Small-x Helicity: Analysis of Fits and Observables

Nicholas Baldonado

New Mexico State University Department of Physics

In collaboration with Matthew Sievert, Yuri Kovchegov, Dan Pitonyak, Nobuo Sato, Wally Melnitchouk, Josh Tawabutr, Andrey Tarasov, and Daniel Adamiak

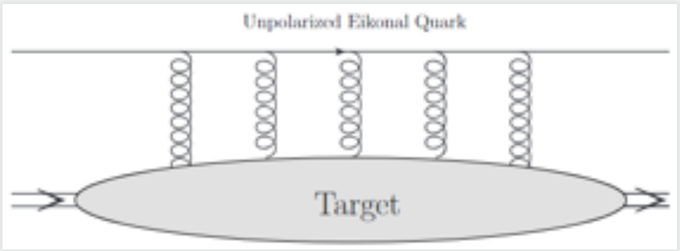
BE BOLD. Shape the Future.®
New Mexico State University

Introduction and Review

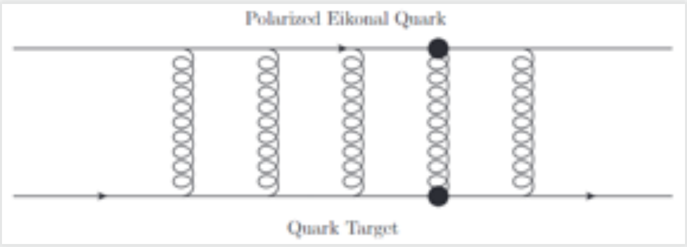
POLARIZED DIPOLE AMPLITUDES
BASIS FUNCTIONS
LINEAR COMBINATIONS

Wilson Lines, Dipole Amplitudes, Observables

Unpolarized Wilson Line



Polarized Wilson Line



- Note the sub-eikonal vertex that transfers polarization information.
- We deconstruct the **polarized Wilson lines** and **dipole amplitudes** (F. Cougoulic, et al. 2022):

$$V_{\underline{x},\underline{y};\sigma',\sigma}\Big|_{\text{sub-eikonal}} \equiv \sigma\,\delta_{\sigma,\sigma'}\,V_{\underline{x}}^{\text{pol}[1]}\,\delta^2(\underline{x}-\underline{y}) + \delta_{\sigma,\sigma'}\,V_{\underline{x},\underline{y}}^{\text{pol}[2]}$$

$$(U_{\underline{x},\underline{y};\lambda',\lambda})^{ba}\Big|_{\text{sub-eikonal}} \equiv \lambda\,\delta_{\lambda,\lambda'}\,(U_{\underline{x}}^{\text{pol}[1]})^{ba}\,\delta^2(\underline{x}-\underline{y}) + \delta_{\lambda,\lambda'}\,(U_{\underline{x},\underline{y}}^{\text{pol}[2]})^{ba}$$

$\rightarrow$

$$Q_{10}(zs) \equiv \frac{1}{2N_c}\,\mathrm{Re}\left\langle\!\left\langle \mathrm{T\,tr}\left[V_{\underline{0}}\,V_{\underline{1}}^{\text{pol}[1]\,\dagger}\right] + \mathrm{T\,tr}\left[V_{\underline{1}}^{\text{pol}[1]}\,V_{\underline{0}}^{\dagger}\right]\right\rangle\!\right\rangle(zs)$$

$$G^{i}_{10}(zs) \equiv \frac{1}{2N_c}\left\langle\!\left\langle \mathrm{tr}\left[V_{\underline{0}}^{\dagger}\,V_{\underline{1}}^{i\,G[2]} + \left(V_{\underline{1}}^{i\,G[2]}\right)^{\dagger} V_{\underline{0}}\right]\right\rangle\!\right\rangle(zs).$$

(Kovhegov, Pitonyak, Sievert, 2018)

$$\Delta G = \frac{2N_c}{\alpha_s \pi^2}\left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2}\right) G_2\right]_{x_{10}^2 = \frac{1}{Q^2}}$$

$$\Delta q_f^{+} = -\frac{N_c}{2\pi^3}\int_{\frac{\Lambda^2}{s}}^{1}\frac{dz}{z}\int_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}}\frac{dx_{10}^2}{x_{10}^2}\left[Q_f + 2G_2\right]$$

BE BOLD. Shape the Future.®

Evolution and Basis Functions

*
$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\frac{z}{z'}x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\widetilde{G}(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's)\right]$$

(F. Cougoulic, et al. 2022)

- Dipole evolution → Helicity Evolution.
- EE: Coupled, Closed, **Linear**

5 Undetermined Initial Conditions:

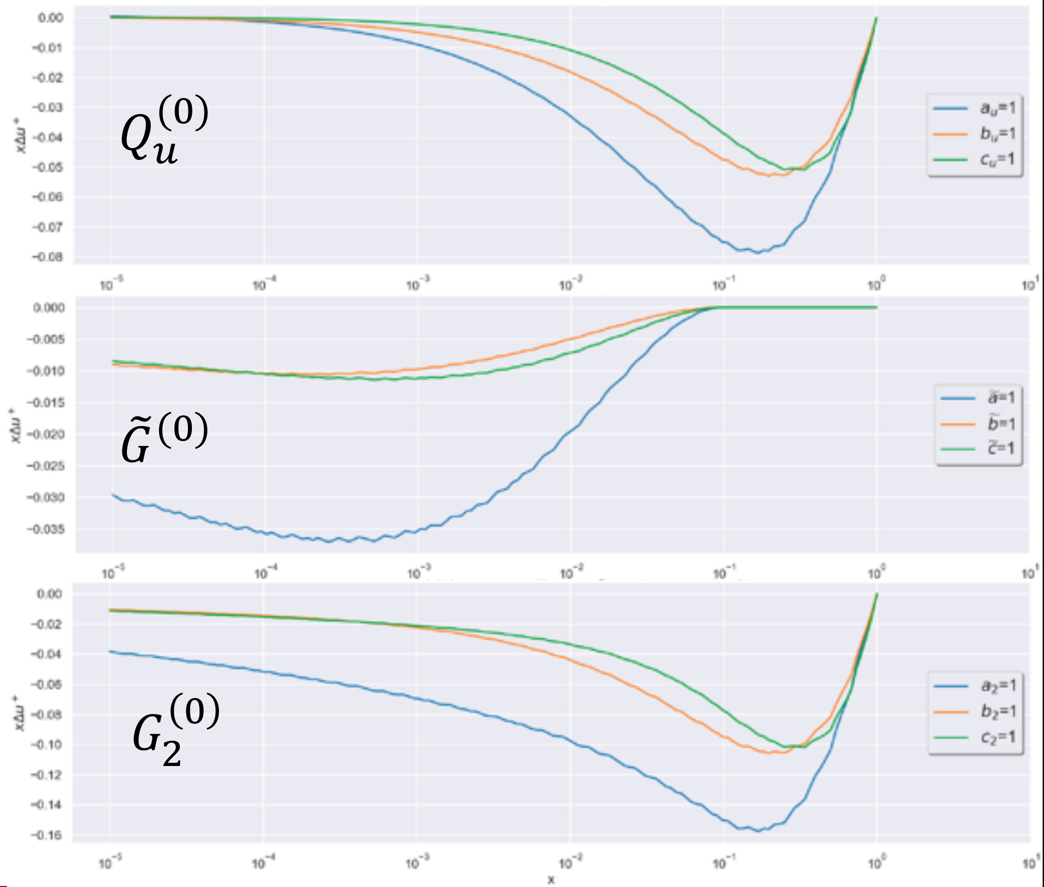
$$Q_f^{(0)} = a_f\, \eta + b_f\, s_{10} + c_f$$

$$\tilde{G}^{(0)} = \tilde{a}\, \eta + \tilde{b}\, s_{10} + \tilde{c}$$

$$G_2^{(0)} = a_2\, \eta + b_2\, s_{10} + c_2$$

Turn these into 15 basis functions!

