Diffractive Physics at the EIC

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Outline

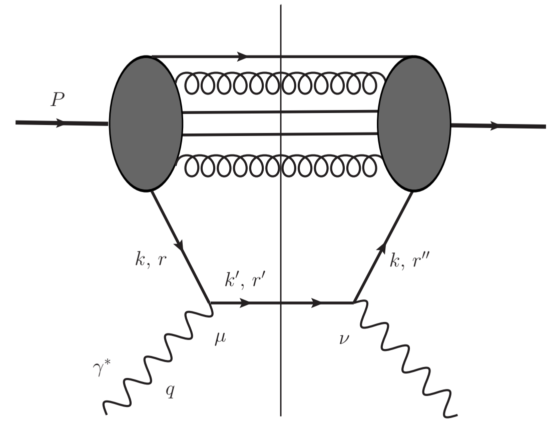
- DIS at small *x* (introduction to color dipoles as degrees of freedom).

- Diffraction at small *x*:
  - Low-mass diffraction and elastic vector meson production.
  - Elastic vector meson production in UPCs.
  - High-mass diffraction: small-x evolution for diffractive scattering in DIS.

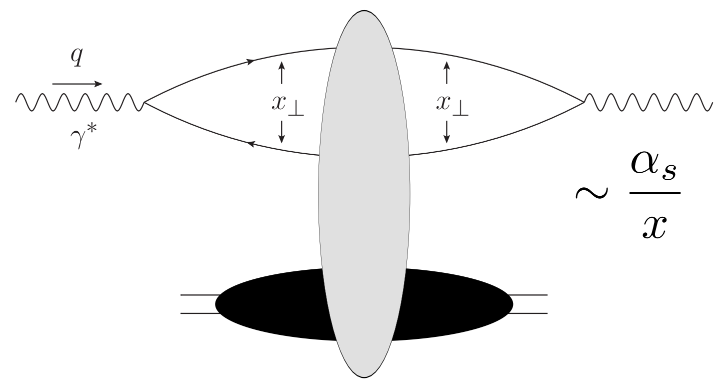
DIS at Small x

Dipole picture of DIS

- At small x, the dominant contribution to DIS structure functions does not come from the handbag diagram.



- Instead, the dominant term comes from the dipole picture of DIS, where the virtual photon splits into a quark-antiquark pair, which then interacts with the target.



$$\sim \frac{\alpha_s}{x}$$

Dipole picture of DIS

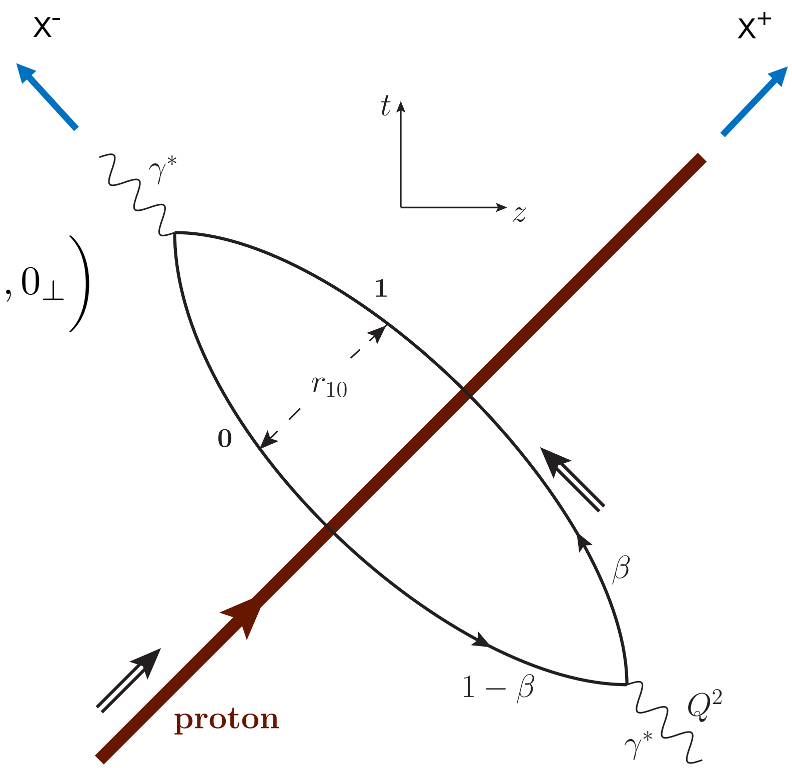
$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x\, e^{iq\cdot x} \langle P|j^\mu(x)\, j^\nu(0)|P\rangle$$

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp\right)$$

Large q^- → large x^- separation

$$e^{iq\cdot x} = e^{i\frac{Q^2}{2q^-}x^- + iq^- x^+}$$

$$x^\pm = \frac{t \pm z}{\sqrt{2}}$$

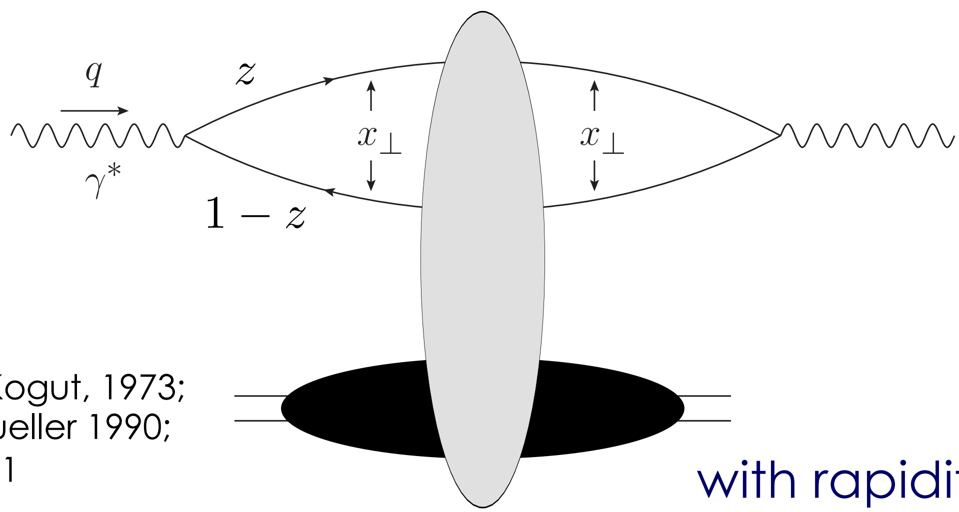


aka the "shock wave"

Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\pi} d^2 b_\perp \int_0^1 \frac{dz}{z(1-z)} \left|\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)\right|^2 N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



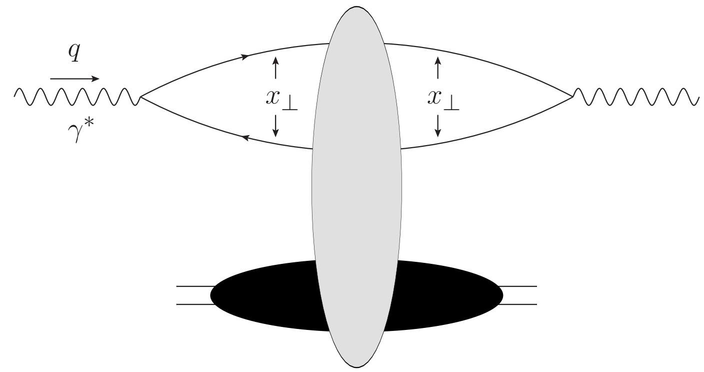
b is the Fourier conjugate to q with t = - q^2, making the dipole amplitude N similar to the GPDs at zero skewedness.

