Longitudinal Double-Spin Asymmetry at Small x in Polarized Proton-Proton Collisions

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M. Li, arXiv: 2304.12842

Work in progress with Daniel Adamiak and Yuri Kovchegov

Outline

- Introduction and Motivation
- Small x Effective Hamiltonian
- Longitudinal double-spin asymmetry for soft gluon production
- Summary

Origin of Nucleon Spin

Jaffe-Manohar spin sum rule for proton

The RHIC Spin Collaboration (2015)

$$S_q + L_q + S_G + L_G = \frac{1}{2}$$

Quark Spin

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$$S_q(Q^2 = 10\text{GeV}^2) \approx [0.15,\ 0.20]$$

$$x \in [0.001, 0.7]$$

Gluon Spin

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

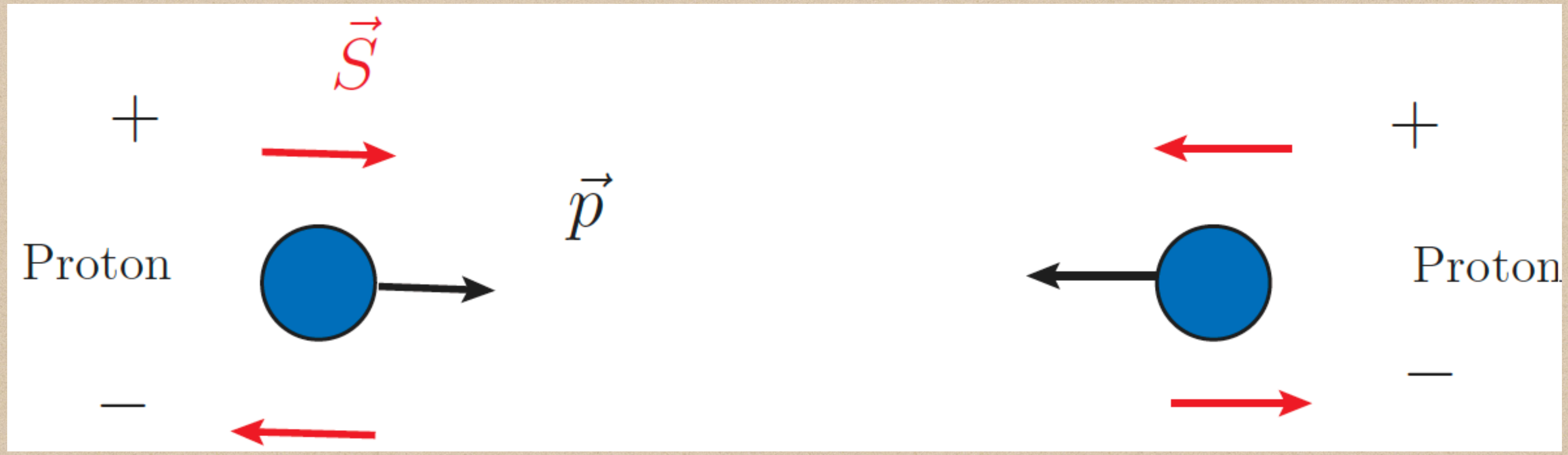
$$S_G(Q^2 = 10\text{GeV}^2) \approx [0.13,\ 0.26]$$

$$x \in [0.05, 0.7]$$

Missing spin of the proton maybe in Quark and Gluon Orbital Angular Momentum L_q and L_G and/or smaller values of x

Longitudinal Double-Spin Asymmetry

How to measure quark and gluon intrinsic spin inside a proton?



$$A_{\mathrm{LL}} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

RHIC has measured A_{LL} for the productions of jets, dijets, π^0, $\pi^{\pm}$, direct photons…

RHIC Spin Collaboration, arXiv: 2302.00605

Longitudinal Double-Spin Asymmetry

Longitudinal double-spin asymmetry is related to parton helicity distribution.

$$A_{\rm LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

Collinear Factorization (also parity invariance)

Babcock, Monsay and Sivers (1979),
de Florian, Sassot, Stratmann and Vogelsang (2008)(2014)

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \, \Delta f_a(x_a, Q^2)\, \Delta f_b(x_b, Q^2)\, d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

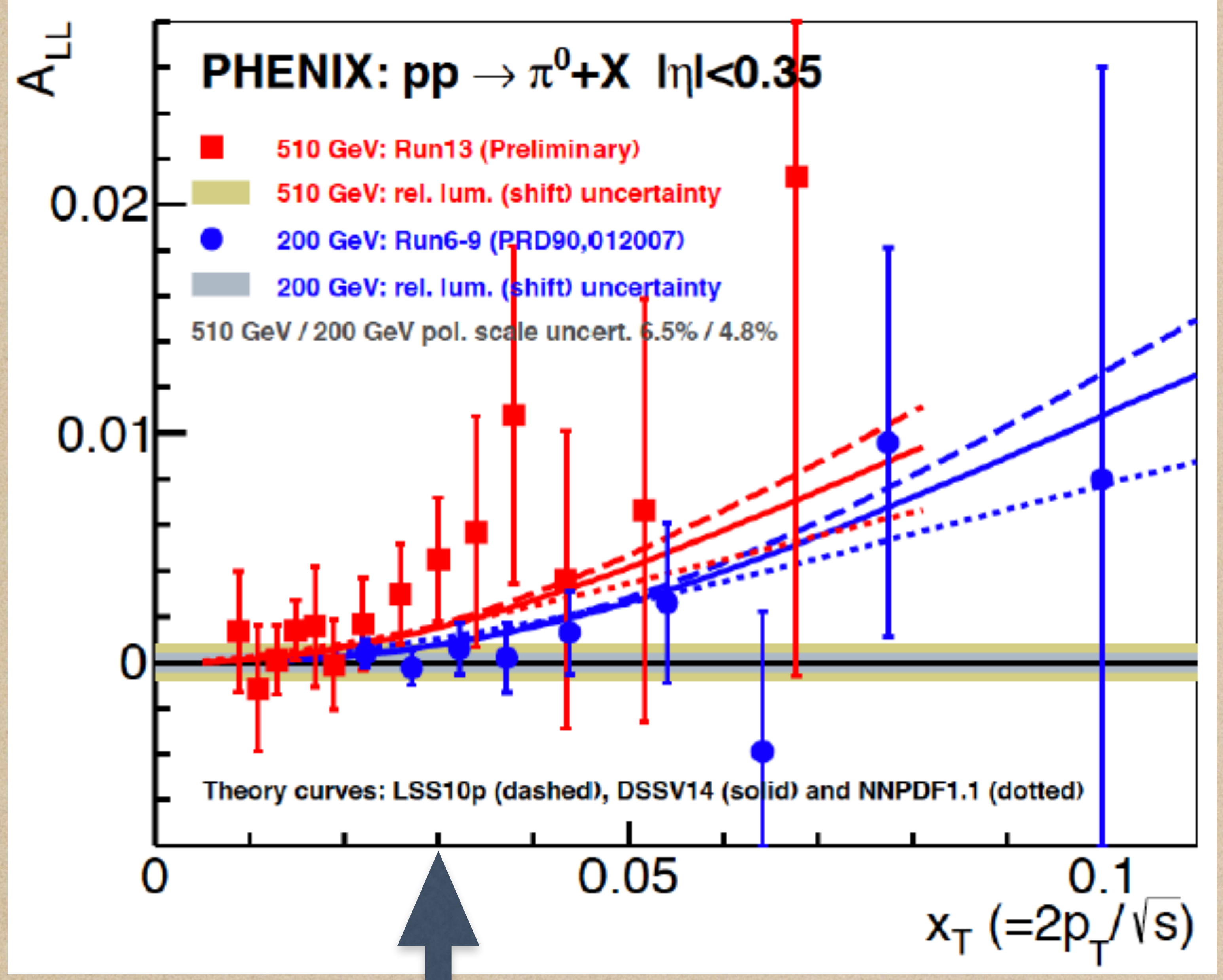
(Anti) quark and gluon helicity distribution $\Delta f_j(x, Q^2) \equiv f_j^+(x, Q^2) - f_j^-(x, Q^2)$

Partonic level double-spin asymmetry $d\Delta\hat{\sigma} = d\hat{\sigma}^{++} - d\hat{\sigma}^{+-}$

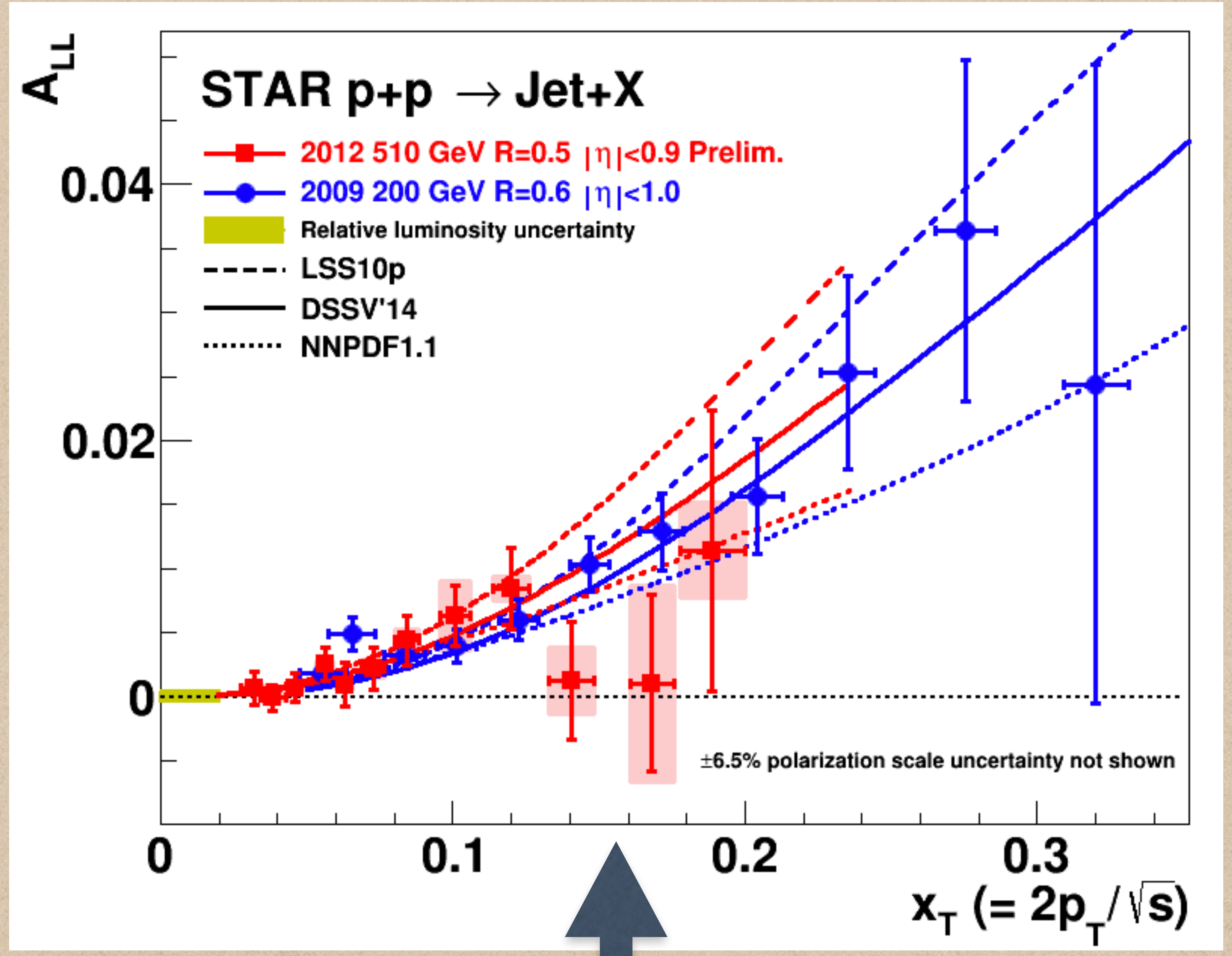
Longitudinal Double-Spin Asymmetry at small x

Collinear Factorization (applicable for large transverse momentum)

$$d\Delta\sigma = \sum_{ab}\int dx_a \int dx_b\, \Delta f_a(x_a, Q^2)\, \Delta f_b(x_b, Q^2)\, d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$



Low transverse momentum region, sensitive to small x gluons, collinear factorization probably breaks down



Large transverse momentum region, collinear factorization successful, but not sensitive to small x gluons.

