

# SURGE Collaboration Meeting and Workshop

Hosted by Brookhaven National Laboratory  
June 28–30, 2023



## eHIJING updates: final-state interactions in $e-A$

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**Weiyao Ke** (Los Alamos National Laboratory)

June 30, 2023 (remote)

[1] WK, Y. Zhang, H. Xing, and X.-N. Wang 2304.10779

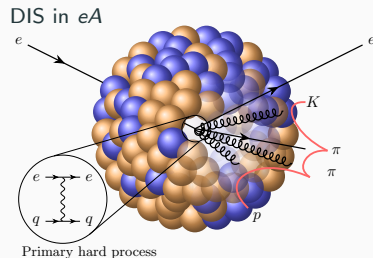
# Nuclear effects in deep inelastic scattering

**Nuclear non-perturbative input:** Nuclear structures, nuclear parton distribution [e.g. extract transverse momentum dependent PDF & FF, Alrashed, Anderle, Kang, Terry, Xing PRL129(2022)242001]

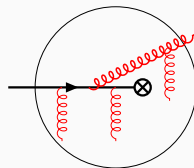
## Dynamical effects:

- Dynamical shadowing, in-medium parton shower, hadronic interactions, target dynamics, ...
- Dynamical medium effects can be process dependent.

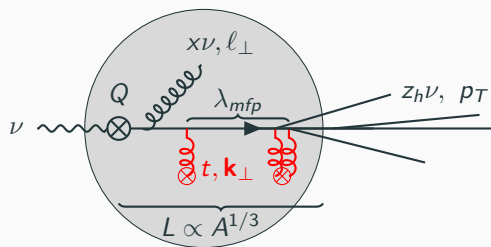
**Understanding dynamical effects is critical to define what are the NP inputs.**



## Drell-Yan in $pA$



# The DIS limit of $e$ - $A$ collisions



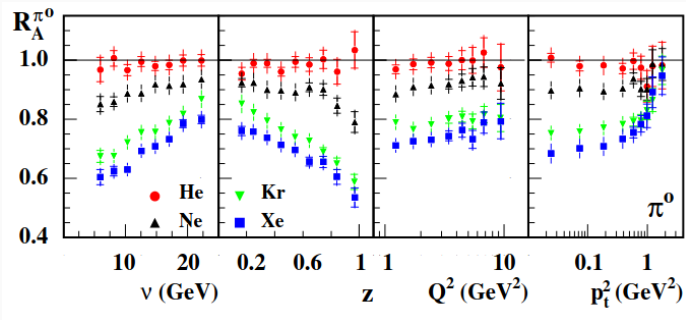
- DIS: asymptotically large  $Q^2$  and fixed  $x_B$ .
- Localized hard production in the nucleus

$$\Delta r_{\perp} \sim \frac{1}{Q} \ll r_0 A^{1/3},$$

$$\Delta r^+ \sim \frac{\nu}{Q^2} = \frac{1}{2x_B m_p} \approx \frac{0.1}{x_B} \text{ fm} < r_0 A^{1/3}.$$

- Meaningful to talk about final-state interactions after the hard process with a given path length  $L$ :  $\frac{d\sigma_{eA \rightarrow h}}{dx_B dQ^2 dz_h} = \frac{2\pi\alpha_e^2}{Q^4} \sum_{i,j} f_{i/A} \otimes e_q^2 H_{ij} \otimes (d_{h/j} + \Delta d_{h/j})$ .
- Partonic final-state interactions mediated by Glauber gluons  $k^+ k^- \ll k_{\perp}^2 \Leftrightarrow$  transport properties ( $\hat{q} = \frac{d\langle p_T^2 \rangle}{dL}$ ) of cold nuclear matter (CNM).

# Sizeable modifications to fragmentation observed at EMC, HERMES, CLAS



$$R_A^h = \frac{N_{eA \rightarrow \pi^0}(z_h, p_T^2; \nu, Q^2)}{N_{ed \rightarrow \pi^0}(z_h, p_T^2; \nu, Q^2)}$$

$$N_{eX \rightarrow h} = \frac{d\sigma_{eX \rightarrow h}}{d\nu dQ^2 dz_h dp_T^2} / \frac{d\sigma_{eX}}{d\nu dQ^2}$$

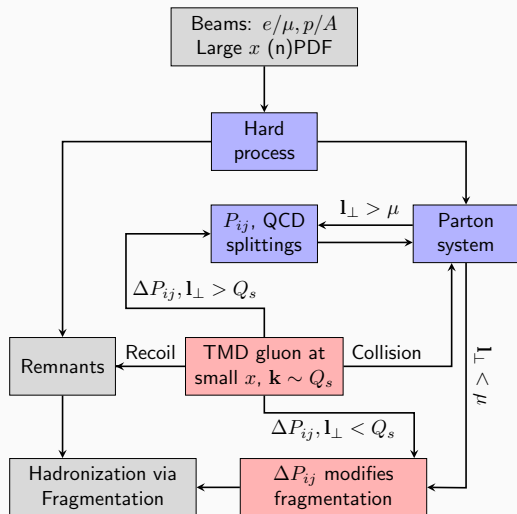
◁ HERMES NPB780(2007)1-27

EMC ZPC52(1991)1-11

CLAS PRC105(2022)015201

**eHIJING** (electron-Heavy-Ion-Jet-Interaction-Generator) aims to model the development of parton shower in the cold nuclear matter.

