Precise charm- and bottom-quark mass determinations & multi-loop calculations to polarization functions

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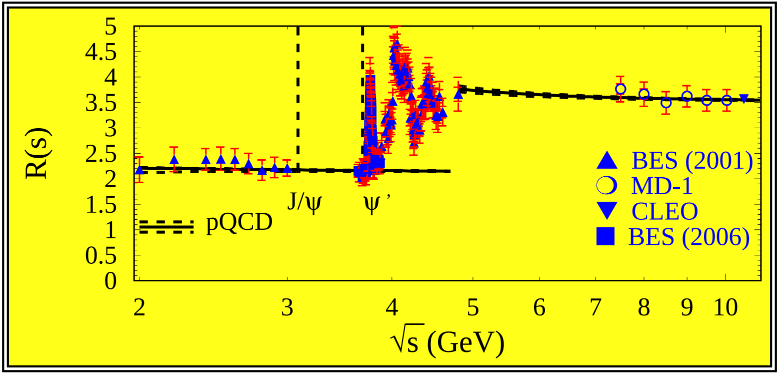
Precise determination of the charm- and bottom-quark masses important:

- Quark masses are fundamental parameters of the Standard Model $\rightsquigarrow$ enter in many physical observables
- Quark masses play an important role in Higgs physics:
  e.g. Higgs decays:
  SM Higgs boson light $\rightsquigarrow$ dominant decay into $b\bar{b}$
  $\Gamma(H \to b\bar{b}) = \frac{G_f M_h}{4\sqrt{2}\pi} m_b^2 (1 + \mathcal{O}(\alpha_s) + \ldots), \quad \Gamma(H \to c\bar{c}) \sim m_c^2$
- Quark masses relevant in flavor physics:
  e.g. B meson decays: $\Gamma \propto m_b^5, \quad B \to X_u \ell \bar{\nu}, B \to X_c \ell \bar{\nu}$
  Virtual charm quarks: $K \to \pi \nu \bar{\nu}$, $B \to X_s \gamma$
- Comparison with other methods, e.g. lattice methods $\rightsquigarrow$ valuable, mutual cross-checks

II. Method

Experiment: *R*-ratio

$$R(s) = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

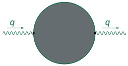


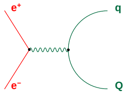
- Heavy quark correlator

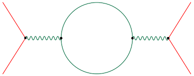
$$\Pi^{\mu\nu}(q,j) = i \int dx\, e^{iqx} \langle 0|T j^{\mu}(x) j^{\nu}(0)|0\rangle$$

Here: $j^{\mu}(x)$ electromagnetic heavy quark vector current

$$\Pi^{\mu\nu}(q) = (-g^{\mu\nu} + q^{\mu}q^{\nu}/q^2)\Pi(q^2) \sim$$



- $\int d\Pi \Big|$ 

$\Big|^2 = 2\,\mathrm{Im}\Big($  $\Big)$

$$R(s) = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\left[\Pi(q^2 = s + i\varepsilon)\right]$$

- With the help of dispersion-relations:

$$\Pi(q^2) = \Pi(q^2 = 0) + \frac{q^2}{12\pi^2} \int ds \frac{R(s)}{s(s - q^2)}$$

- Exp. moments are related to derivatives of $\Pi(q^2)$ at $q^2 = 0$

II. Method

Relation: Theory $\Longleftrightarrow$ Experiment

- Exp. moments are related to derivatives of $\Pi(q^2)$ at $q^2 = 0$:

$$\frac{12\pi}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)\Big|_{q^2=0} = \boxed{\mathcal{M}_n^{\text{exp}} = \int \frac{ds}{s^{n+1}} R^{\text{exp}}(s)}$$

- In terms of expansion coefficients:

$$\Pi(q^2) = \frac{3Q_f^2}{16\pi^2}\sum_n \overline{C}_n^v \left(\frac{q^2}{4m^2}\right)^n ,$$

Q_f: charge of quark
$m = m(\mu)$: $\overline{\text{MS}}$ mass

$\overline{C}_n^v$ can be calculated perturbatively

- First and higher derivatives of $\Pi(q^2)$ allow direct determination of the $\overline{\text{MS}}$ charm- and bottom-quark mass:

$$\boxed{\overline{m}(\mu) = \frac{1}{2}\left(Q_f^2 \frac{9}{4} \frac{\overline{C}_n^v}{\mathcal{M}_n^{\text{exp}}}\right)^{1/(2n)}}$$

← Theory ($\overline{C}_n^v$)

← Experiment ($\mathcal{M}_n^{\text{exp}}$)

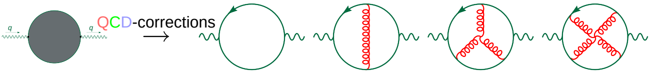
c-quarks: Novikov et al. '78; b-quarks: Reinders et al. '85