RHIC Spin Collaboration (2015)

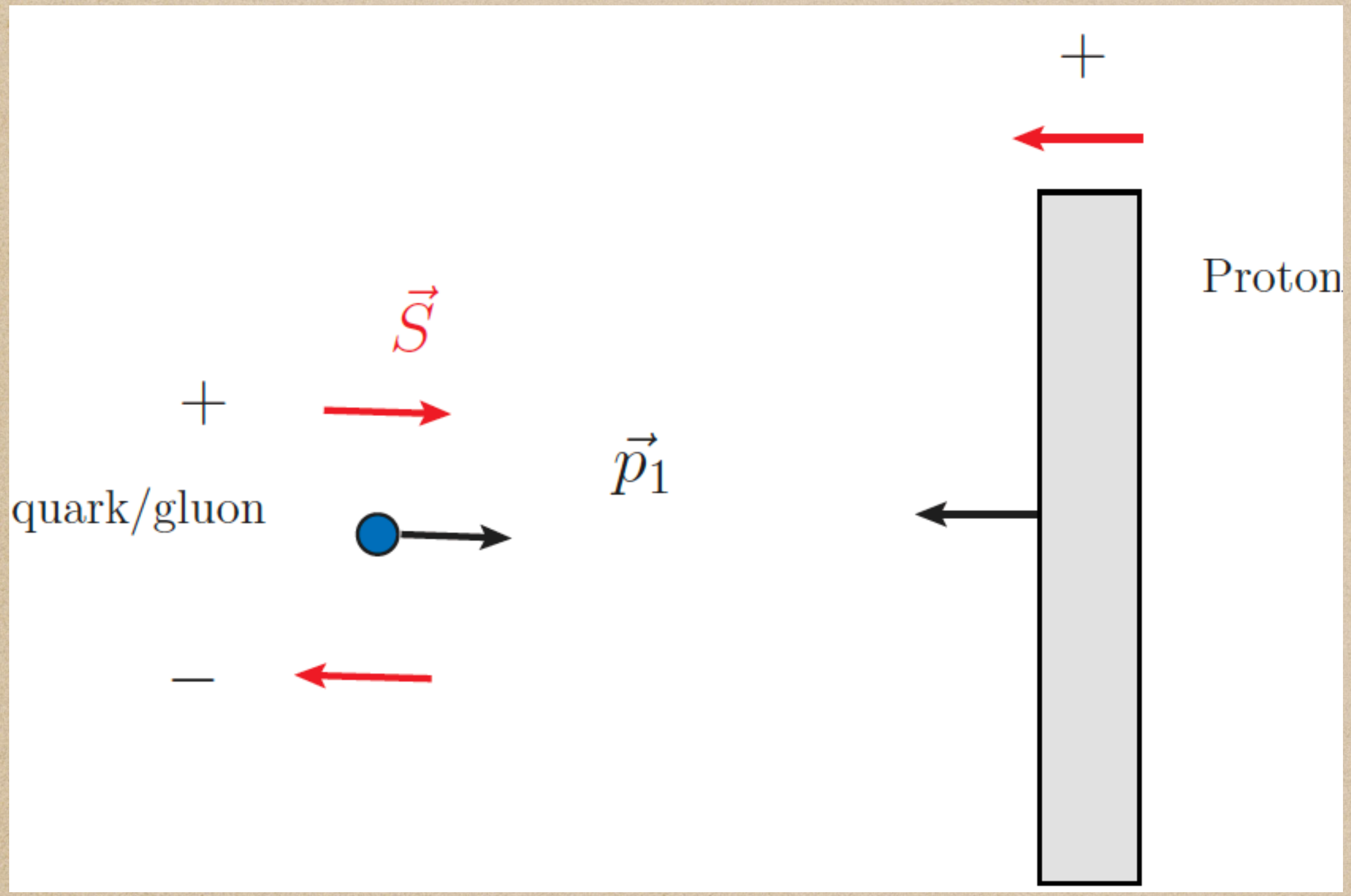
Small x gluon saturation regime

$\mathbf{p}_T \sim Q_s$

We need transverse momentum dependent framework to describe A_{LL} in low transverse momentum region

Double-spin asymmetry at small x: gluon production at mid-rapidity

Treating projectile proton and target proton on equal footing at small x is challenging.



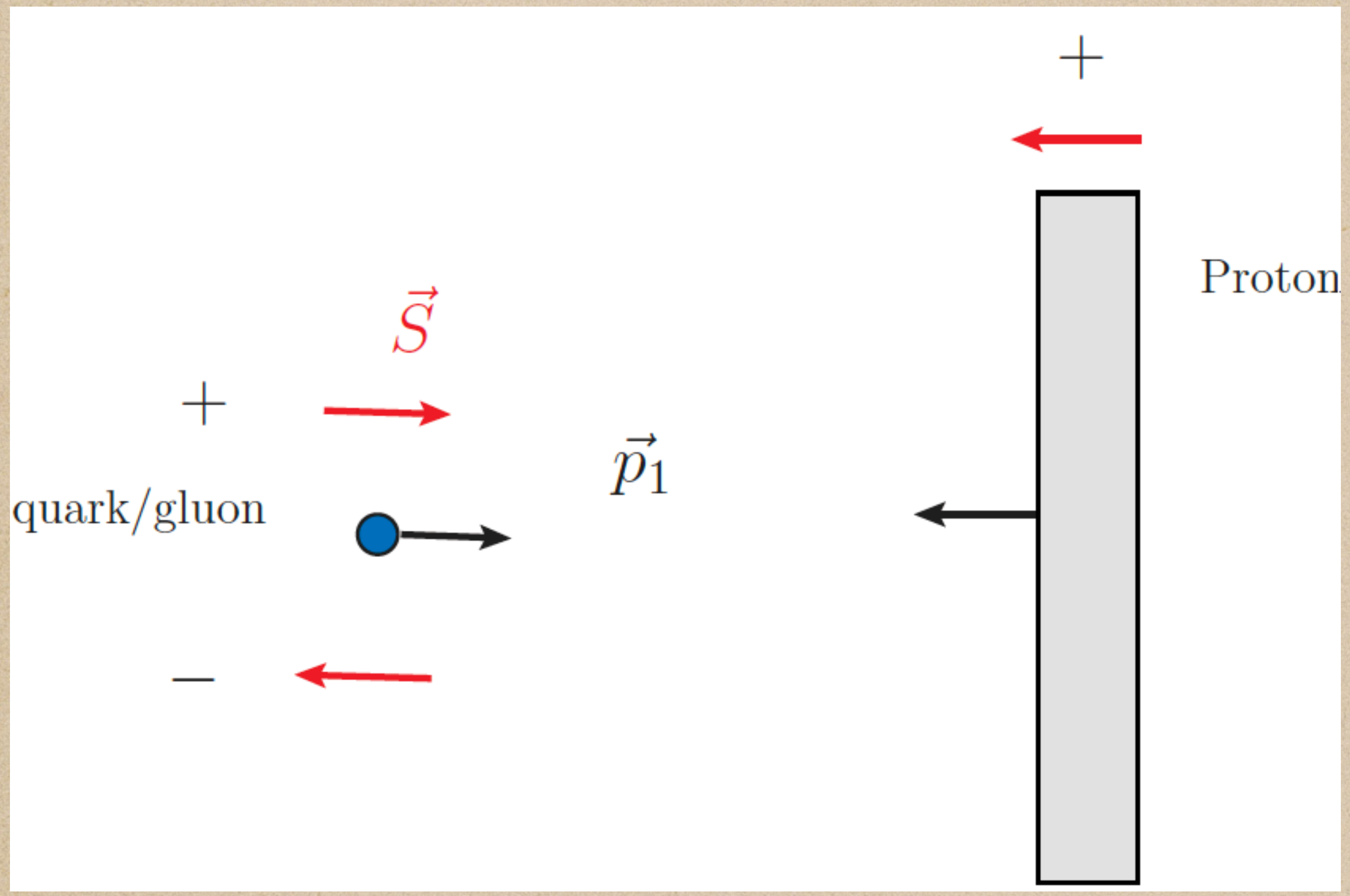
$$\text{quark/gluon} + \text{proton} \longrightarrow \text{gluon} + X$$

$$A_{\rm LL}^{g} \equiv \frac{d\Delta\sigma}{d^2\mathbf{p}dy} = \frac{d\sigma^+}{d^2\mathbf{p}dy} - \frac{d\sigma^-}{d^2\mathbf{p}dy}$$

Goal: $A_{\rm LL}^{g}$ for Gluon production at mid-rapidity

Convention: proton travels along negative-z direction.

The Formalism: Small-x Effective Hamiltonian



Need to go beyond the eikonal approximation. The Shockwave Picture of High energy Scatterings: proton is treated as background gluon and quark fields

S-matrix element for highly boosted states *Bjorken, Kogut and Soper (1971)*

$$S_{\mathrm{fi}} = \langle \phi_{\mathrm{f}} | \, e^{i\omega \hat{K}^3} \mathscr{P}\exp\left\{ -i \int_{-\infty}^{+\infty} dz^+ V_{\mathrm{I}}(z^+) \right\} e^{-i\omega \hat{K}^3} \, | \phi_{\mathrm{i}} \rangle$$

$$= \langle \phi_{\mathrm{f}} | \, \mathscr{P}\exp\left\{ -i \int_{-\infty}^{+\infty} dz^+ e^{i\omega \hat{K}^3} V_{\mathrm{I}}(z^+) e^{-i\omega \hat{K}^3} \right\} | \phi_{\mathrm{i}} \rangle \, .$$

S-matrix element of boosted interaction for unboosted states

Light-cone Hamiltonian in the background fields → **Boosting the background fields, expanding in powers of $\xi = e^{-\omega}$** → **Small-x effective Hamiltonian up to linear order in ξ**

The Formalism: Small-x Effective Hamiltonian

Light-Cone Gauge $A^+ = 0$.

