Chiral and trace anomalies in DVCS

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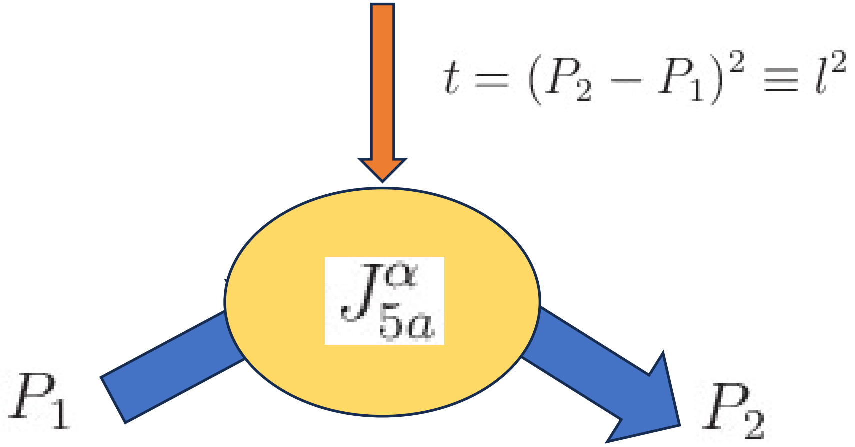
based on 2210.13419, 2305.09431
with Shohini Bhattacharya and Werner Vogelsang

Isovector axial form factors



Nucleon form factor of $J_{5a}^{\alpha} = \sum_q \bar{q}\gamma^{\alpha}\gamma_5 \frac{\tau^a}{2} q$

$$\langle P_2 | J_{5a}^{\alpha} | P_1 \rangle = \bar{u}(P_2) \left[\gamma^{\alpha}\gamma_5 F_A(t) + \frac{l^{\alpha}\gamma_5}{2M} F_P(t) \right] \frac{\tau^a}{2} u(P_1)$$

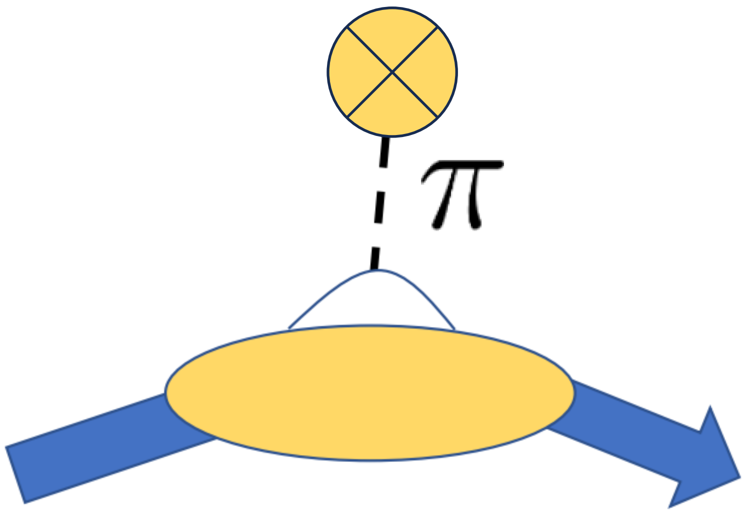
pseudovector pseudoscalar

In massless QCD, the current is exactly conserved $\partial_{\alpha} J_{5a}^{\alpha} = 0$

$$2MF_A(t) + \frac{tF_P(t)}{2M} = 0 \quad \Rightarrow \quad F_P(t) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

Pole at $t=0$ due to massless pion exchange

Pion pole in GPD



Generalized Parton Distributions
= x-dependent form factors

$$F_P(t) = \int_{-1}^{1} dx \left(\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \right) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

Massless pole already in GPD

$$\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \sim \theta(\xi - |x|) \frac{g_A^{(3)}}{t}$$

Penttinen, Polyakov, Goeke (1999)

First indication from lattice QCD? Bhattacharya, et al. (2023)

Singlet axial form factors

Form factor of $J_5^\alpha = \sum_q \bar{q}\gamma^\alpha\gamma_5 q$

$$\langle P_2|J_5^\alpha|P_1\rangle = \bar{u}(P_2)\left[\gamma^\alpha\gamma_5 g_A(t) + \frac{l^\alpha\gamma_5}{2M}g_P(t)\right]u(P_1)$$

In massless QCD, the current is classically conserved

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = 0 \quad \Rightarrow \quad \frac{g_P(t)}{2M} \approx -\frac{2M\Delta\Sigma}{t}$$

Pole at $t=0$ due to massless η_0 meson exchange?

