Gluon Polarization from Global Analyses

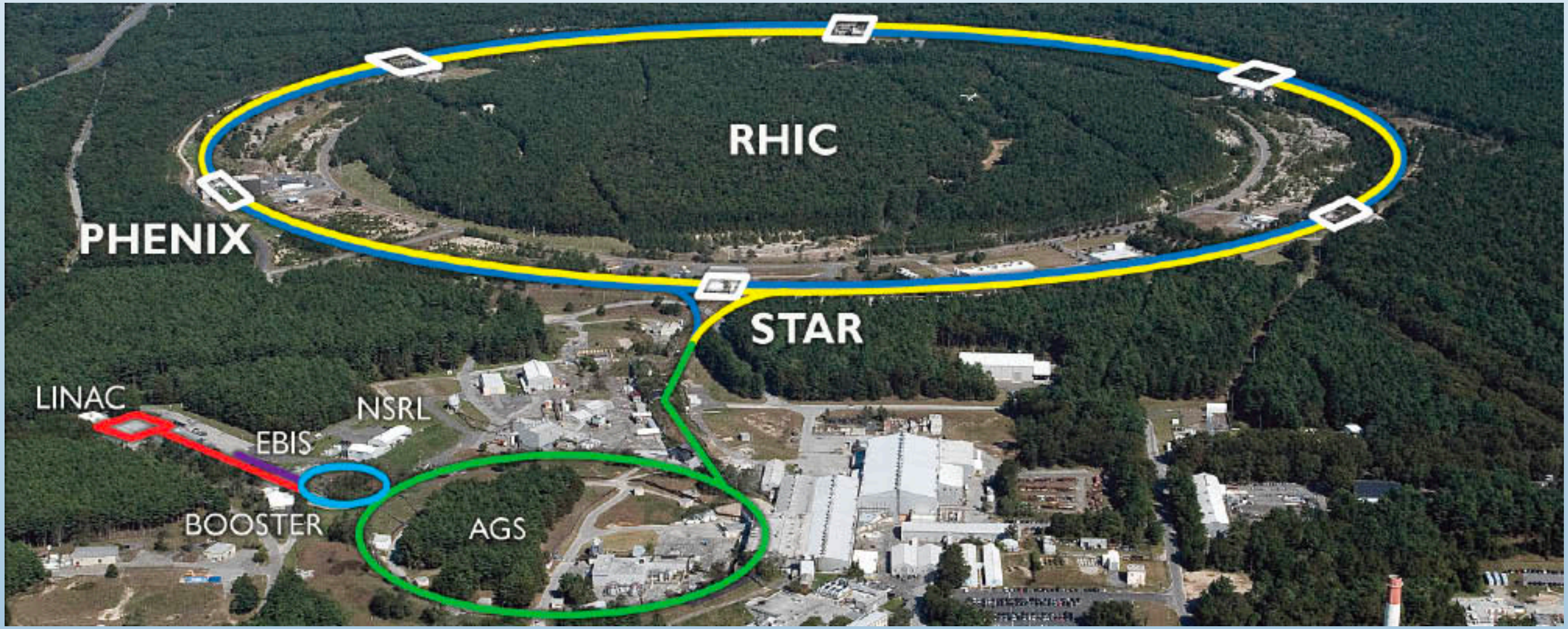
Christopher Cocuzza

August 1, 2023

2023 RHIC/AGS ANNUAL USERS' MEETING

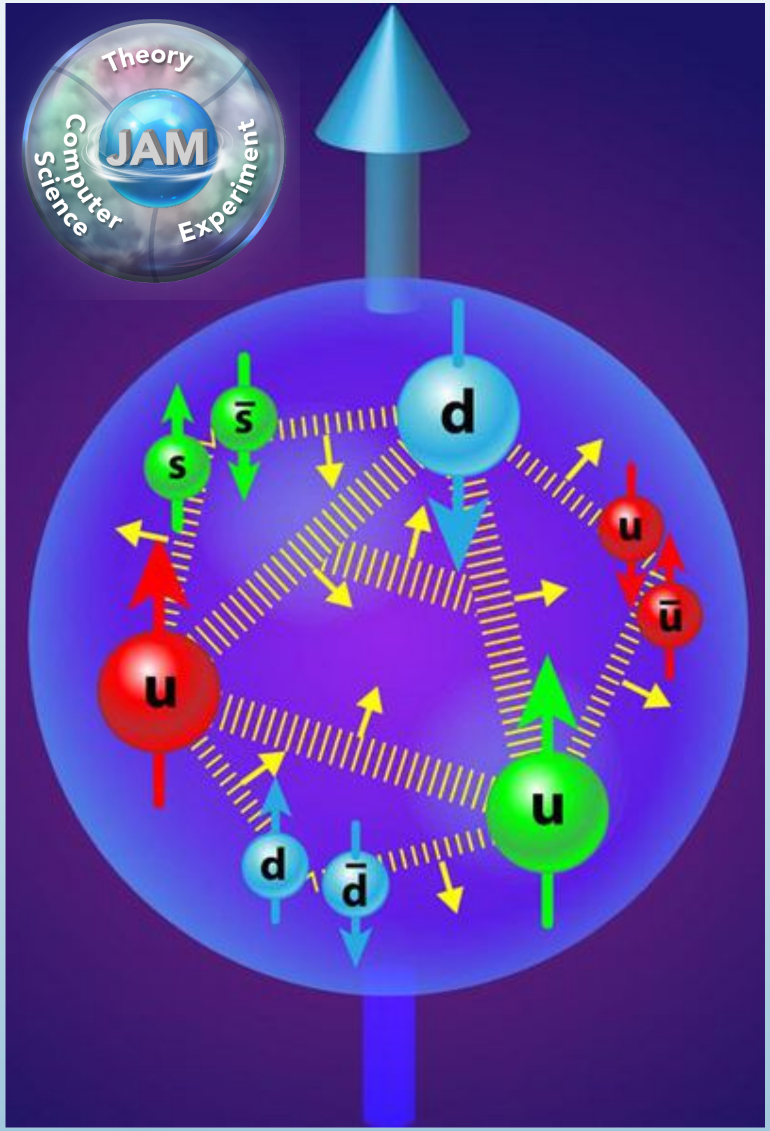
CELEBRATING NEW BEGINNINGS AT RHIC and EIC

August 1-4, 2023



JAM Collaboration

3-dimensional structure of nucleons:
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)



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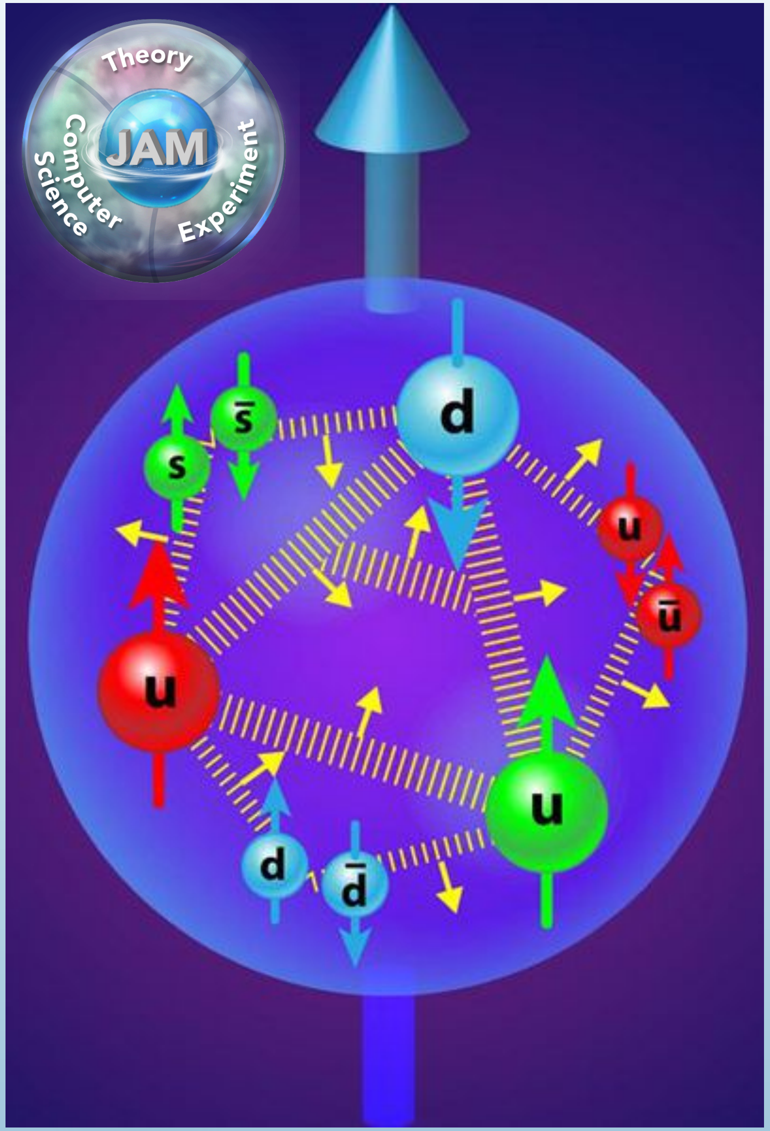
3-dimensional structure of nucleons:
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Hadron Structure

Global QCD Analysis

Introduction

Hadron Structure

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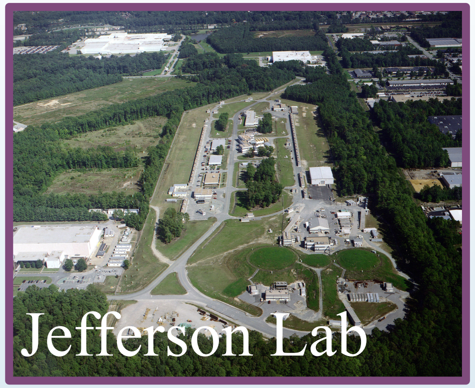

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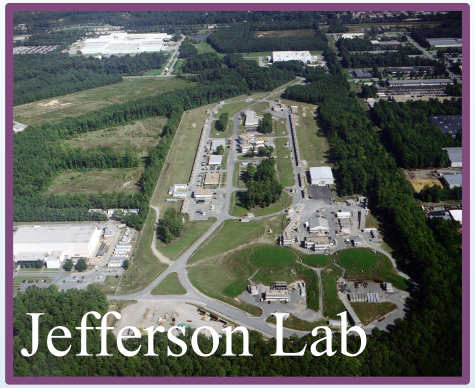

Global QCD Analysis

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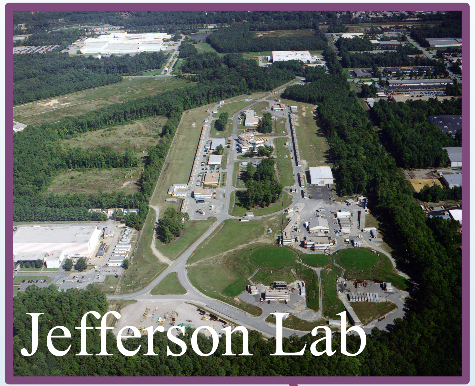
USQCD

Extended Twisted Mass Collaboration

Introduction



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$$\frac{d}{d\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

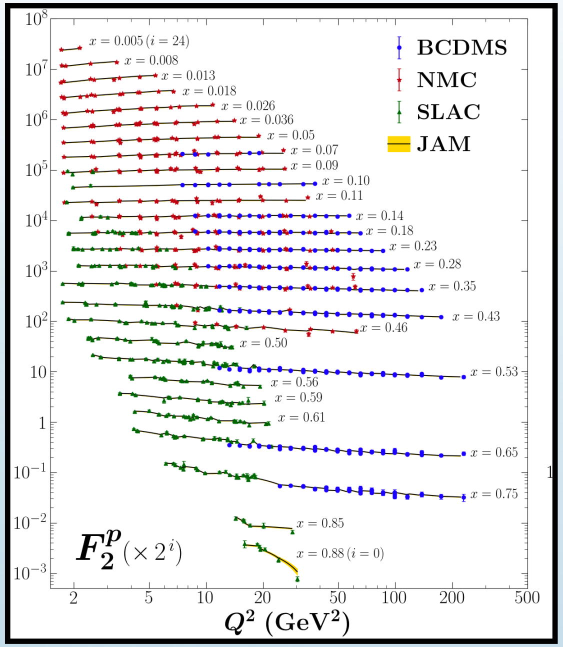
Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

χ^2 Minimization



$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$

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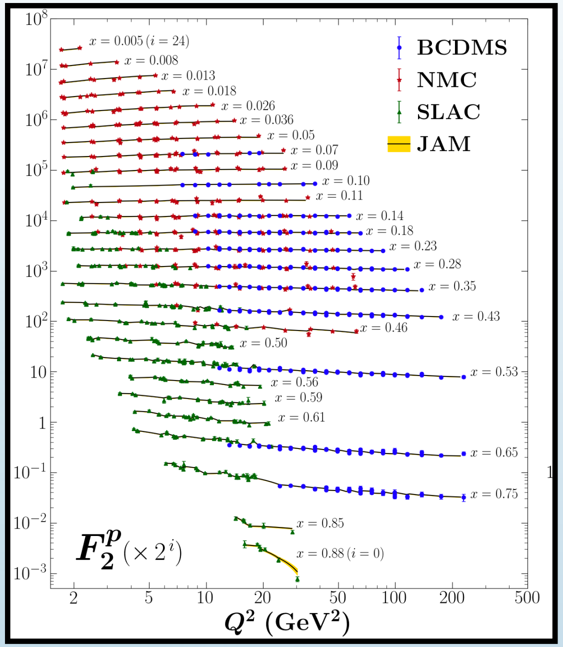

Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

USQCD

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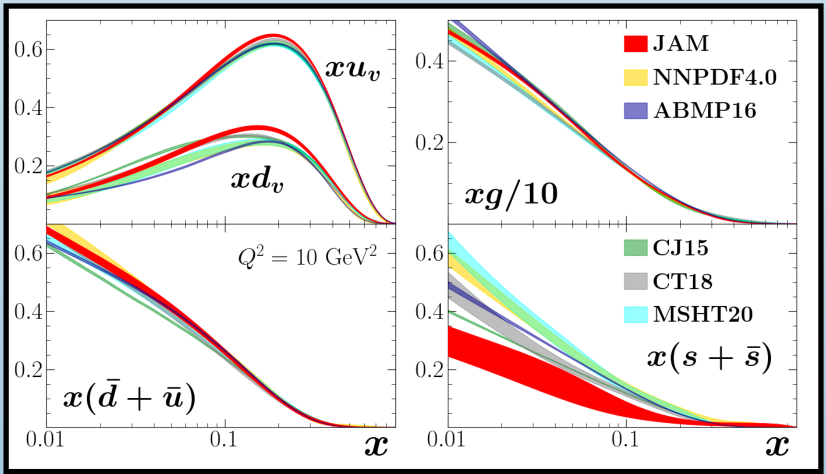

Current State of Helicity PDFs

Proton spin puzzle:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \int_0^1 dx \sum_q \Delta q^+$$

$$\Delta G = \int_0^1 dx \Delta g$$

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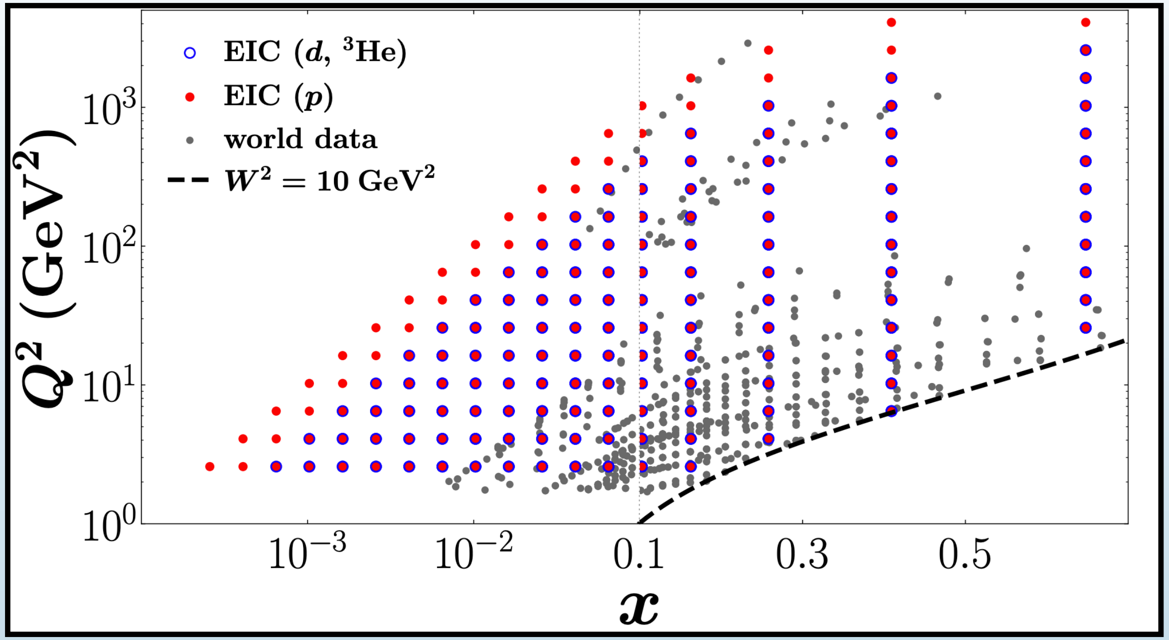

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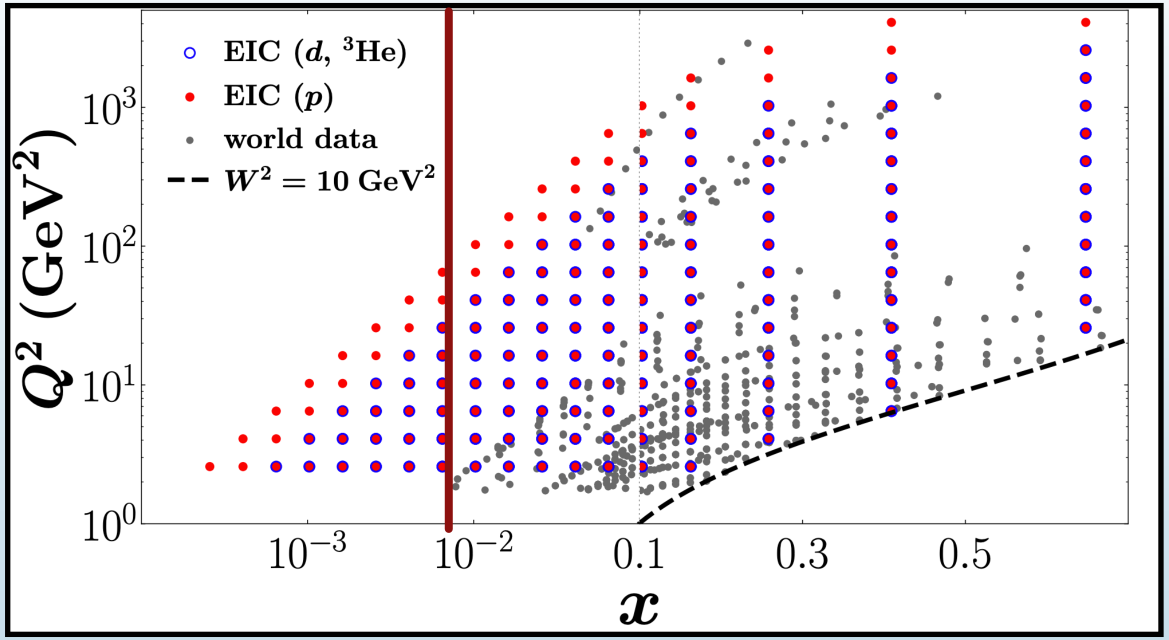

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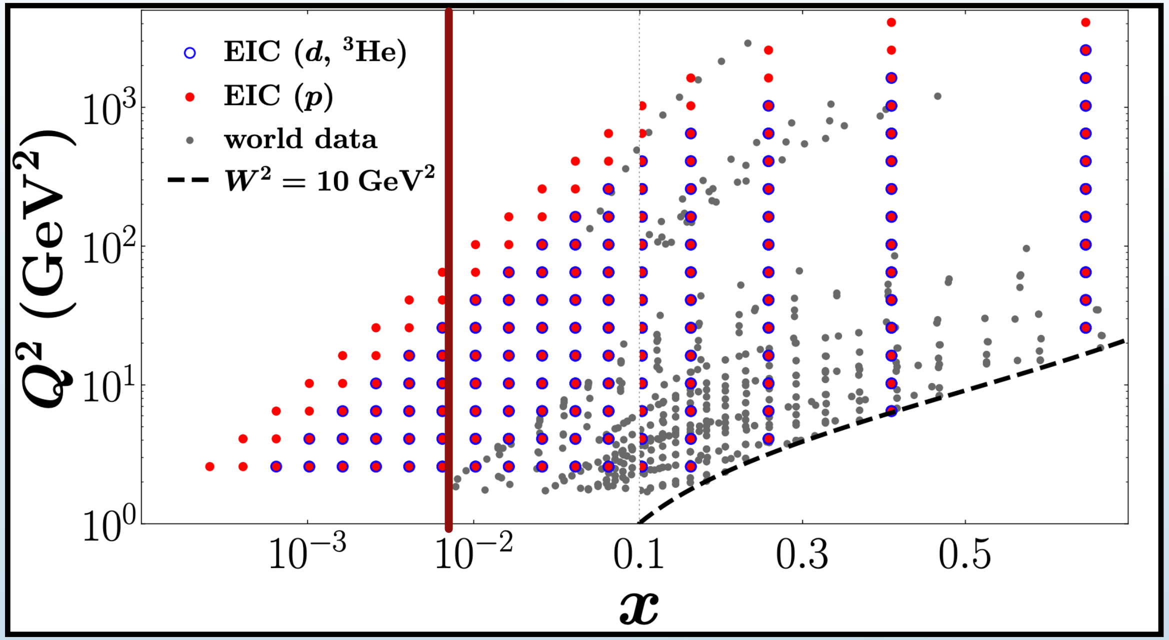

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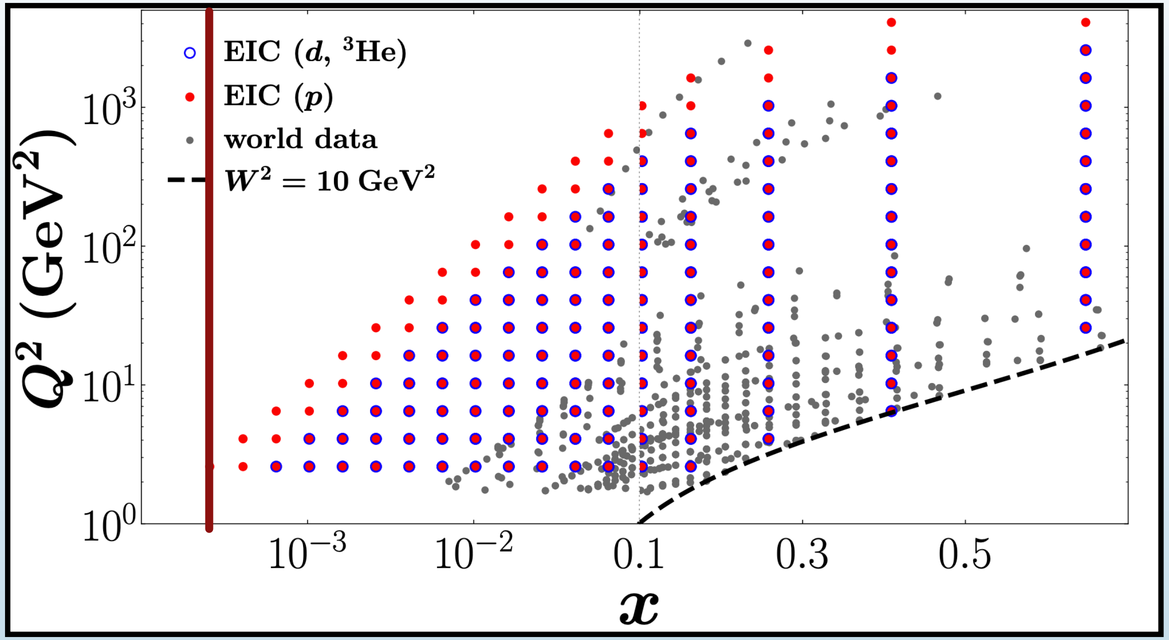

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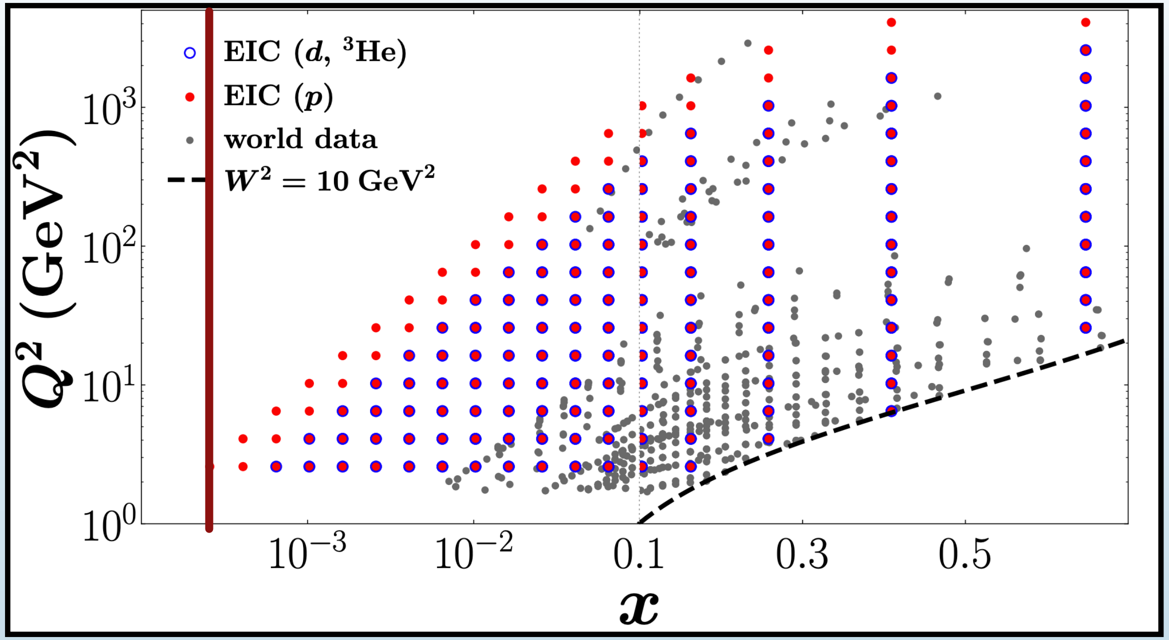