# eHIJING (electron-Heavy-Ion-Jet-Interaction-Generator)



A Monte Carlo model for jet shower in  $e$ - $A$  (DIS limit):

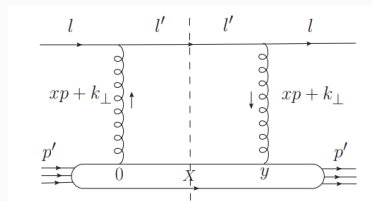
- $e$ - $p$  (and hard event in  $e$ - $A$ ) generation from Pythia8 [T. Sjöstrand et al, Comput.Phys.Commun.191(2015)159].
- $e$ - $A$  (including changes to Pythia8):
  - 1) multiple scatterings
  - 2) modified splitting functions
  - 3) parton shower in CNM
  - 4) hadronization

# Modelling jet-medium interactions

- Glauber gluon carries a small nucleon momentum fraction  $x_g = \frac{k_\perp^2}{Q^2} x_B$ .

Relate jet-medium cross section to the unintegrated gluon distribution  $\phi_g(x, \mathbf{k}_\perp^2)$  (or TMD distribution  $\phi_g/(4\pi x)$ ) [J. Casalderrey-Solana, X.-N. Wang, PRC77(2008)024902]

$$\frac{d\sigma_G}{d\mathbf{k}_\perp^2} = \frac{C_R}{d_A} \frac{\alpha_s \phi_g(x, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2}$$



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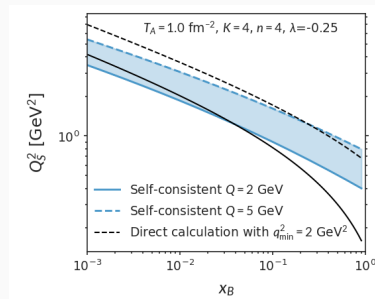
$$\frac{d\sigma_G}{d\mathbf{k}_{\perp}^2} = \frac{C_R}{d_A} \frac{\alpha_s \phi_g(x, \mathbf{k}_{\perp}^2)}{\mathbf{k}_{\perp}^2}$$

- A saturation-motivated parametrization for  $\phi_g(x, \mathbf{k}_{\perp}^2)$

$$\alpha_s \phi_g(x, \mathbf{k}_{\perp}^2) = K \frac{(1-x)^n x^{\lambda}}{\mathbf{k}_{\perp}^2 + Q_s^2(x_B, Q^2)}$$

$Q_s$  calculated self-consistently [Y-Y Zhang, X-N Wang PRD105(2022)034015; A. Mueller NPB558(1999)285-303]

$$Q_s^2(x_B, Q^2) = \rho_N L \frac{C_A}{d_A} \int_{\Lambda^2}^{Q^2/x_B} \alpha_s \phi_g\left(\frac{x_B \mathbf{k}_{\perp}^2}{Q^2}, \mathbf{k}_{\perp}^2\right) d^2 \mathbf{k}_{\perp}$$



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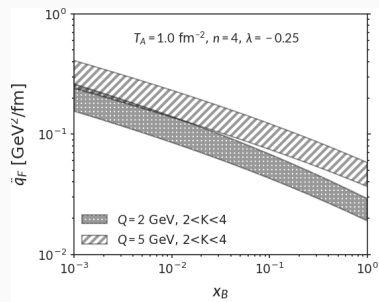
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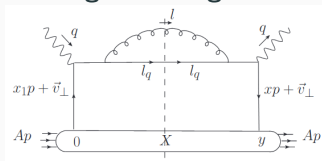
The resulting transport parameter in CNM  $\hat{q}_R = \frac{C_R}{C_A} \frac{Q_s^2}{L}$



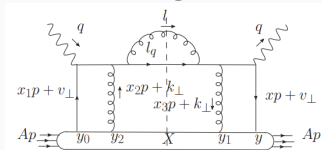
# Medium-modified QCD splitting functions: $q \rightarrow q$

Theory inputs: (generalized) twist-four calculation of medium-induced radiative correction [Y-Y Zhang, X-N Wang PRD105(2022)034015].

