Exploring quantum corrections to the initial condition for high energy evolution

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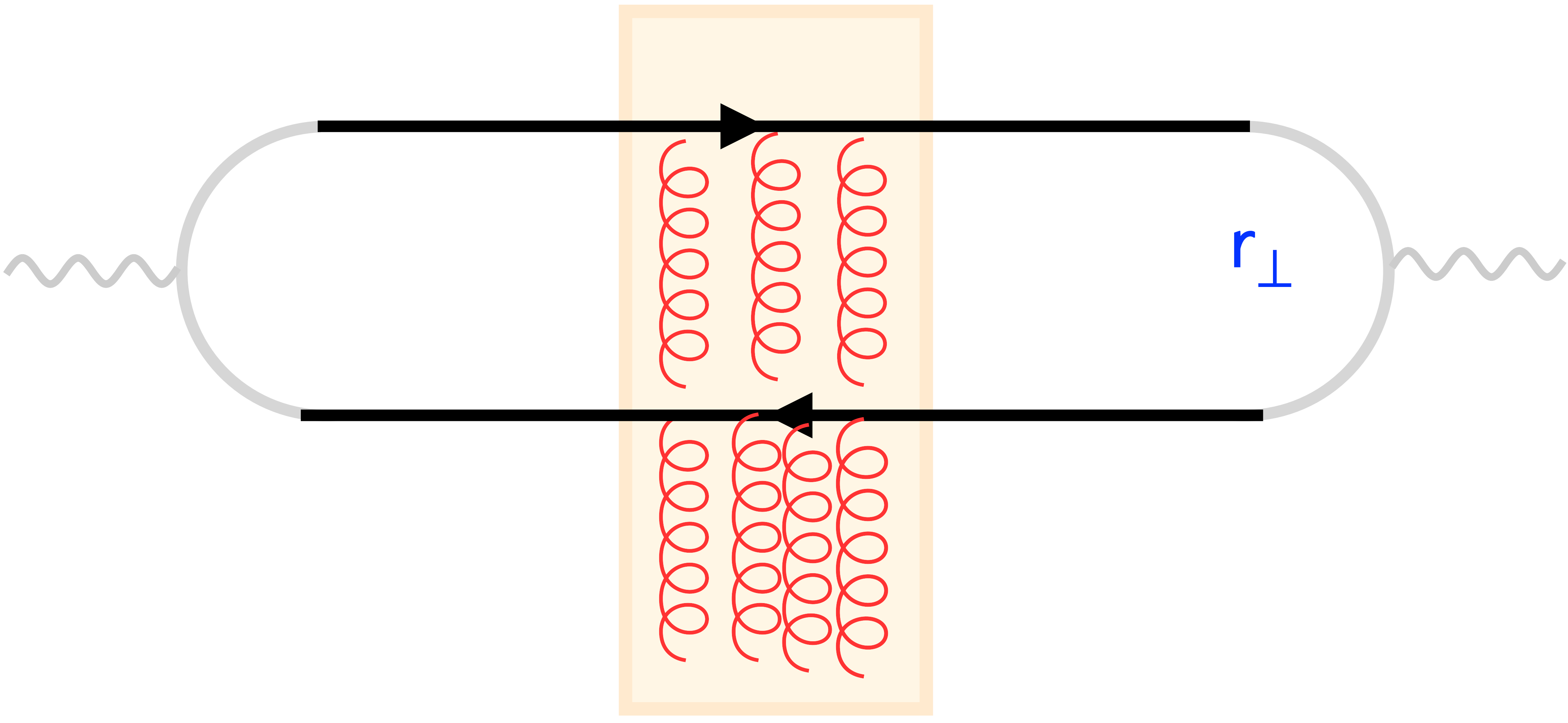
Initial condition of small x evolution



Nucleus/Proton

Dipole S-matrix

$$S_{GBW}(x_\perp) \simeq \exp\left[-\frac{1}{4}Q_s^2 r_\perp^2\right]$$

$$S_{MV}(x_\perp) \simeq \exp\left[-\frac{1}{4}Q_s^2 r_\perp^2 \ln\frac{1}{r_\perp \mu}\right]$$

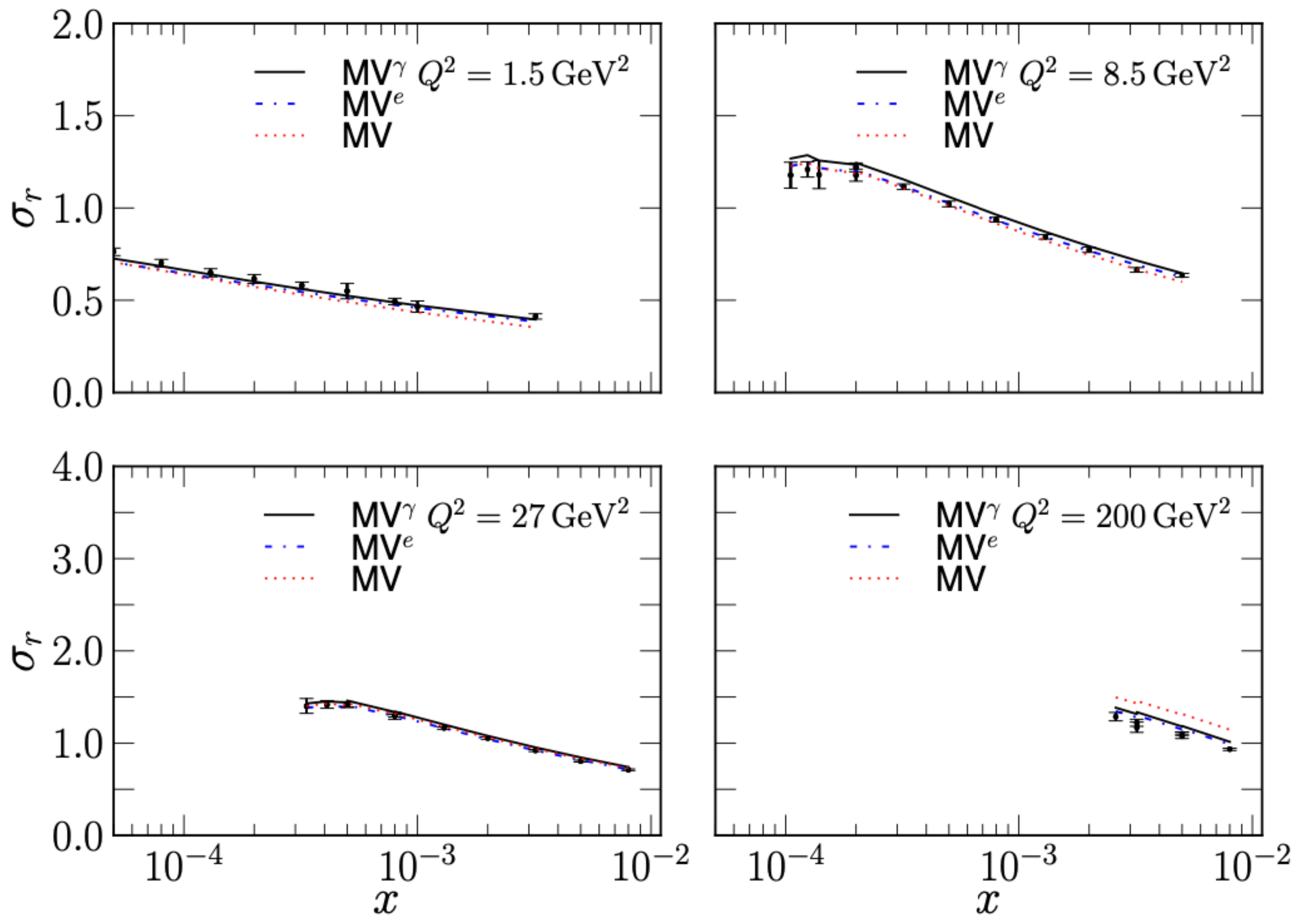
$$Q_s^2 = Q_0^2 A^{1/3}$$

Mueller, McLerran-Venugopalan (MV), Golec-Biernat-Wüsthoff (GBW)

