

# Extraction of TMD distributions

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June 30, 2023, SURGE Collaboration Meeting, BNL

# Introduction

🍏 The  $q_T$  distribution of a generic **high-mass** ( $Q$ ) system has two main regimes:

🍏 for  $q_T \gtrsim Q$  **collinear factorisation** at *fixed perturbative order* is appropriate:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{f.o.}} = \int_0^1 dx_1 \int_0^1 dx_2 f_1(x_1, Q) f_2(x_2, Q) \frac{d\hat{\sigma}}{dq_T} + \mathcal{O}\left[\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^n\right]$$

🍏 for  $q_T \ll Q$  **transverse-momentum-dependent (TMD) factorisation** at *fixed logarithmic order* is appropriate:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \stackrel{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2) + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

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**Main subject of this talk**

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🍏 Collinear and TMD factorisations may eventually be **matched** to produce accurate results over the the full  $q_T$  spectrum.

# TMD factorisation

🍏 TMD factorisation introduces two independent scales:

🍏 the **renormalisation scale**  $\mu$ , originating from the UV renormalisation,

🍏 the **rapidity scale**  $\zeta$ , originating from the cancellation of rapidity divergences.

🍏 The respective **evolution equations** are:

$$\frac{\partial \ln F}{\partial \ln \sqrt{\zeta}} = K(\mu_0) - \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu'))$$

$$\frac{\partial \ln F}{\partial \ln \mu} = \gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta}}{\mu}$$

🍏 In addition, for small values of  $b_T$ , TMDs can be matched on coll. dists.:

$$F(\mu, \zeta) = C(\mu, \zeta) \otimes f(\mu)$$

🍏 The solution is:

$$F(\mu, \zeta) = \exp \left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} C(\mu_0, \zeta_0) \otimes f(\mu_0)$$

🍏 Anomalous dims. and matching funcs. **perturbatively** computable. 4

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🍏 The solution is:

$$\mu_b = b_0 / b_T$$


$$F(\mu, \zeta) = \exp \left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} C(\mu_0, \zeta_0) \otimes f(\mu_0)$$

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# TMD factorisation

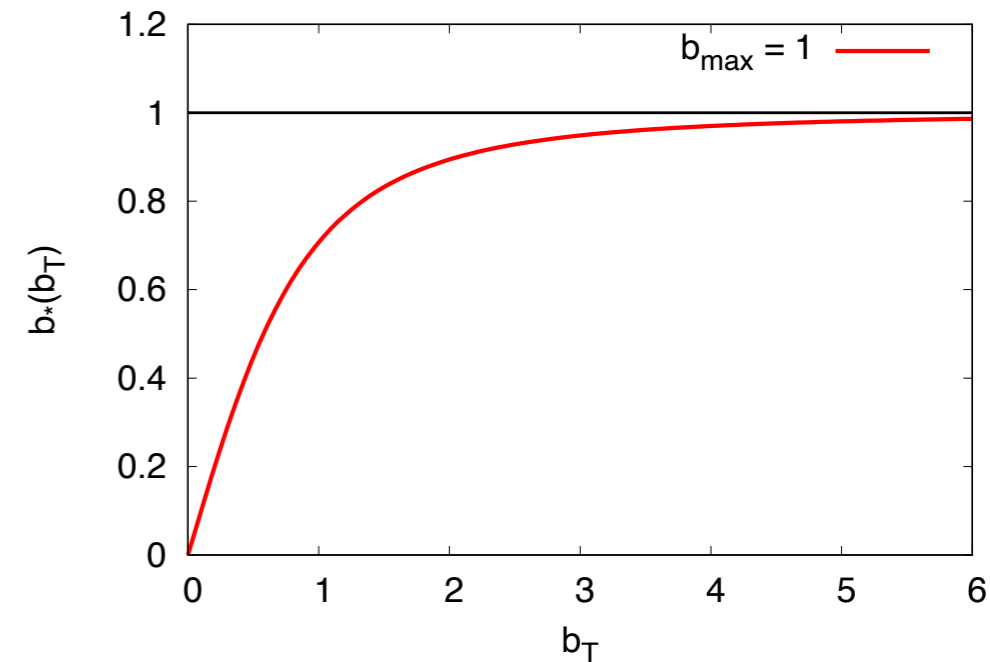
🍏 When integrating over  $b_T$ , **large values of  $b_T$**  give rise to low scales in the **non-perturbative** region.

🍏 Introduce the so-called  **$b_*$ -prescription**:

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

🍏 and rewrite:

$$F(x, b_T, \mu, \zeta) = \left[ \frac{F(x, b_T, \mu, \zeta)}{F(x, b_*(b_T), \mu, \zeta)} \right] F(x, b_*(b_T), \mu, \zeta) \equiv f_{\text{NP}}(x, b_T, \zeta) F(x, b_*(b_T), \mu, \zeta)$$

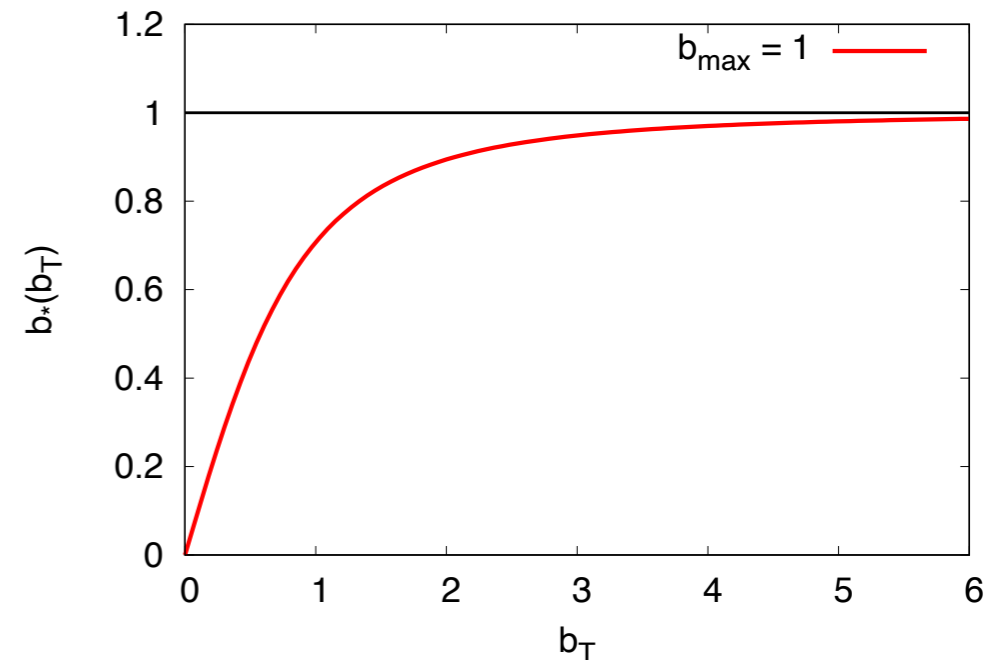


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🍏 Properties of  $f_{\text{NP}}$ :

🍏 has to go to **one** as  $b_T$  goes to zero: reproduce the fully perturbative regime,

🍏 has to go to **zero** as  $b_T$  becomes large: mimic the Sudakov suppression.

🍏 Bottom line: avoidance of the non-perturbative region upon integration in  $b_T$  implies the presence of **both**  $b_*$ -prescription and  $f_{\text{NP}}$ .

# TMD factorisation

 Final expression:

$$\begin{aligned} F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \mu_b^2) \otimes f_{j/P}(x, \mu_b) && : A \\ &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} && : B \\ &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C \end{aligned}$$



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- matching onto the collinear region at  $b_T \ll 1/\Lambda_{\text{QCD}}$ ,
- factorises as *hard* (perturbative) and *longitudinal* (i.e. collinear, non-perturbative).

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- CS and RGE evolution,
- evolution in  $\mu$  and  $\zeta$ ,
- perturbative.

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- matching onto the collinear region at  $b_T \ll 1/\Lambda_{\text{QCD}}$ ,
- factorises as *hard* (perturbative) and *longitudinal* (i.e. collinear, non-perturbative).

- avoid the Landau pole,
- $f_{\text{NP}}$  accounts for the introduction of  $b_*$ ,
- $f_{\text{NP}}$  is non-perturbative thus **fit** to data.

- CS and RGE evolution,
- evolution in  $\mu$  and  $\zeta$ ,
- perturbative.

# Logarithmic counting

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \stackrel{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2)$$

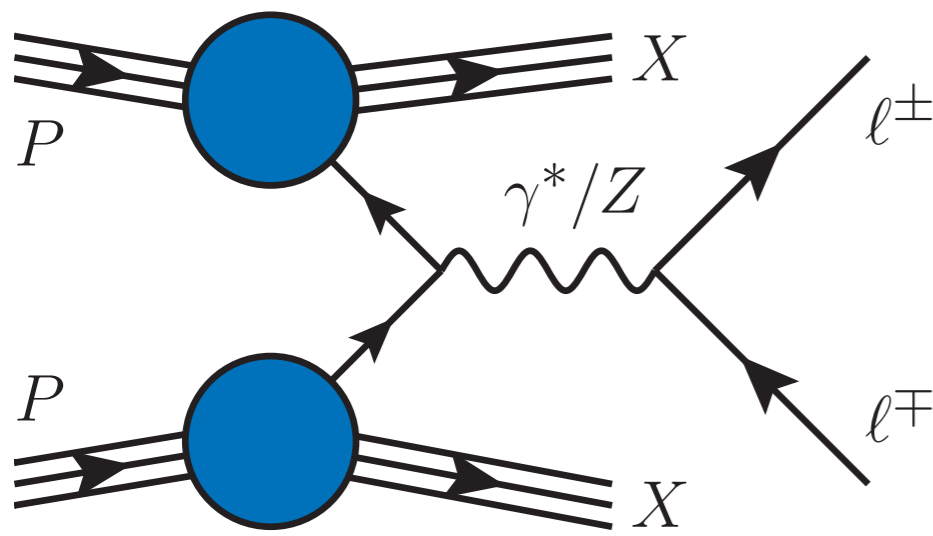
$$F_i = \sum_j (C_{i/j} \otimes f_j) \exp \left\{ K \ln \frac{\sqrt{\zeta}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\}$$

Accuracy	$\gamma_K$	$\gamma_F$	$K$	$C_{fij}$	$H$	FFs/PDFs/ $\alpha_s$
LL	$\alpha_s$	-	-	1	1	-
NLL	$\alpha_s^2$	$\alpha_s$	$\alpha_s$	1	1	LO
NLL'	$\alpha_s^2$	$\alpha_s$	$\alpha_s$	$\alpha_s$	$\alpha_s$	LO
N <sup>2</sup> LL	$\alpha_s^3$	$\alpha_s^2$	$\alpha_s^2$	$\alpha_s$	$\alpha_s$	NLO
N <sup>2</sup> LL'	$\alpha_s^3$	$\alpha_s^2$	$\alpha_s^2$	$\alpha_s^2$	$\alpha_s^2$	NLO
N <sup>3</sup> LL	$\alpha_s^4$	$\alpha_s^3$	$\alpha_s^3$	$\alpha_s^2$	$\alpha_s^2$	NNLO
N <sup>3</sup> LL'	$\alpha_s^4$	$\alpha_s^3$	$\alpha_s^3$	$\alpha_s^3$	$\alpha_s^3$	NNLO

# Factorising processes

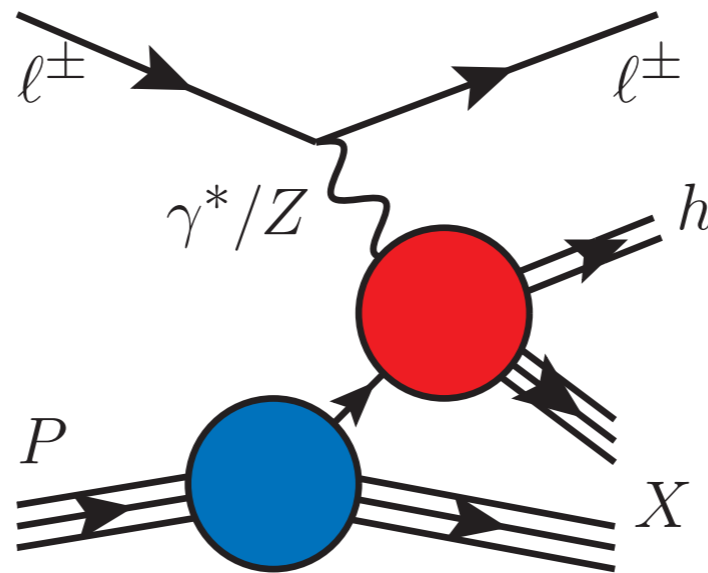
Processes for which leading-power TMD factorisation has been **proven**:

Drell-Yan



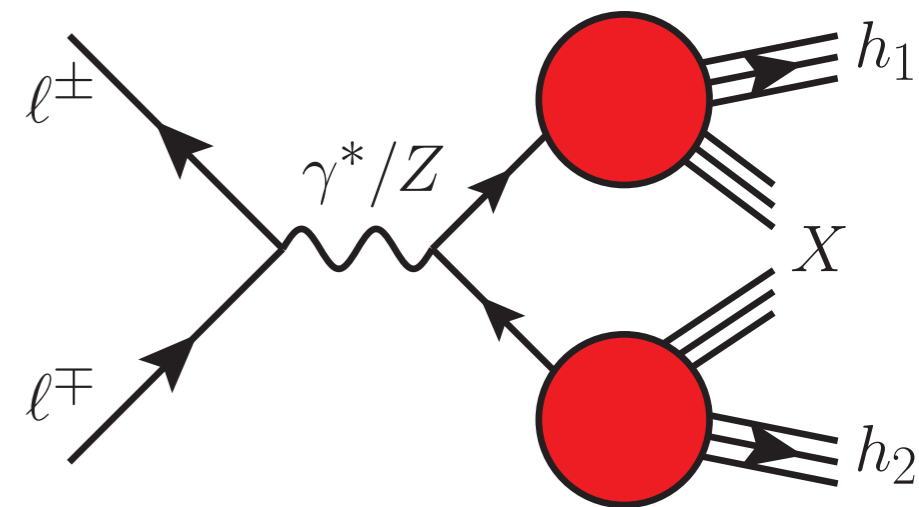
$$PP \longrightarrow l^\pm l^\mp X$$

Semi-inclusive DIS



$$Pl^\pm \longrightarrow l^\pm h X$$

$e^+e^-$  annihilation



$$l^\pm l^\mp \longrightarrow h_1 h_2 X$$

Two TMD PDFs:

Lots of data:

low-energy: FNAL,

mid-energy: RHIC,

high-energy: Tevatron, LHC.

One TMD PDF one FF:

many precise data points:

HERMES at DESY,

COMPASS at CERN.

Two TMD FFs:

di-hadron prod. from:

BELLE at KEK,

BABAR at SLAC.

Examples of other processes:

thrust and  $p_{hT}$  distributions in single-hadron production in  $e^+e^-$ ,

hadron-in-jet production,

...

# Unpolarised TMD extractions

## *A selection of fits*

	Accuracy	SIDIS	Drell-Yan	N. of points
DWS 1984, <a href="#">CERN-TH.3987/84</a>	NLL	✗	✓	a few
BLNY 2003, <a href="#">hep-ph/0212159</a>	NLL'-NNLL	✗	✓	116
Pavia 2013, <a href="#">1309.3507</a>	No evolution	✓	✗	1538 (HERMES)
Torino 2014, <a href="#">1312.6261</a>	No evolution	✓	✗	576 (H) 6284 (C)
DEMS 2014, <a href="#">1407.3311</a>	NNLL	✗	✓	223
Pavia 2017, <a href="#">1703.10157</a>	NLL	✓	✓	8059
SV 2017, <a href="#">1706.01473</a>	N <sup>3</sup> LL	✗	✓ (LHC)	309
BSV 2019, <a href="#">1902.08474</a>	N <sup>3</sup> LL	✗	✓ (LHC)	457
SV 2019, <a href="#">1912.06532</a>	N <sup>3</sup> LL(-)	✓	✓ (LHC)	1039
Pavia 2019, <a href="#">1912.07550</a>	N <sup>3</sup> LL	✗	✓ (LHC)	353
SV+ 2022, <a href="#">2201.07114</a>	N <sup>3</sup> LL	✗	✓ (LHC)	507/309
MAPTMD 2022, <a href="#">2206.07598</a>	N <sup>3</sup> LL(-)	✓	✓ (LHC)	2031

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# Unpolarised TMD extractions

*Many more studies and extractions...*

- 🍏 TMD fragmentation functions from  $e^+e^-$  data [[2108.04182](#), [1704.08882](#)]
- 🍏  $W$  production in  $pp$  collisions [[2011.05351](#)]
- 🍏 Dijet and heavy-meson pair production in DIS [[2008.07531](#), [2111.03703](#)]
- 🍏 Dijet production in  $pp$  collisions [*e.g.* [1807.07573](#)]
- 🍏 hadron-in-jet production [[1612.04817](#)]
- 🍏 Model-independent prescription to extract TMDs [[2201.07237](#)]
- 🍏 Parton-branching methods [*e.g.* [1804.11152](#)]
- 🍏  $q_T$ -resummation based extractions [[2203.05394](#)]
- 🍏 Study of the Sivers TMDs [[1308.5003](#), [2004.14278](#), [2009.10710](#), [2103.03270](#),...]
- 🍏 Pion TMDs [[1907.10356](#), [2210.01733](#)]
- 🍏 TMD flavour dependence [[1807.02101](#), [2201.07114](#)]
- 🍏 ...