**Leading twist, e.g.**



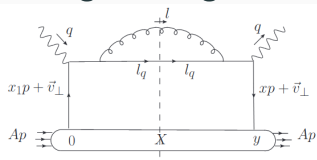
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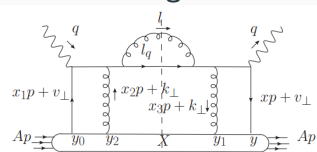
## Leading twist, e.g.



$$\frac{d\sigma_{eA}^D}{dx_B dQ^2 dz d^2l_{\perp} d^2l_{q\perp}} = \frac{2\pi\alpha_{em}^2}{Q^4} \sum_q e_q^2 [1 + (1 - \frac{Q^2}{x_{BS}})^2] \frac{\alpha_s}{2\pi} \frac{1+z^2}{1-z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_{\perp}}{(2\pi)^2} \int d^2b_{\perp} dy_0^- dy_1^- \times \rho_A(y_0^-, b_{\perp}) \rho_A(y_1^-, b_{\perp}) q_N(x_B, \vec{v}_{\perp}, b_{\perp}) \frac{\phi_N(x_G, \vec{k}_{\perp})}{k_{\perp}^2} [\mathcal{N}_g^{qLPM} + \mathcal{N}_g^{gLPM} + \mathcal{N}_g^{nonLPM}]. \quad (19)$$

$$\mathcal{N}_g^{qLPM} = \frac{1}{N_c} \left( \frac{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp}] \cdot [\vec{l}_{\perp} - (1-z)(\vec{l}_{\perp} + \vec{l}_{q\perp})]}{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp}]^2 [\vec{l}_{\perp} - (1-z)(\vec{l}_{\perp} + \vec{l}_{q\perp})]^2} - \frac{1}{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp}]^2} \right) \times (1 - \cos[(x_L + x_E - x_F)p^+(y_1^- - y_0^-)]), \quad \text{Suppressed by } 1/N_c^2 \quad (20)$$

## Twist four, e.g.



$$\mathcal{N}_g^{gLPM} = C_A \left( \frac{2}{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp} - \vec{k}_{\perp}]^2} - \frac{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp} - \vec{k}_{\perp}] \cdot [\vec{l}_{\perp} - (1-z)(\vec{l}_{\perp} + \vec{l}_{q\perp})]}{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp} - \vec{k}_{\perp}]^2 [\vec{l}_{\perp} - (1-z)(\vec{l}_{\perp} + \vec{l}_{q\perp})]^2} \right. \\ \left. - \frac{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp}] \cdot [\vec{l}_{\perp} - (1-z)\vec{v}_{\perp} - \vec{k}_{\perp}]}{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp}]^2 [\vec{l}_{\perp} - (1-z)\vec{v}_{\perp} - \vec{k}_{\perp}]^2} \right) \times (1 - \cos[(x_L + \frac{z}{1-z}x_D + x_S - x_F)p^+(y_1^- - y_0^-)]), \quad \text{Main contributions, medium enhanced} \quad (21)$$

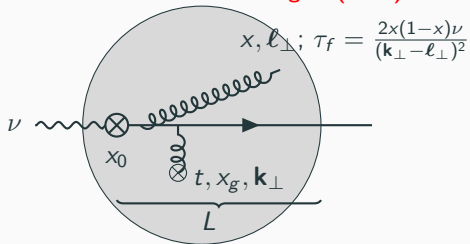
$$\mathcal{N}_g^{nonLPM} = C_F \left( \frac{1}{[\vec{l}_{\perp} - (1-z)(\vec{l}_{\perp} + \vec{l}_{q\perp})]^2} - \frac{1}{[\vec{l}_{\perp} - (1-z)\vec{v}_{\perp}]^2} \right), \quad (22)$$

Power suppressed by UV cut off, not enhanced by medium

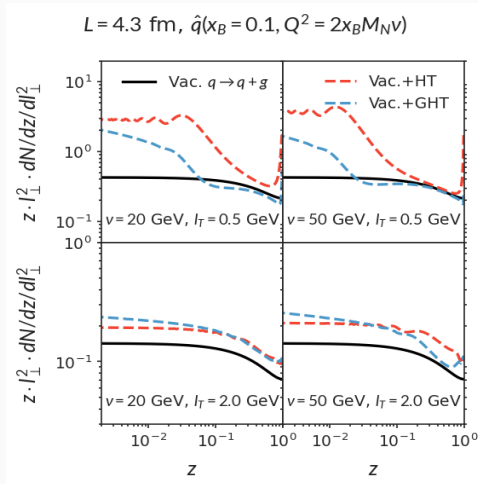
# The soft gluon emission approximation

$$\frac{dN_{gq}^{(1)}}{dx d^2\ell_{\perp}} = \frac{\alpha_s C_F}{2\pi^2} \frac{P_{qq}(x)}{\ell_{\perp}^2} \int d^2\mathbf{k}_{\perp} \frac{C_A}{d_A} \frac{\alpha_s \phi_g(\mathbf{k}_{\perp}^2)}{\mathbf{k}_{\perp}^2} \frac{2\mathbf{k}_{\perp} \cdot \ell_{\perp}}{(\ell_{\perp} - \mathbf{k}_{\perp})^2} \int_0^L dt \rho_N(x_0 + t) \left[ 1 - \cos \frac{t}{\tau_f} \right]$$