$b_\perp, \; Y$

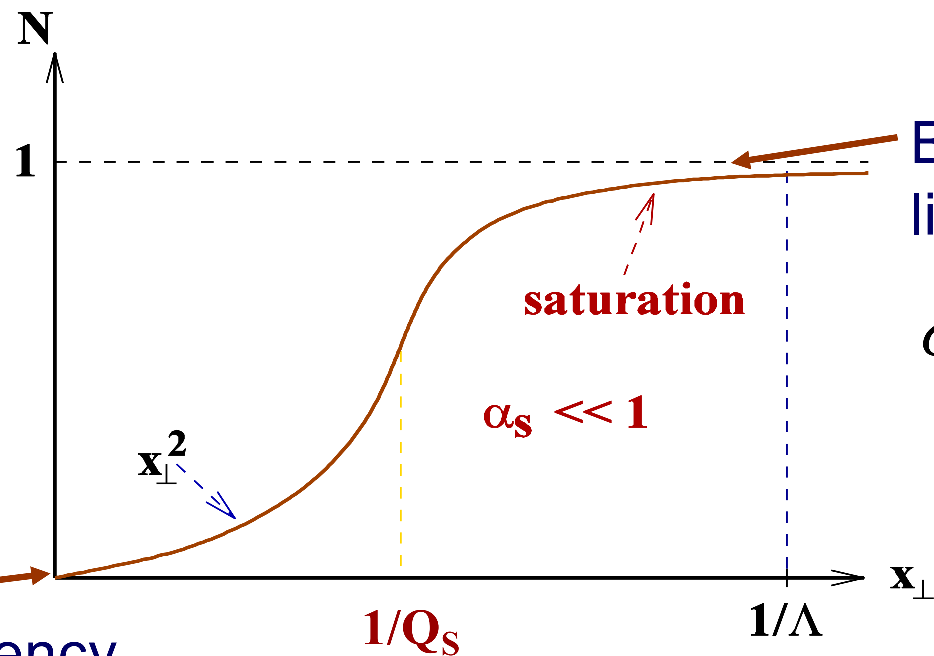
Gribov, 1970; Bjorken and Kogut, 1973; Frankfurt, Strikman 1988; Mueller 1990; Nikolaev and Zakharov 1991

with rapidity Y=ln(1/x)

Dipole Amplitude



The dipole-nucleus amplitude
as a function of the dipole size is



Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

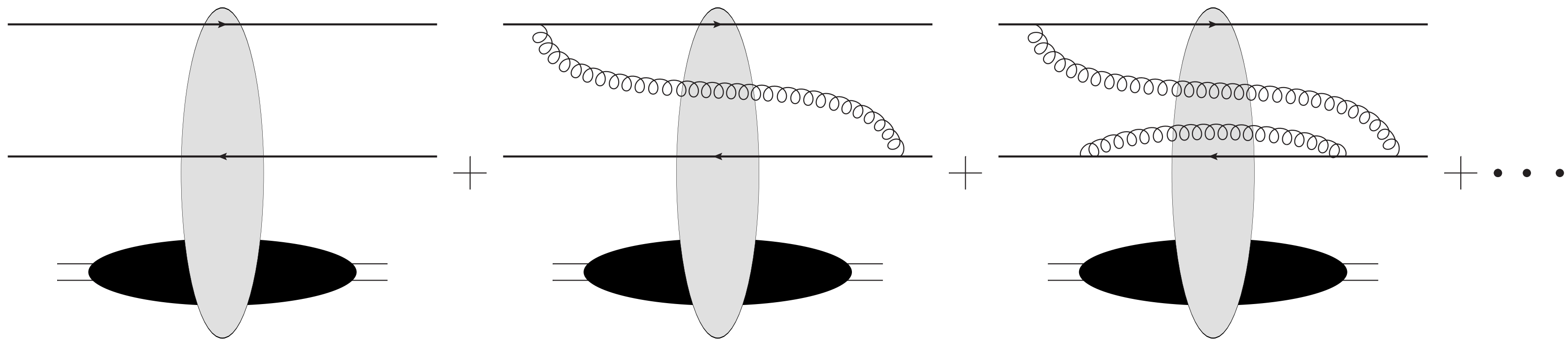
$$\sigma^{q\bar{q}A} = 2\int d^2b\, N(x_\perp, b_\perp, Y)$$

Color transparency

Small-x Evolution

- Energy dependence comes in through the long-lived *s*-channel gluon corrections (higher Fock states):

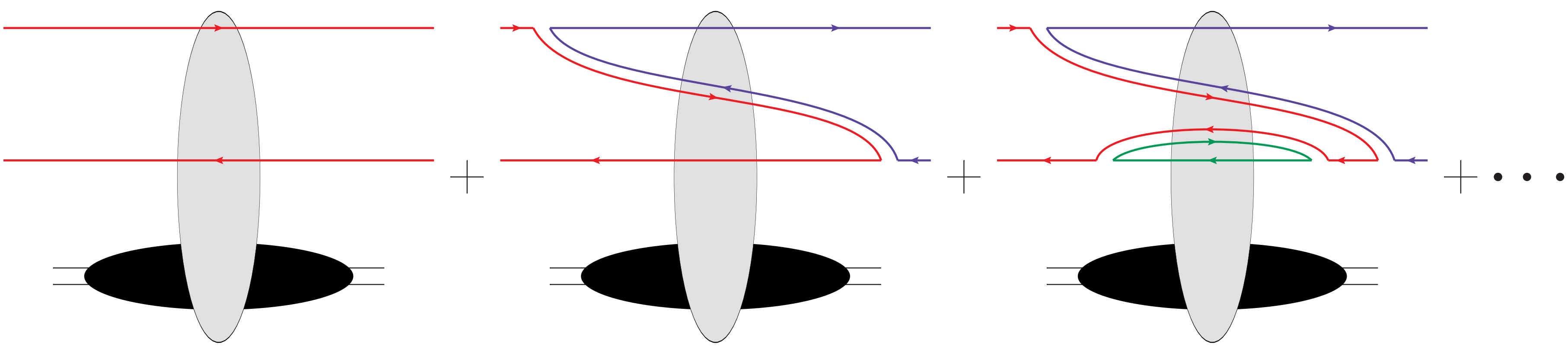
$$\alpha_s \ \ln s \sim \alpha_s \ \ln \frac{1}{x} \sim 1$$



These extra gluons bring in powers of α_S ln s, such that when α_S << 1 and ln s >>1 this parameter is α_S ln s ~ 1 (leading logarithmic approximation, LLA).

