Hydrodynamic Fluctuations near the QCD critical point

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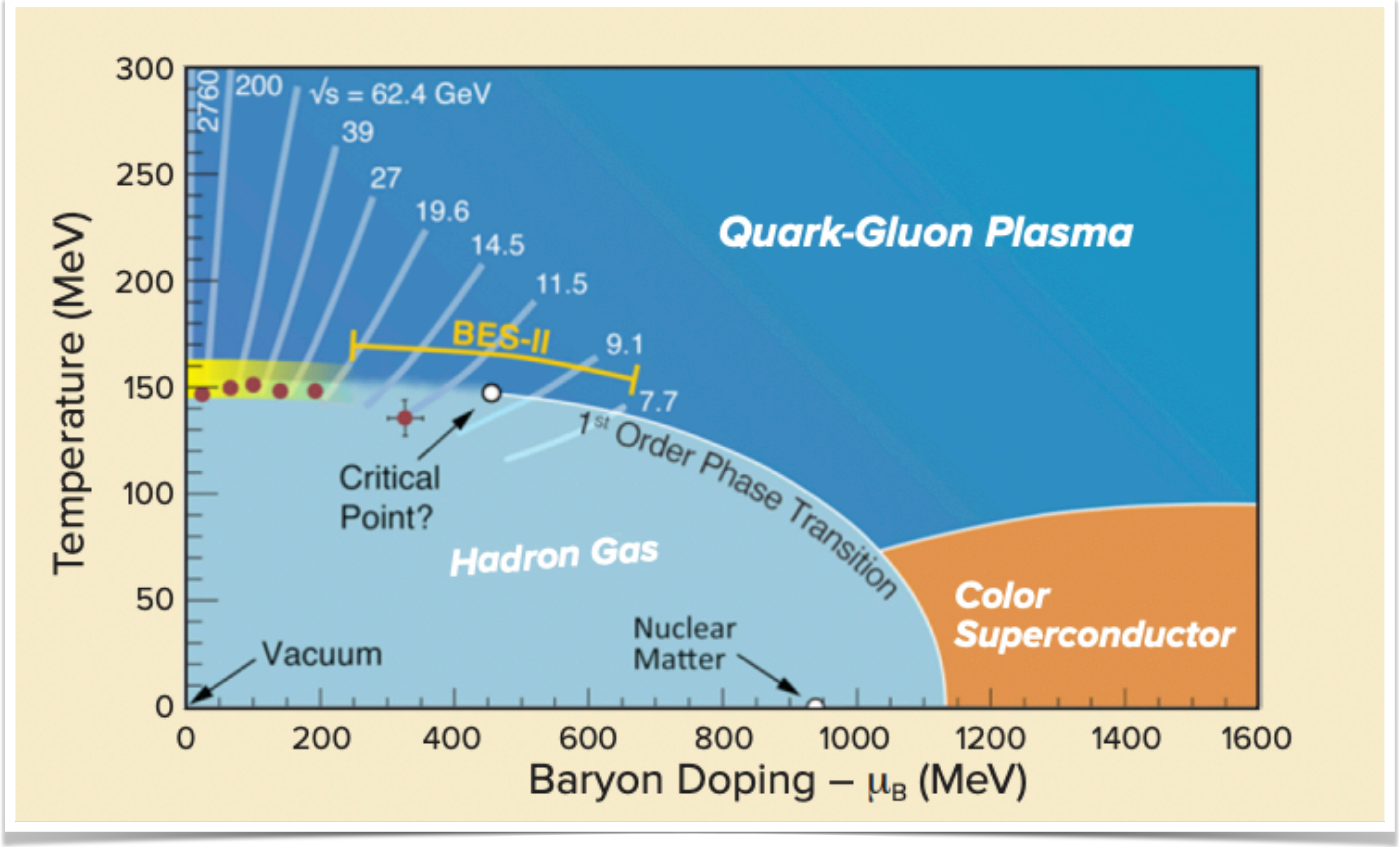
QCD Phase Diagram



- Map of singularities of the Equation of State , $P(\mu, T)$
- At $\mu_B = 0$: It is a cross-over.
- At T= 0 : Models predict a first order phase transition

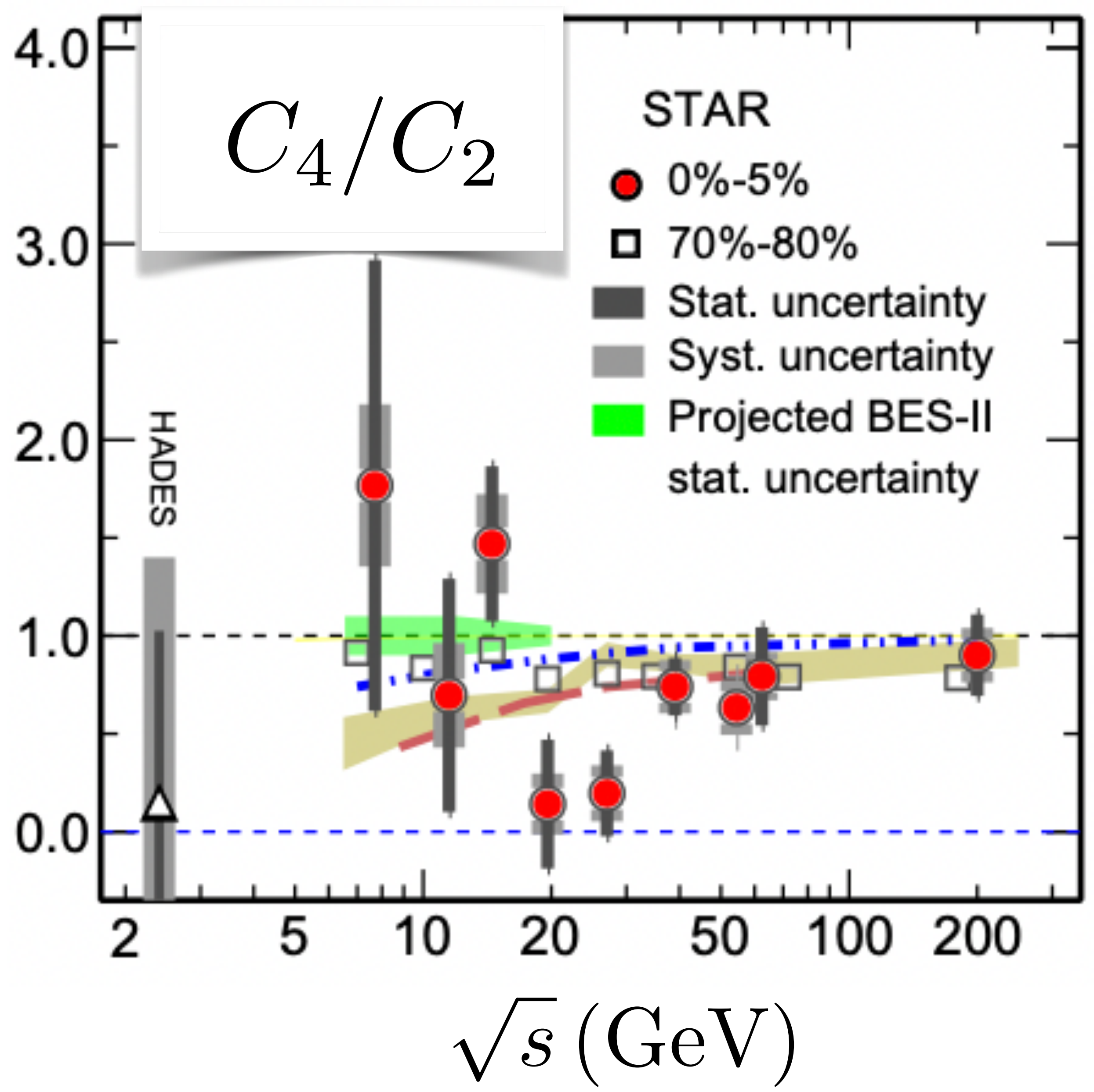
Heavy-ion collisions at varying center of mass energies can scan the phase diagram

Non-monotonic deviation of the cumulants of particle multiplicities from the baseline

Fluctuations are enhanced near CP

Diverges at CP

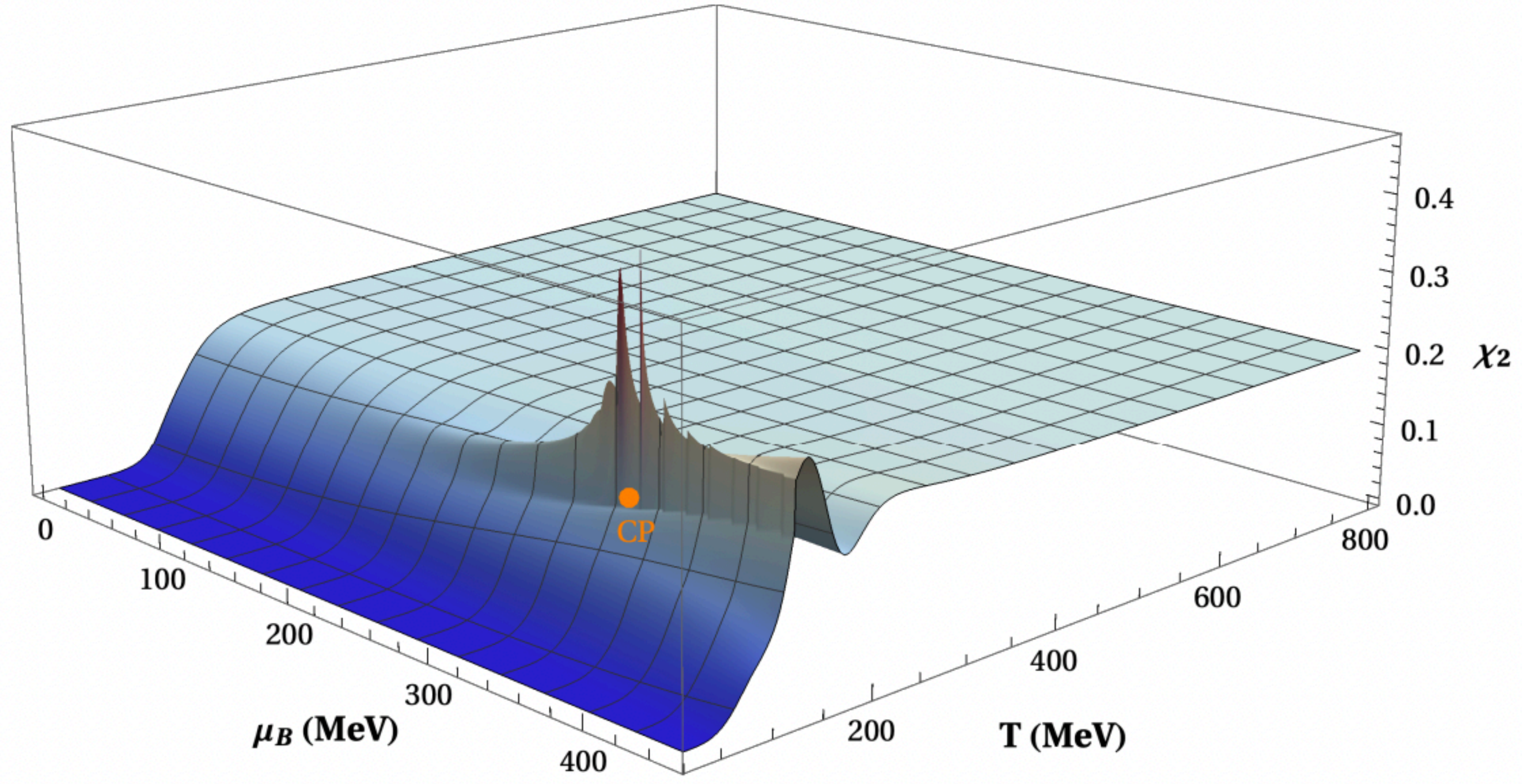
$$C_k \equiv \left\langle \delta N_B^k \right\rangle_c \overset{\text{in eq.}}{=} V T^{k-1} \frac{\partial^k P}{\partial \mu^k}$$



STAR Collaboration, 21

Beam Energy Scan- I results; BES-II results with better precision anticipated in near future

(For net proton multiplicity)



Karthein et al, 21

Higher cumulants are more sensitive to critical point.