Chiral anomaly

Quantum mechanically, the current is not conserved

$$\partial_\alpha J_5^\alpha = \frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\Rightarrow \quad \frac{g_P(t)}{2M} = \frac{1}{t}\left(i\frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F\tilde{F} | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2M g_A(t) \right)$$

anomaly pole $\qquad$ η_0 pole

In real QCD, there is no massless pole in $g_P(t)$ due to pole cancellation

Gravitational form factors

Nucleon matrix element of the energy momentum tensor $\Theta^{\alpha\beta}$

$$\langle P_2|\Theta^{\alpha\beta}|P_1\rangle = \bar{u}(P_2)\left[A(t)\frac{P^\alpha P^\beta}{M} + (A(t)+B(t))\frac{P^{(\alpha}i\sigma^{\beta)\lambda}l_\lambda}{2M} + D(t)\frac{l^\alpha l^\beta - g^{\alpha\beta}t}{4M}\right]u(P_1)$$

In massless QCD, $\Theta^{\alpha\beta}$ is classically traceless

$$A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t = 0 \qquad \frac{3}{4}D(t) \approx \frac{M^2}{t}A(t) \qquad (t \to 0)$$

Pole at $t=0$ due to massless spin-0 glueball exchange?

Trace anomaly

Quantum mechanically, the trace is nonzero

$$(\Theta)^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

$$\Rightarrow \quad \frac{3}{4} D(t) \approx -\frac{M}{t} \left(\frac{\langle P_2 | \frac{\beta(g)}{2g} F^2 | P_1 \rangle}{\bar{u}(P_2) u(P_1)} - M A(t) \right)$$

anomaly pole glueball pole

In real QCD, there is no massless pole in $D(t)$ due to pole cancellation

Anomalies relate form factors

Chiral anomaly

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

Trace anomaly

$$M\left(A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t\right)\bar{u}(P_2)u(P_1) = \langle P_2|\frac{\beta(g)}{2g}F^{\mu\nu}F_{\mu\nu}|P_1\rangle$$

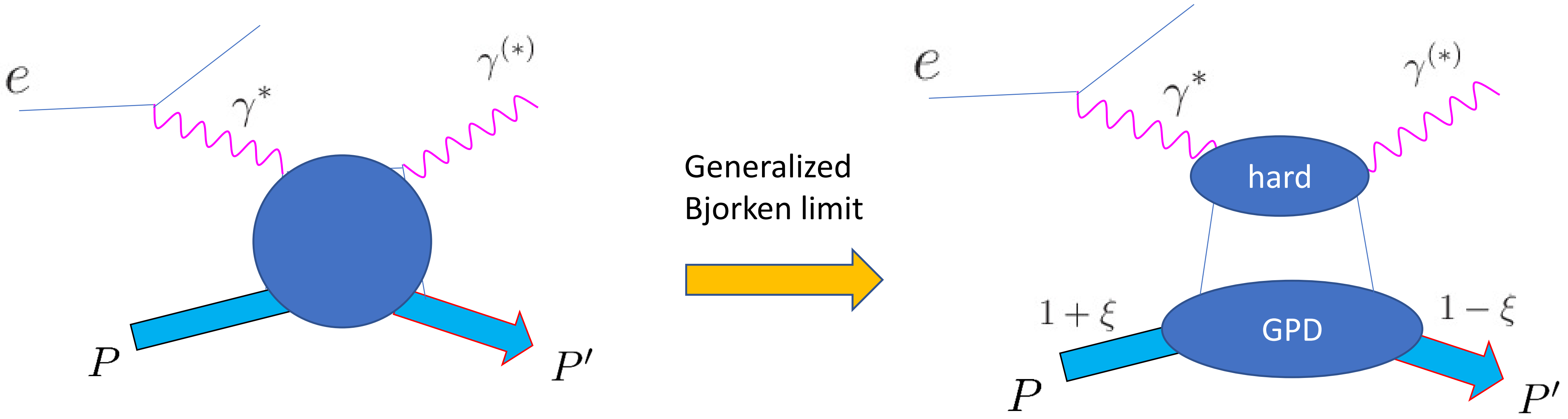
Form factors are moments of GPDs

$$g_P(t) = \sum_q \int_{-1}^{1} dx \tilde{E}_q(x,\xi,t)$$

$$A_q(t) + \xi^2 D_q(t) = \int_{-1}^{1} dx\, x H_q(x,\xi,t)$$

→ Anomalies relate GPDs!