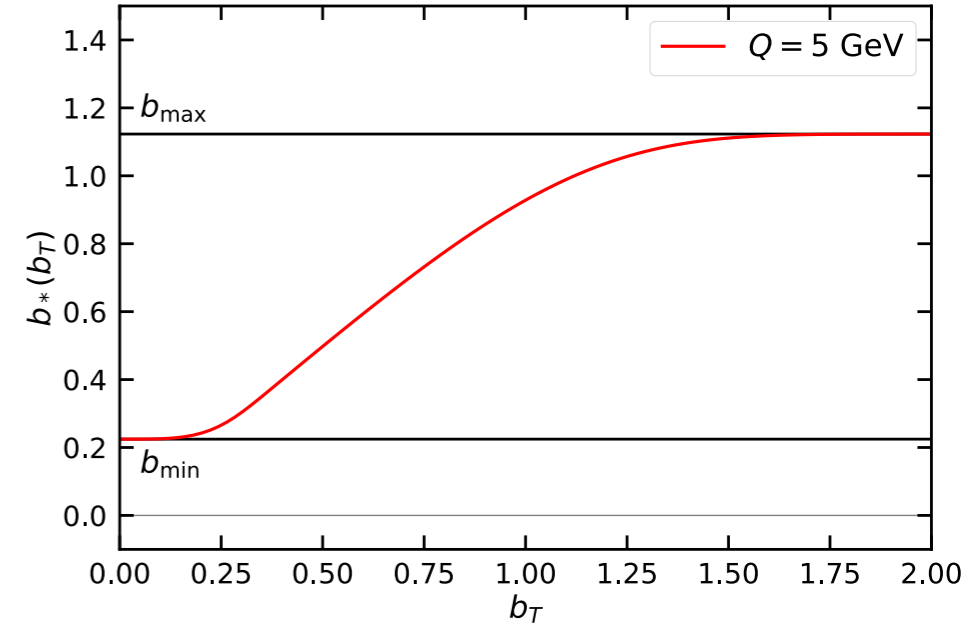


# MAPTMD 2022

## Main settings

🍏  $b^*$  prescription:

$$b_*(b_T) = b_{\max} \left( \frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4} \quad \text{with} \quad \begin{cases} b_{\max} = 2e^{-\gamma_E} \\ b_{\min} = b_{\max}/Q \end{cases}$$



🍏 Non-perturbative function  $f_{NP}$ :

🍏 evolution:  $g_K(\mathbf{b}_T^2) = -g_2^2 \frac{\mathbf{b}_T^2}{2}$

🍏 PDFs:

$$f_{1NP}(x, \mathbf{b}_T^2; \zeta, Q_0) = \frac{g_1(x) e^{-g_1(x) \frac{\mathbf{b}_T^2}{4}} + \lambda^2 g_{1B}^2(x) \left[ 1 - g_{1B}(x) \frac{\mathbf{b}_T^2}{4} \right] e^{-g_{1B}(x) \frac{\mathbf{b}_T^2}{4}} + \lambda_2^2 g_{1C}(x) e^{-g_{1C}(x) \frac{\mathbf{b}_T^2}{4}}}{g_1(x) + \lambda^2 g_{1B}^2(x) + \lambda_2^2 g_{1C}(x)} \left[ \frac{\zeta}{Q_0^2} \right]^{g_K(\mathbf{b}_T^2)/2}$$

🍏 FFs:

$$D_{1NP}(z, \mathbf{b}_T^2; \zeta, Q_0) = \frac{g_3(z) e^{-g_3(z) \frac{\mathbf{b}_T^2}{4z^2}} + \frac{\lambda_F}{z^2} g_{3B}^2(z) \left[ 1 - g_{3B}(z) \frac{\mathbf{b}_T^2}{4z^2} \right] e^{-g_{3B}(z) \frac{\mathbf{b}_T^2}{4z^2}}}{g_3(z) + \frac{\lambda_F}{z^2} g_{3B}^2(z)} \left[ \frac{\zeta}{Q_0^2} \right]^{g_K(\mathbf{b}_T^2)/2}$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha_{\{1,2,3\}}^2}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha_{\{1,2,3\}}^2}} \quad g_{\{3,3B\}}(z) = N_{\{3,3B\}} \frac{(z^{\beta_{\{1,2\}}} + \delta_{\{1,2\}}^2)(1-z)^{\gamma_{\{1,2\}}^2}}{(\hat{z}^{\beta_{\{1,2\}}} + \delta_{\{1,2\}}^2)(1-\hat{z})^{\gamma_{\{1,2\}}^2}}$$

🍏 11 (PDFs) + 9 (FFs) + 1 (evol): **21 free parameters** to fit to data.

🍏 Perturbative accuracies: **N<sup>3</sup>LL(-)**.

🍏 **Monte Carlo** method for the experimental error propagation.

# MAPTMD 2022

## Dataset



DY data:



fixed-target low-energy DY,



RHIC data,



LHC and Tevatron data,



selection cut  $q_T / Q < 0.2$ ,



484 data points.



SIDIS data:



HERMES and COMPASS,



$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$



$Q > 1.4 \text{ GeV}, 0.2 < z < 0.7$ ,



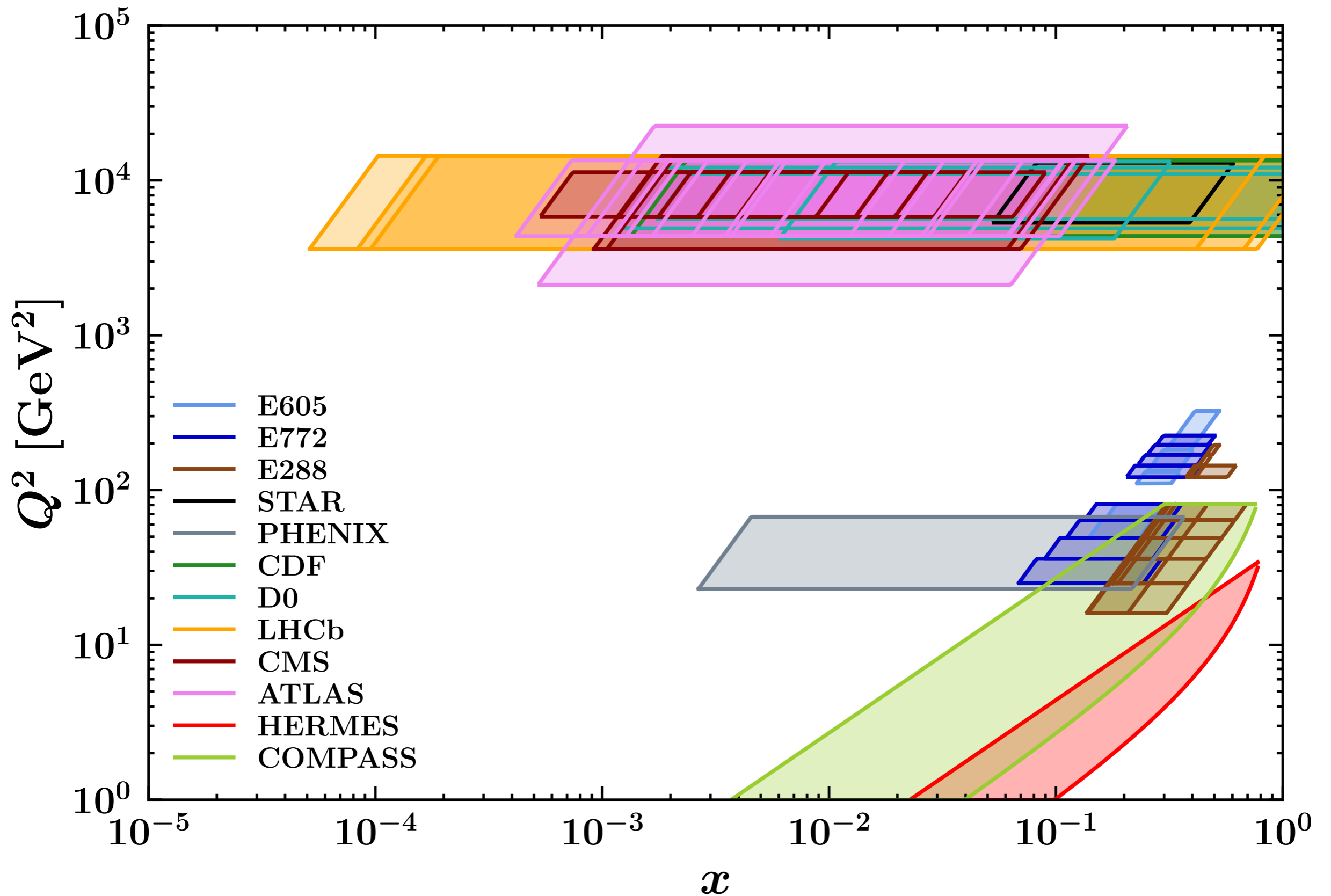
1547 points.

Experiment	$N_{\text{dat}}$	Observable	$\sqrt{s}$ [GeV]	$Q$ [GeV]	$y$ or $x_F$	Lepton cuts	Ref.
E605	50	$Ed^3\sigma/d^3\mathbf{q}$	38.8	7 - 18	$x_F = 0.1$	-	[55]
E772	53	$Ed^3\sigma/d^3\mathbf{q}$	38.8	5 - 15	$0.1 < x_F < 0.3$	-	[51]
E288 200 GeV	30	$Ed^3\sigma/d^3\mathbf{q}$	19.4	4 - 9	$y = 0.40$	-	[56]
E288 300 GeV	39	$Ed^3\sigma/d^3\mathbf{q}$	23.8	4 - 12	$y = 0.21$	-	[56]
E288 400 GeV	61	$Ed^3\sigma/d^3\mathbf{q}$	27.4	5 - 14	$y = 0.03$	-	[56]
STAR 510	7	$d\sigma/d \mathbf{q}_T $	510	73 - 114	$ y  < 1$	$p_{T\ell} > 25 \text{ GeV}$ $ \eta_\ell  < 1$	-
PHENIX200	2	$d\sigma/d \mathbf{q}_T $	200	4.8 - 8.2	$1.2 < y < 2.2$	-	[52]
CDF Run I	25	$d\sigma/d \mathbf{q}_T $	1800	66 - 116	Inclusive	-	[57]
CDF Run II	26	$d\sigma/d \mathbf{q}_T $	1960	66 - 116	Inclusive	-	[58]
D0 Run I	12	$d\sigma/d \mathbf{q}_T $	1800	75 - 105	Inclusive	-	[59]
D0 Run II	5	$(1/\sigma)d\sigma/d \mathbf{q}_T $	1960	70 - 110	Inclusive	-	[60]
D0 Run II ( $\mu$ )	3	$(1/\sigma)d\sigma/d \mathbf{q}_T $	1960	65 - 115	$ y  < 1.7$	$p_{T\ell} > 15 \text{ GeV}$ $ \eta_\ell  < 1.7$	[61]
LHCb 7 TeV	7	$d\sigma/d \mathbf{q}_T $	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20 \text{ GeV}$ $2 < \eta_\ell < 4.5$	[62]
LHCb 8 TeV	7	$d\sigma/d \mathbf{q}_T $	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20 \text{ GeV}$ $2 < \eta_\ell < 4.5$	[63]
LHCb 13 TeV	7	$d\sigma/d \mathbf{q}_T $	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20 \text{ GeV}$ $2 < \eta_\ell < 4.5$	[64]
CMS 7 TeV	4	$(1/\sigma)d\sigma/d \mathbf{q}_T $	7000	60 - 120	$ y  < 2.1$	$p_{T\ell} > 20 \text{ GeV}$ $ \eta_\ell  < 2.1$	[65]
CMS 8 TeV	4	$(1/\sigma)d\sigma/d \mathbf{q}_T $	8000	60 - 120	$ y  < 2.1$	$p_{T\ell} > 15 \text{ GeV}$ $ \eta_\ell  < 2.1$	[66]
CMS 13 TeV	70	$d\sigma/d \mathbf{q}_T $	13000	76 - 106	$ y  < 0.4$ $0.4 <  y  < 0.8$ $0.8 <  y  < 1.2$ $1.2 <  y  < 1.6$ $1.6 <  y  < 2.4$	$p_{T\ell} > 25 \text{ GeV}$ $ \eta_\ell  < 2.4$	[53]
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/d \mathbf{q}_T $	7000	66 - 116	$ y  < 1$ $1 <  y  < 2$ $2 <  y  < 2.4$	$p_{T\ell} > 20 \text{ GeV}$ $ \eta_\ell  < 2.4$	[67]
ATLAS 8 TeV on-peak	6 6 6 6 6	$(1/\sigma)d\sigma/d \mathbf{q}_T $	8000	66 - 116	$ y  < 0.4$ $0.4 <  y  < 0.8$ $0.8 <  y  < 1.2$ $1.2 <  y  < 1.6$ $1.6 <  y  < 2$ $2 <  y  < 2.4$	$p_{T\ell} > 20 \text{ GeV}$ $ \eta_\ell  < 2.4$	[68]
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/d \mathbf{q}_T $	8000	46 - 66 116 - 150	$ y  < 2.4$	$p_{T\ell} > 20 \text{ GeV}$ $ \eta_\ell  < 2.4$	[68]
ATLAS 13 TeV	6	$(1/\sigma)d\sigma/d \mathbf{q}_T $	13000	66 - 113	$ y  < 2.5$	$p_{T\ell} > 27 \text{ GeV}$ $ \eta_\ell  < 2.5$	[54]
Total	484						

Experiment	$N_{\text{dat}}$	Observable	Channels	$Q$ [GeV]	$x$	$z$	Phase space cuts	Ref.
HERMES	344	$M(x, z,  \mathbf{P}_{hT} , Q)$	$p \rightarrow \pi^+$ $p \rightarrow \pi^-$ $p \rightarrow K^+$ $p \rightarrow K^-$ $d \rightarrow \pi^+$ $d \rightarrow \pi^-$ $d \rightarrow K^+$ $d \rightarrow K^-$	$1 - \sqrt{15}$	$0.023 < x < 0.6$ (6 bins)	$0.1 < z < 1.1$ (8 bins)	$W^2 > 10 \text{ GeV}^2$ $0.1 < y < 0.85$	[46]
COMPASS	1203	$M(x, z, \mathbf{P}_{hT}^2, Q)$	$d \rightarrow h^+$ $d \rightarrow h^-$	1 - 9 (5 bins)	$0.003 < x < 0.4$ (8 bins)	$0.2 < z < 0.8$ (4 bins)	$W^2 > 25 \text{ GeV}^2$ $0.1 < y < 0.9$	[72]
Total	1547							

# MAPTMD 2022

## *Kinematic coverage*



# MAPTMD 2022

## *Fit quality*

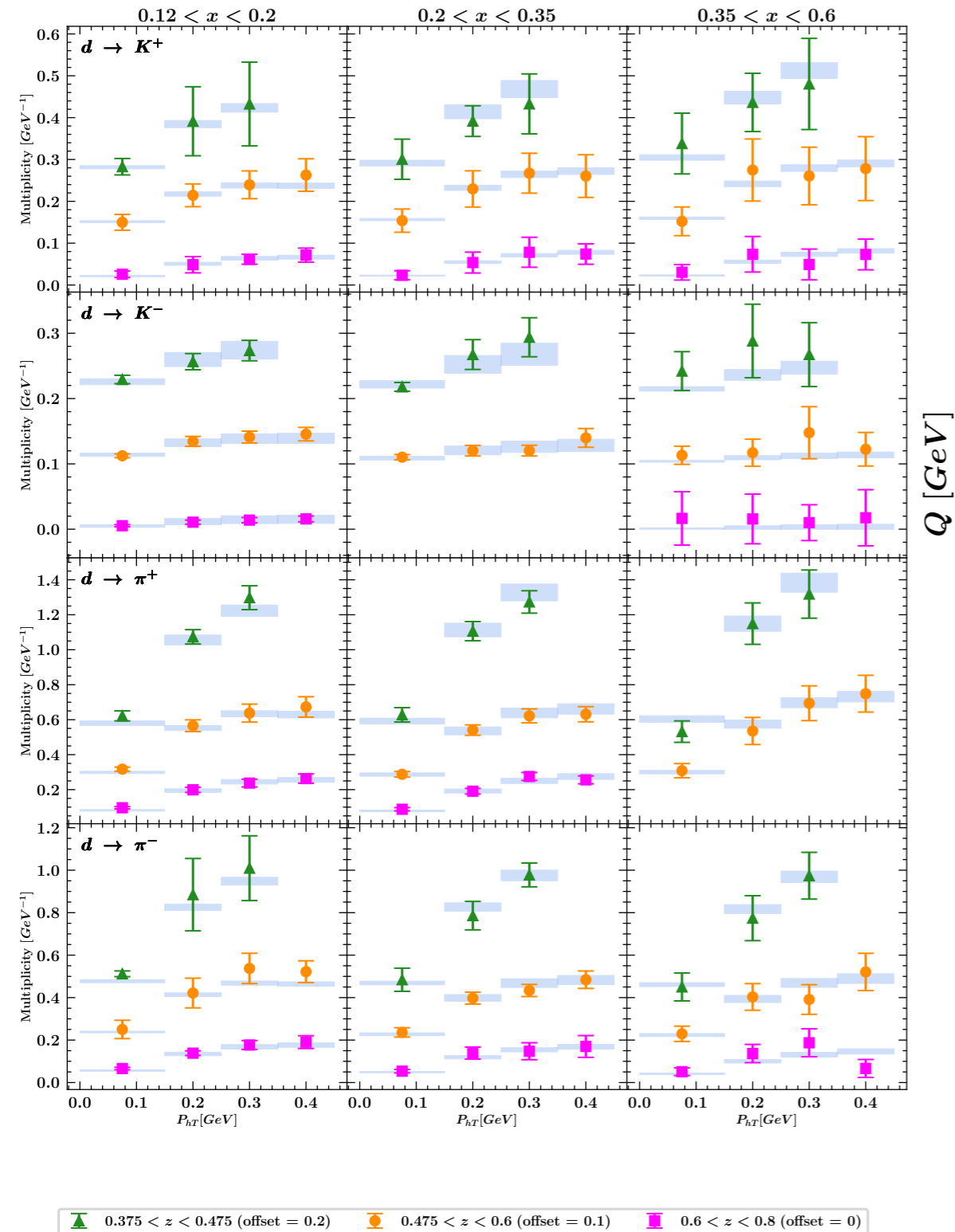
	$N^3LL^-$			
Data set	$N_{\text{dat}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi_0^2$
CDF Run I	25	0.45	0.09	0.54
CDF Run II	26	0.995	0.004	1.0
D0 Run I	12	0.67	0.01	0.68
D0 Run II	5	0.89	0.21	1.10
D0 Run II ( $\mu$ )	3	3.96	0.28	4.2
<i>Tevatron total</i>	71	0.87	0.06	0.93
LHCb 7 TeV	7	1.24	0.49	1.73
LHCb 8 TeV	7	0.78	0.36	1.14
LHCb 13 TeV	7	1.42	0.06	1.48
<i>LHCb total</i>	21	1.15	0.3	1.45
ATLAS 7 TeV	18	6.43	0.92	7.35
ATLAS 8 TeV	48	3.7	0.32	4.02
ATLAS 13 TeV	6	5.9	0.5	6.4
<i>ATLAS total</i>	72	4.56	0.48	5.05
CMS 7 TeV	4	2.21	0.10	2.31
CMS 8 TeV	4	1.938	0.001	1.94
CMS 13 TeV	70	0.36	0.02	0.37
<i>CMS total</i>	78	0.53	0.02	0.55
PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
DY collider total	251	1.86	0.2	2.06