Factors that affect the magnitude of the cumulants

- Non-monotonicity comes from the thermal fluctuations near the critical point
- Factors that affect the magnitude of critical contribution : EoS, dynamical suppression and freeze-out parameters
- Non-critical sources: Fluctuating Initial State, Resonance Decays, Detector related etc..

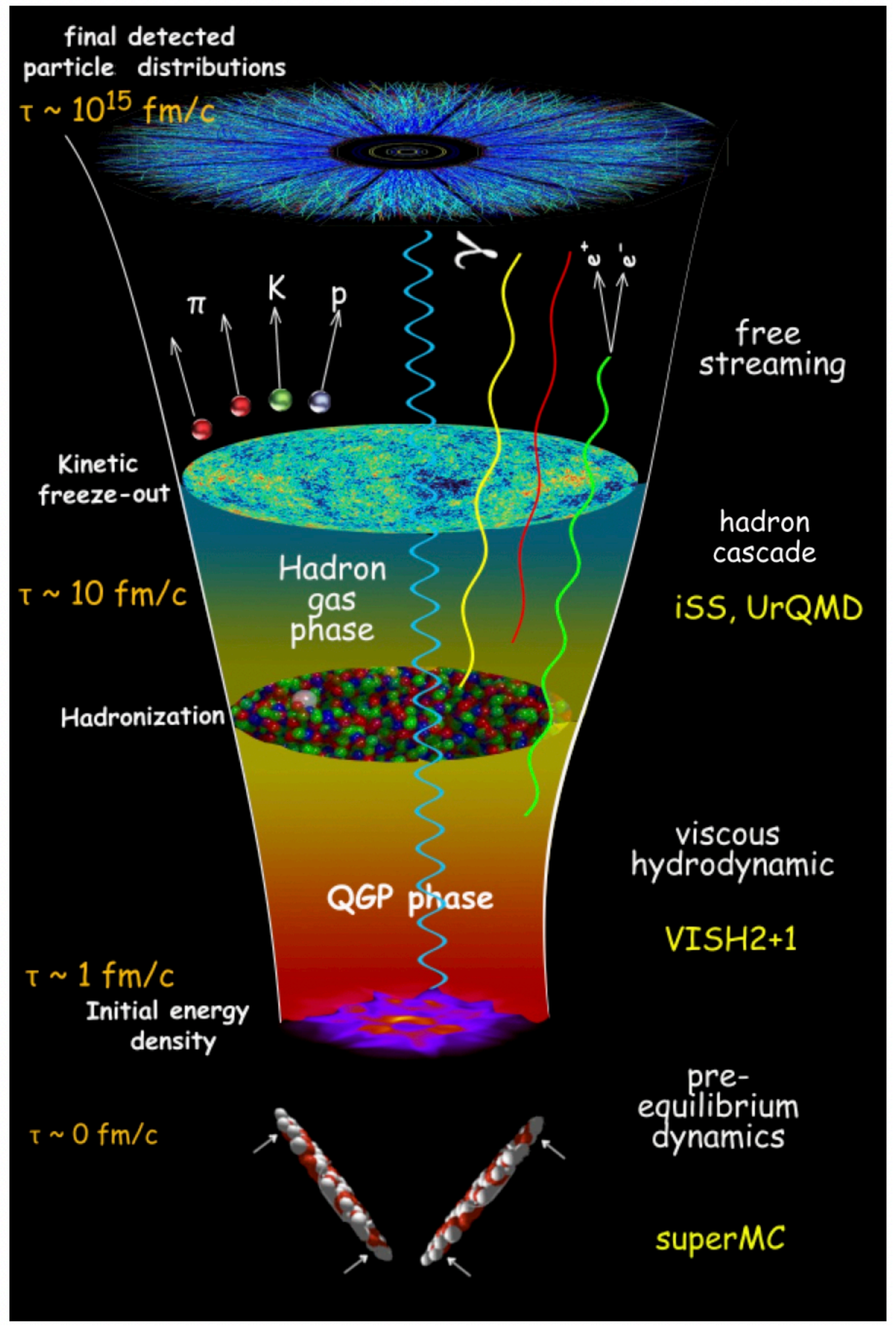
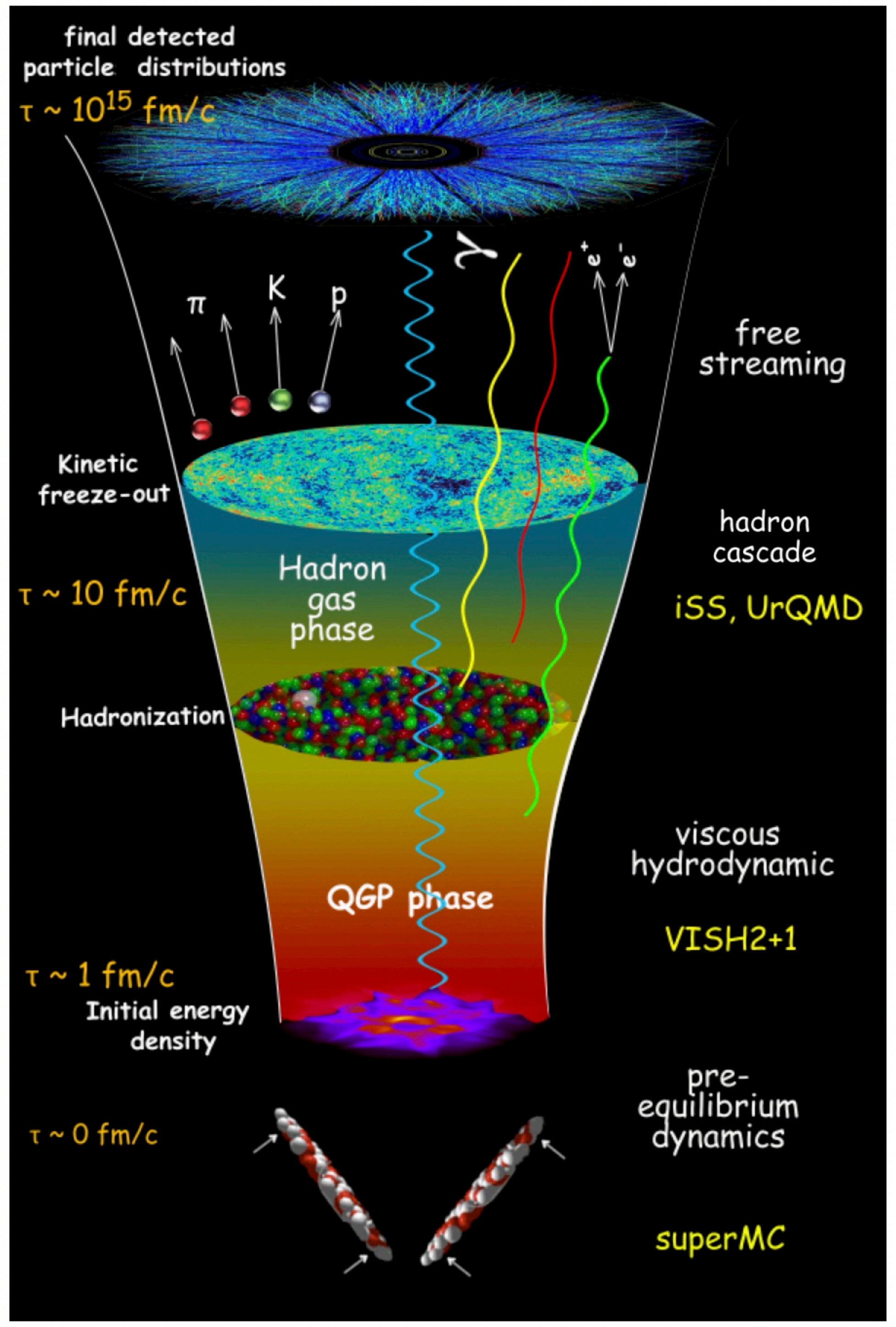


Figure by Chun Shen

Need a theoretical framework for describing fluctuations in HICs.

Overview of this talk

Figure by Chun Shen



I will talk about how fluctuations evolve during the hydrodynamic regime and eventually freeze-out .

1. Equilibrium estimates for cumulants of proton multiplicities
2. Evolution of thermodynamic fluctuations near the critical point
3. Particle multiplicity correlations from hydrodynamic fluctuations

Equilibrium estimates for cumulants of proton multiplicities

1. Cumulants of baryon-multiplicity are proportional to thermodynamic derivatives of pressure
2. Cumulants of particle multiplicities of a certain kind - we need a model
3. Critical fluctuations in particle multiplicities are incorporated via the interaction of the particle fields with a critical σ field