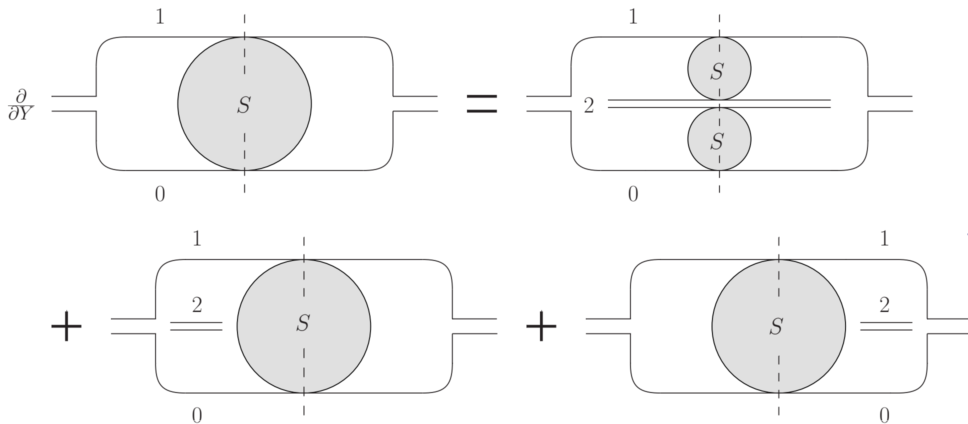
Small-x Evolution: Large N_c Limit

- How do we resum this cascade of gluons?
- The simplification comes from the large-Nc limit, where each gluon becomes a quark-antiquark pair: $3 \otimes \bar{3} = 1 \oplus 8 \quad \Rightarrow \quad N_c \otimes \bar{N}_c = 1 \oplus (N_c^2 - 1) \approx N_c^2 - 1$
- Gluon cascade becomes a dipole cascade (each color outlines a dipole):



Nonlinear Evolution

To sum up the gluon cascade at large-N_C we write the following equation for the dipole S-matrix:



dashed line = all interactions with the target

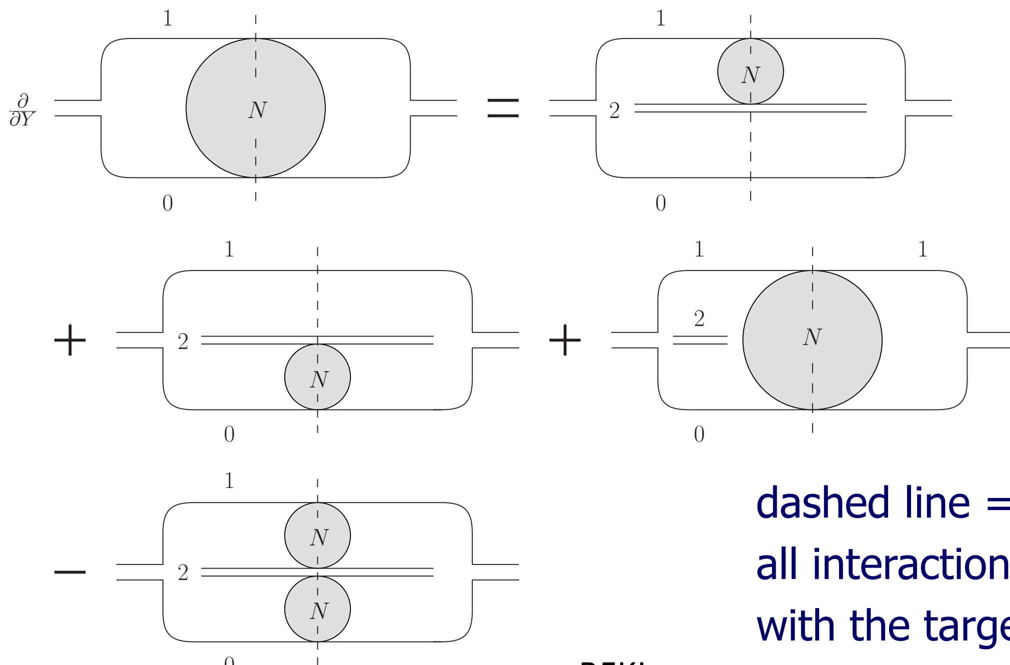
$$Y = \ln \frac{1}{x} \sim \ln s$$

$$\partial_Y S_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \left[S_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, S_{\mathbf{x}_2,\mathbf{x}_1}(Y) - S_{\mathbf{x}_0,\mathbf{x}_1}(Y)\right]$$

Remembering that S=1 + i T = 1 - N where N = Im(T) we can rewrite this equation in terms of the dipole scattering amplitude N.

Nonlinear evolution at large N_c

As N=1-S we write

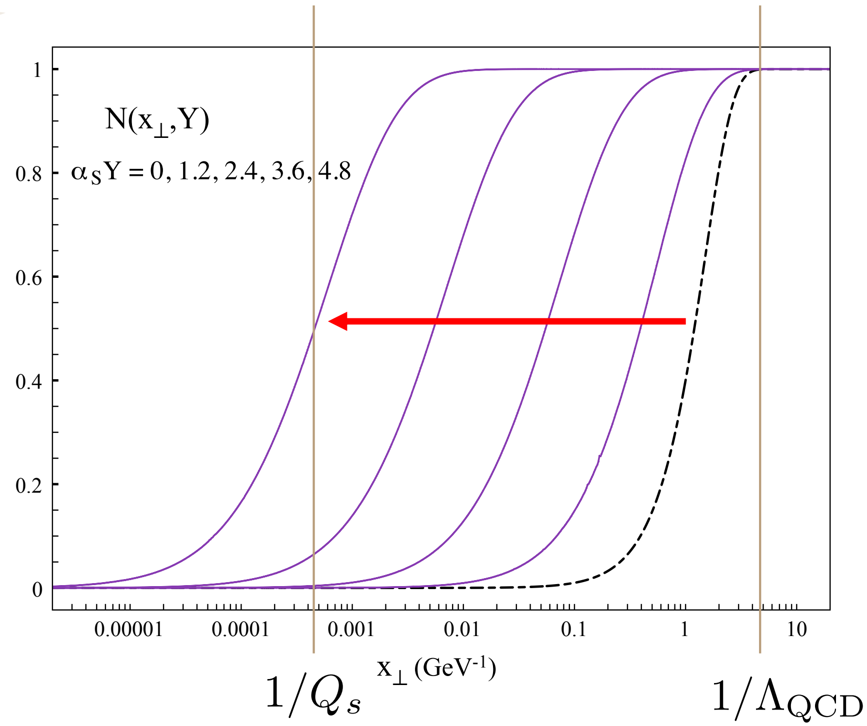


dashed line =
all interactions
with the target

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \, [\overbrace{N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y)}^{\text{BFKL}} - N_{\mathbf{x}_0,\mathbf{x}_2}(Y) \, N_{\mathbf{x}_2,\mathbf{x}_1}(Y)]$$