E288 200 GeV	30	0.35	0.19	0.54
E288 300 GeV	39	0.33	0.09	0.42
E288 400 GeV	61	0.5	0.11	0.61
E772	53	1.52	1.03	2.56
E605	50	1.26	0.44	1.7
DY fixed-target total	233	0.85	0.4	1.24
HERMES ( $p \rightarrow \pi^+$ )	45	0.86	0.42	1.28
HERMES ( $p \rightarrow \pi^-$ )	45	0.61	0.31	0.92
HERMES ( $p \rightarrow K^+$ )	45	0.49	0.04	0.53
HERMES ( $p \rightarrow K^-$ )	37	0.18	0.13	0.31
HERMES ( $d \rightarrow \pi^+$ )	41	0.68	0.45	1.13
HERMES ( $d \rightarrow \pi^-$ )	45	0.63	0.35	0.97
HERMES ( $d \rightarrow K^+$ )	45	0.2	0.02	0.22
HERMES ( $d \rightarrow K^-$ )	41	0.14	0.08	0.22
<i>HERMES total</i>	344	0.48	0.23	0.71
COMPASS ( $d \rightarrow h^+$ )	602	0.55	0.31	0.86
COMPASS ( $d \rightarrow h^-$ )	601	0.68	0.3	0.98
<i>COMPASS total</i>	1203	0.62	0.3	0.92
SIDIS total	1547	0.59	0.28	0.87
<b>Total</b>	<b>2031</b>	<b>0.77</b>	<b>0.29</b>	<b>1.06</b>

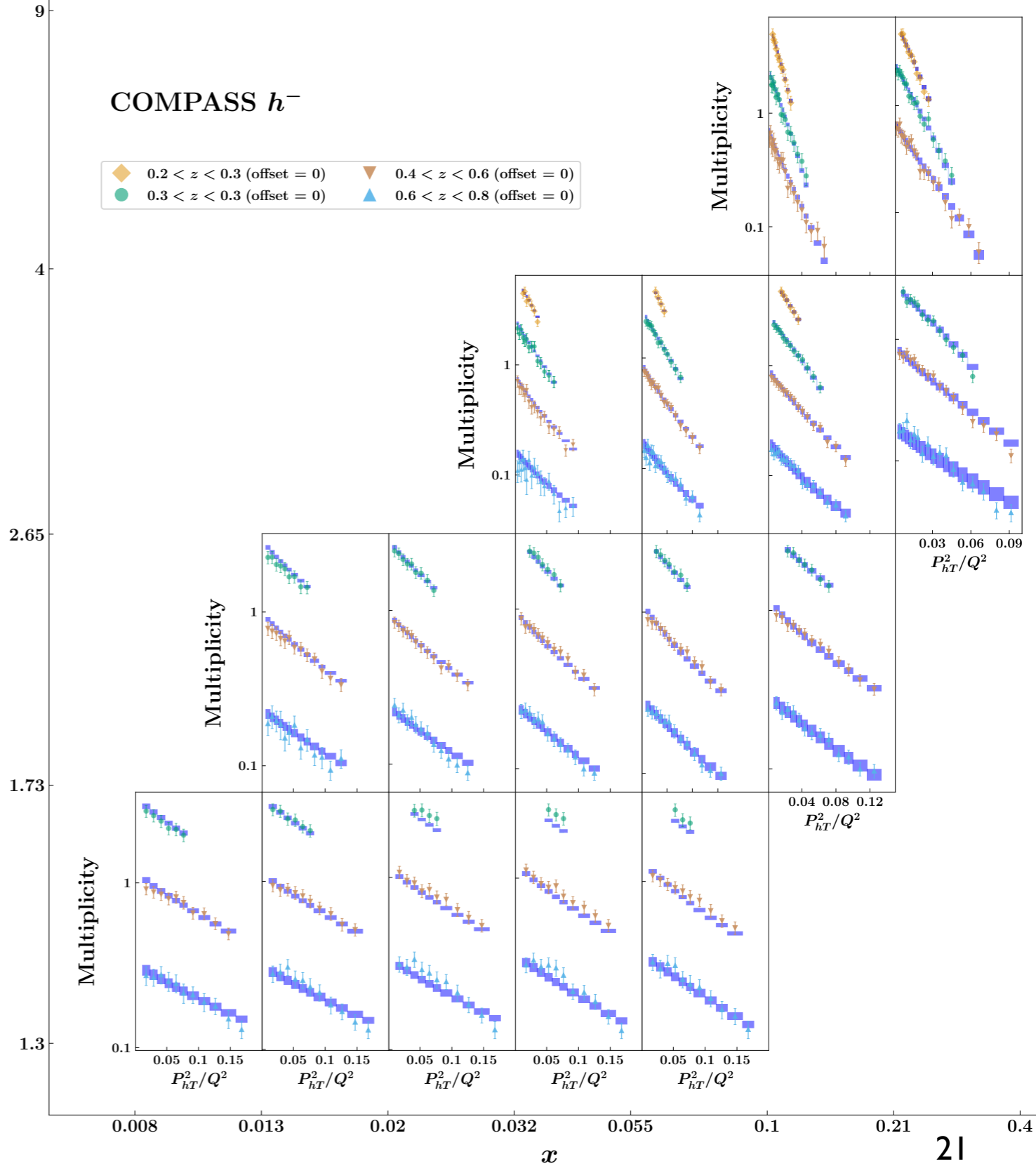
# MAPTMD 2022

## Fit quality: SIDIS

HERMES

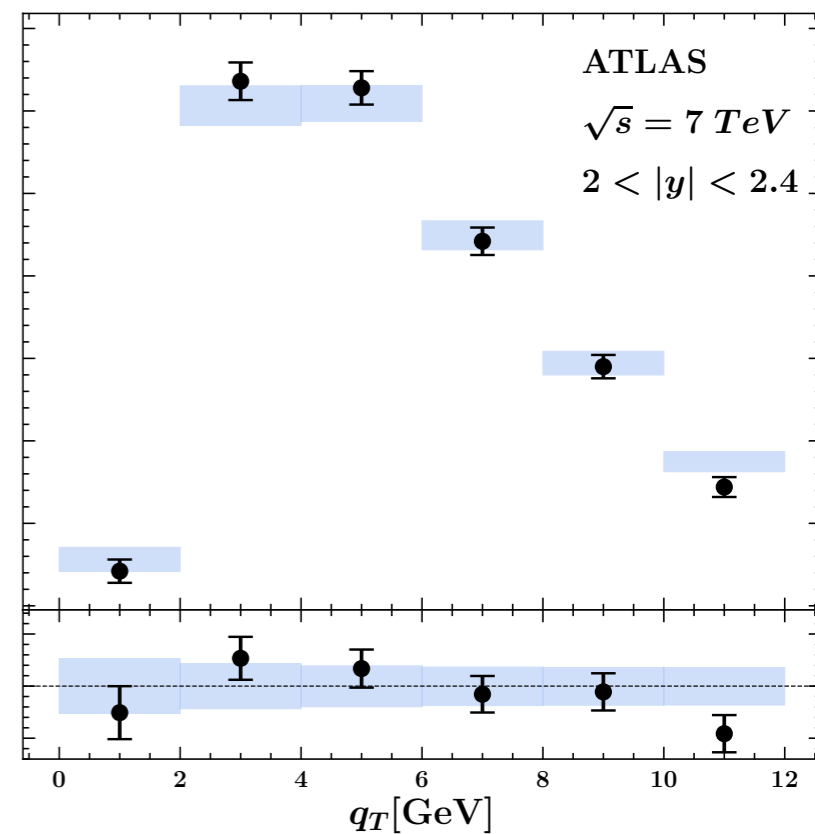
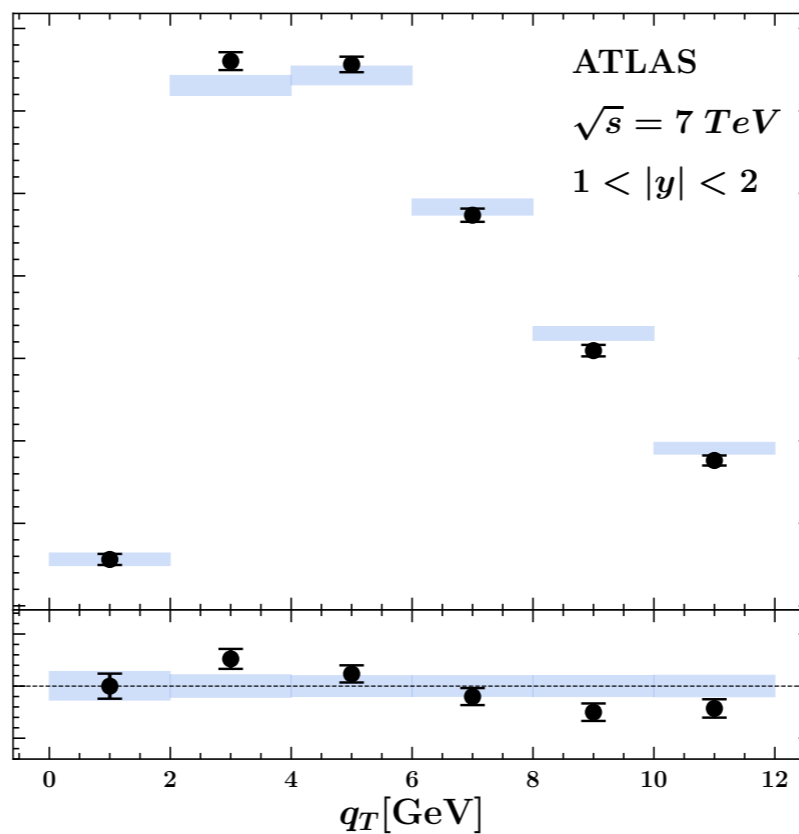
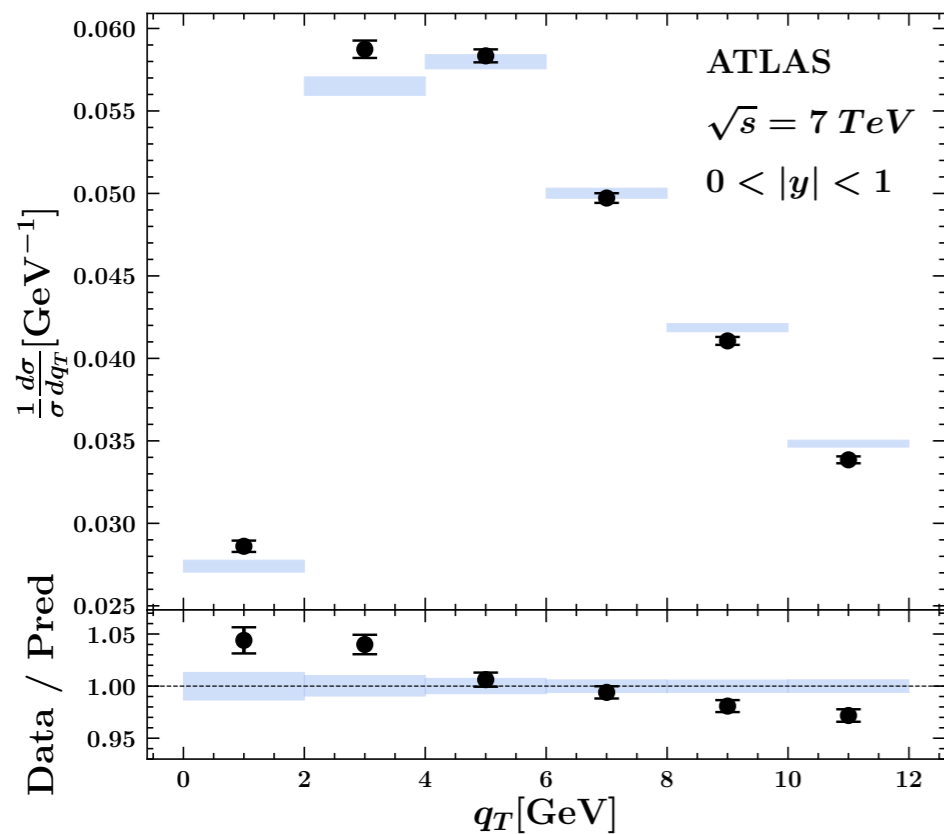
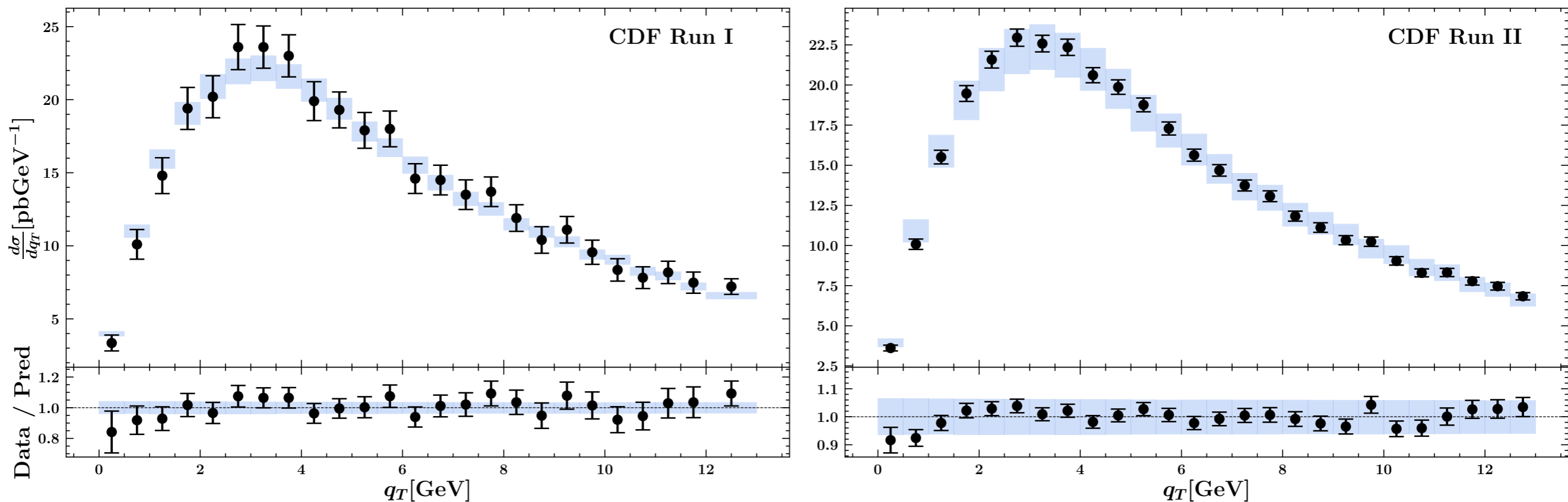