The Landau-Pomeranchuk-Migdal (LPM) effect

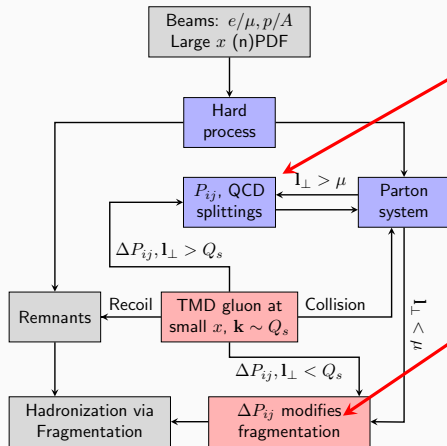


- “Higher-Twist” (HT): in the past, one expands it in  $1/\ell_{\perp}^2$  before  $\mathbf{k}_{\perp}$  integration.
- ★ Generalized Higher-Twist (GHT): one should perform twist expansion after  $\mathbf{k}_{\perp}$  integration.



# Multiple emissions

Multiple emissions follow the modified DGLAP evolution [Chang, Deng, Wang PRC89(2014)034911].



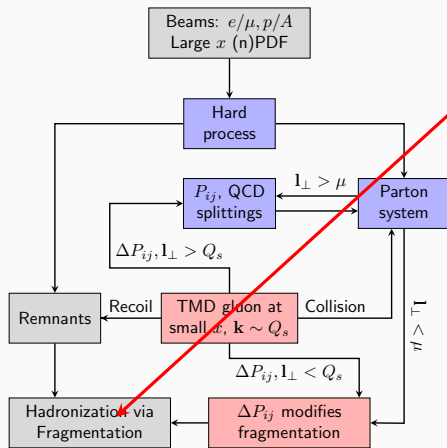
- At high virtuality, eHIJING modifies vacuum splitting functions in Pythia8's  $p_T$ -ordered shower,  $(\ell_\perp)_{\min} < (\ell_\perp)_n < \dots < (\ell_\perp)_1 < Q/2$

$$\frac{dP}{dx d^2 \ell_\perp} = \frac{\alpha_s}{2\pi^2} \frac{1}{\ell_\perp^2} P^{(0)}(x) + \frac{dN^{(1)}}{dx d^2 \ell_\perp} \Theta(\ell_\perp^2 - Q_s^2)$$

- At low virtuality, multiple emissions ordered in formation time  $1/Q_s \sim \tau_{f,1} < \dots < \tau_{f,n} < \frac{p}{\Lambda^2}$

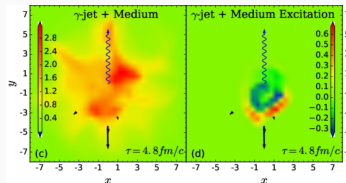
$$\frac{dP}{dx d^2 \ell_\perp} = \frac{dN^{(1)}}{dx d^2 \ell_\perp} \Theta(Q_s^2 - \ell_\perp^2)$$

# Hadronization



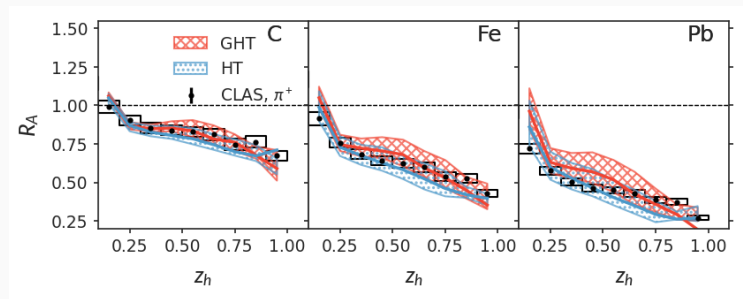
- Shower + remnants hadronization from Lund string model.
- Remnants from multiple collisions: quark + diquark. For future: soft energy-momentum deposition & medium excitation?

Similar practices in heavy-ion collisions.