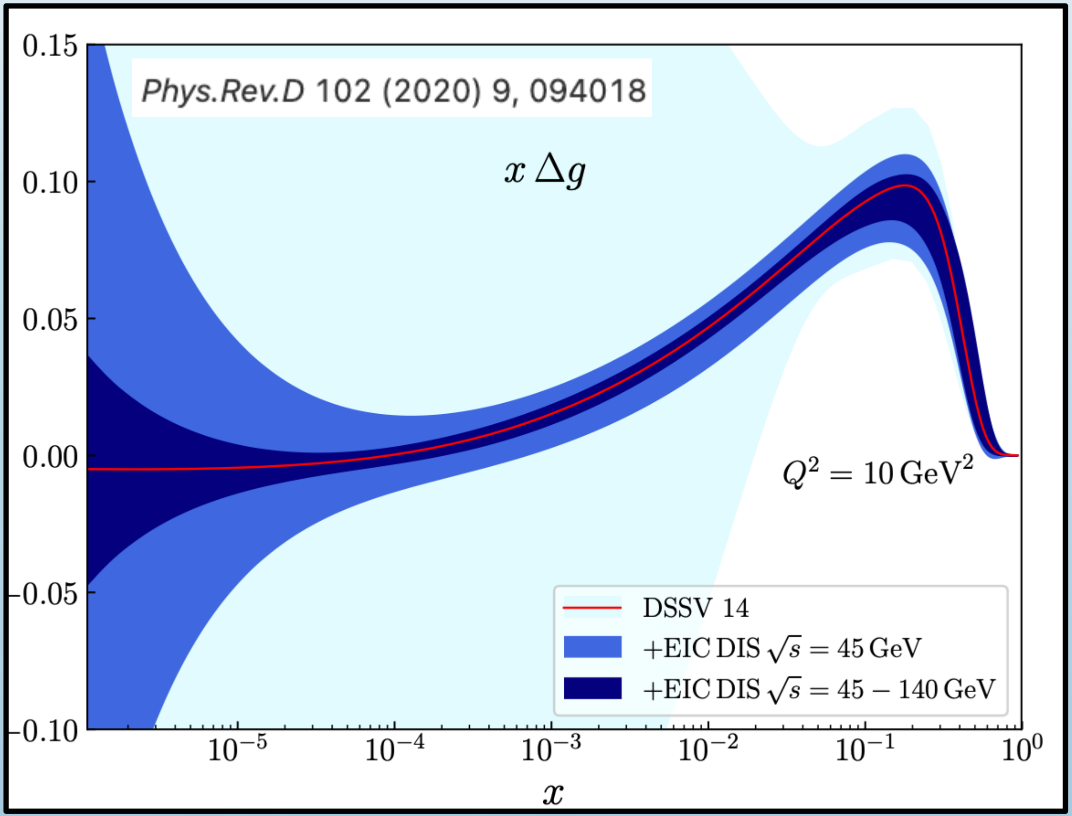

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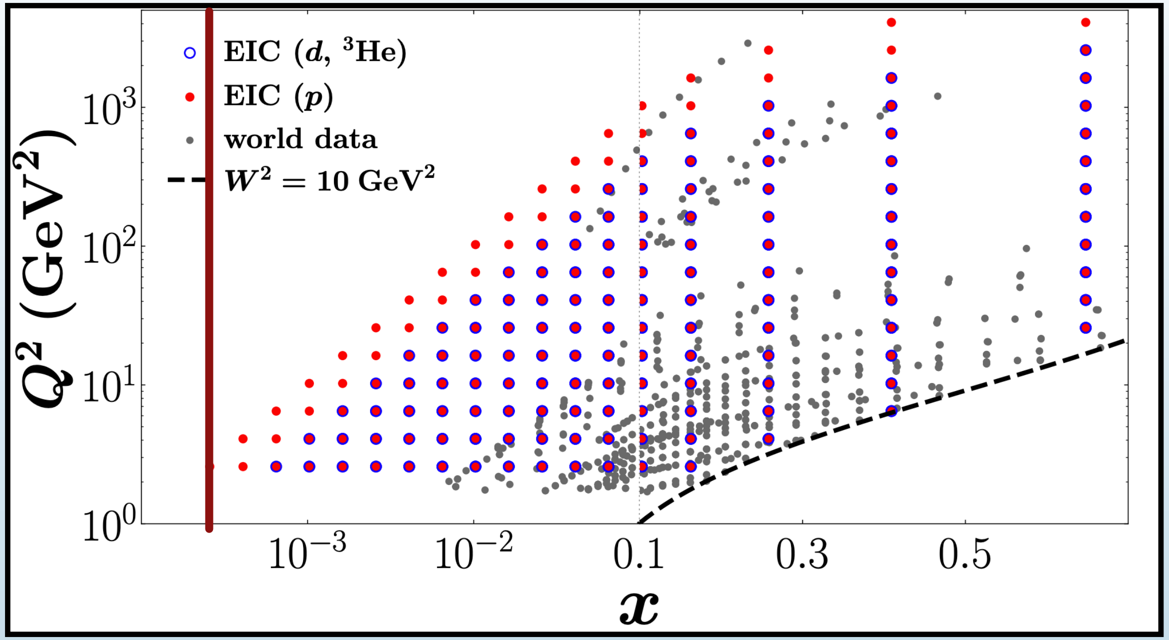

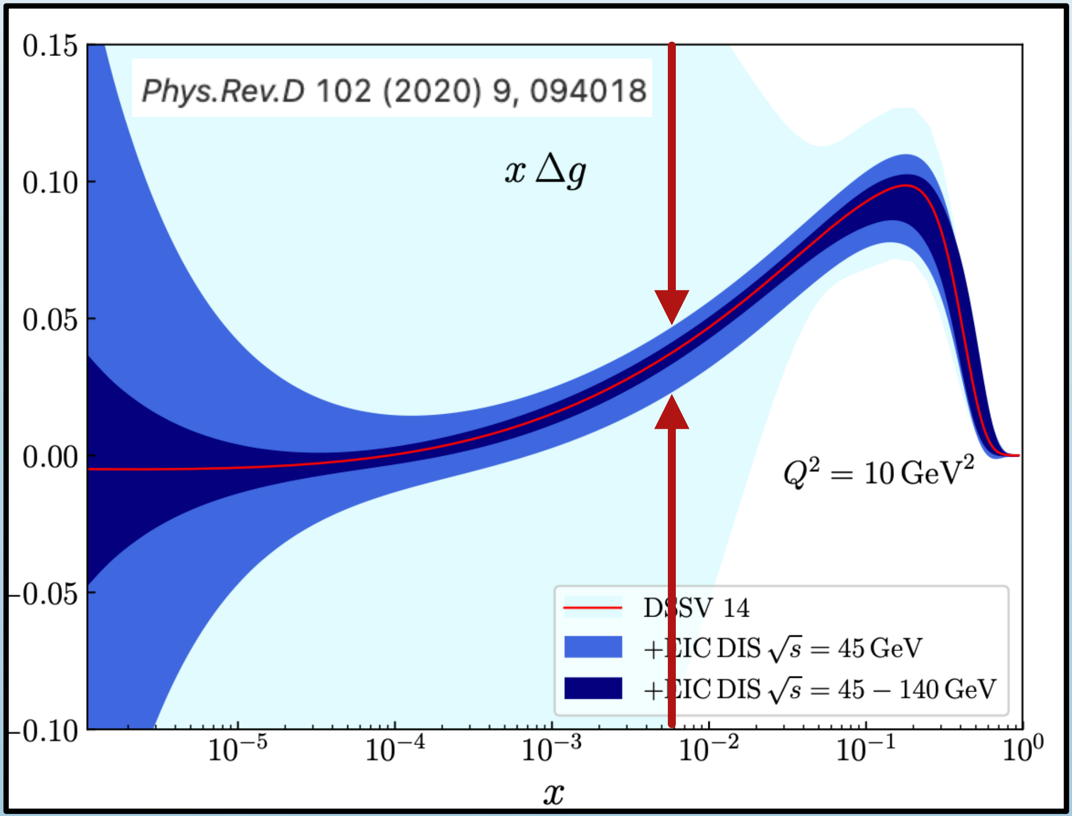

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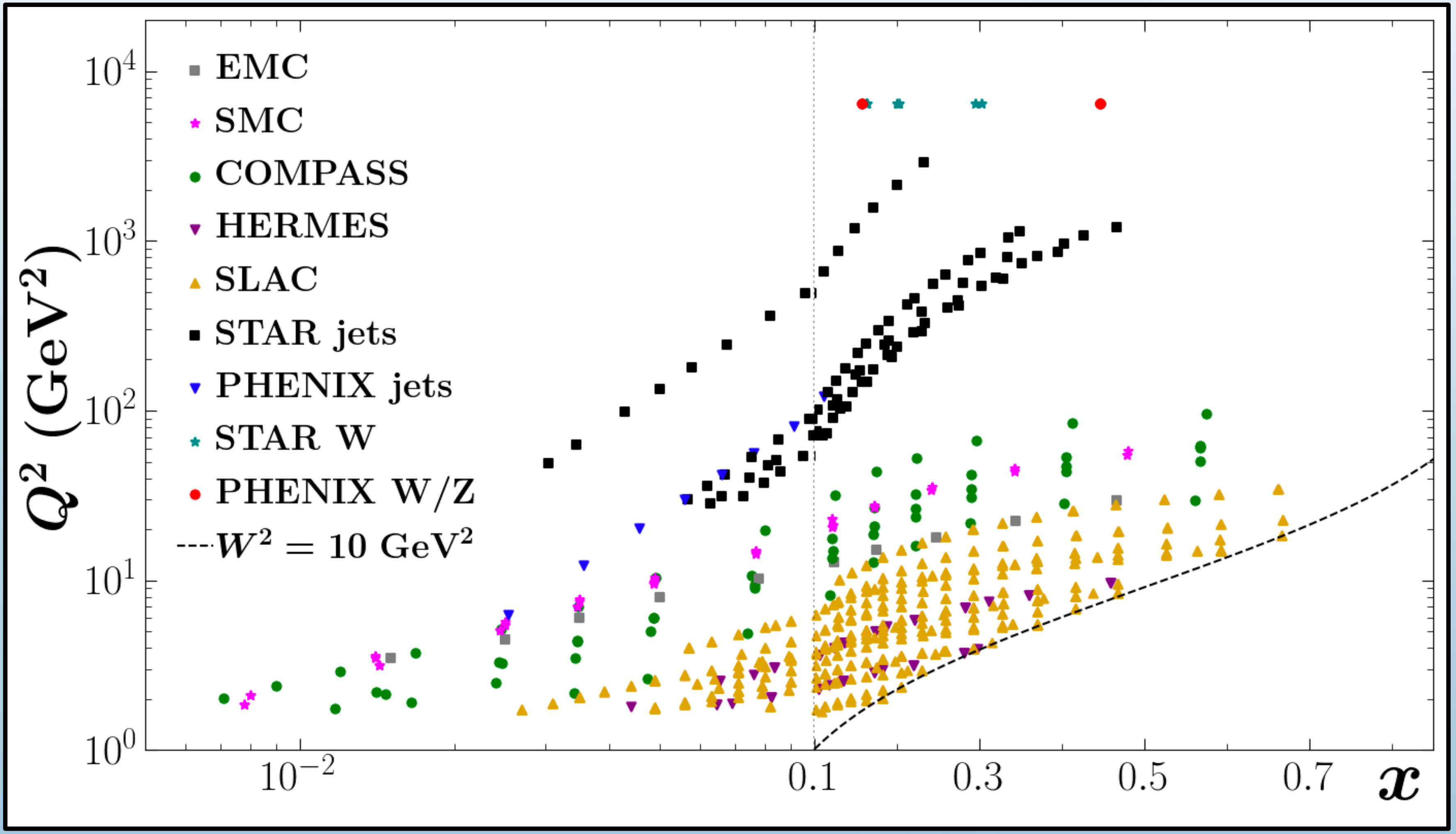
JAM Baseline Analysis

Deep Inelastic Scattering	COMPASS, EMC, HERMES, SLAC, SMC	365 points
Semi-Inclusive DIS	COMPASS, HERMES, SMC	231 points
W/Z Boson Production	STAR, PHENIX	18 points
Jets	STAR, PHENIX	61 points



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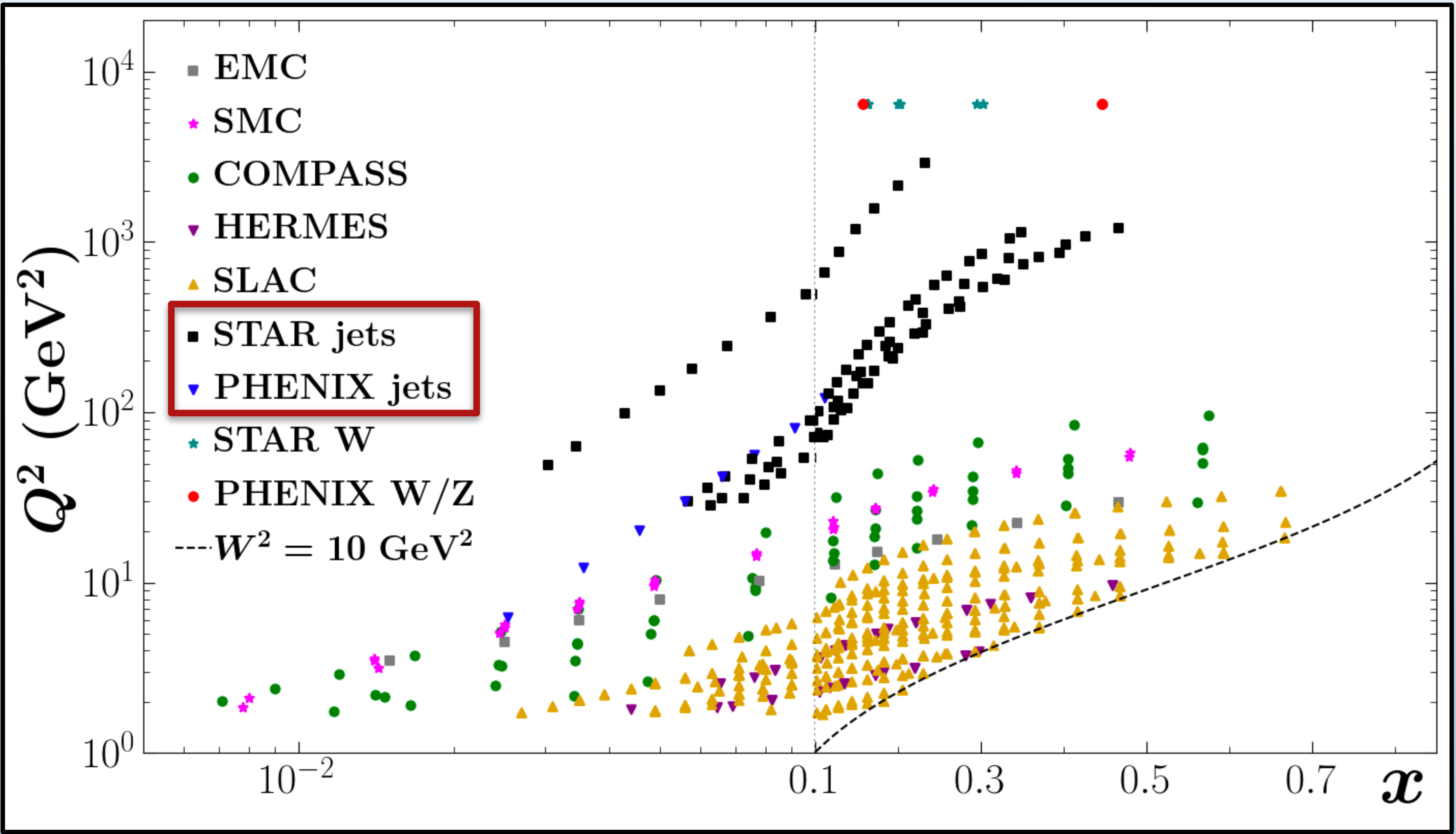
Jets provide most direct constraints on gluon distribution



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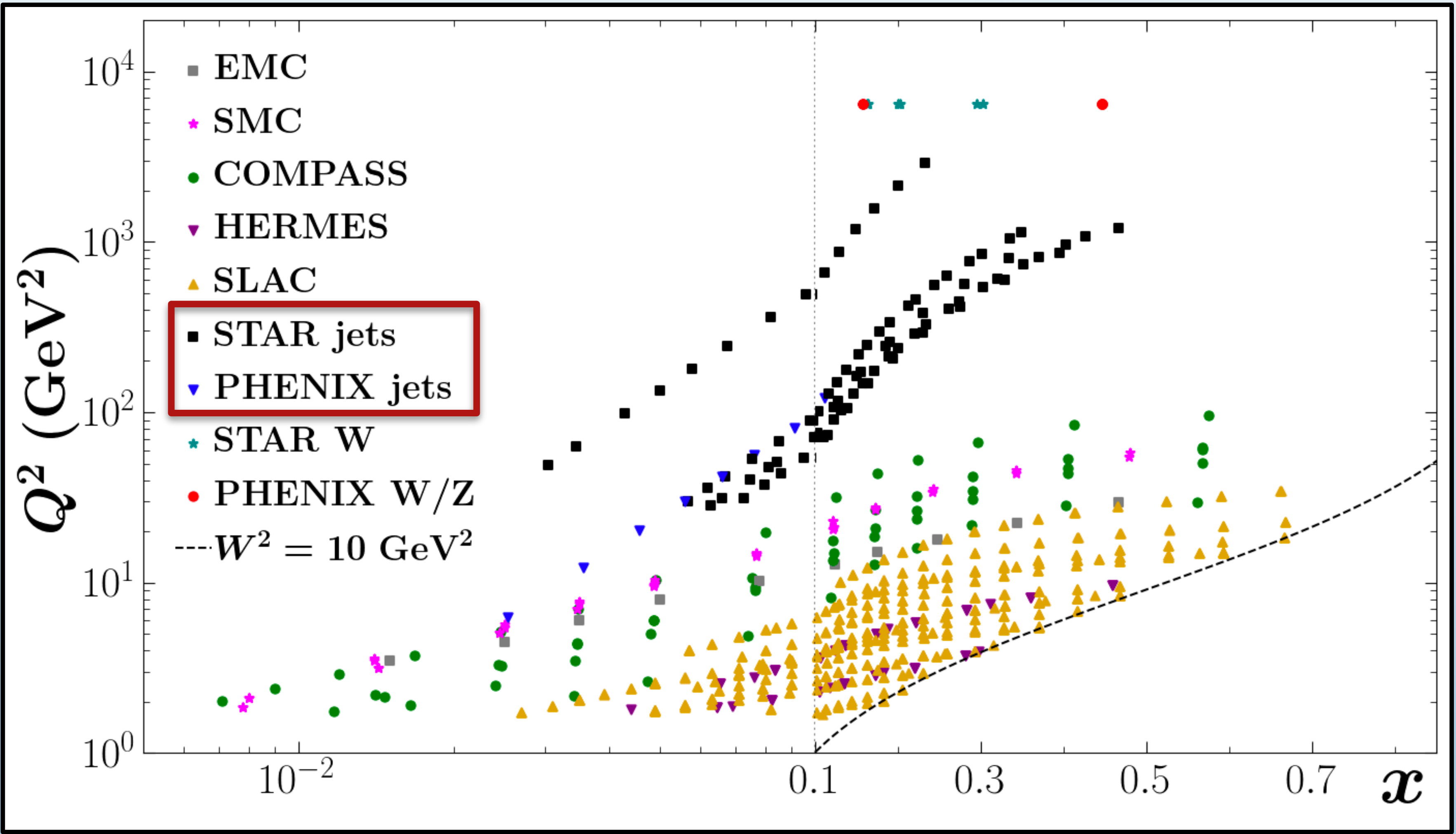
process	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
polarized		
inclusive DIS	365	0.95
SIDIS (π^+, π^-)	64	1.05
SIDIS (K^+, K^-)	57	0.42
SIDIS (h^+, h^-)	110	0.95
inclusive jets	83	0.84
STAR $W^\pm$	12	0.65
PHENIX $W^\pm/Z$	6	0.50
total	**697**	**0.89**
unpolarized		
inclusive DIS	3908	1.17
SIDIS (π^+, π^-)	498	0.94
SIDIS (K^+, K^-)	494	1.31
SIDIS (h^+, h^-)	498	0.71
inclusive jets	198	1.28
Drell-Yan	205	1.21
W/Z production	153	1.01
total	**5954**	**1.12**
SIA $(\pi^\pm)$	231	0.91
SIA $(K^\pm)$	213	0.70
SIA $(h^\pm)$	120	1.07
total	**7215**	**1.08**



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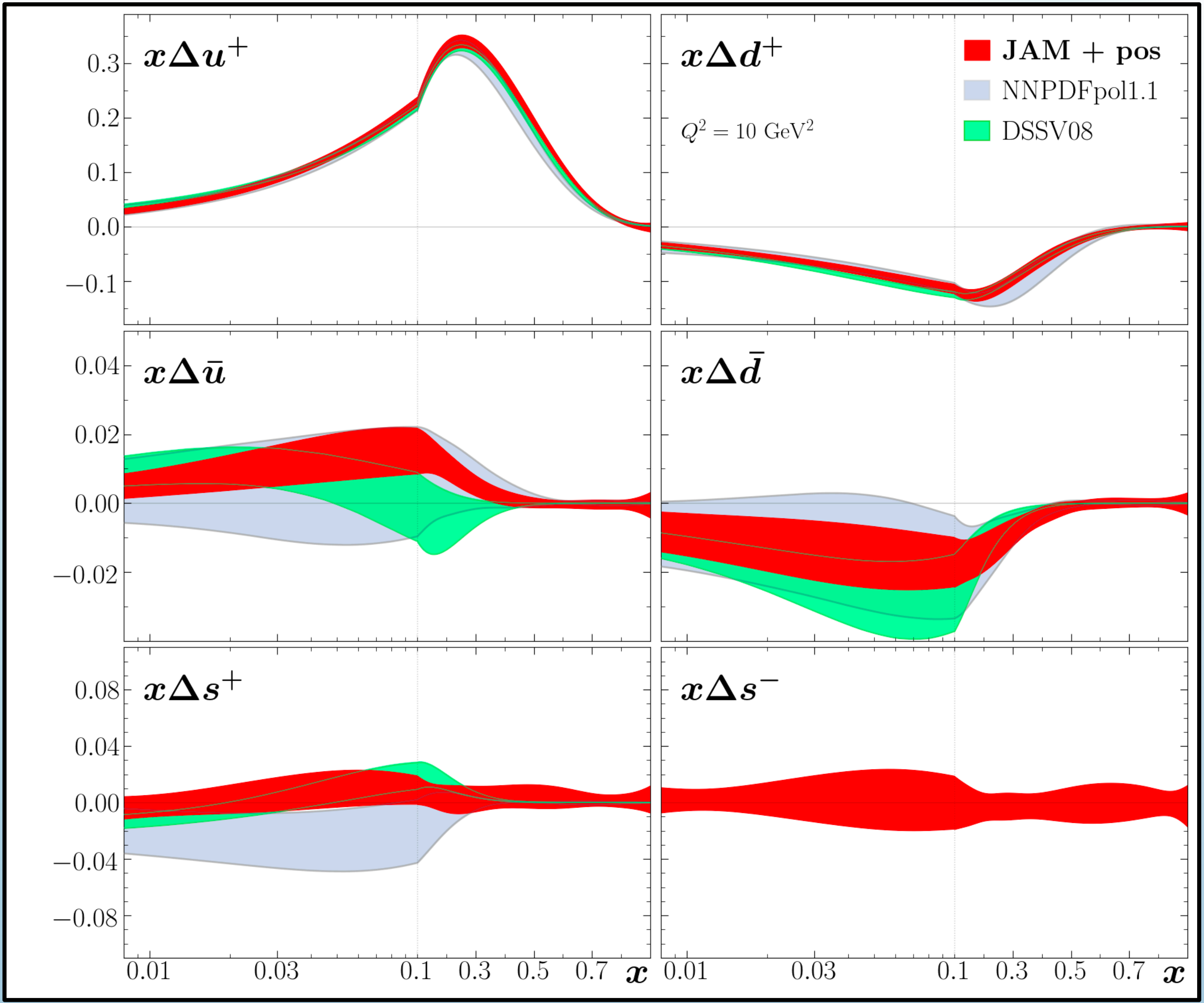
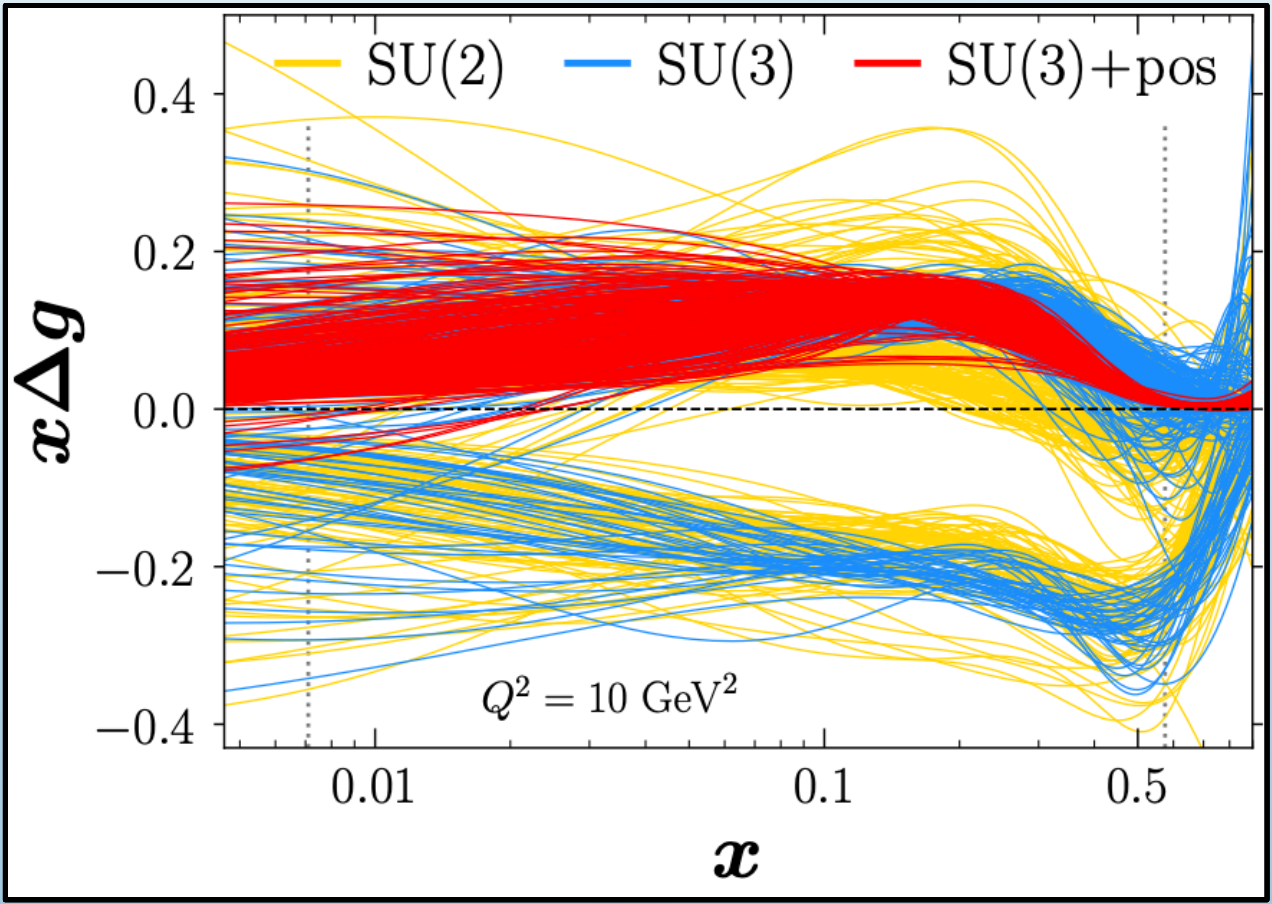