Expansion in ξ **is equivalent to expansion in** x: $\xi = e^{-|Y_P - Y_T|} \sim x e^{-m_N/Q}$

Order ξ^0: $V_{(0)} = a_b^- J_b^+ = a_b^- \left(g\bar{\Psi}\gamma^+ t^b \Psi - ig[A^i, F^{+i}]^b \right)$

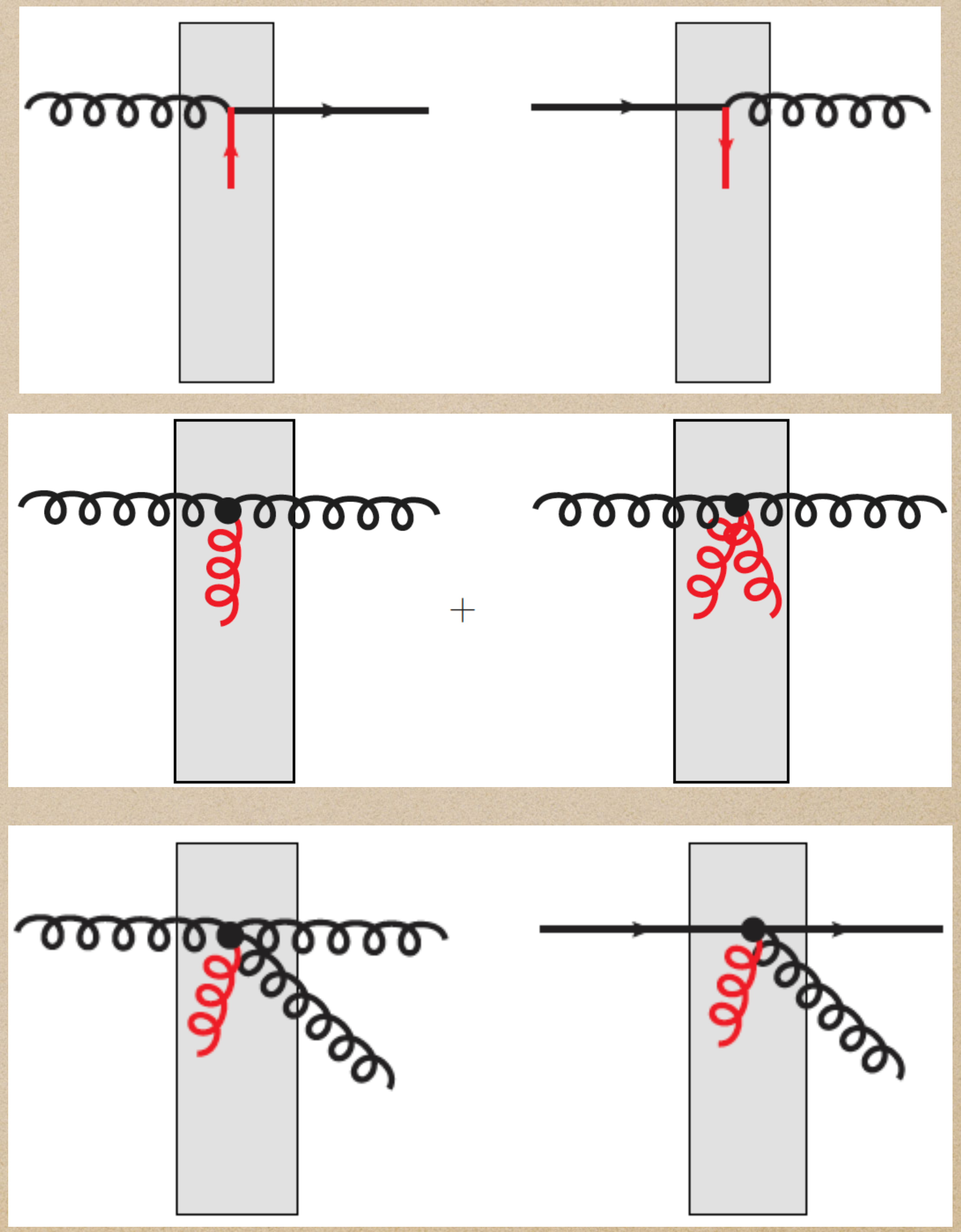
Order $\xi^{\frac{1}{2}}$: $V_{(\frac{1}{2})} = g\bar{\Psi}_G \gamma^i A_i \psi_B + g\bar{\psi}_B \gamma^i A_i \Psi_G$

Order ξ^1:

$$V_{(1)} = -\frac{1}{2}A_a^i\left((\mathcal{D}_l\mathcal{D}^l)^{ab}g_{ij} + 2ig(f_{ij})^{ab}\right)A_b^j + \frac{i}{\sqrt{2}}\Psi_G^\dagger\left(gf_{ji}S^{ij} - \mathcal{D}_l\mathcal{D}^l\right)\frac{1}{\partial_-}\Psi_G$$
$$+ig\left[A_i, A_j\right]_b(\mathcal{D}^iA^j)_b + (\mathcal{D}_iA^i)_b\frac{1}{\partial_-}\left(-ig\left[\partial_-A^j, A_j\right]^b + \sqrt{2}g\Psi_G^\dagger t^b\Psi_G\right)$$
$$+\frac{1}{\sqrt{2}}g\Psi_G^\dagger A_j\gamma^j\gamma^i\mathcal{D}_i\frac{1}{\partial_-}\Psi_G + h.c.$$

$\Psi = \Psi_G + \Psi_B \qquad \mathcal{D}^i = \partial^i + ig[a^i, .]$

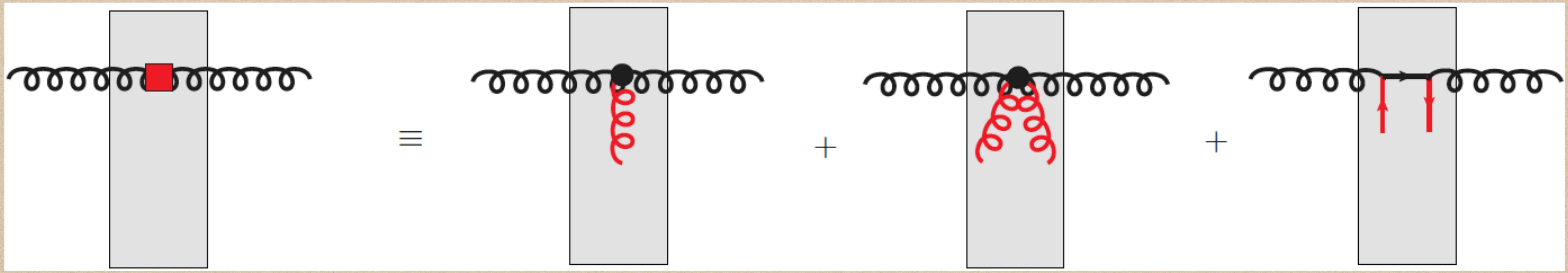
Background fields: a^-, a^i, ψ_B **Quantum fields:** Ψ_G, A^i



The Formalism: Polarized Wilson Lines

Single gluon scattering amplitude up to sub-eikonal order

$$M^{g\to g} = (2\pi)2k^+\delta(k^+ - k'^+)\delta_{\lambda'\lambda}\Big[\delta(\mathbf{x}-\mathbf{y})U_{\mathbf{x}} + \xi\lambda\delta(\mathbf{x}-\mathbf{y})U_{\mathbf{x}}^{\mathrm{pol}[1]}(k^+) + \xi U_{\mathbf{y},\mathbf{x}}^{\mathrm{pol}[2]}(k^+)\Big]^{c'c}$$



$$U_{\mathbf{x}}^{\mathrm{G}[1]} = -\frac{2ig}{2k^+}\int_{-\infty}^{+\infty} dw^+ U_{\mathbf{x}}(+\infty, w^+) f_{12}(w^+, \mathbf{x})\, U_{\mathbf{x}}(w^+, -\infty)$$

$$U_{\mathbf{x}}^{\mathrm{q}[1]} = -\frac{g^2}{2k^+}\int_{-\infty}^{+\infty} dw_2^+ \int_{-\infty}^{w_2^+} dw_1^+\, U_{\mathbf{x}}^{c'h'}(+\infty, w_2^+)\bar{\psi}^{\alpha}_{B,n'}(w_2^+, \mathbf{x})\left[t^{h'}V_{\mathbf{x}}(w_2^+, w_1^+)t^{h}\right]^{n'n}\left[\frac{\gamma^-\gamma^5}{2}\right]^{\alpha\beta}\psi^{\beta}_{B,n}(w_1^+, \mathbf{x})\, U_{\mathbf{x}}^{hc}(w_1^+, -\infty) + c.c.$$

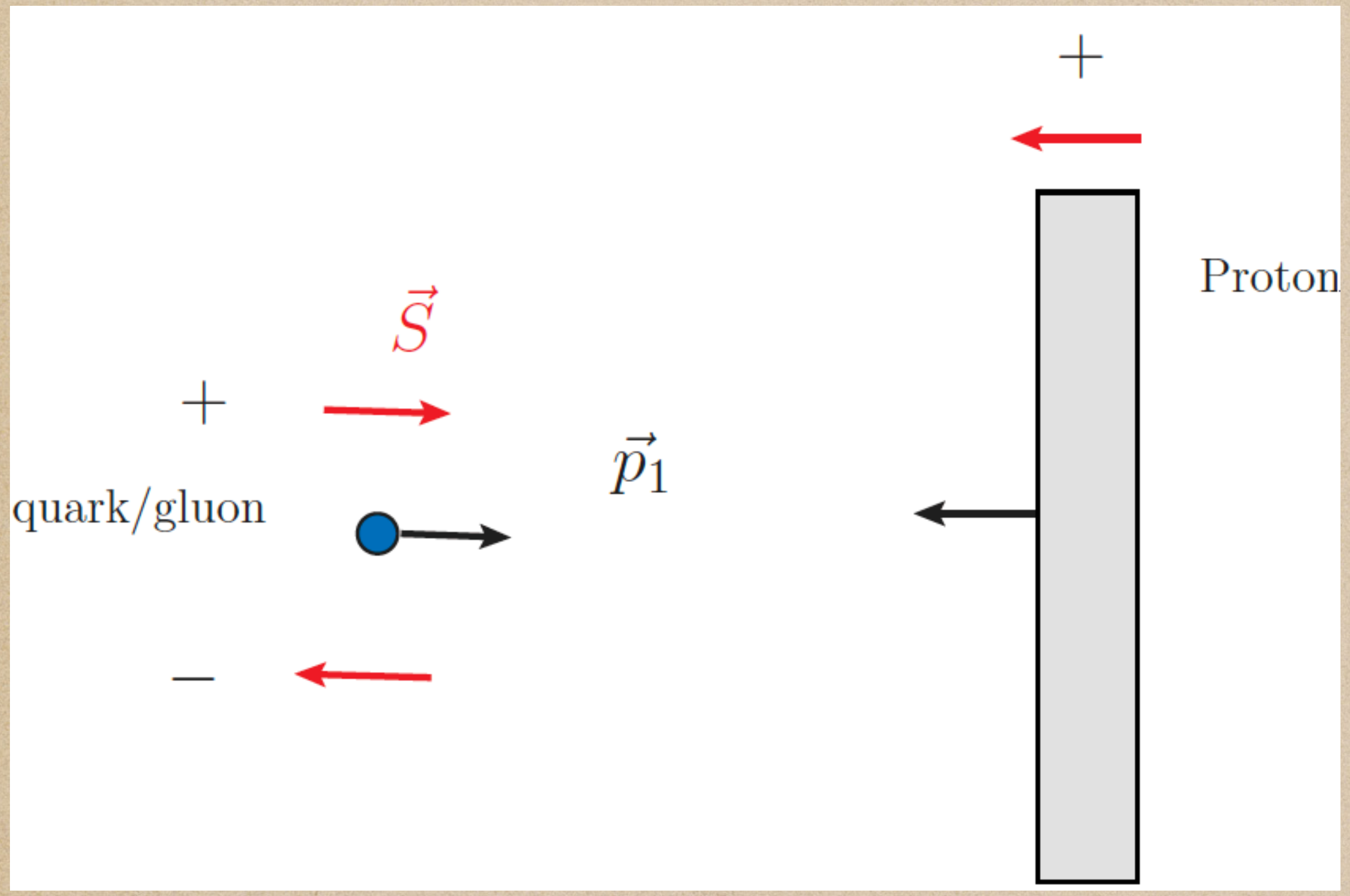
$$U_{\mathbf{y},\mathbf{x}}^{\mathrm{G}[2]} = \frac{i}{2k^+}\int_{-\infty}^{+\infty} dw^+\, U_{\mathbf{y}}^{c'a}(+\infty, w^+)\int_{\mathbf{z}} \delta(\mathbf{y}-\mathbf{z})\left[\overleftarrow{\mathscr{D}}_l\overrightarrow{\mathscr{D}}^l\right]^{ab}(w^+, \mathbf{z})\delta(\mathbf{z}-\mathbf{x})\, U_{\mathbf{x}}^{bc}(w^+, -\infty),$$

$$U_{\mathbf{x}}^{\mathrm{q}[2]} = -g^2\frac{1}{2k^+}\int_{-\infty}^{+\infty} dw_2^+ \int_{-\infty}^{w_2^+} dw_1^+ U_{\mathbf{x}}^{c'h'}(+\infty, w_2^+)\bar{\psi}^{\alpha}_{B,n'}(w_2^+, \mathbf{x})\left[t^{h'}V_{\mathbf{x}}(w_2^+, w_1^+)t^{h}\right]^{n'n}\left[\frac{\gamma^-}{2}\right]^{\alpha\beta}\psi^{\beta}_{B,n}(w_1^+, \mathbf{x})U_{\mathbf{x}}^{hc}(w_1^+, -\infty) + c.c.$$

