NLO corrections:
<u>single, double hadron production and total cross sections</u>
in DIS at small x

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SURGE collaboration meeting

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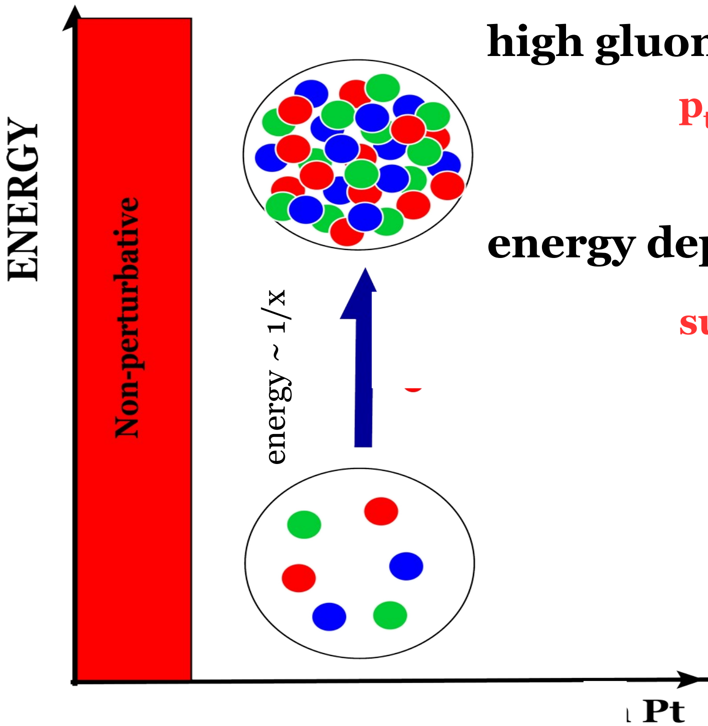
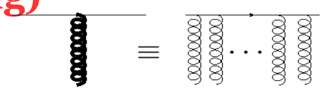
QCD at high energy/small x: gluon saturation



high gluon density: **Eikonal multiple scattering**

p_t broadening (generic to multiple scattering)



energy dependence: **x-evolution via JIMWLK/BK**

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

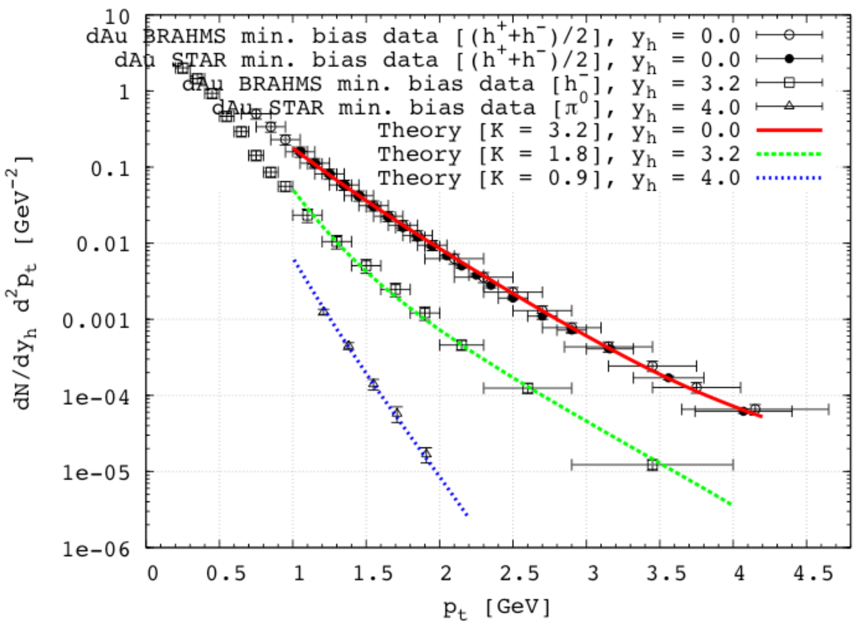
CGC: a framework for multi-particle production
in high energy collisions at small x (low p_t)

