NLO corrections to back-to-back dijet electroproduction at small x

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based on 2304.03304 and work in progress

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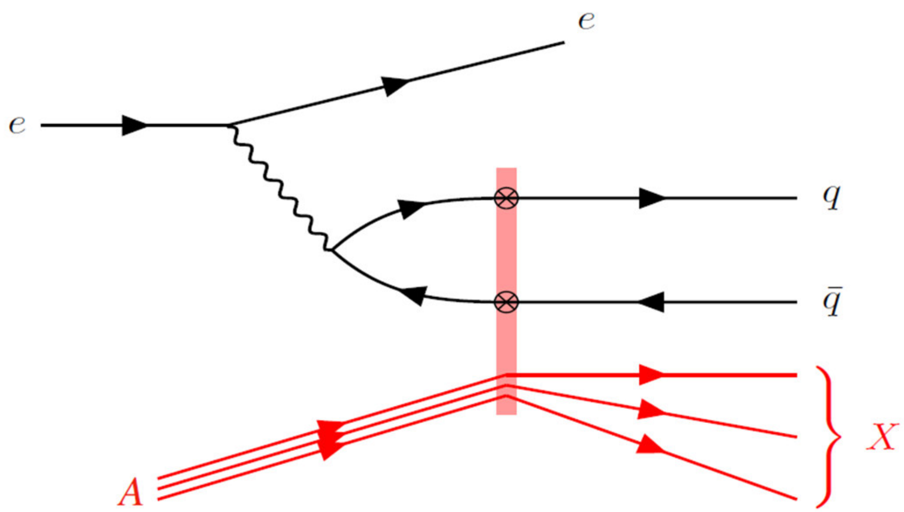
Outline

- Dijet production in DIS
- CGC calculation at LO and NLO
- Back-to-back limit
- Numerical results

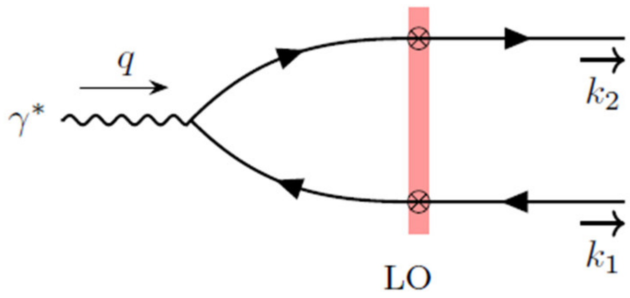
Dijet production in DIS



$$\frac{\mathrm{d}\sigma^{e+A\to e'+q\bar{q}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{k}_{\mathbf{1}\perp}\mathrm{d}^2\boldsymbol{k}_{\mathbf{2}\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = \sum_{\lambda=\mathrm{L,T}} f_\lambda(x_{\mathrm{Bj}},Q^2)\,\frac{\mathrm{d}\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{\mathbf{1}\perp}\mathrm{d}^2\boldsymbol{k}_{\mathbf{2}\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2}\,.$$

$$f_{\lambda=\mathrm{L}}(x_{\mathrm{Bj}},Q^2) = \frac{\alpha_{\mathrm{em}}}{\pi Q^2 x_{\mathrm{Bj}}}(1-y)\,,$$
$$f_{\lambda=\mathrm{T}}(x_{\mathrm{Bj}},Q^2) = \frac{\alpha_{\mathrm{em}}}{2\pi Q^2 x_{\mathrm{Bj}}}[1+(1-y)^2]\,,$$

CGC calculation at LO



$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^\star + A \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot \boldsymbol{r}_{xx'}} e^{-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} \times \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp)\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'})\,.$$

Impact factor:

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) = 8 z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{xy}) K_0(\bar{Q} r_{x'y'})\,,$$
$$\mathcal{R}^{\mathrm{T}}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) = 2 z_1 z_2 \left[z_1^2 + z_2^2\right] \frac{\boldsymbol{r}_{xy}\cdot \boldsymbol{r}_{x'y'}}{r_{xy} r_{x'y'}} \bar{Q}^2 K_1(\bar{Q} r_{xy}) K_1(\bar{Q} r_{x'y'})\,,$$