A_{LL}^{g} at small x: gluon production at mid-rapidity

Large-x Gluon initiated processes: $g \to g + g, \quad g \to q + g, \quad g \to \bar{q} + g\,.$

Large-x Quark (antiquark) initiated processes: $q \to q + g, \quad q \to g + g;$
$\bar{q} \to \bar{q} + g, \quad \bar{q} \to g + g\,.$



The talk focuses on large-x gluon initiated processes.

$$M = M_{(0)}^{g\to gg} + \xi^{\frac{1}{2}} M_{(\frac{1}{2})}^{g\to qg} + \xi M_{(1)}^{g\to gg} + \dots$$

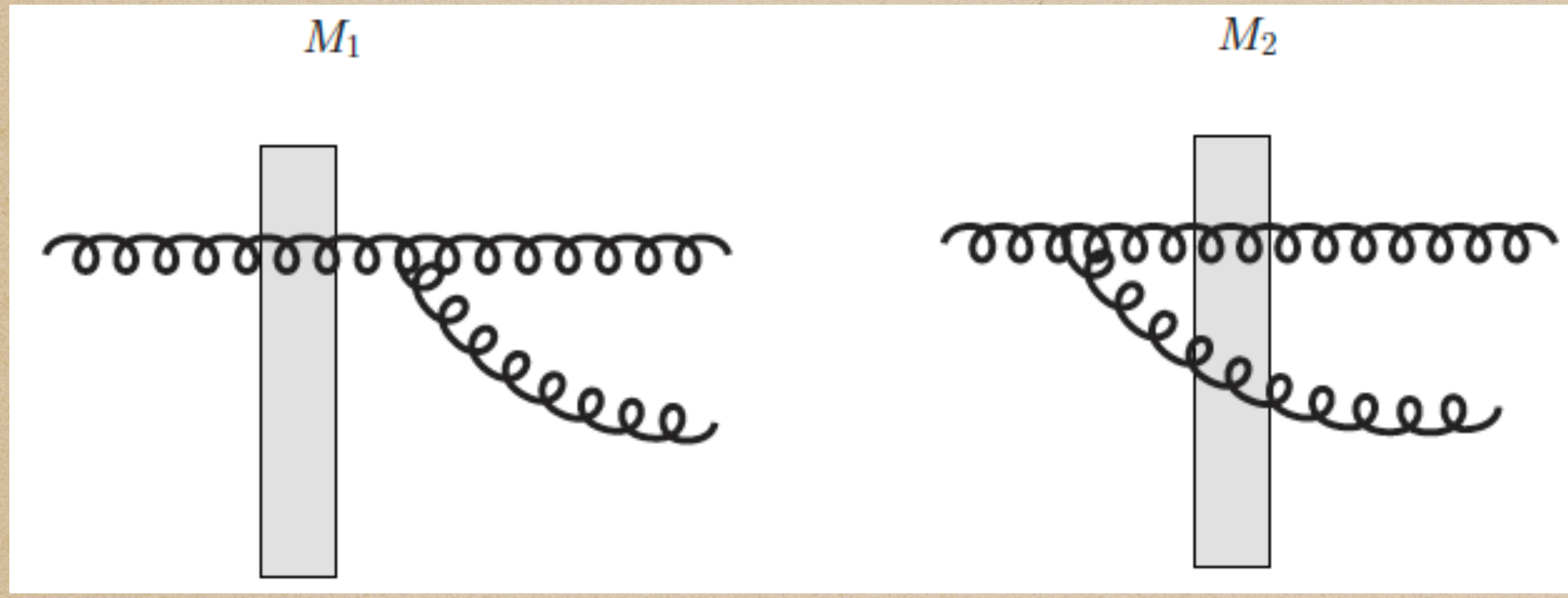
$$|M|^2 = |M_{(0)}^{g\to gg}|^2 + \xi\Big(|M_{(\frac{1}{2})}^{g\to qg}|^2 + M_{(1)}^{g\to gg}(M_{(0)}^{g\to gg})^* + c.c. \Big) + \dots$$

Isolate the part that is proportional to the polarization of incoming gluon.

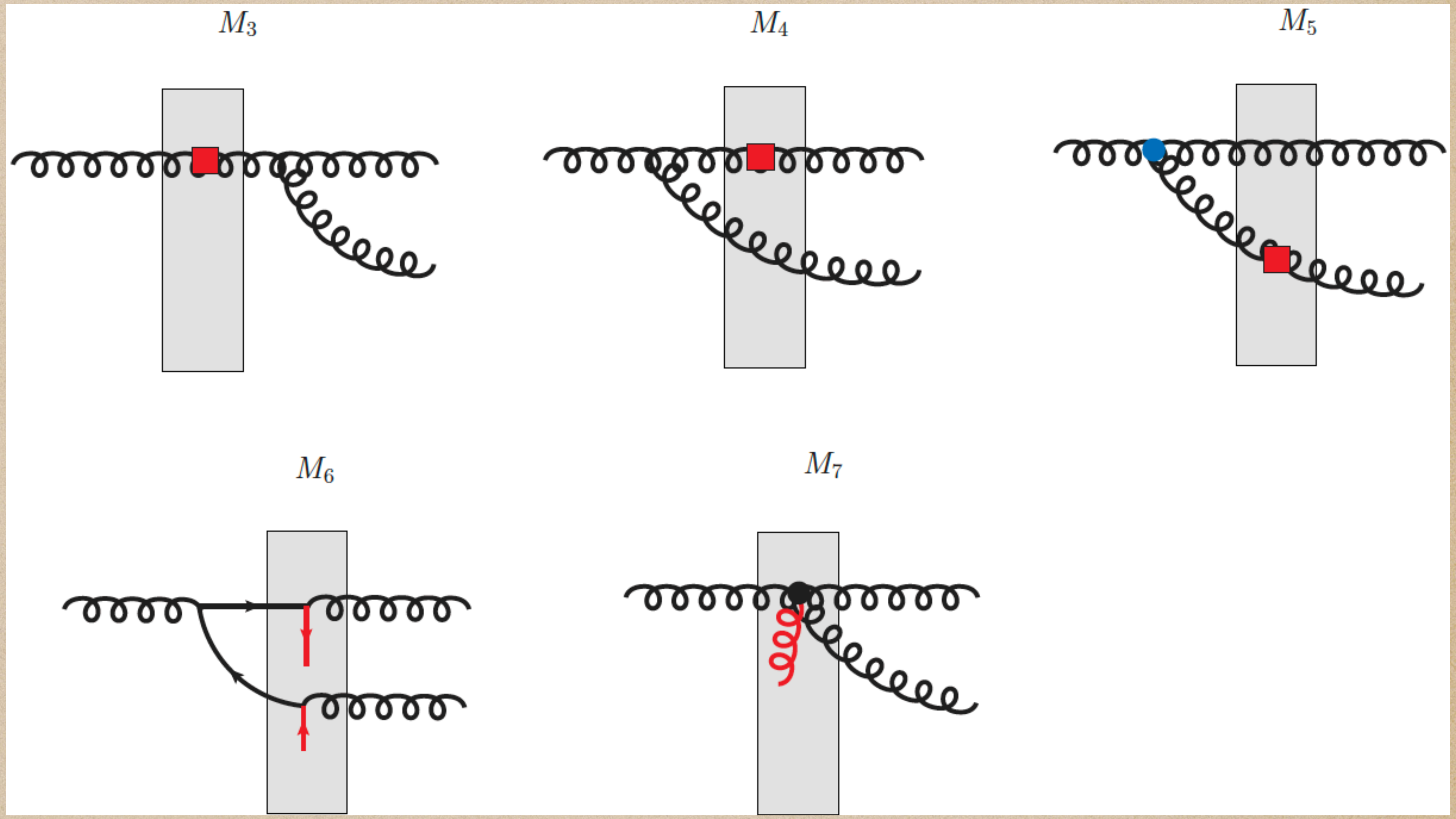
A_{LL}^{g} at small x: gluon production at mid-rapidity

