Spin physics at the EIC

Yoshitaka Hatta
BNL/RIKEN BNL

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The proton spin problem

The proton has spin $\frac{1}{2}$.

The proton is not an elementary particle.

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g
\]

Jaffe-Manohar sum rule

Ji sum rule

\[
\Delta \Sigma = 1 \quad \text{in the naïve quark model}
\]
Longitudinal double spin asymmetry in polarized DIS

\[ A_{LL} = \frac{\mu^+ p^+ - \mu^+ p^-}{\mu^+ p^+ + \mu^+ p^-} \]
\[ \approx \left( 1 + \frac{\sigma_L}{\sigma_T} \right) \frac{2x g_1}{F_2} \]

\[ \int_0^1 dx g_1(x) = \frac{1}{9} (\Delta u + \Delta d + \Delta s) \]
\[ + \frac{1}{12} (\Delta u - \Delta d) \]
\[ + \frac{1}{36} (\Delta u + \Delta d - 2\Delta s) + O(\alpha_s) \]
Helicity pQCD precision frontier

4-loop evolution of $\Delta \Sigma$
De Florian, Vogelsang (2019)

NNLO jet production in polarized DIS
Borsa, de Florian, Pedron (2020)

NNLO longitudinal spin asymmetry of W at RHIC
Boughezal, Li, Petriello (2021)

3-loop Wilson coefficients for $g_1(x)$
Blumlein, Marquard, Schneider, Schonwald (2022)
Evidence of nonzero gluon helicity  \[ \Delta G = \int_0^1 dx \Delta G(x) \]

A major achievement of the RHIC spin program!

\[
\begin{align*}
\int_{0.05}^{1} dx \Delta G(x, Q^2 = 10\text{GeV}^2) &= 0.20^{+0.06}_{-0.07} \quad \text{DSSV} \\
\int_{0.05}^{0.2} dx \Delta G(x, Q^2 = 10\text{GeV}^2) &= 0.17 \pm 0.06 \quad \text{NNPDF} \\
\int_{0.05}^{1} dx \Delta G(x, Q^2 = 10\text{GeV}^2) &= 0.23 \pm 0.03 \quad \text{JAM}
\end{align*}
\]

Huge uncertainty from the small-\(x\) region

Does the remaining spin (\(~30\%)\) come from the small-\(x\) region of \(\Delta G(x)\)?
Direct Photons Point to Positive Gluon Polarization

Results from 'golden measurement' at RHIC's PHENIX experiment show the spins of gluons align with the spin of the proton they're in

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Helicity evolution at small-x

All-order resummation of small-x **double** logarithms for helicity distributions

\[ (\alpha_s \ln^2 1/x)^n \]

Unlike BFKL, we need to include quark ladders
Unlike BFKL, we need to include non-ladder diagrams

Resummation very hard, but can be done!

Bartels, Ermolaev, Ryskin (1996)

\[ \Delta q(x), \Delta G(x) \sim \frac{1}{x^\alpha} \]

\[ \alpha \approx 3.664 \sqrt{\frac{\alpha_s N_c}{2\pi}} \]
Regge intercept at small-\(x\), revisited

Based on Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)

\[
\Delta q(x), \Delta G(x) \sim \frac{1}{x^\alpha}
\]

\[
\alpha_{BER} \approx 3.664 \sqrt{\frac{\alpha_s N_c}{2\pi}}
\]

\[
\alpha_{BK} \approx 3.661 \sqrt{\frac{\alpha_s N_c}{2\pi}}
\]
An elephant in the room: Orbital angular momentum

It’s an undeniable fact that experimentally we know nothing about OAM.

Of course, this doesn’t mean that OAM is unimportant.

At small-x, helicity and OAM cancel.

\[
\frac{d}{d \ln Q^2} L_g(x) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} (2C_F + \cdots) \Delta q(x/z)
\]

\[
\frac{d}{d \ln Q^2} \Delta G(x) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} (2C_F + \cdots) \Delta q(x/z)
\]
Helicity-OAM cancellation at small-\(x\)

If \(\Delta G(x) \approx \frac{1}{x^\alpha}\), then \(L_g(x) \approx -\frac{2}{1 + \alpha} \Delta G(x)\)

There might be a sizable contribution to \(\Delta G\) from the small-\(x\) region.

If so, there will be even larger \(L_g\) from the same \(x\)-region with an opposite sign.