Balitsky '96, Yu.K. '99; beyond large N_c, JIMWLK evolution, 0.1% correction for the dipole amplitude

Solution of BK equation



numerical solution
by J. Albacete ‘03

We conclude that

$$Q_s^2 \sim \left(\frac{1}{x}\right)^{\lambda}$$

Energy increases → Q_s increases
moving further away from Λ_{QCD}

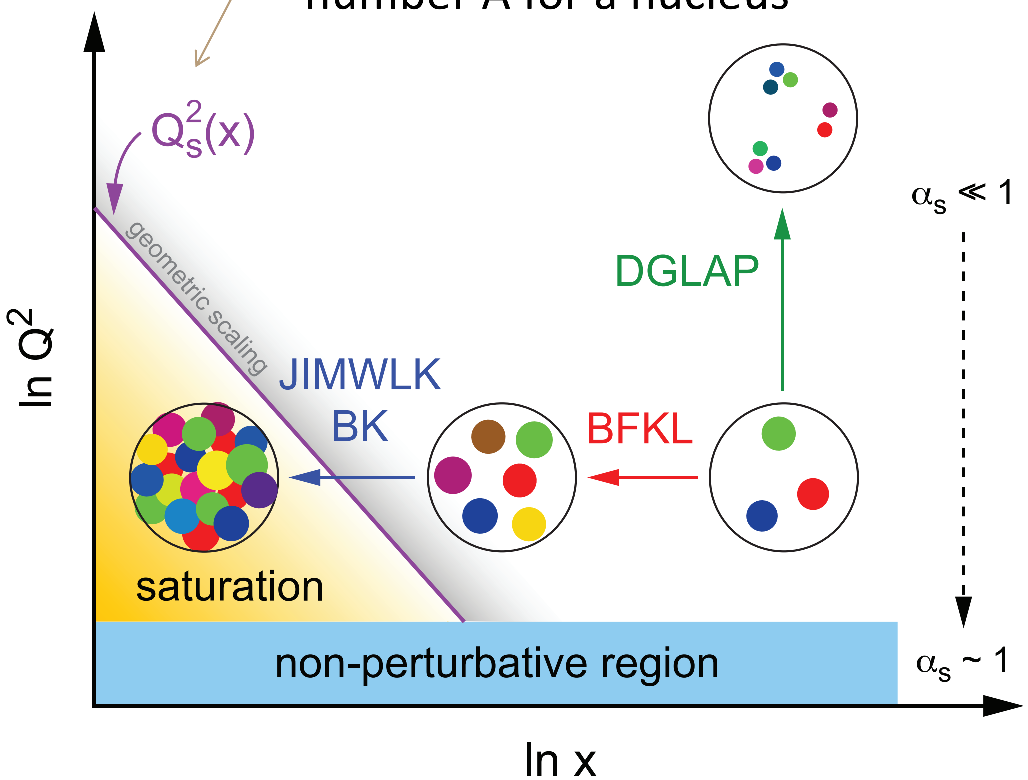
BK solution preserves the black disk limit, N<1 always
(unlike the linear BFKL equation)

$$\sigma^{q\bar{q}A} = 2 \int d^2b\, N(x_\perp, b_\perp, Y)$$

Map of High Energy QCD

Saturation Scale
grows with energy and atomic number A for a nucleus

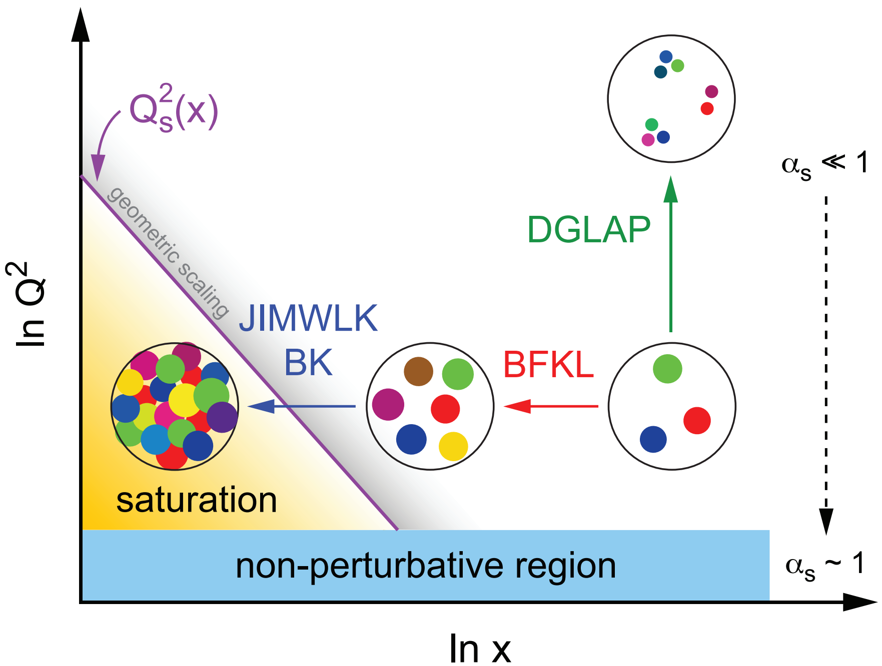
$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3}$$