Particle multiplicity fluctuations

Athanasiou, Rajagopal, Stephanov, 10

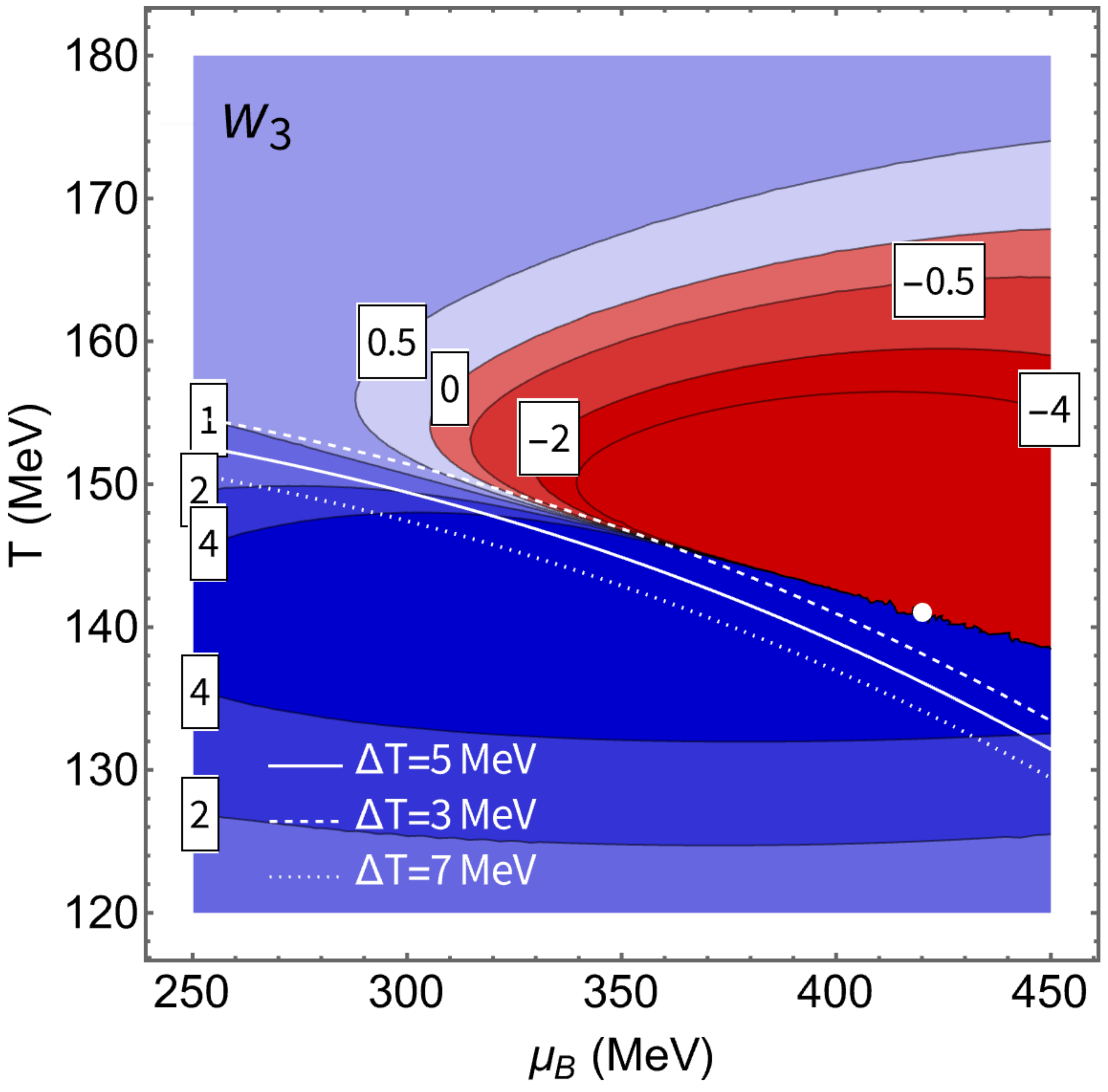
Unknown constants

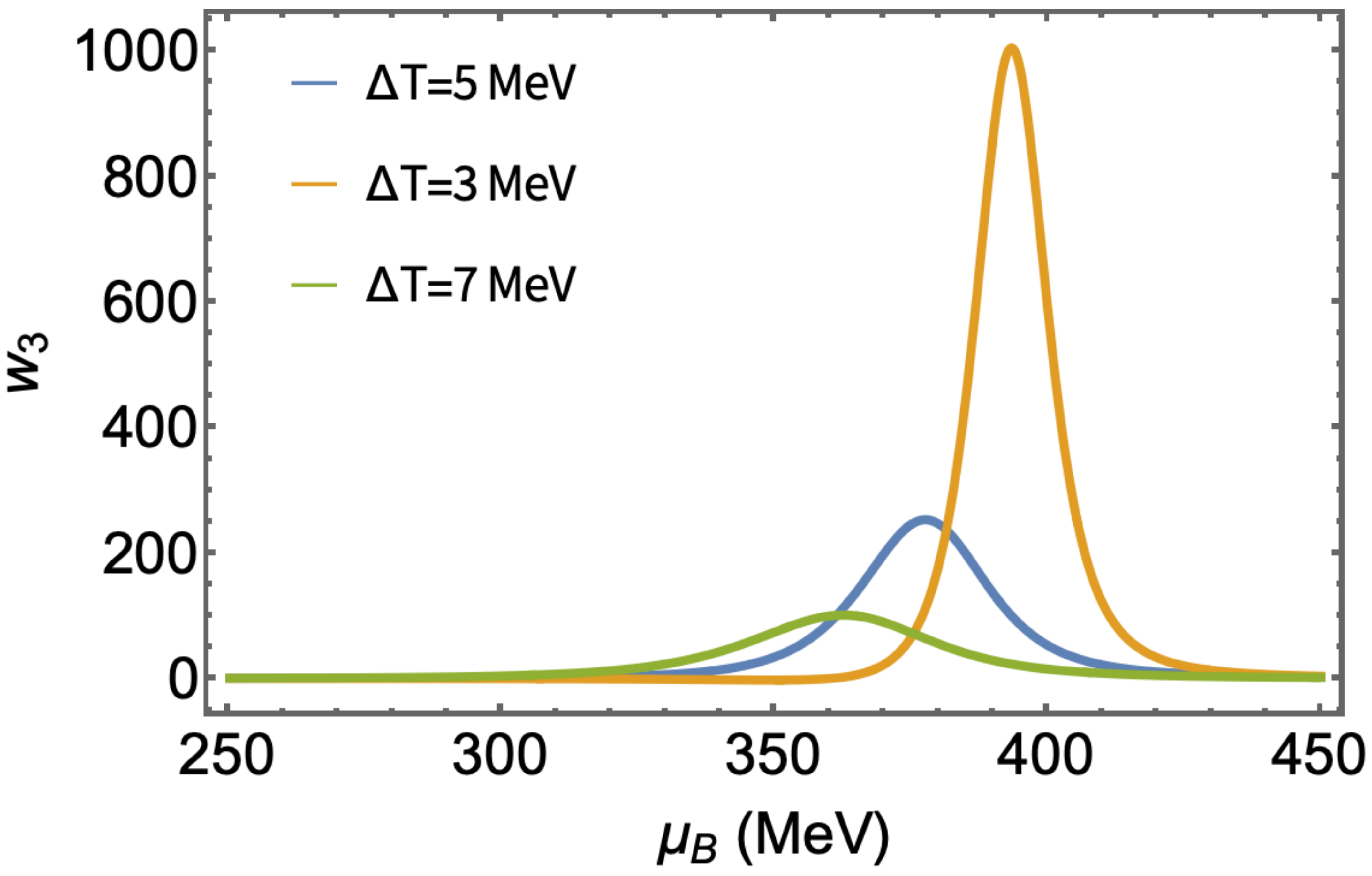
Update :Karthein, MP, Rajagopal, Stephanov, Yin (in progress)

$$\Delta G_{ABC} \equiv \langle \delta f_A \delta f_B \delta f_C \rangle = \frac{g_A g_B g_C}{T^3} \frac{m_A}{E_A} \frac{m_B}{E_B} \frac{m_C}{E_C} f_A f_B f_C \langle \delta\sigma\delta\sigma\delta\sigma \rangle$$