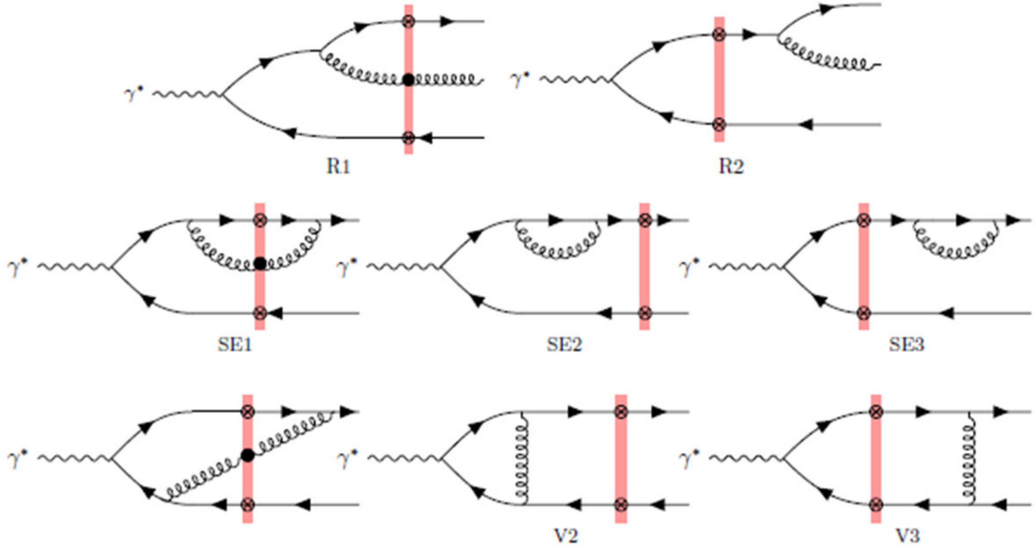
Color correlator:

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\left(V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right)\left(V(\boldsymbol{y}'_\perp)V^\dagger(\boldsymbol{x}'_\perp) - \mathbb{1}\right)\right]\right\rangle_Y$$
$$= \left\langle Q_{xy,y'x'} - D_{xy} - D_{y'x'} + 1\right\rangle_Y\,,$$

where Wilson line: $V(\boldsymbol{x}_\perp) = P\exp\left(ig\int \mathrm{d}x^- A^+_{\mathrm{cl}}(\boldsymbol{x}_\perp, x^-)\right)$

NLO corrections

P. Caucal, F. Salazar and R. Venugopalan *(2021)*



NLO impact factor:

$$\mathrm{d}\sigma_{\mathrm{R_2\times R_2,sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times C_F \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{\alpha_s}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{r}_{xx'}^2R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R_2\times R_2',sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times \Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{(-\alpha_s)}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(4\alpha_sC_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}}{(\boldsymbol{k}_{g\perp}-\frac{z_g}{z_1}\boldsymbol{k}_{1\perp})^2}\left\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2}}r_{x'y'})\delta_z^{(3)}\right.$$
$$\left.-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow2)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2'}}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right]\right.$$
$$\left.\times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\}$$
$$+(1\leftrightarrow2)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}$$
$$\alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO,1}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)\right.$$
$$+\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO,1}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)$$
$$+\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO,4}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$-\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO,4}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$\left.+(1\leftrightarrow2)+c.c.\right\}-\frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{\alpha_sC_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{k}_{2\perp}{}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})$$
$$\times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}}r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy})\right.\right.$$
$$\left.-\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right]$$
$$\left.+K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow2)\right\}\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

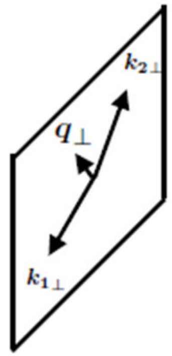
$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}$$
$$\times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO,1}}\right.$$
$$-\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}}$$
$$-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)\right.$$
$$\left.\left.-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO,1}}+(1\leftrightarrow2)\right\}+c.c..$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi}$$
$$\times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right.$$
$$\left.+\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

Very hard to do numerics…

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)

Back-to-back limit (LO)



$$\boldsymbol{q}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$
$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