COMPASS  $h^-$



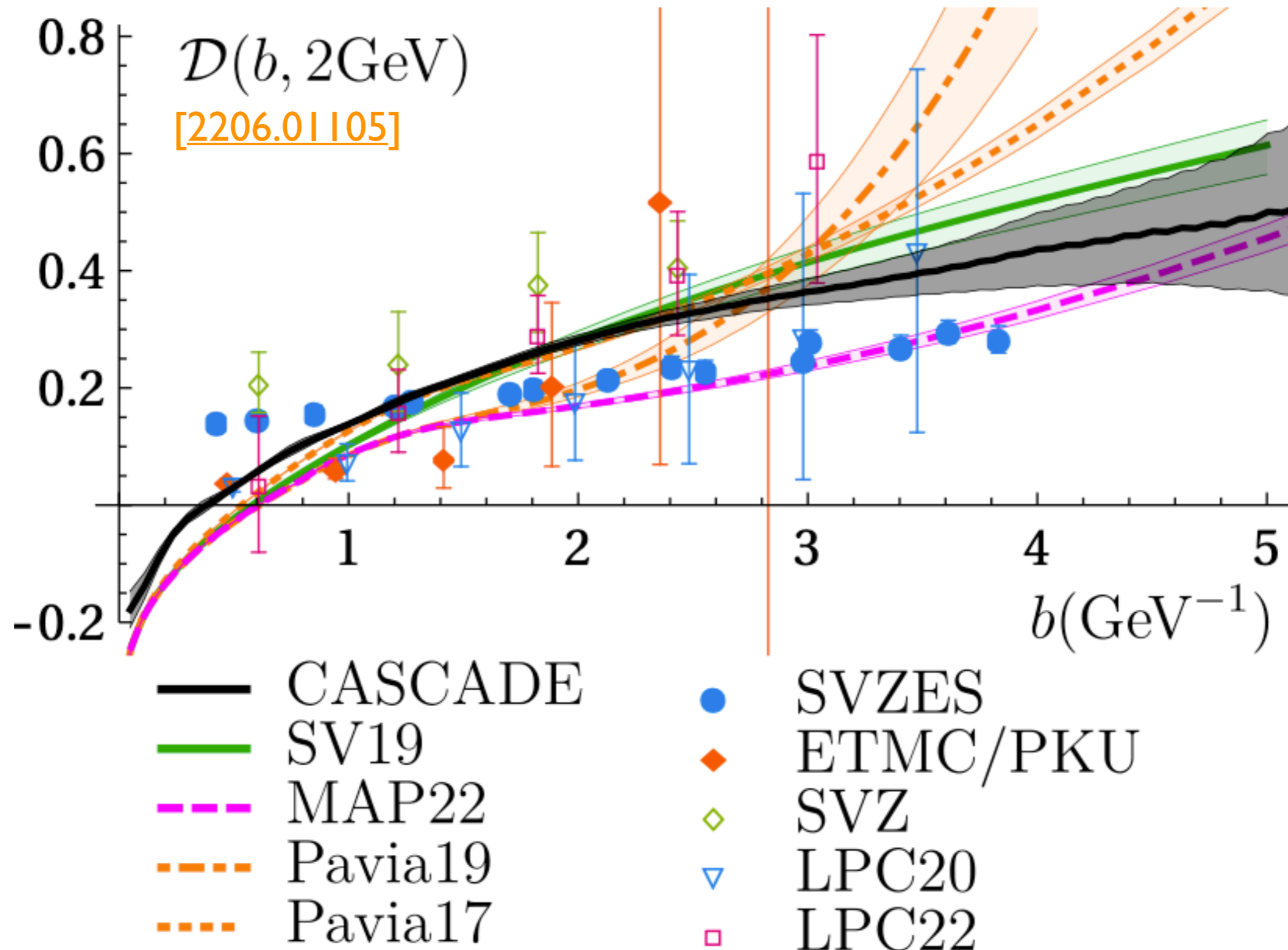
# MAPTMD 2022

*Fit quality: DY*



# MAPTMD 2022

## *Collins-Soper kernel*



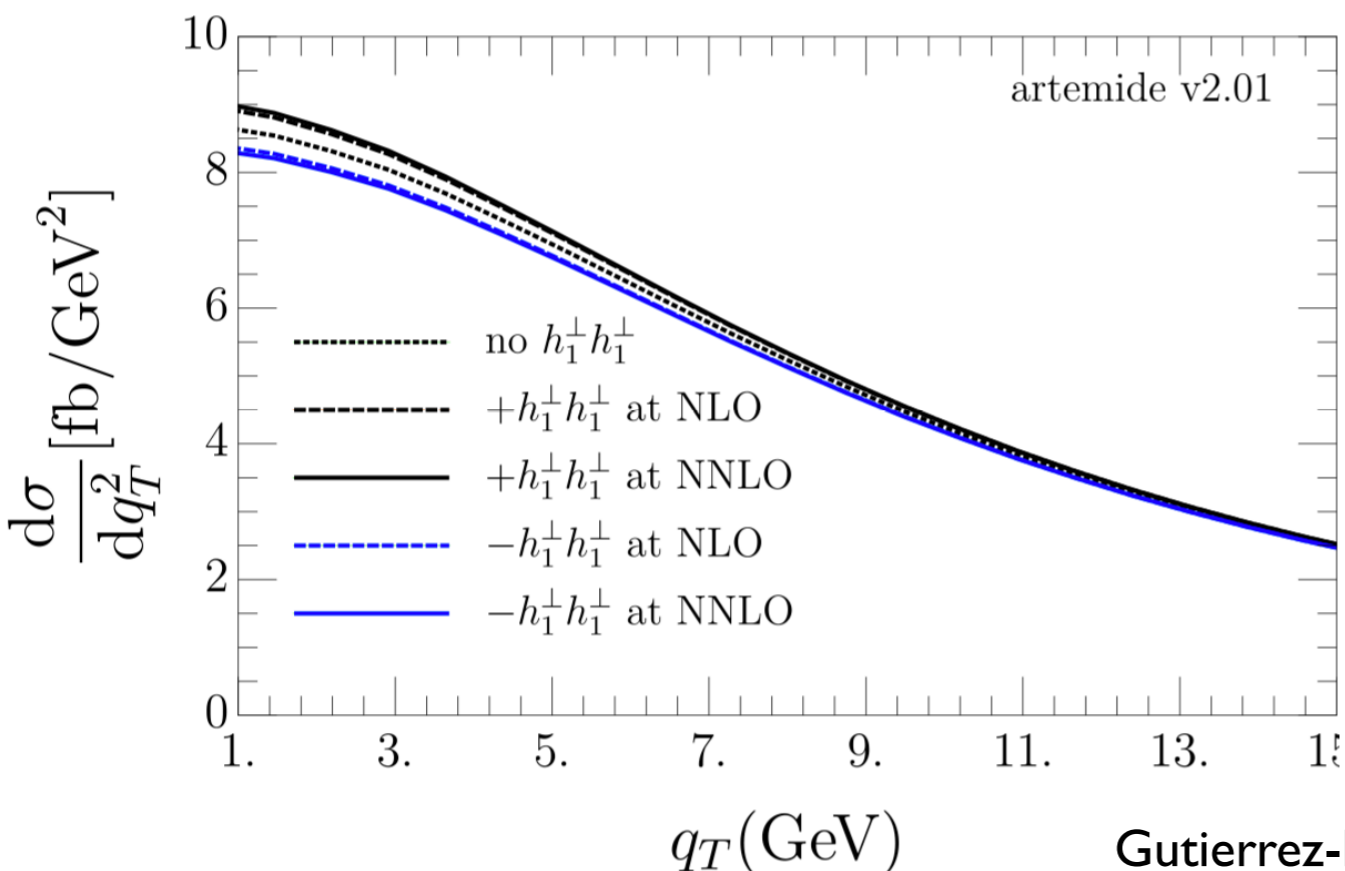
# Gluon TMDs

- 🍏 The *linearly polarised* ( $h_T$ ) and *unpolarised* ( $f$ ) gluon TMD PDFs contribute to the low- $q_T$  spectrum of Higgs production in gluon fusion:

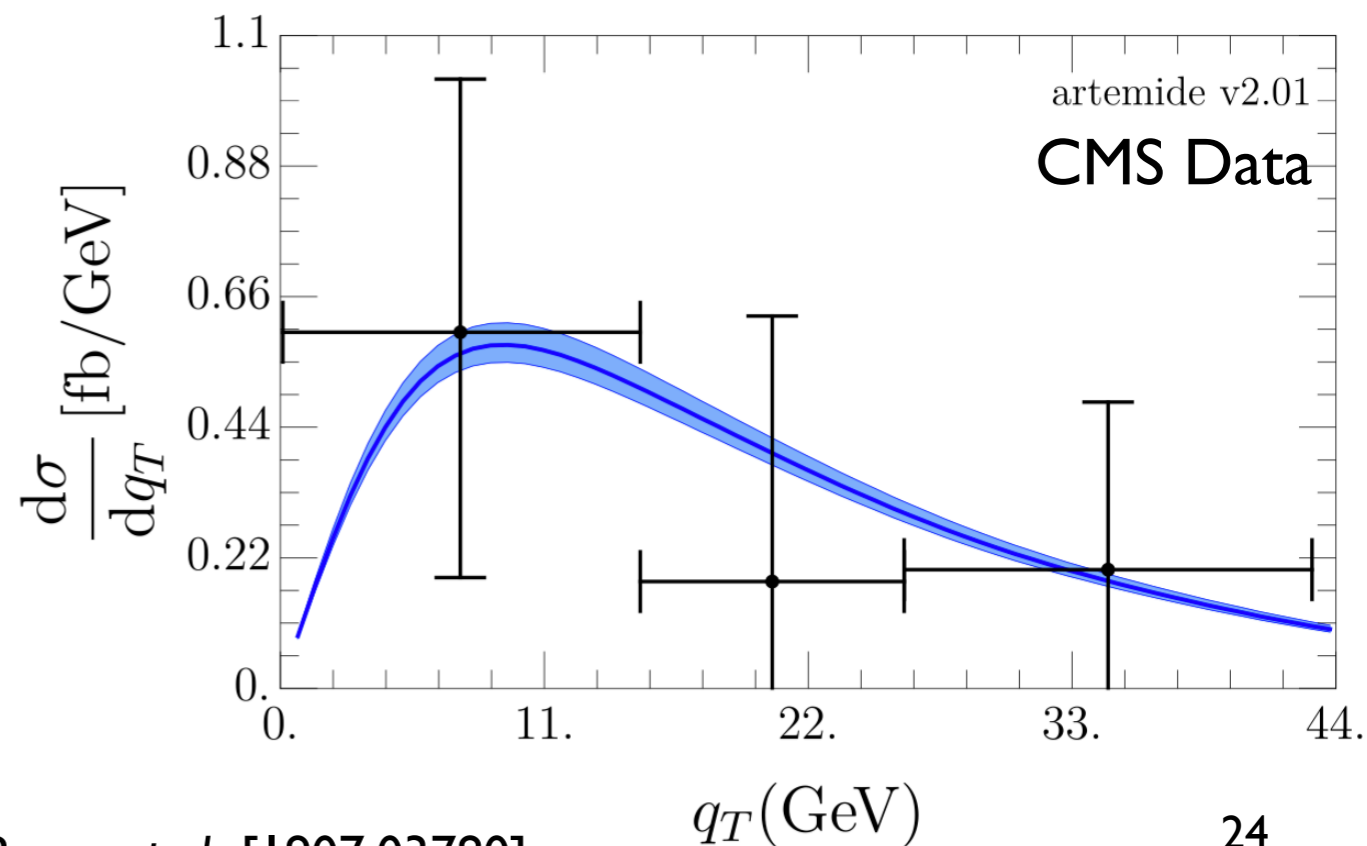
$$\frac{d\sigma}{dyd^2\mathbf{q}_T} = \frac{\sigma_{gg\rightarrow H}}{(2\pi)^2} \int d^2\mathbf{b} e^{-i(\mathbf{b}\mathbf{q}_T)} \left( f_{1,g}(x_A, \mathbf{b})f_{1,g}(x_B, \mathbf{b}) + h_{1,g}^\perp(x_A, \mathbf{b})h_{1,g}^\perp(x_B, \mathbf{b}) \right)$$

- 🍏 Despite linearly polarised gluons enter at NNLL, their effect on the cross section can be assessed, particularly at low  $q_T$ , but **below current LHC data accuracy**:

Influence of  $h_1^\perp h_1^\perp$  in  $pp \rightarrow H + X$



$pp \rightarrow H(\rightarrow \gamma\gamma) + X$





# Conclusions

- 🍏 **TMD factorisation** provides a valuable tool to describe  $q_T$  distributions at small values of  $q_T$  (resummation of large logs),
  - 🍏 written in terms of **TMD distributions**,
- 🍏 Non-perturbative component of TMDs is to be determined from **data**.
- 🍏 A lot of effort is being invested on the extraction of TMD PDFs and FFs:
  - 🍏 tremendous progress made over the past few years,
  - 🍏 wide and precise **datasets** (COMPASS, HERMES, LHC and Tevatron exps.),
  - 🍏 more data to come from the LHC,
  - 🍏 state-of-the-art **theoretical computation** moving to even higher accuracy,
  - 🍏 looking forward to the **EIC** to pin down TMDs to unprecedented accuracy.

**Backup**

# Logarithmic counting

- 🍏 TMD factorisation provides **resummation** of large logs  $L = \log(q_T/Q)$ :
  - 🍏 implemented through the **Sudakov** form factor  $R$ .

- 🍏 A **perturbative expansion** in powers of  $\alpha_s$  of  $R$  would give:

One Sudakov for each TMD  $\longrightarrow R^2 = \sum_{n=0}^{\infty} \alpha_s^n \sum_{k=1}^{2n} \tilde{S}^{(n,k)} L^k$  Double-log expansion

- 🍏 that can be rearranged as:

$$R^2 = \sum_{m=0}^{\infty} R_{N^m LL}^2 \quad \text{with} \quad R_{N^m LL}^2 = \sum_{n=[m/2]}^{\infty} \tilde{S}^{(n, 2n-m)} \alpha_s^n L^{2n-m}$$

Integer part of  $m/2$

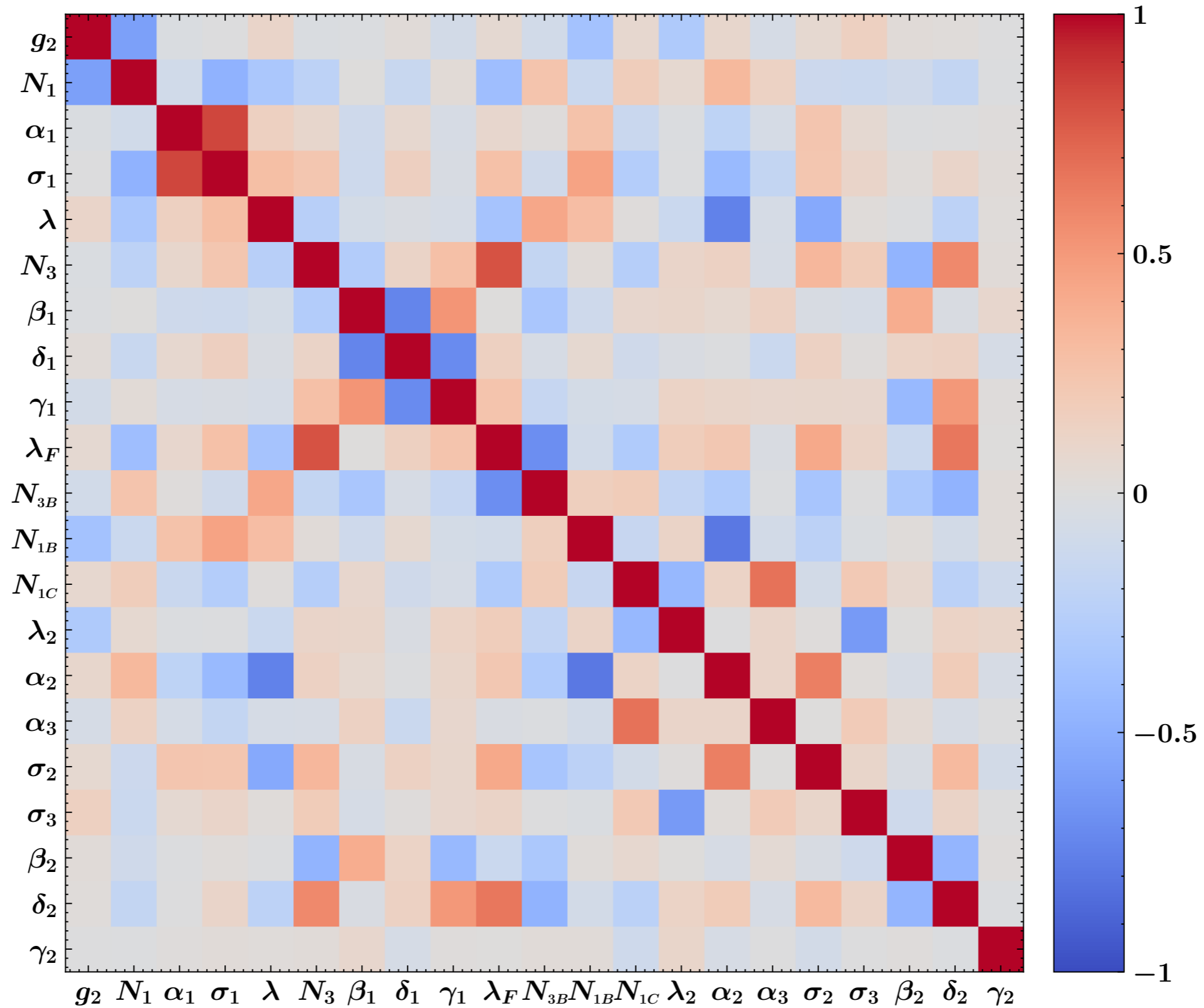
- 🍏 Therefore, multiplying  $R$  by a power  $p$  of  $\alpha_s$  gives:

$$\alpha_s^p R_{N^m LL}^2 = \sum_{j=[(m+2p)/2]}^{\infty} \tilde{S}^{(j-p, 2j-(m+2p))} \alpha_s^j L^{2j-(m+2p)} \sim R_{N^{m+2p} LL}^2$$

- 🍏 Bottom line: any additional power of  $\alpha_s$  causes a shift of **two units** in the logarithmic ordering.

