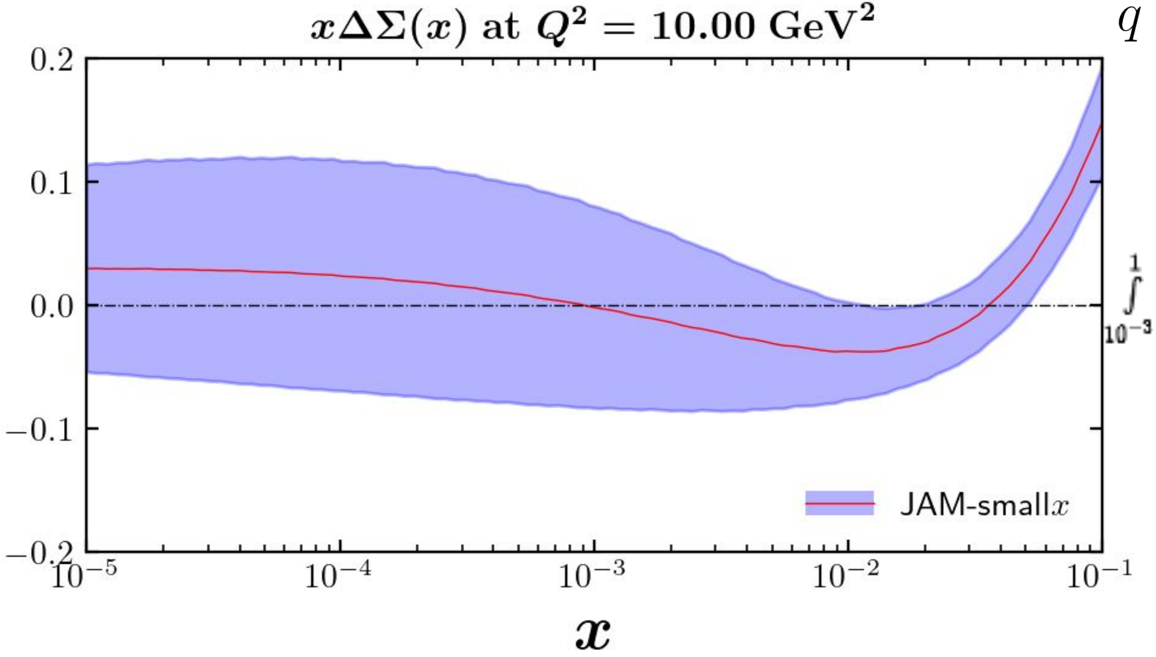

Large-x region

$$\int_{10^{-3}}^{1} dx\, \Delta\Sigma(x, Q^2 = 10Gev^2) = 0.4 \pm 0.05$$

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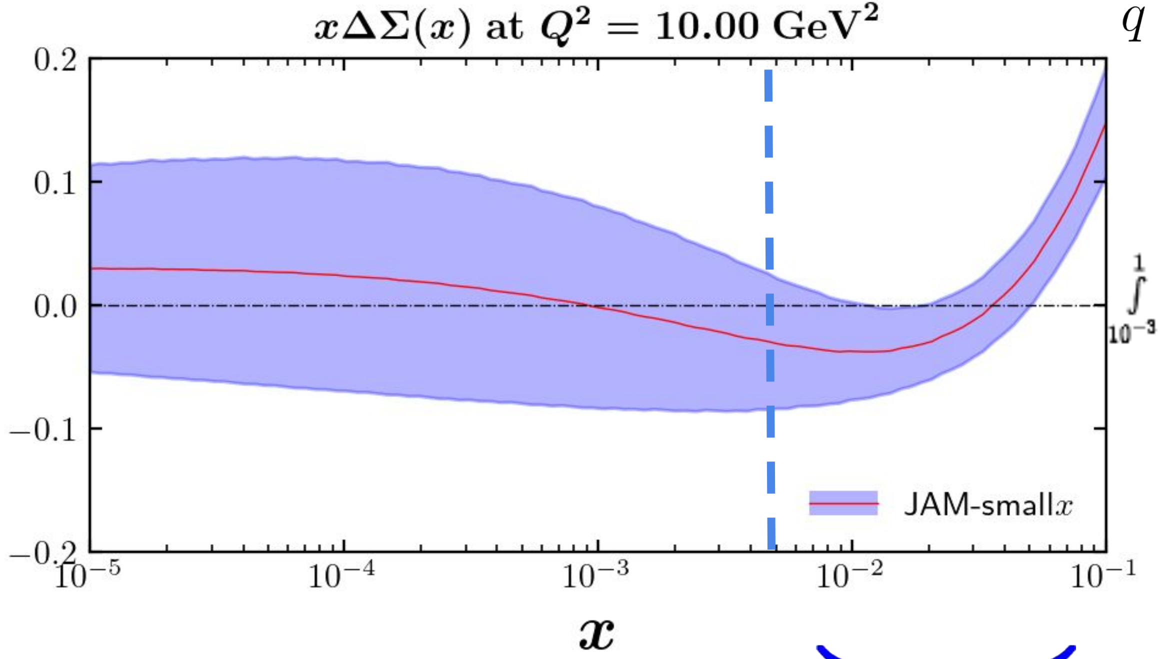