Deeply Virtual Compton Scattering



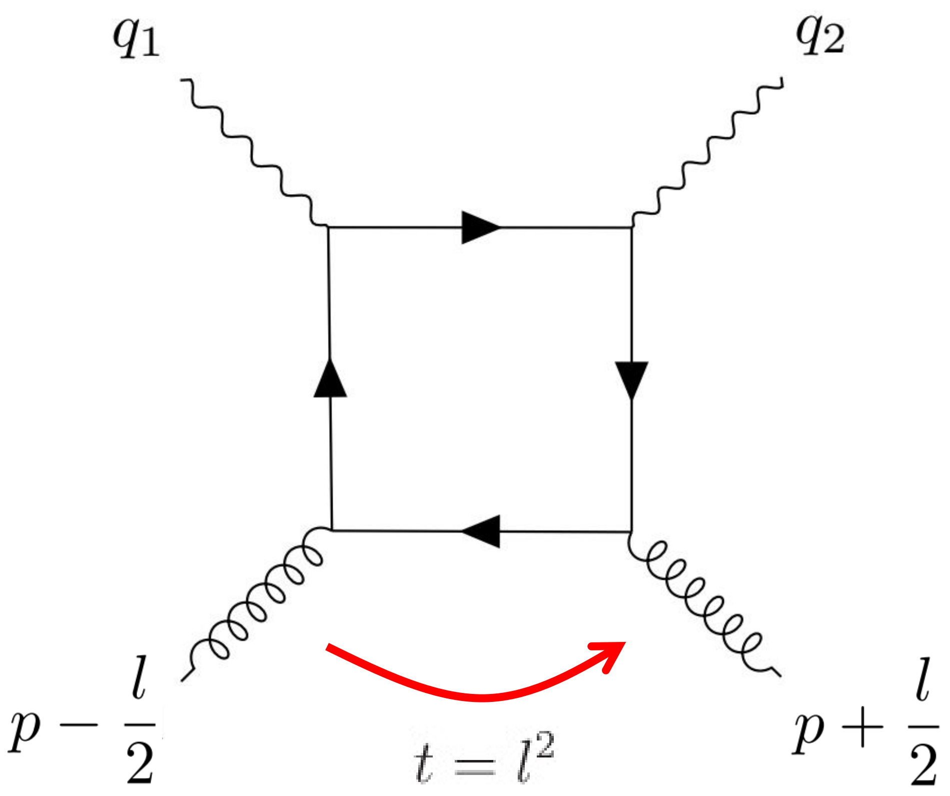
Factorization proof Collins, Freund (1998); Ji, Osborne (1998)

$$T^{\mu\nu}(x_B,\xi,t) = \sum_{a=q,g} \int \frac{dx}{x} C_a^{\mu\nu}\left(\frac{x_B}{x},\frac{\xi}{x}\right) f_a(x,\xi,t) + \mathcal{O}(1/Q^2)$$

GPD

Skewness $\xi = \frac{P^+ - P'^+}{P^+ - P'^+}$

The box diagram



In all the previous works on DVCS, the hard part is computed at $\xi \neq 0$ and $t = 0$

Naively, introducing $t \neq 0$ only produces higher twist corrections of order t/Q^2

However, calculations with $t \neq 0$ can reveal anomaly poles. Tarasov, Venugopalan (2019,2021)
Beware the box diagram.

$t \neq 0$ also naturally cuts off the collinear singularity → alternative regularization scheme

$$\frac{1}{\epsilon} \to \ln \frac{Q^2}{-t}$$

One-loop result

(Showing only the pole terms. For a complete result, see 2305.09431)

$$T^{\mu\nu} = \frac{g_\perp^{\mu\nu}}{2P^+}\bar{u}(P_2)\left[\gamma^+\mathcal{H} + \frac{i\sigma^{+\nu}l_\nu}{2M}\mathcal{E}\right]u(P_2) - i\frac{\epsilon_\perp^{\mu\nu}}{2P^+}\bar{u}(P_2)\left[\gamma^+\gamma_5\tilde{\mathcal{H}} + \frac{\gamma^5 l^+}{2M}\tilde{\mathcal{E}}\right]u(P_2)$$

$$\mathcal{H},\mathcal{E} \sim \frac{\alpha_s}{t}A\otimes\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)WF_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

$$\tilde{\mathcal{E}} \sim \frac{\alpha_s}{t}\tilde{A}\otimes\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)W\tilde{F}_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