The channel $g \to g + g$

Eikonal Order ξ^0



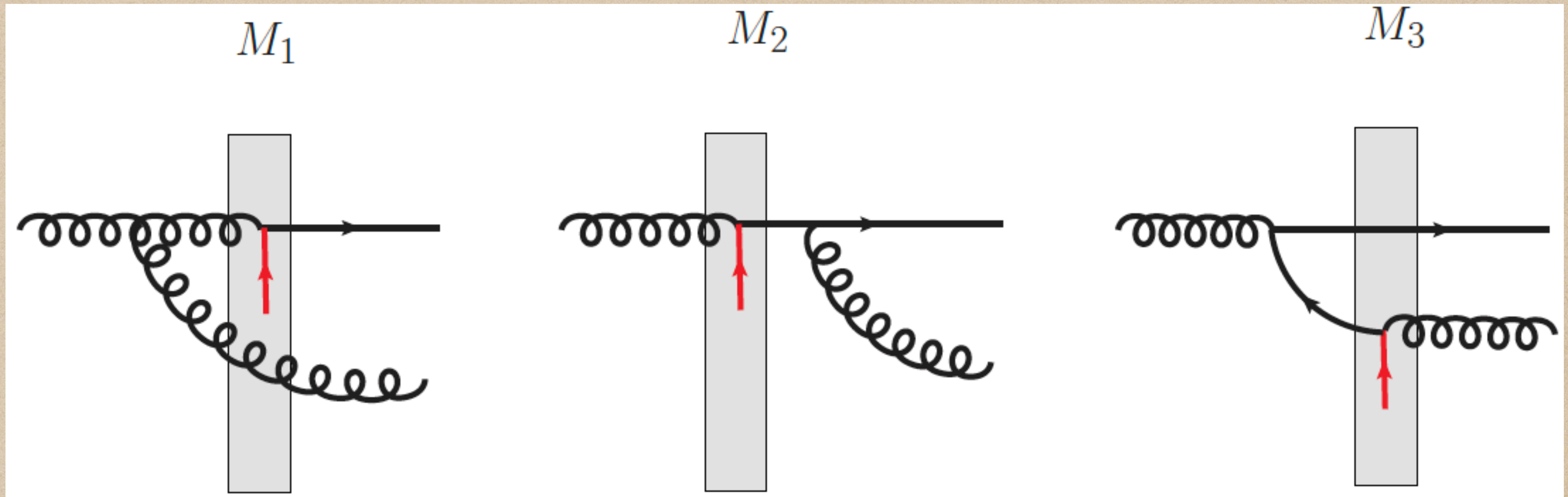
Sub-eikonal Order ξ^1



A_{LL}^{g} at small x: gluon production at mid-rapidity

The channels $g \to q + g, \quad g \to \bar{q} + g$

Sub-eikonal Order $\xi^{\frac{1}{2}}$



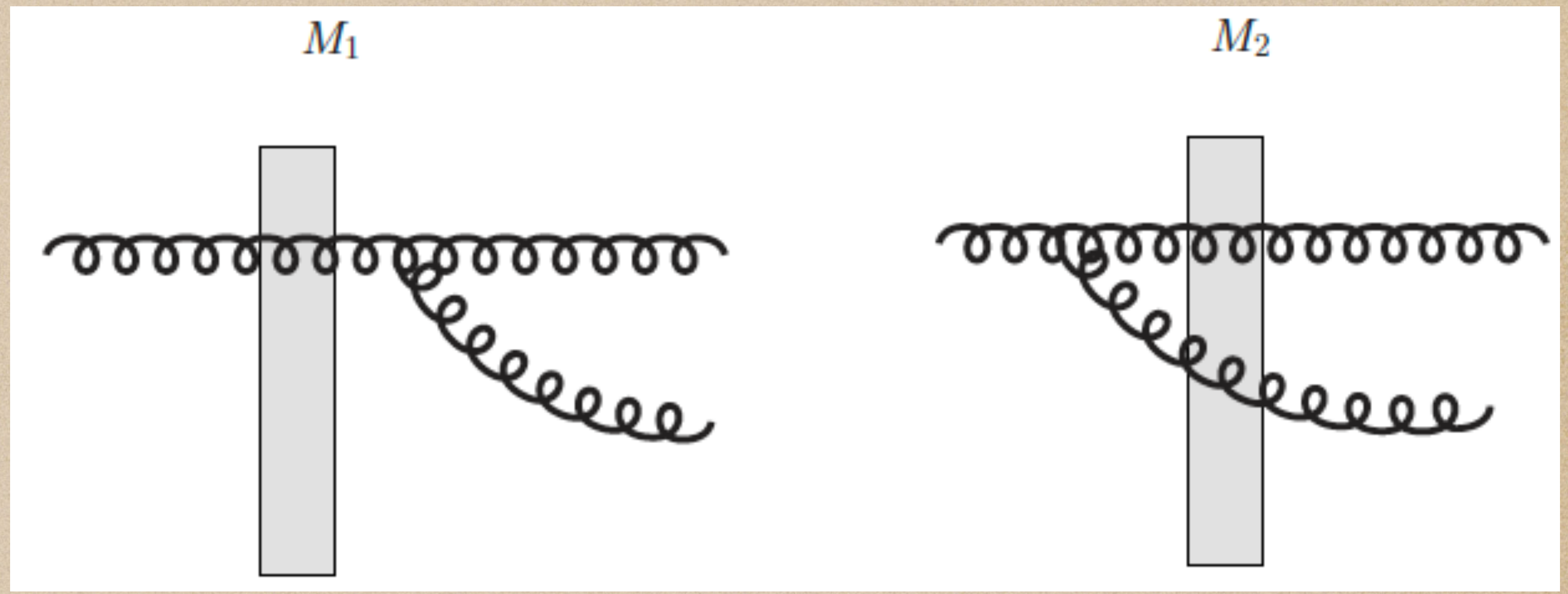
$$|M|^2 = |M_{(0)}^{g\to gg}|^2 + \xi\Big(|M_{(\frac{1}{2})}^{g\to qg}|^2 + M_{(1)}^{g\to gg}(M_{(0)}^{g\to gg})^* + c.c.\Big) + \ldots$$

Various cancellations and combinations between these two channels

Review: Eikonal Order Gluon Production

Eikonal order gluon production at mid-rapidity

$$\frac{d\sigma}{d^2\mathbf{p}_1 dy} = \frac{\alpha_s C_A}{\pi^2} \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}} \frac{1}{(2\pi)^2} \frac{\mathbf{x}_{10}\cdot\mathbf{x}_{1'0}}{|\mathbf{x}_{10}|^2 |\mathbf{x}_{1'0}|^2} \left[D(\mathbf{x}_0,\mathbf{x}_0) - D(\mathbf{x}_0,\mathbf{x}_1) - D(\mathbf{x}_0,\mathbf{x}_1') + D(\mathbf{x}_1,\mathbf{x}_1')\right]$$

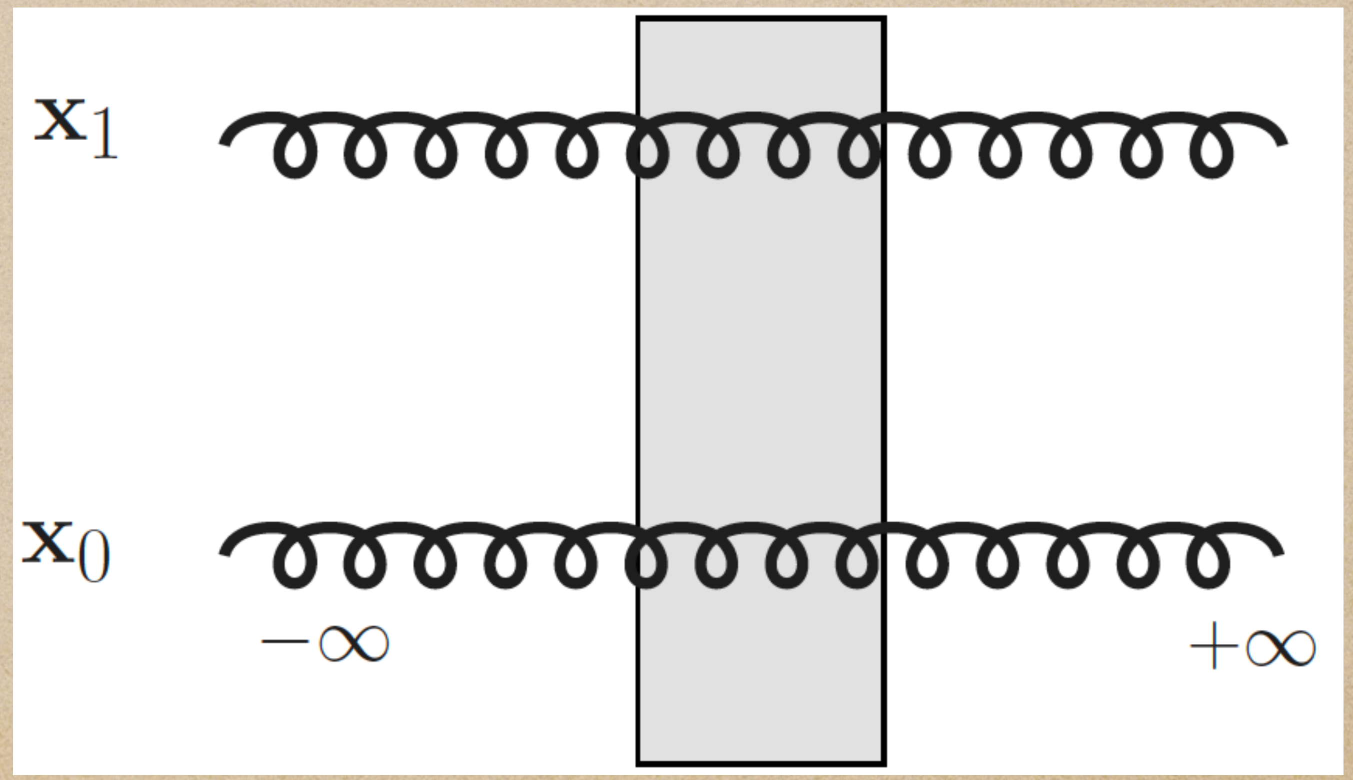


Gluon Dipole Correlator:

$$D(\mathbf{x}_0,\mathbf{x}_1) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U^\dagger_{\mathbf{x}_0} U_{\mathbf{x}_1}\right]\right\rangle$$

Related to the dipole gluon TMD in small x limit

$$D(x,\mathbf{k}^2) = \frac{g^2}{4(N_c^2-1)} \frac{xG(x,\mathbf{k}^2)}{\mathbf{k}^2}$$

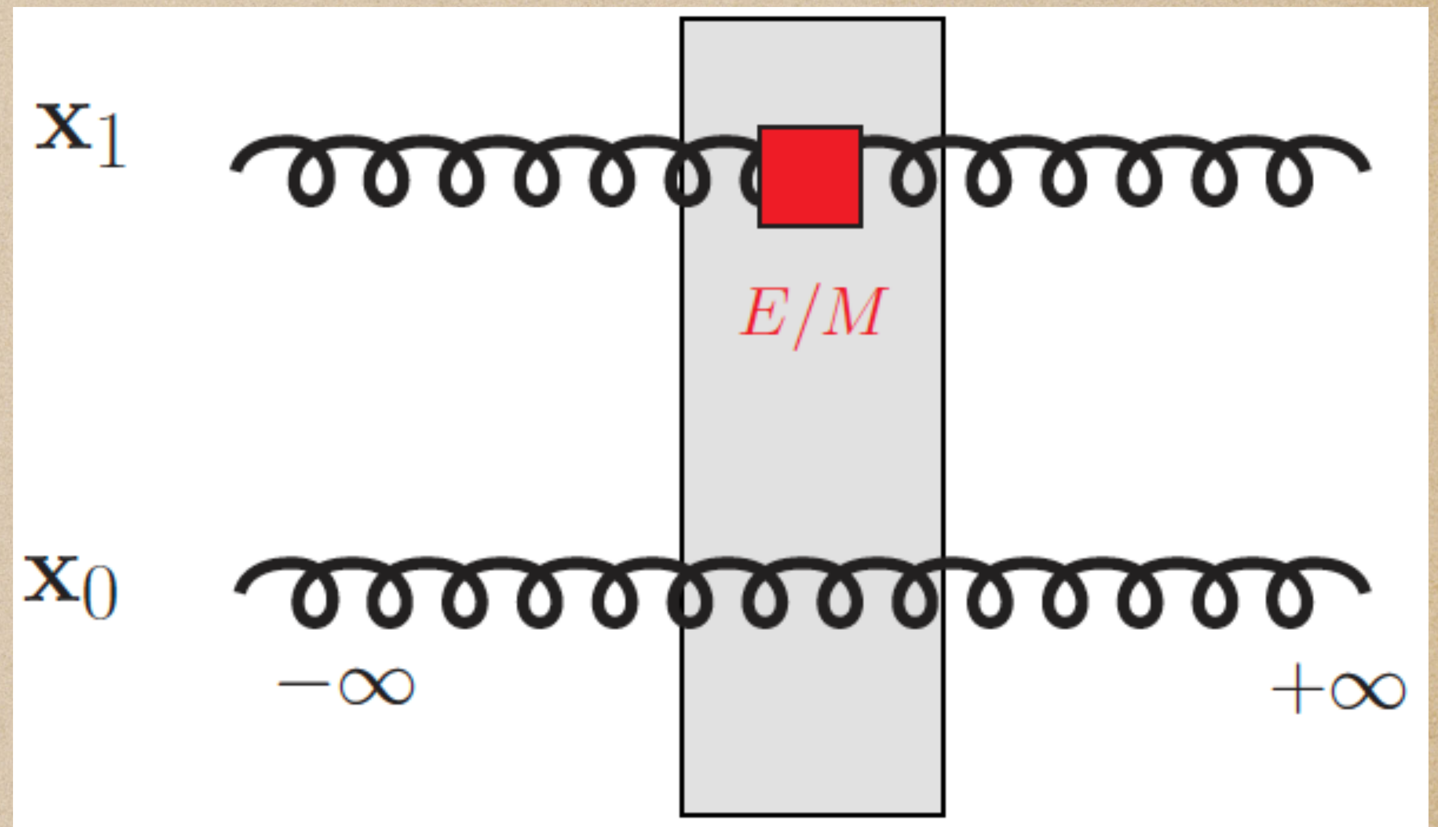


Polarized Wilson Line Correlators

Background gluon field polarized Wilson line correlators.