$\overline{C}_n^v$ depend on the quark mass through $\log(m(\mu)^2/\mu^2)$

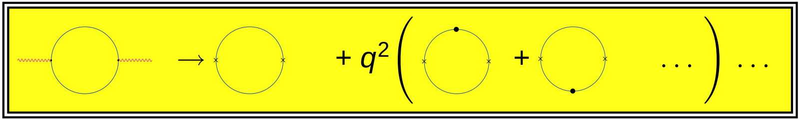
II. Calculation

Pert. calculation of expansion coefficients

- Sample diagrams

$\Pi^{\mu\nu}(q,j) =$ 

- Expansion diagrammatically:



$\hookrightarrow$ One-scale multi-loop integrals in pQCD

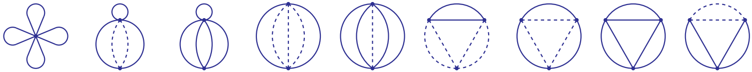
- 3-loop(order α_s^2) coefficients $\overline{C}_n$ up to n=8 Chetyrkin, Kühn, Steinhauser 96
  up to higher moments $n \sim 30$ Czakon et al. 06; Maierhöfer, Maier, Marquard 07
  for correlators VV, AA, PP, SS

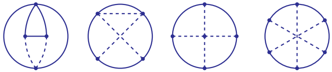
II. Calculation

Techniques, IBP, MI

Computation consists of two steps:

- First step:
  Reduction to a small set of master integrals,
  Integration by parts techniques
- Second step:
  Computation of master integrals
  Here: 13 master integrals





continuous lines: massive propagators
dashed lines: massless propagators

Solution with high precision numeric Y. Schröder, A. Vuorinen
with difference equation method S. Laporta
Subsequently with independent method: ε-finite basis
K.G. Chetyrkin, M. Faisst, C.S., M. Tentyukov

other contributions: D.J. Broadhurst; S. Laporta; B.A. Kniehl, A.V. Kotikov; Y. Schröder, M. Steinhauser

Analytical results in sufficient deep order

II. Calculation

Results at 4-loops

R-ratio method:

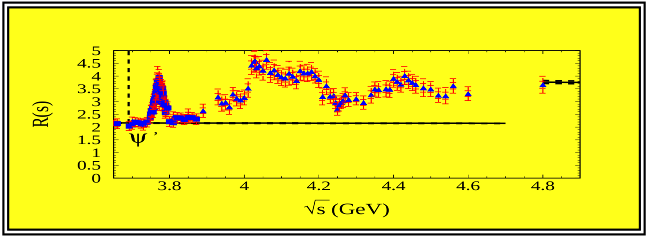
– Vector case:
  - first moments $\overline{C}_0$, $\overline{C}_1$
    K. G. Chetyrkin, J. H. Kühn, C.S.'06; R. Boughezal, M. Czakon, T. Schutzmeier'06
  - second moment $\overline{C}_2$ A. Maier, P. Maierhöfer, P. Marquard'08
  - third moment $\overline{C}_3$ A. Maier, P. Maierhöfer, P. Marquard, A.V. Smirnov '09 ← *new*
  - fourth moment $\overline{C}_{4,\ldots,10}$ Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard '09 ← *new*

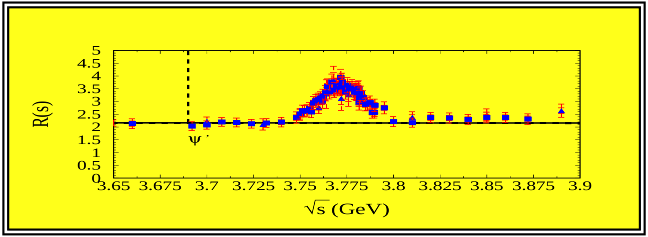
Lattice method: Replace $\mathcal{M}_n^{exp}$ by lattice sim. HPQCD+ K. Chetyrkin, J. Kühn, M. Steinhauser,C.S.

– Pseudoscalar case:
  - first moments $\overline{C}_0$, $\overline{C}_1$, $\overline{C}_2$ I. Allison, E. Dalgic, C.T.H. Davies, E. Follana, R.R. Horgan, K. Hornbostel, G.P. Lepage, C. McNeile, J. Shigemitsu, H. Trottier, R.M. Woloshyn, K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, C.S. 08
  - third moment $\overline{C}_3$ A. Maier, P. Maierhöfer, P. Marquard'08
  - fourth moment $\overline{C}_4$ A. Maier, P. Maierhöfer, P. Marquard, A.V. Smirnov '09
  - fifth moment $\overline{C}_{5,\ldots,10}$ Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard '09

– Axial-vector and scalar case:
  - first moments $\overline{C}_0$, $\overline{C}_1$ C. S.'08
  - third moment $\overline{C}_3$ A. Maier, P. Maierhöfer, P. Marquard, A.V. Smirnov '09
  - fourth moment $\overline{C}_{4,\ldots,10}$ Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard '09

<u>Determine</u>: $\mathcal{M}_n^{exp} = \int \frac{ds}{s^{n+1}} R^{\text{exp}}(s)$





III. Analysis

Extraction of the exp. moments from $R(s)$ (charm quark case)