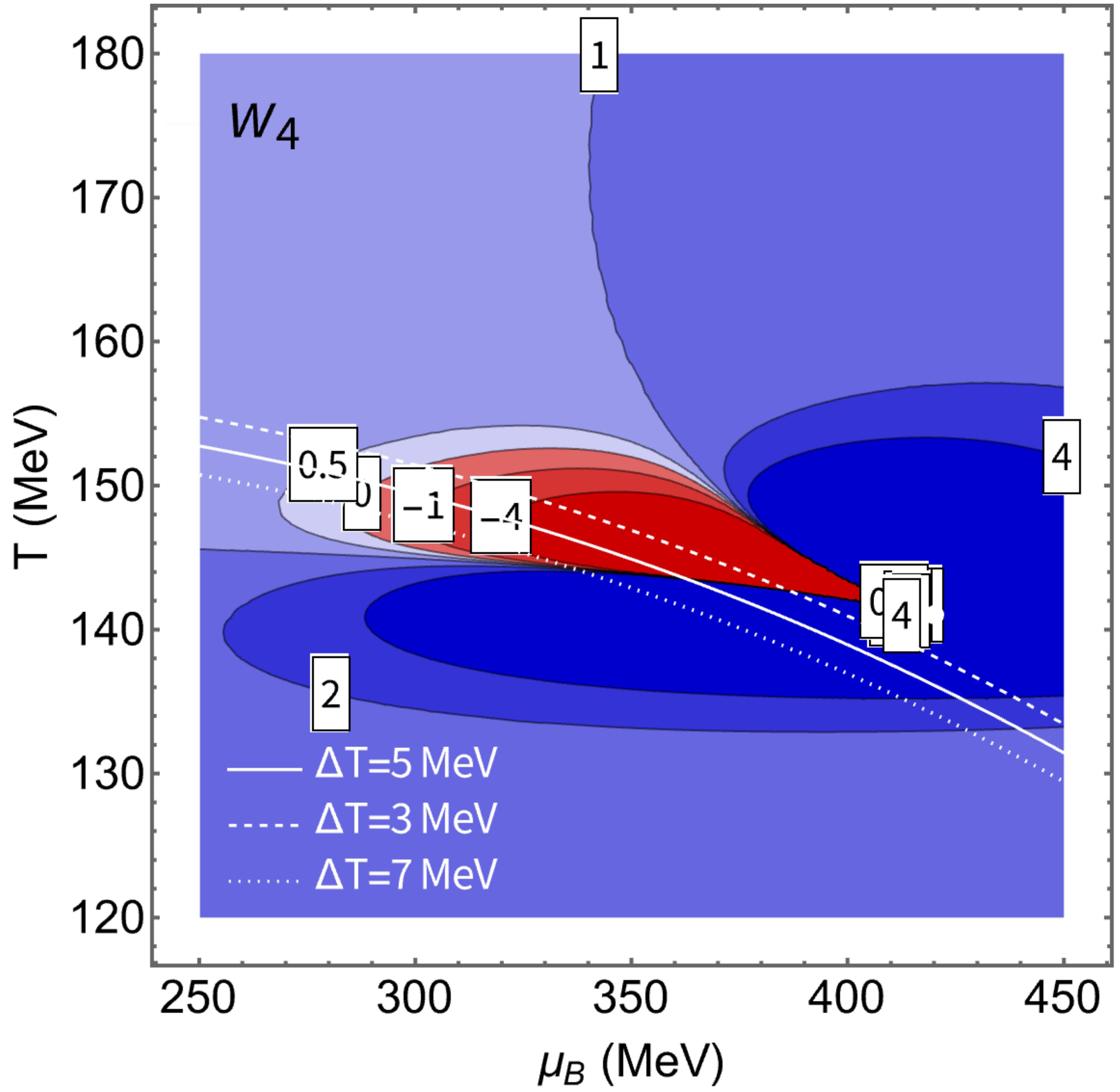
3D Ising model universality

$$w_3 = \frac{\langle \delta N_p^3 \rangle}{\langle N_p \rangle}$$

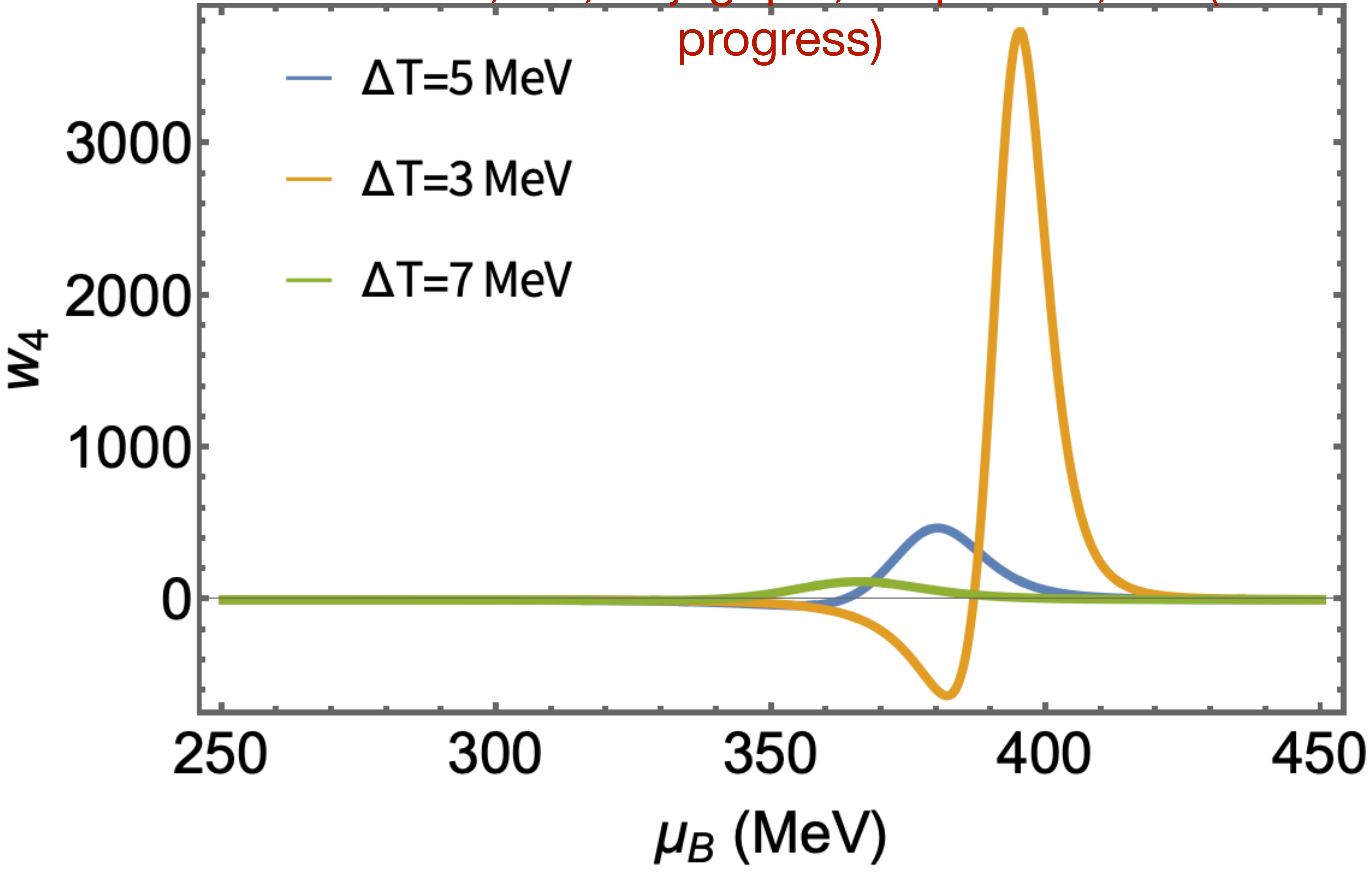




$$w_4 = \frac{\langle \delta N_p^4 \rangle_c}{\langle N_p \rangle} \qquad \Delta G_{ABCD} \equiv \langle \delta f_A \delta f_B \delta f_C f_D \rangle = \frac{g_A g_B g_C g_D}{T^4} \frac{m_A}{E_A} \frac{m_B}{E_B} \frac{m_C}{E_C} \frac{m_D}{E_D} f_A f_B f_C f_D \langle \delta\sigma\delta\sigma\delta\sigma\delta\sigma \rangle$$



Karthein, MP, Rajagopal, Stephanov, Yin (in progress)



Magnitude of the cumulants of baryon multiplicity in equilibrium also depends on the the choice of mapping parameters from QCD to 3D Ising model

Dynamics of correlation functions near critical point

Critical slowing down and effects of charge conservation

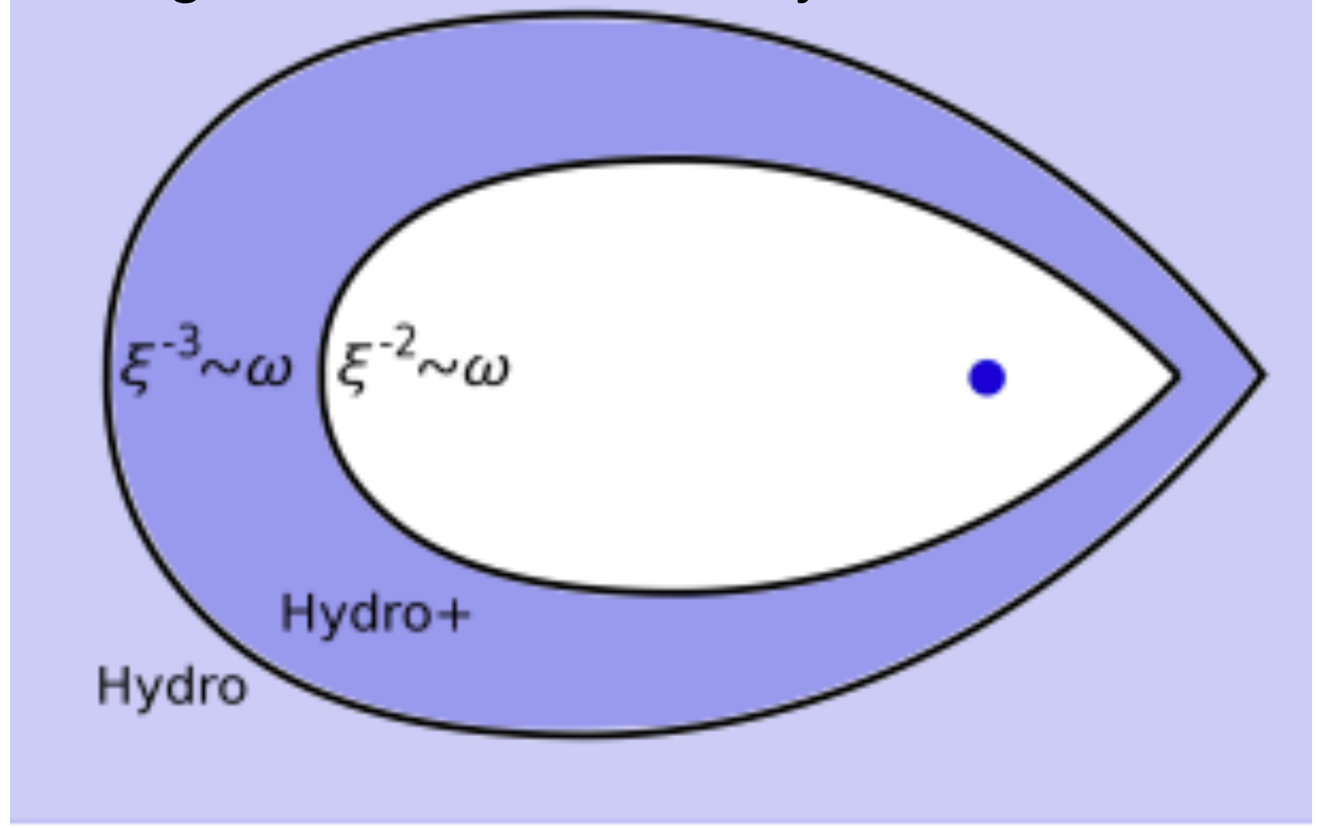
Based on M.P, K.Rajagopal, M.Stephanov, Y.Yin, *Phys.Rev.D* 106 (2022) 3, 036017

Hydro+

Stephanov and Yin, 17

EFT to describe the coupled evolution of the hydrodynamic densities & out-of-equilibrium fluctuations

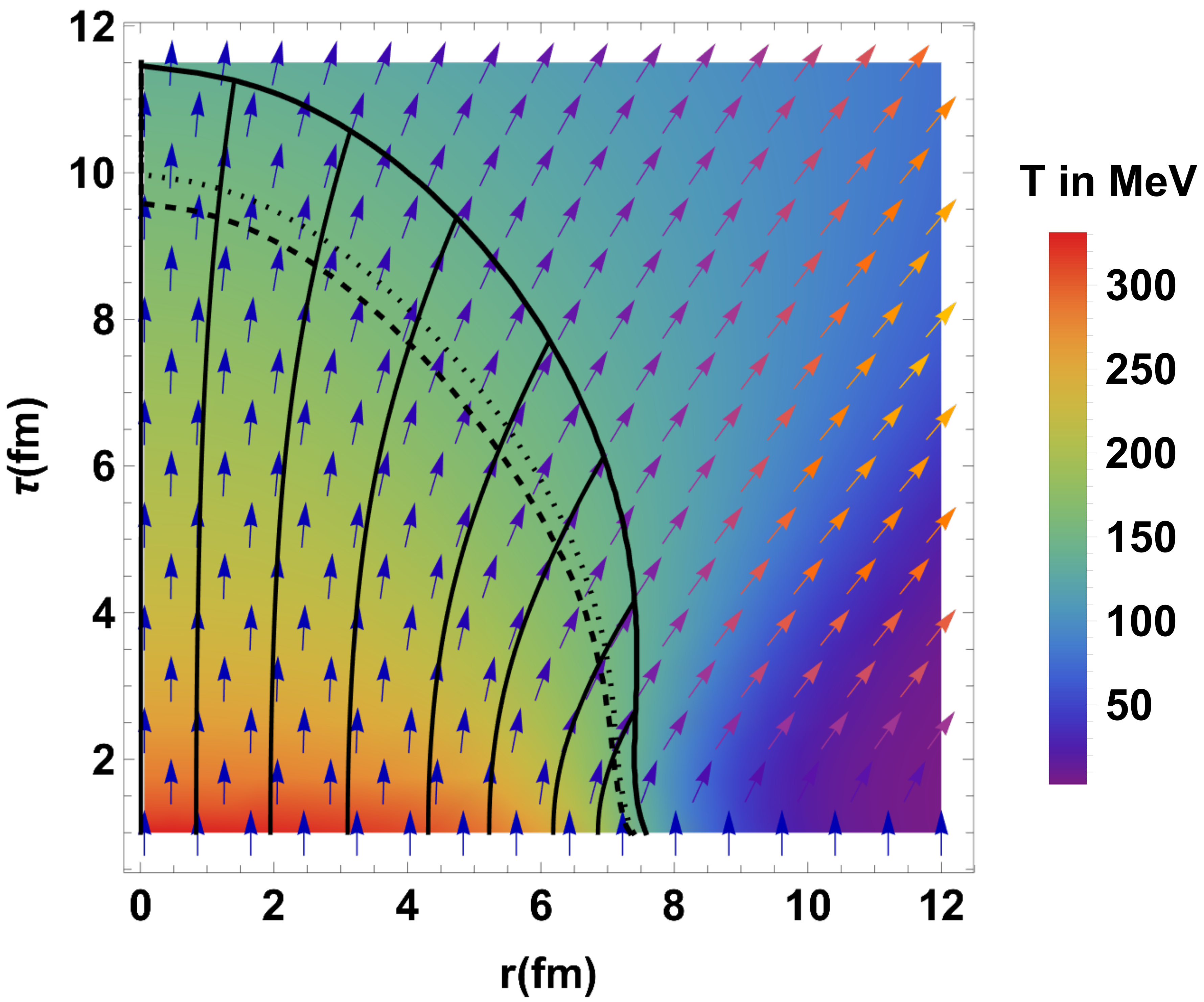
Figure taken from talk by Xin An, 2019



In practice, it may suffice to consider only one of the non-hydro modes to be out of equilibrium.