$x\Delta u^+ \; (Q^2 = 10.00\; GeV^2)$



* Just one flavor-singlet example.

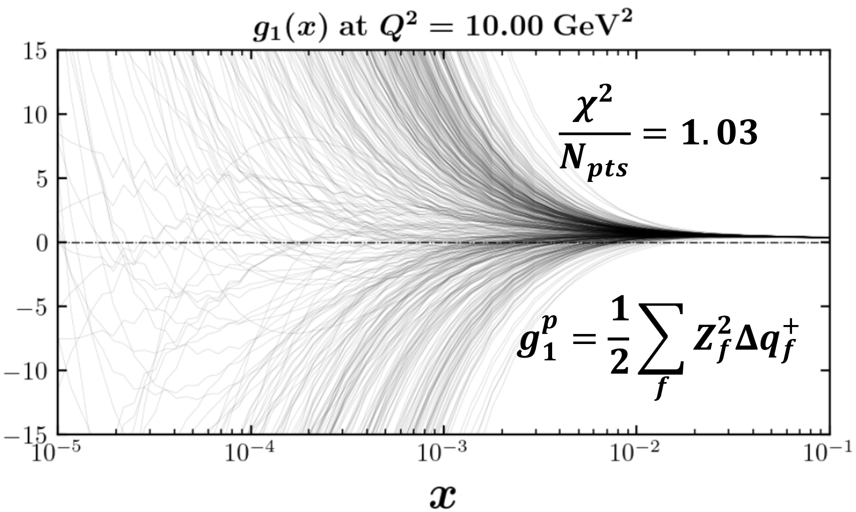
NM STATE BE BOLD. Shape the Future.®

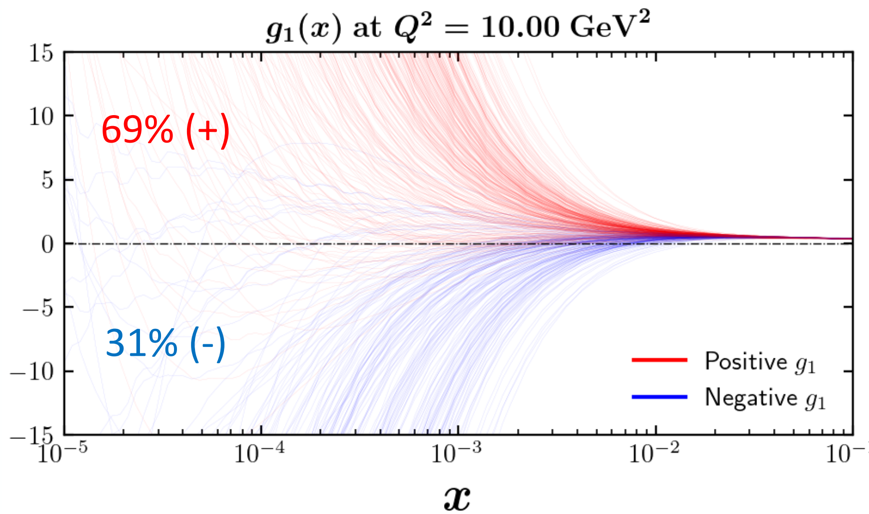
Full Theory Fit: Analyzing Asymptotics

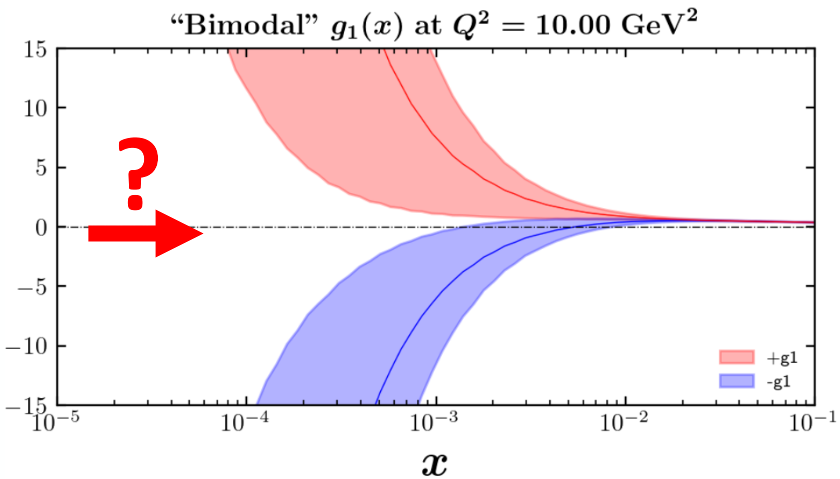
ASYMPTOTIC SIGN

CONFIDENCE IN ASYMPTOTICS

Full Theory Fit: What Can We Know?