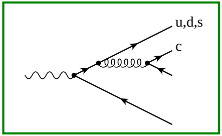
Determine: $\mathcal{M}_n^{exp} = \int \frac{ds}{s^{n+1}} R^{\text{exp}}(s) = \mathcal{M}_n^{res} + \mathcal{M}_n^{thr} + \mathcal{M}_n^{cont}$

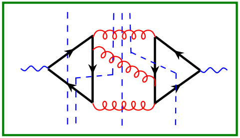
For charm quarks:

$\mathcal{M}_n^{res}$: Contains: $J/\Psi, \Psi(2S)$ treated in narrow width approximation

$$R^{res}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$$

$\mathcal{M}_n^{thr}$: BES data ($\sqrt{s} \geq 3.73$ GeV) subtract background from R_{uds},





$\bar{R}$ from data below 3.73 GeV, $\sqrt{s}$-dependence from theory

$\mathcal{M}_n^{cont}$: pQCD above $\sqrt{s} \geq 4.8$ GeV ,

spare data,

$R(s)$ with full quark mass dependence rhad: R. Harlander, M. Steinhauser '02

- Charm quark mass:

$\mu = (3 \pm 1)$ GeV $\quad \alpha_s(M_Z) = 0.1189 \pm 0.002$

n	m_c(3 GeV)	exp	α_s	μ	np	total
1	0.986	0.009	0.009	0.002	0.001	0.013
2	0.976	0.006	0.014	0.005	0.000	0.016
3	0.978	0.005	0.015	0.007	0.002	0.017
4	1.004	0.003	0.009	0.031	0.007	0.033

- Remarkable consistency between $n = 1, 2, 3, 4$
- Result: n=1: m_c(3 GeV)=0.986(13) GeV

  $m_c(m_c)$=1.279(13) GeV

  Theo. uncertainty by truncation error comparable

III. Analysis

Extraction of the exp. moments from $R(s)$ (bottom quark case)

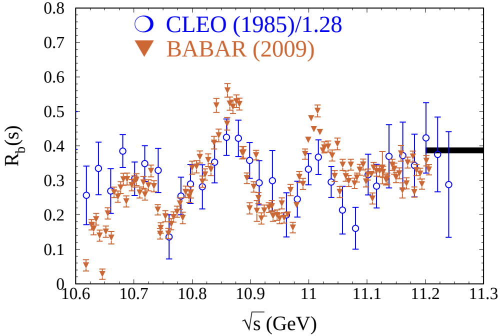
$\mathcal{M}_n^{th}$: analog to charm case, only $n_f = 5$

$\mathcal{M}_n^{np}$: negligible

$\mathcal{M}_n^{res}$: $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$ (PDG)

$\mathcal{M}_n^{thr.}$: BABAR, CLEO data up to 11.24 GeV

Improvements based on recent BABAR results ← *new*



Systematic experimental error $\sim$3.5%

$\mathcal{M}_n^{cont}$: pQCD above 11.24 GeV

III. Analysis & Results

Determination of the bottom quark mass from $R(s)$

- Bottom quark masses:

$\mu = (10 \pm 5)$ GeV; $\alpha_s(M_Z) = 0.1189 \pm 0.002$

n	$m_b(10\ \text{GeV})$	exp	α_s	μ	total
1	3.597	0.014	0.007	0.002	0.016
2	3.610	0.010	0.012	0.003	0.016
3	3.619	0.008	0.014	0.006	0.018
4	3.631	0.006	0.015	0.020	0.026

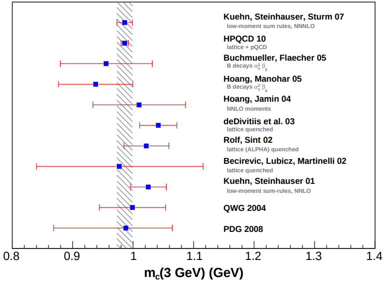
- Consistency and stability between $n = 1, 2, 3, 4$
- Result: n=2: $m_b(10\ \text{GeV})$= 3.610(16) GeV

  $m_b(m_b)$= 4.163(16) GeV

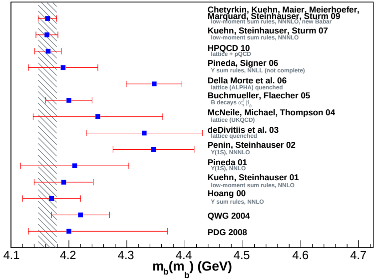
  Theo. uncertainty by truncation error comparable
- Well consistent with KSS 2007

III. Comparison

charm-quarks



bottom-quarks



IV. Summary & Conclusion

- Precise determination of the charm- and bottom-quark mass can be obtained from the experimentally measured R-ratio in combination with heavy quark current correlators computed in continuum perturbation theory
- Calculation of expansion coefficients of polarization functions up to NNNLO
- Analysis of the R-ratio and extraction of charm- and bottom-quark masses
- Final results

  quark masses :

  - Charm-mass: $m_c(3\text{ GeV}) = 0.986(13)\text{ GeV}$ e^+e^- + pQCD
  - Bottom-mass: $m_b(10\text{ GeV}) = 3.610(16)\text{ GeV}$ e^+e^- + pQCD