$$\Delta D^{g}_{M}(p^+;\mathbf{x}_0,\mathbf{x}_1)=\frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U^{\dagger}_{\mathbf{x}_0}U^{G[1]}_{\mathbf{x}_1}(p^+)\right]\right\rangle$$

$$\Delta D^{g,j}_{\mathrm{E}}(p^+;\mathbf{x}_0,\mathbf{x}_1)=\frac{1}{2(N_c^2-1)}\left\langle \mathrm{Tr}\left[U^{\dagger}_{\mathbf{x}_0}U^{j,\mathrm{G}[2]}_{\mathbf{x}_1}(p^+)-U^{j,\mathrm{G}[2]\dagger}_{\mathbf{x}_0}(p^+)U_{\mathbf{x}_1}\right]\right\rangle$$

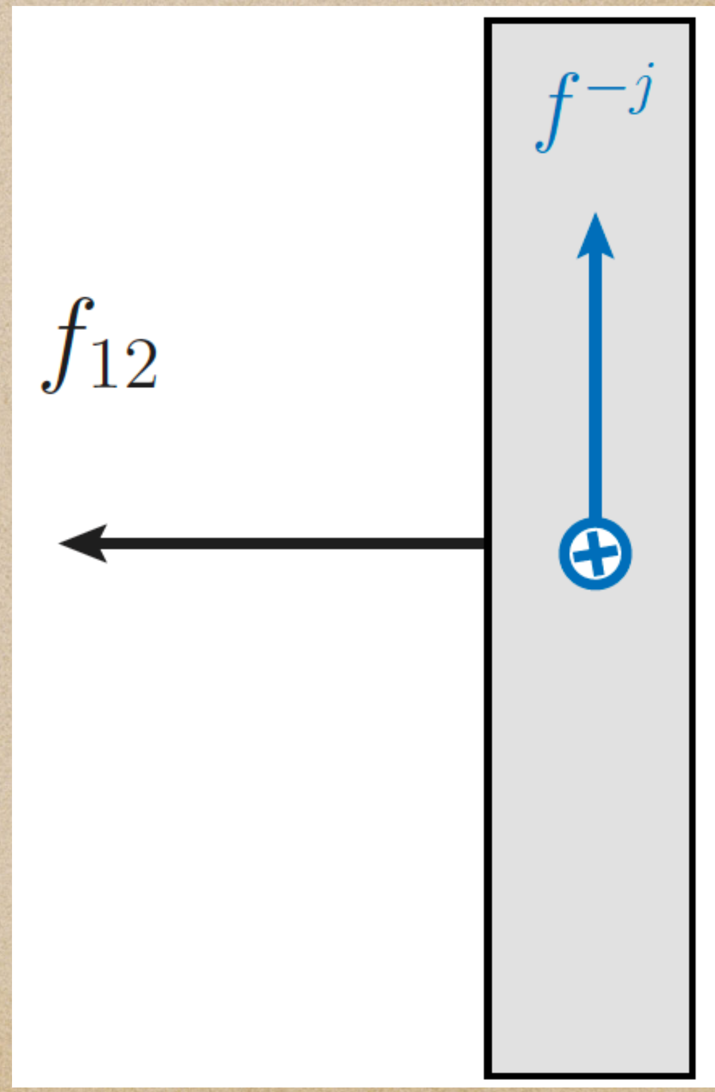


Magnetically Polarized Wilson Line

$$U^{\mathrm{G}[1]}_{\mathbf{x}}(p^+)=-\frac{2ig}{2p^+}\int_{-\infty}^{+\infty}dw^+U_{\mathbf{x}}(+\infty,w^+)f_{12}(w^+,\mathbf{x})\,U_{\mathbf{x}}(w^+,-\infty),$$

Electrically Polarized Wilson Line

$$U^{j,\mathrm{G}[2]}_{\mathbf{x}}(p^+)=\frac{ig}{2p^+}\int_{-\infty}^{+\infty}dw^+\,w^+\,U_{\mathbf{x}}(+\infty,w^+)f^{-j}(w^+,\mathbf{x})\,U_{\mathbf{x}}(w^+,-\infty)\,.$$



No contributions from transverse magnetic field f^{+j} and longitudinal electric field f^{+-}.

High order in eikonality

Longitudinal momentum exchange

Gluon Helicity TMDs at Small x

In the small x limit: *Cougoulic, Kovchegov, Tarasov, and Tawabutr (2022)*

$$\Delta D_E^g(x, \mathbf{k}^2) = \frac{g^2}{2(N_c^2 - 1)} g_{1L}^g(x, \mathbf{k}^2)$$

Gluon Helicity TMD

$$\Delta D_M^g(x, \mathbf{k}^2) = \frac{g^2}{4(N_c^2 - 1)} x\Delta H_{3L}^{\perp}(x, \mathbf{k}^2)$$

Twist-3 Gluon TMD

Gluon correlation functions: *Mulders and Rodrigues (2001)*

$$\Gamma^{\mu\nu;\rho\sigma}(k, P, S) = \int d^4\mathbf{x}\, e^{ik\cdot x} \left\langle P, S \,|\, \mathrm{Tr}\left[F^{\mu\nu}(0)\mathcal{U}^{[+]}(0,x)F^{\rho\sigma}(x)\mathcal{U}^{[-]}(x,0)\right] |\, P, S \right\rangle$$

$$\rightarrow \quad g_{1L}^g(x, \mathbf{k}^2) \subset \Gamma^{-i;-j}(k, P, S), \quad x\Delta H_{3L}^{\perp}(x, \mathbf{k}^2) \subset \Gamma^{ij;l-}(k, P, S)$$

$$\int_{\mathbf{k}} x\Delta H_{3L}^{\perp}(x, \mathbf{k}^2) = 0$$

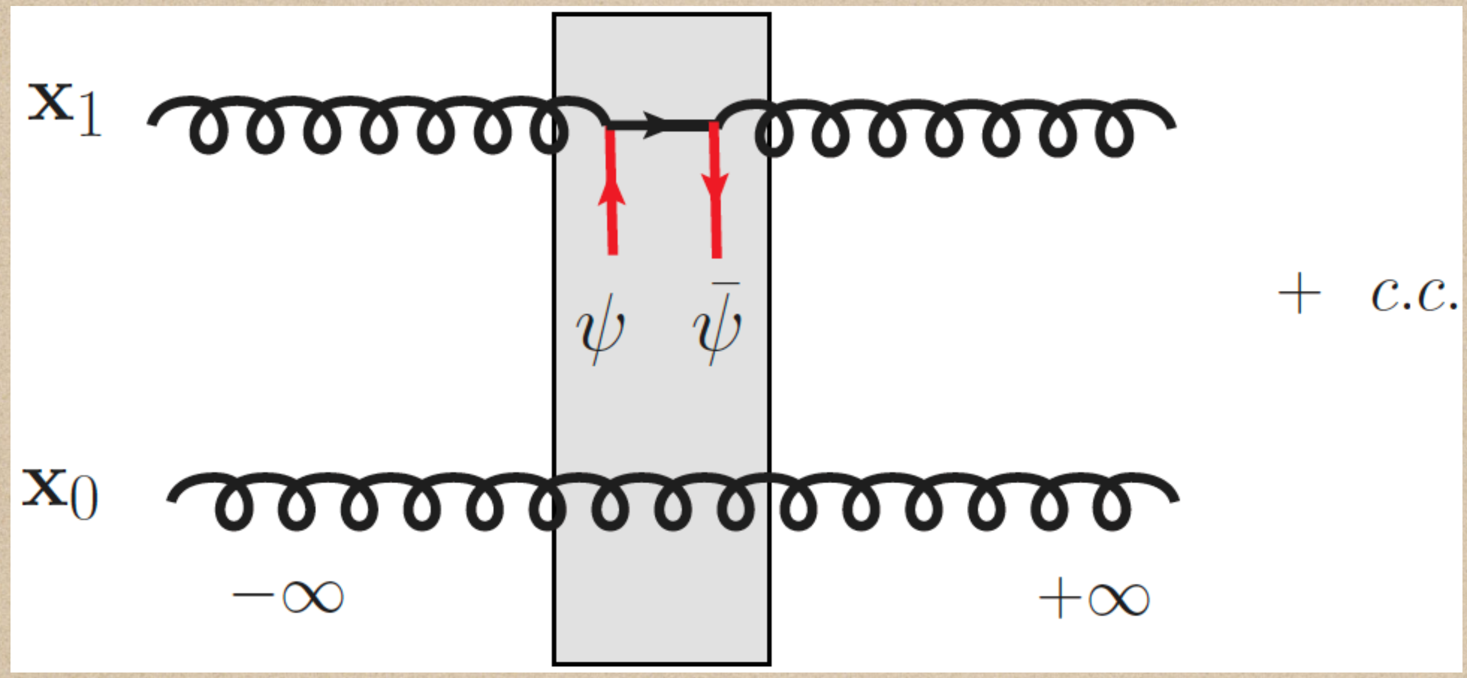
Twist-3 gluon TMD does not contribute to gluon helicity PDF

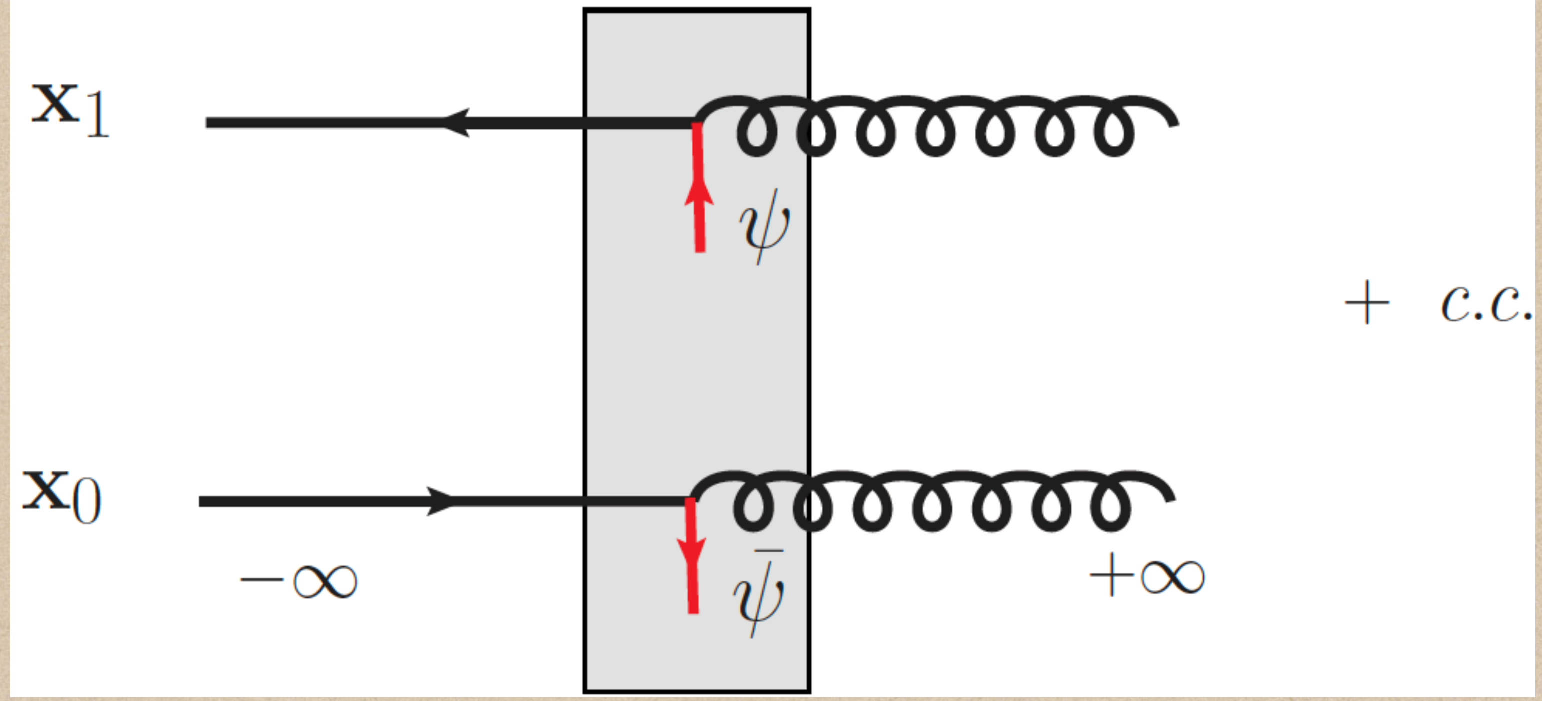
Polarized Wilson Line Correlators and Quark Helicity TMD at small x