An, Basar, Stephanov, Yee, 20

- Hydrodynamics : Only mean conserved densities are dynamical.
- The fluctuations of $\hat{s} \equiv s/n$ which relaxes parametrically as $\Gamma \sim \xi^{-3}$ is the slowest non-hydrodynamic mode
- Dynamics governed by hydrodynamics + relaxation equations for the correlation functions of $\hat{s}$



Rajagopal, Ridgway, Weller, Yin, 19

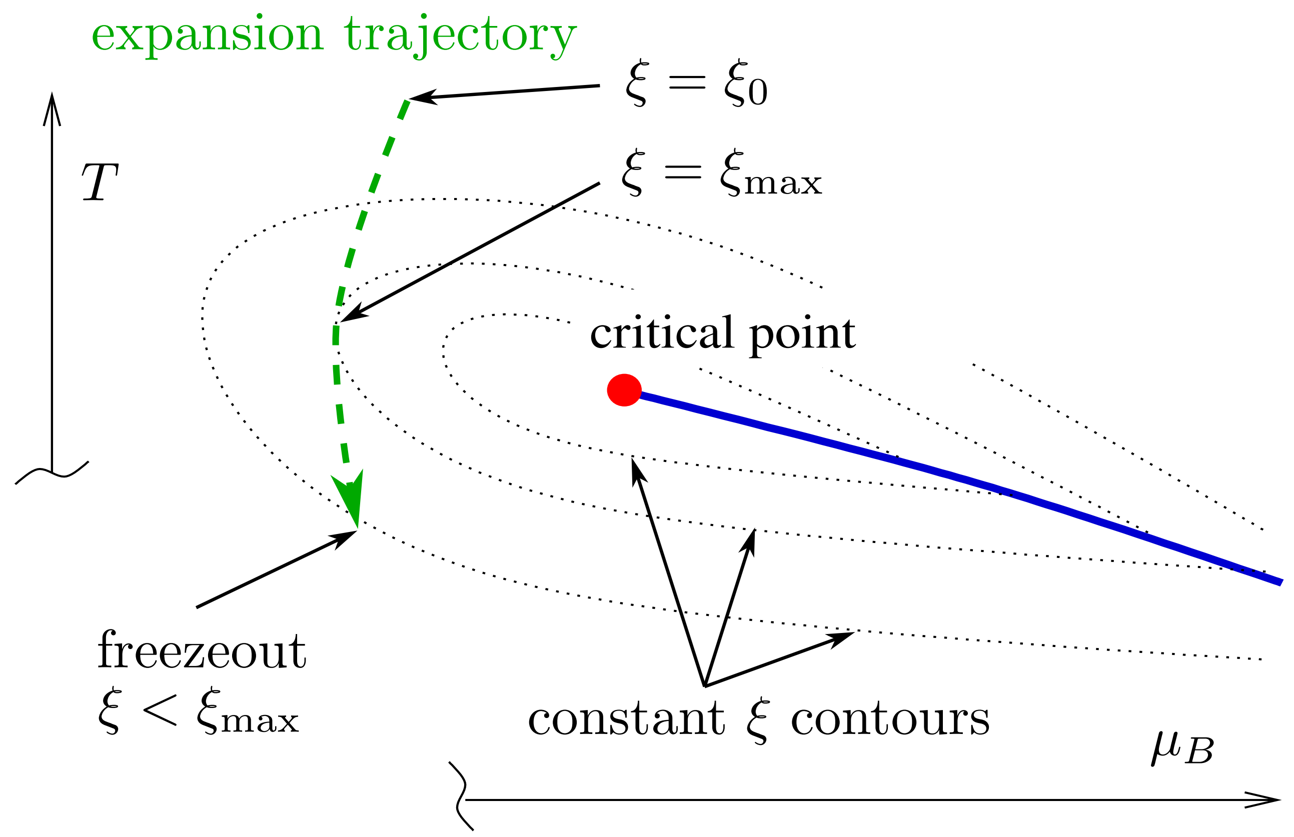
M.P, K.Rajagopal, M.Stephanov, Y.Yin, 22

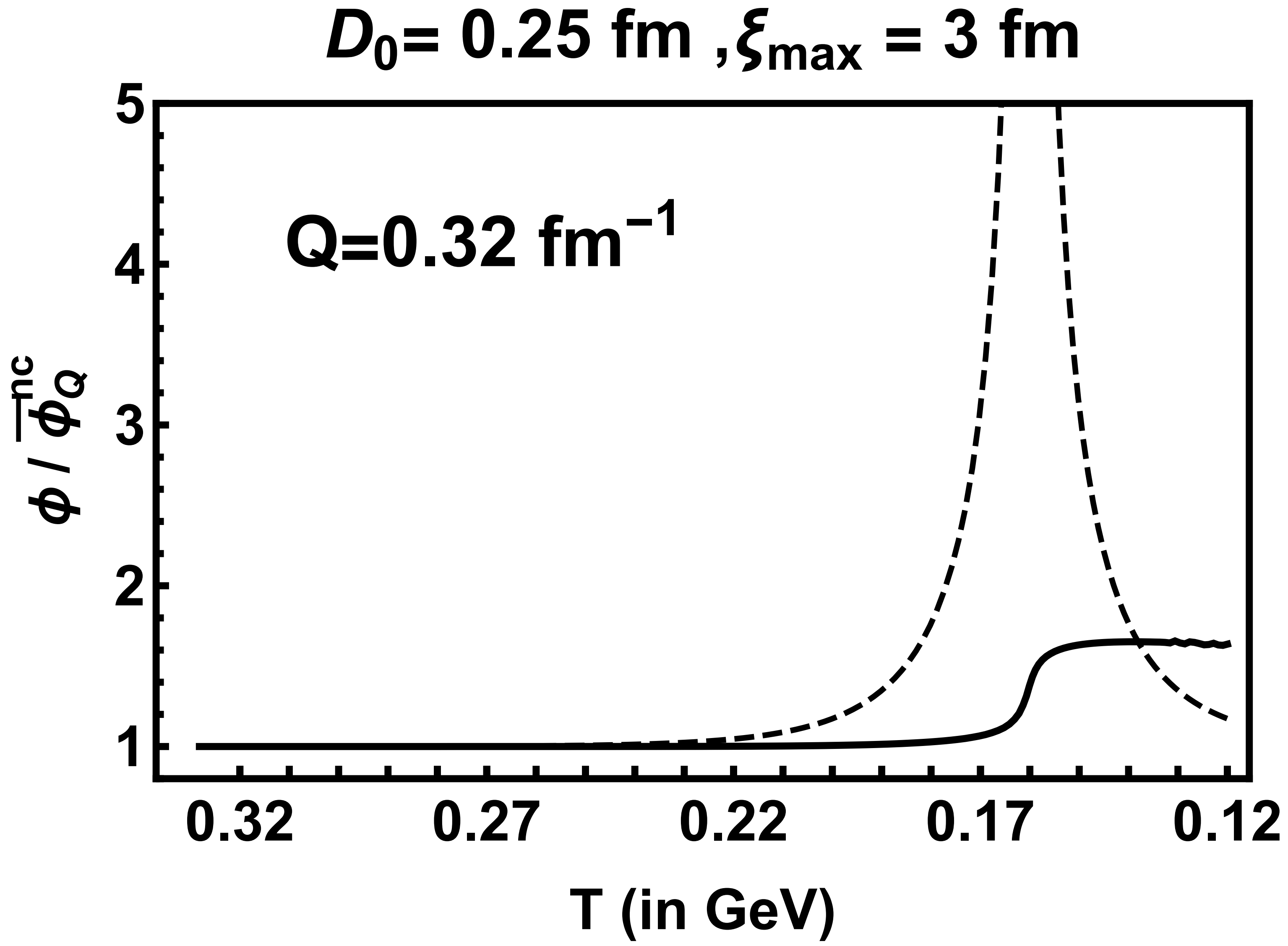
$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int e^{i\mathbf{Q}\cdot\mathbf{\Delta x}}\phi_{\mathbf{Q}}\,, \quad \mathbf{\Delta x} = x_+ - x_-$$

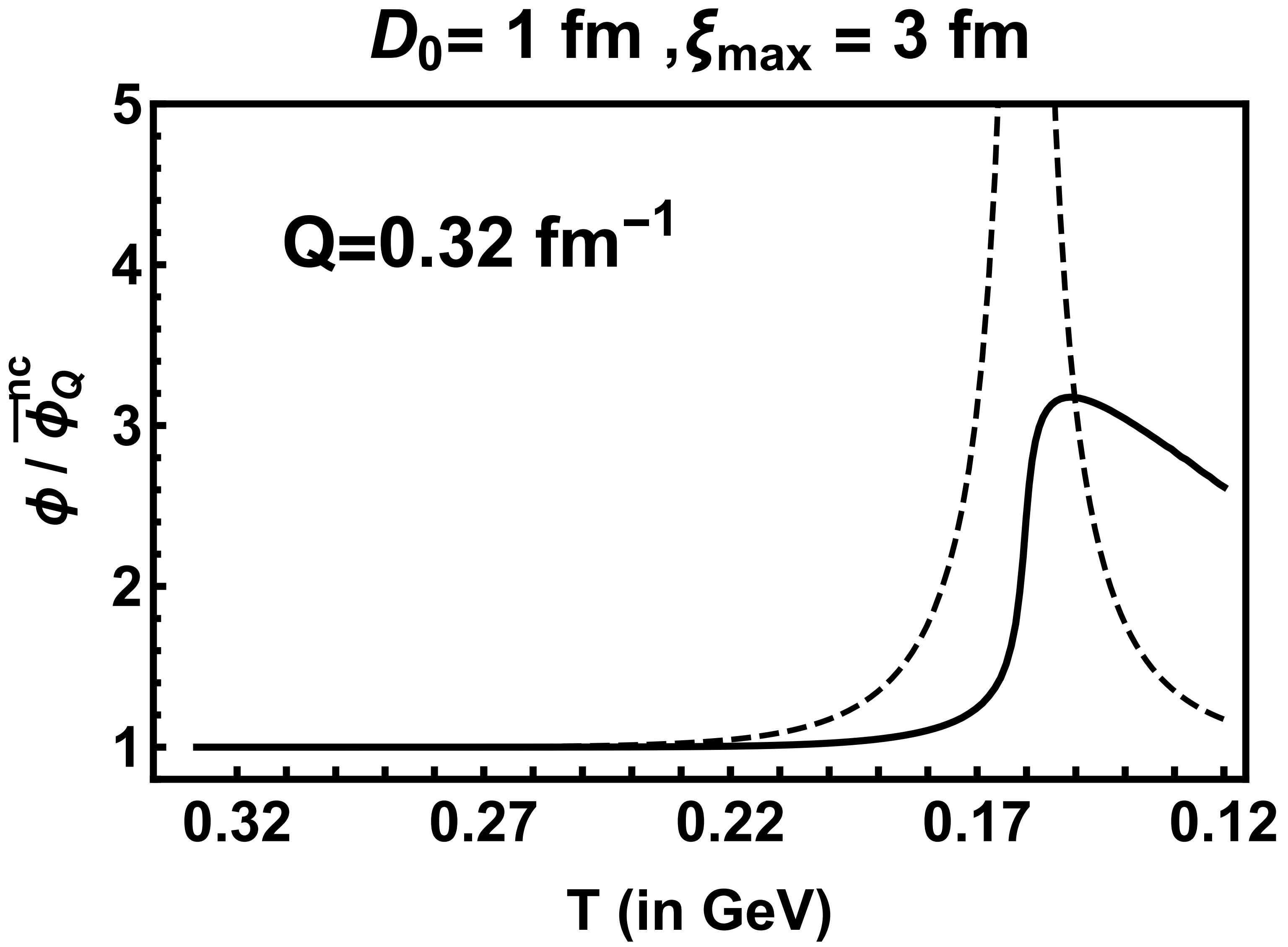
Azimuthally symmetric radially expanding fluid with