Initial condition of small x evolution

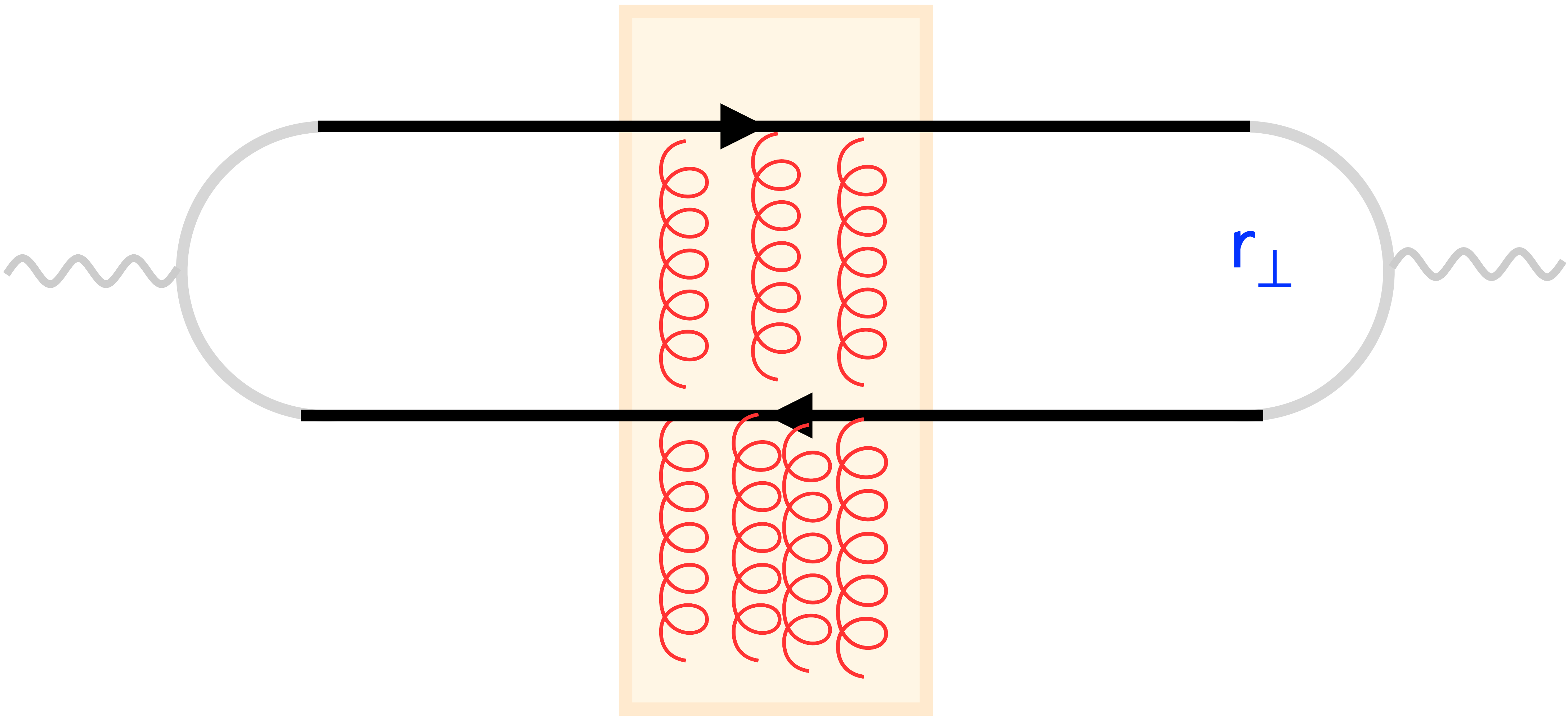
- Parametrization for the proton from HERA fits

$$S(x_\perp) \simeq \exp\left[-\frac{1}{4}(r_\perp^2 Q_{s0}^2)^\gamma \ln\left(\frac{1}{r_\perp \mu} + e\right)\right]$$



- E.g. Lappi, Mäntysaari (2013) , Albacete et al (2010)
- Ad hoc form: ok for the proton?
- This talk: first principles approach for large Nuclei. $A^{1/3} \gg 1$

Initial condition of small x evolution



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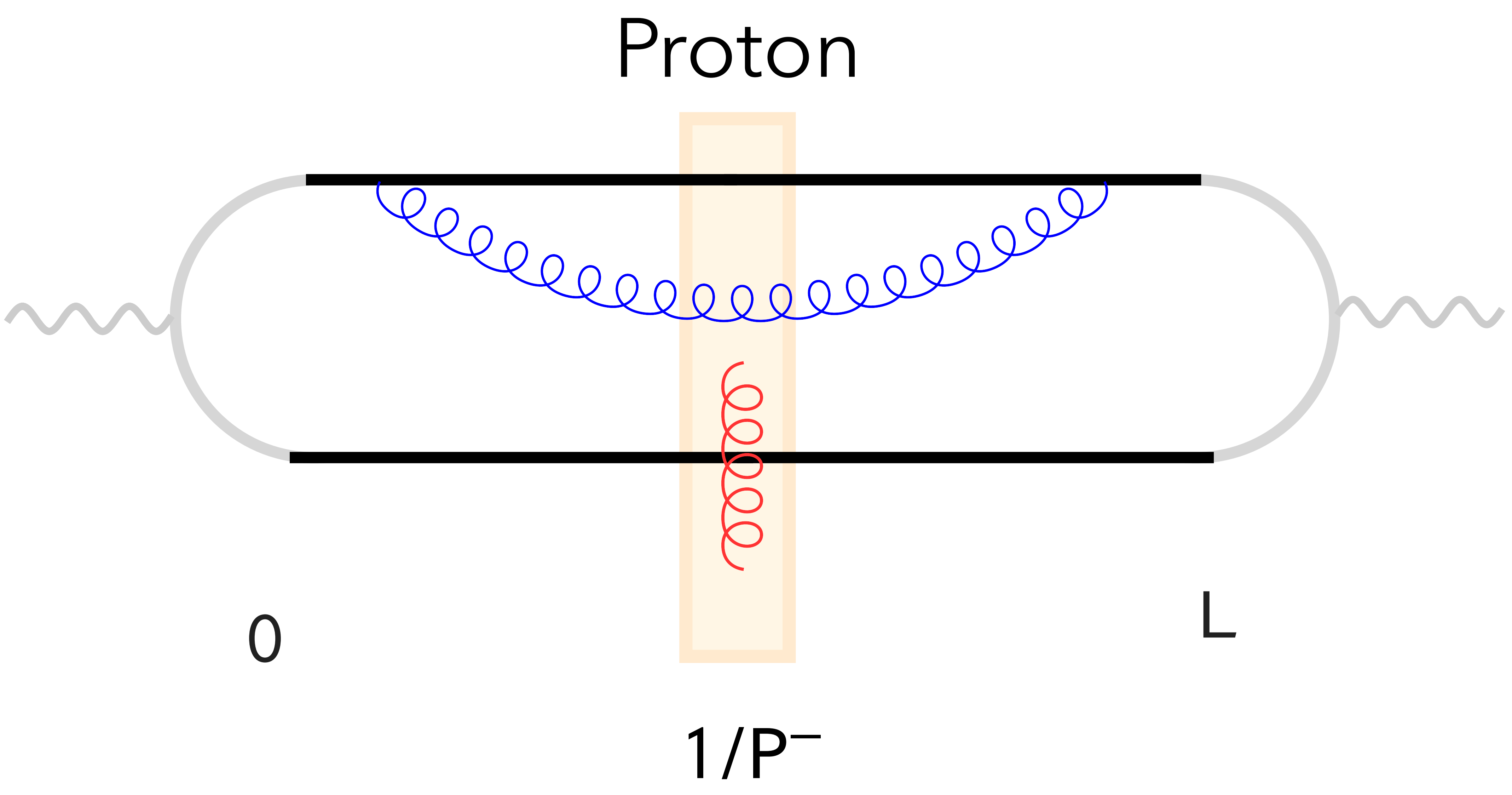
Quantum corrections of two kinds