[Jet wake in HIC, Wei Chen et al PLB777(2018)86]

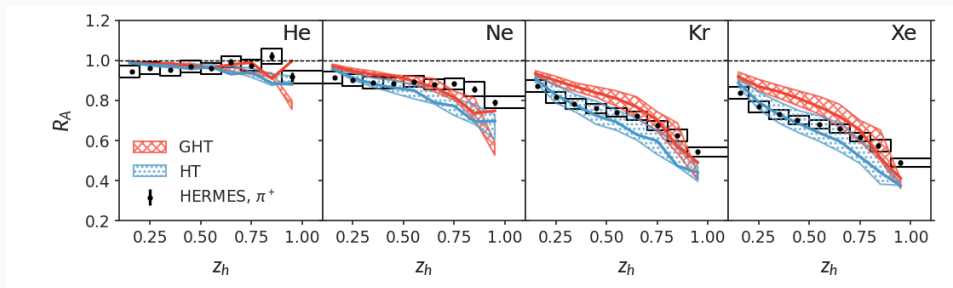
# Test of medium-modified hadronization at CLAS and HERMES



[CLAS PRC105(2022)015201]

- $R_A = [N_h(\nu, Q^2; z_h, p_t)/N_\gamma]_{eA} / [N_h(\nu, Q^2; z_h, p_t)/N_\gamma]_{ed}$ .
- Higher Twist (HT, blue) and Generalized Higher Twist (GHT, red) using the same range of  $\hat{q}_F(x_B = 0.1, Q^2 = 2.25 \text{ GeV}^2) \in [0.027, 0.06] \text{ GeV}^2/\text{fm}$ .

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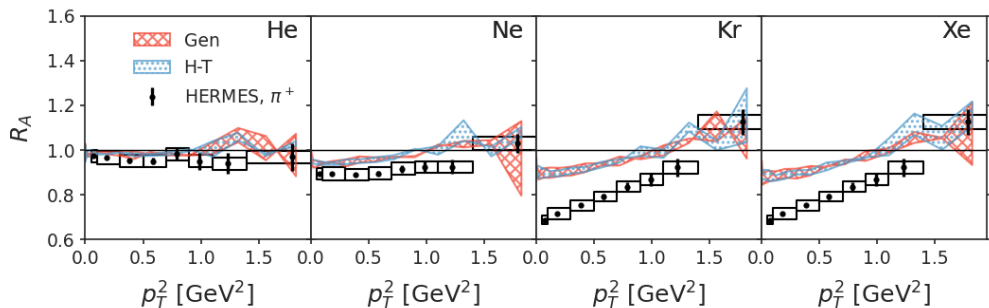


[HERMES, NPB 780, 24 (2007)]

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# Collisional & radiative contribution to momentum broadening

Collisional broadening of the parton shower



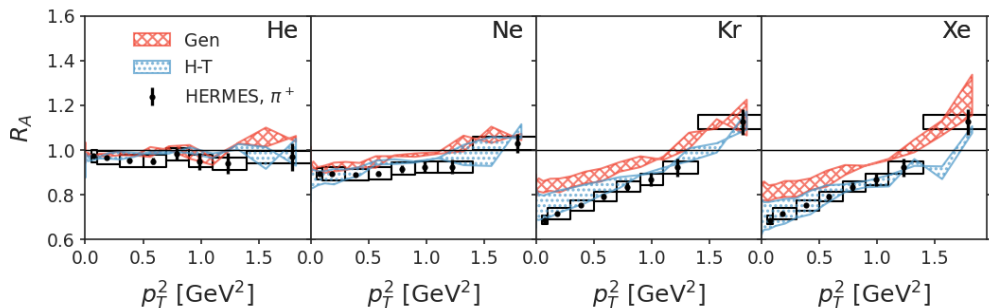
[HERMES, NPB 780, 24 (2007)]

For the same range of  $\hat{q}$  that describes  $R_A(z_h)$ , the pure collisional processes only account for part of the momentum broadening.



# Collisional & radiative contribution to momentum broadening

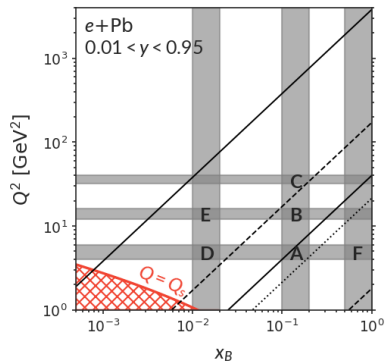
Broadening from both collisions & induced radiations



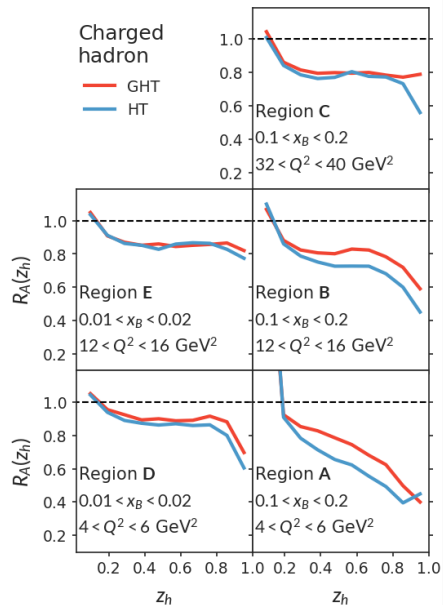
[HERMES, NPB 780, 24 (2007)]

Broadening due to medium-induced radiation is important in large nuclei!