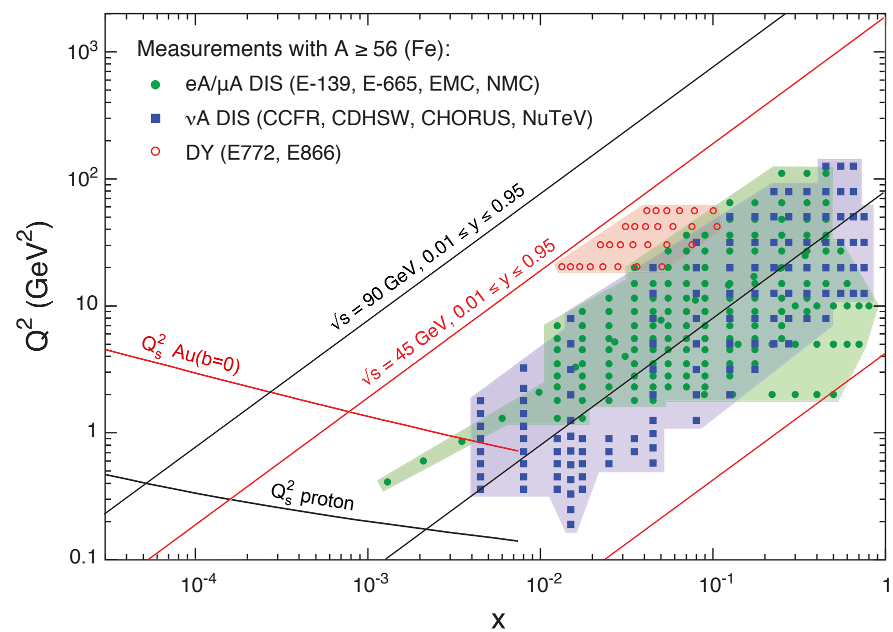


BFKL, DGLAP – linear equations
BK/JIMWLK – nonlinear

Can Saturation be Discovered at EIC?

EIC will have an unprecedented small-x reach for DIS on large nuclear targets, enabling decisive tests of saturation and non-linear evolution:

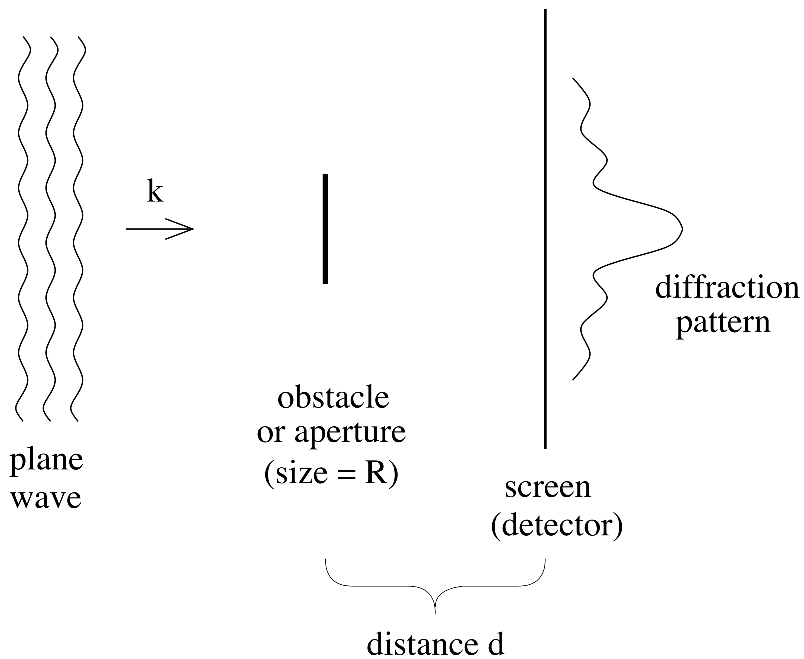




Plots from the EIC White Paper, '12, '14 (2nd ed).

Diffraction at Small x

Diffraction in optics

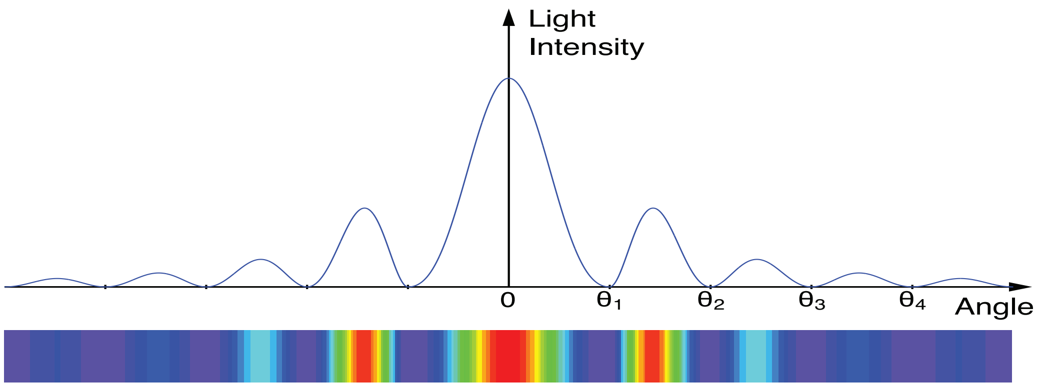


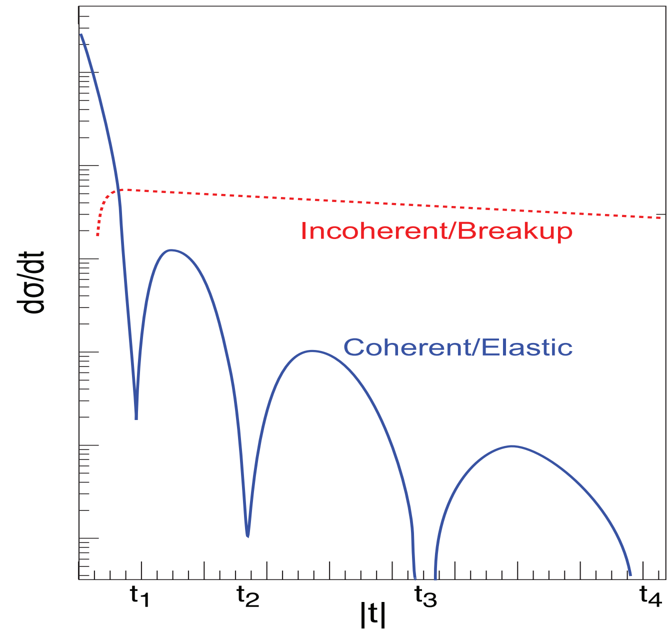
Diffraction pattern contains information about the size R of the obstacle and about the optical "blackness" of the obstacle.

In optics, diffraction pattern is studied as a function of the angle θ. In high energy scattering the diffractive cross sections are plotted as a function of the Mandelstam variable t with $\sqrt{|t|}$ = k sin θ.