- BK evolution: long-lived quantum fluctuations outside the target enhanced by $\ln(1/x_{BJ})$



$$\frac{1}{x_{Bj}P^-} \ll \frac{1}{k^-} \ll \frac{1}{P^-}$$

Target momentum:

$$P^- = \sqrt{s} \sim \gamma m_N$$

Formation time:

$$\tau \sim \frac{1}{k^-}$$

Quantum corrections of two kinds

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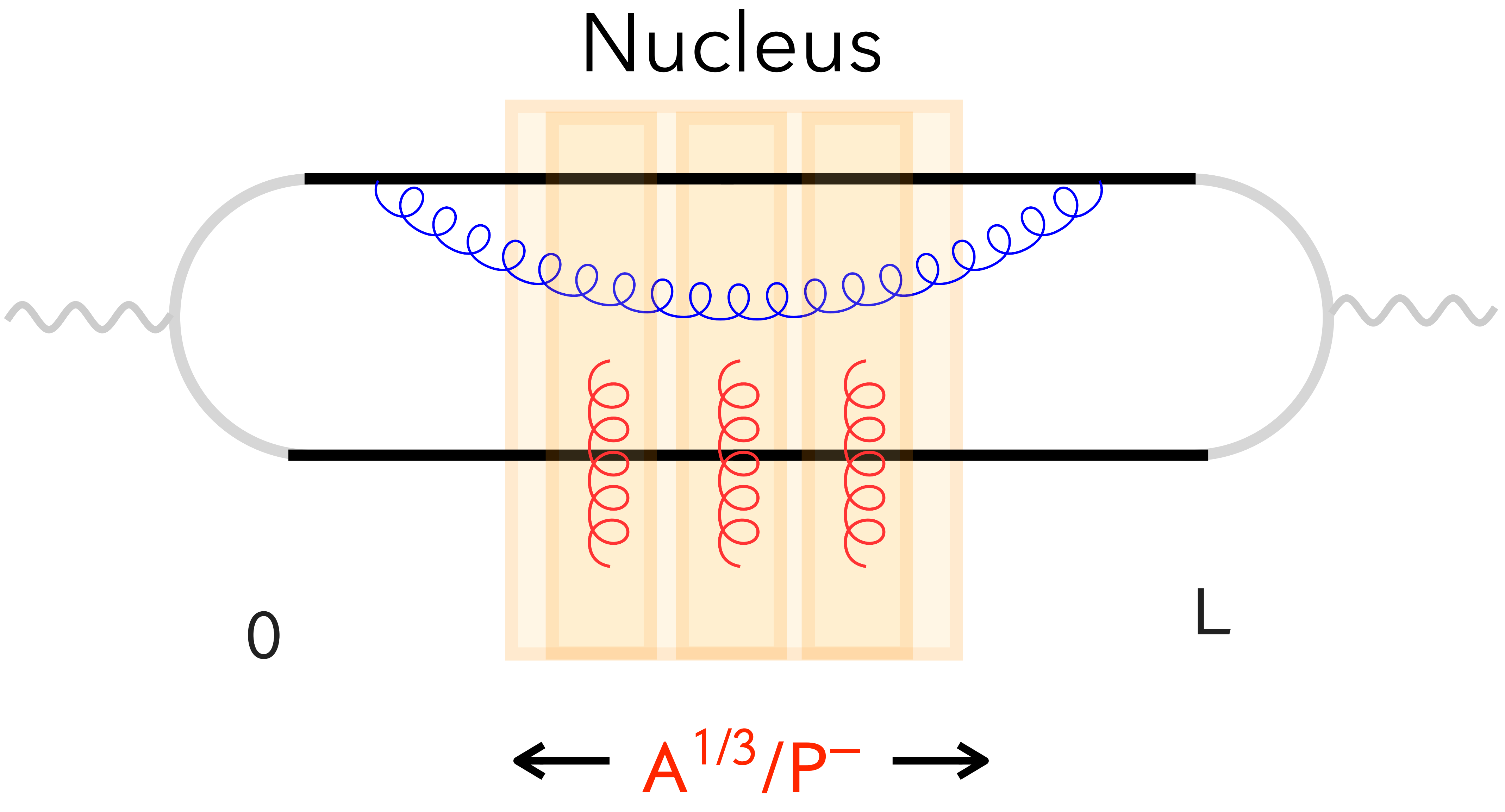


Target momentum:

$$P^- = \sqrt{s} \sim \gamma m_N$$

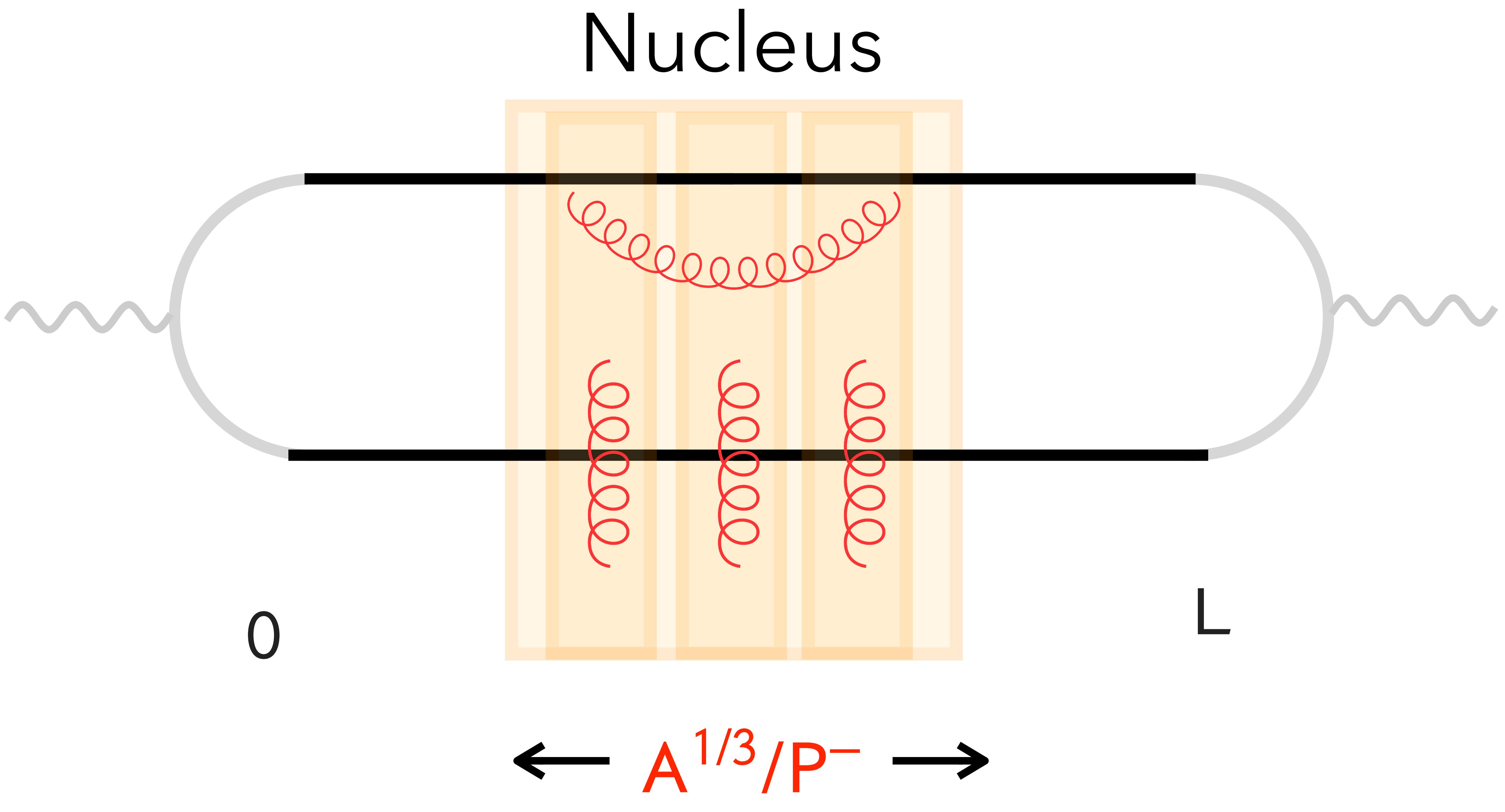
Formation time:

$$\tau \sim \frac{1}{k^-}$$

$$\frac{1}{x_{Bj}P^-} \ll \frac{1}{k^-} \ll \frac{A^{1/3}}{P^-} \ll \frac{1}{P^-}$$