JAM Baseline Analysis

How well do we know the gluon polarization in the proton? #1

Jefferson Lab Angular Momentum (JAM) Collaboration • Y. Zhou (South China Normal U. and UCLA and William-Mary Coll. and Jefferson Lab) et al. (Jan 6, 2022)

Published in: *Phys.Rev.D* 105 (2022) 7, 074022 • e-Print: 2201.02075 [hep-ph]



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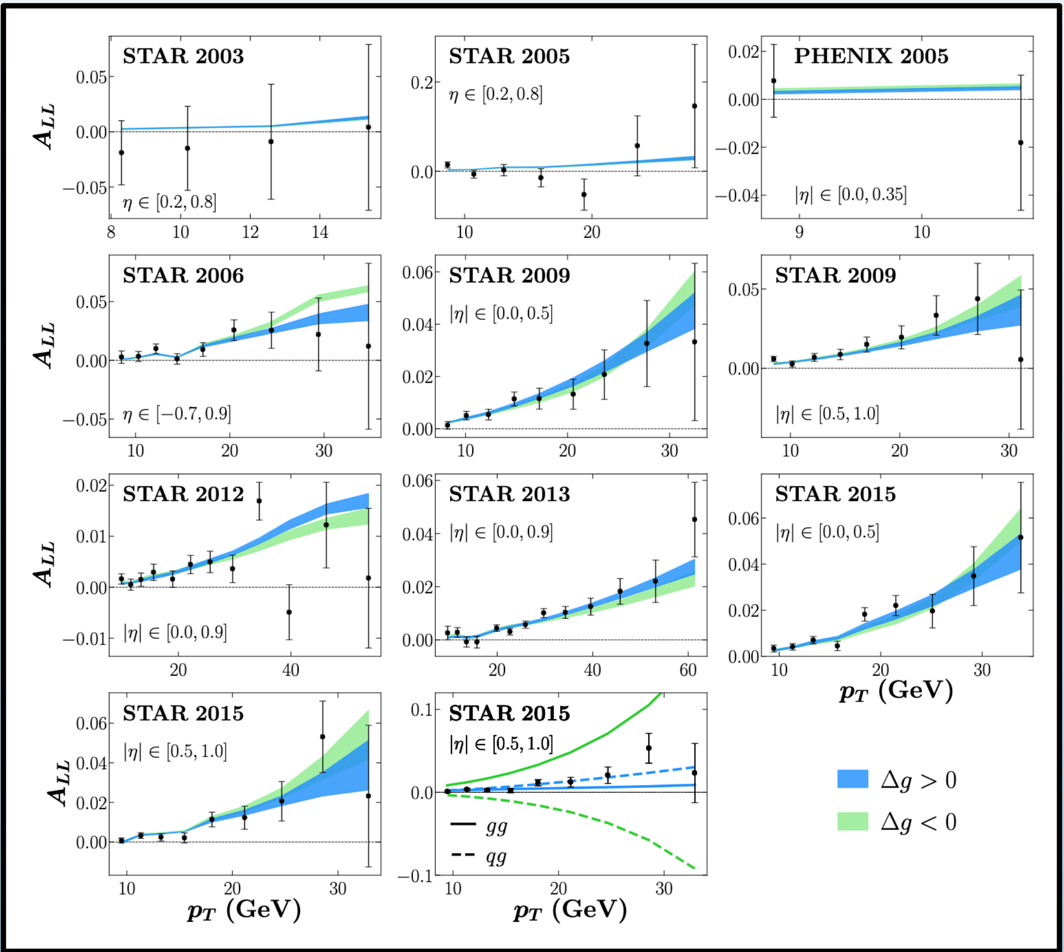
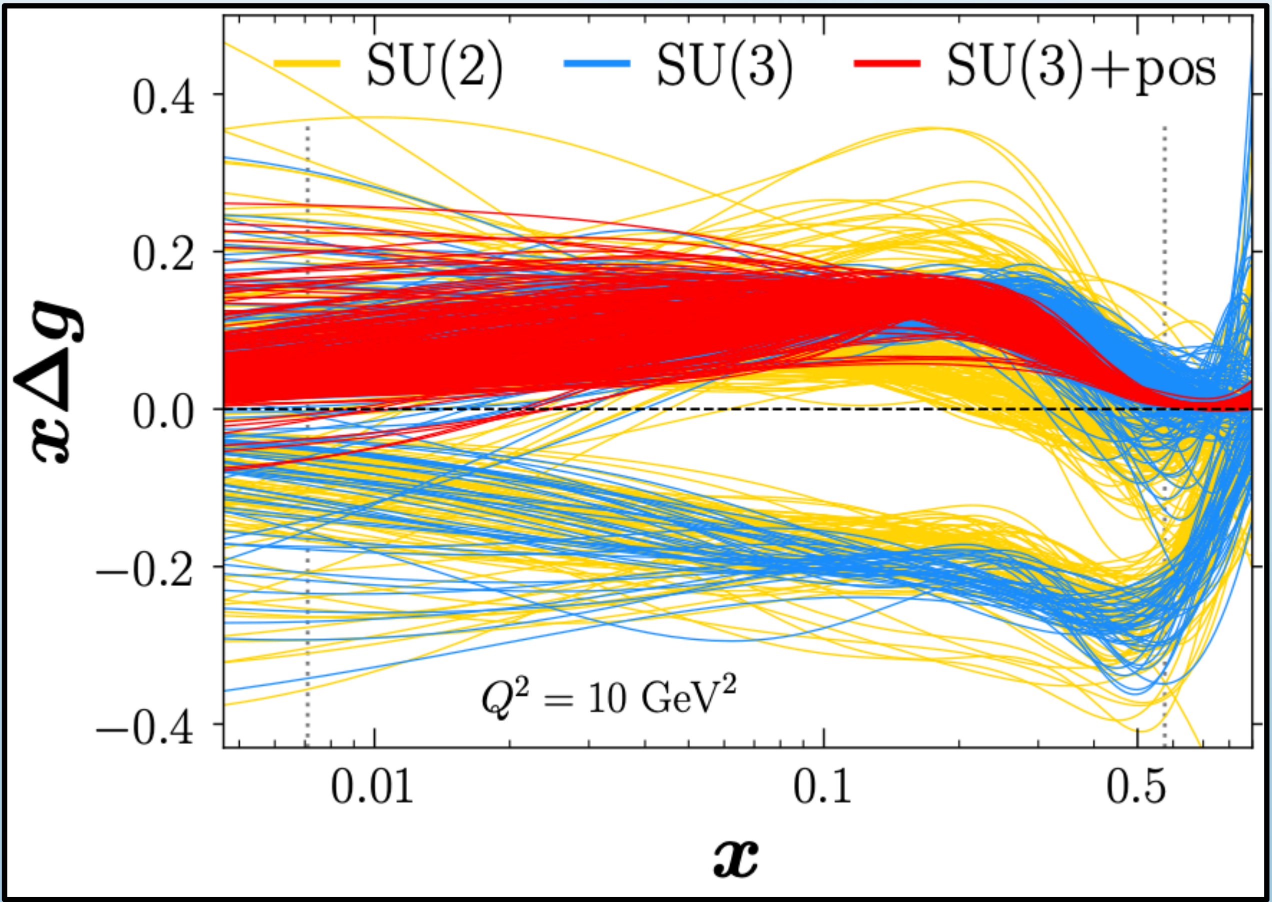


$$A_{LL}^{\text{jet}} \sim (\Delta g)^2 + \Delta q \Delta g + \dots$$



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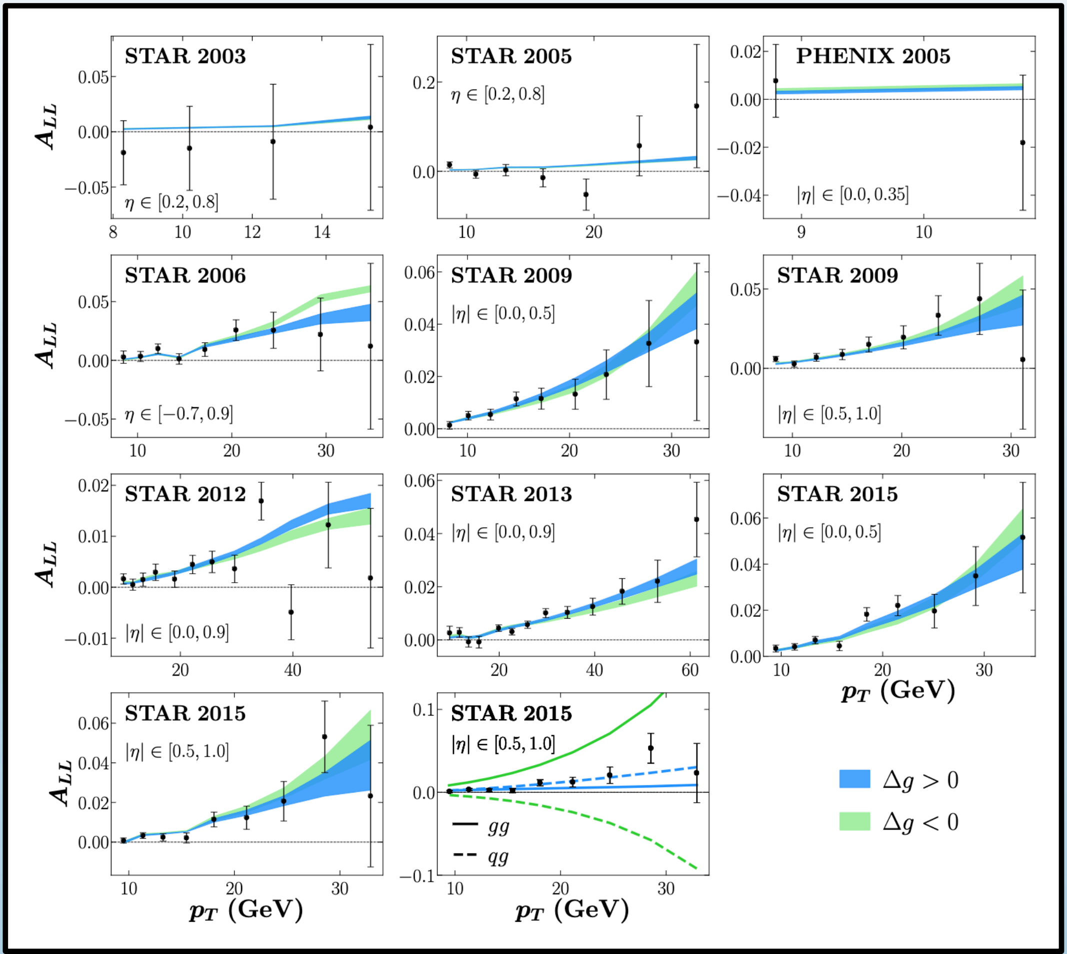
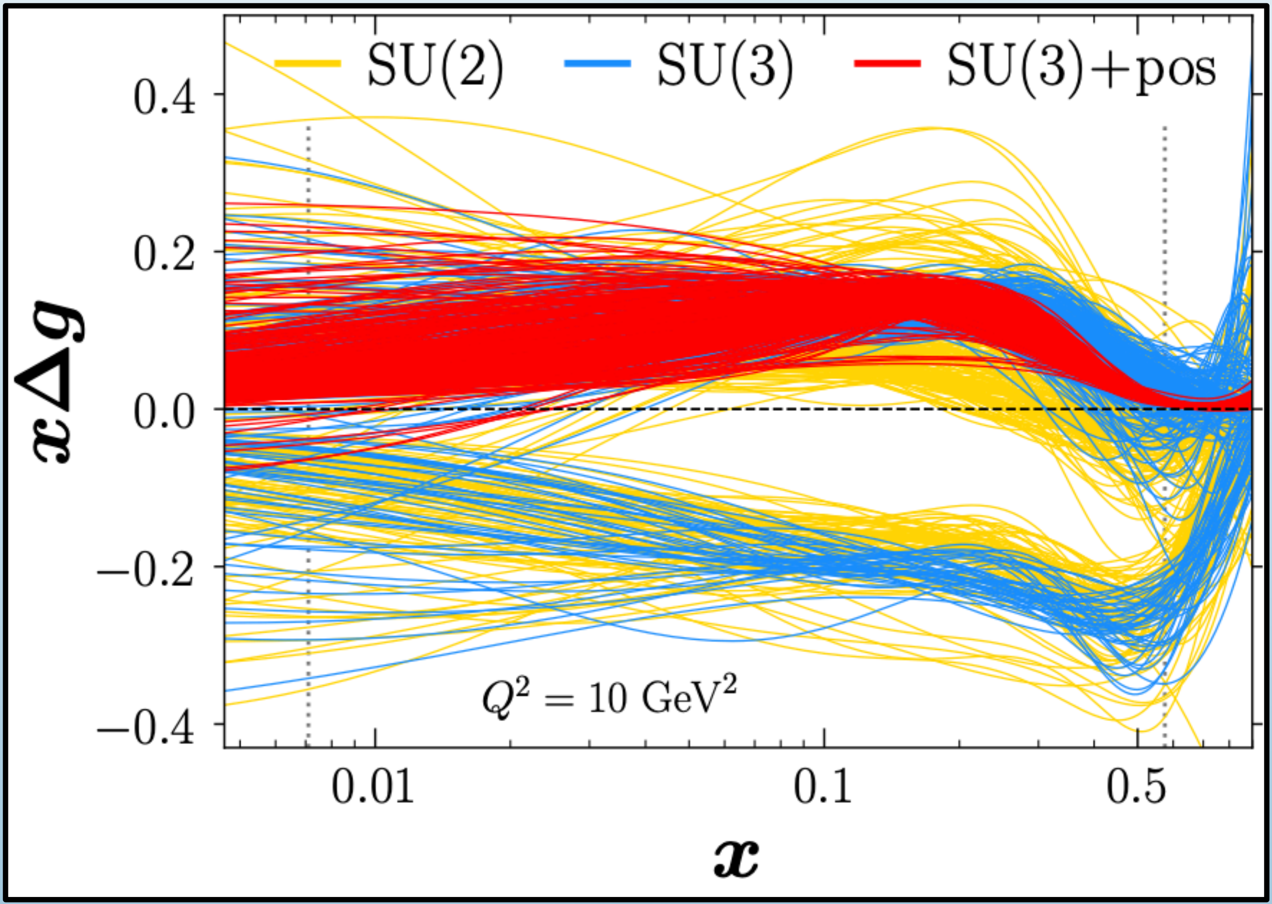


$$A_{LL}^{\text{jet}} \sim (\Delta g)^2 + \Delta q \Delta g + \ldots$$

$$|\Delta f(x, Q^2)| < f(x, Q^2)$$



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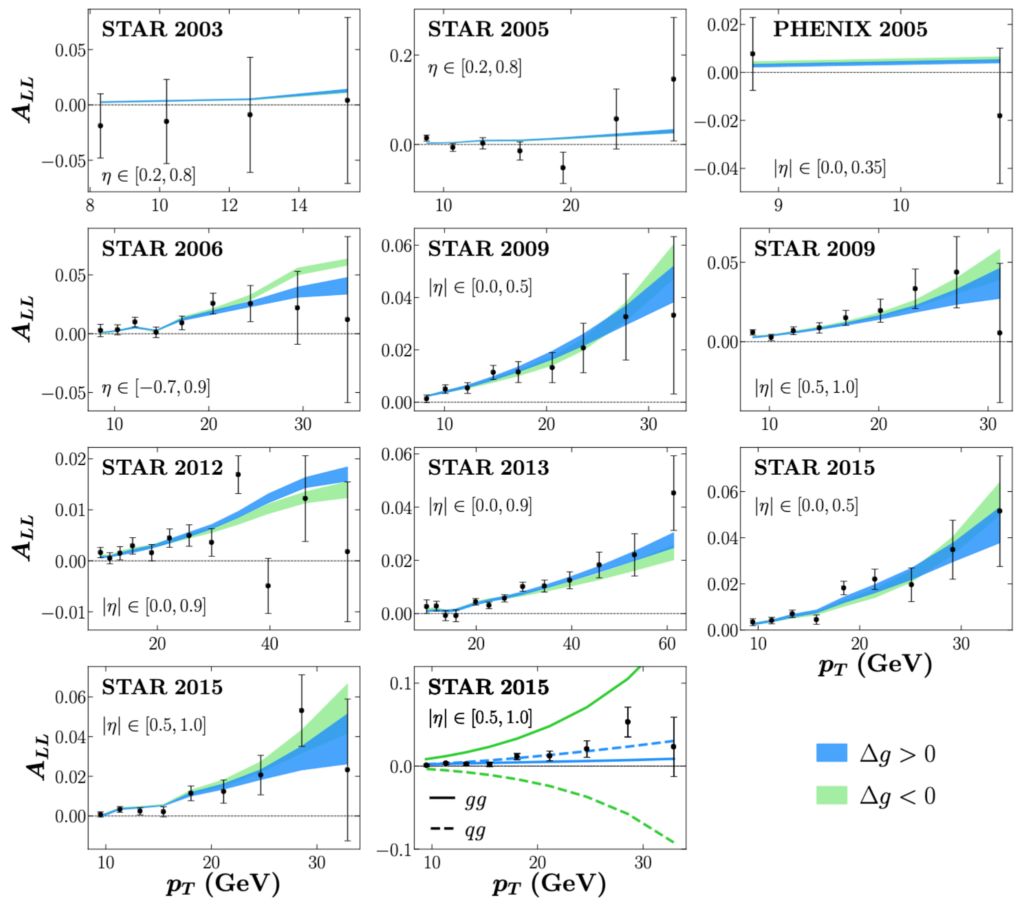


$$A_{LL}^{\text{jet}} \sim (\Delta g)^2 + \Delta q \Delta g + \ldots$$

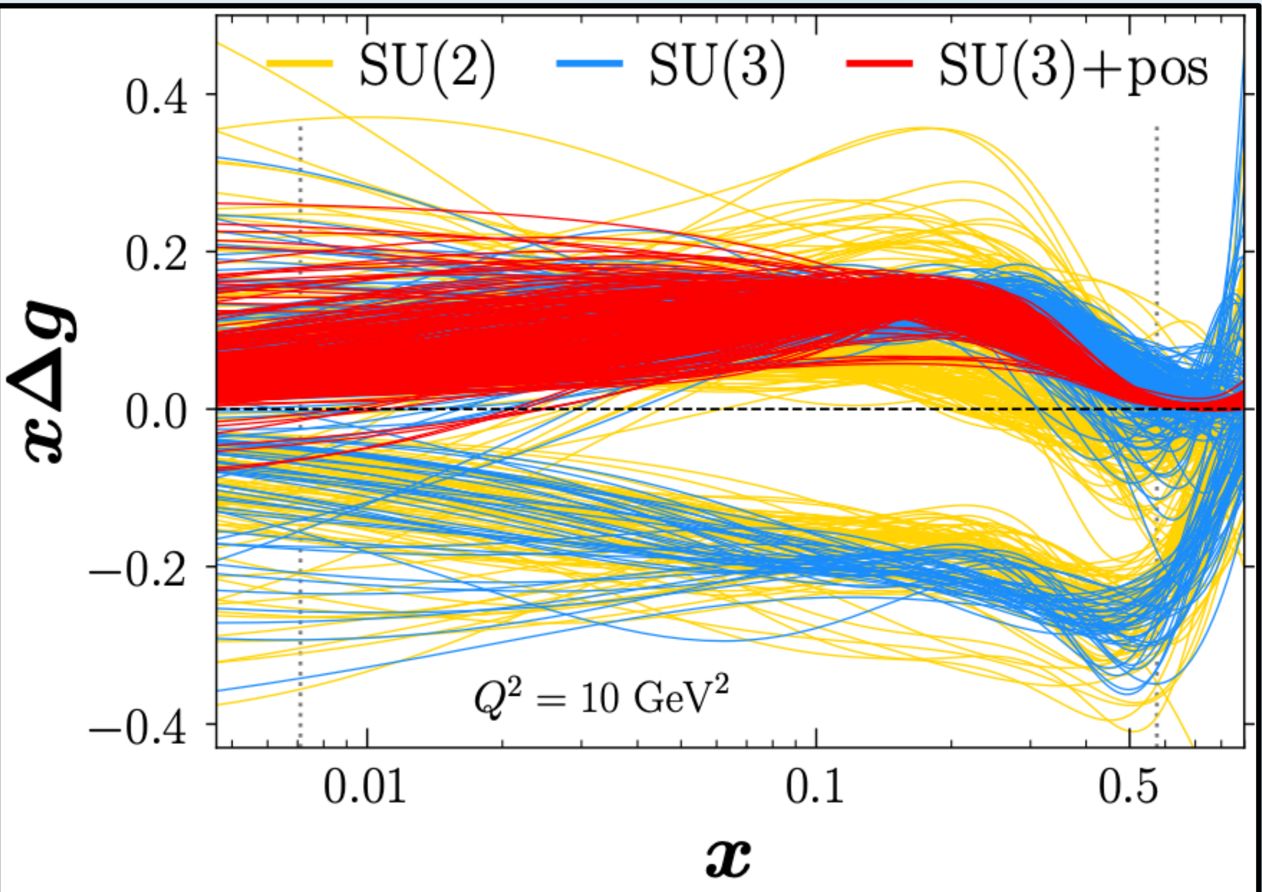
Can $\overline{\text{MS}}$ parton distributions be negative?