Helicity is only half of the story.
Can EIC seriously address OAM?
OAM and the Wigner distribution

Wigner/GTMD distribution
Phase space distribution of partons in QCD

Belitsky, Ji, Yuan (2004);

\[ W(x, \vec{k}_\perp, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \frac{dz^-d^2z_\perp}{16\pi^3} e^{ixP^+z^- -i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(b - z/2)\gamma^+ q(b + z/2) | P + \frac{\Delta}{2} \rangle \]

Define

\[ L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \]

Lorce, Pasquini (2011);
YH (2011);
Ji, Xiong, Yuan (2012)

5D tomography encoded in the Wigner distribution—Holy grail of the nucleon structure
Can we probe it at the EIC?
Gluon OAM from single/double spin asymmetry in diffractive dijet

\[
\int d^2 k_\perp k_\perp^i W_g(k_\perp, \Delta_\perp) \sim \epsilon^{ij} \Delta_\perp^j L_g
\]

Expand the amplitude to linear order in \( k_\perp \) (twist-3 effect)

\[
d\sigma^{h_p h_1} \sim h_p h_1 \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \text{Re} \mathcal{H}_g^{(1)*} \left( \tilde{\mathcal{H}}_g^{(2)} + \frac{q_\perp^2 - \mu^2}{q_\perp^2 + \mu^2} \mathcal{L}_g \right)
\]

Ji, Yuan, Zhao (2016) (single)
Bhattacharya, Boussarie, YH (2022) (double)

Wigner distribution

lepton angle
recoil proton angle

Compton form factors of gluon helicity and OAM
First-ever quantitative prediction for an observable sensitive to OAM

\[ q_{\perp}^2 - \frac{Q^2}{4} \]

Depending on the sign of, helicity and OAM add up/cancel.

In practice, jets at low momenta are hard to reconstruct.

Alternative measurement? Bhattacharya, Boussarie, YH, in preparation
Generalized Parton Distribution (GPD)

Off-forward generalization of PDF

\[ P^+ \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \langle P' S' | \bar{\psi}(0) \gamma^\mu \psi(y^-) | PS \rangle \]

\[ = H_q(x, \Delta) \bar{u}(P' S') \gamma^\mu u(P S) + E_q(x, \Delta) \bar{u}(P' S') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m} u(P S) \]

Second moments relevant to Ji sum rule

\[ J_{q,g} = \frac{1}{2} \int_0^1 dx x (H_{q,g}(x) + E_{q,g}(x)) \]

EIC offers an unprecedented kinematical coverage for DVCS and other exclusive processes. New era of GPD studies.
Deeply Virtual Compton Scattering

\[ i \int d^4y e^{iqy} \langle P' | T \{ J^\mu(y) J^\nu(0) \} | P \rangle = \frac{g_{\mu\nu}^{\pm}}{2p^+} \bar{u}(p_2) \left[ \gamma^+ \mathcal{H} + \frac{i\sigma^{+\nu}}{2M} \mathcal{E} \right] u(p_2) + \cdots \]

Compton form factors

\[
\begin{pmatrix}
\mathcal{H}(\xi) \\
\mathcal{E}(\xi)
\end{pmatrix} = \int_{-1}^{1} dx C_q(x, \xi) \begin{pmatrix} H_q(x, \xi) \\ E_q(x, \xi) \end{pmatrix} + \alpha_s \int_{-1}^{1} dx C_g(x, \xi) \begin{pmatrix} H_g(x, \xi) \\ E_g(x, \xi) \end{pmatrix} + \cdots
\]

Ingredients for NNLO global analysis ready in near future Braun, Ji, Schoenleber (2023)
In practice, NLO global analysis is already a challenge Kumericki,…
Very hard to access GPD E, especially the gluon one.
GPD $E_g$ from $J/\psi$ single spin asymmetry

$A_N \sim \frac{\text{Im}(\mathcal{H}_g^* \mathcal{E}_g)}{|\mathcal{H}_g|^2}$

Will be measured by the STAR collaboration in UPC
Can be continued at the EIC
Gluon GPD \( E_g(x) \) at small-\( x \)

**Nucleon helicity non-flip**

\[
x H_g(x) = x G(x) = \int d^2 k_\perp G(x, k_\perp)
\]

\[\sim \left( \frac{1}{x} \right)^4 \ln 2 \bar{\alpha}_s \]

*BFKL pomeron*

**Nucleon helicity flip**

\[
x E_g(x) = \int d^2 k_\perp E(x, k_\perp)
\]

\[\sim \left( \frac{1}{x} \right)^{??} \]