Quantum corrections of two kinds

- Potentially large phase-space for soft radiation inside the target $\rightarrow \ln^2 A^{1/3}$



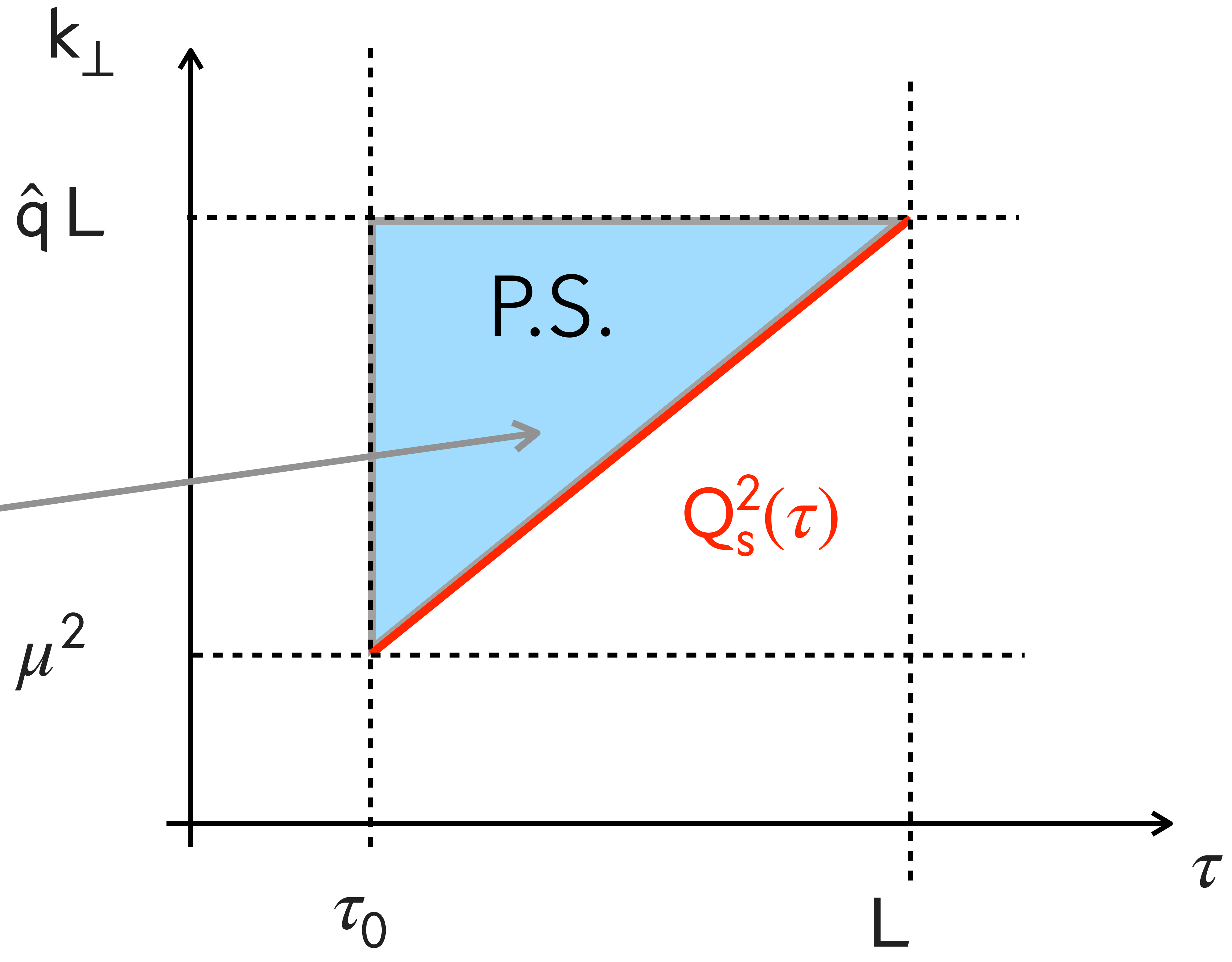
$P^- = \sqrt{s} \sim \gamma m_N$

$$\frac{1}{x_{Bj}P^-} \ll \frac{A^{1/3}}{P^-} \ll \frac{1}{k^-} \ll \frac{1}{P^-}$$

Double log enhancement and saturation

$$Q_s^2(A) = Q_0^2 A^{1/3} \left(1 + \frac{\bar{\alpha}}{2} \log^2 A^{1/3}\right)$$