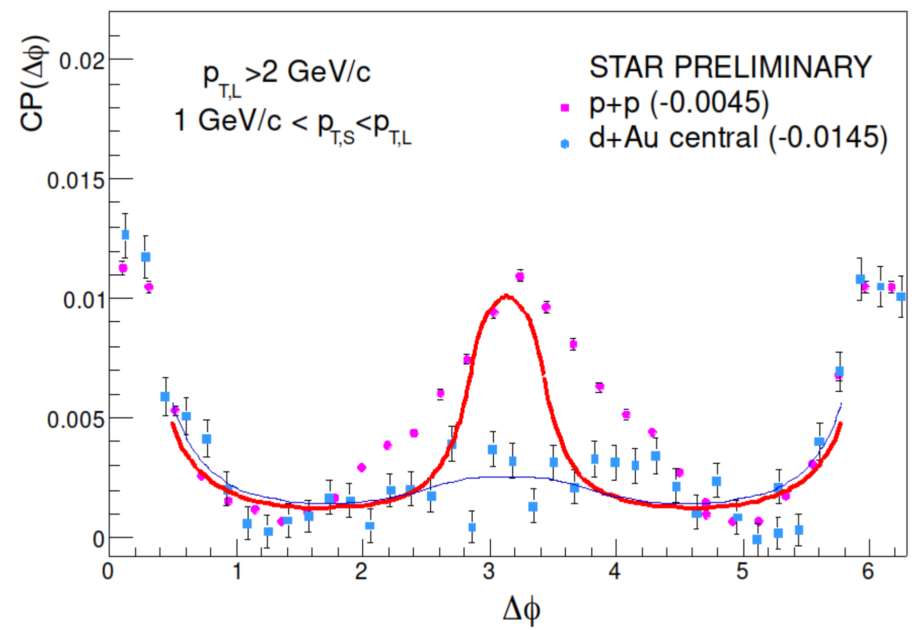
$$\mathbf{x \leq 0.01}$$

$$\alpha_s \ln\left(x_v/x\right) \sim 1$$

CGC at RHIC

Single and double inclusive hadron production in dA collisions





Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

Albacete, Marquet, PRL105 (2010) 162301

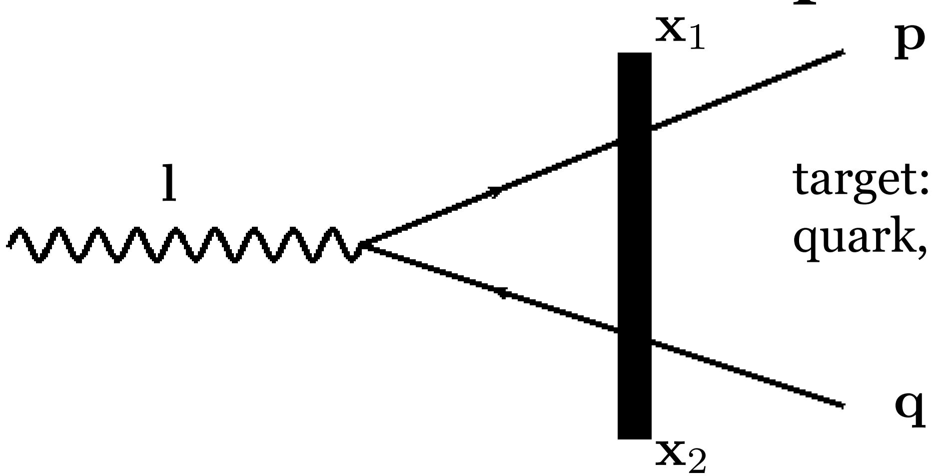
Recent review by A. Morreale and F. Salazar, e-print: 2108.08254

Toward precision: NLO corrections to inclusive observables in DIS at small x

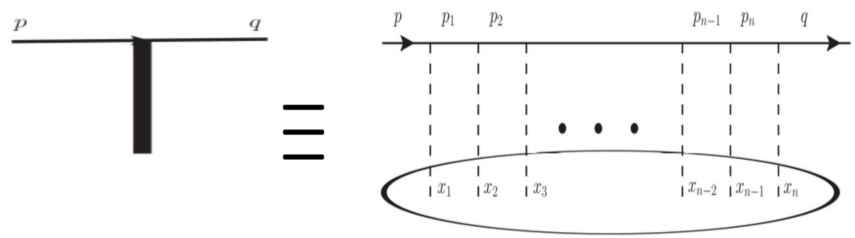
Based on F. Bergabo and JJM:

PRD 107 (2023) 5, 054036 (dihadrons: transverse photon)
JHEP 01 (2023) 095 (single inclusive hadrons: longitudinal photon)
PRD 106 (2022) 5, 054035 (dihadrons: longitudinal photon)
NPA 1018 (2022) 122358 (coherent e-loss in dihadron production)

LO: inclusive dihadron production in DIS at small x



target: a classical color field
quark, anti-quark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1)+$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\,K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

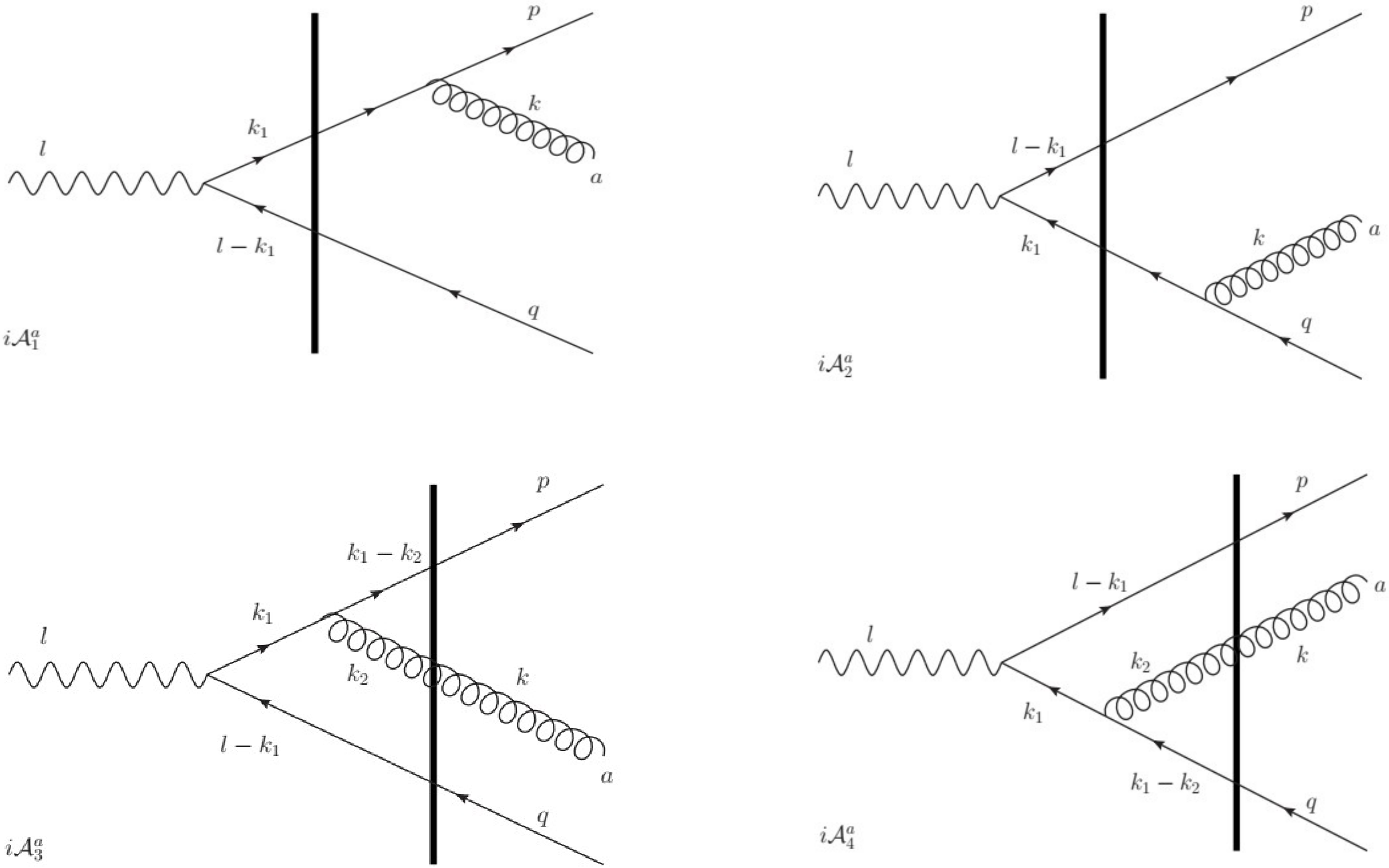
with

$$\mathbf{S}_{12} \equiv \frac{1}{N_c}Tr\,V(x_1)\,V^\dagger(x_2)$$

$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

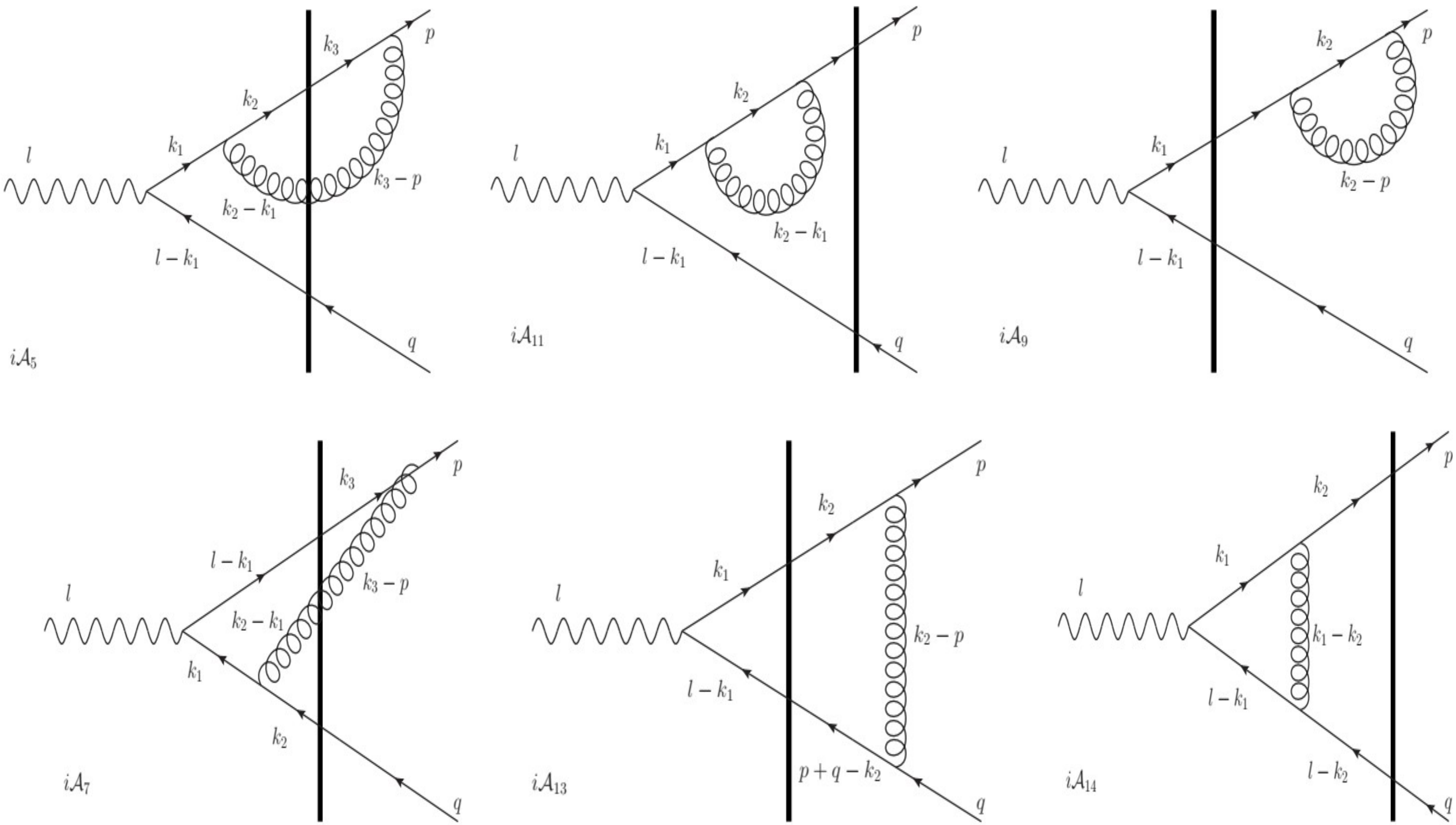
$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c}Tr\,V(\mathbf{x}_1)\,V^\dagger(\mathbf{x}_2)\,V(\mathbf{x}_{2'})\,V^\dagger(\mathbf{x}_{1'})$$