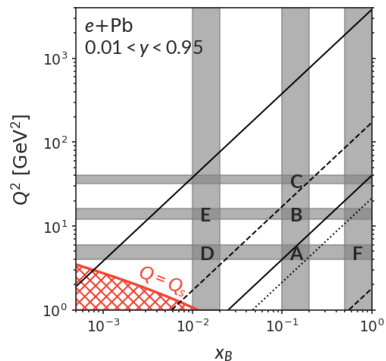
# From fixed target to collider energies



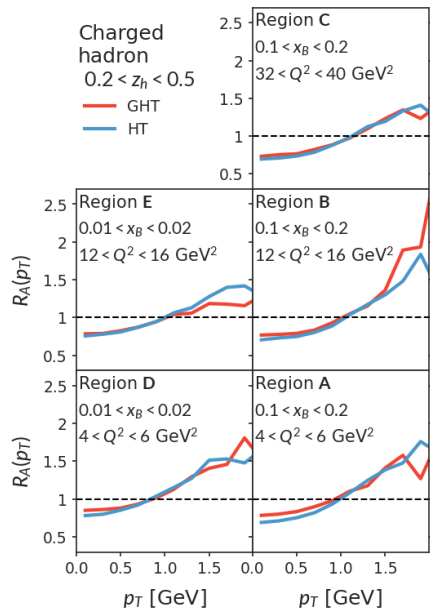
- Coverage from EIC, EicC to CLAS-12.
- Scan regions with  $Q^2 \gg Q_s^2$ ,  $x_B > 0.01$ .



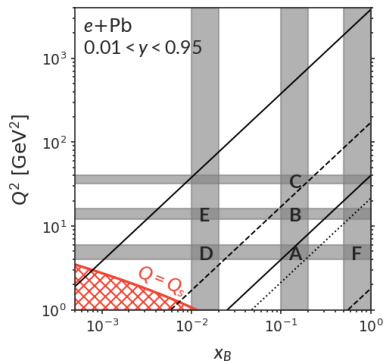
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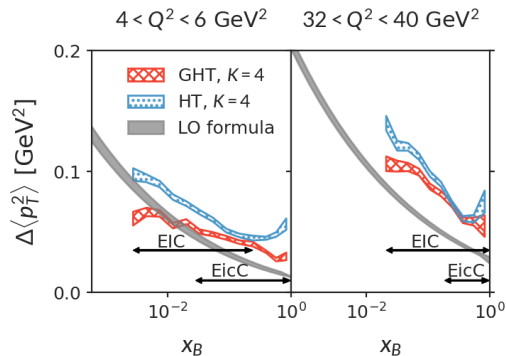
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# From fixed target to collider energies



- Coverage from EIC, EicC to CLAS-12.
- Scan regions with  $Q^2 \gg Q_s^2$ ,  $x_B > 0.01$ .



Transverse momentum broadening

( $0.2 < z_h < 0.5$ )

Simulation v.s. LO expectation

$$\Delta \langle p_T^2 \rangle_{\text{LO}} \approx \frac{C_F}{C_A} Q_s^2(x_B, Q^2) \frac{\int_{0.2}^{0.5} z_h^2 \frac{d\sigma}{dz_h dx_B dQ^2} dz_h}{\int_{0.2}^{0.5} \frac{d\sigma}{dz_h dx_B dQ^2} dz_h}$$

## Towards smaller $x_B$ , maintaining large $Q^2$

- ✓ Large  $x_B$  region  $\tau_H/\text{fm} = \frac{0.1}{x_B} \ll 1.2A^{1/3}$ . Final-state interaction is completely factorized from hard production.
- For  $x_B \sim 0.01$ ,  $\frac{0.1}{x_B} \sim 1.2A^{1/3}$ . Multiple collisions become coherent with the hard process  $\Rightarrow$  dynamical shadowing [J Qiu, I Vitev, PRL93(2004)262301], or parametrized by nuclear PDF.
- Finally,  $\frac{0.1}{x_B} \gg 1.2A^{1/3}$ , the dipole regime.

To extend eHIJING to smaller  $x_B$  is to consider how to generate shower / excited the nucleus while taking coherence into account.