longitudinal boost invariance

$$\Gamma \approx \frac{2D_0\xi_0}{\xi}Q^2$$



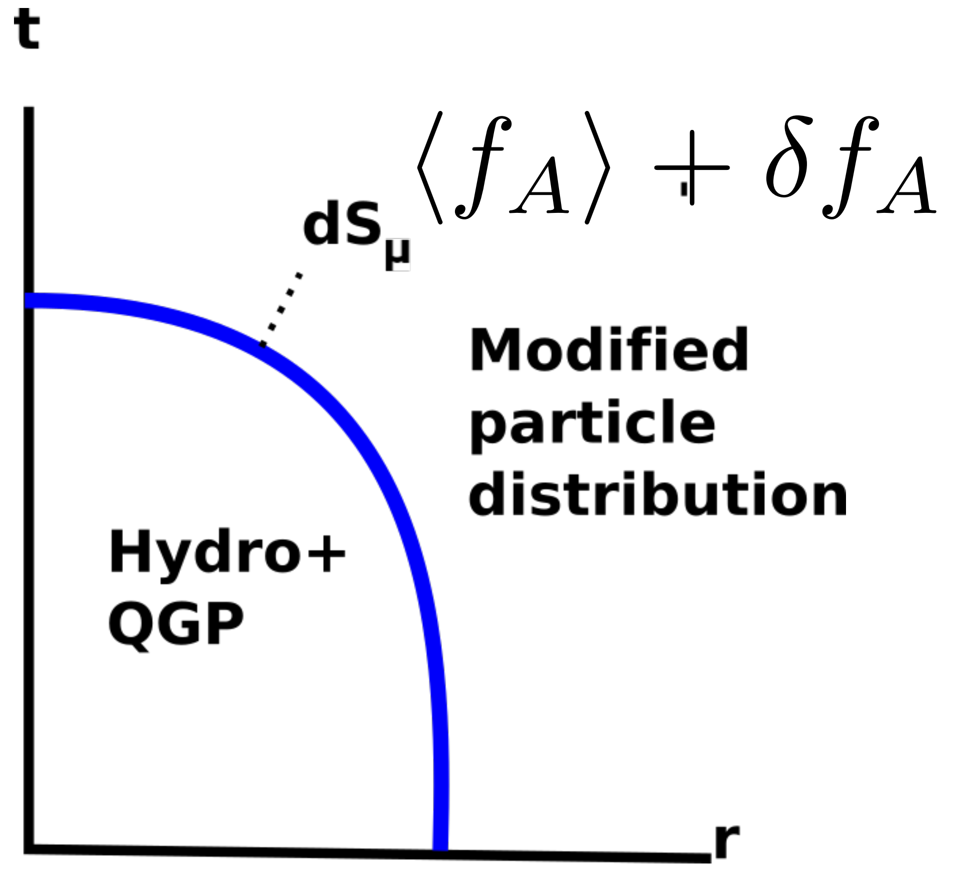




Lower Q modes are suppressed more.

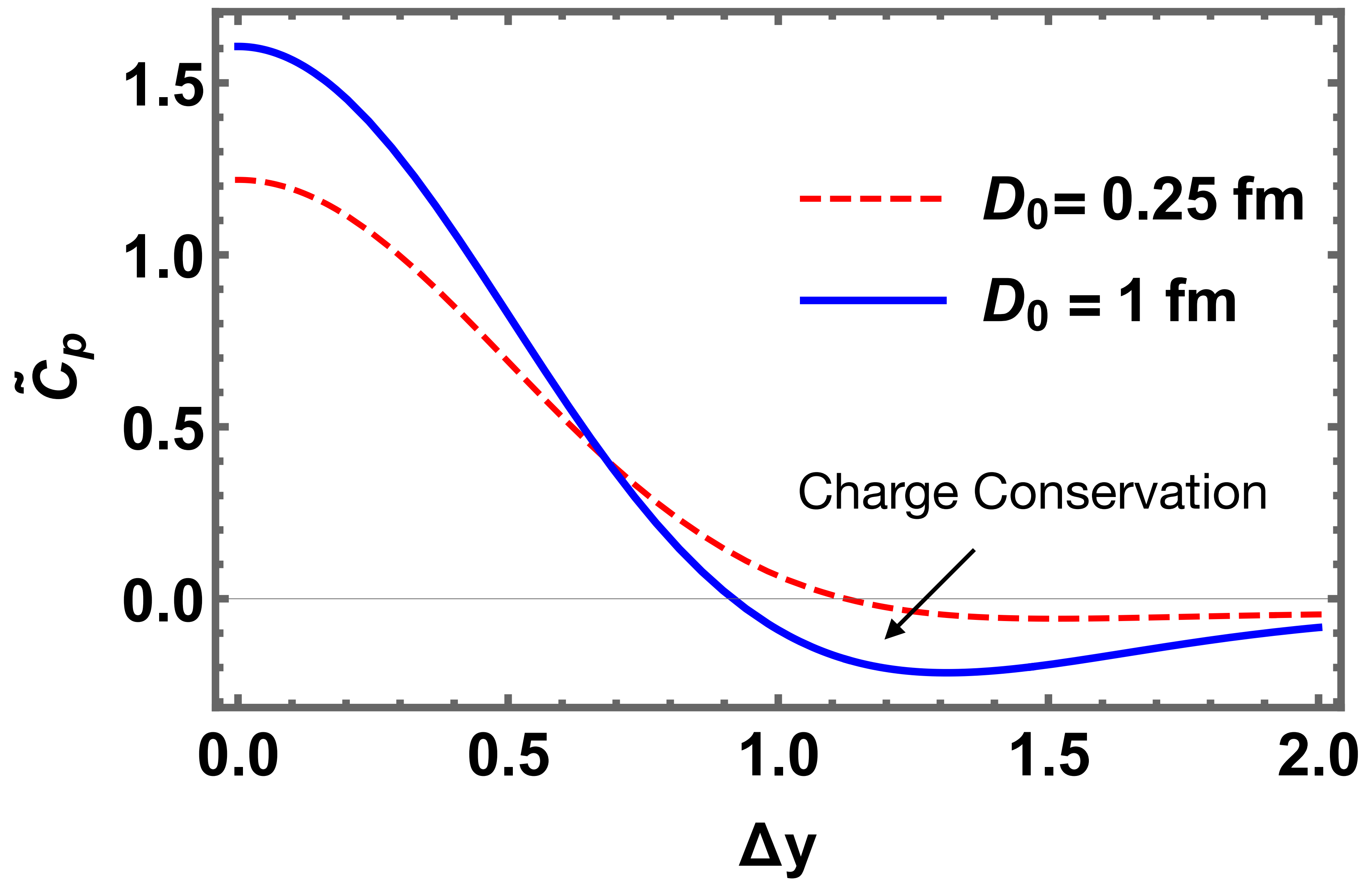
Freeze-out of critical fluctuations

- We used an ansatz inspired by universality near CP in equilibrium :