Optical Analogy

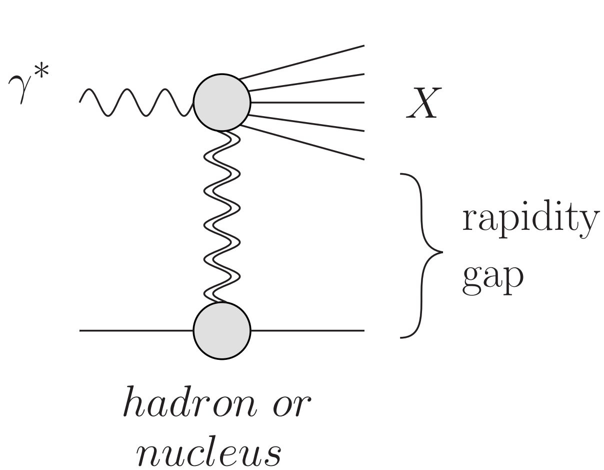
Diffraction in high energy scattering is not very different from diffraction in optics: both have diffractive maxima and minima:



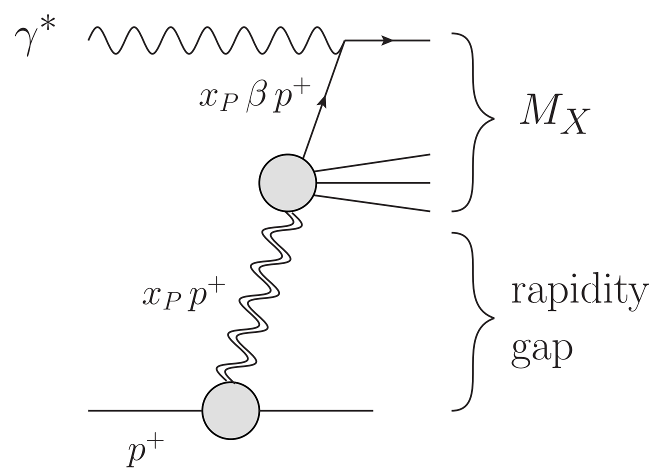


Coherent: target stays intact;
Incoherent: target nucleus breaks up, but nucleons are intact.

Diffraction terminology



(a)



(b)

W^2 = cms energy squared
for the photon+proton/nucleus
system

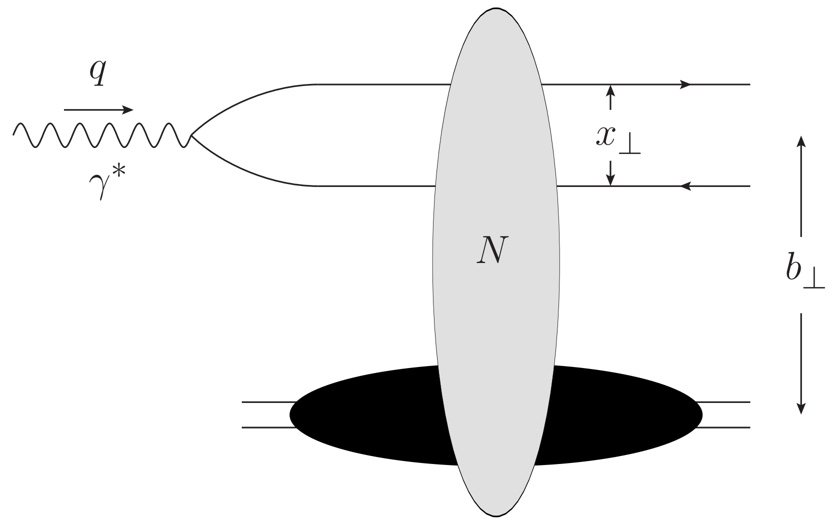
$$x_P = \frac{Q^2 + M_X^2}{Q^2 + W^2} \approx \frac{M_X^2}{W^2} \qquad \beta = \frac{x_{Bj}}{x_P} = \frac{Q^2}{Q^2 + M_X^2} \approx \frac{Q^2}{M_X^2}$$

Low-Mass Diffraction

M_X^2 is comparable to Q^2

Quasi-elastic DIS

Consider the case when nothing but the quark-antiquark pair is produced:



The quasi-elastic cross section is then proportional to the square of the dipole amplitude N:

$$\sigma_{el}^{\gamma^* A} = \int \frac{d^2 x_\perp}{4\,\pi}\, d^2 b_\perp \int\limits_0^1 \frac{dz}{z\,(1-z)}\, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \; N^2(\vec{x}_\perp, \vec{b}_\perp, Y)$$

Buchmuller et al '97, McLerran and Yu.K. '99

Diffraction on a black disk

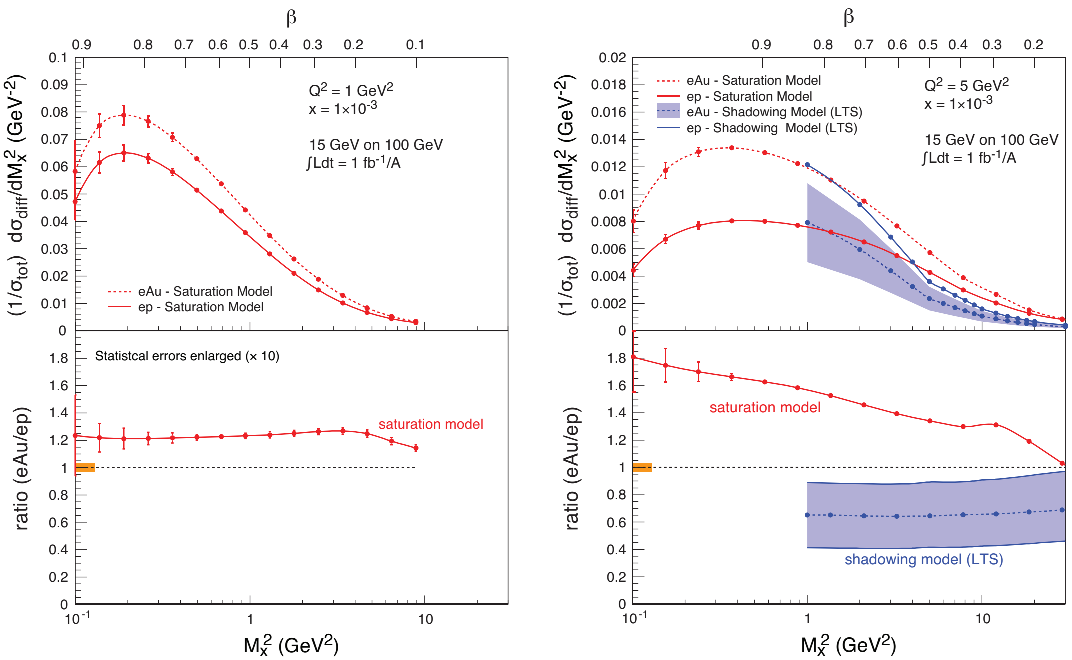
- For low Q^2 (large dipole sizes) the black disk limit is reached with N=1
- Diffraction (elastic scattering) becomes a half of the total cross section