$$\bar{\alpha} \int \frac{d\tau}{\tau} \int \frac{dk_\perp}{k_\perp}$$

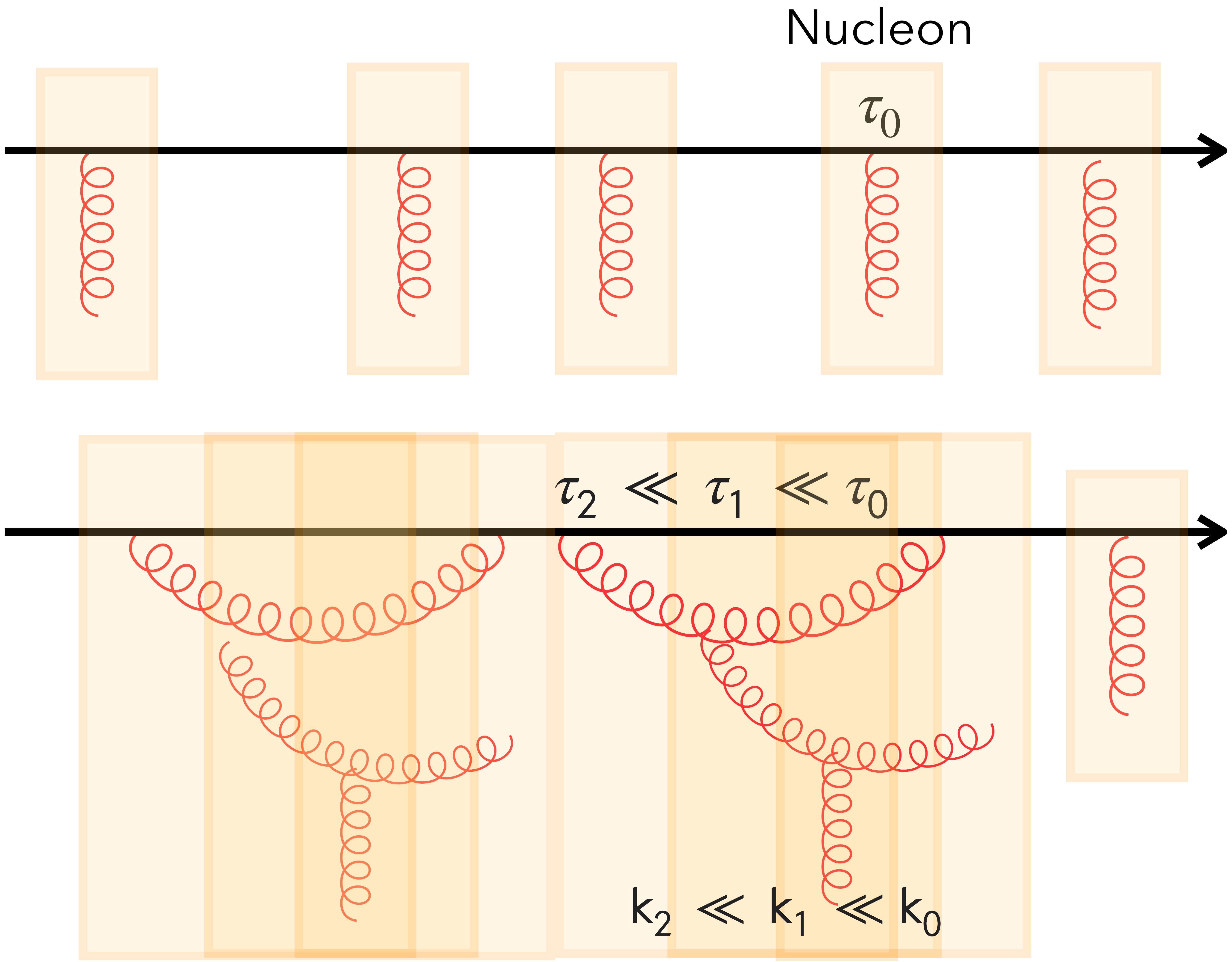


[Liou, Mueller, Wu (2013)
Blaizot, Dominguez, Iancu, MT (2014)]

- Saturation line $Q_s^2(\tau) \equiv \hat{q}(Q_s, \tau)\,\tau$: Multiple-scattering screen mass singularity
- Not the standard DGLAP double log: the factor 1/2 reflects the presence of multiple scattering constraint $\hat{q}\,\tau \ll k_\perp \ll \hat{q}\,L \sim A^{1/3}$

Quantum corrections to MV

- LO: local/instantaneous interactions
- DLA + saturation: quasi-local interactions
- Exponentiation of the double logs with adequate phase space constraints



Emissions strongly ordered in $k_\perp$ and τ

Anomalous diffusion

- Scaling solution for large L: $x = r_\perp^2 Q_s^2(L)$ (akin to geometric scaling at small x)
- From Gaussian to Stretched exponential:

$$S(r_\perp, L) \simeq \simeq \exp\left[-\frac{1}{4}(Q_s^{(0)} + \bar{\alpha}\, Q_s^{(1)} + O(\bar{\alpha}^2))\;\; r_\perp^2\right] \quad \rightarrow \quad \exp\left[-(r_\perp^2 Q_s^2(L))^{\gamma}\right]$$

- Saturation scale:

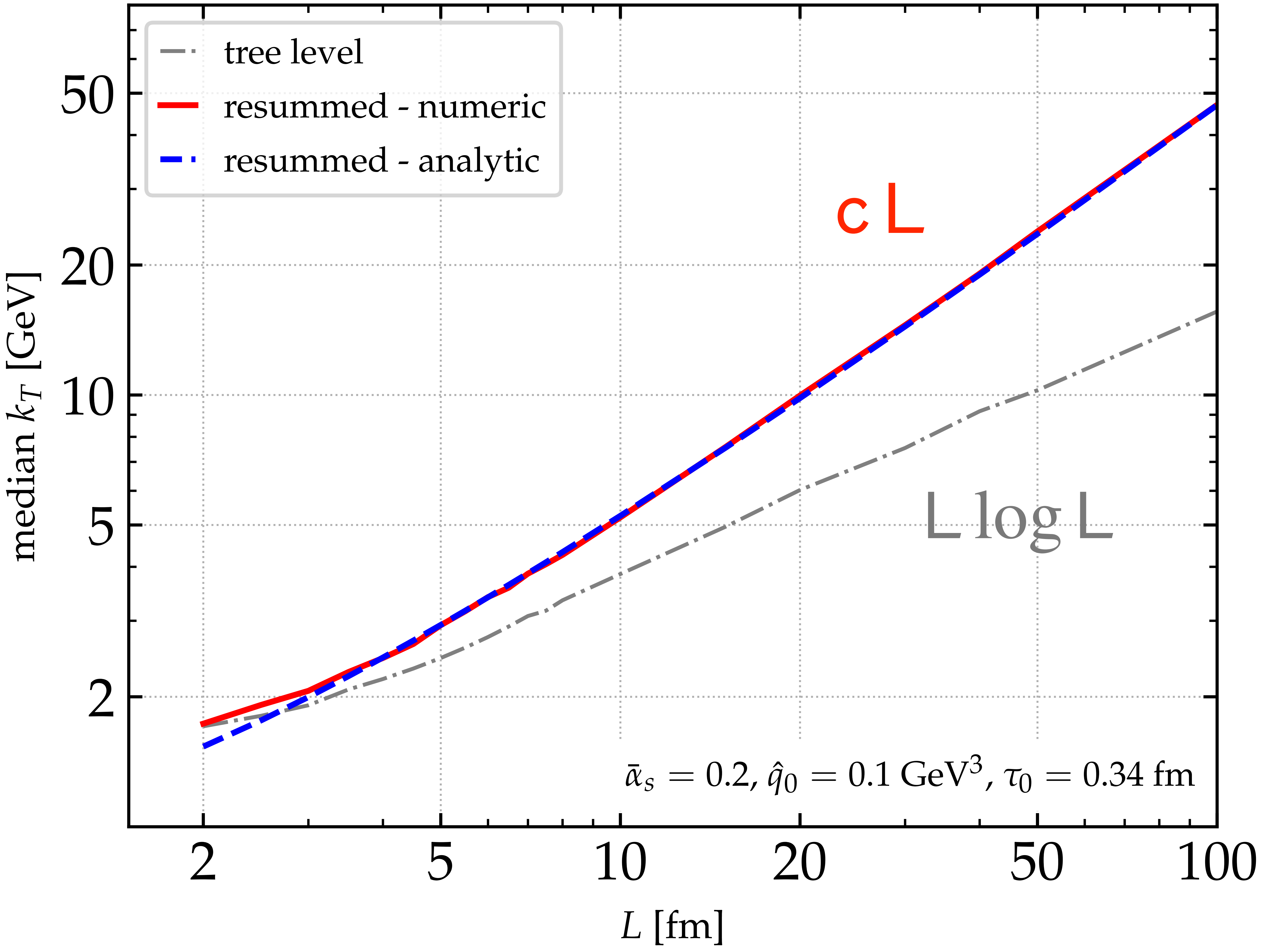
$$\langle k_\perp^2 \rangle_{typ} \sim Q_s^2(L) \sim L^c$$

- Anomalous dimension at DLA:

$$\gamma = 1 - 2\sqrt{\bar{\alpha}} \qquad\qquad c = 1 + 2\sqrt{\bar{\alpha}}$$

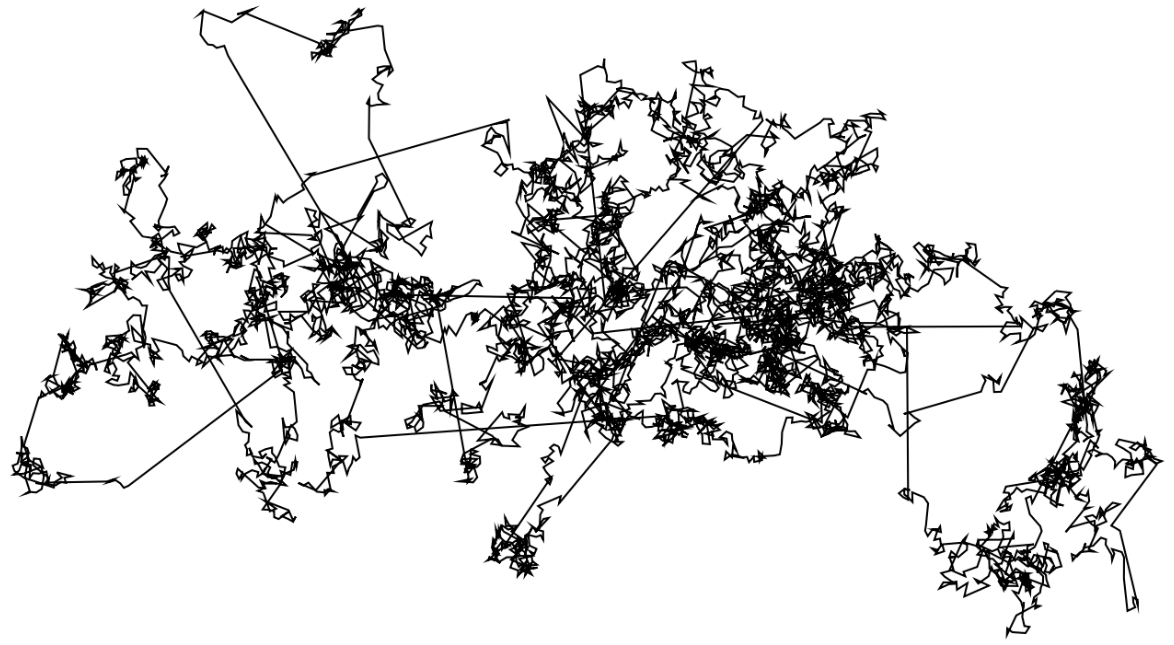
$L \sim A^{1/3}$ dependence of the saturation scale