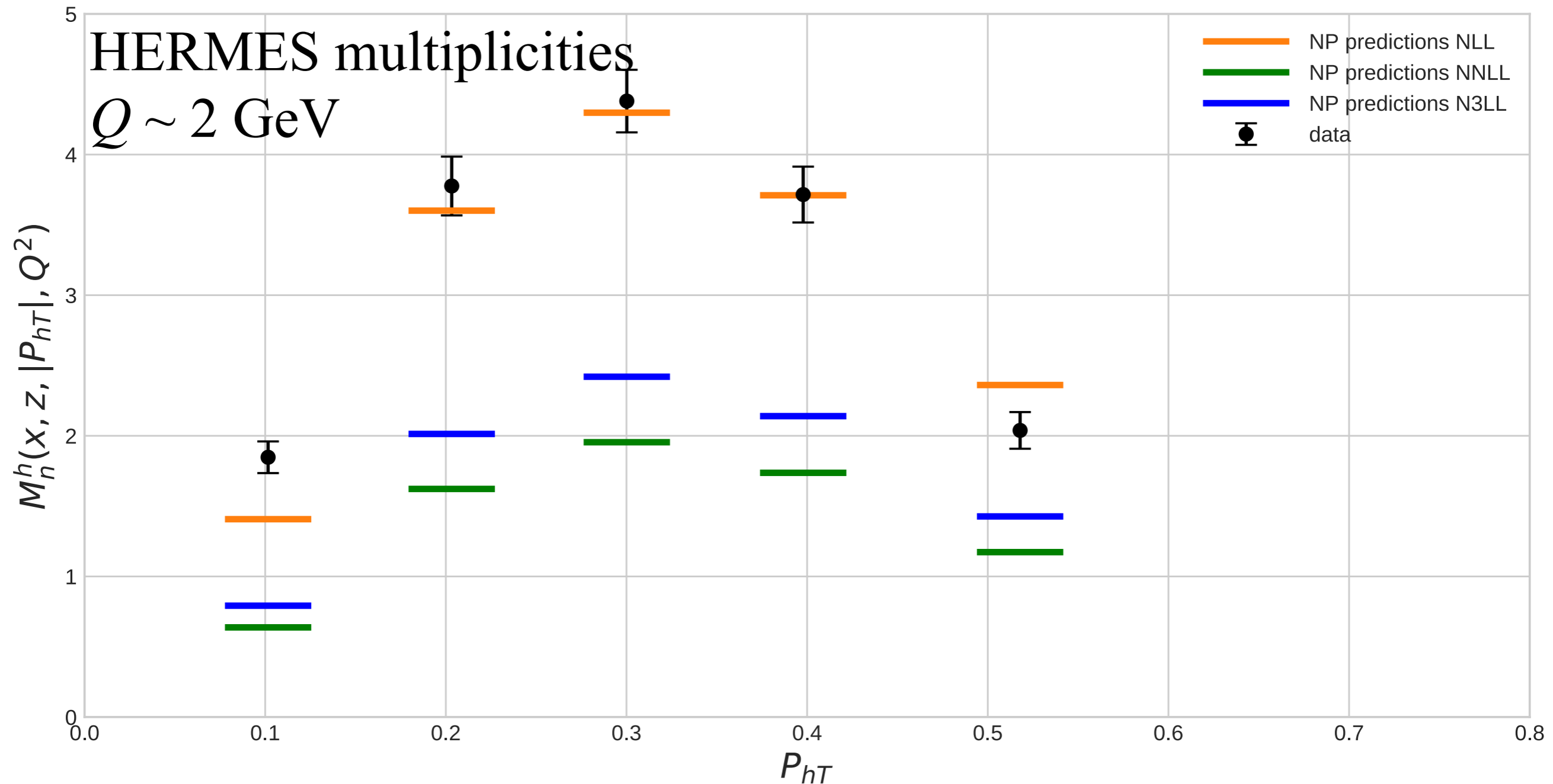
# MAPTMD 2022

## *Correlation between fit parameters*



# MAPTMD 2022

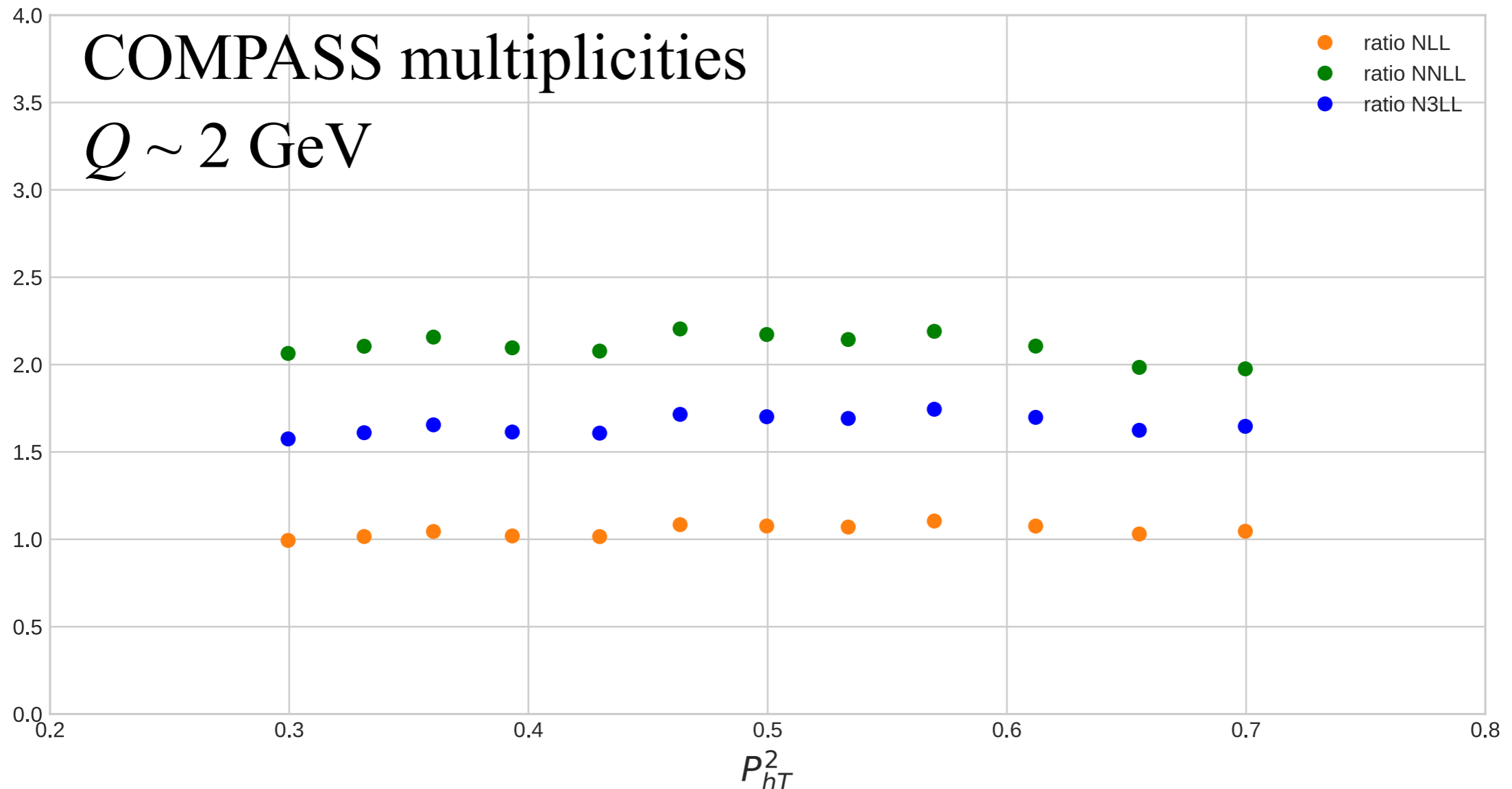
## *Normalisation of SIDIS*



🍏 Description of SIDIS multiplicities considerably worsens moving from NLL to higher perturbative orders.

# MAPTMD 2022

## *Normalisation of SIDIS*



🍏 Normalisation problem already observed in the literature.

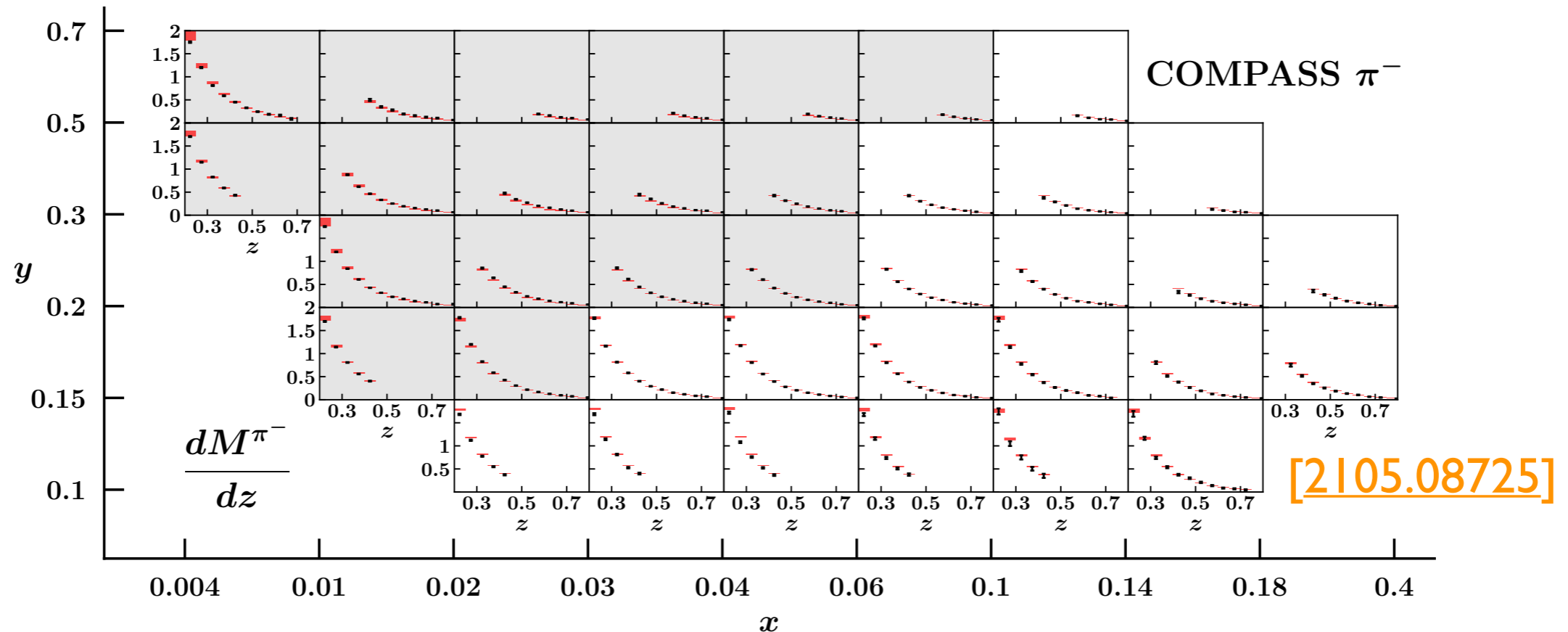
🍏 Large perturbative corrections particularly to the hard function:

$$H_{\text{SIDIS}}(Q) = 1 + \frac{\alpha_s(Q)}{4\pi} \underbrace{C_F \left( -16 + \frac{\pi^2}{3} \right)}_{\sim -17} + \mathcal{O}(\alpha_s^2)$$

# MAPTMD 2022

## Normalisation of SIDIS

- 🍏 SIDIS multiplicity:  $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} / \frac{d\sigma}{dx dQ}$
- 🍏 The SIDIS cross section integrated over  $P_{hT}$  ( $d\sigma/dx dQ dz$ ) is ok.



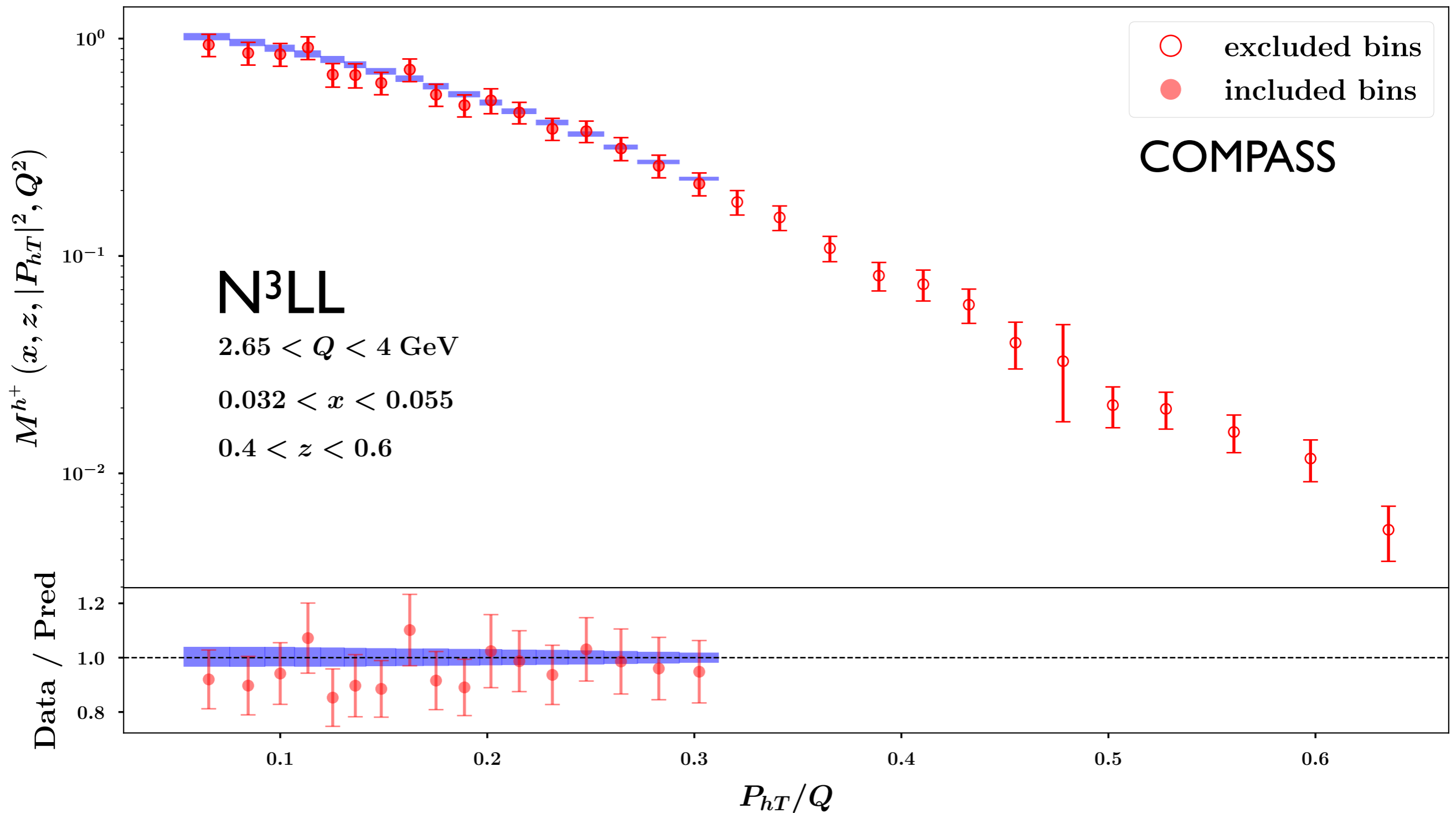
- 🍏 Normalise predictions such that integral over  $P_{hT}$  gives  $d\sigma/dx dQ dz$ :

$$M(x, z, P_{hT}, Q) = \mathcal{N} \frac{\frac{d\sigma}{dx dQ dz dP_{hT}}}{\frac{d\sigma}{dx dQ}} \quad \mathcal{N} = \frac{\frac{d\sigma}{dx dQ dz}}{\int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}}$$

- 🍏 Theoretically determined normalisation, **not fitted**.

# MAPTMD 2022

## *Normalisation of SIDIS*

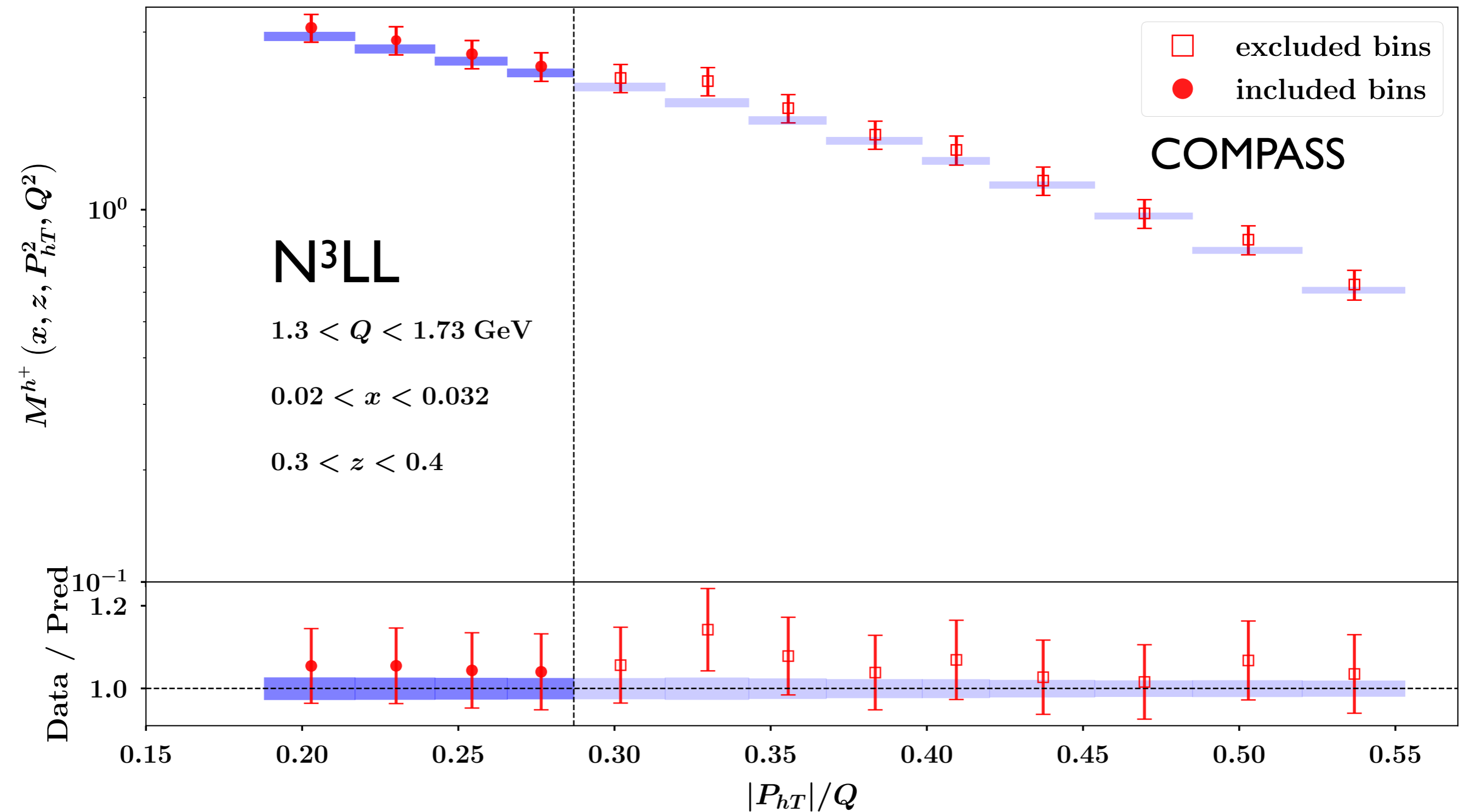


🍏 Excellent agreement upon normalisation.