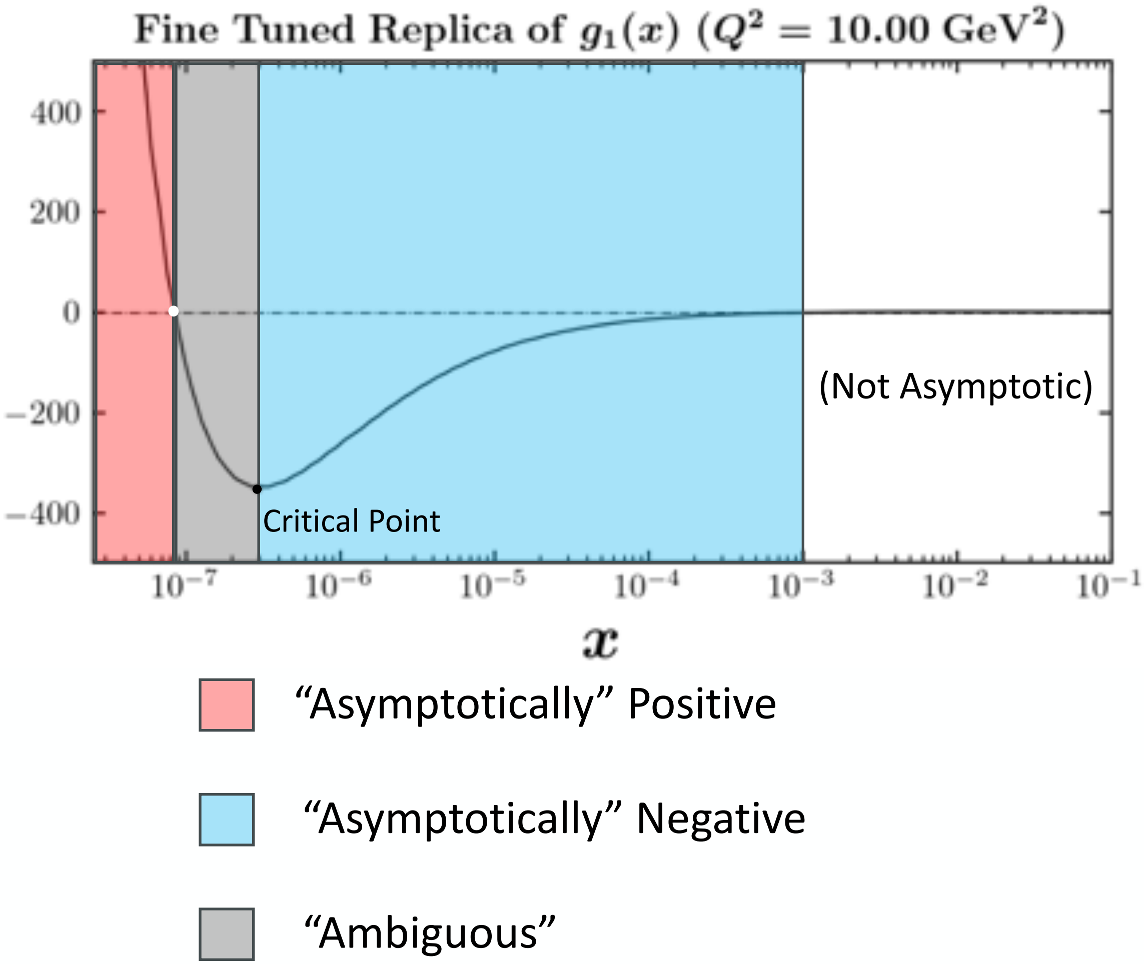






- Prior ranges curated to ensure good χ^2.
- Computational Balance: $x_{\min} = 10^{-7.5}$
- Hints at bimodality?
- Potential Issues:
  - **Uncertainty** in asymptotics?
  - How small is **small enough** $x_{\min}$?

Ambiguities and Critical Points

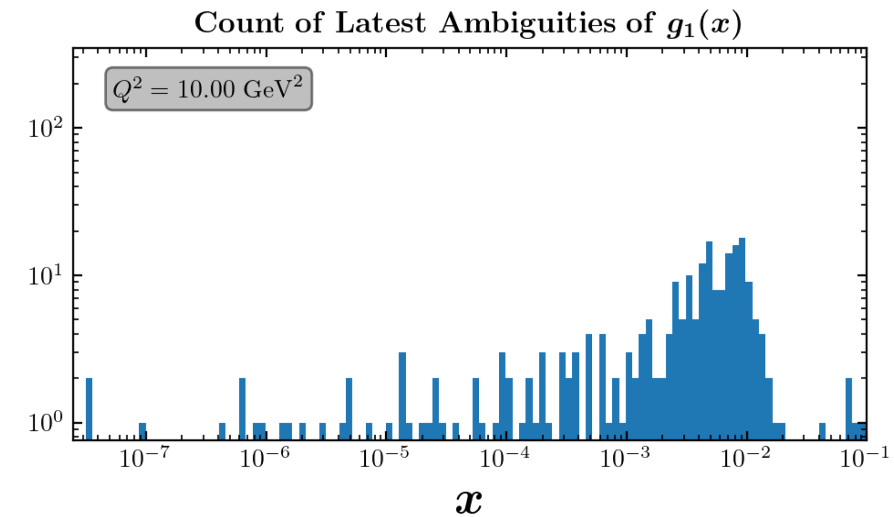


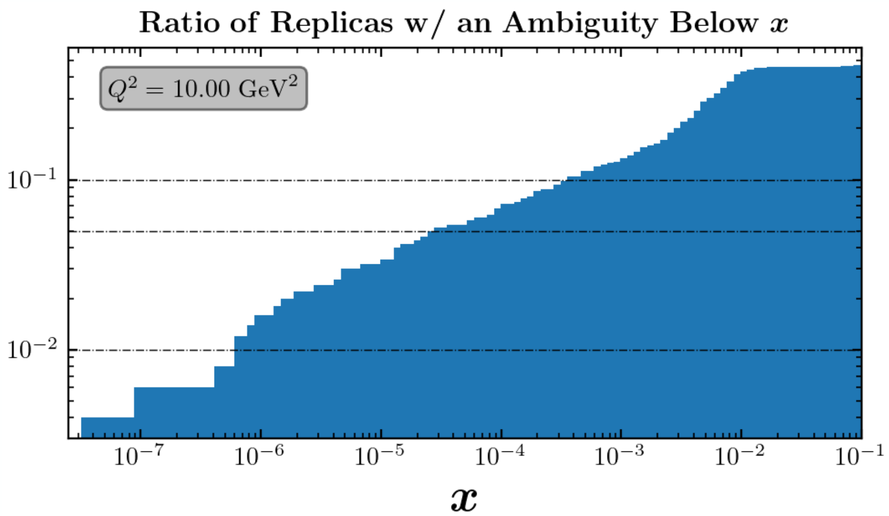
- "Asymptotic" sign depends on $x_{\min}$.
- Different Linear Combinations ➔ Different Critical points
- Quantify ambiguity at some x':

$$\text{Sign}\left(\frac{dg_1}{dx}\Big|_{x'}\right) \neq \text{Sign}\left(g_1\Big|_{x'}\right)$$

- Only consider the smallest-x ambiguity per replica for probability statistics

Confidence of Asymptotics





- Approximately 55% of replicas are unambiguously positive.
- All asymptotically negative replicas have at least one ambiguity.
- 95% of **all replicas** have confirmed asymptotics by $\boldsymbol{x = 10^{-4.62}}$.
- **Conclusion:** Choice to compute replicas as low as $\boldsymbol{x = 10^{-7.5}}$ is justified with only **0.4%** uncertainty.

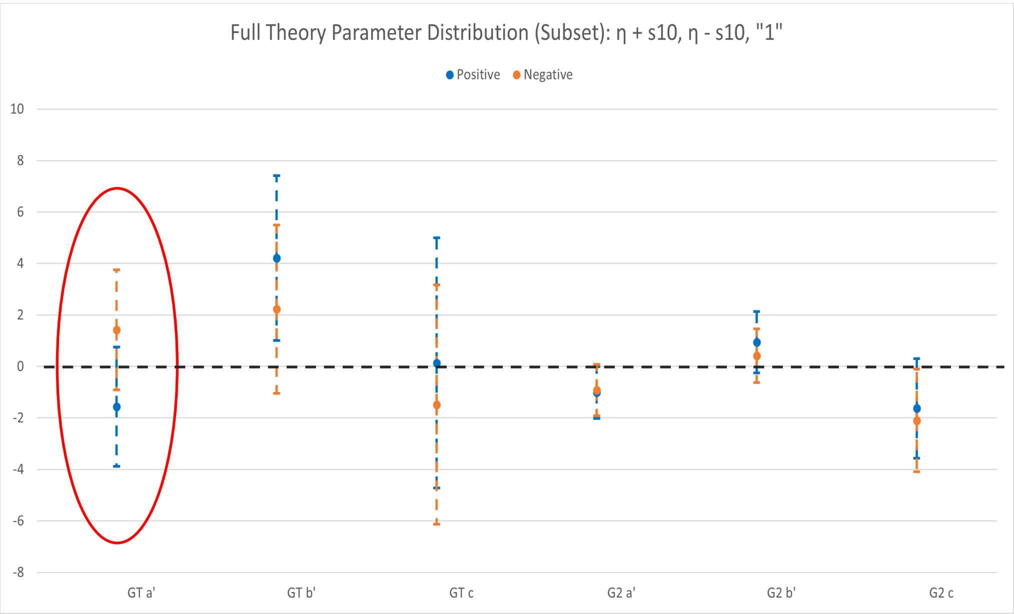
Exploration of Parameters

g_1 STRUCTURE FUNCTION

RISE OF BIMODALITY

SMALL-x BEHAVIOR OF OBSERVABLES

Parameter Distribution: $g_1(x \to 0) = \pm\infty$



For a given polarized dipole amplitude:

$$\boldsymbol{G = a\,\eta + b\,s_{10} + c}$$

- Off all basis functions, only $\widetilde{\boldsymbol{G}}_{\boldsymbol{s10}}$ has a meaningful separation.
- Data is sensitive to **amplitudes** only.