$Q_s(L)$



Super diffusion

Normal diffusion



Brownian motion

c = 1

Lévy flight

c > 1

Universal pre-asymptotic solution at fixed coupling

YMT, P. Caucal 2109.12041 [hep-ph]

- Saturation scale : $\ln Q_s^2(Y) = cY + b\log Y + \text{const}\,.$ $\quad Y \equiv \ln A^{1/3} \qquad x \equiv \ln \frac{1}{Q_s^2 r_\perp^2}$

- Shape of the wave front $r_\perp < 1/Q_s$:

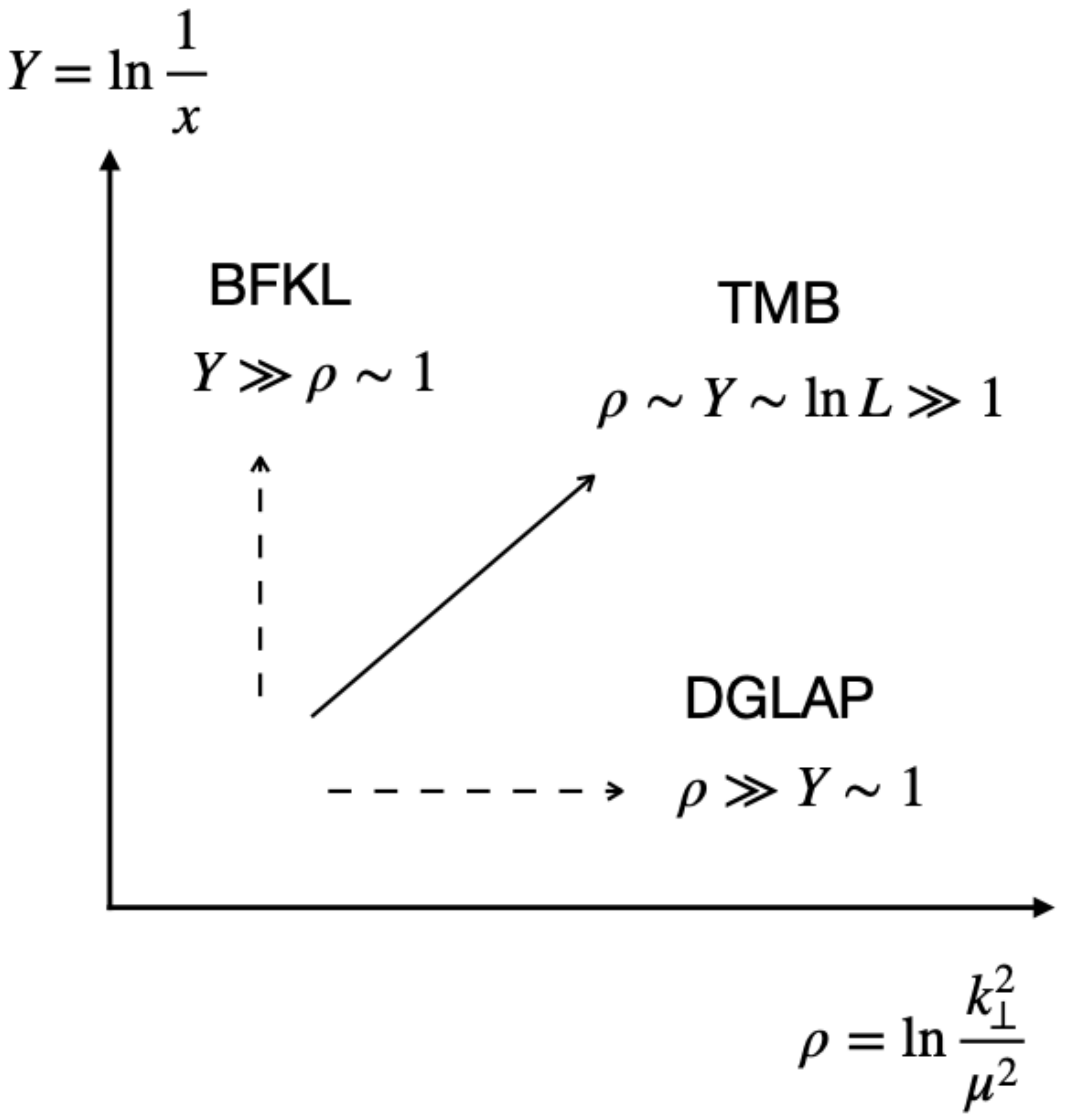
$$\ln S(r_\perp, A^{1/3}) = \frac{1}{4}\exp\left((1+\beta)x - \frac{\beta x^2}{4cY}\right)\left[1 + \beta x + \frac{bx}{c^2 Y}\left(1 + \frac{\beta(c+4)x}{6}\right) + O(Y^{-2})\right]$$

- Velocity of the wave front:

$$c \simeq 1 + 2\sqrt{\bar{\alpha}} \qquad \beta = \frac{c-1}{2c} \qquad b = -\frac{2}{3(1-\beta)}$$

Next-to-double logarithm (NDL) evolution

- Resummation of NDL from BFKL or DGLAP + saturation boundary



- Running coupling scaling variable

$$x \equiv \ln \frac{k_\perp^2}{Q_s^2} \quad \rightarrow \quad x \equiv \frac{\ln \frac{k_\perp^2}{Q_s^2}}{\sqrt{Y}} \sim \sqrt{\bar{\alpha}(Y)} \ln \frac{k_\perp^2}{Q_s^2}$$

NLL BFKL + saturation boundary

Caucal, MT, 2209.08900 [hep-ph]

- Solve linear NLL-BFKL imposing $\rho > \rho_s(Y) = \ln Q_s^2(Y)/\mu^2$ (non-linear effects)

$$\frac{\partial \hat{q}}{\partial Y} = \chi_{\rm LL}(\partial_\rho)\left[\bar{\alpha}_s(\rho)\hat{q}(\rho,Y)\right] + \bar{\alpha}_s^2(\rho)\tilde{\chi}_{\rm NLL}(\partial_\rho)\hat{q}(\rho,Y)$$

$$Y \equiv \ln\frac{L}{\tau_0}$$

$$\rho \equiv \ln\frac{1}{\mu^2 r_\perp^2}$$

[Mueller, Triantafyllopoulos (2002) (Iancu, Itakura, McLerran (2002) Munier, Peschanski (2003)]

- Expansion of BFKL kernel around DLA: $\gamma = 0$

$$\chi_{\rm LL}(\gamma) = \frac{1}{\gamma} + 2\zeta(3)\gamma^2 + \mathcal{O}(\gamma^4)\,, \qquad \tilde{\chi}_{\rm NLL} \sim B_g/\gamma^2 + a_{1,-1}/\gamma$$