# Summary

A Monte Carlo model for jet shower modifications in  $e$ - $A$ :

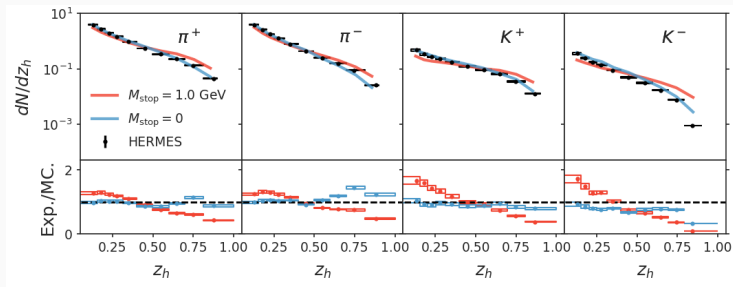
- $e$ - $p$  event generation from Pythia8.
- $e$ - $A$  (c++17, including modifications to Pythia8):  
Requires large  $Q^2$  and large  $x_B$ .
  - 1) Multiple scatterings with a saturation-motivated TMD gluon density.
  - 2) Modified splitting functions from generalized higher-twist approach & the HT limit.
  - 3) In-medium parton shower.
  - 4) Hadronization (Lund string fragmentation).

To do:

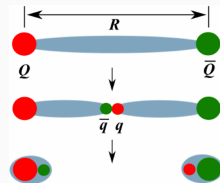
- Hadronic final-state interactions.
- Event generations at smaller  $x_B$  (and relatively large  $Q^2$ ).

Questions?

# The Lund string model in Pythia8: $D(z)$



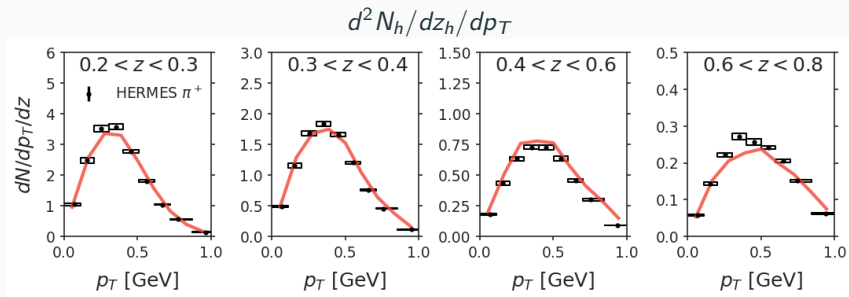
[HERMES, Phys Rev D 87, 074029 (2013)]



- Change a default Pythia8 fragmentation parameter  $M_{\text{stop}}$  from 1 GeV to 0 to fit  $\pi$  and  $K$  spectra in  $e-d$  collisions at HERMES energy.
- $M_{\text{stop}}$  controls the minimum mass of string to break  $W > m_q + m_{\bar{q}} + M_{\text{stop}}$ .



# The Lund string model in Pythia8: $d^2 N_h/dz_h/dp_T$



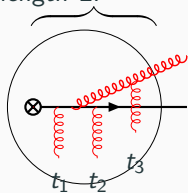
[HERMES, Phys Rev D 87, 074029 (2013)]

Reasonable agreement with Pythia8's non-perturbative modeling

- A primordial quark  $k_T$ ,  $k_T \sim e^{-k_T^2/2\sigma_1^2}$  [T. Sjöstrand and P.Z. Skands, JHEP 03 (2004) 053].
- $k_T$  from Lund string fragmentation,  $k_T \sim e^{-k_T^2/2\sigma_2^2}$  with  $\sigma_2 = 0.335$  GeV as default.

# Multiple collisions in eHIJING

- For each newly generated parton in the shower, compute the in-medium path length  $L$ .  
Currently only implemented for round nuclei.
- Compute average number of multiple collisions  $\langle N \rangle = L/\lambda_G \approx \sigma_G \rho L$
- Number of collisions  $N$  follows a Poisson distribution  $P(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}$
- Sample the location and kinematics of collisions  $(t_i, \mathbf{k}_{\perp i}, x_i)$



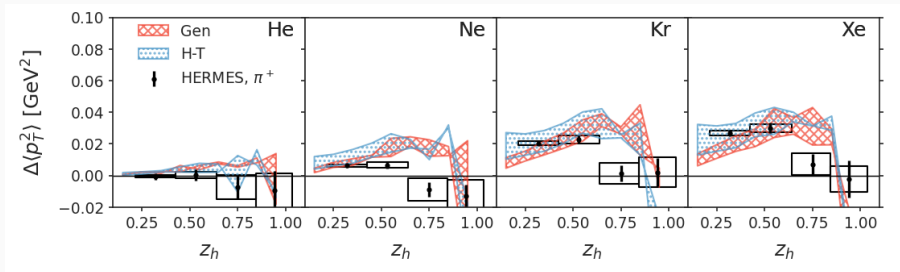
## Stochastic version of the medium-modified splitting functions