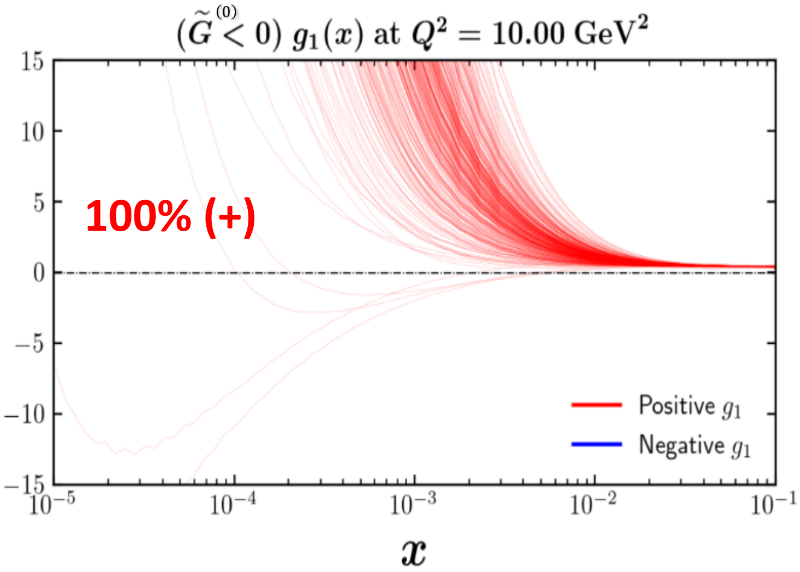
$$a' = \frac{1}{2}(a+b) \qquad b' = \frac{1}{2}(a-b)$$

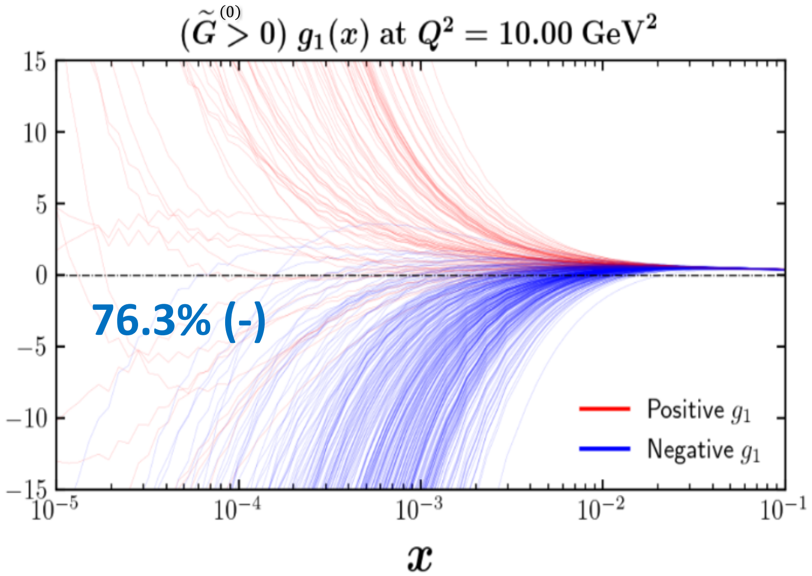
$$\boldsymbol{G = a'(\eta + s_{10}) + b'(\eta - s_{10}) + c}$$

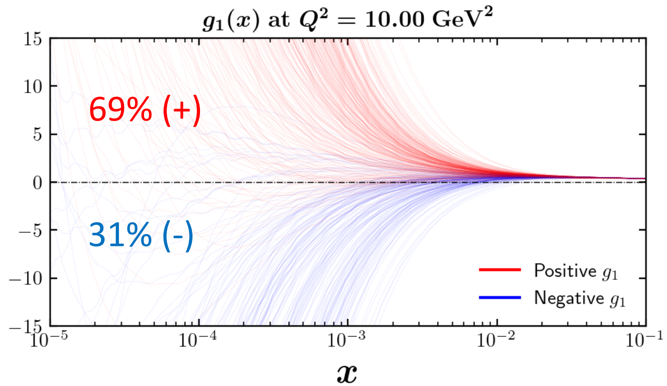
Standout Basis Functions:

- Asymptotically negative solutions prefer $\widetilde{\boldsymbol{G}}_{\boldsymbol{\eta+s_{10}}} > 0$ and vice-versa.
- $\boldsymbol{G_2}$ dipole consistent with zero.