Alessandro Candido, Stefano Forte and Felix Hekhorn

Positivity and renormalization of parton densities

John Collins, Ted C. Rogers, Nobuo Sato

$$|\Delta f(x, Q^2)| < f(x, Q^2)$$



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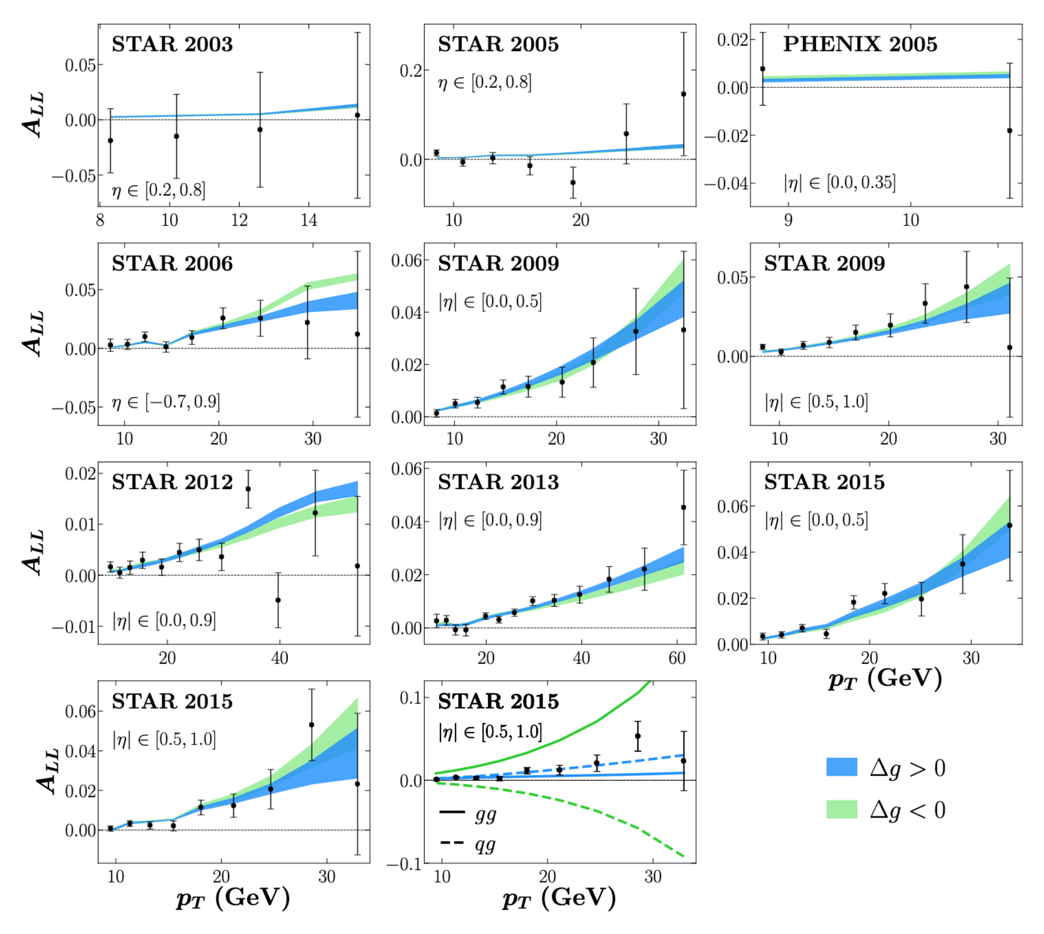
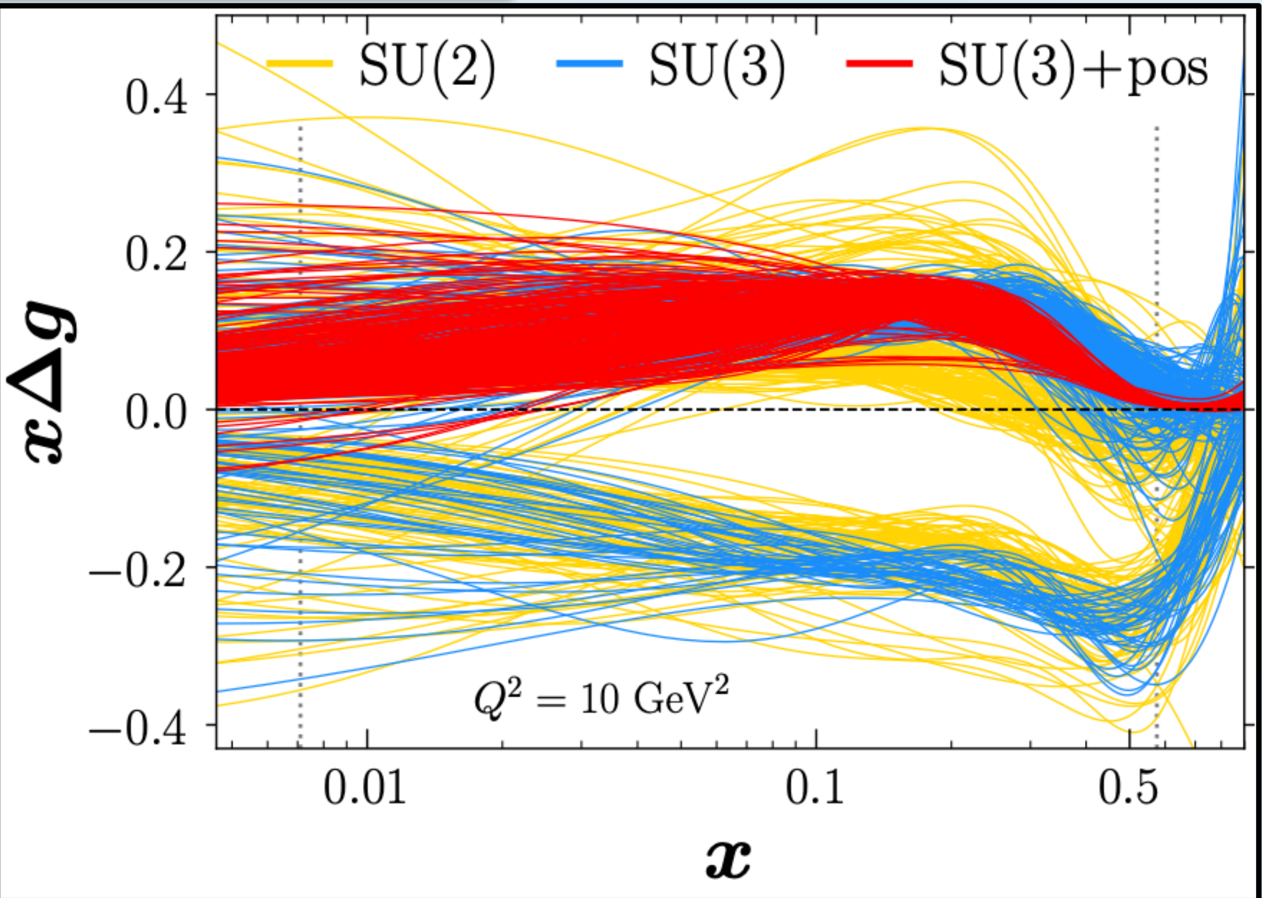


Positivity constraints rule out negative solution

$$|\Delta f(x, Q^2)| < f(x, Q^2)$$



$$A_{LL}^{\text{jet}} \sim (\Delta g)^2 + \Delta q \Delta g + \ldots$$

Measurement of charged pion double spin asymmetries at midrapidity in longitudinally polarized $p+p$ collisions at $\sqrt{s}$ = 510 GeV

PHENIX Collaboration • U.A. Acharya (Georgia State U.) et al. (Apr 6, 2020)

Published in: *Phys.Rev.D* 102 (2020) 3, 032001 • e-Print: 2004.02681 [hep-ex]

Charged-pion cross sections and double-helicity asymmetries in polarized p+p collisions at $\sqrt{s}$=200 GeV

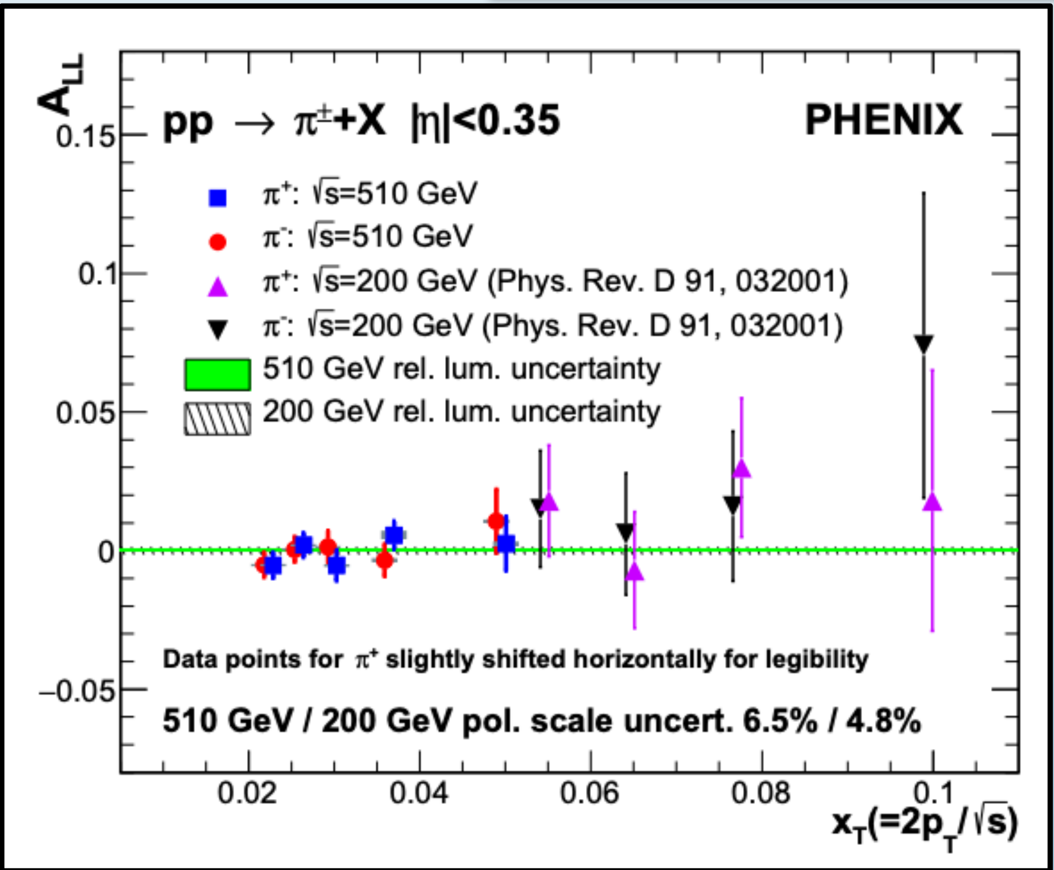
PHENIX Collaboration • A. Adare (Colorado U.) et al. (Sep 5, 2014)

Published in: *Phys.Rev.D* 91 (2015) 3, 032001 • e-Print: 1409.1907 [hep-ex]

$$\vec{p} + \vec{p} \rightarrow \pi^{\pm} + X$$

Charge ordering:

$$\text{If } \Delta g > 0 : A_{LL}^{\pi^+} > A_{LL}^{\pi^0} > A_{LL}^{\pi^-}$$



Current Experiments

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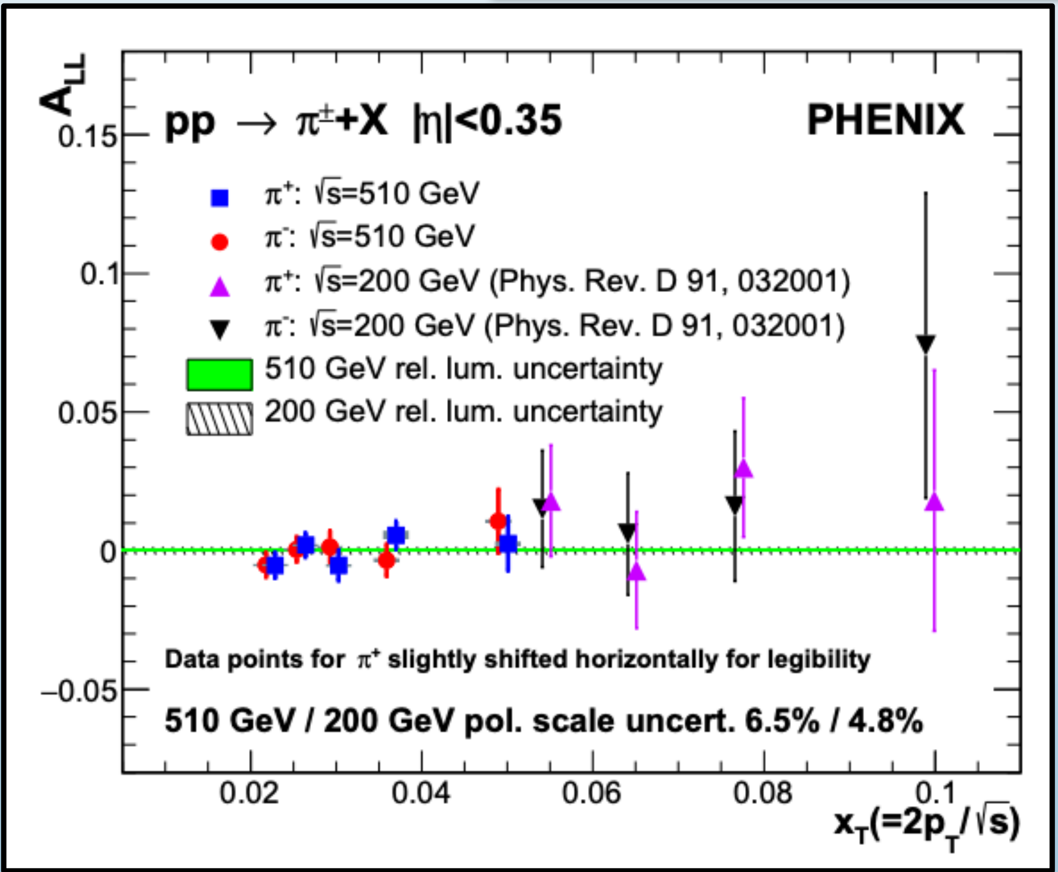
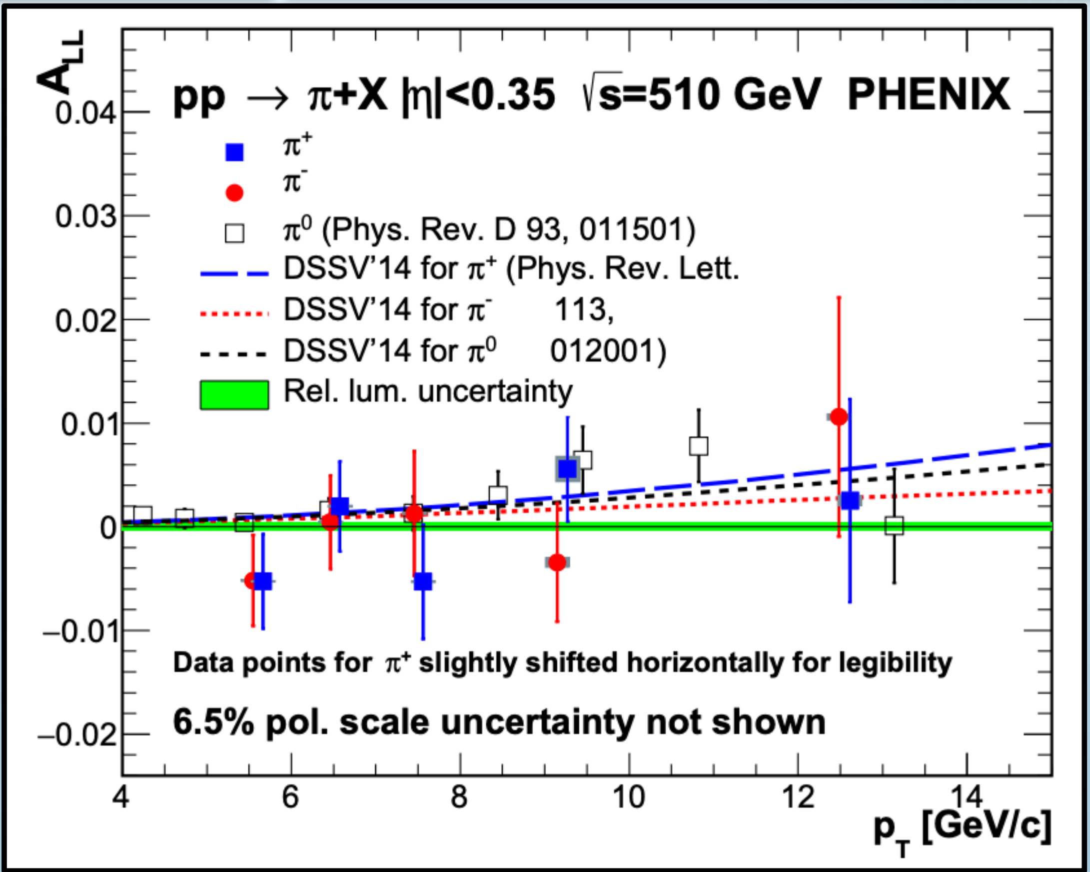
Charge ordering:
If $\Delta g > 0 : A_{LL}^{\pi^+} > A_{LL}^{\pi^0} > A_{LL}^{\pi^-}$

Consistent with DSSV14 analysis (which included 210 GeV data) with $\Delta g > 0$





Phys.Rev.Lett. 113 (2014) 1, 012001

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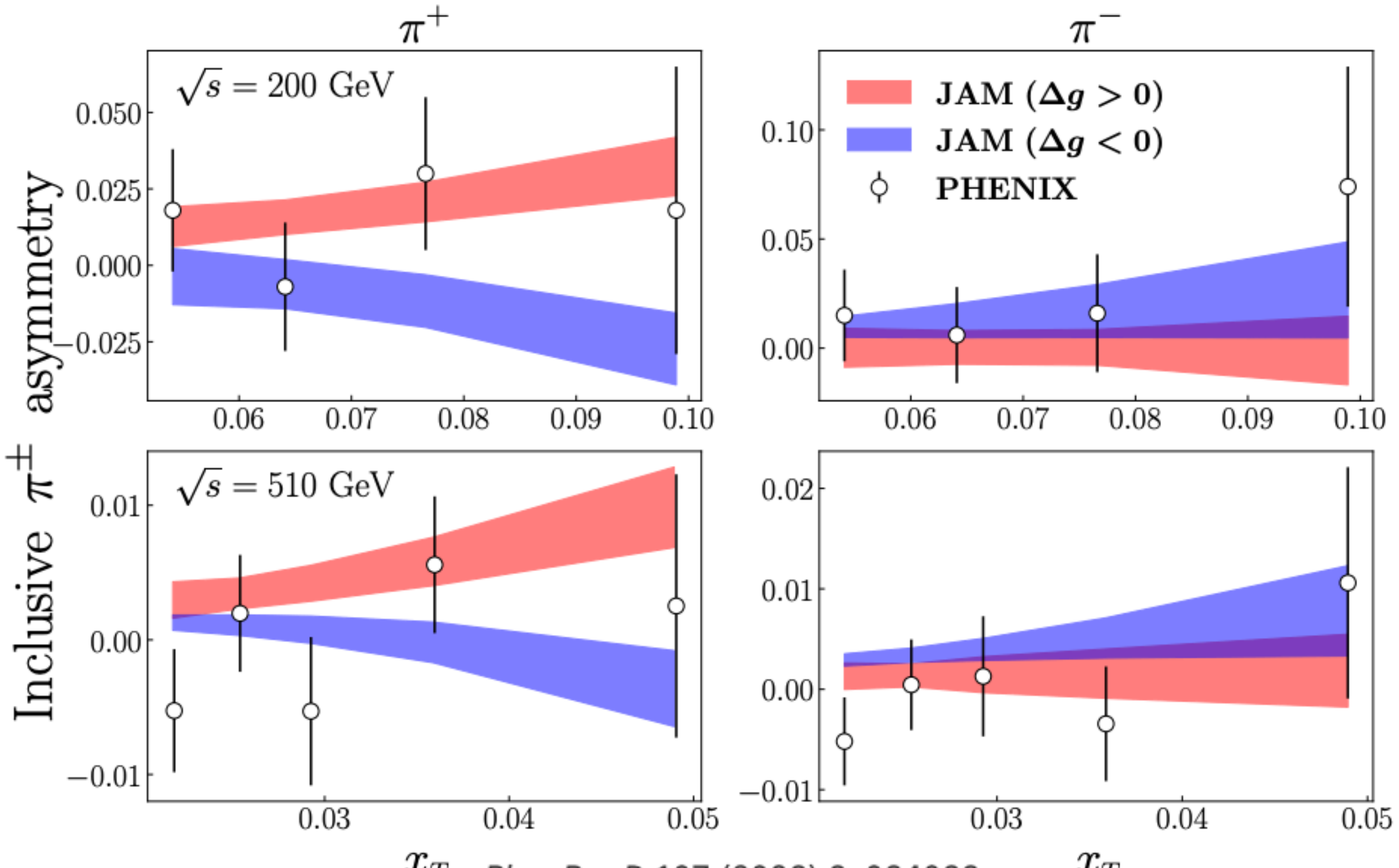

Phys.Rev.D 107 (2023) 3, 034033

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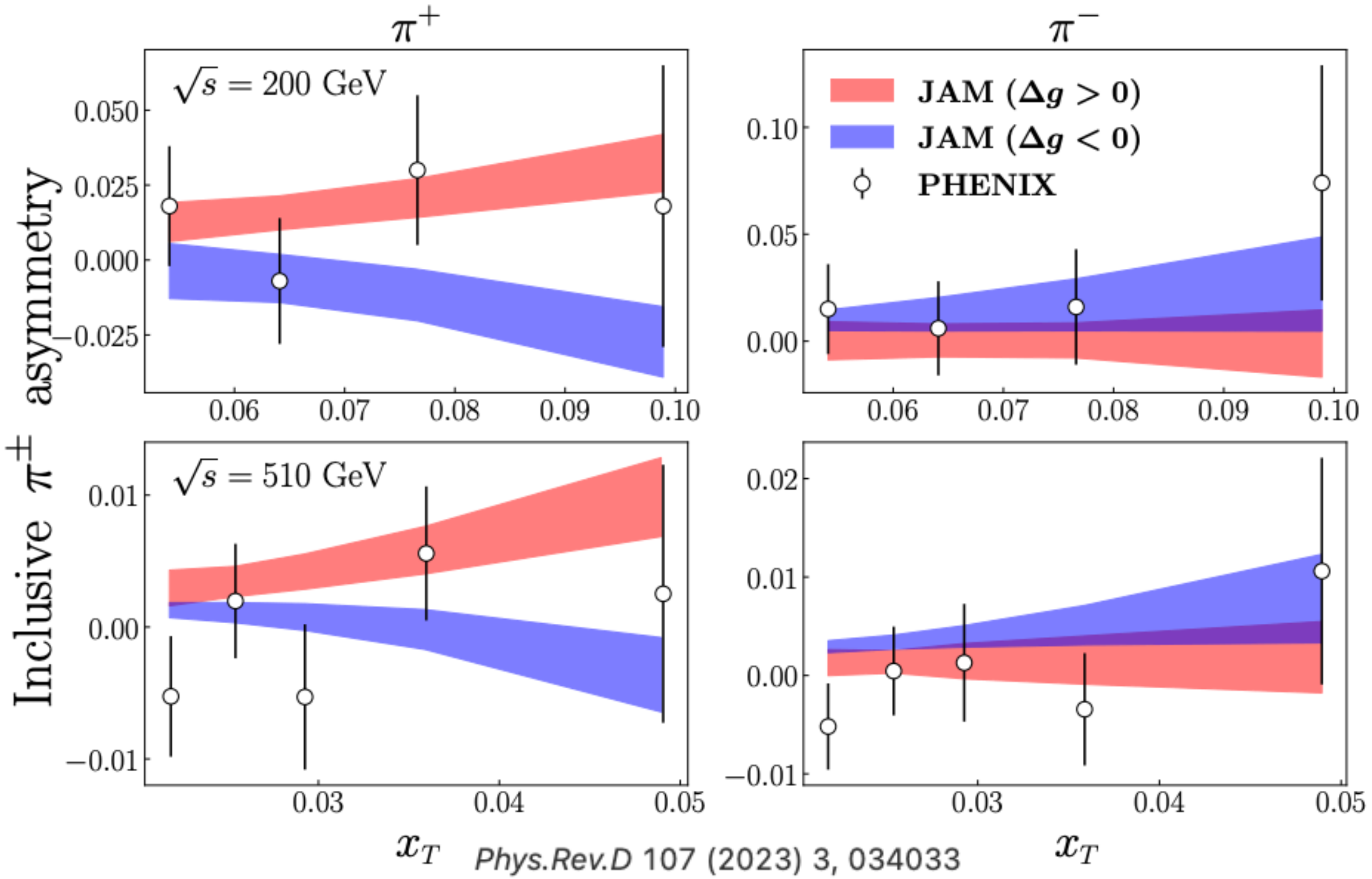

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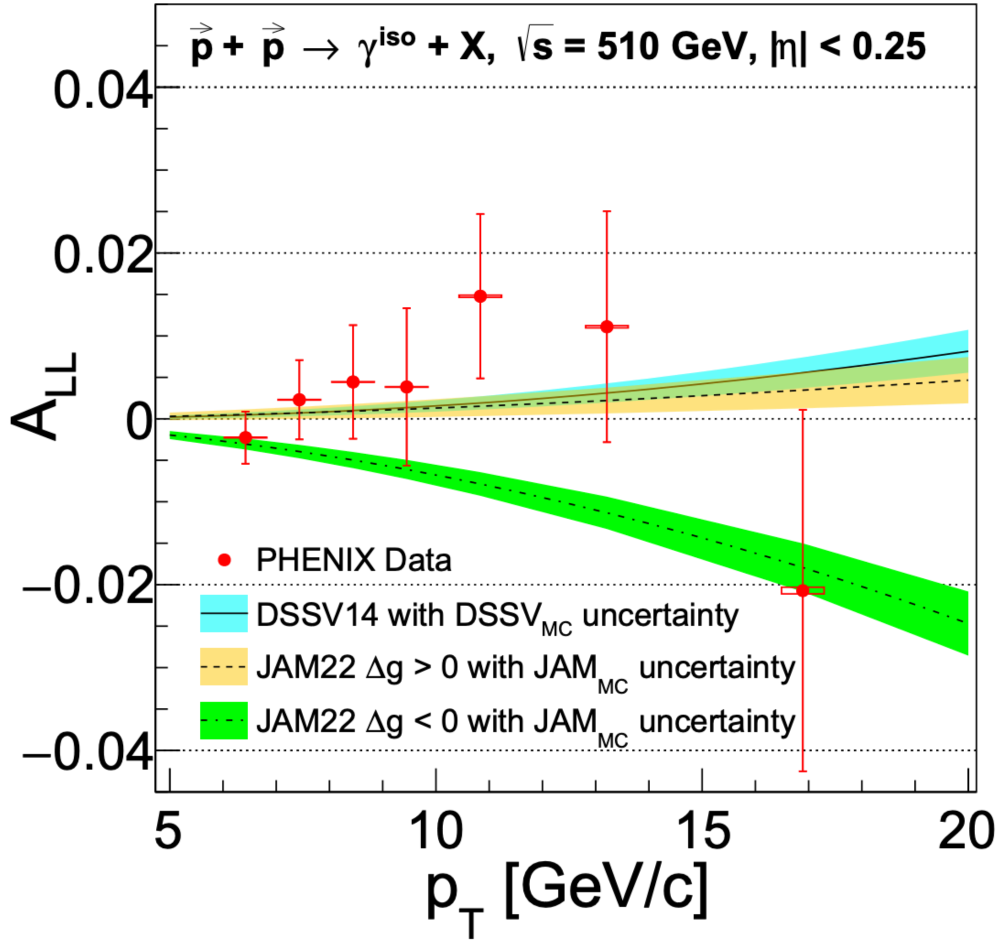
It is inconclusive whether data can distinguish between two solutions

Measurement of Direct-Photon Cross Section and Double-Helicity Asymmetry at $\sqrt{s} = 510$ GeV in $\vec{p} + \vec{p}$ Collisions

PHENIX Collaboration • U. Acharya (Georgia State U., Atlanta) et al. (Feb 16, 2022)

e-Print: 2202.08158 [hep-ex]



Direct sensitivity to the sign of Δg!