twist-four GPDs, not suppressed by $1/Q^2$

$$A(x,x_B,\xi) = \frac{2T_R}{x}\left(1 + \frac{\hat{x}(1-\hat{x})\ln\frac{\hat{x}-1}{\hat{x}} + \hat{x}(\hat{x}-\hat{\xi})\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}} + (\hat{x}\to-\hat{x})}{1-\hat{\xi}^2}\right)$$

$$\tilde{A}(x,x_B,\xi) = \frac{8T_R}{x}\frac{(1-\hat{x})\ln\frac{\hat{x}-1}{x} + (\hat{x}-\hat{\xi})\ln\frac{\hat{x}-\xi}{\hat{x}} - (\hat{x}\to-\hat{x})}{1-\hat{\xi}^2}$$

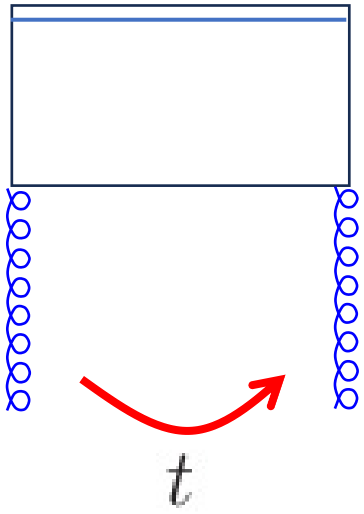
← imaginary part of this agrees with Tarasov, Venugopalan (2019)

Naively, the new terms break QCD factorization. (or OPE is violated, as Ian puts it.)

However, we also computed the quark GPD of a gluon and found a pole

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2 | \bar{q}(-z/2) W \gamma^+ q(z^-/2) | p_1 \rangle$$

$$\sim \quad \frac{1}{t} \langle p_2 | F^{\mu\nu} F_{\mu\nu} | p_1 \rangle$$



Poles from the box diagram, both real and imaginary parts, can be systematically absorbed into twist-two GPDs via an infrared subtraction procedure. Factorization restored.

However, this is an unusual subtraction. Absorb twist-4 GPD into twist-2.

The fate of anomaly poles

After absorbed into twist-2 GPD, the anomaly pole becomes a part of the GPD

$$\sum_q (\tilde{E}_q(x,\xi,t) + \tilde{E}_q(-x,\xi,t)) = \frac{T_R n_f \alpha_s}{\pi} \frac{M^2}{t} \tilde{C}^{\text{anom}} \otimes \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) W \tilde{F}_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} + \cdots$$

integrate over x

$$\frac{g_P(t)}{2M} = \frac{1}{t} \left(i \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F\tilde{F} | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2M g_A(t) \right)$$

exactly reproduce the anomaly pole!

Twist-2 and twist-4 GPDs related by the chiral anomaly

D-term and gluon condensate

Trace anomaly pole induced in the Polyakov-Weiss D-term of unpol GPDs

$$H_q^{\rm PW}(x,\xi,t) = -E_q^{\rm PW}(x,\xi,t) = \theta(\xi-|x|)D_q(x/\xi,t)$$

$$\sum_q D_q(z,t) \approx -\frac{T_R n_f \alpha_s}{\pi} z(1-|z|)\frac{M}{t}\left(\frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - \frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{t=0}\right) + \cdots$$

anomaly pole glueball pole

$$\sum_q D_q(t) \approx -\frac{M}{t}\left(\frac{\langle P_2|\frac{T_R n_f \alpha_s}{6\pi}F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - \frac{\langle P_2|\frac{T_R n_f \alpha_s}{6\pi}F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{t=0}\right) + \cdots$$

Compare to the full relation

$$\frac{3}{4}D(t) \approx -\frac{M}{t}\left(\frac{\langle P_2|\frac{\beta(g)}{2g}F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - MA(t)\right) \qquad \frac{\beta(g)}{2g} = \frac{T_R n_f \alpha_s}{6\pi} + \cdots$$

Summary

Anomalies relate form factors

Form factors are moments of GPDs

→ Anomalies relate GPDs

GPDs encode profound aspects of QCD such as chiral symmetry breaking, origin of mass.

Any implications for small-x physics?

Small-x behavior of $g_1(x)$? ← talk by Andrey

Small-x behavior of twist-2 and twist-4 GPD.
Is there a zero mode?

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2|F^{\mu\nu}(-z^-/2)WF_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)} \sim \delta(x) \quad ??$$

YH, Zhao (2020), Radyushkin, Zhao (2021)