Small-x Behavior: The Story of $\widetilde{G}$ and G_2







$$x\Delta u^{+} \propto x g_1^p$$

Small small-x Contribution

$(x\Delta u^{+} \sim 0)$

Large Contribution in Data range

$(x\Delta u^{+} \sim 0.07)$

Large small-x Contribution

$(x\Delta u^{+} \sim 0.03)$

Small Contribution in Data range

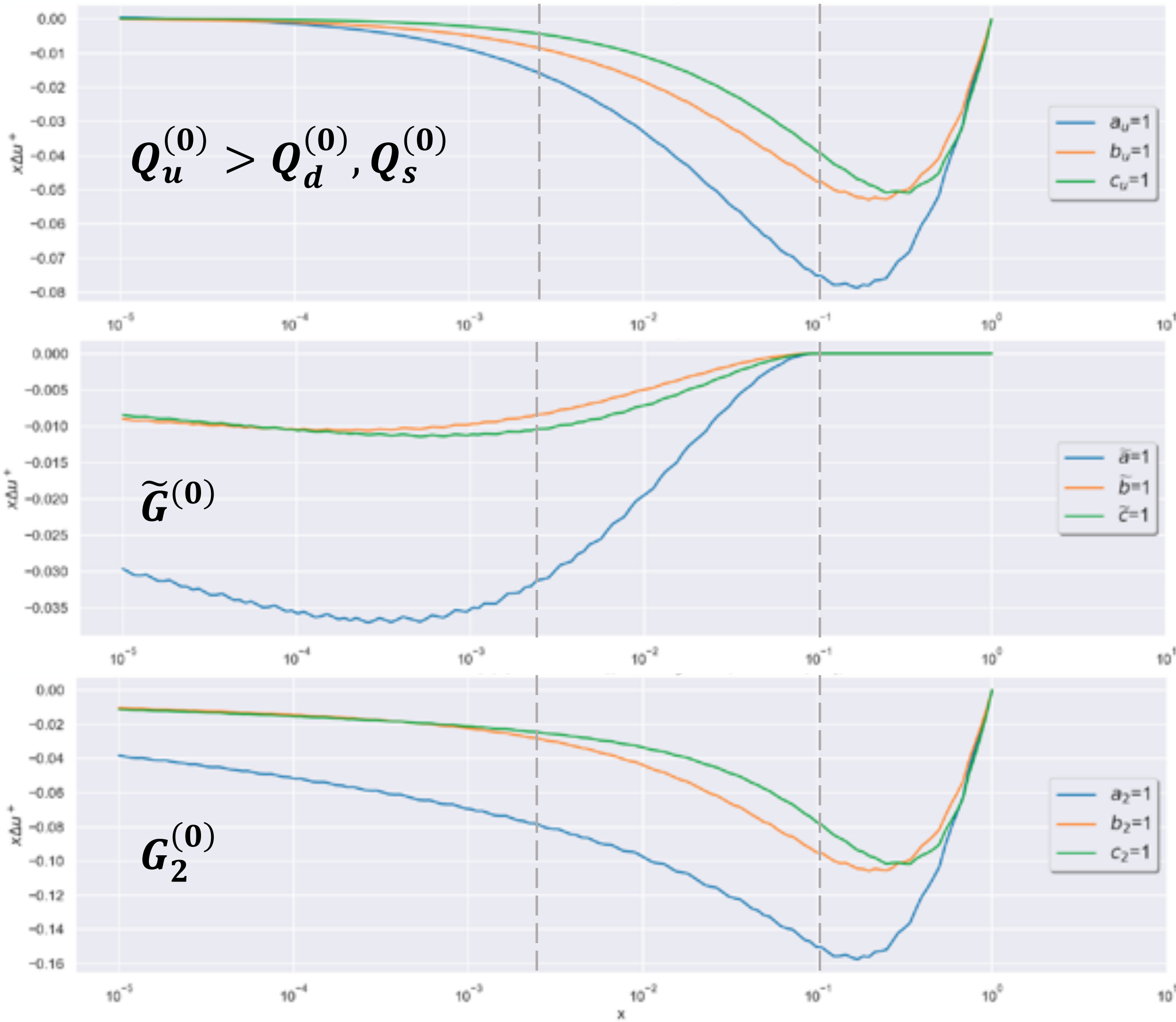
$(x\Delta u^{+} \sim 0.03)$

Large small-x Contribution

$(x\Delta u^{+} \sim 0.04)$

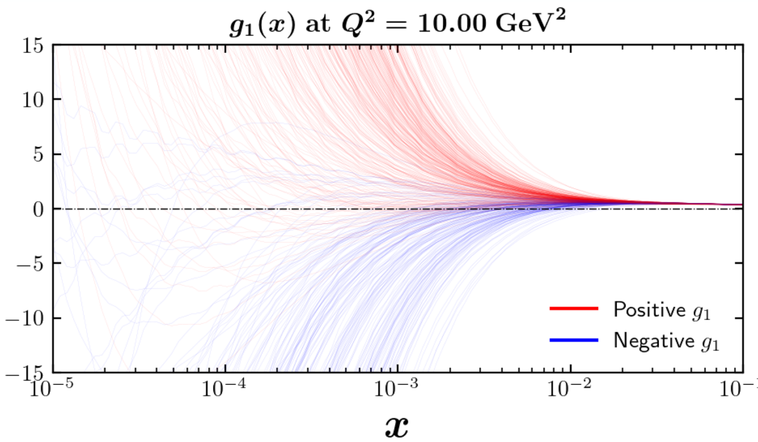
Large Contribution in Data range

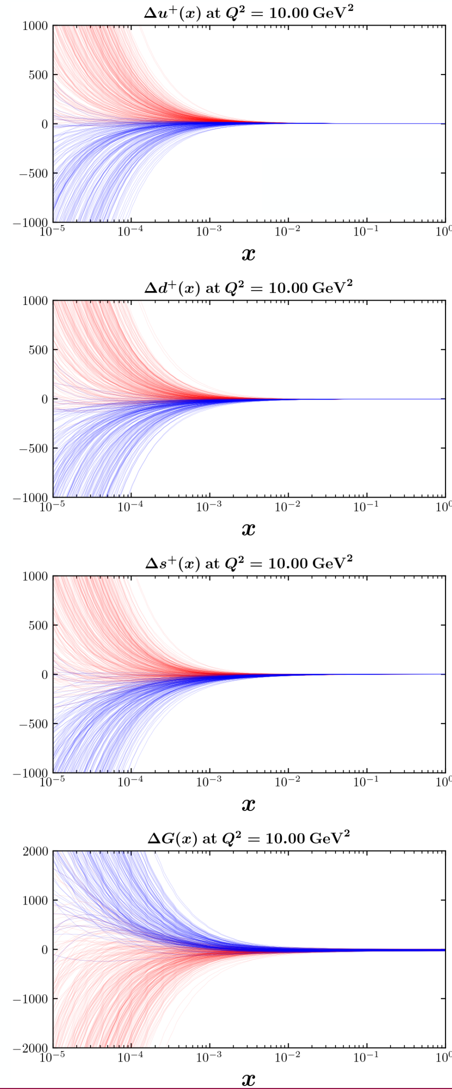
$(x\Delta u^{+} \sim 0.14)$



Small-x Behavior: Novel Correlations

- Each replica is colored according to the asymptotic sign of g_1 only.
- Δq_f^+ and ΔG also show **asymptotic bimodality.**





$$Q_f, G_2\Big|_{x\to 0} \approx \widetilde{G}$$

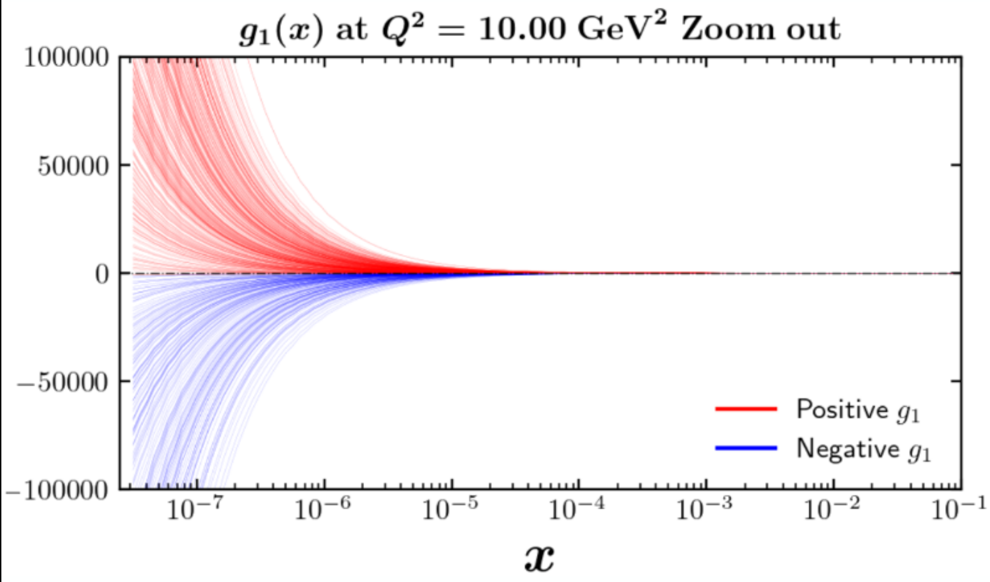
$$g_1 \sim \Delta q_f^+ \sim -\Delta G \sim -\widetilde{G}$$

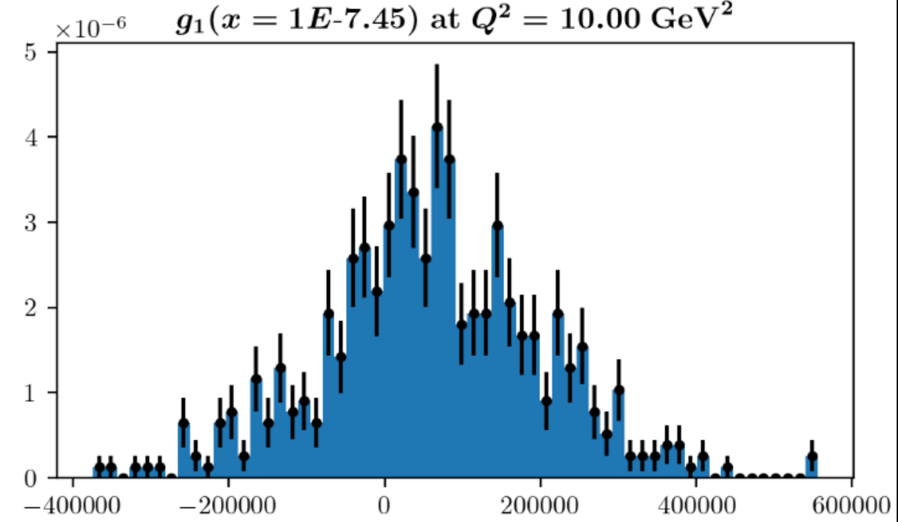
The Fall of Bimodality

NORMAL DISTRIBUTION AND PCA

THE RISE (AGAIN?) OF BIMODALITY

Fall of Bimodality





- Optical Illusion: Replicas are closer to zero than to each other.
- Also confirmed by principal component analysis: No bimodal parameters, all correlations of parameters are equally valid.