NLO corrections – real diagrams



3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

NLO corrections – virtual diagrams



F. Bergabo and JJM, dihadrons, 2207.03606, 2301.03117
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347,.......

$$\frac{\sigma_{1-1}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_2^3(1-z_2)^2(z_1^2+(1-z_2)^2)}{(2\pi)^{10}z_1}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_2)\Delta^{(3)}_{11'}$$
$$[S_{122'1'}-S_{12}-S_{1'2'}+1]\,e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{2-2}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^3(1-z_1)^2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}z_2}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)\Delta^{(3)}_{22'}$$
$$[S_{122'1'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{1-2}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1z_2(1-z_1)(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_1)$$
$$\Delta^{(3)}_{12'}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_2'-\mathbf{x}_3)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{3-3}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1z_2^3(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{11'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{4-4}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^3z_2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{22'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{3-4}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1^2z_2^2(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{12'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{1-3}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_2^3(1-z_2)(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{11'}$$
$$[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{1-4}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1z_2^2(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{12'}$$
$$[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{2-3}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^2z_2(1-z_1)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{21'}$$
$$[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma_{2-4}^{\mathrm{Real},L}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1^3(1-z_1)(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{22'}$$
$$[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

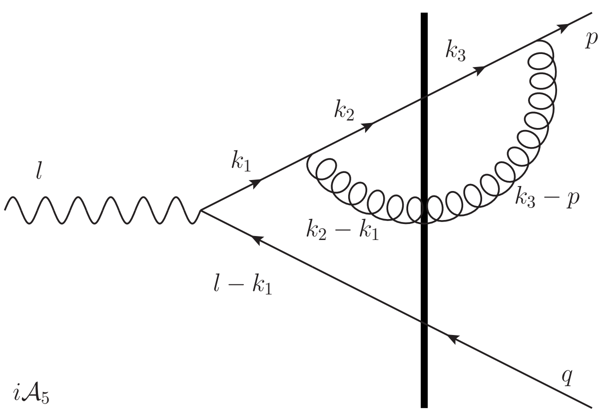
divergences

- ***Ultraviolet:***

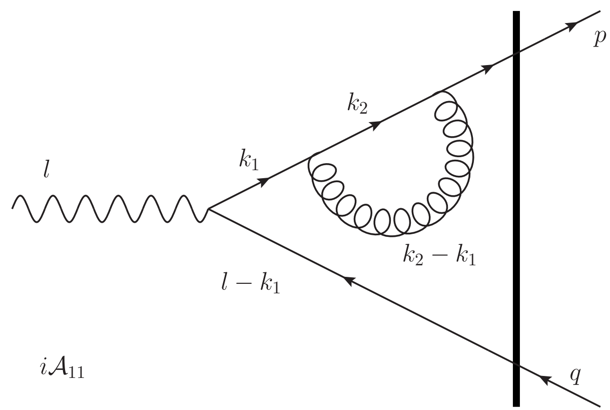
Real corrections are UV finite

UV divergences cancel among virtual corrections $\quad \mathbf{k \to \infty} \quad$ **or** $\quad \mathbf{x_3 \to x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