# MAPTMD 2022

## *Normalisation of SIDIS*

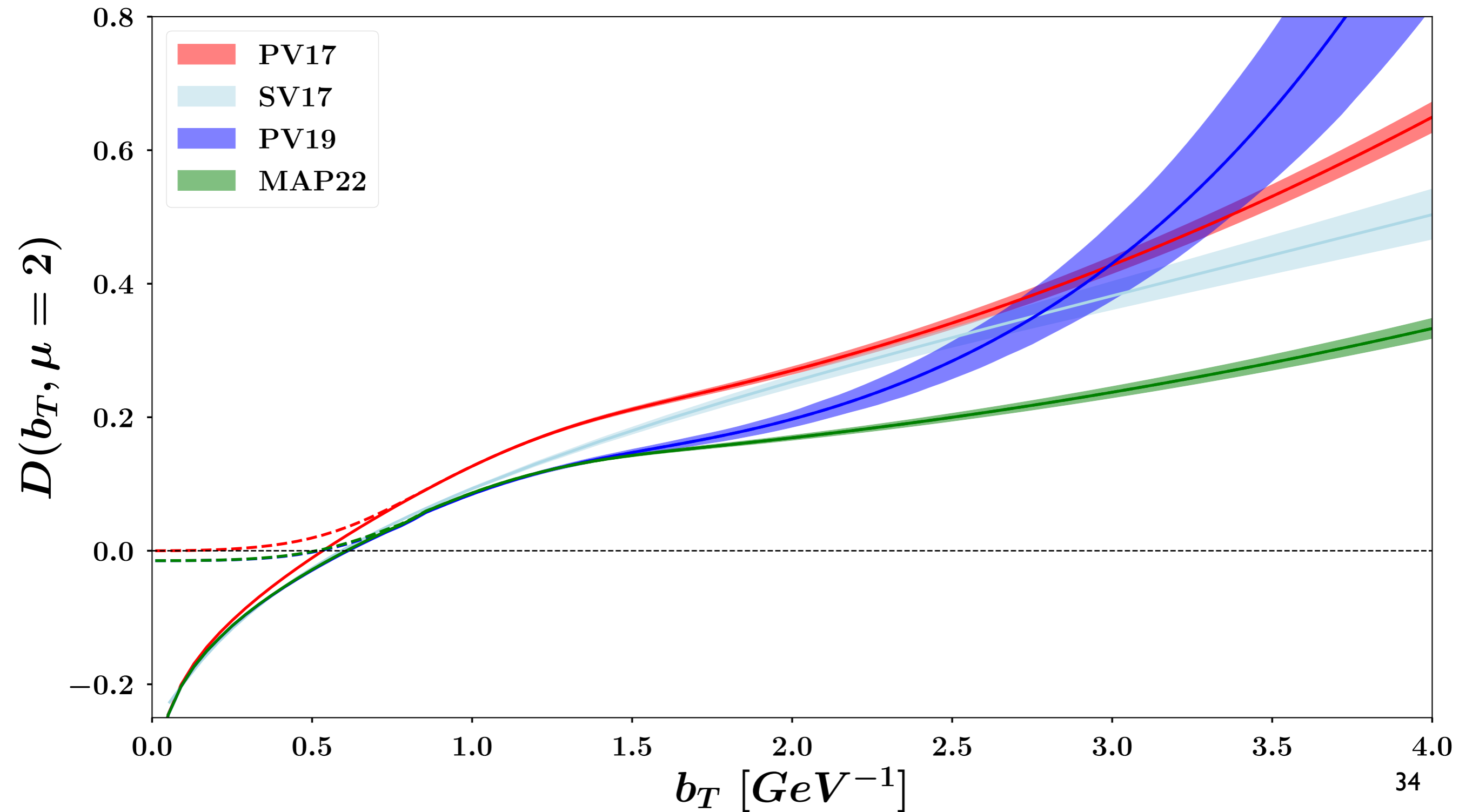


Agreement extends well above the expected validity region. To be clarified.

# MAPTMD 2022

*Collins-Soper kernel*

$$D(b_T, \mu) = -K(b_*(b_T), \mu) - g_K(b_T)$$



# Pavia2019

## Dataset

 DY data only:

 fixed-target low-energy DY,

 STAR data

 LHC and Tevatron data,

 353 data points,

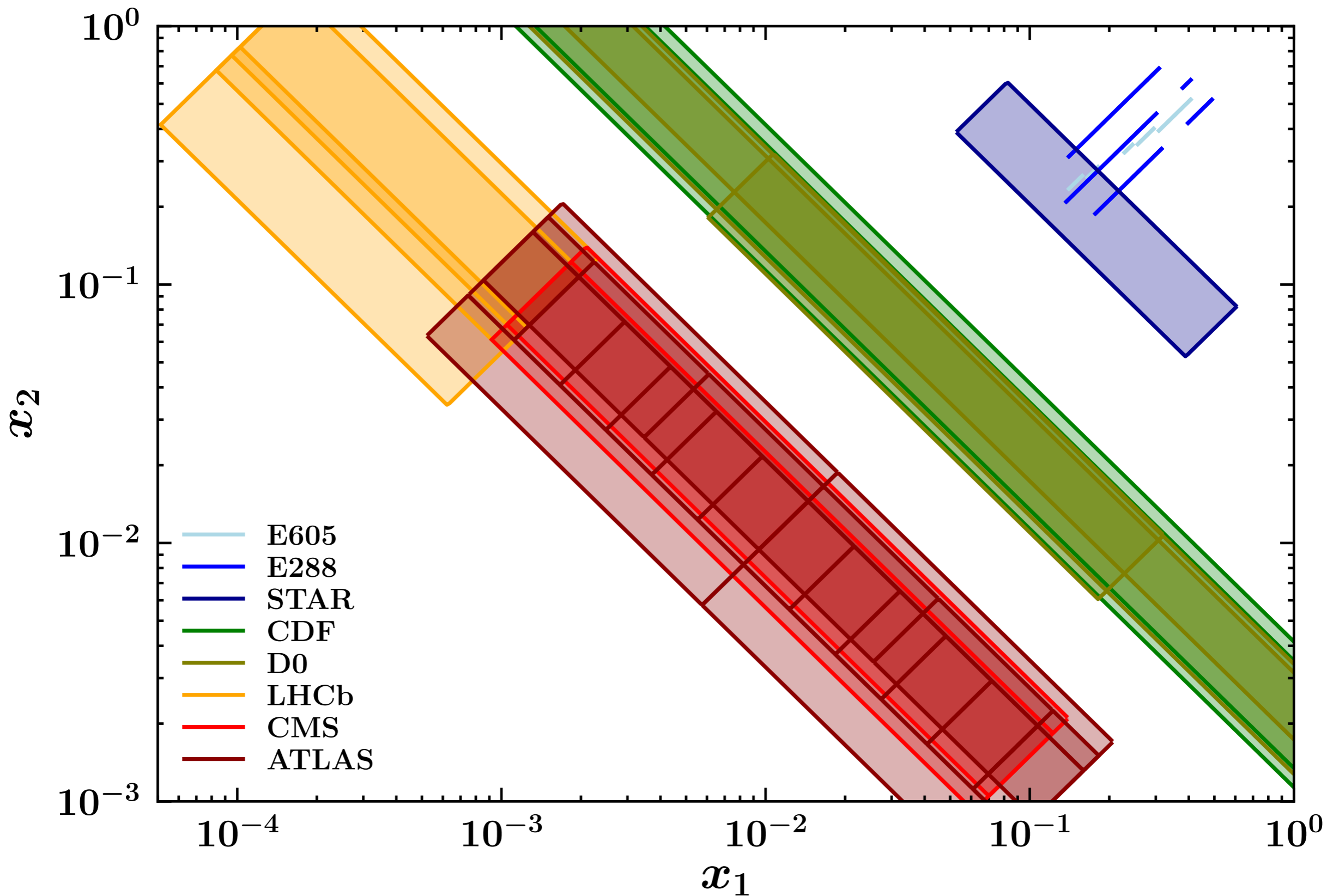
 selection cut  $q_T / Q < 0.2$ .

Experiment	$N_{\text{dat}}$	Observable	$\sqrt{s}$ [GeV]	$Q$ [GeV]	$y$ or $x_F$	Lepton cuts	Ref.
E605	50	$Ed^3\sigma/d^3q$	38.8	7 - 18	$x_F = 0.1$	-	[79]
E288 200 GeV	30	$Ed^3\sigma/d^3q$	19.4	4 - 9	$y = 0.40$	-	[80]
E288 300 GeV	39	$Ed^3\sigma/d^3q$	23.8	4 - 12	$y = 0.21$	-	[80]
E288 400 GeV	61	$Ed^3\sigma/d^3q$	27.4	5 - 14	$y = 0.03$	-	[80]
STAR 510	7	$d\sigma/dq_T$	510	73 - 114	$ y  < 1$	$p_{T\ell} > 25$ GeV $ \eta_\ell  < 1$	-
CDF Run I	25	$d\sigma/dq_T$	1800	66 - 116	Inclusive	-	[81]
CDF Run II	26	$d\sigma/dq_T$	1960	66 - 116	Inclusive	-	[82]
D0 Run I	12	$d\sigma/dq_T$	1800	75 - 105	Inclusive	-	[83]
D0 Run II	5	$(1/\sigma)d\sigma/dq_T$	1960	70 - 110	Inclusive	-	[84]
D0 Run II ( $\mu$ )	3	$(1/\sigma)d\sigma/dq_T$	1960	65 - 115	$ y  < 1.7$	$p_{T\ell} > 15$ GeV $ \eta_\ell  < 1.7$	[85]
LHCb 7 TeV	7	$d\sigma/dq_T$	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[86]
LHCb 8 TeV	7	$d\sigma/dq_T$	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[87]
LHCb 13 TeV	7	$d\sigma/dq_T$	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[92]
CMS 7 TeV	4	$(1/\sigma)d\sigma/dq_T$	7000	60 - 120	$ y  < 2.1$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.1$	[88]
CMS 8 TeV	4	$(1/\sigma)d\sigma/dq_T$	8000	60 - 120	$ y  < 2.1$	$p_{T\ell} > 15$ GeV $ \eta_\ell  < 2.1$	[89]
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/dq_T$	7000	66 - 116	$ y  < 1$ $1 <  y  < 2$ $2 <  y  < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.4$	[93]
ATLAS 8 TeV on-peak	6 6 6 6 6	$(1/\sigma)d\sigma/dq_T$	8000	66 - 116	$ y  < 0.4$ $0.4 <  y  < 0.8$ $0.8 <  y  < 1.2$ $1.2 <  y  < 1.6$ $1.6 <  y  < 2$ $2 <  y  < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.4$	[90]
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/dq_T$	8000	46 - 66 116 - 150	$ y  < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.4$	[90]
Total	353	-	-	-	-	-	-

# Pavia2019

*Kinematic coverage*

$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

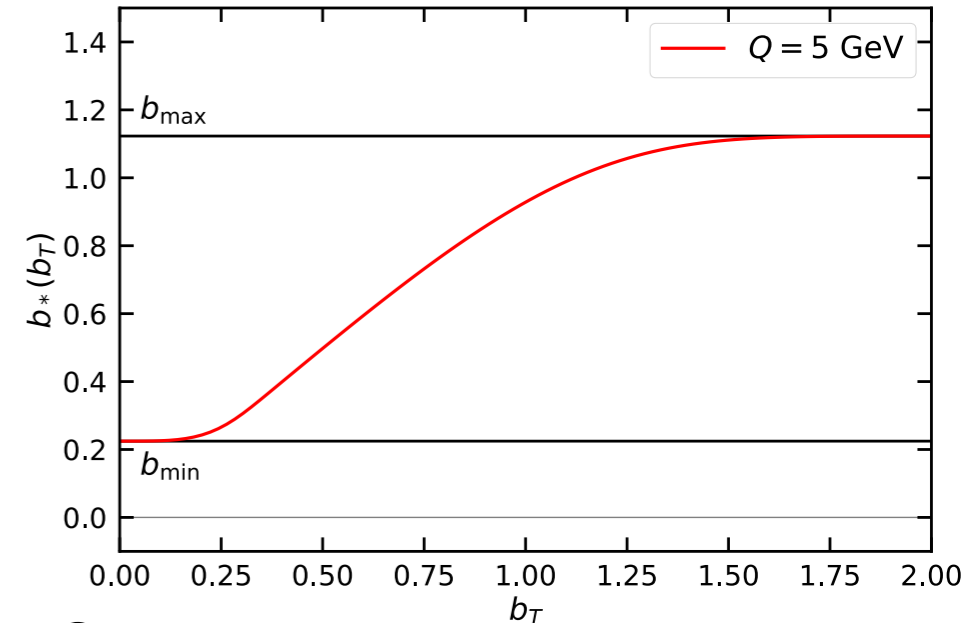


# Pavia2019

## Main settings

🍏  $b^*$  prescription:

$$b_*(b_T) = b_{\max} \left( \frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4} \quad \text{with} \quad \begin{cases} b_{\max} = 2e^{-\gamma_E} \\ b_{\min} = b_{\max}/Q \end{cases}$$



🍏 Non-perturbative function  $f_{\text{NP}}$ :

🍏 evolution:

$$g_K(b_T) = - (g_2 + g_{2B} b_T^2) \frac{b_T^2}{2}$$

🍏 PDFs:

$$\tilde{f}_{\text{NP}}(x, b_T) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right]$$

$$g_1(x) = \frac{N_1}{x\sigma} \exp \left[ -\frac{1}{2\sigma^2} \ln^2 \left( \frac{x}{\alpha} \right) \right] \quad g_{1B}(x) = \frac{N_{1B}}{x\sigma_B} \exp \left[ -\frac{1}{2\sigma_B^2} \ln^2 \left( \frac{x}{\alpha_B} \right) \right]$$