Conclusion:

g_1, Δq_f^+, and ΔG are not bimodal, but their small-x behavior is determined by $\widetilde{G}$

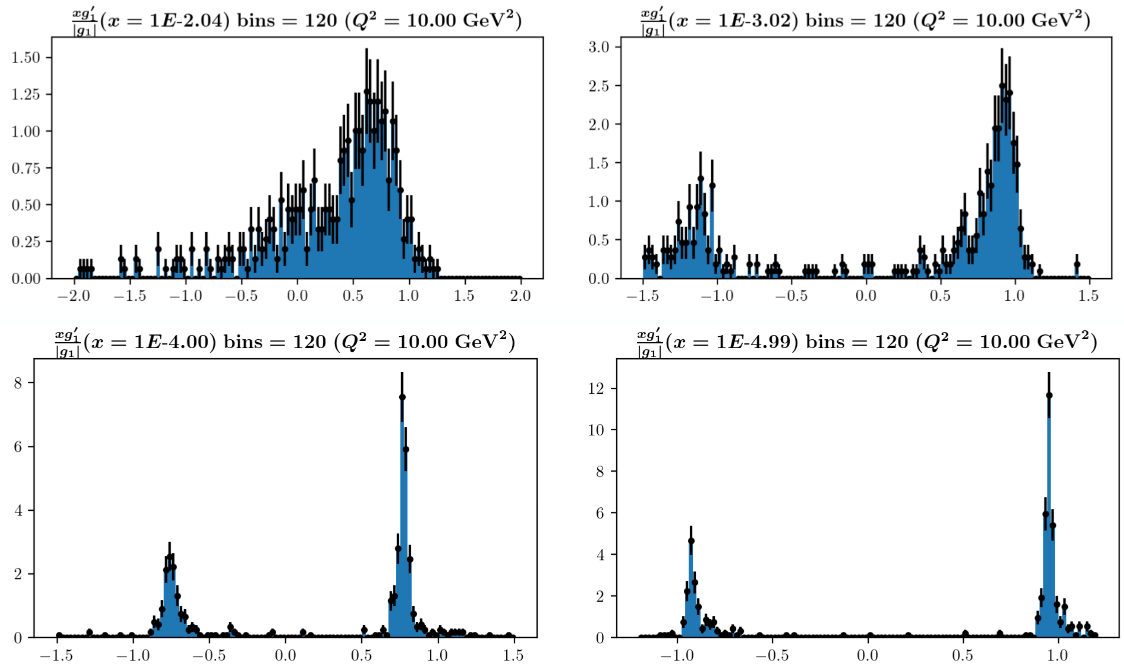
Rise (again) of Bimodality

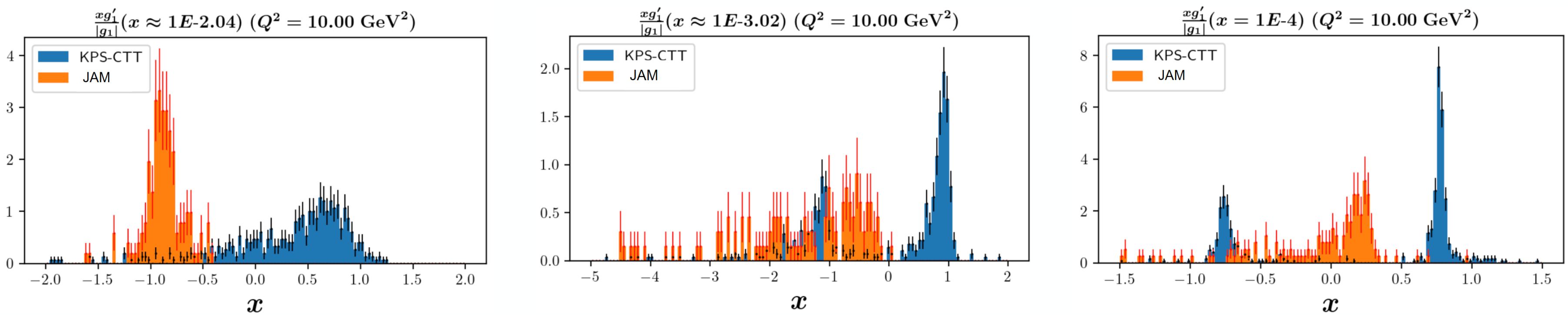
Consider the **curvature** of g_1:

$$g_1 \to \lim_{x\to 0} g_1 = g_1^0\, x^{-\alpha_h}$$

$$\frac{dg_1}{d(\log(x))} = x\,\frac{dg_1}{dx} = -\alpha_h g_1^0 x^{-\alpha_h}$$

$$x\,\frac{g_1'}{|g_1|} = -\mathrm{Sign}(g_1)\alpha_h$$





Conclusions

- Our **Full Theory,** using flavor singlet and flavor non-singlet evolution, can fit global data of both DIS and SIDIS very well, $\frac{\chi^2}{N_{pts}} = 1.03$.
- We must choose the small-x resolution of our g_1, $\Delta q_f^{\pm}$, and ΔG replicas. Statistics suggest asymptotic sign uncertainty of ~5% when generating replicas to $x = 10^{-5}$.
- The asymptotic sign of all observables is dominated (through evolution) by the $\tilde{G}$ polarized dipole amplitude – predicting asymptotic g_1 seemingly relies on $\tilde{G}$-sensitive data.
- g_1 (and the other observables) are not truly bimodal, but there is comparative bimodality in $x \frac{g_1'}{|g_1|}$ between KPS-CTT helicity evolution and DGLAP evolution.

Future Work

- Our future work is to expand the theory to include pp data – this may provide some data sensitive to G_2 but may also require more polarized dipole amplitudes.
- We can also perform a sophisticated matching of our theory onto JAM-DGLAP large-x data to further constrain our parameters.

Bibliography

[1] R. L. Jaffe and A. Manohar,The G(1) Problem: Fact and Fantasy on the Spin of the Proton, Nucl. Phys. B337 (1990) 509–546.

[2] Y. V. Kovchegov, D. Pitonyak, and M. D. Sievert, Helicity Evolution at Small-x, J. High Energy Phys. 2016, 72 (2016).

[3] Y. V. Kovchegov, D. Pitonyak, and M. D. Sievert, Helicity Evolution at Small x: Flavor Singlet and Non-Singlet Observables, Phys. Rev. D 95, 014033 (2017).

[4] Y. V. Kovchegov, D. Pitonyak, and M. D. Sievert, Small-x Asymptotics of the Gluon Helicity Distribution, J. High Energy Phys. 2017, 198 (2017).

[5] Y. V. Kovchegov and M. D. Sievert, Small-x Helicity Evolution: An Operator Treatment, Phys. Rev. D 99, 054032 (2019).

[6] D. Adamiak, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato, and M. D. Sievert, First Analysis of World Polarized DIS Data with Small-x Helicity Evolution, Phys. Rev. D 104, L031501 (2021).

[7] F. Cougoulic, Y. V. Kovchegov, A. Tarasov, and Y. Tawabutr, Quark and Gluon Helicity Evolution at Small x: Revised and Updated, J. High Energy Phys. 2022, 95 (2022).