$+$



$= \mathbf{0}$

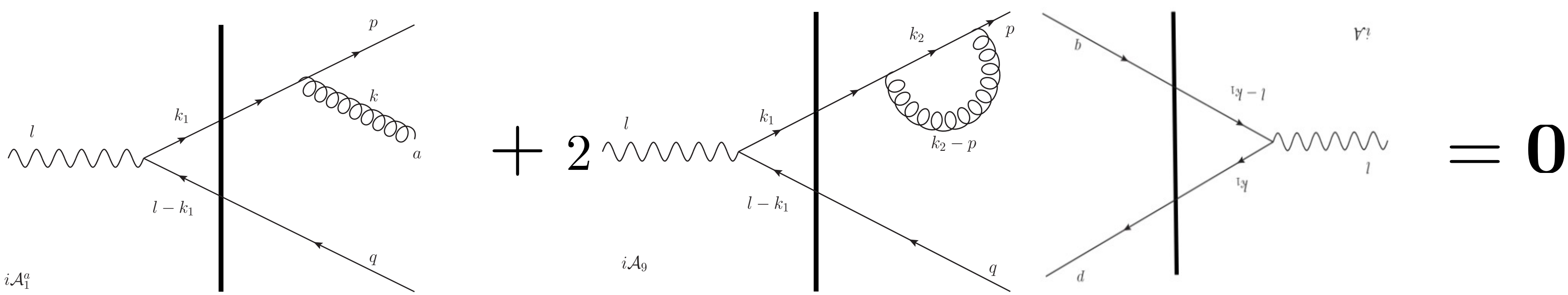
divergences

- ***Soft:*** $\mathbf{k^\mu \to 0 \ \ (x_3 \to \infty \ \ AND \ \ z \to 0)}$

Soft divergences cancel between real and virtual corrections

$$\left(d\sigma_{1-1} + d\sigma_9\right)_{soft} = 0,$$
$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$
$$\left(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4}\right)_{soft} = 0$$
$$\left(d\sigma_{1-3} + d\sigma_{1-4}\right)_{soft} = 0$$
$$\left(d\sigma_{2-3} + d\sigma_{2-4}\right)_{soft} = 0$$
$$\left(d\sigma_5 + d\sigma_7\right)_{soft} = 0$$
$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$

2



divergences

- ***Rapidity:*** $\mathbf{z} \to 0$ ***, but finite*** $\boldsymbol{k_t}$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}}\, \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12} \left(S_{13} S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'} \left(S_{1'3} S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

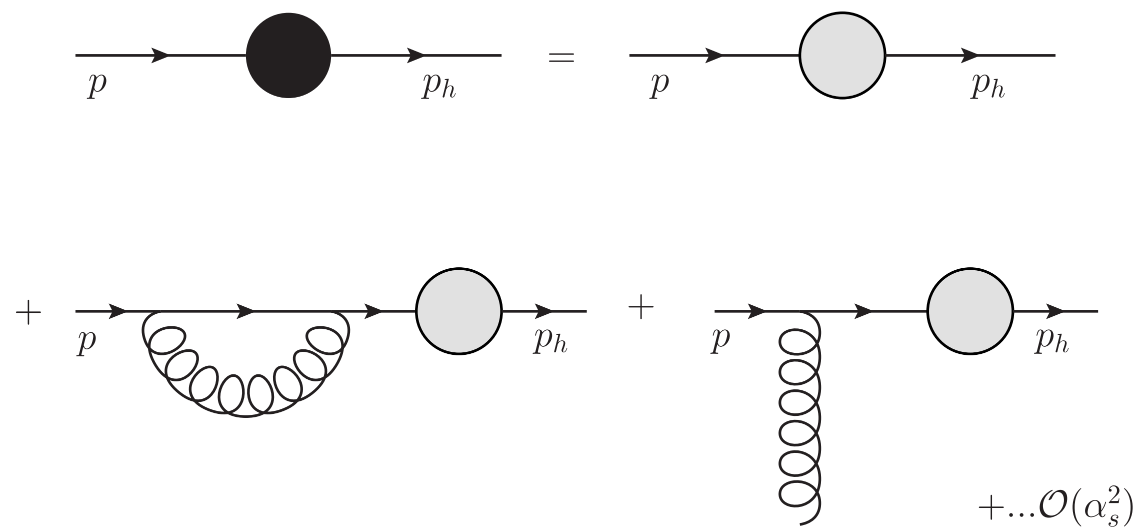
JIMWLK evolution of dipoles

$$\tilde{\boldsymbol{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$

divergences

- ***Collinear:*** $$\frac{1}{(p+k)^2} = \frac{1}{|\overrightarrow{p}||\overrightarrow{k}|(1-cos\,\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



$$D_{h_1/q}(z_{h1},\mu^2) = \int_{z_{h1}}^{1} \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h1}}{\xi}\right) \left[\delta(1-\xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi)\,\log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