Saturation scale $\rho_s(Y) = \ln Q_s^2(Y)/\mu^2$

$Y \equiv \ln A^{1/3}$

Caucal, MT, 2109.12041 [hep-ph] 2203.09407 [hep-ph]

$$\rho_s(Y) = Y + 2\sqrt{4b_0 Y} + 3\xi_1 (4b_0 Y)^{1/6} + \left(\frac{1}{4} - 2b_0\right)\ln(Y) + \kappa$$
$$+ \frac{7\xi_1^2}{180}\frac{1}{(4b_0 Y)^{1/6}} + \xi_1\left(\frac{5}{108} + 18b_0\right)\frac{1}{(4b_0 Y)^{1/3}} + b_0\left(1 - 8b_0\right)\frac{\ln(Y)}{\sqrt{4b_0 Y}} + \mathcal{O}\left(Y^{-1/2}\right)$$

First four terms conjectured by Iancu and Triantafyllopoulos (2015)

- Non universal terms start at order $Y^{-1/2}$ as can be seen by the substitution $Y \to Y + Y_0$
- NNLO BFKL and beyond do not contribute to the universal terms

Early hints in electron-Nucleus data

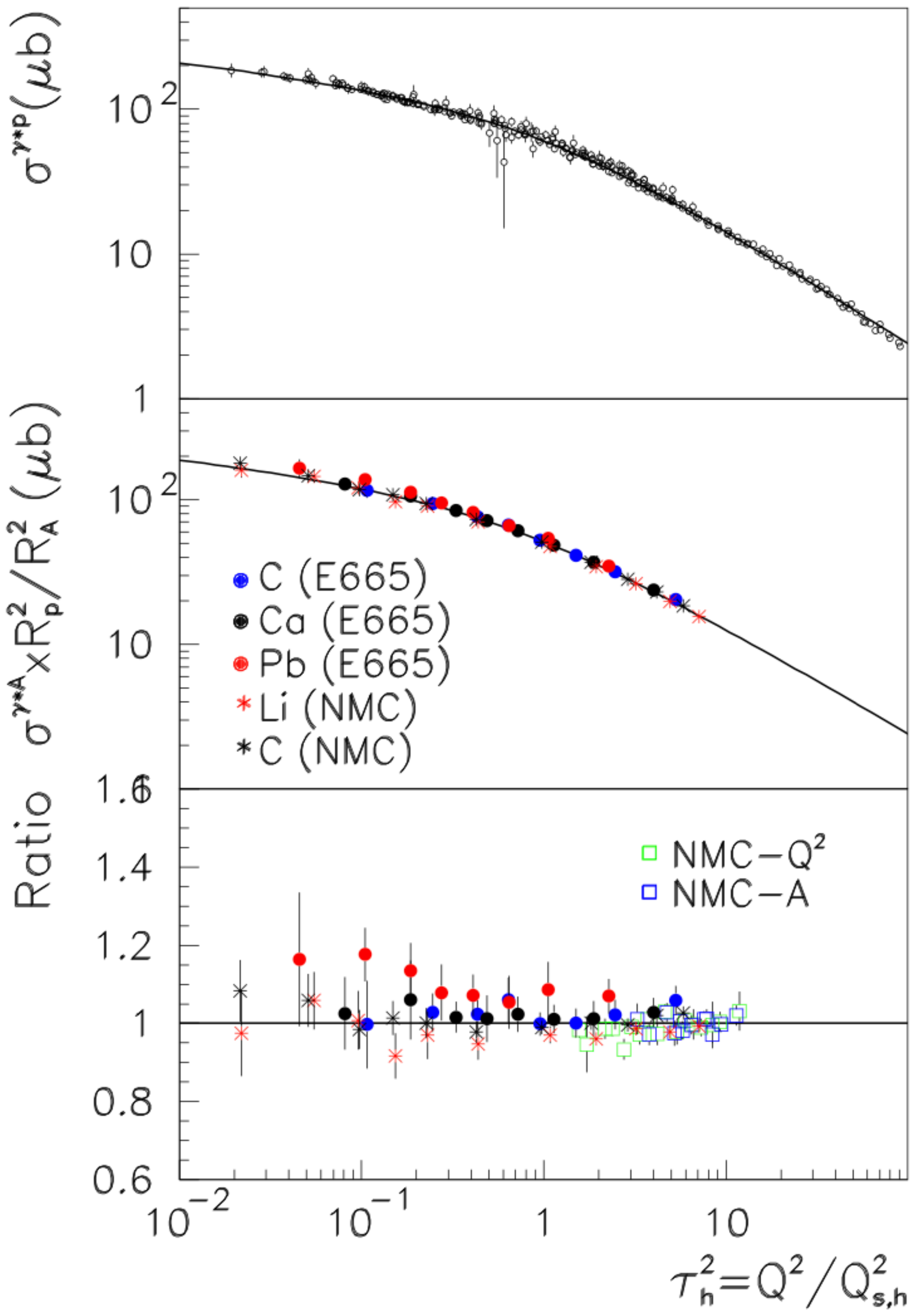
- Geometric scaling (GS) $\sigma(Q^2/Q_s^2(A))$ see in data
  Sasto, Golec-Biernat, Kwiecinski (2001)
- Phenomenological study of GS favors an anomalous scaling of

$$Q_s^2(A) \sim A^{4/9}$$

Armesto, Salgado, Wiedemann (2005)

- First principles: nonlocal gluon fluctuation inside the nucleus

  Liou, Mueller, Wu, Blaizot, Iancu, MT, Dominguez (2013)