Extra Slides!

FOR DISCUSSION AND FOR FUN!

Full Theory Fit: Prior Ranges

- Selection of Priors: Good-Fitting replicas, random distribution of parameters.

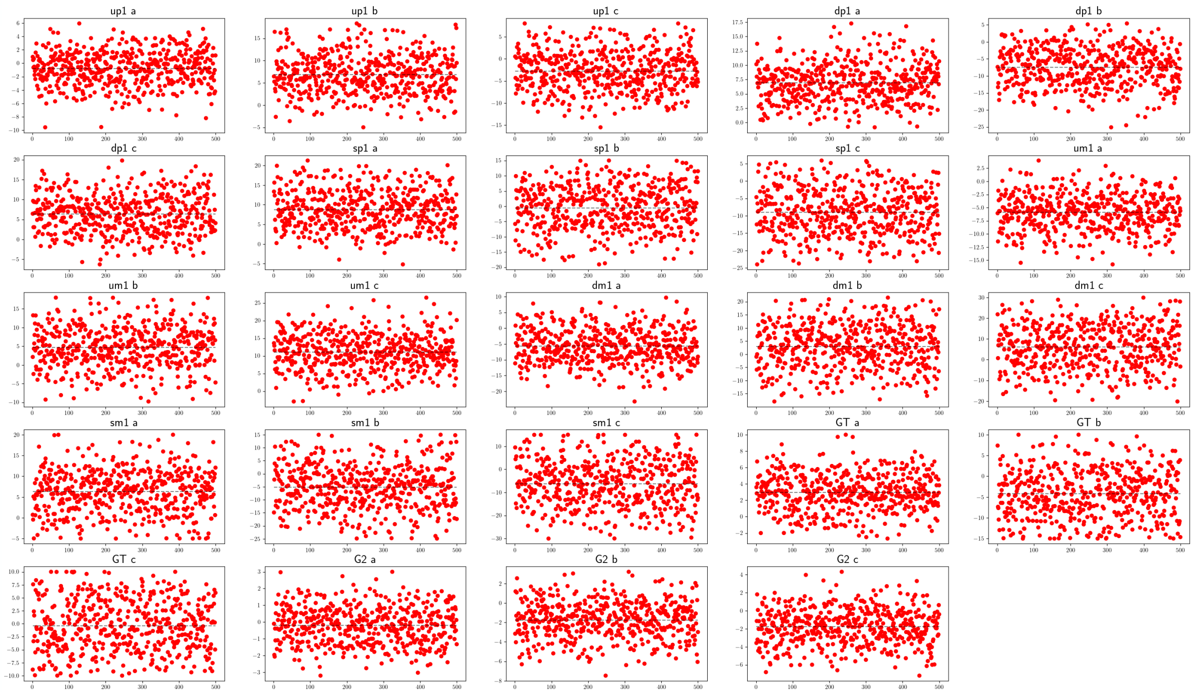
$$g_1^p(x,Q^2) = \frac{1}{2}\sum_f Z_f^2 \Delta q_f^+(x,Q^2)$$

$$\Delta q_f^+(x,Q^2) = -\frac{N_c}{2\pi^3}\int_{\frac{\Lambda^2}{s}}^{1}\frac{dz}{z}\int_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}}\frac{dx_{10}^2}{x_{10}^2}\left[Q_f + 2G_2\right]$$

Basis Initial Condition	Prior Range
$Q_u - \eta$	[-10,6]
$Q_u - s_{10}$	[-5, 20]
$Q_u - "1"$	[-20,8]
$Q_d - \eta$	[-4, 20]
$Q_d - s_{10}$	[-28, 8]
$Q_d - "1"$	[-8, 22]
$Q_s - \eta$	[-6, 24]
$Q_s - s_{10}$	[-20, 15]
$Q_s - "1"$	[-24, 6]
$\tilde{G} - \eta$	[-3, 10]
$\tilde{G} - s_{10}$	[-15, 10]
$\tilde{G} - "1"$	[-10, 10]
$G_2 - \eta$	[-5, 5]
$G_2 - s_{10}$	[-10, 5]
$G_2 - "1"$	[-10, 5]

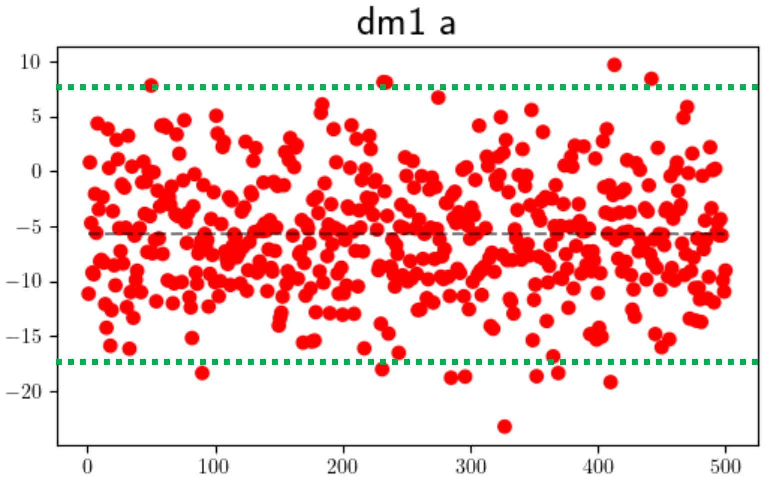
Basis Initial Condition	Prior Range
$G_u^{NS} - \eta$	[-18, 8]
$G_u^{NS} - s_{10}$	[-10, 18]
$G_u^{NS} - "1"$	[-5, 28]
$G_d^{NS} - \eta$	[-25, 10]
$G_d^{NS} - s_{10}$	[-18, 22]
$G_d^{NS} - "1"$	[-25, 30]
$G_s^{NS} - \eta$	[-5, 20]
$G_s^{NS} - s_{10}$	[-25, 15]
$G_s^{NS} - "1"$	[-30, 15]

$$\frac{\chi^2}{N_{pts}} = 1.03$$

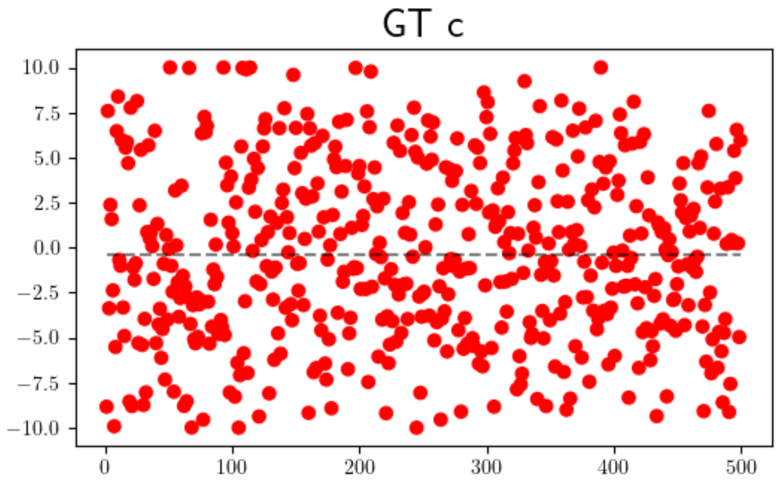


Constraining Parameter Space

<u>Good Priors</u> (Example)