Background quark field polarized Wilson line correlators.

$$\Delta D^q_{[1]}(p^+;\mathbf{x}_0,\mathbf{x}_1) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U^\dagger_{\mathbf{x}_0} U^{q[1]}_{\mathbf{x}_1}(p^+)\right]\right\rangle$$

$$\Delta D^q_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_1) = \frac{1}{2(N_c^2-1)}\frac{g^2}{2p^+}\int_{-\infty}^{+\infty} dw^+ dw'^+\, U^{ce}_{\mathbf{x}_0}(+\infty, w'^+)\,\bar{\psi}_B(w'^+,\mathbf{x}_0)$$
$$\times \left[t^e V_{\mathbf{x}_0}(w'^+,-\infty)V^\dagger_{\mathbf{x}_1}(w^+,-\infty)t^d\right]\left[\frac{\gamma^-\gamma^5}{2}\right]\psi_B(w^+,\mathbf{x}_1)\,U^{cd}_{\mathbf{x}_1}(+\infty,w^+) + c.c.$$





Definition of Quark Helicity TMD $g^q_{1L}(x,\mathbf{k}^2) = \frac{1}{2V^+}\sum_{S_L} S_L \int dz^+ d^2\mathbf{z}\, dy^+ d^2\mathbf{y}\, e^{ixP^-(z^+-y^+)} e^{-i\mathbf{k}\cdot(\mathbf{z}-\mathbf{y})}\langle P,S_L|\bar{\Psi}(y)\mathcal{U}[y,z]\frac{\gamma^-\gamma^5}{2}\Psi(z)|P,S_L\rangle$

In the small x limit:

$$\Delta D^q_{[2]}(k^+;\mathbf{k}^2) = \frac{g^2}{4(N_c^2-1)}\left[g^q_{1L}(x,\mathbf{k}^2) + g^{\bar{q}}_{1L}(x,\mathbf{k}^2)\right]$$

Gauge link: $\mathcal{U}_{\mathbf{x}_0,\mathbf{x}_1} = V_{\mathbf{x}_0}(w'^+,+\infty)t^c V_{\mathbf{x}_0} V^\dagger_{\mathbf{x}_1} t^c V_{\mathbf{x}_1}(+\infty,w^+) = \frac{1}{2}\mathcal{U}^{[+]}_{\mathbf{x}_0,\mathbf{x}_1}\mathrm{tr}[V_{\mathbf{x}_0}V^\dagger_{\mathbf{x}_1}] - \frac{1}{2N_c}\mathcal{U}^{[-]}_{\mathbf{x}_0,\mathbf{x}_1}$

$A_{\rm LL}^{g}$ at small x : Results

In Momentum Space

Sub-Eikonal order:

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy} = \lambda\frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{k}}\left[\frac{2}{\mathbf{p}_1^2} - \frac{6\mathbf{p}_1\cdot(\mathbf{p}_1-\mathbf{k})}{\mathbf{p}_1^2|\mathbf{p}_1-\mathbf{k}|^2} + \frac{4}{|\mathbf{p}_1-\mathbf{k}|^2}\right]\Big(\Delta D_{\rm M}^{g}(p^+;\mathbf{k}^2) + \Delta D_{[1]}^{q}(p^+;\mathbf{k}^2)\Big)$$

$$+\lambda\frac{\alpha_s C_F}{\pi^2}\int_{\mathbf{k}}\left[\frac{1}{(\mathbf{p}_1-\mathbf{k})^2} - \frac{2\mathbf{p}_1\cdot(\mathbf{p}_1-\mathbf{k})}{\mathbf{p}_1^2|\mathbf{p}_1-\mathbf{k}|^2} + \frac{2}{\mathbf{p}_1^2}\right]\Delta D_{[2]}^{q}(p^+;\mathbf{k}^2)$$

$$+\lambda\frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{k}}\left[\frac{2(\mathbf{p}_1\times\mathbf{k})^2}{(\mathbf{p}_1-\mathbf{k})^2\mathbf{k}^2\mathbf{p}_1^2} - \frac{(\mathbf{p}_1-\mathbf{k})\cdot\mathbf{k}}{|\mathbf{p}_1-\mathbf{k}|^2\mathbf{k}^2}\right]\Delta D_{\rm E}^{g}(p^+;\mathbf{k}^2)\,.$$

Eikonal order:

$$\frac{d\sigma_0}{d^2\mathbf{p}_1 dy} = \frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{k}}\left[\frac{1}{\mathbf{p}_1^2} - \frac{2\mathbf{p}_1\cdot(\mathbf{p}_1-\mathbf{k})}{\mathbf{p}_1^2|\mathbf{p}_1-\mathbf{k}|^2} + \frac{1}{|\mathbf{p}_1-\mathbf{k}|^2}\right]D(\mathbf{k}^2)\,.$$

Longitudinal double-spin asymmetry for gluon production at mid-rapidity:

$$A_{\rm LL}^{g} = \frac{\frac{d\sigma_{\lambda=+1}}{d^2\mathbf{p}_1 dy} - \frac{d\sigma_{\lambda=-1}}{d^2\mathbf{p}_1 dy}}{2\frac{d\sigma_0}{d^2\mathbf{p}_1 dy}}$$

A_{LL}^g at Small x in the Limit $\mathbf{p}_1 \to +\infty$

Large transverse momentum limit

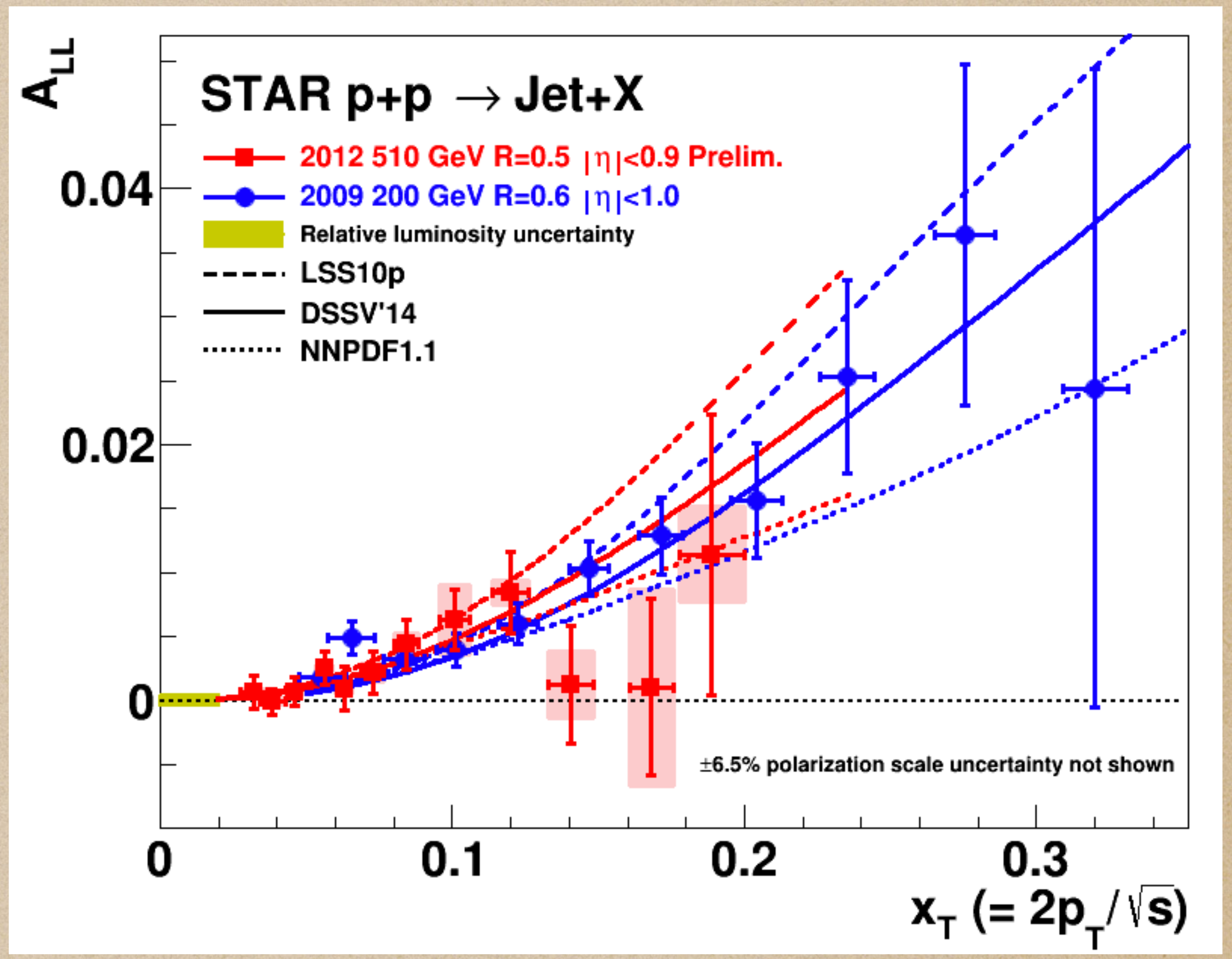
$$\left.\frac{d\sigma_\lambda}{d\mathbf{p}_1 dy}\right|_{\mathbf{p}_1\to+\infty} = \lambda\frac{\alpha_s C_A}{\pi^2}\frac{1}{\mathbf{p}_1^2}\Delta D_{\rm E}^g(x,Q^2) + \lambda\frac{\alpha_s C_F}{\pi^2}\frac{1}{\mathbf{p}_1^2}\Delta D_{[2]}^q(x,Q^2) + \mathcal{O}(\frac{1}{\mathbf{p}_1^4})$$

Reproducing collinear factorization.

Babcock, Monsay and Sivers (1979),

$$\left.\frac{d\sigma_0}{d^2\mathbf{p}_1 dy}\right|_{\mathbf{p}_1\to+\infty} = \frac{\alpha_s C_A}{\pi^2}\frac{1}{\mathbf{p}_1^4}\int_{\mathbf{k}}\mathbf{k}^2 D(\mathbf{k}^2) + \mathcal{O}(\frac{1}{\mathbf{p}_1^6})$$

$$A_{\rm LL}^g\Big|_{\mathbf{p}_1\to+\infty} = \frac{2g_{1L}^g(x,Q^2) + \frac{C_F}{C_A}[g_{1L}^q(x,Q^2) + g_{1L}^{\bar{q}}(x,Q^2)]}{xG(x,Q^2)}\mathbf{p}_1^2 + \mathcal{O}(|\mathbf{p}_1|^0)$$



Parabolic behavior in the large transverse momentum limit with the coefficient determined by the ratio of gluon and quark helicity PDFs over gluon PDF.