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Existing small-x data

Contribution from Quark Spin

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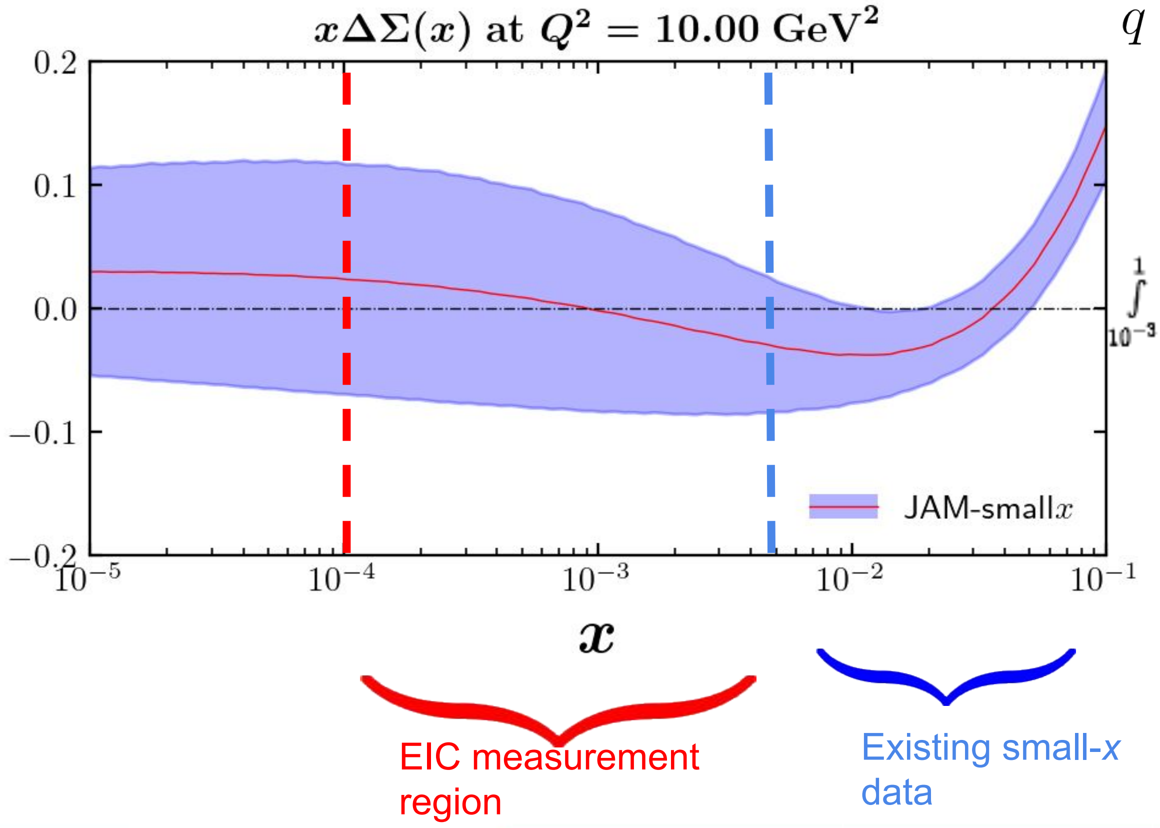

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Contribution from Quark Spin