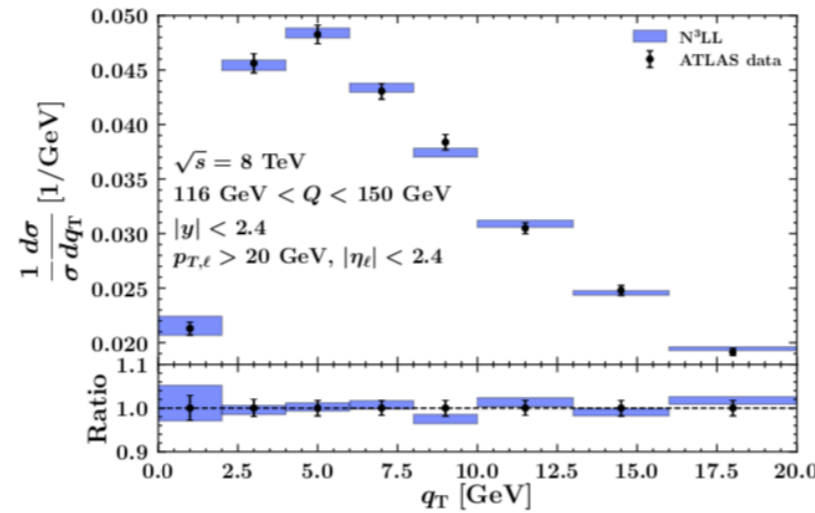
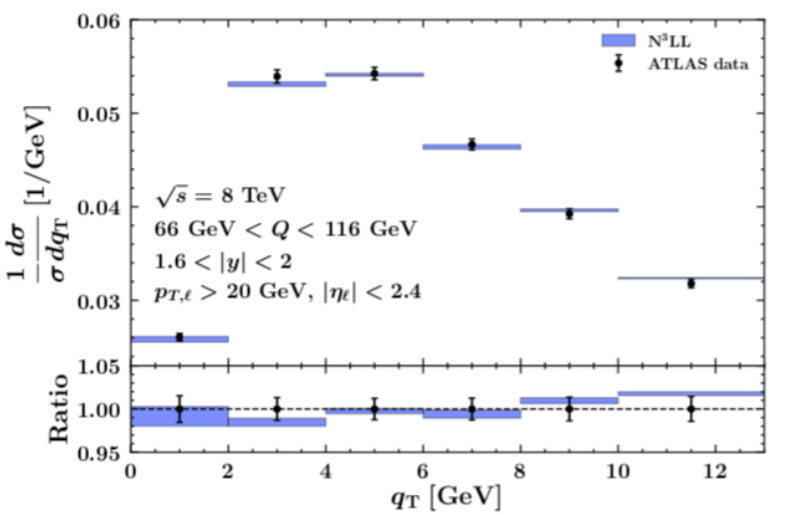
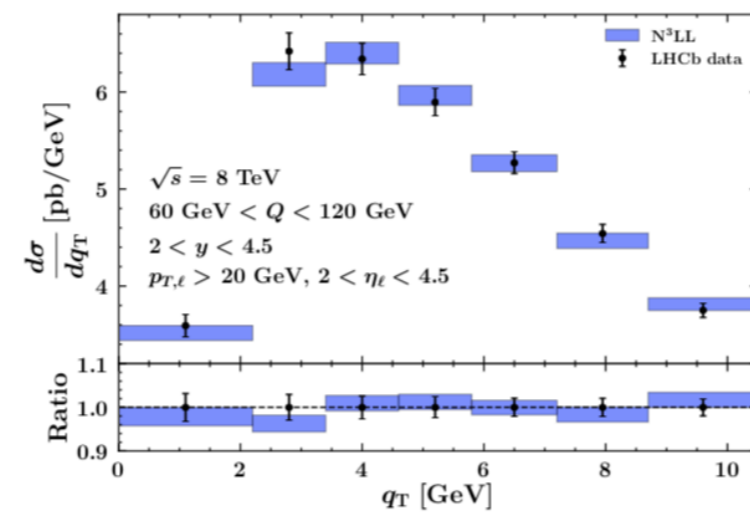
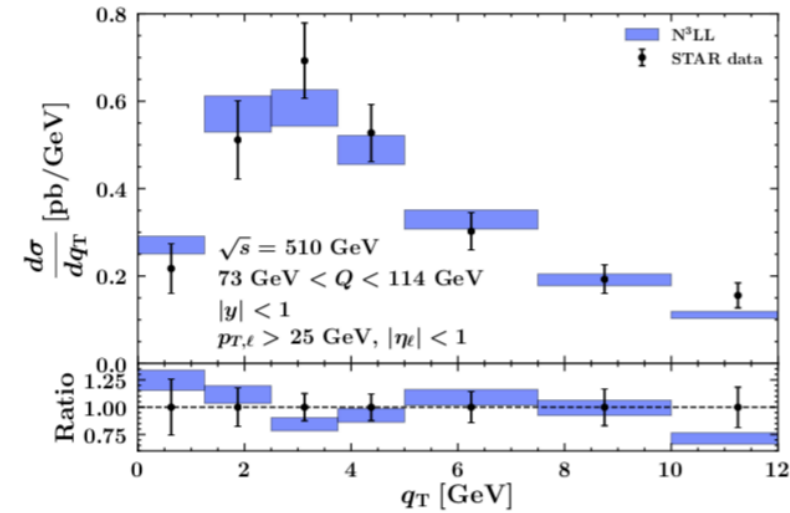
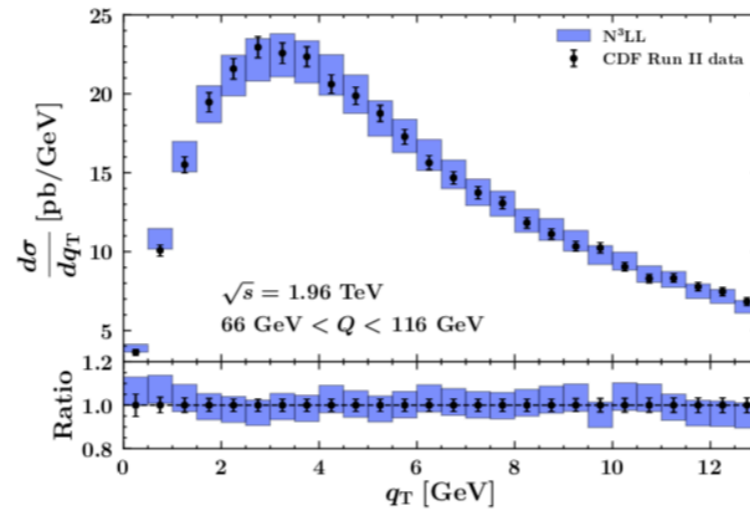
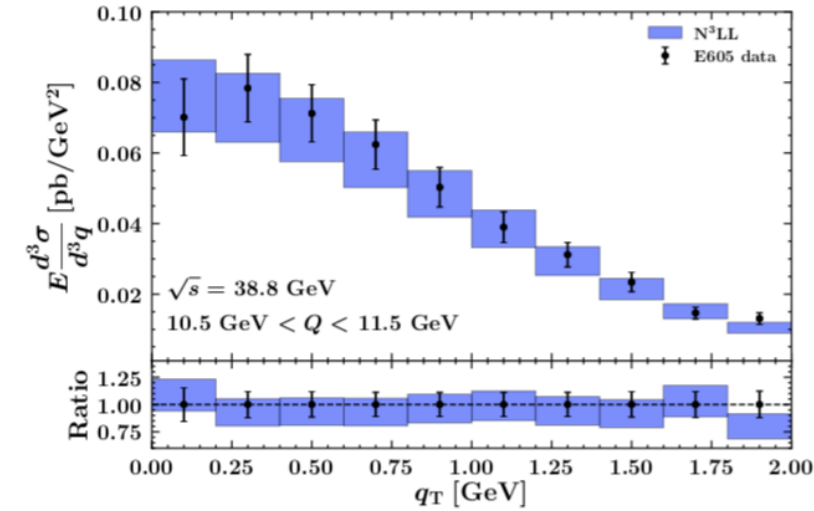
🍏 **9 free parameters** to fit to data.

🍏 Perturbative accuracies: **NLL'**, **NNLL**, **NNLL'**, **N<sup>3</sup>LL**

🍏 **Monte Carlo** method for the experimental error propagation.

# Pavia2019

## Fit quality

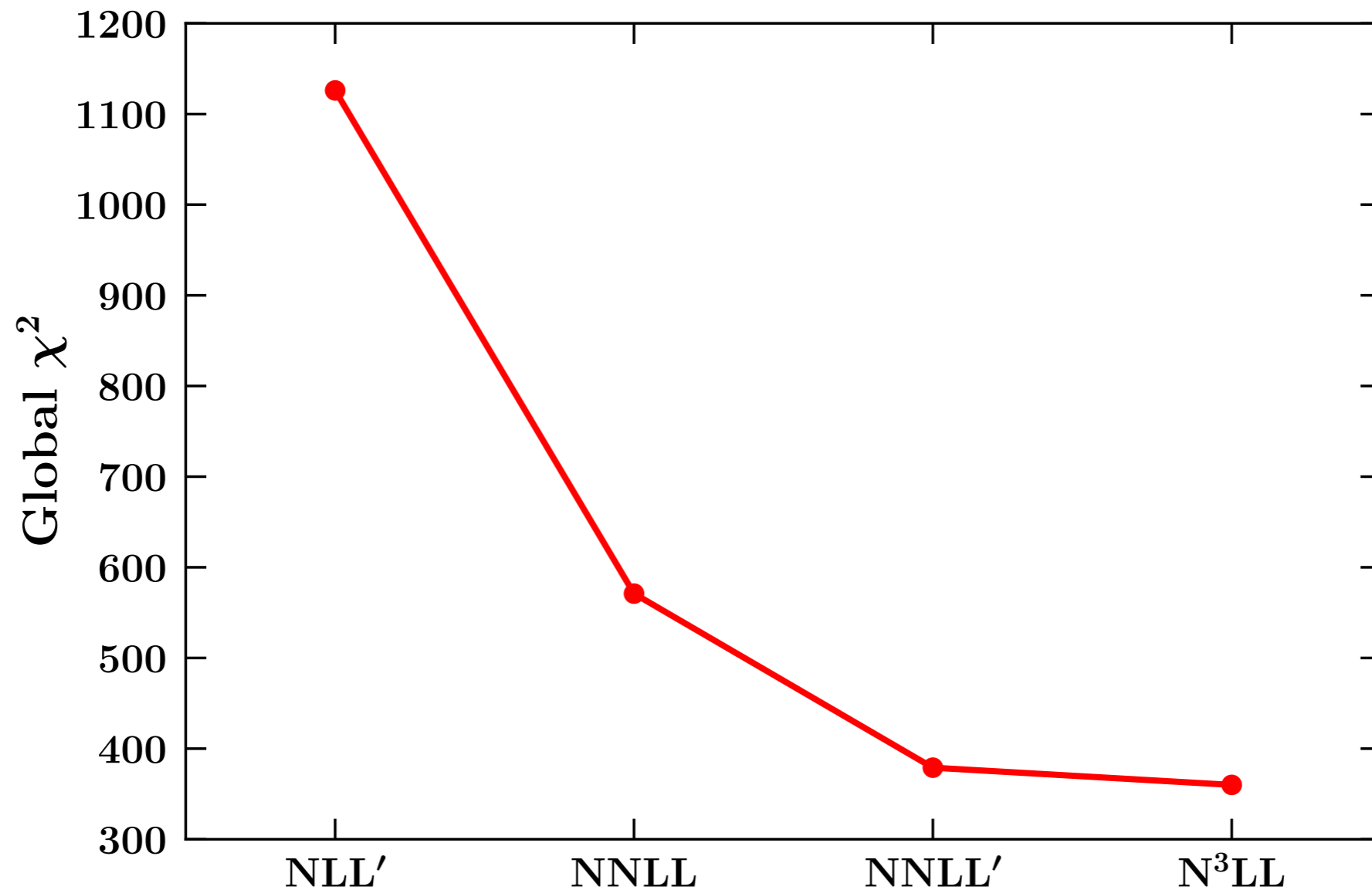


Experiment		$\chi_D^2/N_{\text{dat}}$	$\chi_\lambda^2/N_{\text{dat}}$	$\chi^2/N_{\text{dat}}$	
E605	7 GeV < Q < 8 GeV	0.419	0.068	0.487	
	8 GeV < Q < 9 GeV	0.995	0.034	1.029	
	10.5 GeV < Q < 11.5 GeV	0.191	0.137	0.328	
	11.5 GeV < Q < 13.5 GeV	0.491	0.284	0.775	
	13.5 GeV < Q < 18 GeV	0.491	0.385	0.877	
E288 200 GeV	4 GeV < Q < 5 GeV	0.213	0.649	0.862	
	5 GeV < Q < 6 GeV	0.673	0.292	0.965	
	6 GeV < Q < 7 GeV	0.133	0.141	0.275	
	7 GeV < Q < 8 GeV	0.254	0.014	0.268	
	8 GeV < Q < 9 GeV	0.652	0.024	0.676	
E288 300 GeV	4 GeV < Q < 5 GeV	0.231	0.555	0.785	
	5 GeV < Q < 6 GeV	0.502	0.204	0.706	
	6 GeV < Q < 7 GeV	0.315	0.063	0.378	
	7 GeV < Q < 8 GeV	0.056	0.030	0.086	
	8 GeV < Q < 9 GeV	0.530	0.017	0.547	
E288 400 GeV	11 GeV < Q < 12 GeV	1.047	0.167	1.215	
	5 GeV < Q < 6 GeV	0.312	0.065	0.377	
	6 GeV < Q < 7 GeV	0.100	0.005	0.105	
	7 GeV < Q < 8 GeV	0.018	0.011	0.029	
	8 GeV < Q < 9 GeV	0.437	0.039	0.477	
STAR	11 GeV < Q < 12 GeV	0.637	0.036	0.673	
	12 GeV < Q < 13 GeV	0.788	0.028	0.816	
	13 GeV < Q < 14 GeV	1.064	0.044	1.107	
STAR		0.782	0.054	0.836	
CDF Run I		0.480	0.058	0.538	
CDF Run II		0.959	0.001	0.959	
D0 Run I		0.711	0.043	0.753	
D0 Run II		1.325	0.612	1.937	
D0 Run II ( $\mu$ )		3.196	0.023	3.218	
LHCb 7 TeV		1.069	0.194	1.263	
LHCb 8 TeV		0.460	0.075	0.535	
LHCb 13 TeV		0.735	0.020	0.755	
CMS 7 TeV		2.131	0.000	2.131	
CMS 8 TeV		1.405	0.007	1.412	
ATLAS 7 TeV	0 <  y  < 1	2.581	0.028	2.609	
	1 <  y  < 2	4.333	1.032	5.365	
	2 <  y  < 2.4	3.561	0.378	3.939	
ATLAS 8 TeV	0 <  y  < 0.4	1.924	0.337	2.262	
	0.4 <  y  < 0.8	2.342	0.247	2.590	
	0.8 <  y  < 1.2	0.917	0.061	0.978	
	on-peak	1.2 <  y  < 1.6	0.912	0.095	1.006
		1.6 <  y  < 2	0.721	0.092	0.814
		2 <  y  < 2.4	0.932	0.348	1.280
ATLAS 8 TeV	46 GeV < Q < 66 GeV	2.138	0.745	2.883	
off-peak	116 GeV < Q < 150 GeV	0.501	0.003	0.504	
<b>Global</b>		<b>0.88</b>	<b>0.14</b>	<b>1.02</b>	

# Pavia2019

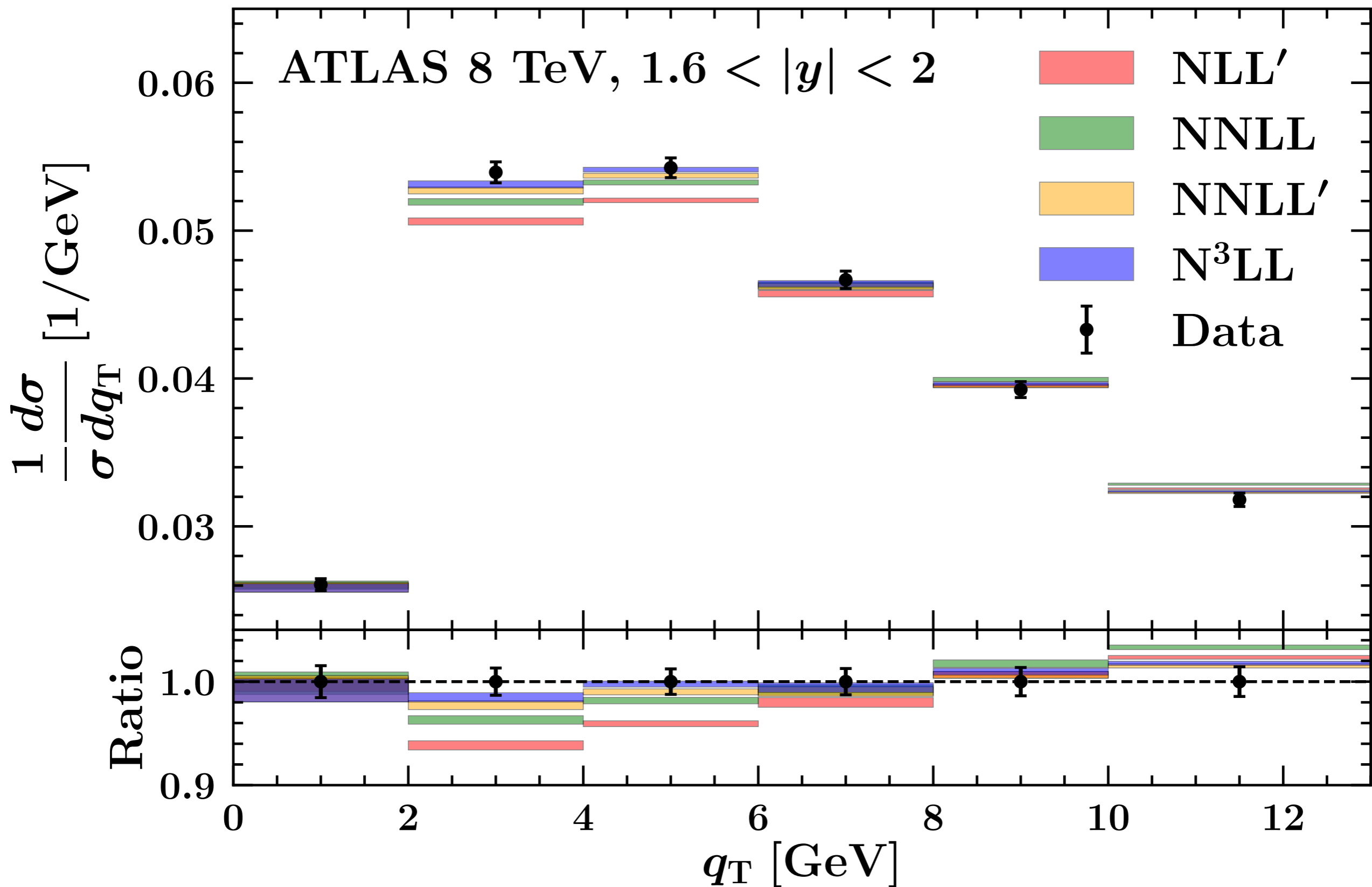
## *Perturbative convergence*

	NLL'	NNLL	NNLL'	N <sup>3</sup> LL
Global $\chi^2$	1126	571	379	360



# Pavia2019

## *Perturbative convergence*





# SV2019

## Dataset

- 🍏 Both DY and SIDIS data:
  - 🍏 fixed-target low-energy DY,
  - 🍏 PHENIX data,
  - 🍏 LHC and Tevatron data,
  - 🍏 HERMES and COMPASS,
  - 🍏 457 + 582 = 1039 data points.