Divergences

•*Ultraviolet*

Real corrections are UV finite
UV divergences cancel among virtual corrections

•*Soft*

Soft divergences cancel between real and virtual corrections

•*Collinear*

Collinear divergences are absorbed into hadron fragmentation functions

•*Rapidity*

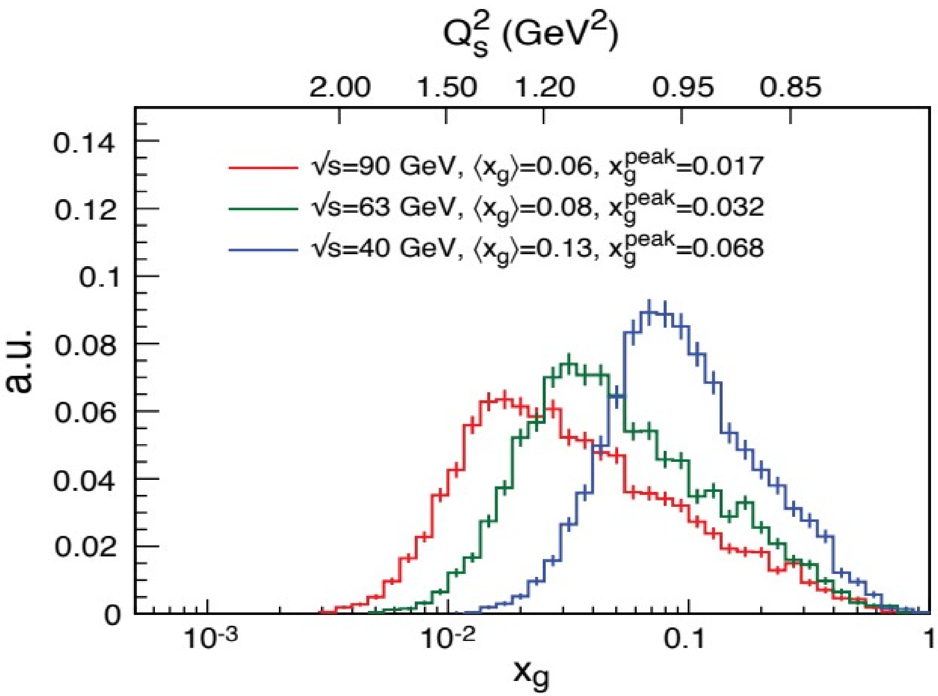
rapidity divergences are absorbed into JIMWLK evolution of dipoles, quadrupoles

$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h_1/q}(z_1, \mu^2) D_{h_2/\bar{q}}(z_2, \mu^2) + \sigma_{NLO}^{\text{finite}}$$

connecting CGC to TMDs,....,
phenomenology: EIC, UPC at the LHC,...

EIC

kinematics of inclusive dihadron production



Aschenauer et al. arXiv:1708.01527

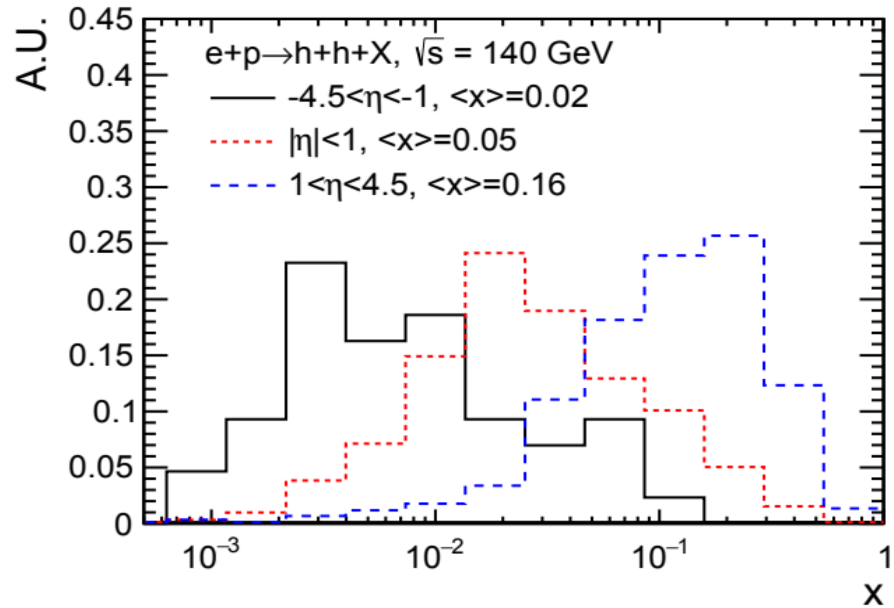


Fig. courtesy of Xiaoxuan Chu

transition region: from large x to small x

Single inclusive hadron production in DIS at small x: NLO

JHEP 01 (2023) 095 (longitudinal photon)