Back-to-back limit:

$$q_\perp, Q_s \ll P_\perp$$

High energy limit: $P_\perp \ll W$

Factorization at the LO:

$$\mathrm{d}\sigma_{\mathrm{LO}}^{\gamma_\lambda^\star + A \to q\bar{q} + X} = \alpha_{\mathrm{em}} e_f^2 \alpha_s \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{\lambda,ij}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}_Y^{ij}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}, \frac{Q_s}{P_\perp}\right)$$

where the hard factors:

$$\mathcal{H}_{\mathrm{LO}}^{\lambda=\mathrm{L},ij}(\boldsymbol{P}_\perp) = 16 z_1^3 z_2^3 Q^2 \frac{\boldsymbol{P}_\perp^i \boldsymbol{P}_\perp^j}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^4},$$

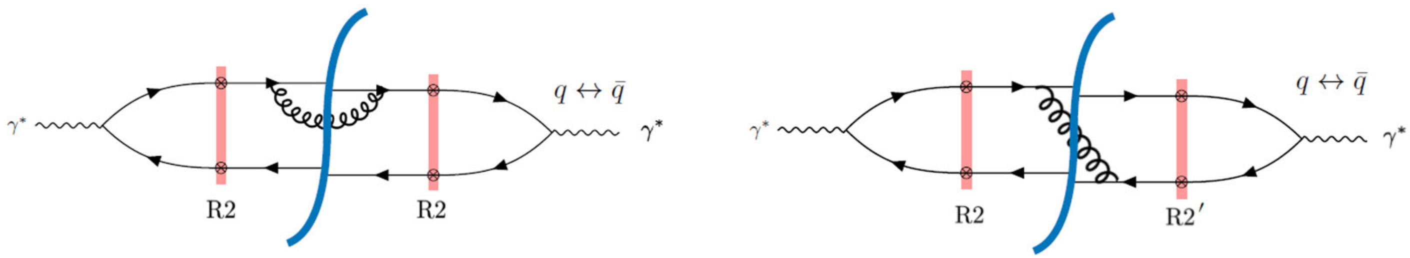
$$\mathcal{H}_{\mathrm{LO}}^{\lambda=\mathrm{T},ij}(\boldsymbol{P}_\perp) = z_1 z_2 (z_1^2 + z_2^2) \left\{ \frac{\delta^{ij}}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^2} - \frac{4\bar{Q}^2 \boldsymbol{P}_\perp^i \boldsymbol{P}_\perp^j}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^4} \right\}$$

and Weizsäcker-Williams (WW) distribution:

$$\hat{G}_Y^{ij}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp) \equiv \frac{-2}{\alpha_s} \left\langle \mathrm{Tr}\left[V(\boldsymbol{b}_\perp) \left(\partial^i V^\dagger(\boldsymbol{b}_\perp)\right) V(\boldsymbol{b}'_\perp) \left(\partial^j V^\dagger(\boldsymbol{b}'_\perp)\right) \right] \right\rangle_Y$$

Caucal, Salazar, Schenke, and Venugopalan (2022)

Back-to-back limit (NLO)



One obtains Sudakov logs:

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \mathrm{d}^2\mathbf{r}_{bb'} e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}} \left[1 + \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \mathbf{r}_{bb'}^2}{c_0^2}\right) + \cdots + \alpha_s \ln\left(\Lambda_f^-/\Lambda^-\right) \mathcal{K}_{LL}\otimes\right] \widetilde{G}_Y(\mathbf{r}_{bb'})$$

But with the wrong (+) sign.

Should be compared to result by Mueller, Xiao, Yuan (2013) for joint small-x and soft gluon resummation:

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\mathbf{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}} \widetilde{G}_Y^0(\mathbf{r}_{bb'}) e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'},\boldsymbol{P}_\perp)}$$

Sudakov factor: $S_{\mathrm{Sud}}(\mathbf{r}_{bb'}, P_\perp) = \frac{\alpha_s N_c}{\pi} \int_{c_0^2/\mathbf{r}_{bb'}^2}^{P_\perp^2} \frac{1}{2} \ln\left(\frac{P_\perp^2}{\mu^2}\right)$

Back-to-back limit (NLO)

- Solution by Taels, Altinoluk, Marquet, Beuf (2022):

One needs to impose kinematical contraint for small-x evolution and get correct Sudakov double log:

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \mathrm{d}^2\mathbf{r}_{bb'} e^{-i\boldsymbol{q}_\perp \cdot \mathbf{r}_{bb'}}$$

$$\left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \mathbf{r}_{bb'}^2}{c_0^2}\right) - \frac{\alpha_s}{\pi} s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \mathbf{r}_{bb'}^2}{c_0^2}\right) + \alpha_s \mathcal{K}_{LL,\mathrm{coll}} \otimes\right] \widetilde{G}_Y(\mathbf{r}_{bb'}) + \mathcal{O}(\alpha_s)$$

Correct Sudakov double log

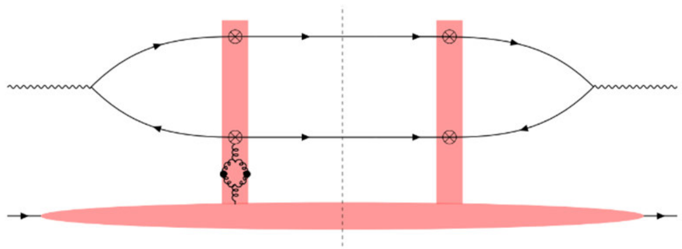
Kinematically improved small-x evolution

Caucal, Salazar, Schenke, TS and Venugopalan (2023)

Back-to-back limit (NLO)

$$\left\langle d\sigma_{\rm LO}^{(0),\lambda} + \alpha_s d\sigma_{\rm NLO}^{(0),\lambda} \right\rangle_{\eta_f} = \frac{1}{2}\mathcal{H}_{\rm LO}^{\lambda,ii} \int \frac{d^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{\eta_f}(\boldsymbol{r}_{bb'},\mu_0) \left\{1 + \frac{\alpha_s(\mu_R)}{\pi}\left[-\frac{N_c}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right) - s_L \ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0 \ln\left(\frac{\mu_R^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \frac{N_c}{2} f_1^\lambda(\chi,z_1,R) + \frac{1}{2N_c} f_2^\lambda(\chi,z_1,R)\right]\right\} + \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{\rm LO}^{\lambda,ii} \int \frac{d^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{\eta_f}(\boldsymbol{r}_{bb'},\mu_0) \left\{\frac{N_c}{2}\left[1+\ln(R^2)\right] - \frac{1}{2N_c}\ln(z_1 z_2 R^2)\right\}$$

- Factorized expression even at NLO!



- Single log calculated: coefficient,

$$C_F \log\left(\frac{1}{z_1 z_2 R^2}\right) + N_c \log\left(1 + \frac{Q^2}{M_{q\bar{q}}^2}\right) - \beta_0$$

agrees with result obtained in the collinear Collins, Soper, Sterman (CSS) resummation.

Hatta, Xiao, Yuan, Zhou (2021)

Back-to-back limit (NLO)

$$\left\langle d\sigma_{\rm LO}^{(0),\lambda} + \alpha_s d\sigma_{\rm NLO}^{(0),\lambda}\right\rangle_{\eta_f} = \frac{1}{2}\mathcal{H}_{\rm LO}^{\lambda,ii}\int \frac{d^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}}\hat{G}^0_{\eta_f}(\boldsymbol{r}_{bb'},\mu_0)\left\{1 + \frac{\alpha_s(\mu_R)}{\pi}\left[-\frac{N_c}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right)\right.\right.$$

$$\left.\left. - s_L\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0\ln\left(\frac{\mu_R^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \frac{N_c}{2}f_1^\lambda(\chi,z_1,R) + \frac{1}{2N_c}f_2^\lambda(\chi,z_1,R)\right]\right\}$$

$$+ \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{\rm LO}^{\lambda,ii}\int \frac{d^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}}\hat{h}^0_{\eta_f}(\boldsymbol{r}_{bb'},\mu_0)\left\{\frac{N_c}{2}\left[1+\ln(R^2)\right] - \frac{1}{2N_c}\ln(z_1z_2R^2)\right\}$$

Assumption: we can resum the large logs exponenting them:

$$\mathcal{S} = \exp\left(-\int_{\frac{c_0^2}{\boldsymbol{r}_{bb'}^2}}^{\mu_h^2} \frac{d\mu^2}{\mu^2}\frac{\alpha_s N_c}{\pi}\left[\frac{1}{2}\ln\left(\frac{\mu_h^2}{\mu^2}\right) + \frac{s_L - \beta_0}{N_c}\right]\right),$$