**SIDIS**  $\langle Q \rangle \geq 2\text{GeV}$   $\delta \equiv \frac{\langle q_T \rangle}{\langle Q \rangle} < 0.25$

Experiment	Reaction	ref.	Kinematics	$N_{\text{pt}}$ after cuts	
HERMES	$p \rightarrow \pi^+$	[58]	$0.023 < x < 0.6$ (6 bins)	24	
	$p \rightarrow \pi^-$			24	
	$p \rightarrow K^+$			24	
	$p \rightarrow K^-$			24	
	$D \rightarrow \pi^+$		$0.2 < z < 0.8$ (6 bins)	$1.0 < Q < \sqrt{20}\text{GeV}$	24
	$D \rightarrow \pi^-$				24
	$D \rightarrow K^+$				24
	$D \rightarrow K^-$				24
COMPASS	$d \rightarrow h^+$	[59]	$W^2 > 10\text{GeV}^2$	195	
	$d \rightarrow h^-$		$0.1 < y < 0.85$	195	
Total				582	

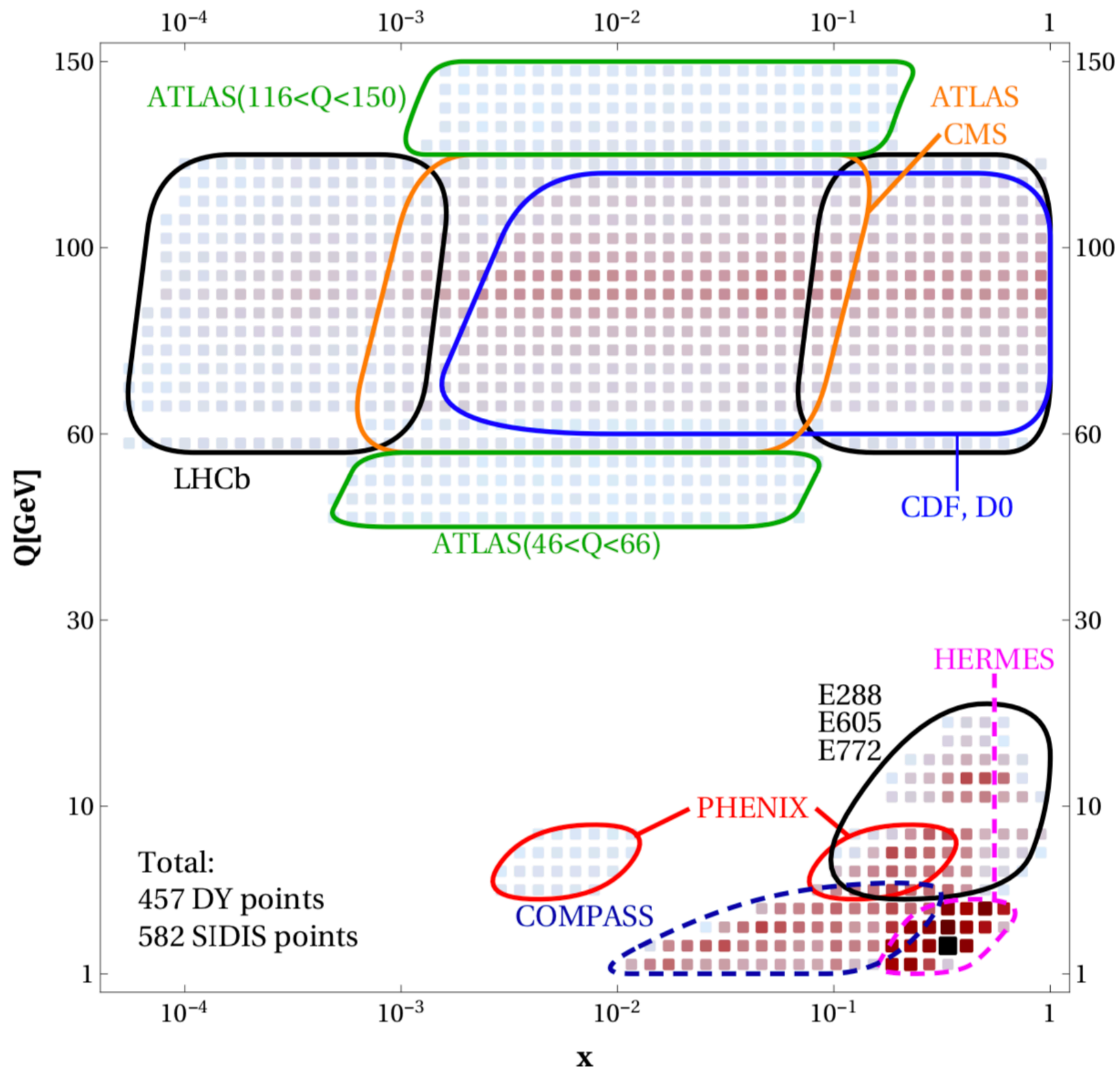
**DY**  $\delta \equiv \frac{\langle q_T \rangle}{\langle Q \rangle} < 0.1$   $\delta < 0.25$  if  $\delta^2 < \sigma$

Experiment	ref.	$\sqrt{s}$ [GeV]	$Q$ [GeV]	$y/x_F$	fiducial region	$N_{\text{pt}}$ after cuts
E288 (200)	[64]	19.4	4 - 9 in 1 GeV bins*	$0.1 < x_F < 0.7$	-	43
E288 (300)	[64]	23.8	4 - 12 in 1 GeV bins*	$-0.09 < x_F < 0.51$	-	53
E288 (400)	[64]	27.4	5 - 14 in 1 GeV bins*	$-0.27 < x_F < 0.33$	-	76
E605	[65]	38.8	7 - 18 in 5 bins*	$-0.1 < x_F < 0.2$	-	53
E772	[66]	38.8	5 - 15 in 8 bins*	$0.1 < x_F < 0.3$	-	35
PHENIX	[67]	200	4.8 - 8.2	$1.2 < y < 2.2$	-	3
CDF (run1)	[68]	1800	66 - 116	-	-	33
CDF (run2)	[69]	1960	66 - 116	-	-	39
D0 (run1)	[70]	1800	75 - 105	-	-	16
D0 (run2)	[71]	1960	70 - 110	-	-	8
D0 (run2) $_{\mu}$	[72]	1960	65 - 115	$ y  < 1.7$	$p_T > 15\text{ GeV}$ $ \eta  < 1.7$	3
ATLAS (7TeV)	[45]	7000	66 - 116	$ y  < 1$ $1 <  y  < 2$ $2 <  y  < 2.4$	$p_T > 20\text{ GeV}$ $ \eta  < 2.4$	15
ATLAS (8TeV)	[46]	8000	66 - 116	$ y  < 2.4$ in 6 bins	$p_T > 20\text{ GeV}$ $ \eta  < 2.4$	30
ATLAS (8TeV)	[46]	8000	46 - 66	$ y  < 2.4$	$p_T > 20\text{ GeV}$ $ \eta  < 2.4$	3
ATLAS (8TeV)	[46]	8000	116 - 150	$ y  < 2.4$	$p_T > 20\text{ GeV}$ $ \eta  < 2.4$	7
CMS (7TeV)	[47]	7000	60 - 120	$ y  < 2.1$	$p_T > 20\text{ GeV}$ $ \eta  < 2.1$	8
CMS (8TeV)	[48]	8000	60 - 120	$ y  < 2.1$	$p_T > 20\text{ GeV}$ $ \eta  < 2.1$	8
LHCb (7TeV)	[73]	7000	60 - 120	$2 < y < 4.5$	$p_T > 20\text{ GeV}$ $2 < \eta < 4.5$	8
LHCb (8TeV)	[74]	8000	60 - 120	$2 < y < 4.5$	$p_T > 20\text{ GeV}$ $2 < \eta < 4.5$	7
LHCb (13TeV)	[75]	13000	60 - 120	$2 < y < 4.5$	$p_T > 20\text{ GeV}$ $2 < \eta < 4.5$	9
Total						457

\*Bins with  $9 \lesssim Q \lesssim 11$  are omitted due to the  $\Upsilon$  resonance.

# SV2019

## *Kinematic coverage*



# SV2019

## Main settings

🍏  $b_*$  prescription:

$$b_*(b_T) = \sqrt{\frac{b_T^2 B_{\text{NP}}^2}{b_T^2 + B_{\text{NP}}^2}}$$

🍏 Non-perturbative function  $f_{\text{NP}}$ :

🍏 evolution:

$$g_K(b_T) = -c_0 b_T b_*(b_T) \rightarrow \begin{cases} -c_0 b_T^2 & \text{for } b_T \rightarrow 0 \\ -c_0 B_{\text{NP}} b_T & \text{for } b_T \rightarrow \infty \end{cases}$$

🍏 PDFs and FFs:

$$f_{\text{NP}}(x, b) = \exp\left(-\frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4}} b^2} b^2\right)$$

$$D_{\text{NP}}(x, b) = \exp\left(-\frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1 + \eta_3(\mathbf{b}/z)^2}} \frac{\mathbf{b}^2}{z^2}\right) \left(1 + \eta_4 \frac{\mathbf{b}^2}{z^2}\right)$$

🍏 **11 free parameters** to fit to data.

🍏 Perturbative accuracies: **NNLL'** (**NNLO**), **N<sup>3</sup>LL(-)** (**N<sup>3</sup>LO**)

🍏 **Monte Carlo** method for the experimental error propagation.

# SV2019

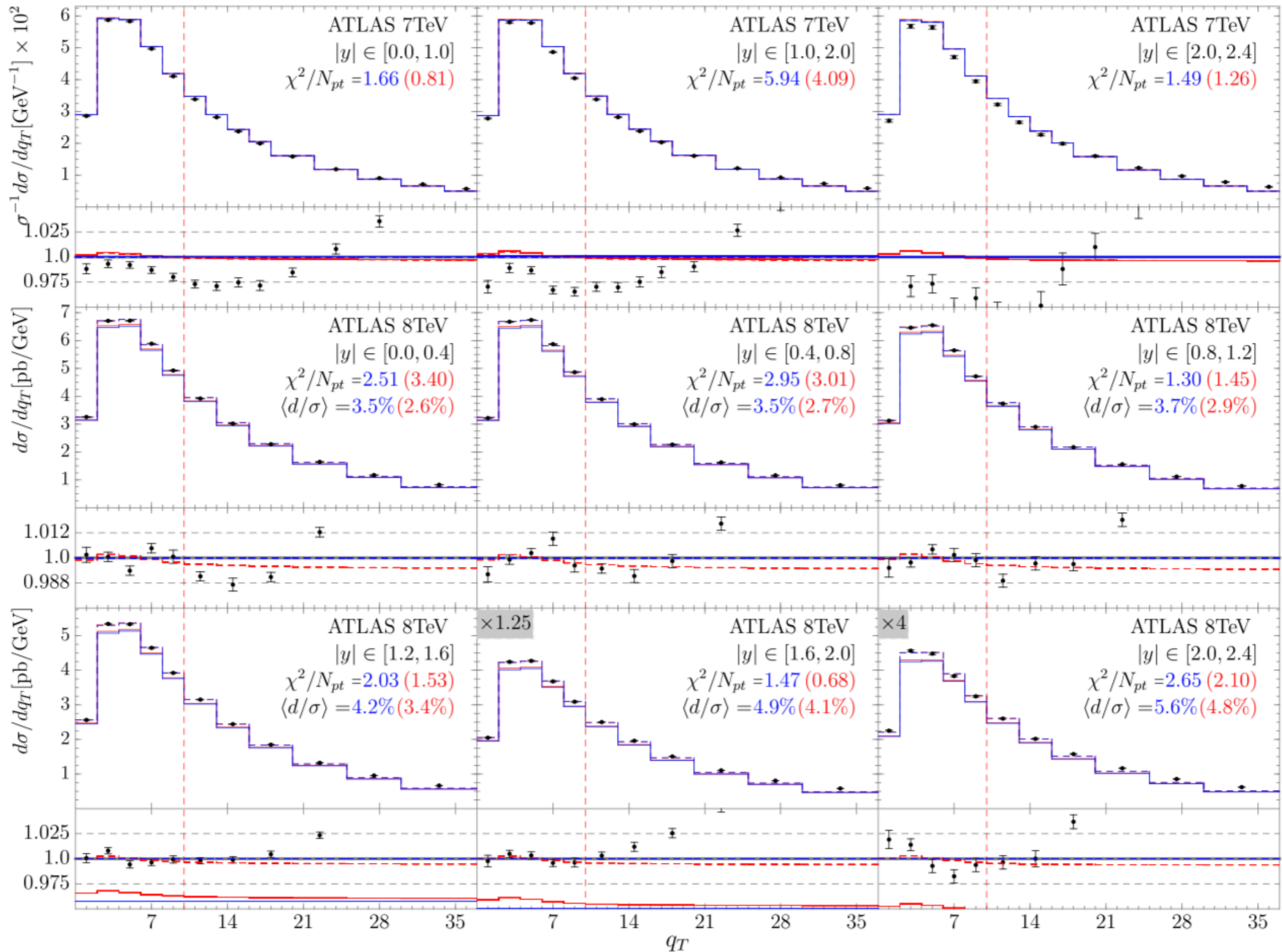
## *Fit quality*

- 🍏 Remarkably good total  $\chi^2$ ,
- 🍏 DY and SIDIS data are separately well described,
- 🍏 **Important achievement:**
  - 🍏 simultaneous description of SIDIS and DY data within the same fit at high perturbative order.

Data set	$N_{pt}$	NNLO		N <sup>3</sup> LO	
		$\chi^2/N_{pt}$	$\langle d/\sigma \rangle$	$\chi^2/N_{pt}$	$\langle d/\sigma \rangle$
CDF run1	33	0.66	8.4%	0.67	7.8%
CDF run2	39	1.28	2.8%	1.41	2.1%
D0 run1	16	0.72	0.1%	0.78	-0.5%
D0 run2	8	1.38	-	1.64	-
D0 run2 ( $\mu$ )	3	0.62	-	0.69	-
Tevatron total	99	0.97		1.06	
ATLAS 7TeV 0.0< y <1.0	5	1.66	-	0.81	-
ATLAS 7TeV 1.0< y <2.0	5	5.94	-	4.09	-
ATLAS 7TeV 2.0< y <2.4	5	1.49	-	1.26	-
ATLAS 8TeV 0.0< y <0.4	5	2.51	3.5%	3.40	2.8%
ATLAS 8TeV 0.4< y <0.8	5	2.95	3.5%	3.03	2.7%
ATLAS 8TeV 0.8< y <1.2	5	1.30	3.7%	1.45	2.9%
ATLAS 8TeV 1.2< y <1.6	5	2.03	4.2%	1.53	3.4%
ATLAS 8TeV 1.6< y <2.0	5	1.47	4.9%	0.70	4.1%
ATLAS 8TeV 2.0< y <2.4	5	2.64	5.6%	2.10	4.8%
ATLAS 8TeV 46<Q<66GeV	3	0.31	1.1%	0.31	0.2%
ATLAS 8TeV 116<Q<150GeV	7	0.84	1.9%	0.97	1.2%
ATLAS total	55	2.12		1.82	
CMS 7TeV	8	1.25	-	1.24	-
CMS 8TeV	8	0.77	-	0.76	-
CMS total	16	1.01		1.00	
LHCb 7TeV	8	2.68	5.8%	2.37	5.2%
LHCb 8TeV	7	4.81	5.8%	4.16	5.1%
LHCb 13TeV	9	0.91	6.4%	0.81	5.7%
LHCb total	24	2.63		2.31	
<b>High energy DY total</b>	<b>194</b>	<b>1.51</b>		<b>1.42</b>	
PHE200	3	0.28	0.2%	0.29	-0.3%
E228-200	43	1.00	35.7%	1.12	35.0%
E228-300	53	0.90	29.2%	1.01	28.3%
E228-400	76	0.86	20.6%	0.96	19.5%
E772	35	1.84	9.5%	1.91	8.5%
E605	53	0.57	21.3%	0.60	20.1%
<b>Low energy DY total</b>	<b>263</b>	<b>0.96</b>		<b>1.04</b>	
HERMES ( $p \rightarrow \pi^+$ )	24	2.20	1.7%	3.06	2.2%
HERMES ( $p \rightarrow \pi^-$ )	24	1.12	0.6%	1.45	0.9%
HERMES ( $p \rightarrow K^+$ )	24	0.71	-0.1%	0.66	0.0%
HERMES ( $p \rightarrow K^-$ )	24	0.69	0.0%	0.66	0.0%
HERMES ( $d \rightarrow \pi^+$ )	24	0.57	0.3%	0.78	0.8%
HERMES ( $d \rightarrow \pi^-$ )	24	0.74	0.5%	0.96	0.7%
HERMES ( $d \rightarrow K^+$ )	24	0.52	-0.1%	0.53	0.0%
HERMES ( $d \rightarrow K^-$ )	24	1.27	0.0%	1.17	0.1%
HERMES total	192	0.98		1.16	
COMPASS ( $d \rightarrow h^+$ )	195	0.61	3.3%	0.76	5.1%
COMPASS ( $d \rightarrow h^-$ )	195	0.68	-2.3%	0.92	-0.5%
COMPASS total	390	0.65		0.84	
<b>SIDIS total</b>	<b>582</b>	<b>0.76</b>		<b>0.95</b>	
<b>Total</b>	<b>1039</b>	<b>0.95</b>		<b>1.06</b>	

# SV2019

## Fit quality



# SV2019

## Fit quality

$$z^2 \times M(z, p_T)$$

$$d \rightarrow h^+$$

# COMPASS