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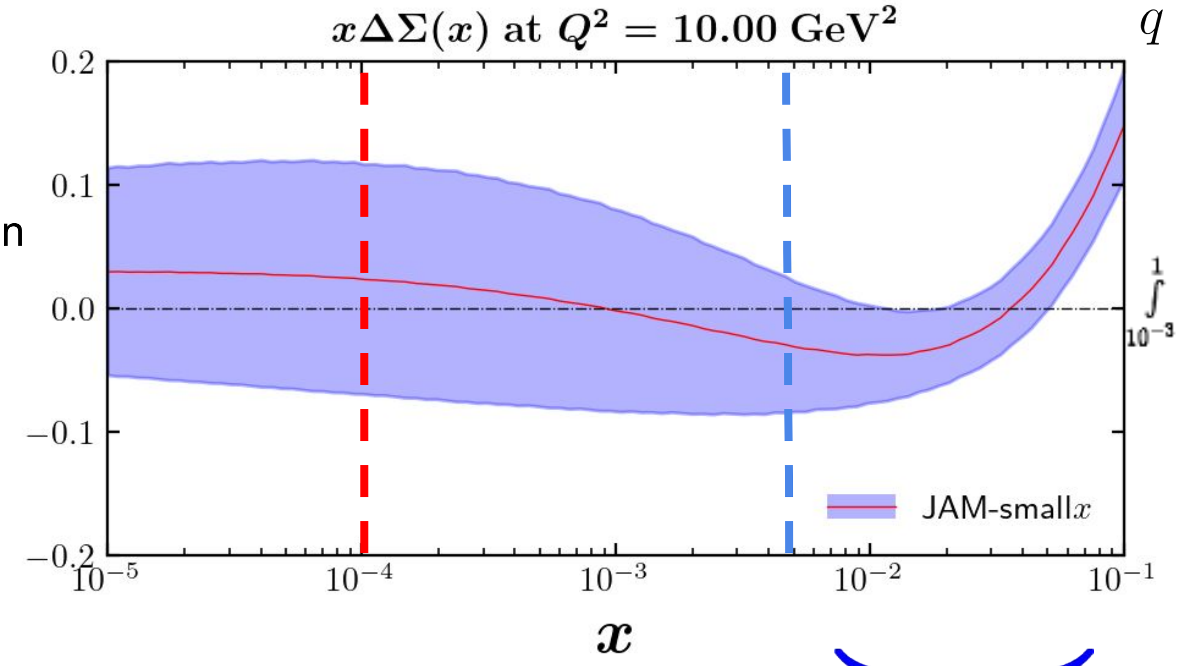
Saturation region



Large-x region

$$\int_{10^{-3}}^{1} dx\ \Delta\Sigma(x, Q^2 = 10Gev^2) = 0.4 \pm 0.05$$

Compare with:

$$\int_{10^{-5}}^{10^{-3}} dx\Delta\Sigma(x, Q^2) = 0.1 \pm 0.4$$

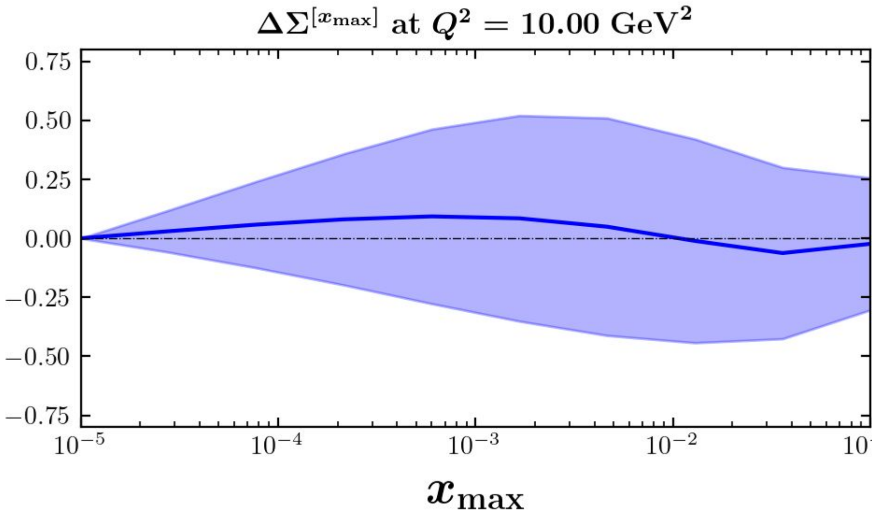
EIC measurement region

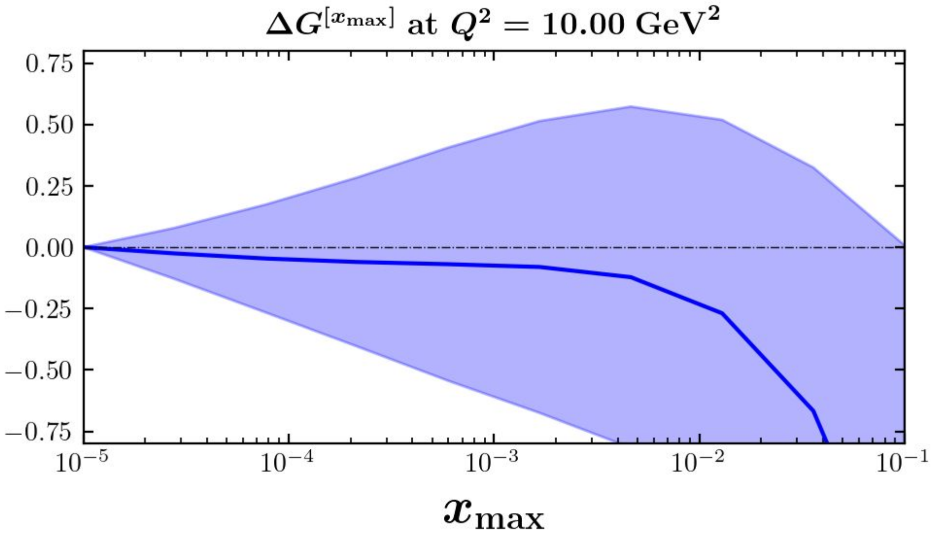
Existing small-x data

Integrated Distributions

$$\Delta\Sigma^{[x_{max}]} = \int_{10^{-5}}^{x_{max}} dx\, \Delta\Sigma(x)$$

$$\Delta G^{[x_{max}]} = \int_{10^{-5}}^{x_{max}} dx\, \Delta G(x)$$

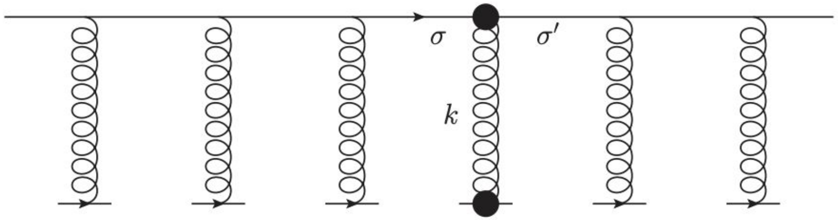


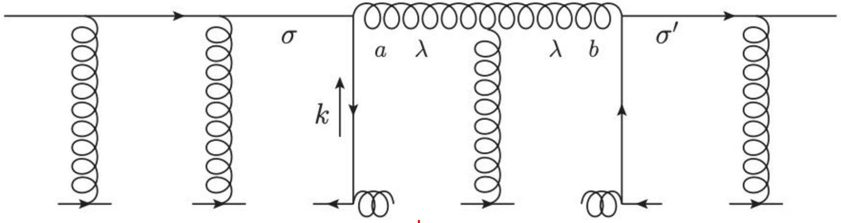


Conclusions

- In order to resolve the spin puzzle, the small-*x* behaviour of the hPDFs need to be understood
- This is accomplished using small-*x* evolution
- Along with fitting to data
- Potentially a significant amount if spin is hiding in the small-*x* region
- More work needs to be done to constrain small-*x* behavior of the various polarized dipoles - especially G_2 and $\tilde{G}$
- Could be constrained by studying particle production in *pp* collisions

Polarized Wilson Lines





$$\vec{\mu} \cdot \vec{B}$$

- Chromo-magnetic field

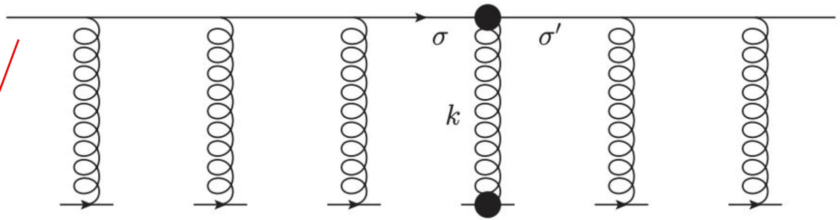
$$\mu B_z \sim F_{12}$$

$$\bar{\psi}\gamma^+\gamma^5\psi$$

- Axial Current

Polarized Dipole Amplitudes: $\text{``}Q_q\text{''}$ , $\text{``}\tilde{G}\text{''}$

Polarized Wilson Lines



- Quark propagator

$$\int \frac{dk^+}{2\pi} e^{ikx} \frac{\not{k}}{2k^+k^- - k_\perp^2}$$

- Sub-eikonal phase expansion

$$A_\perp$$

- Polarized gluon vertex

$$e^{-ix^- \frac{k_\perp^2}{2k^-}} \approx 1 - ix^- \frac{k_\perp^2}{2k^-} \Rightarrow \partial_\perp^2 \rightarrow D_\perp^2 \rightarrow \text{``}G_2\text{''}$$

Constraining the rest of the Polarized Dipole Amplitudes

$$g_1^{p,n} \sim Q_u, Q_d, Q_s, G_2$$

- 2 observables, 4 polarized dipole amplitudes. Under constrained system

$$g_1^h \sim Q_q, G_2, Q_q^{NS}$$

- 8 new observables, 3 new polarized dipole amplitudes. Exactly constrained - but $\tilde{G}$ does not enter directly into observables

$$pp \to jets \sim G_2, \tilde{G}$$

- Particle production might provide final constraints