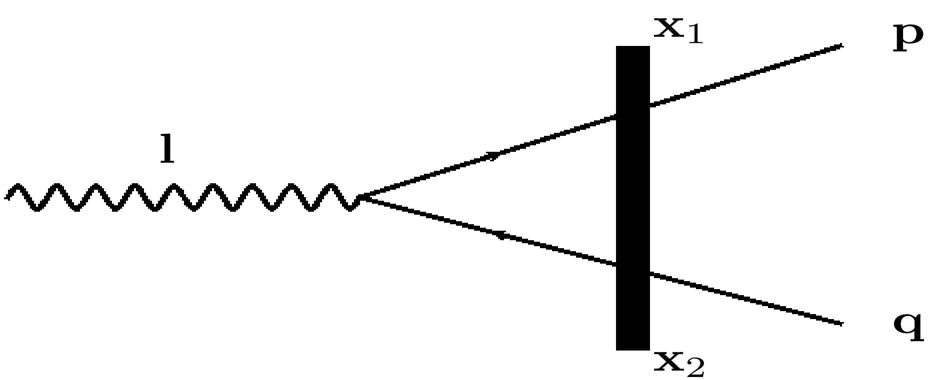
Larger kinematic phase space at EIC

Sudakov (can it be avoided?)

Dipoles only

Forward rapidity: quark or antiquark production

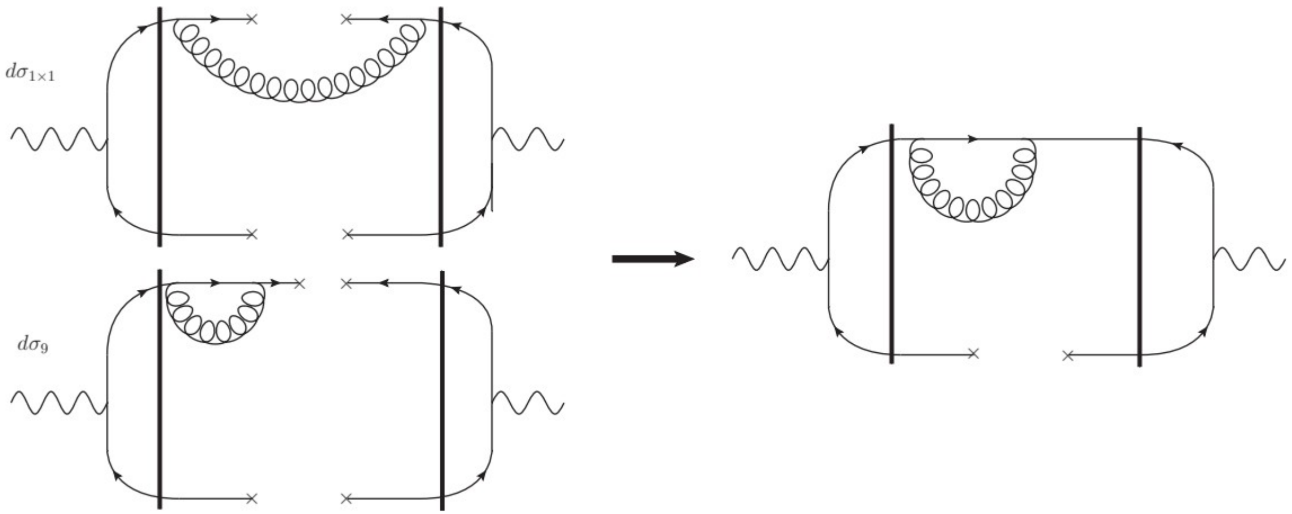
LO: integrate out quark



Single inclusive hadron production in DIS at small x: NLO

start with NLO corrections to dihadron production and integrate out quark

cancellations among diagrams



Single inclusive hadron production in DIS at small x: NLO

all terms with quadrupoles cancel; only dipoles contribute to the cross section

cancellations of divergences as before

$$\sigma^{\gamma^* A \to hX} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/\bar{q}}(z_h, \mu^2) + \sigma_{NLO}^{\text{finite}}$$

phenomenology: need to consider hadronization of any of the 3 partons

relation to TMD,...? negative cross sections?

DIS structure functions at small x: NLO

G. Beuf, arXiv:1708.06557
Beuf, Lappi, Paatelainen, arXiv:2112.03158

integrate out all produced partons

compare with results derived using LC perturbation theory

F. Bergabo, JJM, in progress

TO DO (?):

structure functions:

compare with earlier results based on LCPT

SIDIS:

include transverse photons,...., include gluon-hadron fragmentation, TMDs,...

Inclusive dihadrons:

BtoB limit, Sudakov, TMDs,..., include quark-gluon production,...

SUGGESTIONS WELCOME!