Conclusion

- **We developed an effective Hamiltonian approach to study small-x physics beyond Eikonal approximation with a focus on spin-related processes.**
- **For the first time, we calculated A_{LL}^{g} at small x for gluon production at mid-rapidity in a transverse momentum dependent framework. We found four polarized Wilson line correlators which are related to the small x limit of quark and gluon helicity TMDs.**
- **At the large momentum limit, the transverse momentum dependence is parabolic with the coefficient determined by quark and gluon helicity PDFs and unpolarized gluon PDF.**
- **At the low momentum limit, all four polarized dipole correlators contribute comparably and transverse momentum dependent framework beyond the collinear factorization formalism has to be used to extract gluon helicity distribution at small x.**

Backup: A_{LL}^{g} at small x

In Coordinate Space

Sub-Eikonal order:

$$\frac{d\sigma_\lambda}{d^2\mathbf{p}_1 dy} = \lambda\frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}}\frac{1}{(2\pi)^2}\frac{\mathbf{x}_{1'0}\cdot\mathbf{x}_{10}}{|\mathbf{x}_{1'0}|^2|\mathbf{x}_{10}|^2}\Big[2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_0,\mathbf{x}_0) - 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_0,\mathbf{x}_1) - 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_0,\mathbf{x}_1')$$

$$-\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1,\mathbf{x}_0) - \Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1',\mathbf{x}_0) + 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1',\mathbf{x}_1) + 2\Delta D^{gq}_{\mathrm{M}[1]}(p^+;\mathbf{x}_1,\mathbf{x}_1')\Big]$$

$$+\lambda\frac{\alpha_s C_F}{\pi^2}\int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}}\frac{1}{(2\pi)^2}\frac{\mathbf{x}_{10}\cdot\mathbf{x}_{1'0}}{|\mathbf{x}_{10}|^2|\mathbf{x}_{1'0}|^2}\Big[\Delta D^{q}_{[2]}(p^+;\mathbf{x}_1',\mathbf{x}_1) - \Delta D^{q}_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_1) - \Delta D^{q}_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_1') + 2\Delta D^{q}_{[2]}(p^+;\mathbf{x}_0,\mathbf{x}_0)\Big]$$

$$+\lambda\frac{\alpha_s C_A}{\pi^2}\left(\delta^{kj} - \frac{2\mathbf{p}_1^k\mathbf{p}_1^j}{\mathbf{p}_1^2}\right)\int_{\mathbf{x}_0,\mathbf{x}_1} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{10}}\frac{1}{2\pi}\frac{\epsilon^{kl}\mathbf{x}_{10}^l}{|\mathbf{x}_{10}|^2}\Big[4\Delta D^{g,j}_{\mathrm{E}}(p^+;\mathbf{x}_0,\mathbf{x}_1)\Big].$$

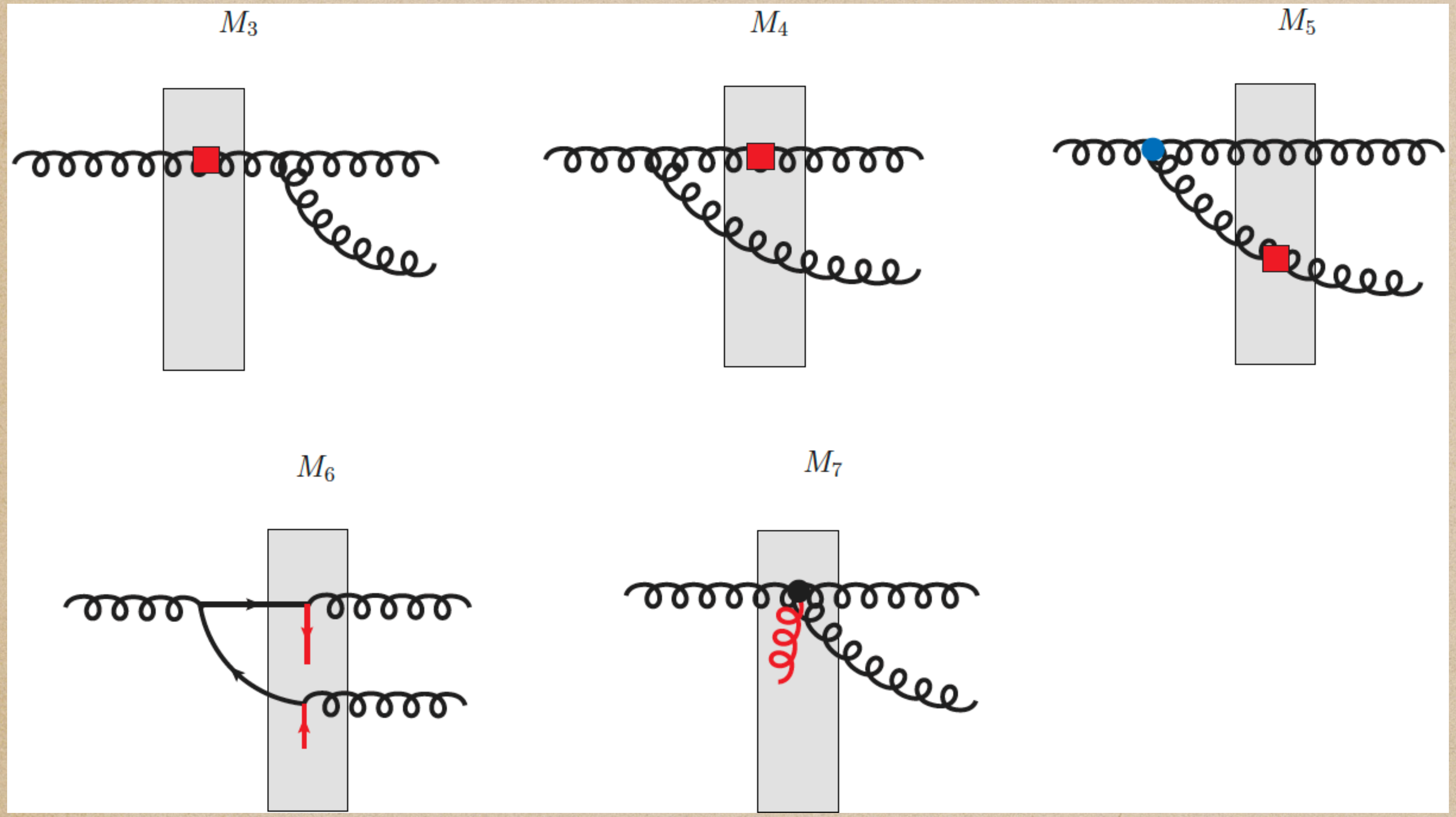
Eikonal order:

$$\frac{d\sigma_0}{d^2\mathbf{p}_1 dy} = \frac{\alpha_s C_A}{\pi^2}\int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_1'} e^{-i\mathbf{p}_1\cdot\mathbf{x}_{11'}}\frac{1}{(2\pi)^2}\frac{\mathbf{x}_{10}\cdot\mathbf{x}_{1'0}}{|\mathbf{x}_{10}|^2|\mathbf{x}_{1'0}|^2}\Big[D(\mathbf{x}_0,\mathbf{x}_0) - D(\mathbf{x}_0,\mathbf{x}_1) - D(\mathbf{x}_0,\mathbf{x}_1') + D(\mathbf{x}_1,\mathbf{x}_1')\Big]$$

Backup: Gluon Radiation Inside the Shockwave

The channel $g \to g + g$

Sub-eikonal Order ξ^1



PHYSICAL REVIEW D VOLUME 19, NUMBER 5 1 MARCH 1979

Quantum-chromodynamic predictions for inclusive spin-spin asymmetries at large transverse momentum

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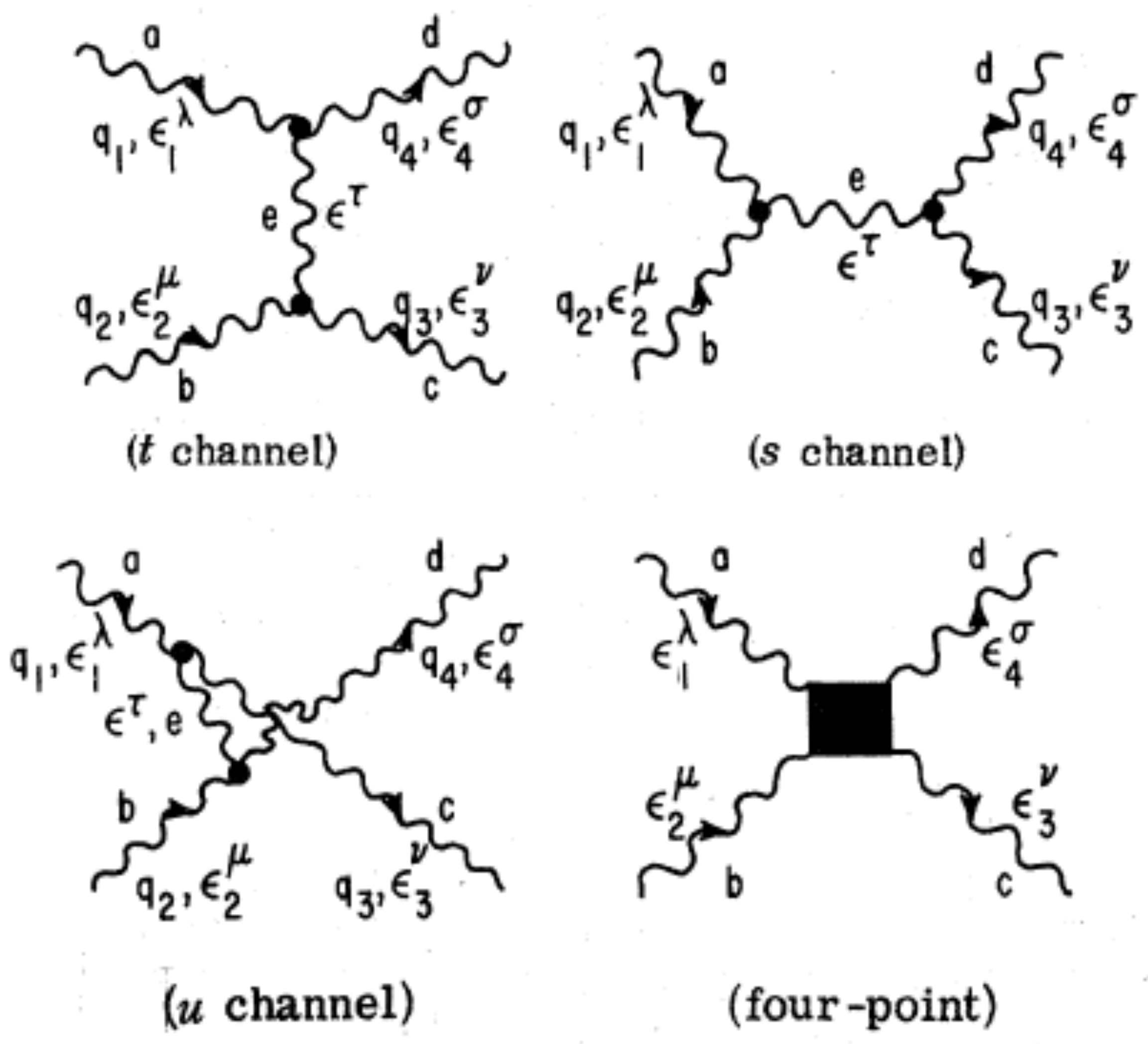
We discuss predictions for the asymmetry A_{LL} in the inclusive production at large p_T of charged and neutral pions by longitudinally polarized protons. We work in the framework of a hard-scattering model based on perturbative quantum chromodynamics. Various assumptions for the distribution of the proton's spin among its constituents—quarks, ocean antiquarks, and gluons—and the effects of scaling violations on the parton distributions are considered.

F. The process $VV \to VV$

The diagrams for gluon-gluon scattering are



Backup: Corresponding Partonic Processes

The channels $g \to q + g, \quad g \to \bar{q} + g$

Babcock, Monsay and Sivers (1979),

Sub-eikonal Order $\xi^{\frac{1}{2}}$