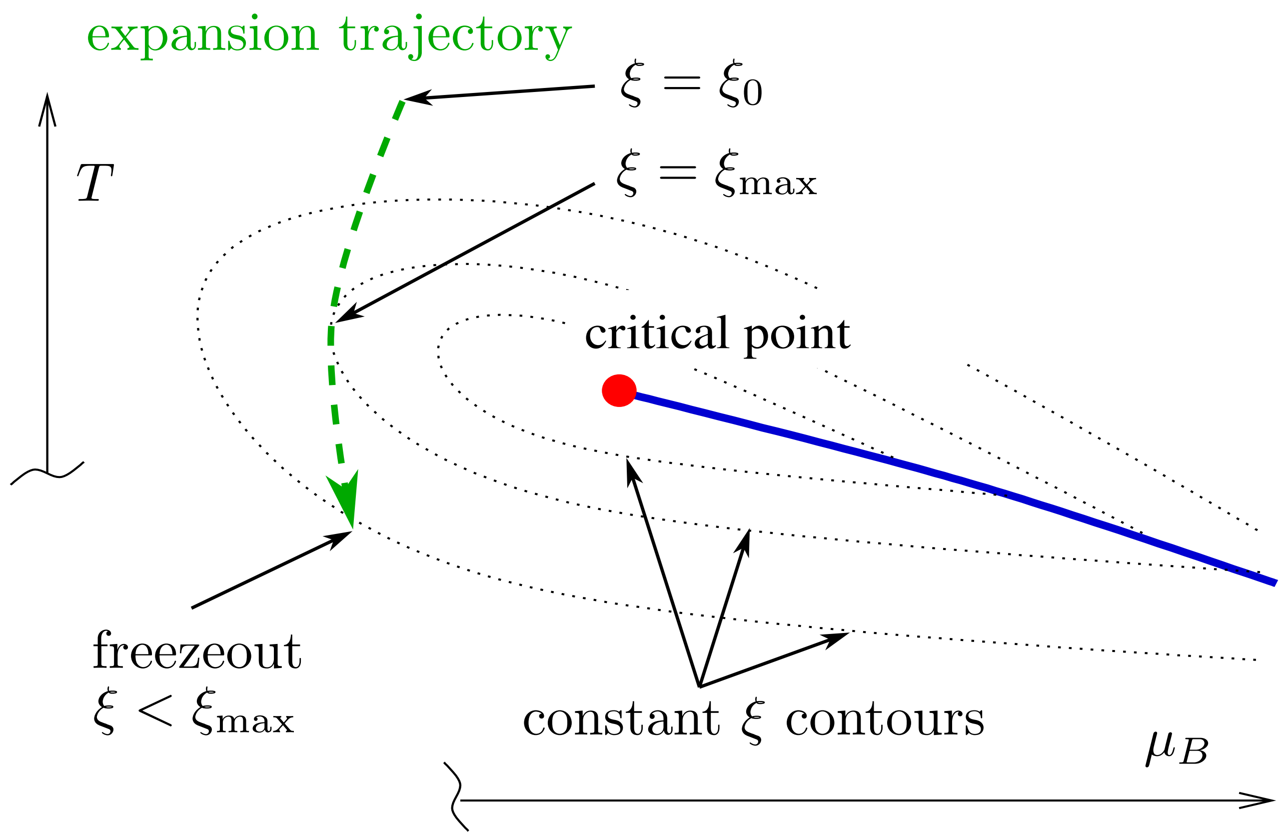


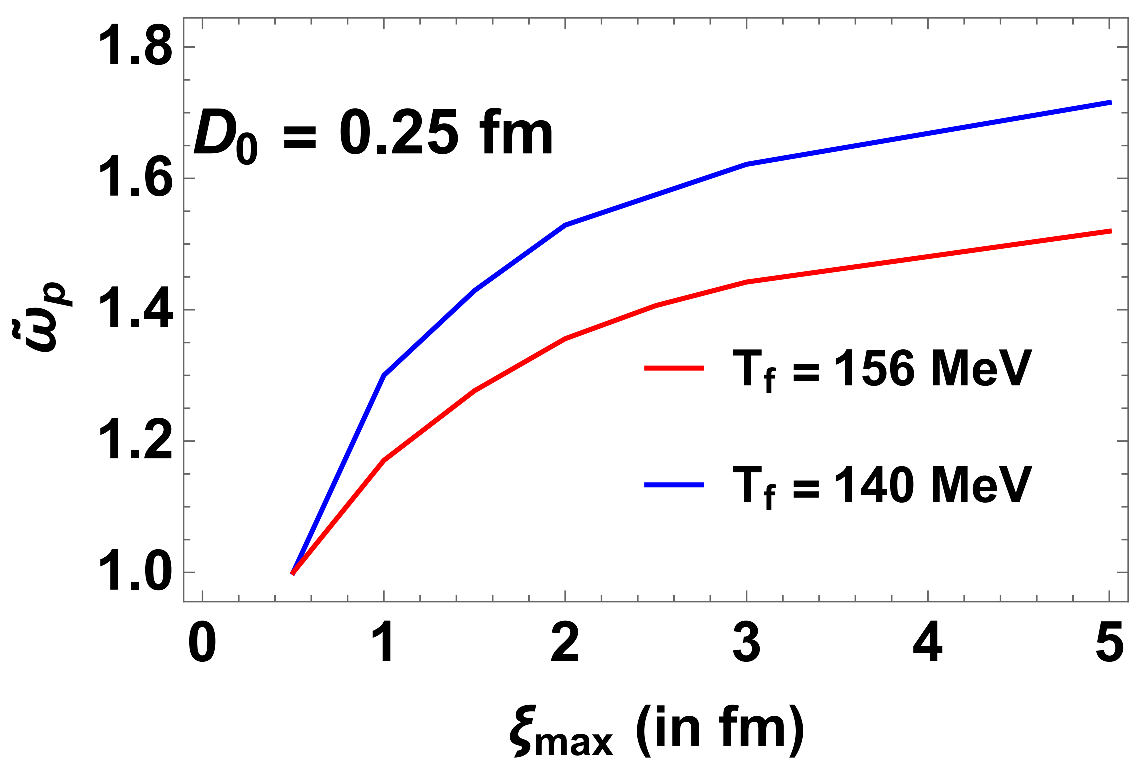
$$\langle \sigma\sigma \rangle \propto \langle \delta\hat{s}\delta\hat{s} \rangle$$

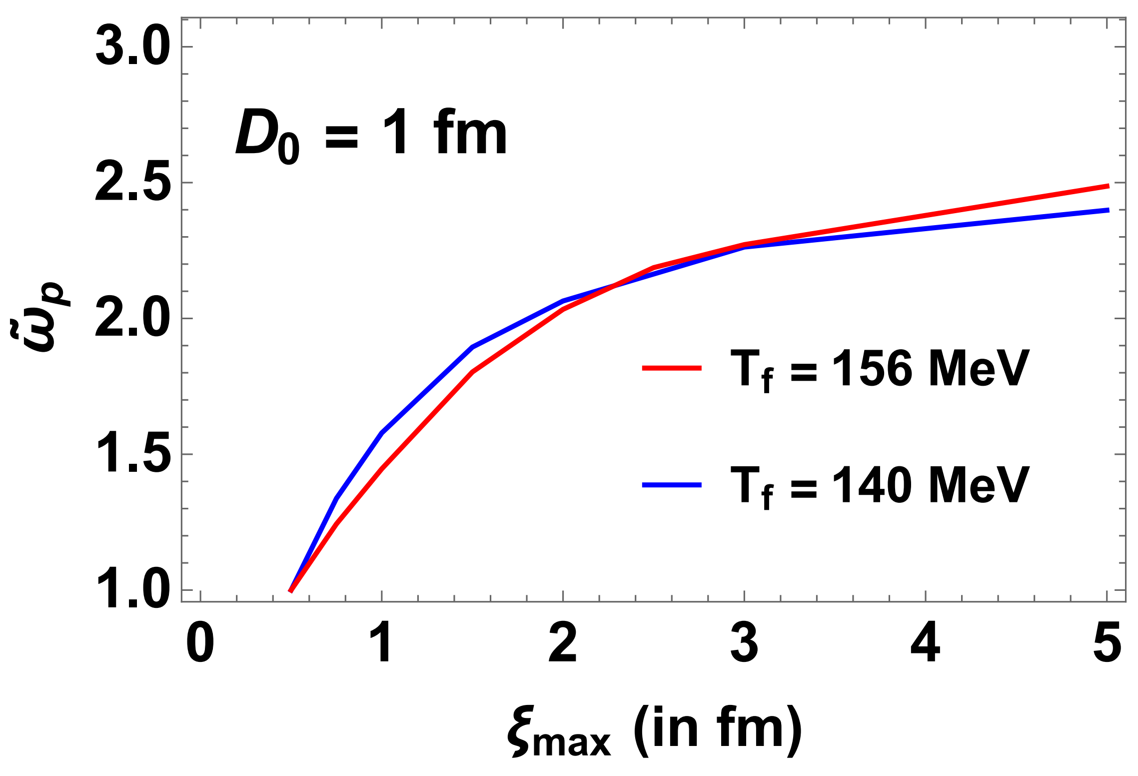
- Correlations between proton multiplicities at large momentum separation can be less than Poisson expectation
- The contribution of lower Q modes of hydrodynamic fluctuations was more dominant

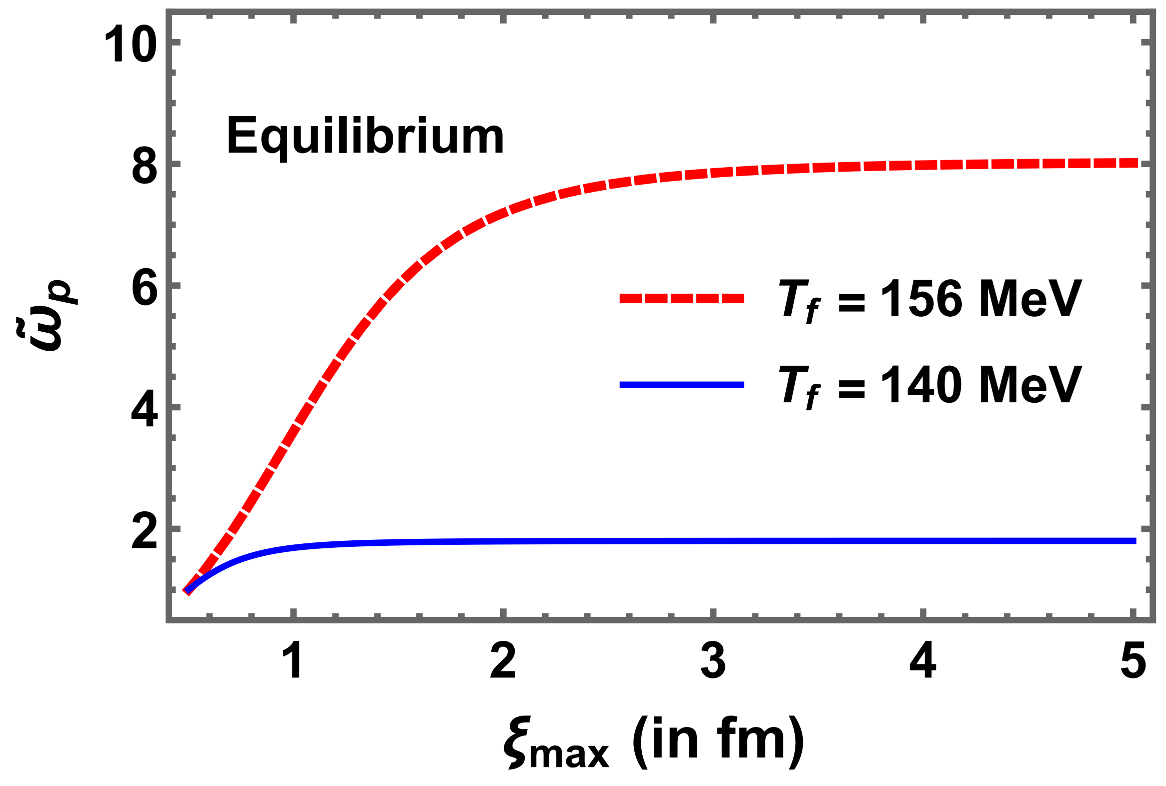


Critical contribution to variance of proton multiplicity









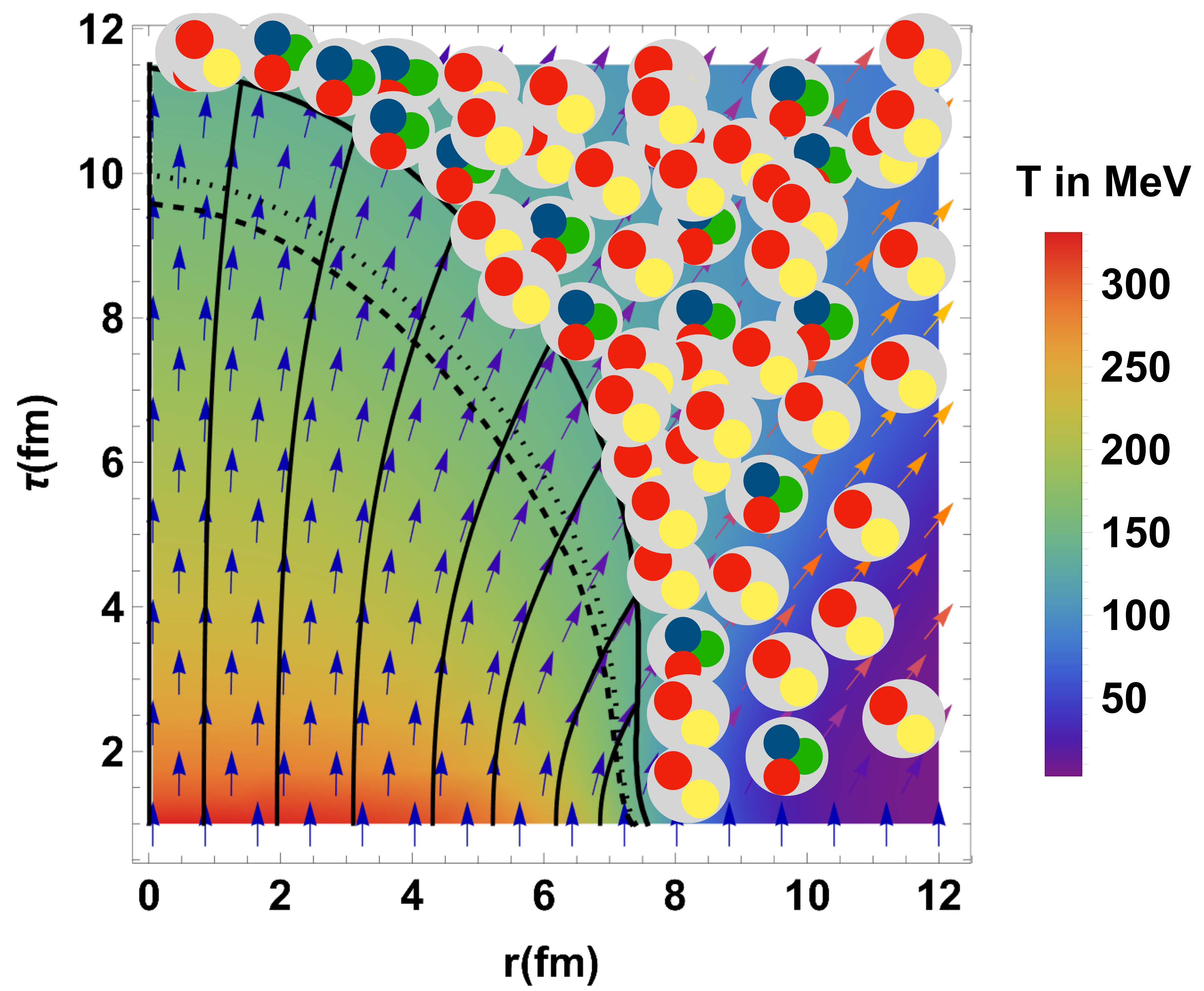
$\xi_{\max}$ Proximity of the trajectory to critical point