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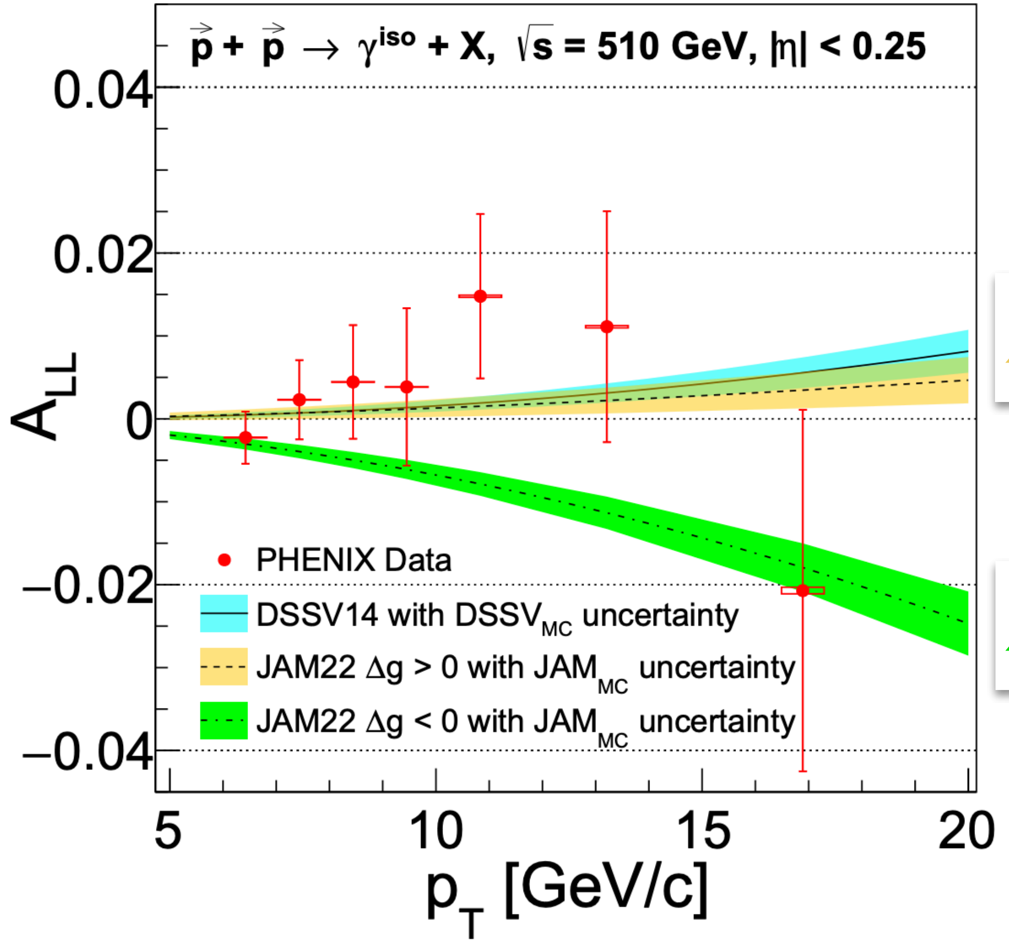

$\chi^2 = 4.7$

$\chi^2 = 12.6$

Direct sensitivity to the sign of Δg!

Current Experiments

Measurement of Direct-Photon Cross Section and Double-Helicity Asymmetry at $\sqrt{s} = 510$ GeV in $\vec{p} + \vec{p}$ Collisions

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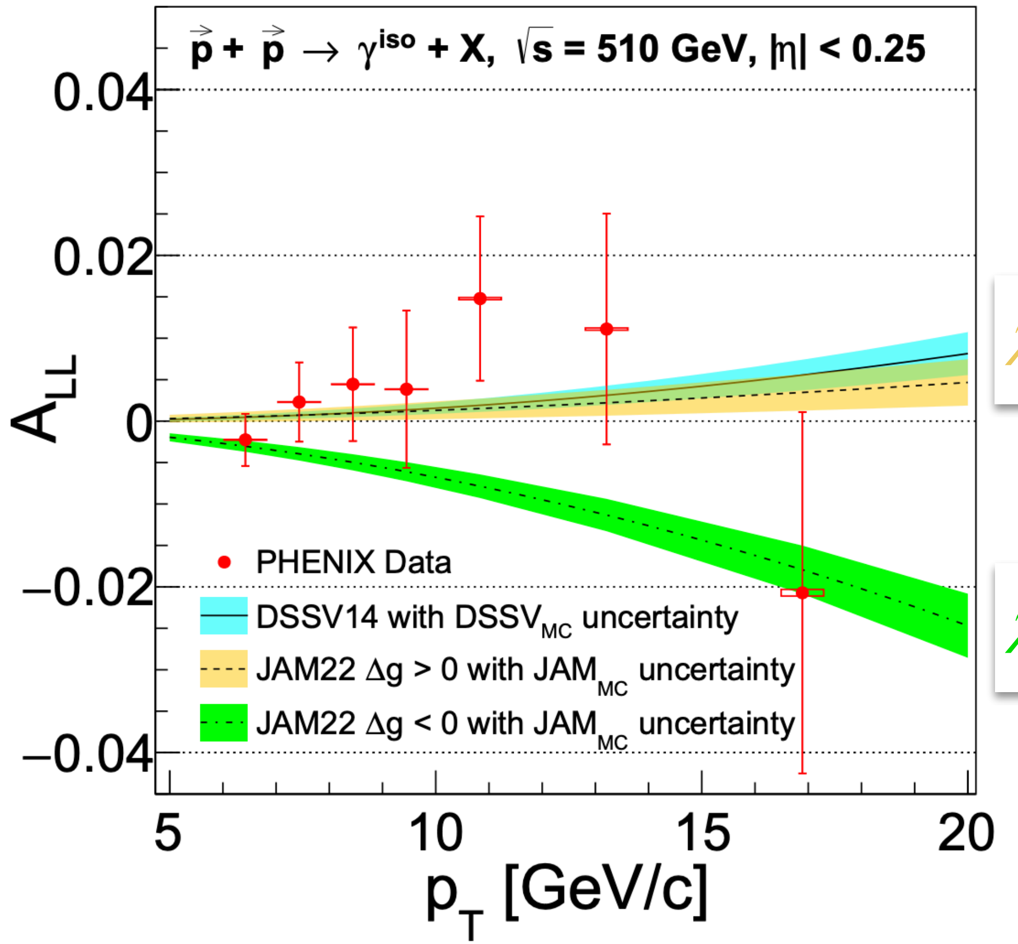

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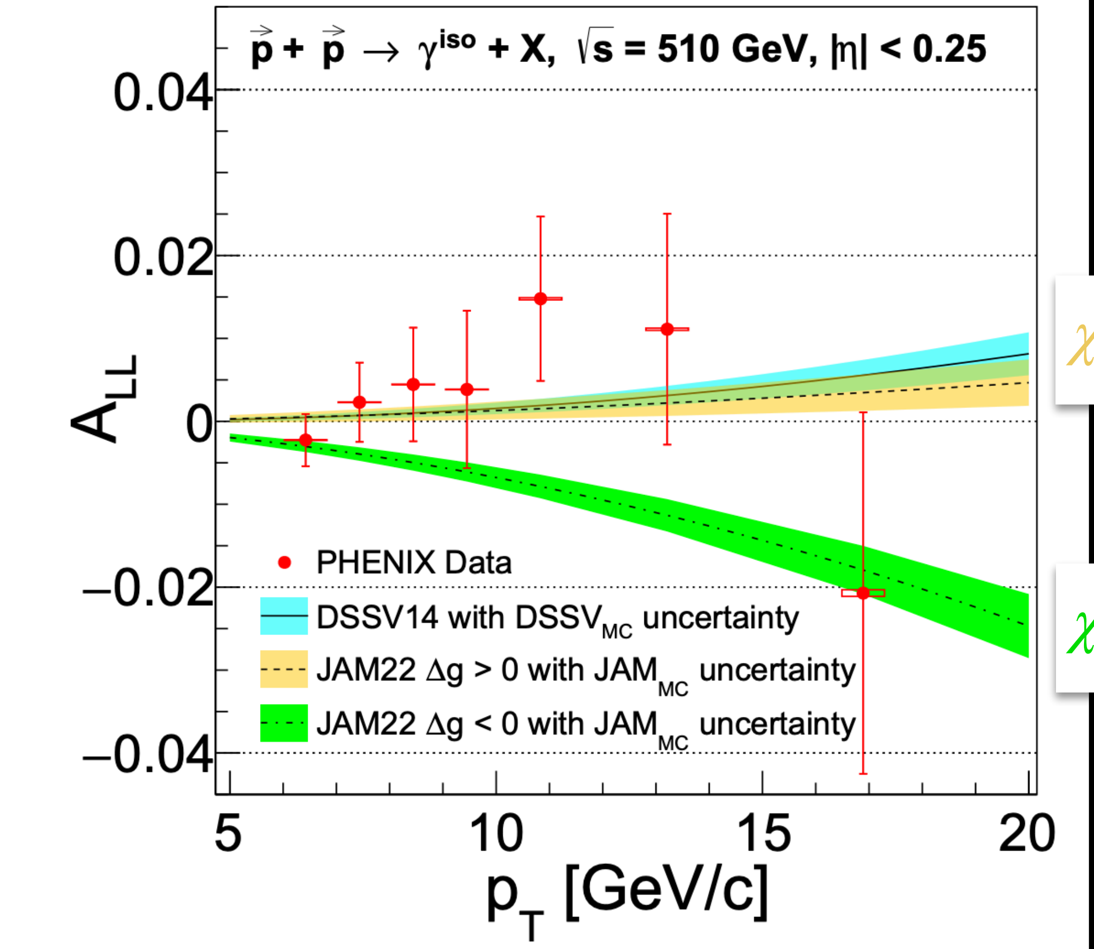

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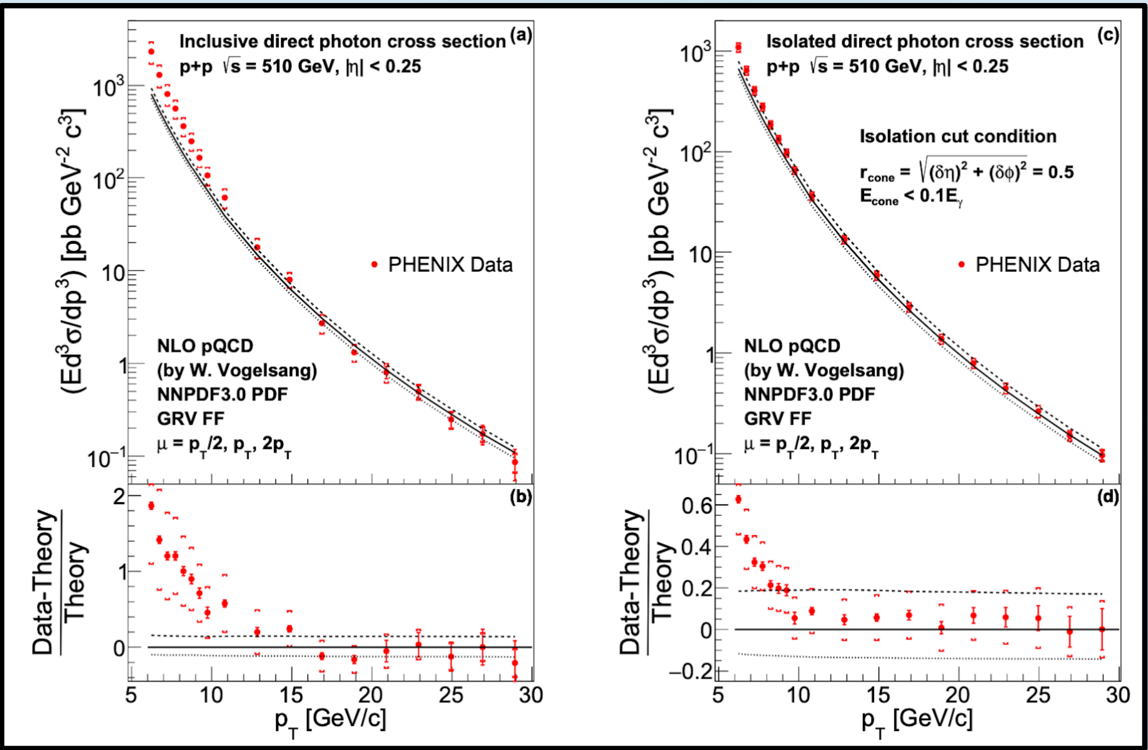

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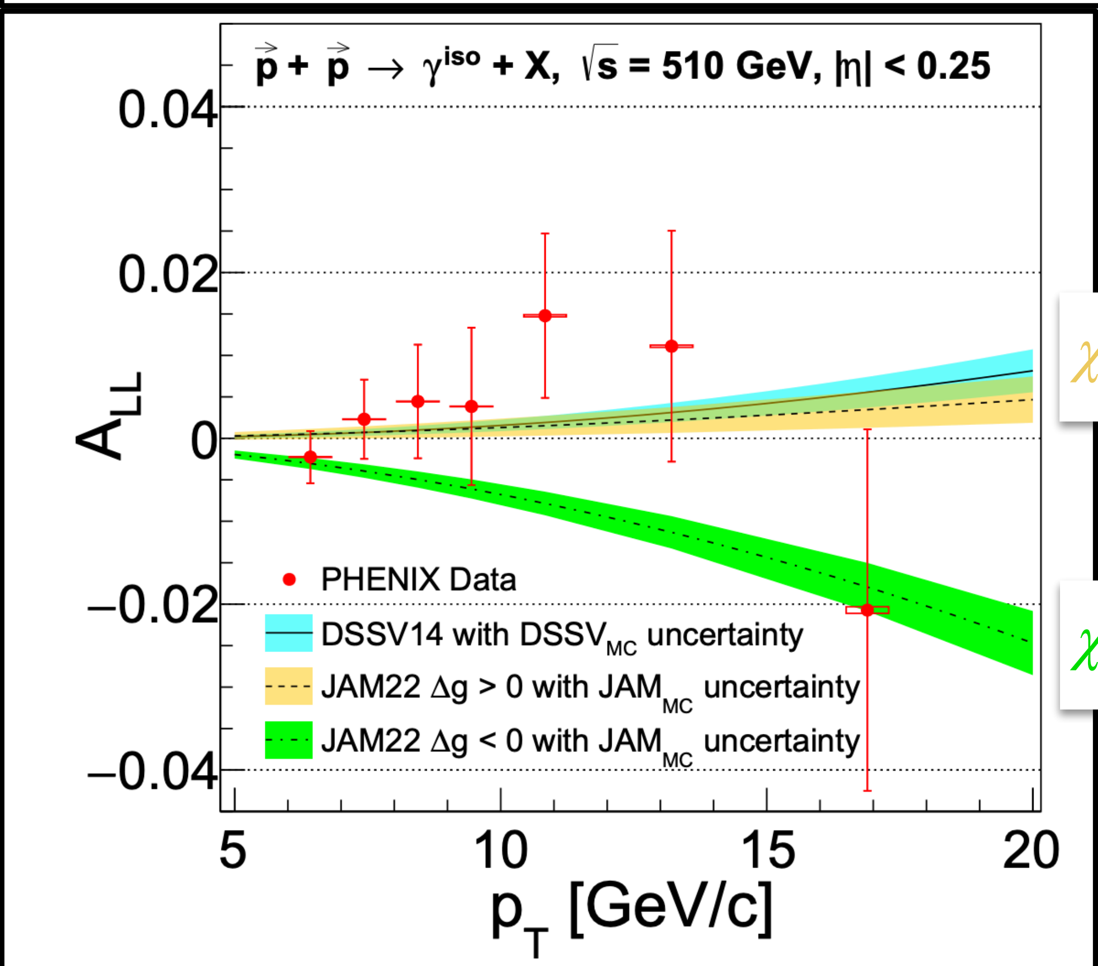

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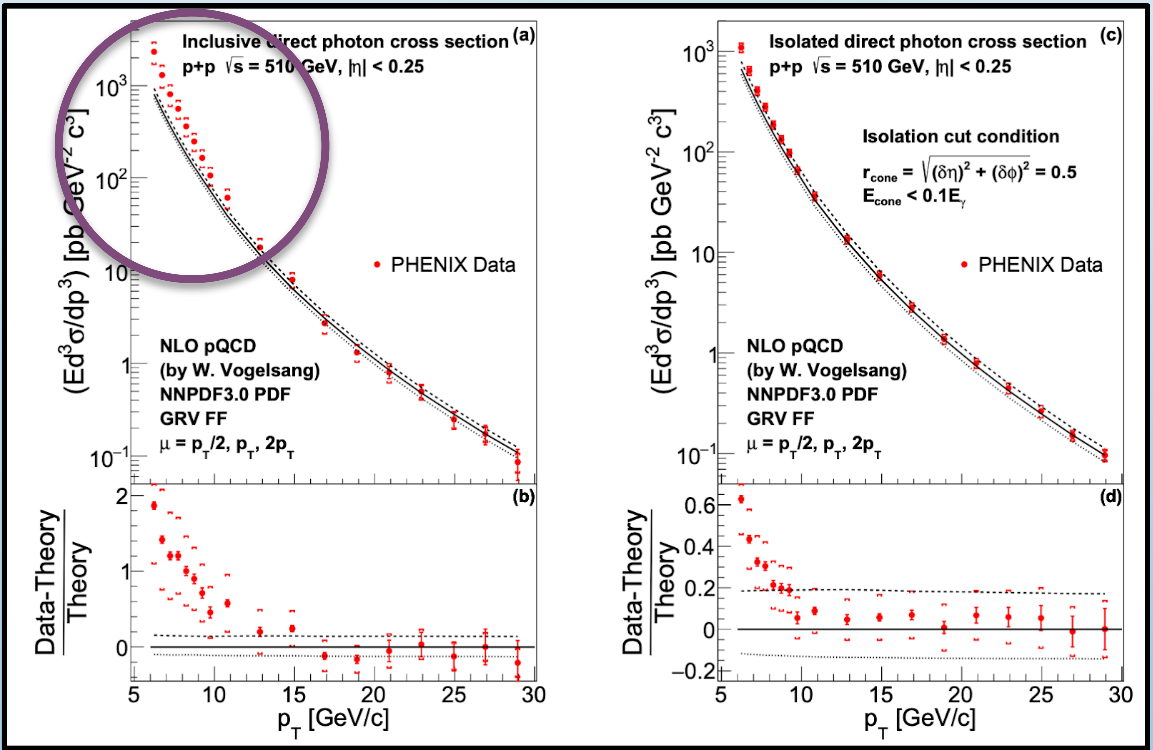

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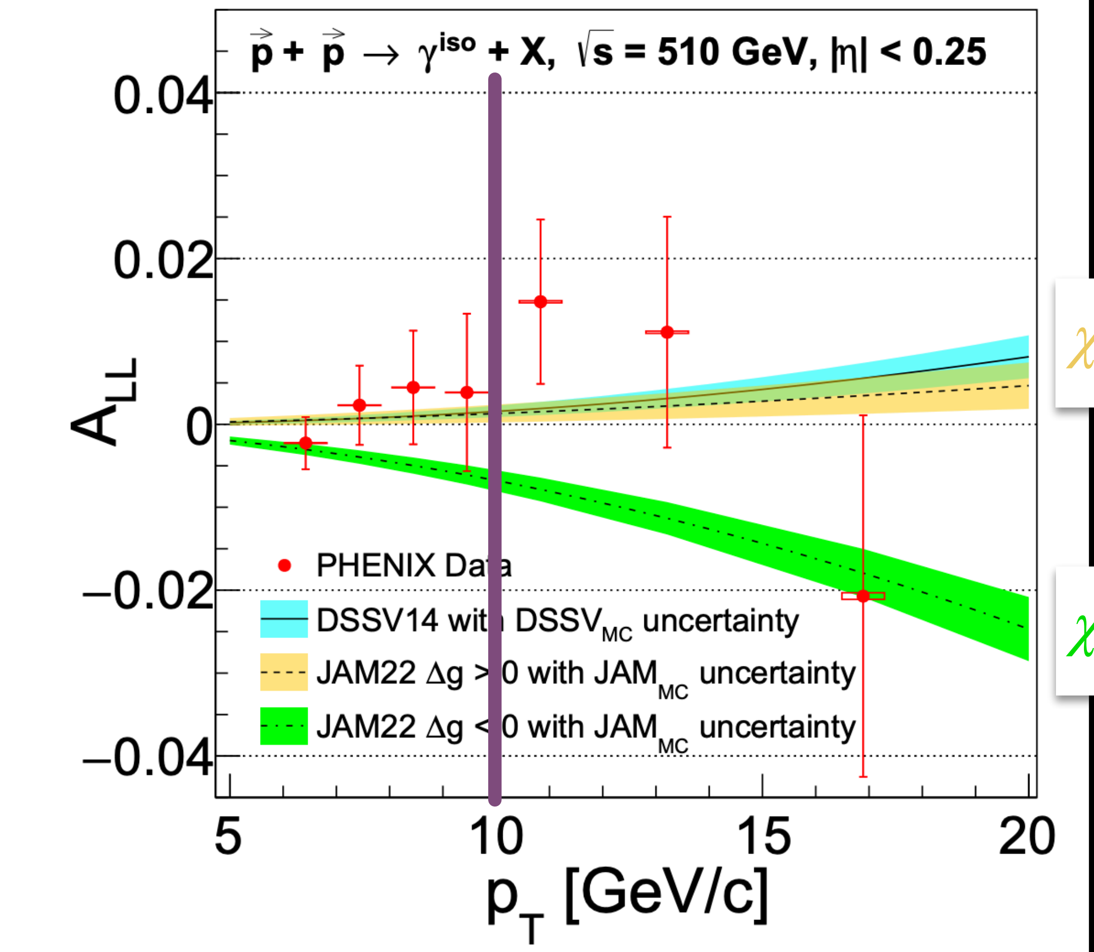