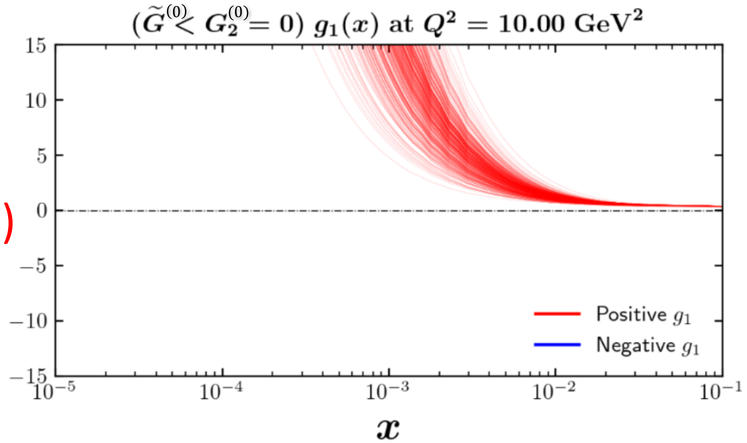


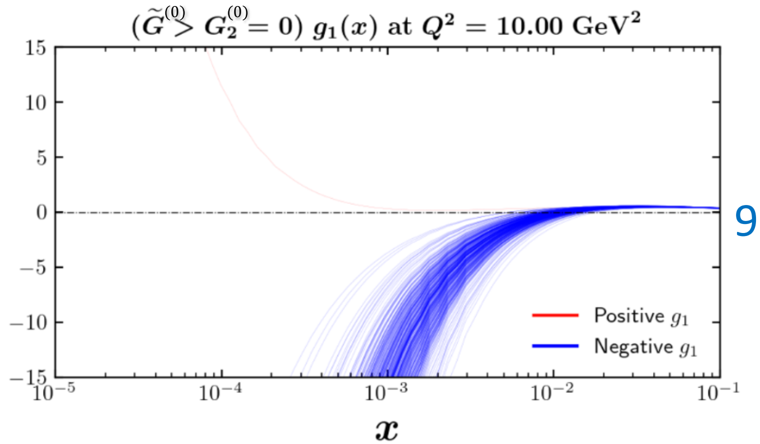
<u>Improved Priors</u> (Example)



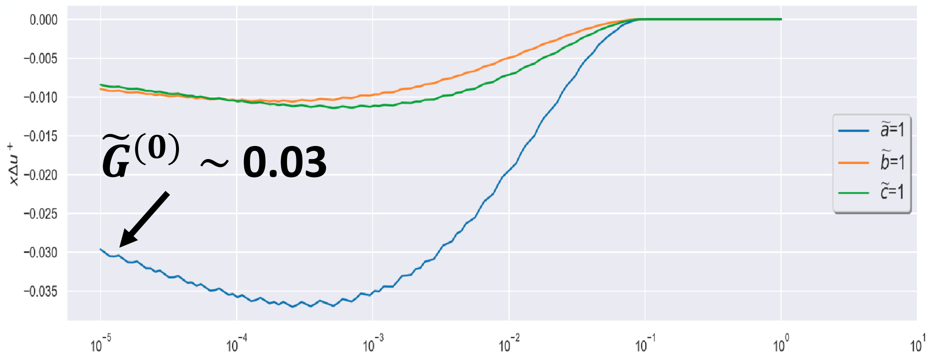
Small-x Dominance: $\widetilde{G}$ vs G_2

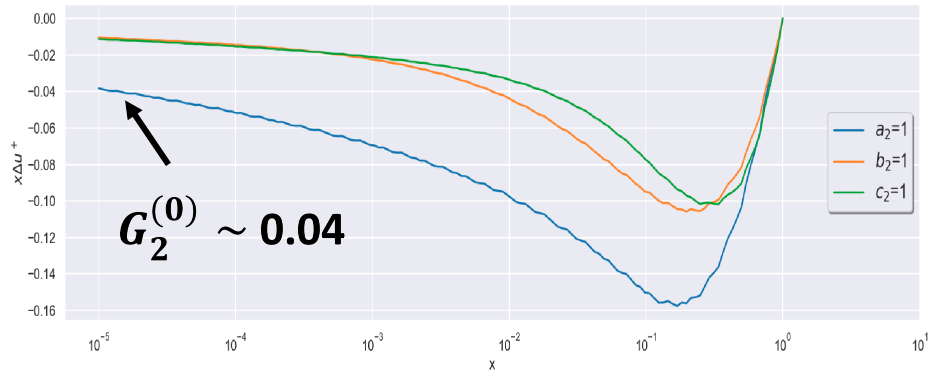
100% (+)



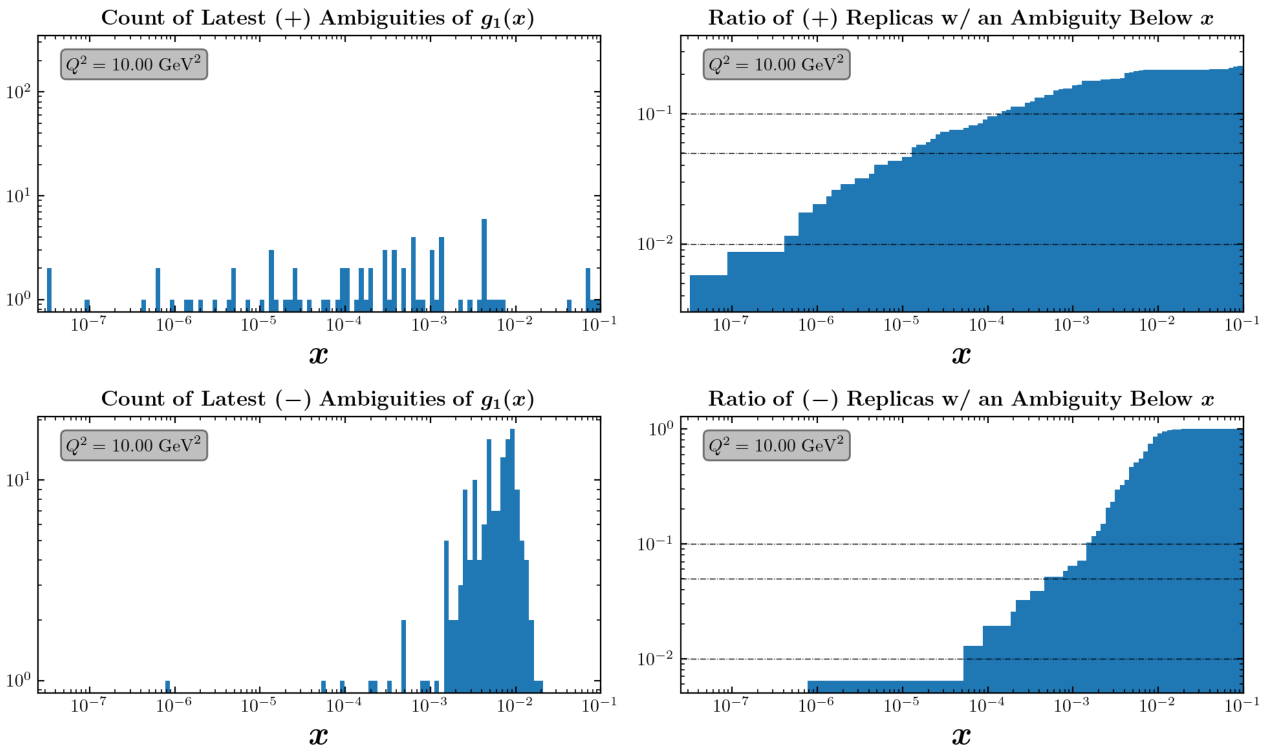


99.8% (+)





Latest Ambiguities: Positive vs Negative



- 95% of **asymptotically positive** replicas are confirmed by $x = 10^{-4.97}$.
- 95% of **asymptotically negative** replicas are confirmed by $x = 10^{-3.37}$.