T_f Proximity of freeze-out point to critical region

$T_{\max} = 160\,\text{MeV}$ (T when $\xi = \xi_{\max}$)

$$\tilde{\omega}_p = \#_{\rm nc}^{-1}(C_2 - C_1)$$

* **The fluctuations are reduced relative to equilibrium value (conservation laws)**
* **Fluctuations increase with D_0 (faster diffusion)**
* **Compared to the equilibrium scenario, the fluctuations are less sensitive to freeze-out temperature**



Freeze-out in a general context

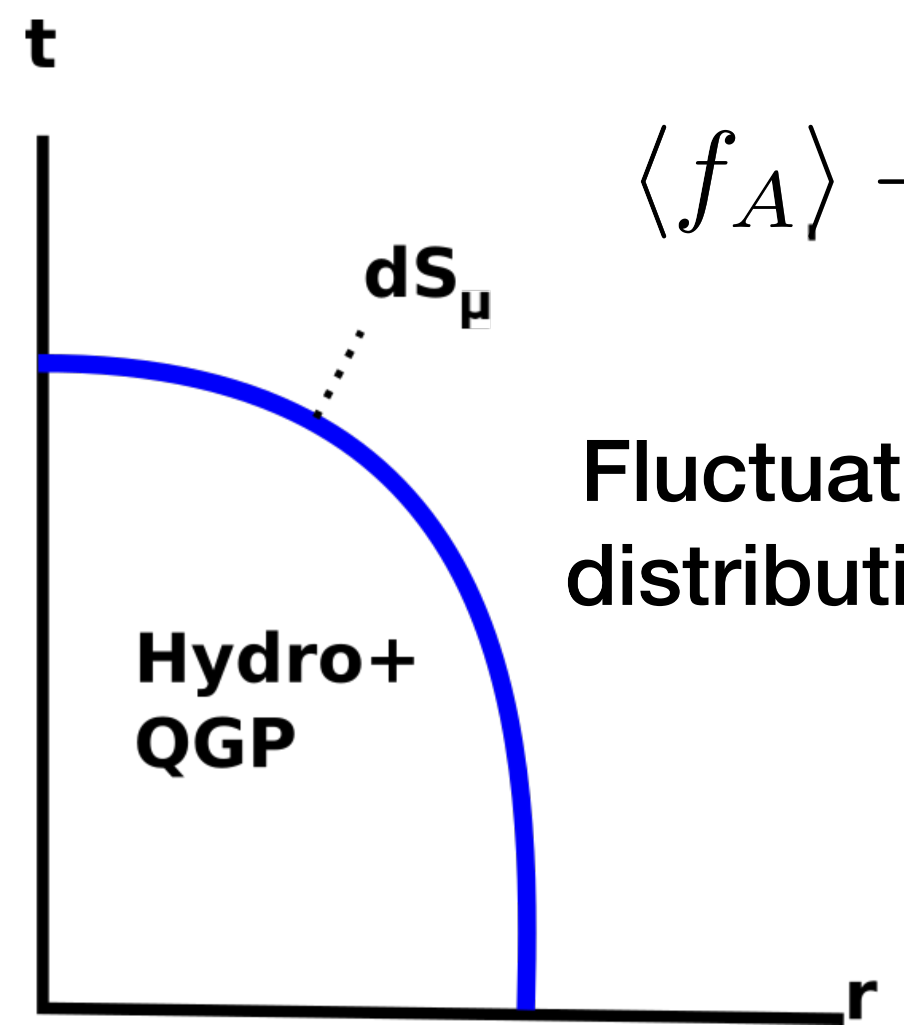
Freeze-out based on a universal maximum entropy principle

Based on M.P, M.Stephanov, *Phys.Rev.Lett.* 130 (2023) 16, 162301

Variables at freeze-out

Hydrodynamic mean densities

$$\{\langle \epsilon u^\mu \rangle \, , \, \langle n \rangle\} \equiv \Psi^a$$



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution function

Hydrodynamic correlations

$$\Psi^a, \, \left\langle \delta\Psi^a \delta\Psi^b \right\rangle \equiv H^{ab}, \ldots \, H^{abc\ldots}$$

Particle distribution function at freeze-out

$$\langle f_A \rangle = \bar{f}_A \, , \, \langle \delta f_A \delta f_B \rangle = G_{AB} \, , \, \langle \delta f_A \delta f_B \delta f_C \rangle = G_{ABC} \, \ldots$$

Matching conditions at freeze-out

Energy-momentum densities

Charge densities

$$\langle \epsilon \, u^\mu \rangle = \sum_A \int_{p_A} \bar{f}_A \, p_A^\mu \,, \quad \langle n \rangle = \sum_A q_A \int_{p_A} \bar{f}_A$$

$$\Psi^a = \sum_A \int_{p_A} \bar{f}_A P_A^a$$

$$P_A = \begin{bmatrix} p_A^\mu \\ q_A \end{bmatrix}$$

Fluctuations

$$H^{abc\ldots} = \sum_{A,B,C,\ldots} \int_{p_A p_B p_C \ldots} G_{ABC\ldots} P_A^a P_B^b P_C^c \cdots$$

- Infinitely many sets of distribution functions that satisfy these matching conditions
- Which of these solutions is the most probable?

Maximum entropy freeze-out

The ***most probable*** ensemble of free streaming particles after freeze-out that obeys the matching conditions?

It is the one which ***maximizes the entropy*** of the fluctuating particle distribution function, subject to the constraints of the matching conditions.

Maximum entropy freeze-out of ordinary hydrodynamics

- Results in a thermal gas of hadrons specified by the local temperature and chemical potential. Coincides with Cooper-Frye, 74
- Recent work on extension to viscous hydrodynamics : Everett-Heinz-Chattopadhyay, 21

Correlations from maximum entropy freeze-out

$$P_A = \begin{bmatrix} p^\mu_A \\ q_A \end{bmatrix}$$

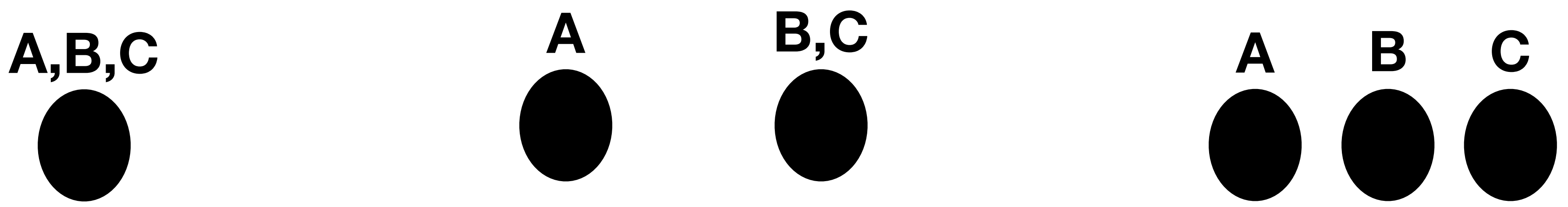
IRC   IRC   Equilibrium

$$\widehat{\Delta} G_{AB\ldots} = \widehat{\Delta} H_{ab\ldots} (\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B \ldots,$$

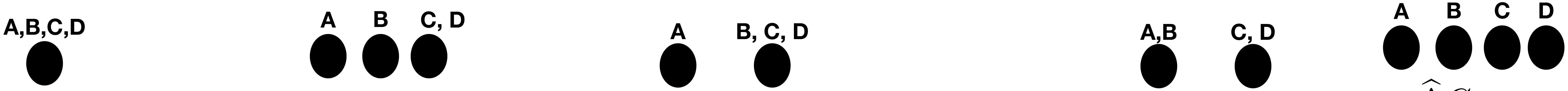
For classical gas, irreducible relative cumulants (IRCs) reduce to factorial cumulants.

Systematically subtracts terms containing self correlations

$$G_{AB} = \bar{G}_{AB} + \widehat{\Delta} G_{AB}$$



$$G_{ABC} = \bar{G}_{ABC} + 3 \widehat{\Delta} G_{AD} \left(\bar{G}_2^{-1} \bar{G}_3 \right)_{DBC} + \widehat{\Delta} G_{ABC}$$



$$G_{ABCD} = \bar{G}_{ABCD} + 6 \widehat{\Delta} G_{ABF} \left(\bar{G}_2^{-1} \bar{G}_3 \right)_{FCD} + 4 \widehat{\Delta} G_{AF} \left(\bar{G}_2^{-1} \bar{G}_4 \right)_{FBCD} + 3 \widehat{\Delta} G_{EF} \left(\bar{G}_2^{-1} \bar{G}_3 \right)_{FCD} \left(\bar{G}_2^{-1} \bar{G}_3 \right)_{EAB} + \widehat{\Delta} G_{ABCD}$$

Applying *maximum-entropy freeze-out* to a *Hydro+* setup :

Depends quadratically on E_A

$$g_A \equiv \hat{g}_A \frac{\sin\alpha_1}{w\sin(\alpha_1 - \alpha_2)}$$

Directly relatable to thermodynamic quantities obtained using EoS

Estimates using BEST EoS

$\mu_c = 350\,\mathrm{MeV}$

$$\hat{g}_{p,\mathbf{0}} \approx -3.1,\ \hat{g}_{\pi,\mathbf{0}} \approx 0.18,\ \hat{g}_{\bar{p},\mathbf{0}} \approx 5.5$$

Mixed correlations of protons and pions can have negative sign

The maximum entropy procedure is being employed in calculation of estimates for non-Gaussian cumulants of particle multiplicities.

Summary and Outlook

BEST COLLABORATION

- EoS with critical point in 3D Ising universality class that matches with lattice QCD is ready. (Parotto et al, 18, Karthein et al, 21)
- Affect of critical slowing down for non-Gaussian fluctuations work in progress
- General freeze-out procedure for hydrodynamic fluctuations based on universal maximum entropy freeze-out is ready

- "EoS with CP" + "Hydro+" +ME freeze-out+ Resonance decays has to be applied to more realistic scenarios to compare with experiments
- The parameters in the paradigm are the mapping parameters to Ising model, the non-critical correlation length and the freeze-out location
- Bayesian analysis can aid in determining these parameters

Thank you!