With a large fluctuation in the number of collisions, we constructed fluctuating in-medium splitting functions

$$P_{qq}(x, \ell_{\perp}) = \frac{\alpha_s C_F}{2\pi} \frac{P_{qq}(x)}{\ell_{\perp}^2} \left\{ 1 + \int dL \rho(L) \int_{\mathbf{k}_{\perp}} \frac{C_F}{dA} \frac{\alpha_s \phi_g(x, \mathbf{k}_{\perp}^2)}{\mathbf{k}_{\perp}^2} \frac{C_A}{C_F} \frac{2\mathbf{k}_{\perp} \cdot \ell_{\perp}}{(\ell_{\perp} - \mathbf{k}_{\perp})^2} \left[ 1 - \cos \frac{L}{\tau_f} \right] \right\}$$
$$\implies \frac{\alpha_s C_F}{2\pi} \frac{P_{qq}(x)}{\ell_{\perp}^2} \left\{ 1 + \sum_i \frac{C_A}{C_F} \frac{2(\mathbf{k}_{\perp})_i \cdot \ell_{\perp}}{[\ell_{\perp} - (\mathbf{k}_{\perp})_i]^2} \left[ 1 - \cos \frac{L_i}{(\tau_f)_i} \right] \right\}$$

The usual average over the medium sources is replaced by the summation over the multiple collisions of the shower parton.

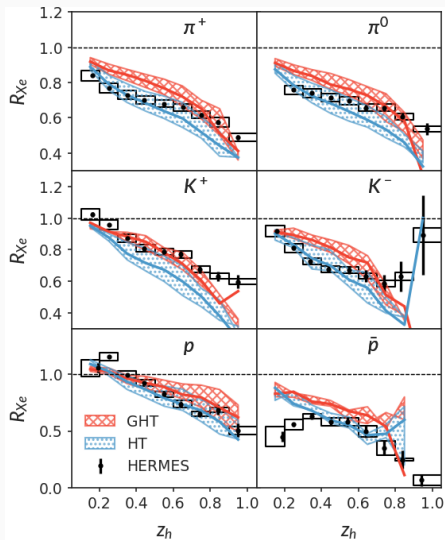
# The net broadening $\Delta\langle p_T^2 \rangle = \langle p_T^2 \rangle_{eA} - \langle p_T^2 \rangle_{ed}$



[HERMES, PLB 684 (2010) 114-118]

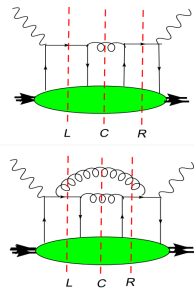
- Qualitatively similar  $z$ -dependence from simulation.
- Data drop more abruptly for  $z_h > 0.7$ . This region shrinks at higher colliding energies.

# Hadron specie dependence: $\pi^\pm, \pi^0, K^\pm, p, \bar{p}$

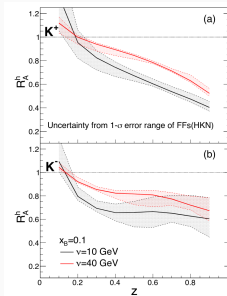


[HERMES, Nuclear Physics B 780, 24 (2007)]

- Notable difference between  $R_A(K^+)$  vs  $R_A(K^-)$ , and  $R_A(p)$  vs  $R_A(\bar{p})$ .
- Importance of medium-induced conversion of  $g \rightarrow q$  and hadronic transport for future.

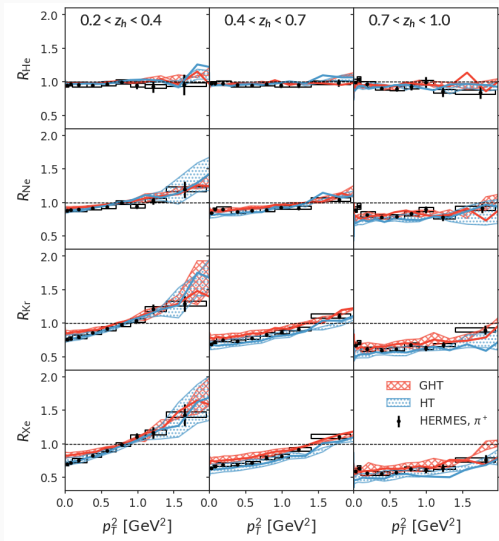


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# $p_T$ -dependent modified fragmentation function $D(z_h, p_T)$



$$R_A = \frac{(N_h(\nu, Q^2; z_h, p_t)/N_\gamma)_{eA}}{(N_h(\nu, Q^2; z_h, p_t)/N_\gamma)_{ed}}$$

- Large  $z$ : suppression due to parton energy loss of leading particles.
- Intermediate to small  $z$ : interplay of  $\mathbf{k}_\perp$  broadening and energy loss.

[HERMES, Nuclear Physics B 780, 24 (2007)]