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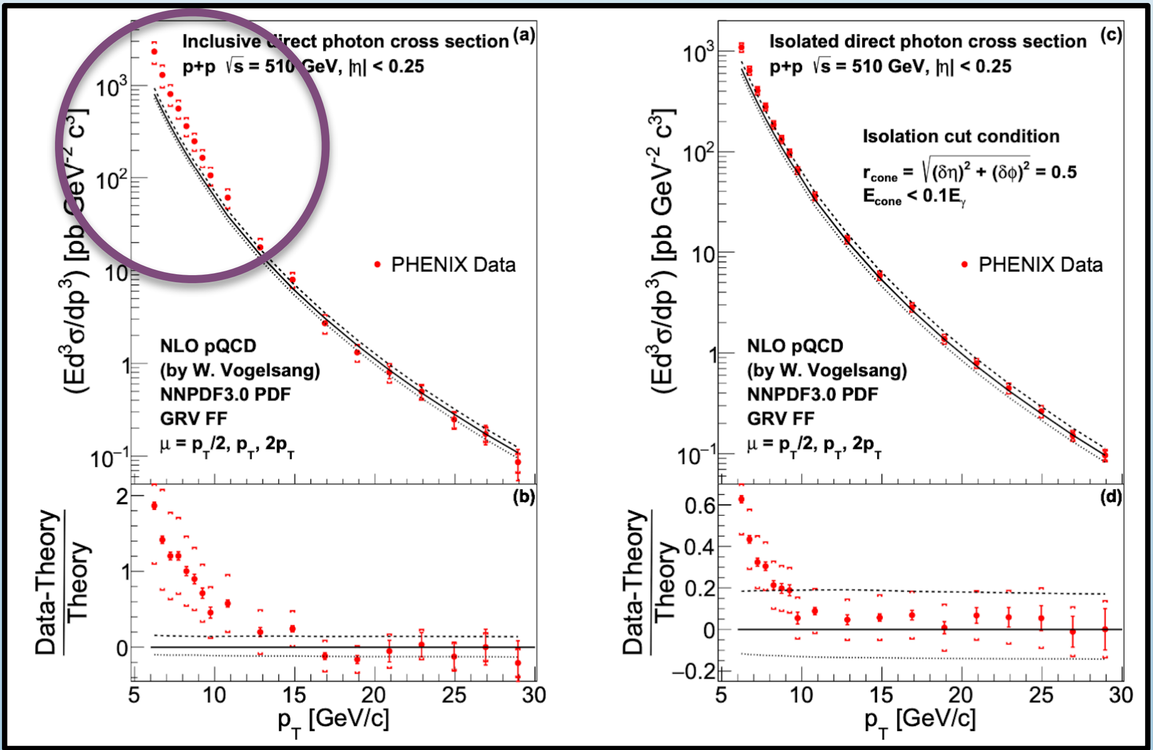


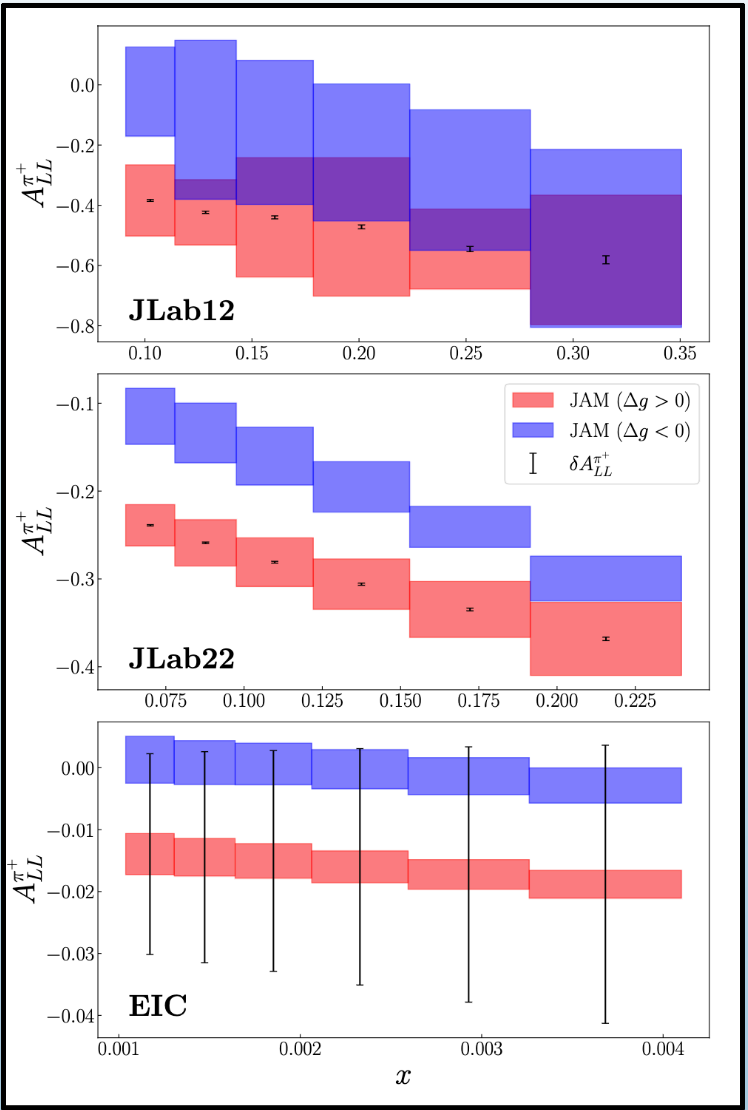
Future Experiments

Accessing gluon polarization with high-P_T hadrons in SIDIS

Jefferson Lab Angular Momentum (JAM) Collaboration • R.M. Whitehill (Wichita State U.) et al. (Oct 21, 2022)

Published in: *Phys.Rev.D* 107 (2023) 3, 034033 • e-Print: 2210.12295 [hep-ph]

$$\vec{l} + \vec{N} \to l' + h + X$$



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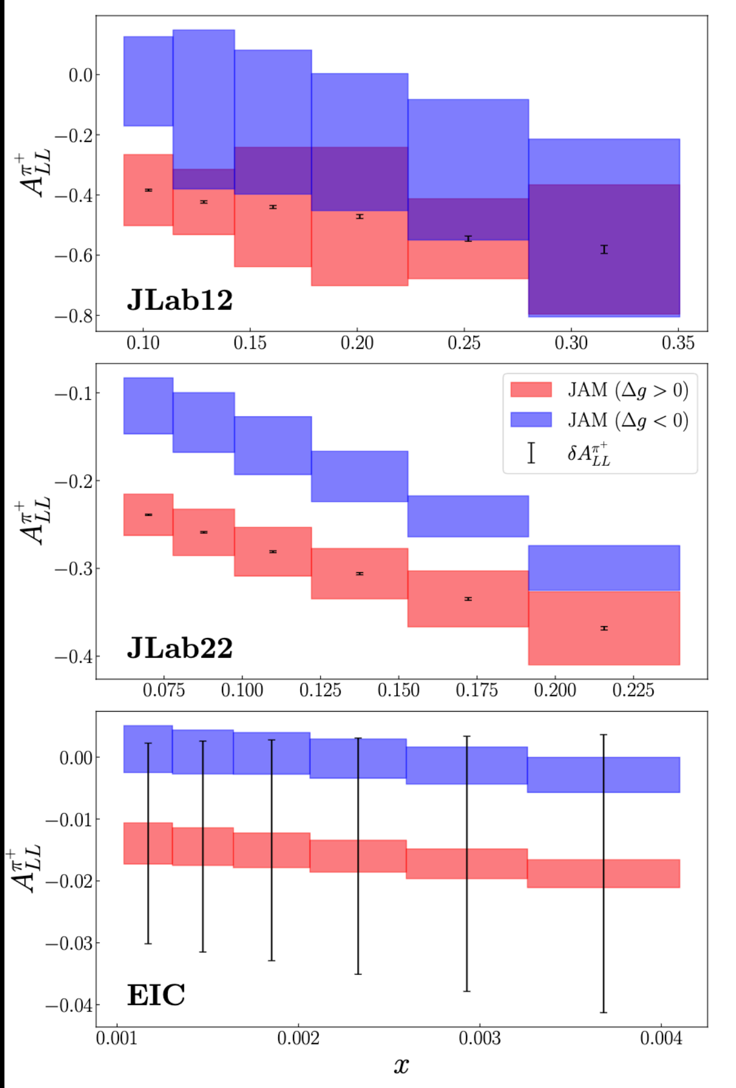

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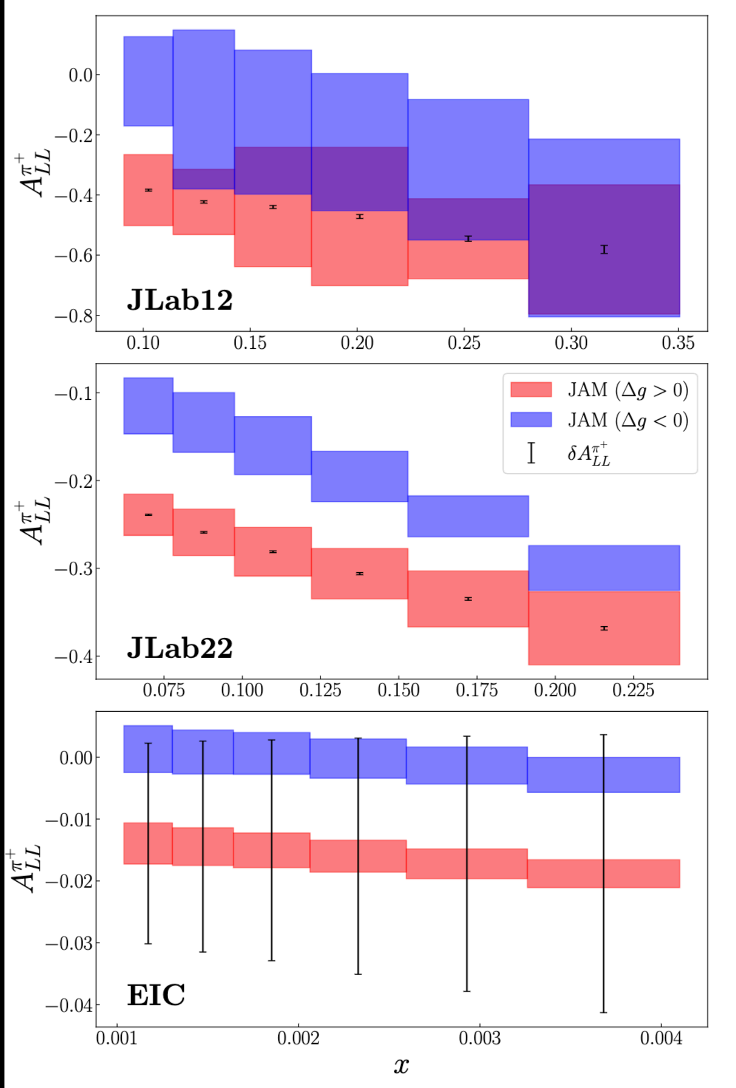

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EIC asymmetry is small due to scaling behavior of unpolarized cross section

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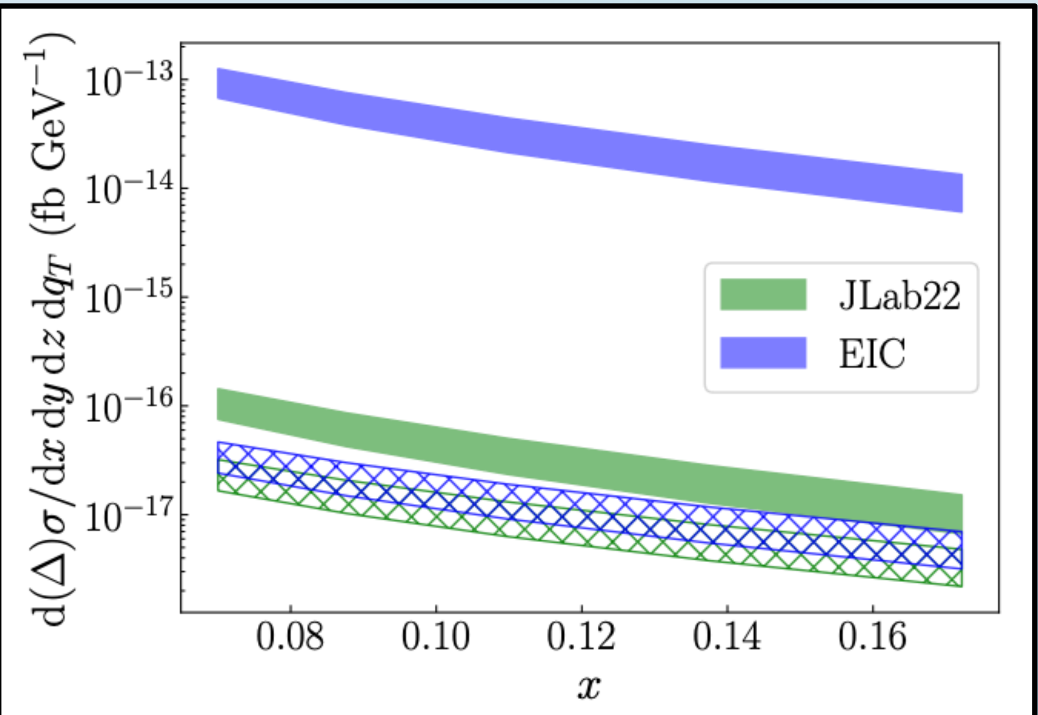

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Revisiting quark and gluon polarization in the proton at the EIC

Jefferson Lab Angular Momentum (JAM) Collaboration • Y. Zhou (William-Mary Coll.) et al. (May 10, 2021)

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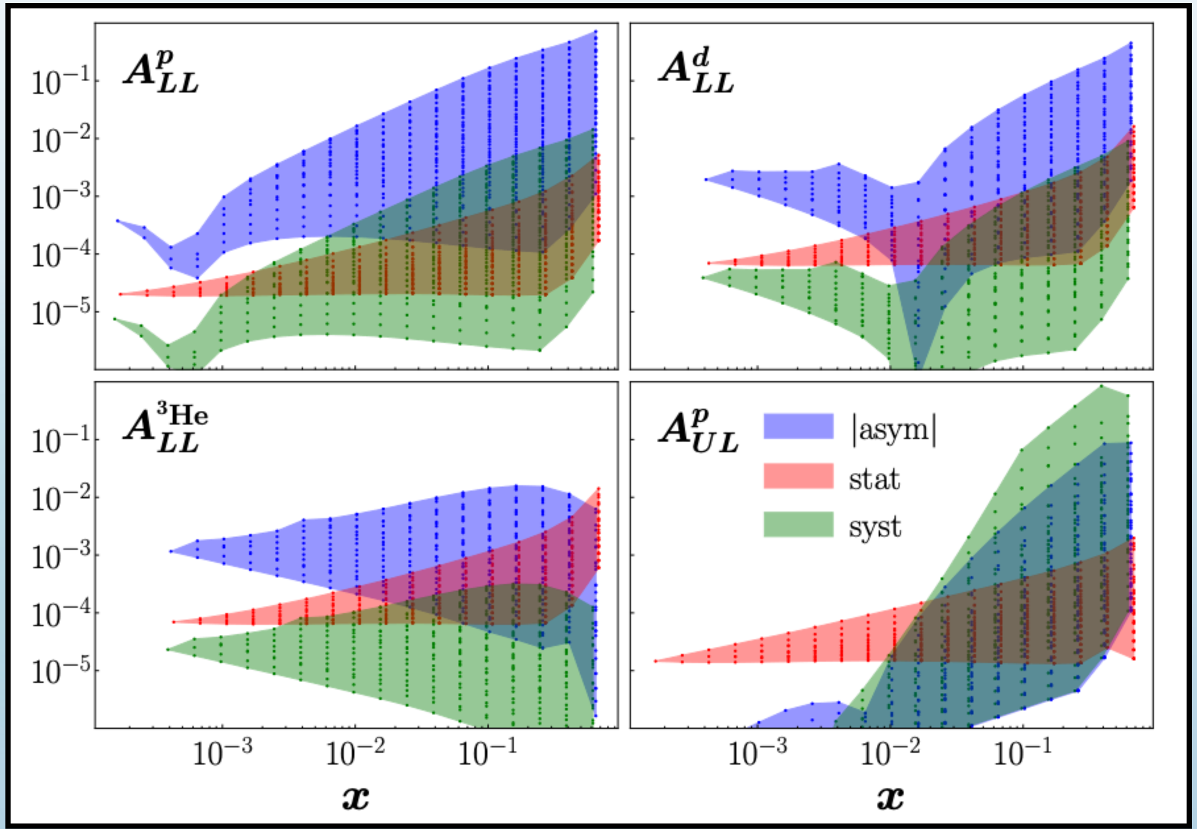

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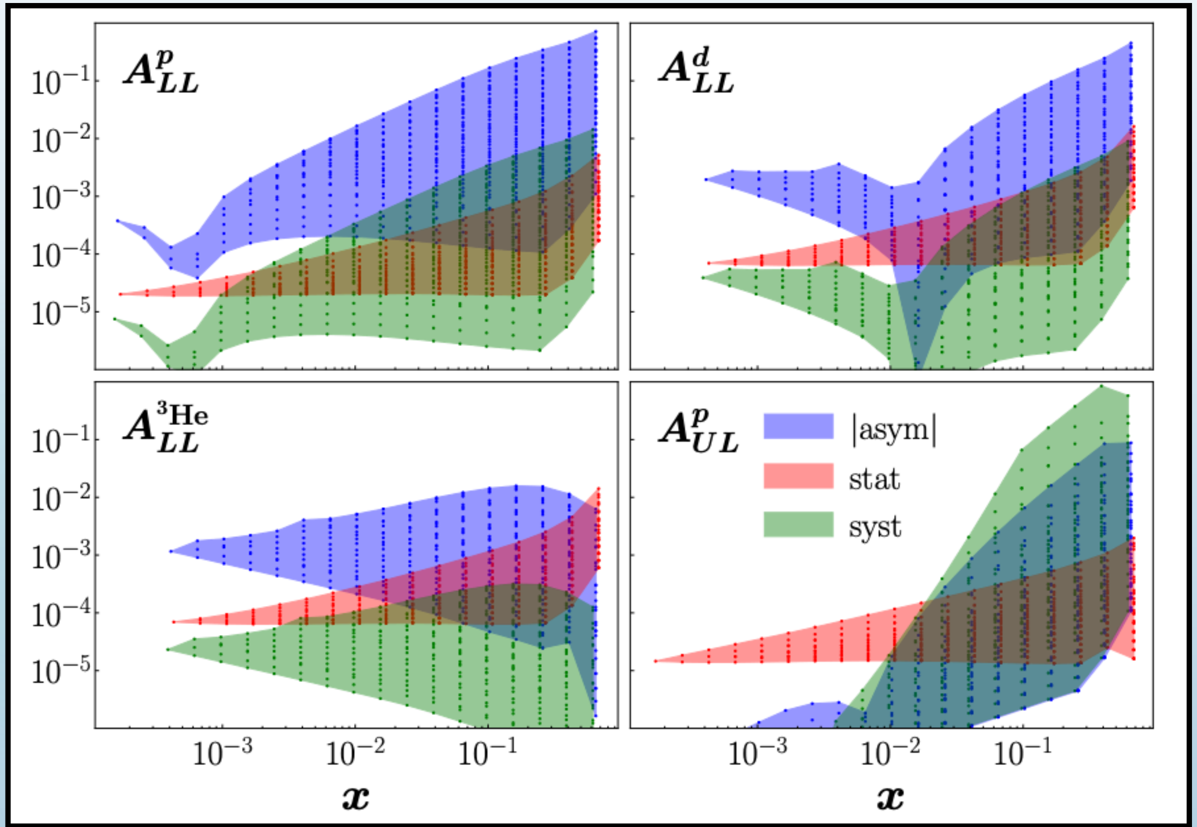
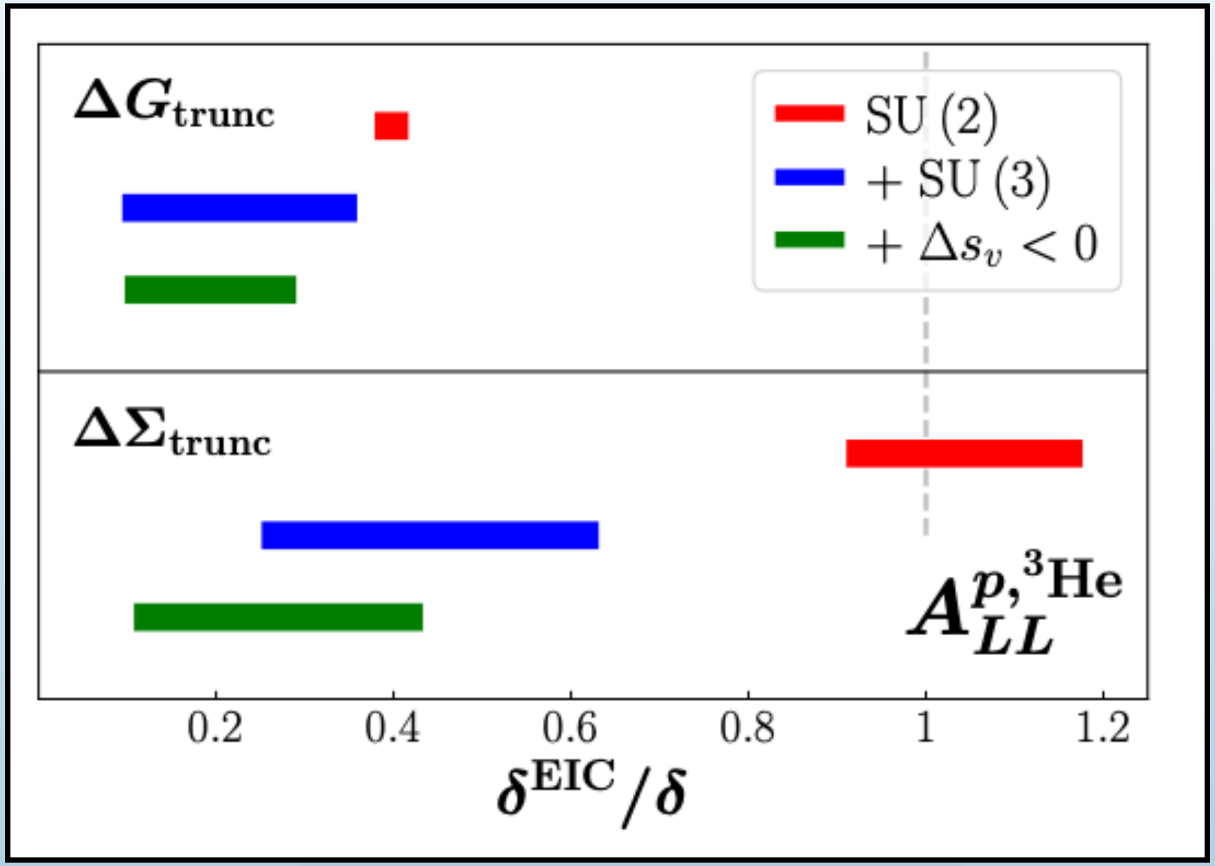
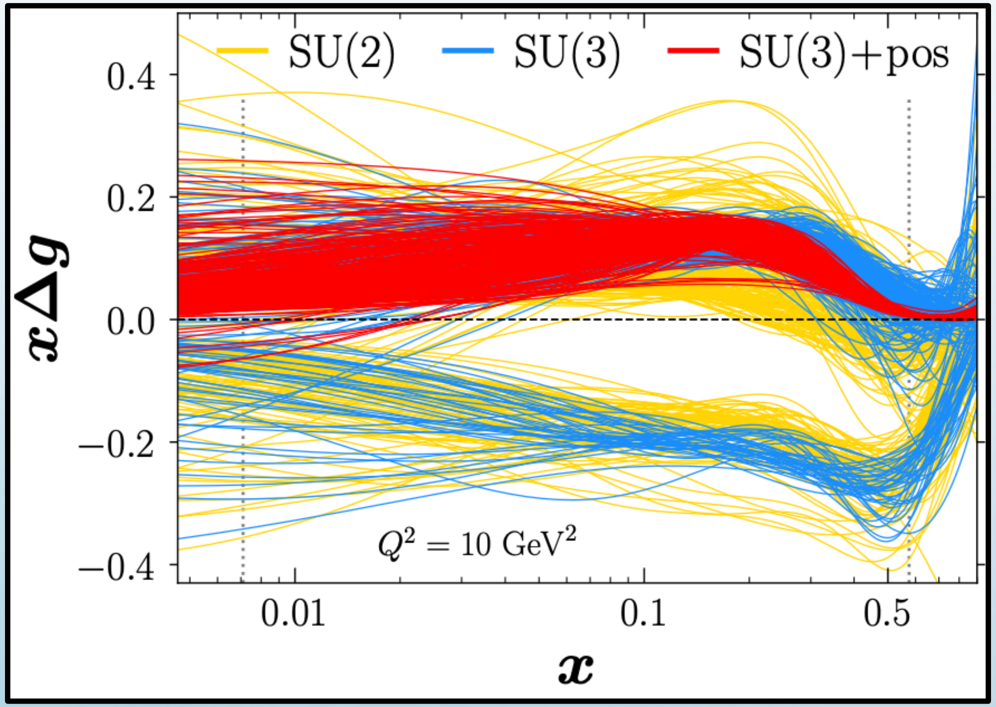