NLO hard coefficient functions

Longitudinal polarization of photon:

$$f_1^{\lambda=\mathrm{L}}(\chi, z_1, R) = 9 - \frac{3\pi^2}{2} - \frac{2\pi^2}{27} - \frac{3}{2}\ln\left(\frac{z_1 z_2 R^2}{\chi^2}\right) - \ln(z_1)\ln(z_2) - \ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1 z_2}\right)$$
$$+ \left\{ \mathrm{Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2(1+\chi^2)}\right) - \frac{1}{4(z_2 - z_1\chi^2)} \right.$$
$$\left. + \frac{(1+\chi^2)(z_2(2z_2 - z_1) + z_1(2z_1 - z_2)\chi^2)}{4(z_2 - z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) + (1 \leftrightarrow 2) \right\}$$

$$f_2^{\lambda=\mathrm{L}}(\chi, z_1, R) = -8 + \frac{19\pi^2}{12} + \frac{3}{2}\ln(z_1 z_2 R^2) - \frac{3}{4}\ln^2\left(\frac{z_1}{z_2}\right) - \ln(\chi)$$
$$+ \left\{ \frac{1}{4(z_2 - z_1\chi^2)} + \frac{(1+\chi^2)z_1(z_2 - (1+z_1)\chi^2)}{4(z_2 - z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) \right.$$
$$\left. + \frac{1}{2}\mathrm{Li}_2(z_2 - z_1\chi^2) - \frac{1}{2}\mathrm{Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2}\right) + (1 \leftrightarrow 2) \right\}$$

where $\chi = \frac{Q}{M_{q\bar{q}}}$

Tranverse polarization: similar expressions.

WW TMD's evolution

TMD satisfies kinematically constrained evolution equation which is not closed (involves other than WW-type correlators).

For numerical evaluation we assume Gaussian approximation:

$$\hat{G}^{ij}(\boldsymbol{r}_{bb'}) = \frac{2C_F S_\perp}{\alpha_s} \frac{\partial^i \partial^j \Gamma(\boldsymbol{r}_{bb'})}{\Gamma(\boldsymbol{r}_{bb'})} \left[1 - \exp\left(-\frac{C_A}{C_F}\Gamma(\boldsymbol{r}_{bb'})\right)\right]$$

WW TMD

$$\Gamma(\boldsymbol{r}_{bb'}) = -\ln\left(S(\boldsymbol{r}_{bb'})\right)$$

$$S = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

S satisfies kinematically constrained BK equation

$$\frac{\partial \mathcal{S}_\eta(\mathbf{r}_{bb'})}{\partial \eta} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta(\eta - \delta_{bb'z}) \frac{\mathbf{r}_{bb'}^2}{\mathbf{r}_{zb}^2 \mathbf{r}_{zb'}^2} \left[\mathcal{S}_{\eta-\delta_{zb}}(\mathbf{r}_{zb}) \mathcal{S}_{\eta-\delta_{zb'}}(\mathbf{r}_{zb'}) - \mathcal{S}_\eta(\mathbf{r}_{bb'})\right]$$

Iancu, Mueller, Soyez, Triantafyllopolous (2019)

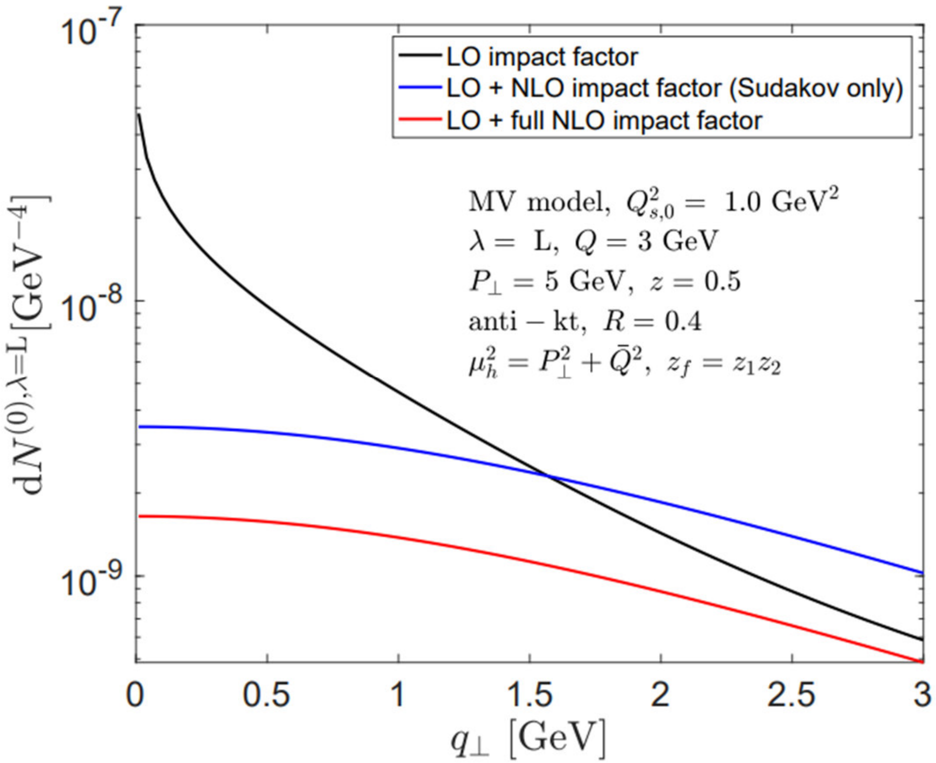
Caucal, Salazar, Schenke, TS and Venugopalan (2023)