Introduce \( k_\perp \)-dependence in GPD \( \rightarrow \) GTMD

Recent theory developments in GTMD help solve this problem
Small-x evolution equation for $E_g(x)$

\[
\partial_Y E(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2} \left[ E(k'_\perp) - \frac{k^2_{\perp}}{2k'^2_{\perp}} E(k_\perp) \right] - 4\pi^2 \alpha_s^2 F_{1,1}(k_\perp) E(k_\perp)
\]

\[Y = \ln 1/x\]

\[x E_g(x) \sim x G(x) \propto \left(\frac{1}{x}\right) \bar{\alpha}_s \ln 2\]

BFKL Pomeron behavior, the same as unpolarized gluon PDF
Eventually reaches gluon saturation.

\[J_g = \frac{1}{2} \int_0^1 dx \, x [H_g(x, \xi) + E_g(x, \xi)]\]
Nucleon electric dipole moment (EDM)

If nonvanishing, both P and CP are violated. CKM mechanism gives a too small value of nucleon EDM,

CP violation from BSM physics?
Various CP-violating operators studied

• Theta term
\[ \frac{\theta \alpha_s}{8\pi} F \tilde{F} \]

• Quark EDM operator
\[ m_q \bar{\psi}_q F^{\mu \nu} \sigma_{\mu \nu} i \gamma_5 \psi_q \]

• Weinberg operator
\[ f_{abc} \tilde{F}_{\mu \nu}^a F_{b \rho}^{\mu \rho} F_{c \rho}^{\nu} \]

• ...

EDM is a vector, must be proportional to nucleon spin
Any connection to high energy QCD spin physics at EIC?
Global analysis of SSA

At the moment, the only viable way to generate $O(10\%)$ asymmetry seems to be twist-3 FFs convoluted with the transversity distribution.

→ Constraints on the nucleon tensor charge.
→ Constraints on the quark EDM operator

Simultaneous fit of

$e^+e^-$ (BELLE, BaBar, BESIII)
SIDIS (COMPASS, HERMES, Jlab) ← input from EIC in future
Drell-Yan (COMPASS, STAR)
pp (STAR, PHENIX, BRAHMS)
Connecting Weinberg operator to higher-twist effect in polarized DIS

Exact identity

\[ g f_{abc} \tilde{F}_{\mu \nu}^a F_{\nu}^{\mu \alpha} F_{\alpha}^c = -\partial^\mu (\tilde{F}_{\mu \nu} \rightarrow \bar{D} \alpha \rightarrow F_{\nu \alpha}^c) - \frac{1}{2} \tilde{F}_{\mu \nu} \rightarrow \bar{D}^2 \rightarrow F_{\mu \nu}^c \]

\[ \langle p' | O_W | p \rangle \approx i \Delta^\mu \langle p | \bar{\psi} g \tilde{F}_{\mu \nu} \gamma^\nu \psi | p \rangle + \cdots \]

This matrix element enters the twist-4 correction in polarized DIS

First moment of g1

\[ \int_0^1 g_1^{p,n}(x, Q^2) dx = (\pm \frac{1}{12} g_A + \frac{1}{36} a_s)(1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)) + \frac{1}{9} \Delta \Sigma(1 - \frac{33 - 8 N_f}{33 - 2 N_f} \frac{\alpha_s}{\pi} + O(\alpha_s^2)) \]

\[ - \frac{8}{9 Q^2} \left\{ \left[ \pm \frac{1}{12} f_3 + \frac{1}{36} f_8 \right] \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-\frac{\gamma_{NS}^0}{2 \beta_0}} + \frac{1}{9} f_0 \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-\frac{1}{2 \beta_0} (\gamma_{NS}^0 + \frac{4}{3} N_f)} \right\}, \]

Shuryak, Vainshtein (1982)
An estimate of EDM

`One-nucleon reducible’ contribution to EDM

\[ d \sim \mu \frac{\langle p' | w \mathcal{O}_W | p \rangle}{m_N \bar{u}(p') i \gamma_5 u(p)} \equiv 4 \mu m_N^2 E \]

teighten magnetic moment

Vary \( E \) in the window \( 0.5 f_0 < E < 1.3 f_0 \)

\( f_0 \) from instanton model.

\[ -12 w' \text{ e MeV} < d_p < -32 w' \text{ e MeV} \quad 22 w' \text{ e MeV} < d_n < 8.4 w' \text{ e MeV}. \]

Can we measure \( f_0 \) at the EIC?

New connection between EIC spin and BSM physics
Summary

• Spin is one of the core science cases of EIC
• OAM is the key to fulfill the JM spin sum rule. Lagging far behind in both theory and experiment, but a glimmer of hope.
• EIC offers great opportunities for GPD studies. From spin physics perspective, extraction of GPD E is a major challenge.