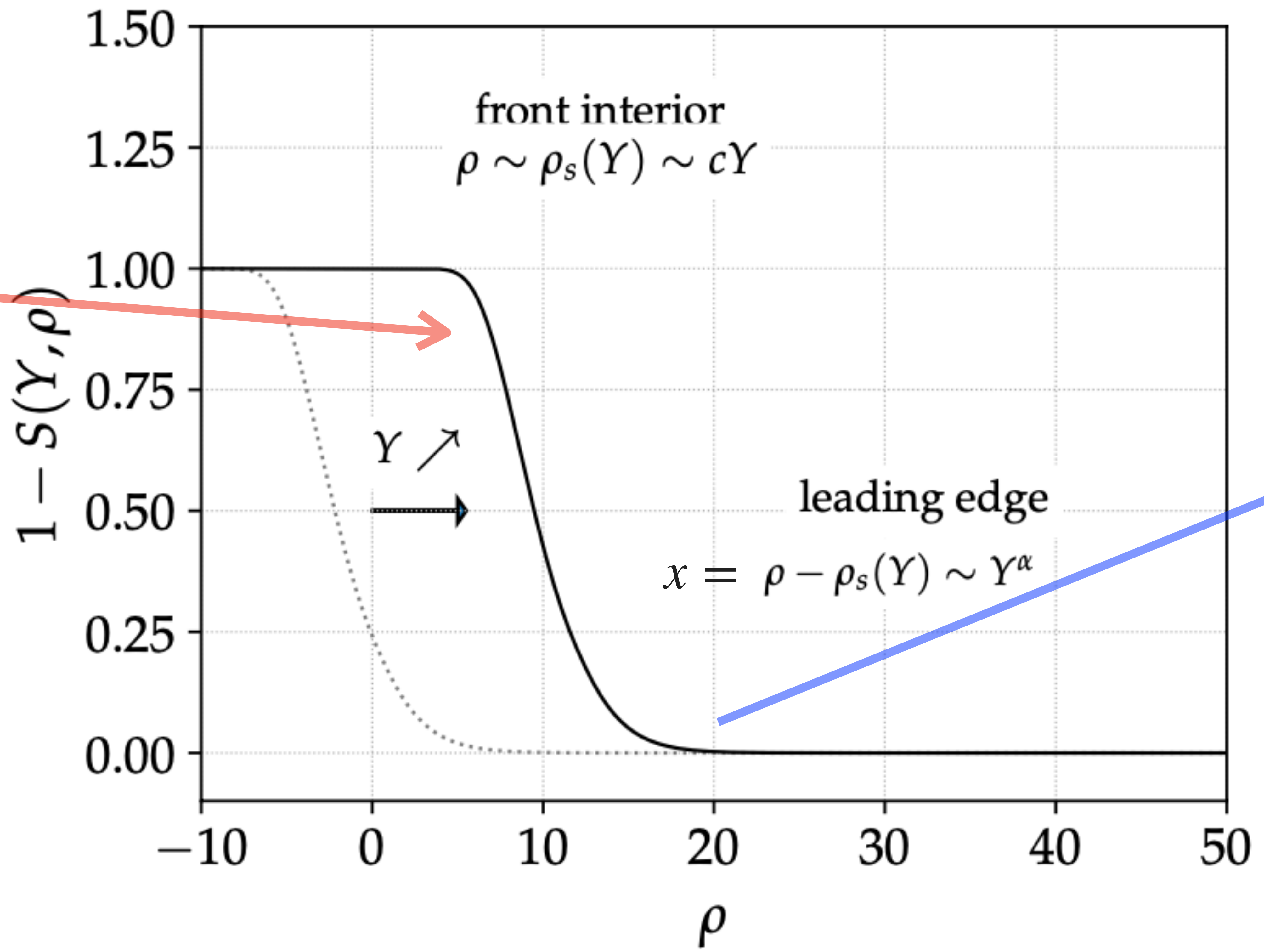
Conclusion

- Potentially large quantum corrections to the initial condition for small x evolution
- Another saturation effect: Anomalous scaling of the saturation scale: $Q_s^2(A) \sim A^{1/3(1+\gamma)}$
- Non-Gaussian initial condition: The dipole S-matrix described by a stretched exponential
- Work in progress: numerical analysis of BK equation (Initial State WG)

Backup

Front interior and leading edge expansions

Front interior expansion (Saturation line)



Leading edge: growth of perturbations around the unstable state $S = 0$ (diffusion of the wave front)

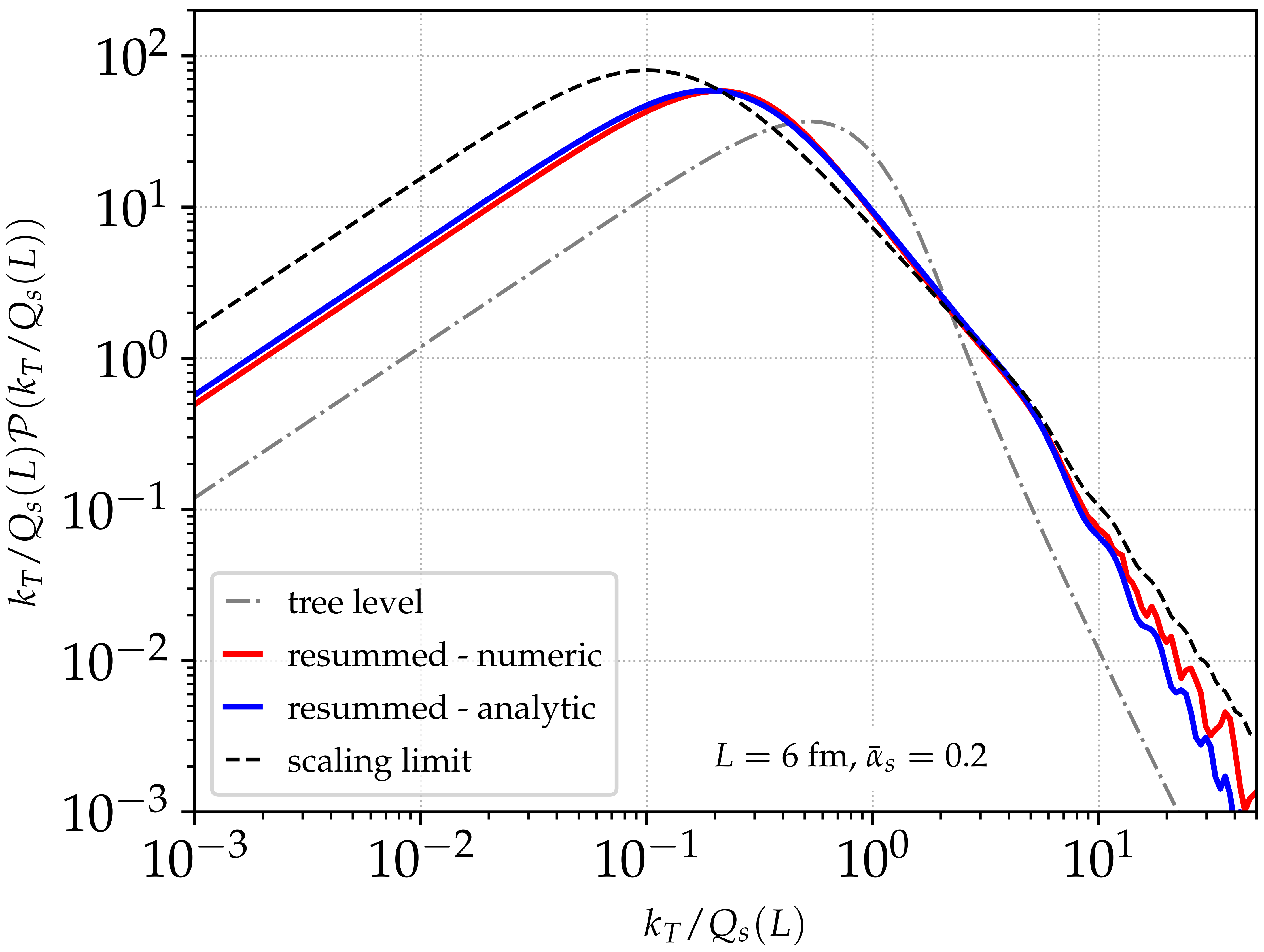
U. Ebert and W. van Saarloos (2000)

$$\frac{\hat{q}(Y,k_\perp)L}{Q_s^2} = e^{\beta x}\left[f_0(x) + \frac{1}{Y^{1/2}}f_1(x) + \frac{1}{Y}f_2(x) + \ldots\right]$$

$$\frac{\hat{q}(Y,k_\perp)L}{Q_s^2} = e^{\beta x}\left[Y^{1/2}G_1\left(\frac{x}{Y^{1/2}}\right) + G_0\left(\frac{x}{Y^{1/2}}\right) + \frac{1}{Y^{1/2}}G_{-1}\left(\frac{x}{Y^{1/2}}\right) + \ldots\right]$$

Boundary conditions at $\rho = \ln k_\perp^2/\mu^2 \to \infty$ constrain the saturation line $Q_s(Y)$

Analytic vs numerics



⇝ wider distribution due to heavy Lévy tail

⇝ Universal pre-asymptotic solution provides a good description of numerical simulations for $L = 6\,\mathrm{fm}$ and $\bar{\alpha} = 0.2$

Renormalization of $\hat{q}$

- DL resummed to all orders in the saturation scale $Q_s^2(\tau) \sim \hat{q}(\tau)\tau$

$$\frac{\partial}{\partial \ln \tau}\hat{q}(k_\perp, \tau) = \bar{\alpha}\int_{Q_s^2(\tau)}^{k_\perp^2} \frac{dk_\perp'^2}{k_\perp'^2}\hat{q}(k_\perp', \tau)$$

[Blaizot, MT (2014), Iancu (2014)]

Liou, Mueller, Wu (2013)
Iancu, Triantafyllopoulos (2015)

- Linear solution $\rightarrow$ $Q_s^2(\tau) \sim \hat{q}_0 \tau$

$$Q_s^2(L) \equiv \hat{q}(Q_s(L), L)\, L = \hat{q}_0 \frac{I_0\left(2\sqrt{\bar{\alpha}}Y\right)}{\sqrt{\bar{\alpha}}Y} \simeq L^{2\sqrt{\bar{\alpha}}}$$

$$Y \equiv \log \frac{L}{\tau_0} \sim \ln A^{1/3}$$