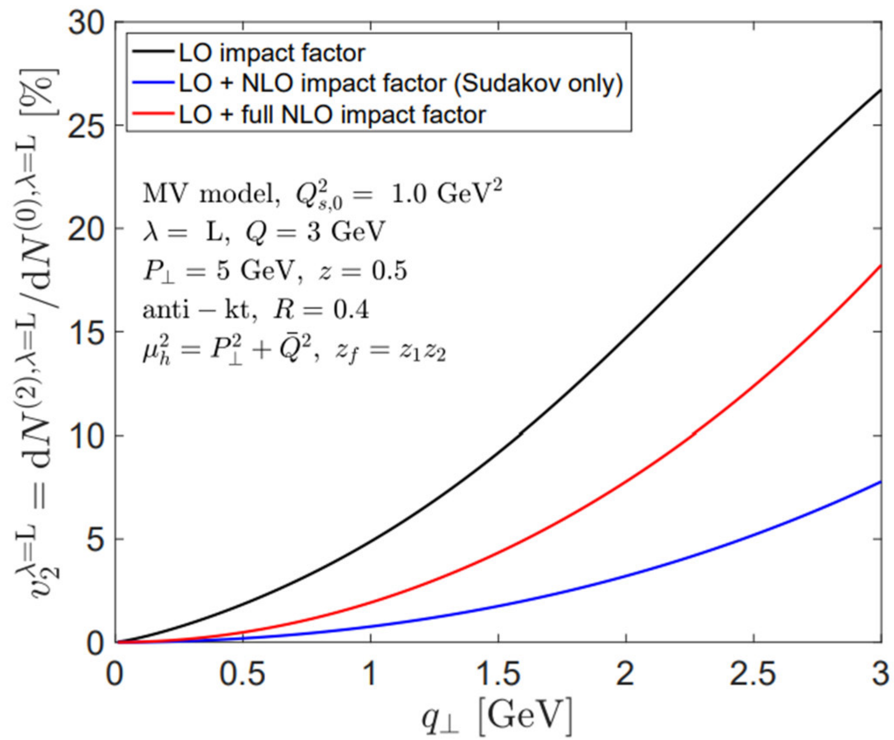
Numerical results for L polarization (without small-x evolution)

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda + A \to \mathrm{dijet} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = \mathrm{d}\sigma^{(0),\lambda}(P_\perp, q_\perp, \eta_1, \eta_2) + 2\sum_{n=1}^{\infty} \mathrm{d}\sigma^{(n),\lambda}(P_\perp, q_\perp, \eta_1, \eta_2)\cos(n\phi)\,,$$

Azimuthally averaged cross-section:

$v_2 = \frac{d\sigma^{(2)}}{d\sigma^{(0)}}$ anisotropy:





Summary

- We calculated back-to-back inclusive dijets cross-section up to NLO accuracy.
- We identify large Sudakov log (both double and single).
- To obtain correct sign of Sudakov double log we need kinematical constraint on small-x evolution (BK/JIMWLK).
- Hard coefficient functions are given by analytic expressions.
- We working on numerical results.

Thank you