$$\Delta G_{\mathrm{trunc}} = \int_{10^{-4}}^{1} \mathrm{d}x \Delta g(x)$$

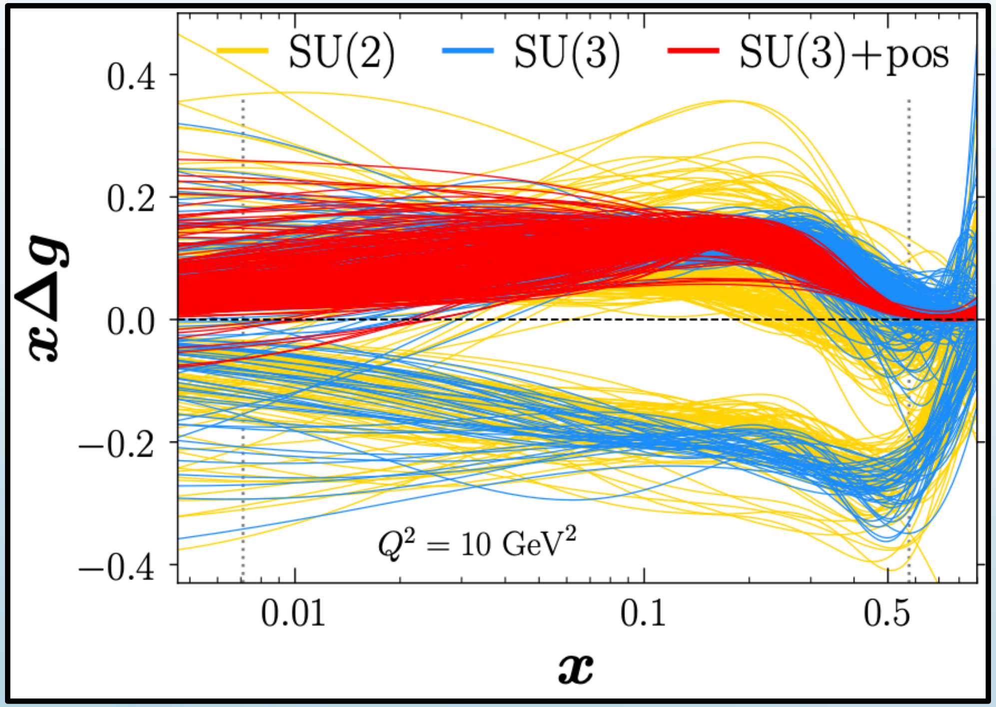
Current JAM analyses have two solutions

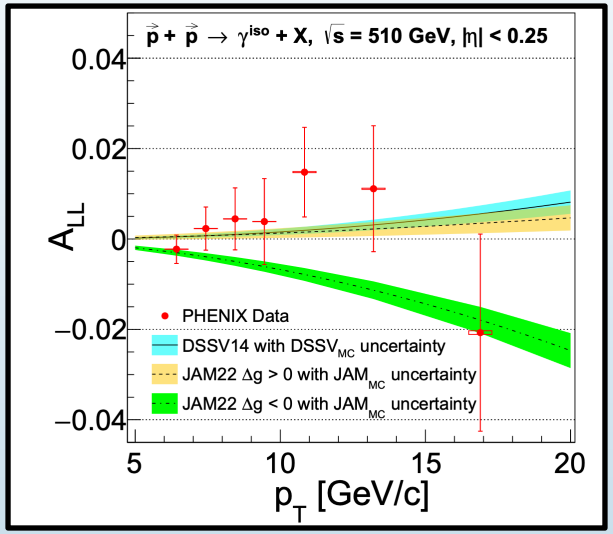


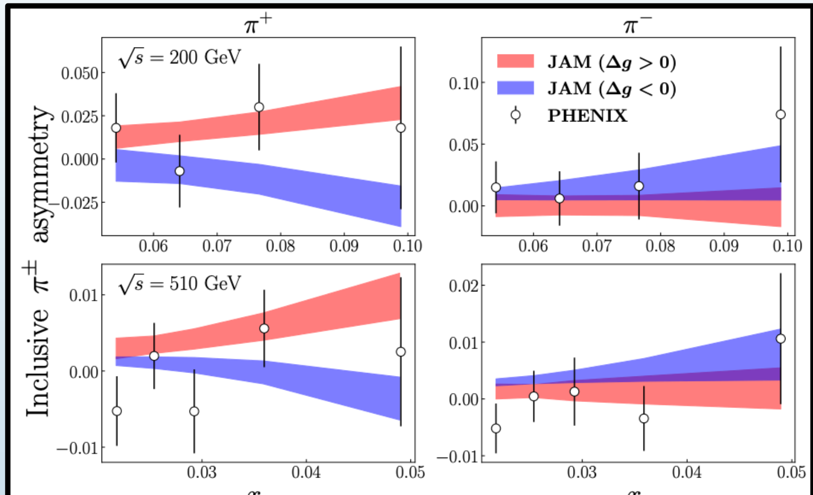
Conclusions and Outlook

Current JAM analyses have two solutions

New data from RHIC may help distinguish them







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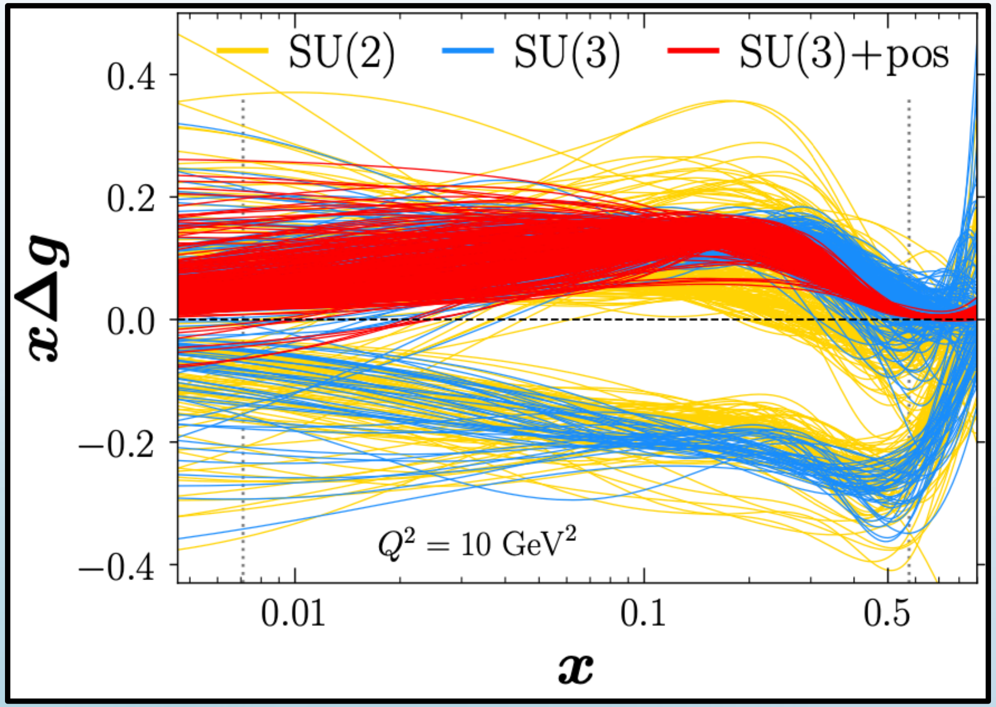

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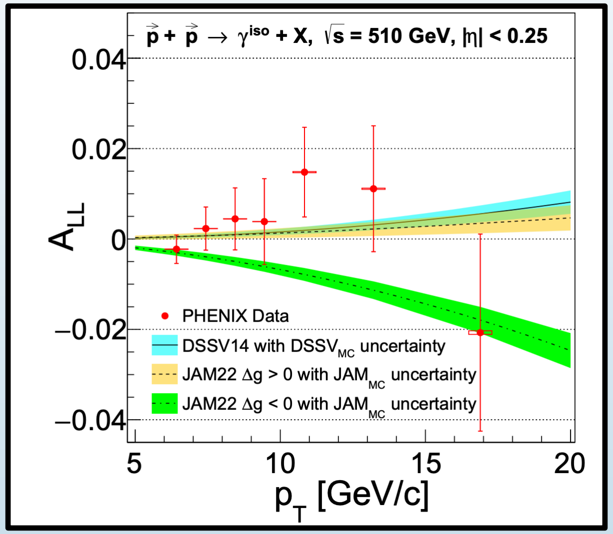
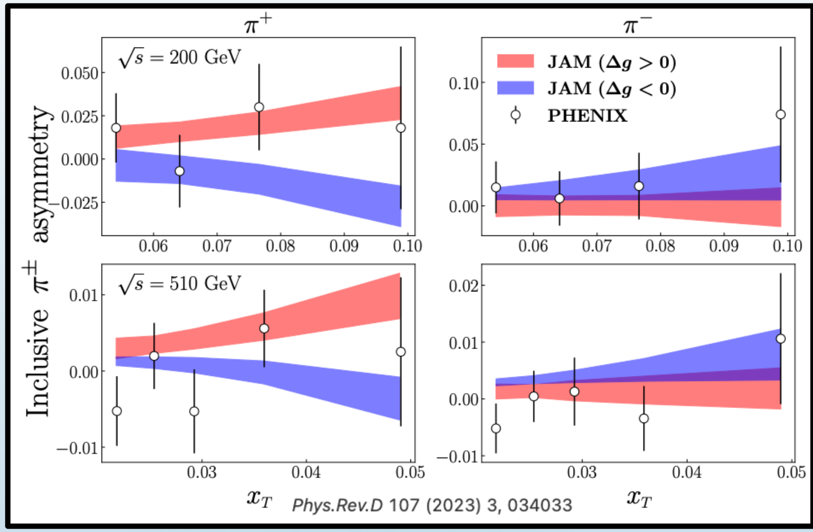
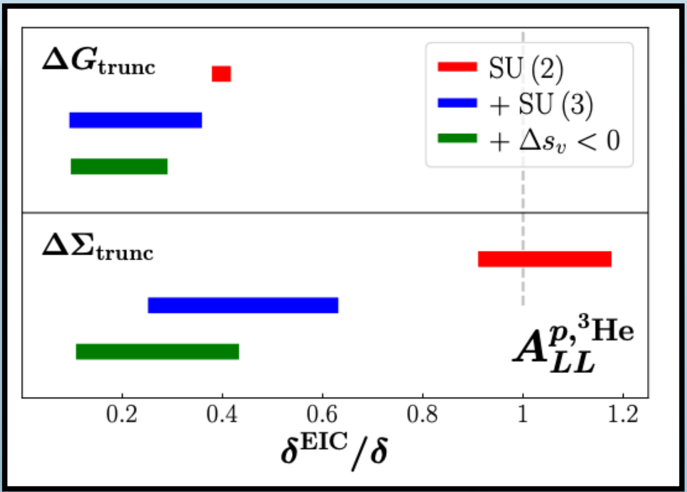
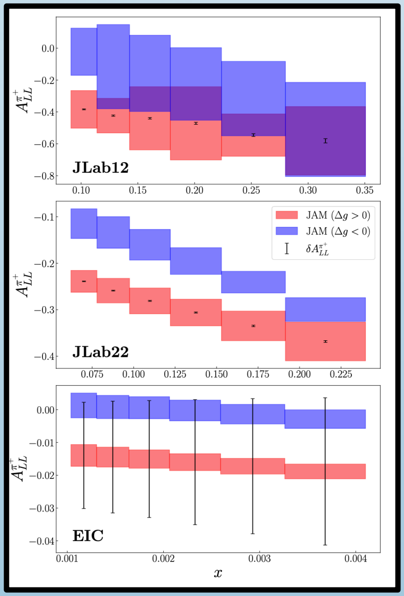




Future data from the EIC and JLab should provide tons of new information on the gluon's helicity





Extra Slides

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

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Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

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Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

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Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M\, \tilde{f}_j(N,\mu_0)\tilde{f}_l(M,\mu_0) \otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0)\right]$$

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$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

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"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k \right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

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Data → $d_{i,e}$

Theory → $T_{i,e}(\boldsymbol{a})$

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Data

Theory

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Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

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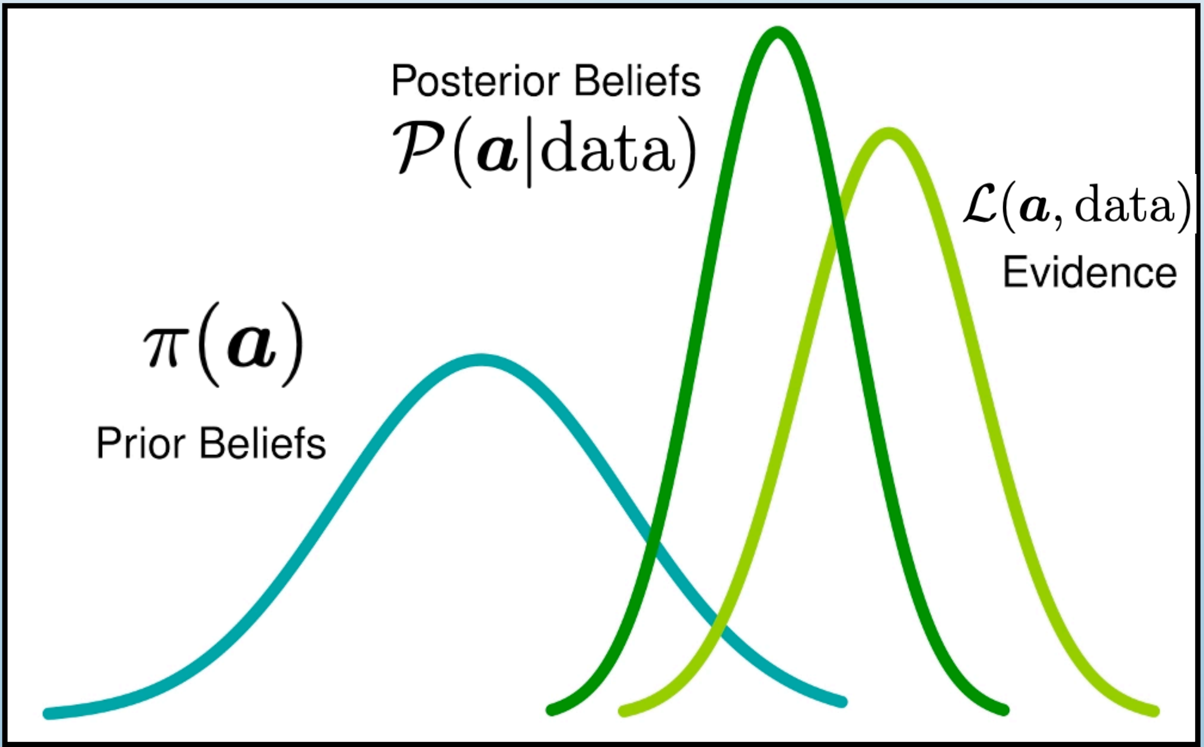
Likelihood Function

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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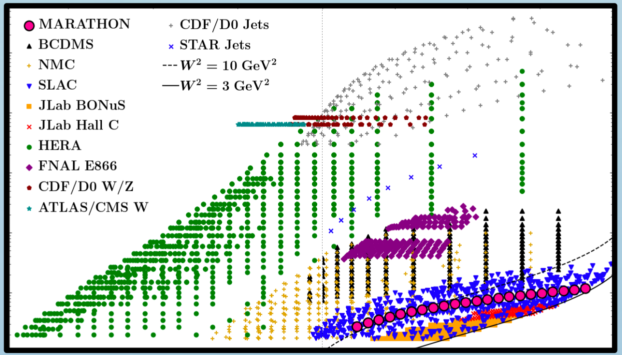
Data

Original Data



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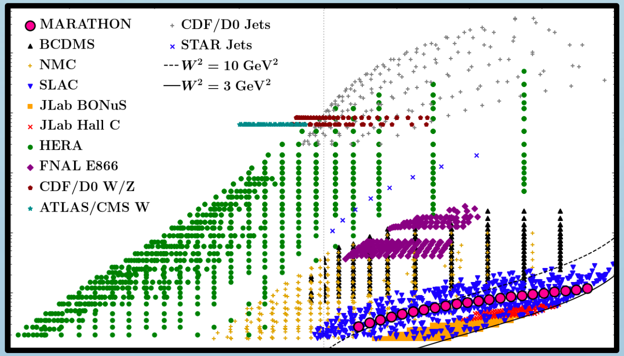


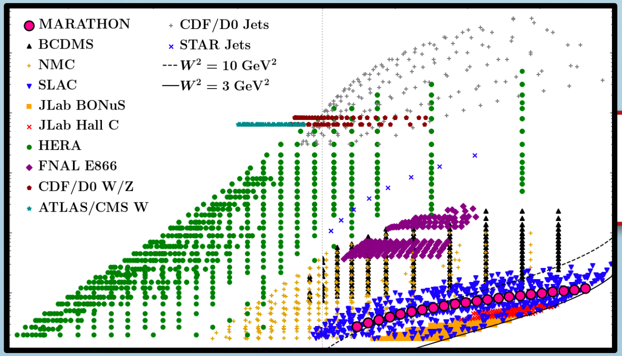
Pseudo-Data →

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

← Uncorrelated Uncertainties

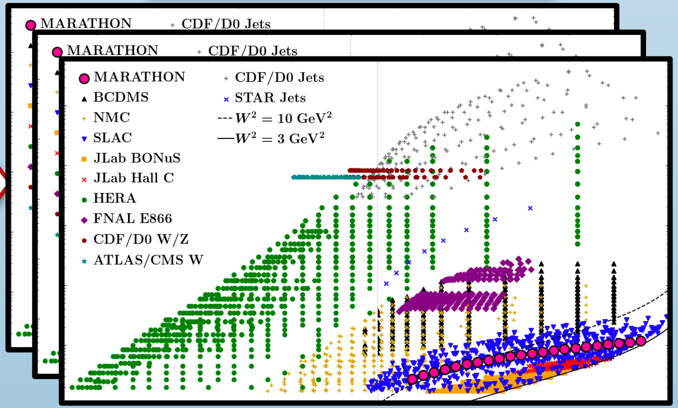
↑ Data

Original Data



DR

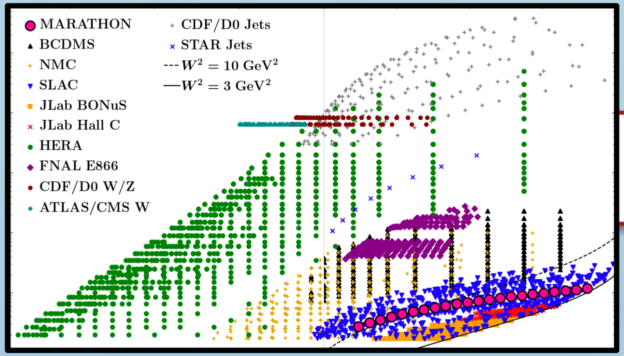
Replica Data



Pseudo-Data → $$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$ ← Uncorrelated Uncertainties

↑ Data

Original Data



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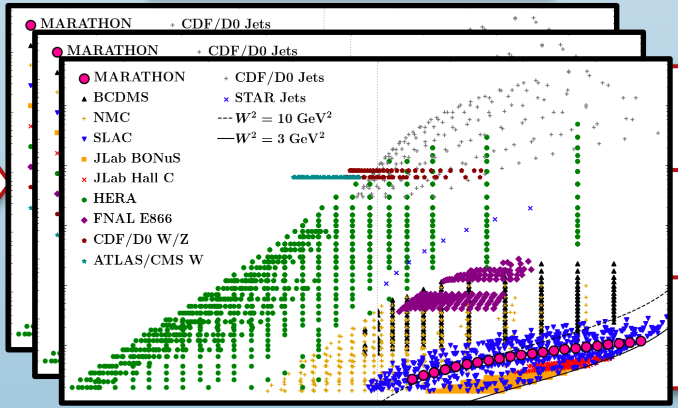
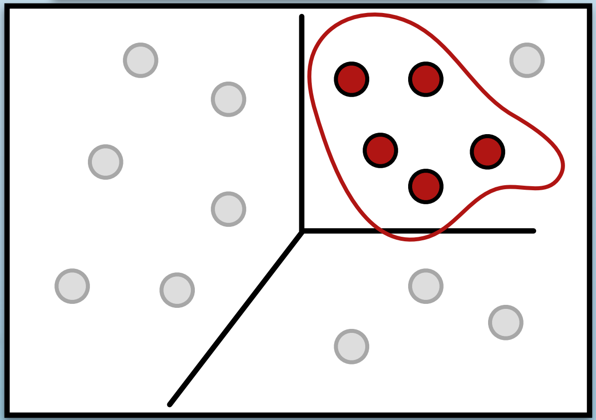


Maximum Likelihood →

Maximum Likelihood →

Maximum Likelihood →

Parameter Space



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; O(\boldsymbol{a})$$

$$V[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but

$n = \mathcal{O}(100)$!

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Build an MC ensemble

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Average over k sets
of the parameters
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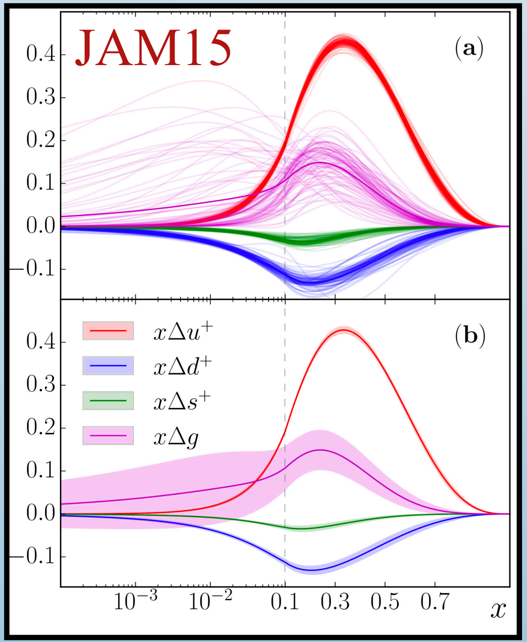
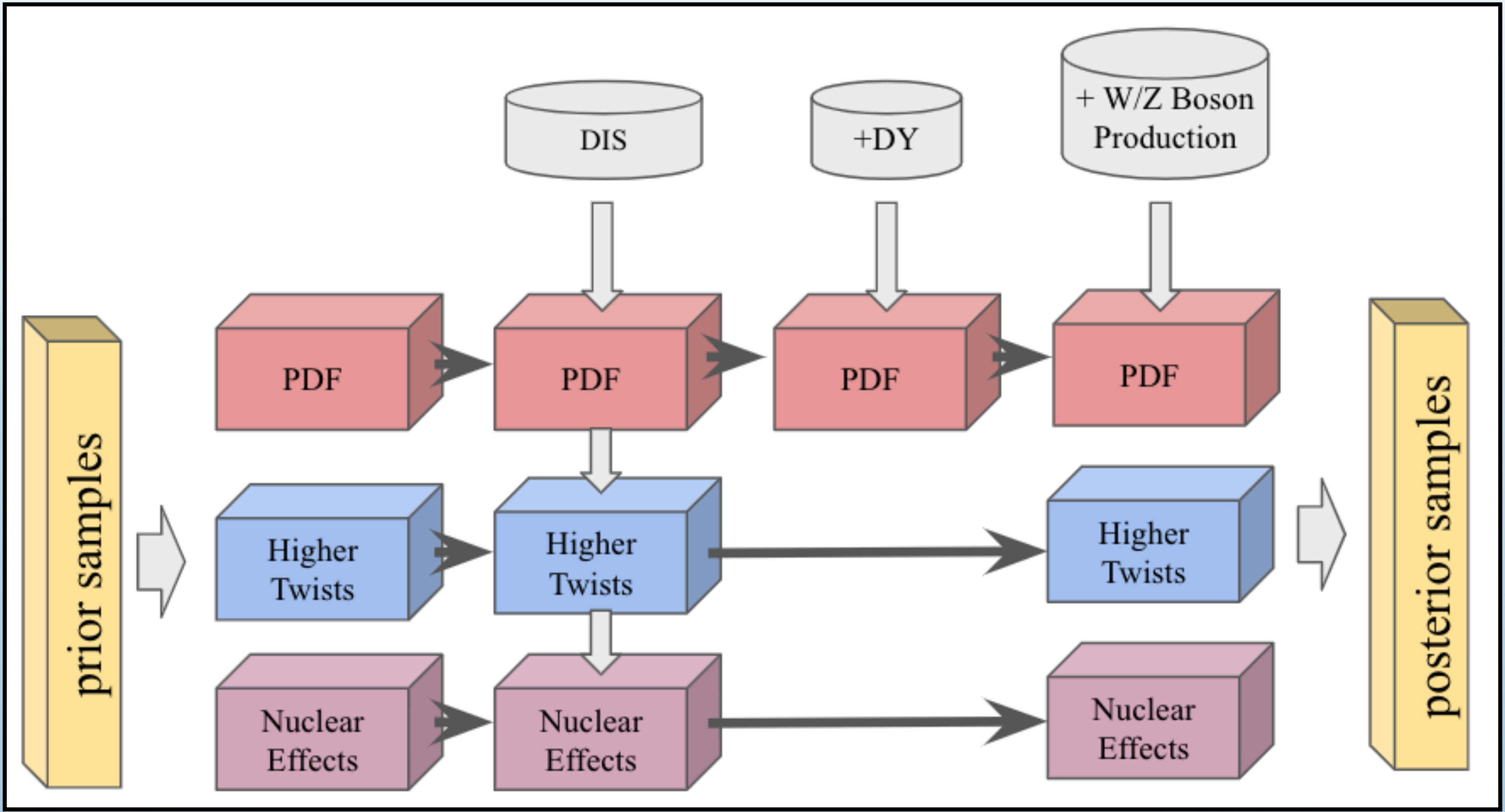
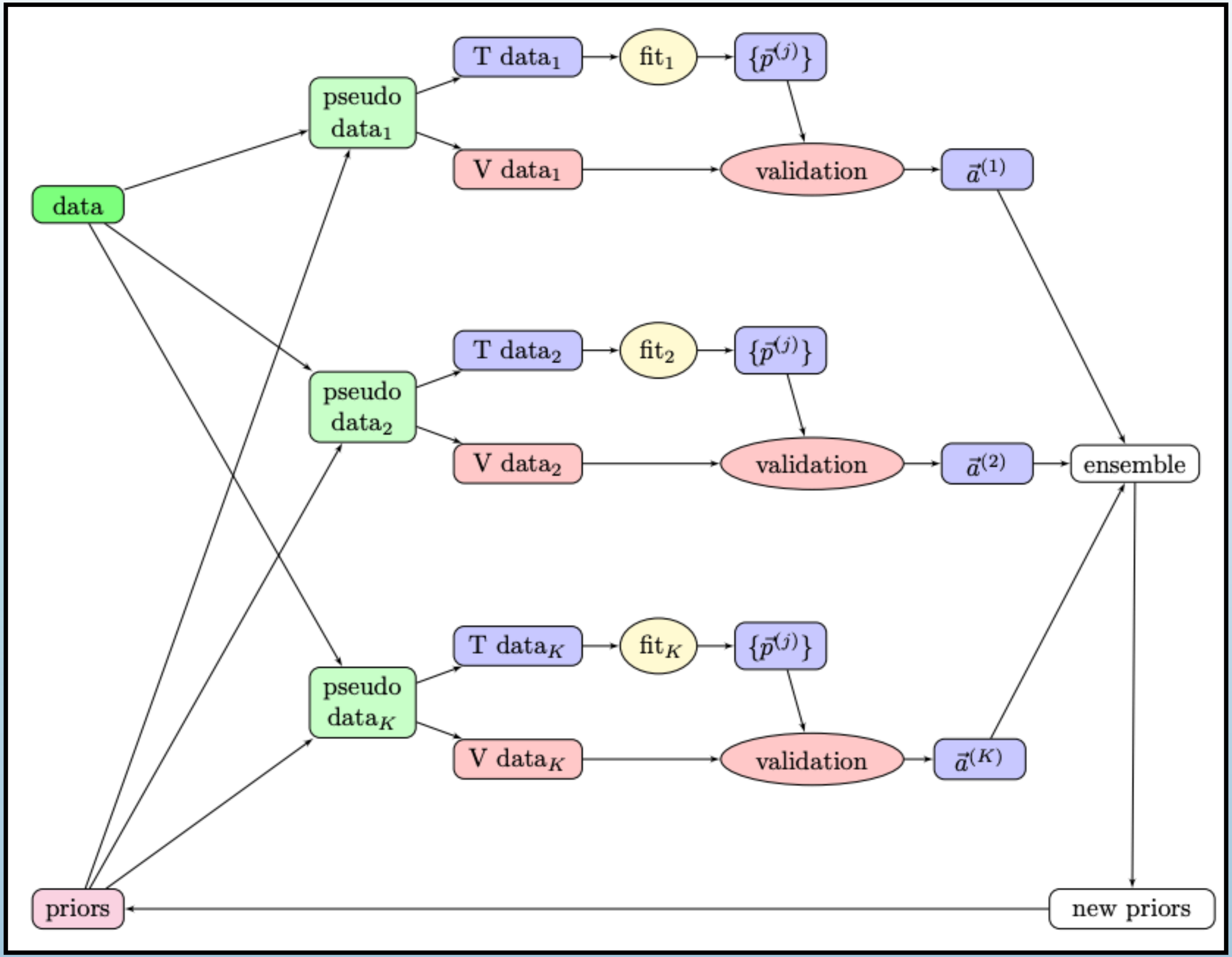
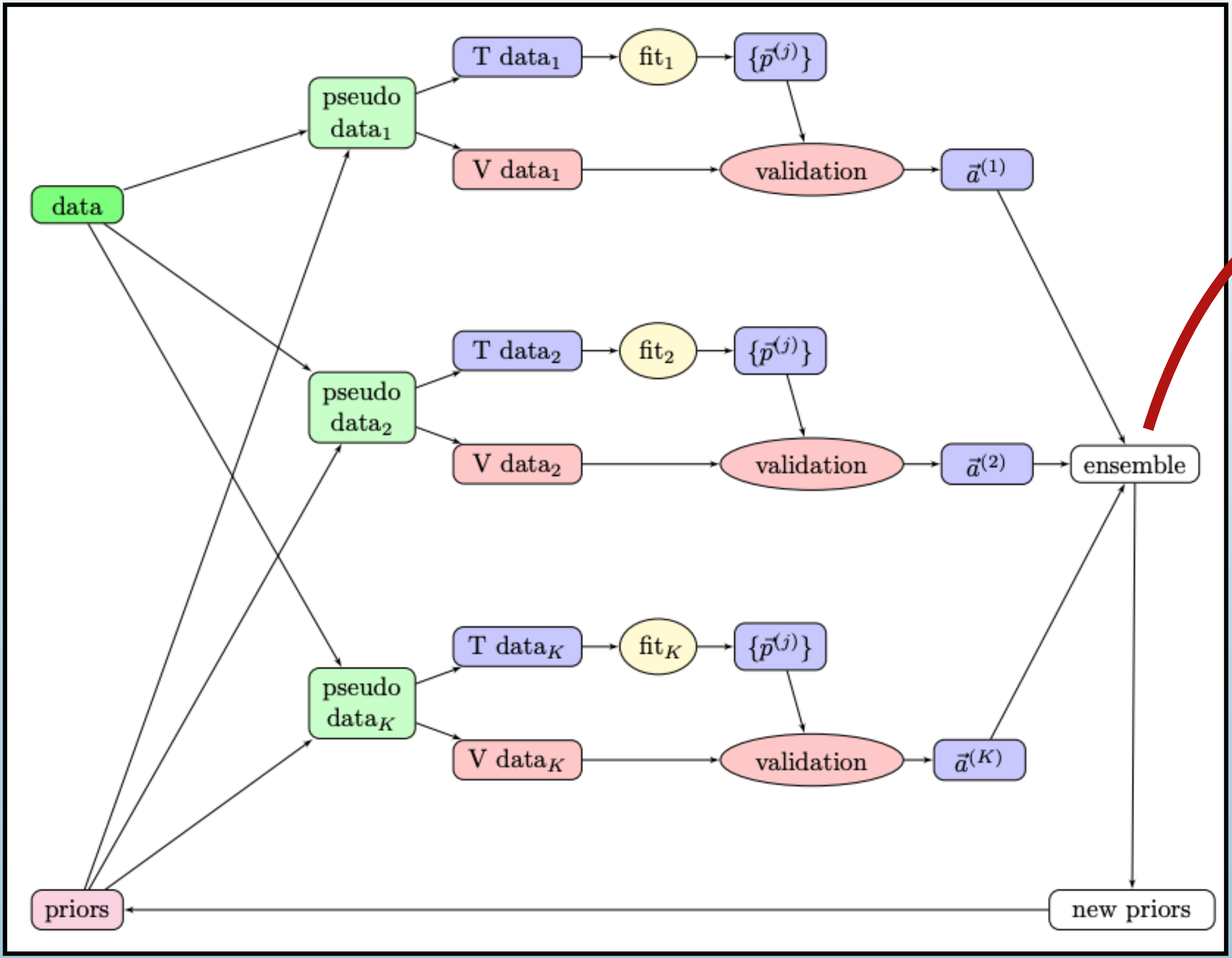
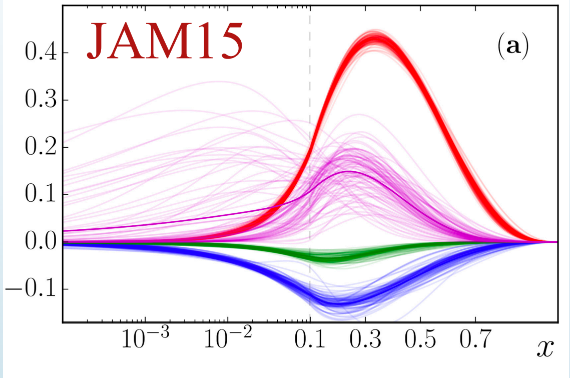
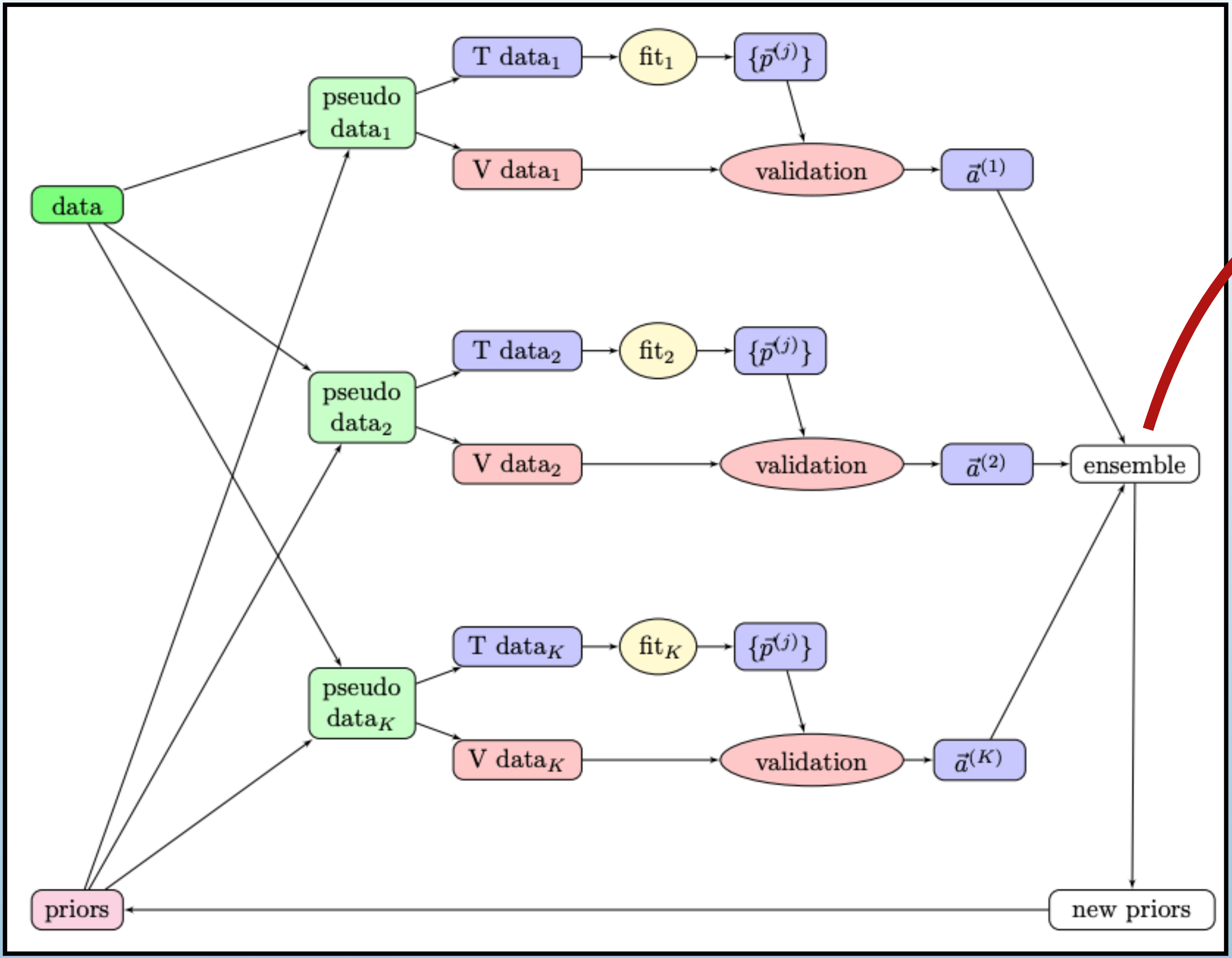


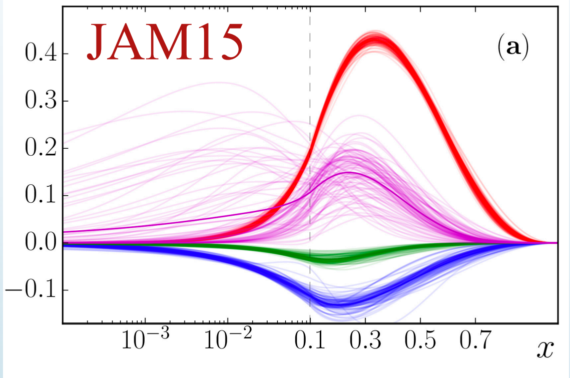

DIS
+DY
+ W/Z Boson Production
prior samples
PDF
PDF
PDF
PDF
Higher Twists
Higher Twists
Higher Twists
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples


data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
V data$_2$
T data$_K$
V data$_K$
fit$_1$
fit$_2$
fit$_K$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
validation
validation
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
new priors
priors


data
pseudo data_1
pseudo data_2
pseudo data_K
T data_1
V data_1
T data_2
V data_2
T data_K
V data_K
fit_1
fit_2
fit_K
$\{\vec{p}^{(j)}\}$
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
priors
new priors


JAM15
(a)
0.4
0.3
0.2
0.1
0.0
−0.1
10^{-3}
10^{-2}
0.1
0.3
0.5
0.7
x

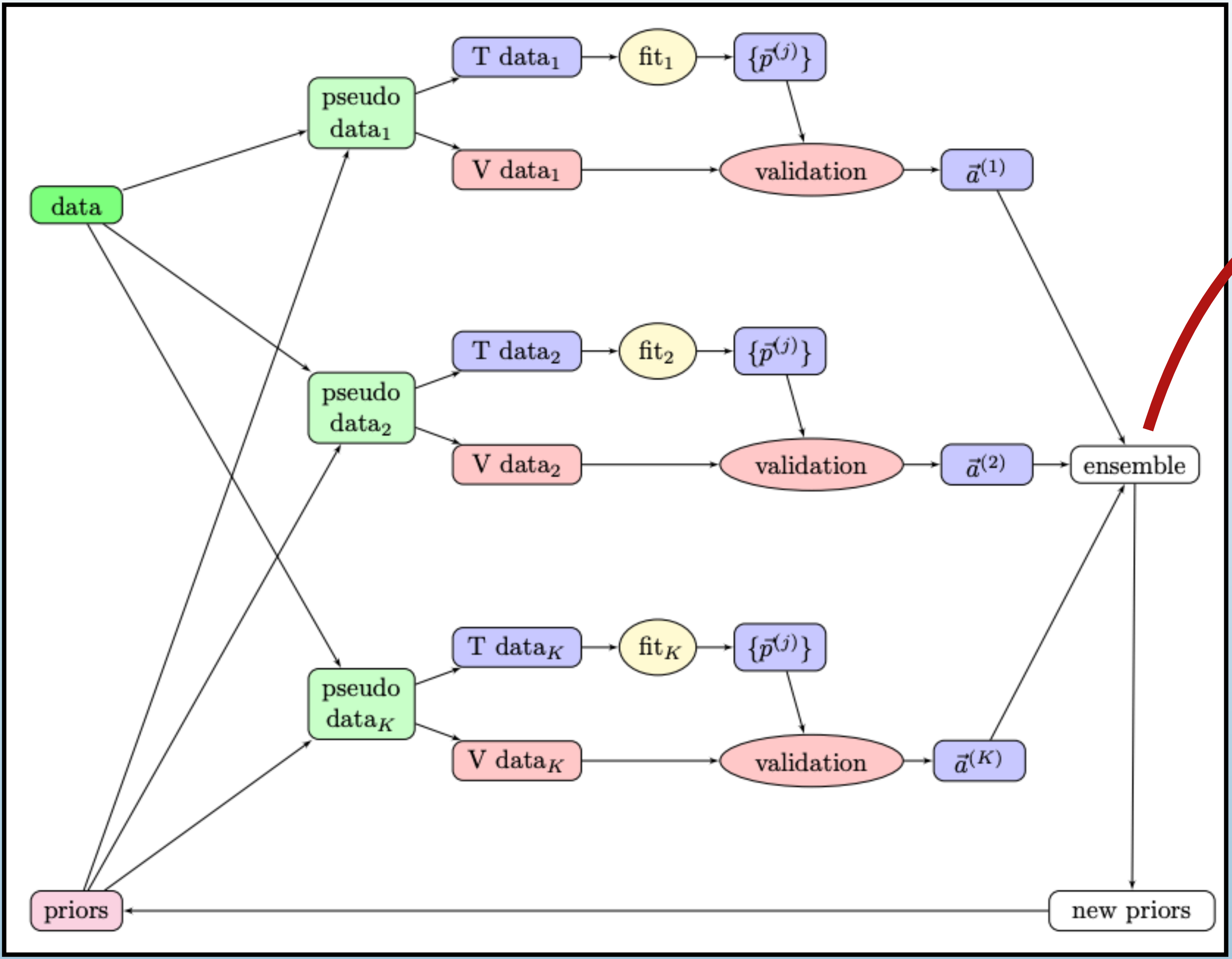


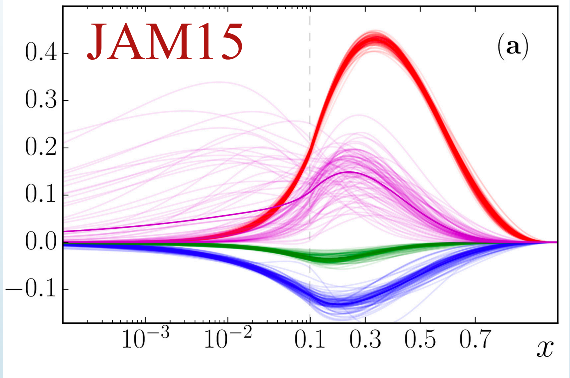


+

$$E[O] \approx \frac{1}{N}\sum_k O(\boldsymbol{a}_k)$$

$$V[O] \approx \frac{1}{N}\sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$





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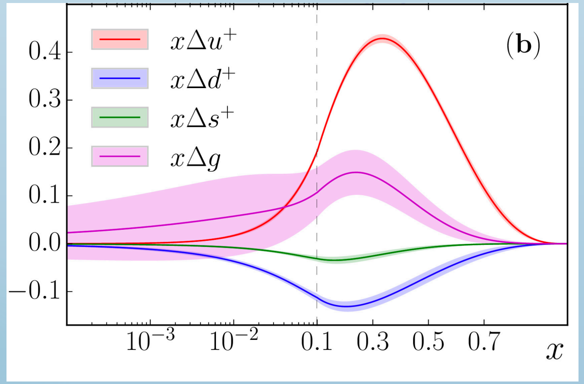


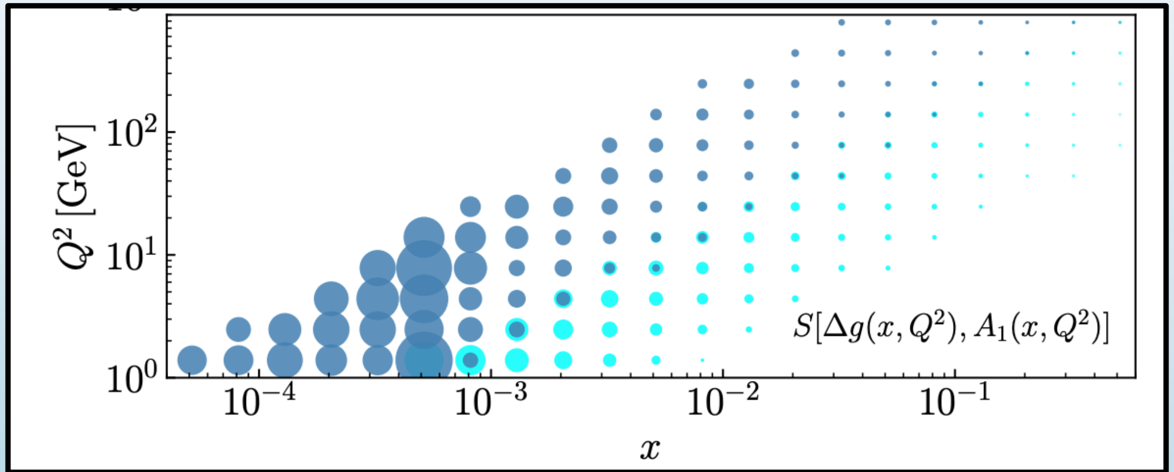
Future Experiments

Revisiting helicity parton distributions at a future electron-ion collider #2

Ignacio Borsa (U. Buenos Aires), Gonzalo Lucero (U. Buenos Aires), Rodolfo Sassot (U. Buenos Aires), Elke C. Aschenauer (Brookhaven Natl. Lab.), Ana S. Nunes (Brookhaven Natl. Lab.) (Jul 16, 2020)

Published in: *Phys.Rev.D* 102 (2020) 9, 094018 • e-Print: 2007.08300 [hep-ph]

$$\vec{l} + \vec{N} \rightarrow l' + X$$



Sensitivity of A_1 to Δg

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Ignacio Borsa (U. Buenos Aires), Gonzalo Lucero (U. Buenos Aires), Rodolfo Sassot (U. Buenos Aires), Elke C. Aschenauer (Brookhaven Natl. Lab.), Ana S. Nunes (Brookhaven Natl. Lab.) (Jul 16, 2020)

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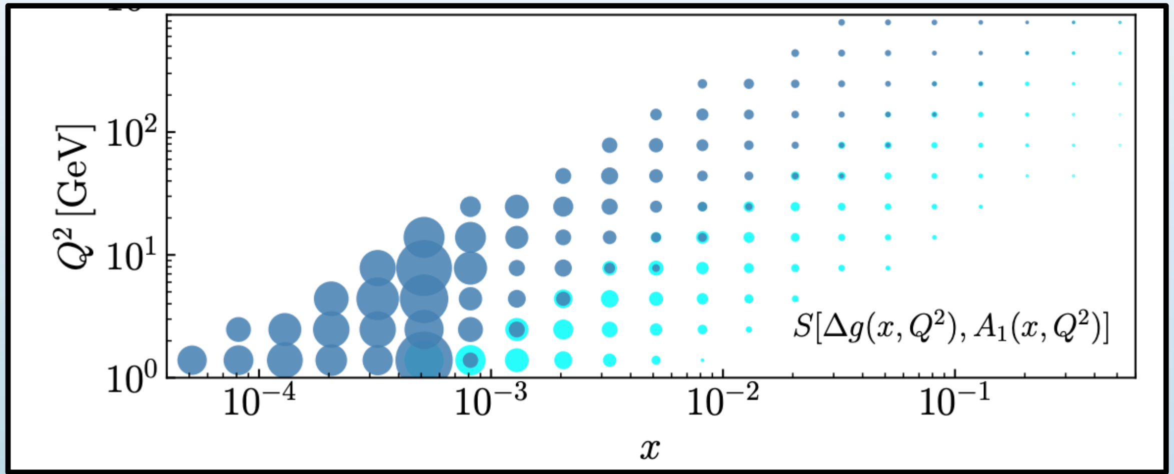
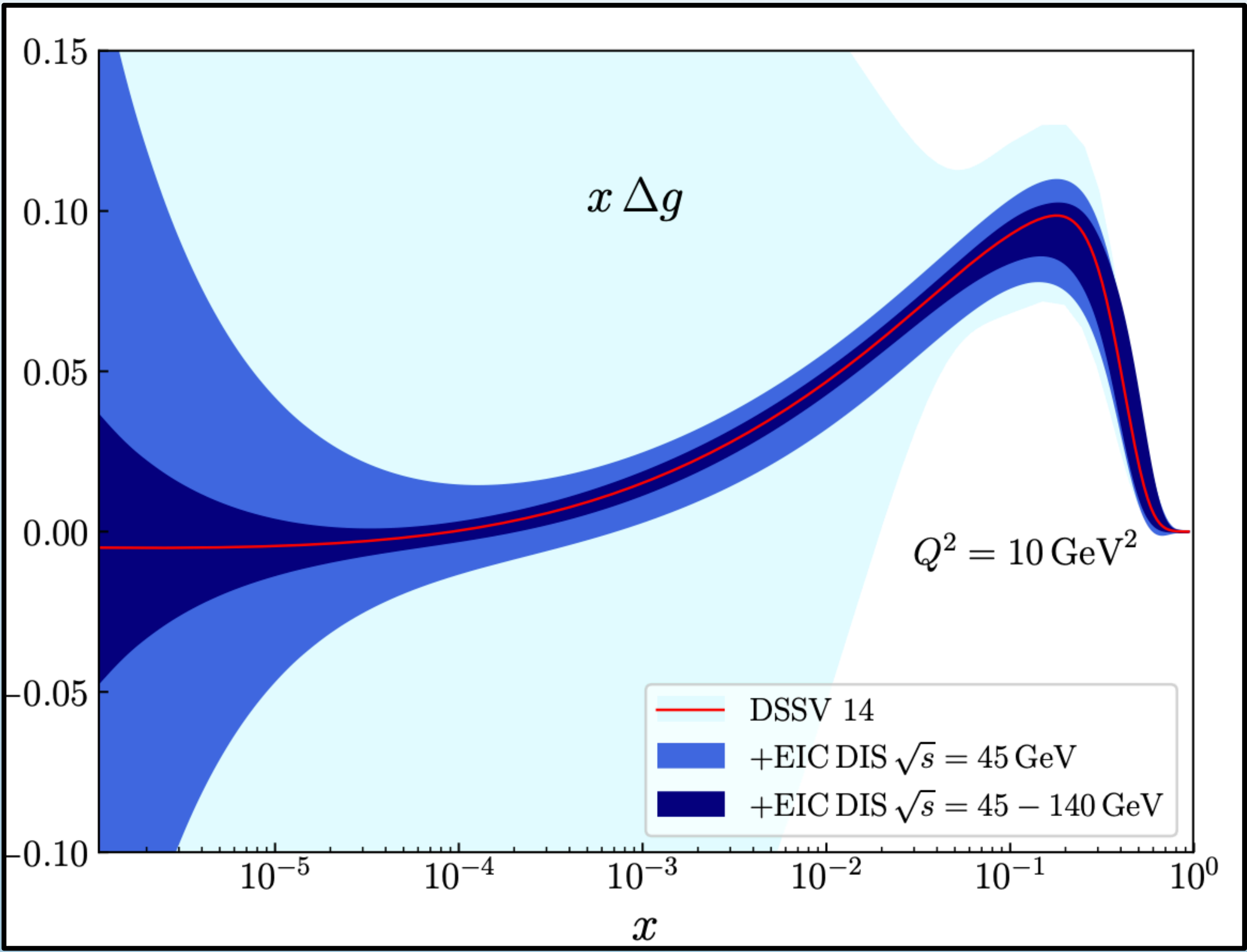


Sensitivity of A_1 to Δg



Large impact on Δg predicted, especially below $x \approx 0.01$