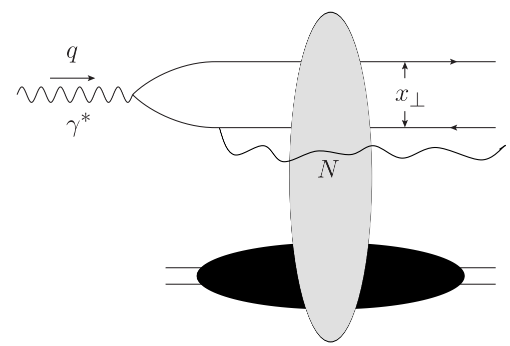
$$\frac{\sigma_{el}^{q\bar{q}A}}{\sigma_{tot}^{q\bar{q}A}} = \frac{\int d^2b\, N^2}{2\,\int d^2b\, N} \longrightarrow \frac{1}{2}$$

- Large fraction of diffractive events in DIS is a signature of reaching the black disk limit!
- HERA: ~15% (unexpected!) ; EIC: ~25% expected from saturation

Diffractive over total cross sections

- Here's an EIC measurement which may **distinguish saturation from non-saturation** approaches (from the 2012 EIC White Paper):

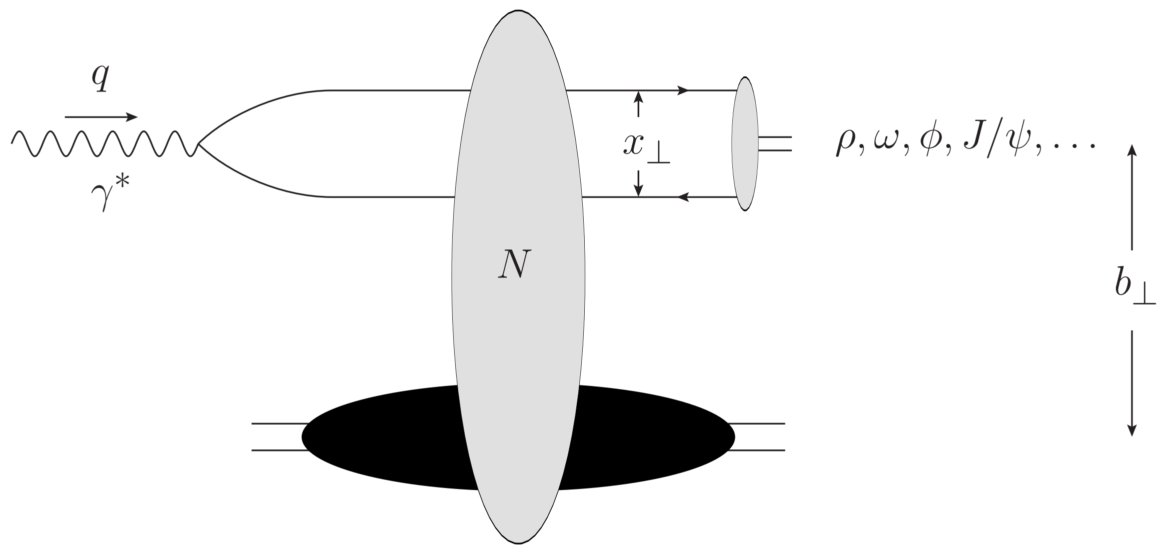




sat = Kowalski et al '08, plots generated by Marquet
no-sat = Leading Twist Shadowing (LTS), Frankfurt, Guzey, Strikman '04, plots by Guzey

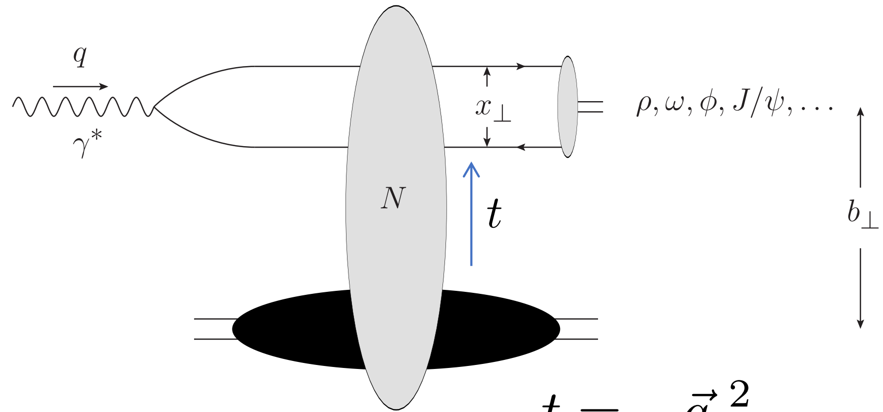
Exclusive Vector Meson Production

- An important diffractive process which can be measured at EIC is exclusive vector meson production:



Exclusive VM Production:
Probe of Spatial Gluon Distribution

- Differential exclusive VM production cross section is



$$\frac{d\sigma^{\gamma^* + A \to V + A}}{dt} = \frac{1}{4\pi} \left| \int d^2 b \, e^{-i \, \vec{q}_\perp \cdot \vec{b}_\perp} \, T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) \right|^2$$

$$t = -\vec{q}_\perp^{\,2}$$

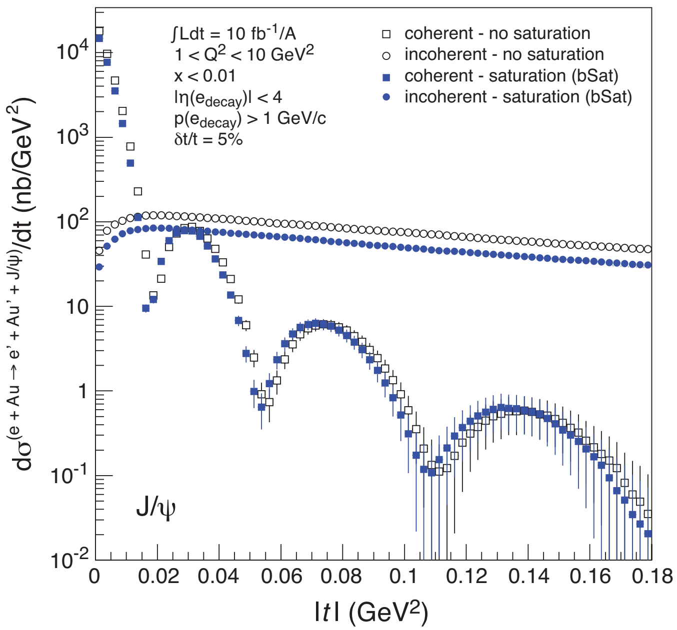
- the T-matrix is related to the dipole amplitude N:

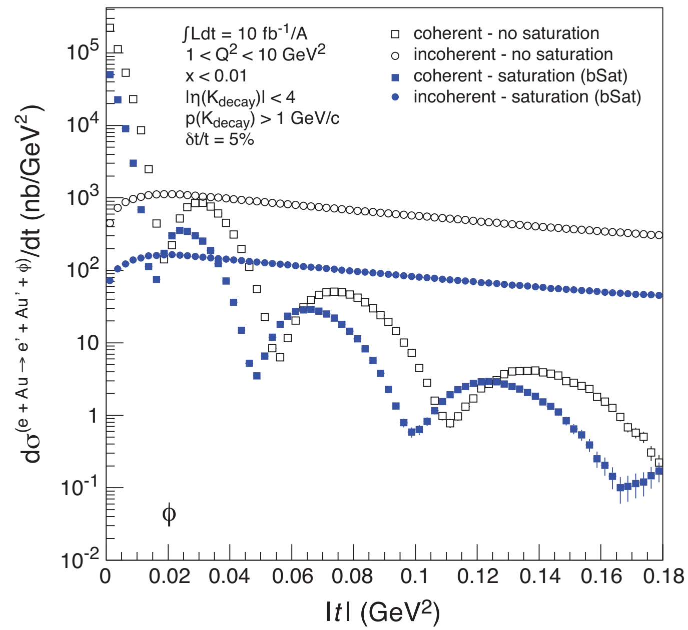
$$T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) = i \int \frac{d^2 x_\perp}{4\,\pi} \int_0^1 \frac{dz}{z\,(1-z)} \; \Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z) \; N(\vec{x}_\perp, \vec{b}_\perp, Y) \, \Psi^V(\vec{x}_\perp, z)^*$$

Brodsky et al '94, Ryskin '93

- Can study t-dependence of the dσ/dt and look at different mesons **to find the dipole amplitude N(x,b,Y)** (Munier, Stasto, Mueller '01).
- Learn about the **gluon distribution in space**. This is similar to GPDs.

Exclusive VM Production as a Probe of Saturation





Plots by T. Toll and T. Ullrich using the Sartre event generator
(b-Sat (=GBW+b-dep+DGLAP) + WS + MC, from the 2012 EIC White Paper).

- J/psi is smaller, less sensitive to saturation effects
- Phi meson is larger, more sensitive to saturation effects

Connection to UPCs at RHIC and LHC

Ultra-peripheral collisions

- Consider elastic vector meson production in ultra-peripheral A+A collisions at RHIC and LHC.

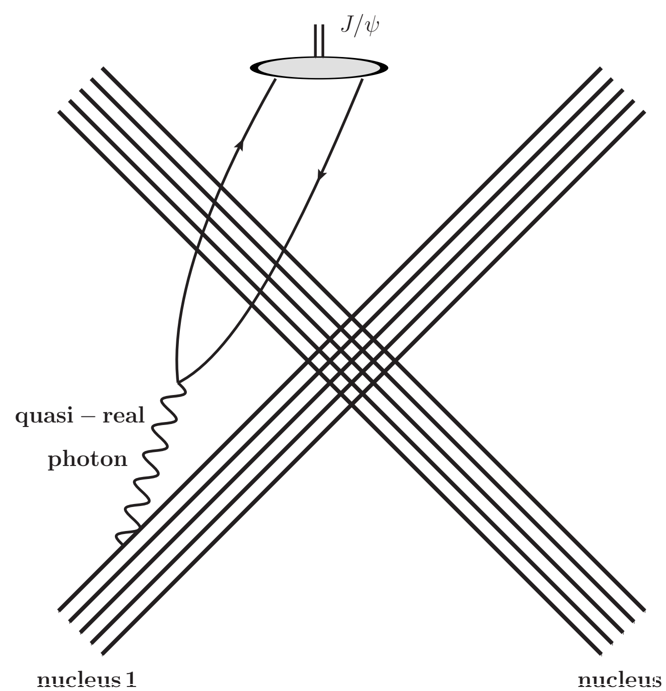
- The dipole structure of the interaction is clear from the diagram:

- Again, the cross section depends on the dipole amplitude!

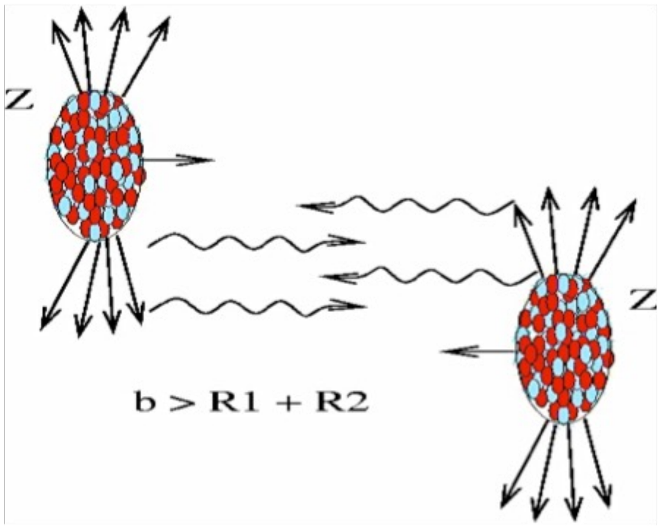
$$\frac{d\sigma}{dt} \sim \left| \int d^2 b_\perp e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \ \Psi_V \otimes N(\vec{x}_\perp, \vec{b}_\perp, Y) \otimes \Psi_\gamma \right|^2$$

$$t = -q_\perp^2$$

$$Q^2 \approx 0$$



Ultra peripheral collision (Ap or AA)



Fast moving highly-charged ions carry strong electromagnetic fields that act as a beam of photons.

$Q^2 \approx 20\ \mathrm{MeV}^2$ small virtuality

We define R_1, the ratio of elastic vector meson production cross section to the inelastic cross section

$$R_1 = \frac{\sigma^{\gamma^* A \to VA}}{\frac{d\sigma_{inel}}{d^2 p_T}}$$

and R_2, the double ratio between pA scattering and AA scattering,

$$R_2 = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)}$$

Huachen (Brian) Sun,
Zhoudunming Tu,
YK, in preparation

Double ratio for Au+p and Au+Au for J/ψ production

Inelastic quark production

$$\frac{d\sigma}{d^2p_T} \sim A \text{ for } p_T > Q_s; \qquad \frac{d\sigma}{d^2p_T} \sim A^{2/3} \text{ for } p_T < Q_s$$

Elastic J/ψ production (assuming no nuclear effects):

$$\sigma_{el}^{J/\psi} \propto A^{4/3}$$

Elastic ρ production (approximately):

$$\sigma_{el}^{\rho} \propto A^{2/3}$$

- $R_1(J/\psi) = \frac{\sigma^{\gamma^* A \to J/\psi A}}{\frac{d\sigma_{inel}}{d^2p_T}} = \begin{cases} f_1(p_T)A^{\frac{1}{3}} & , \quad p_T > Q_s \\ f_2(p_T)A^{\frac{2}{3}} & , \quad p_T < Q_s \end{cases}$

- $R_1(\rho) = \begin{cases} f_1(p_T)A^{-\frac{1}{3}} & , \quad p_T > Q_s \\ f_2(p_T)A^0 & , \quad p_T < Q_s \end{cases}$

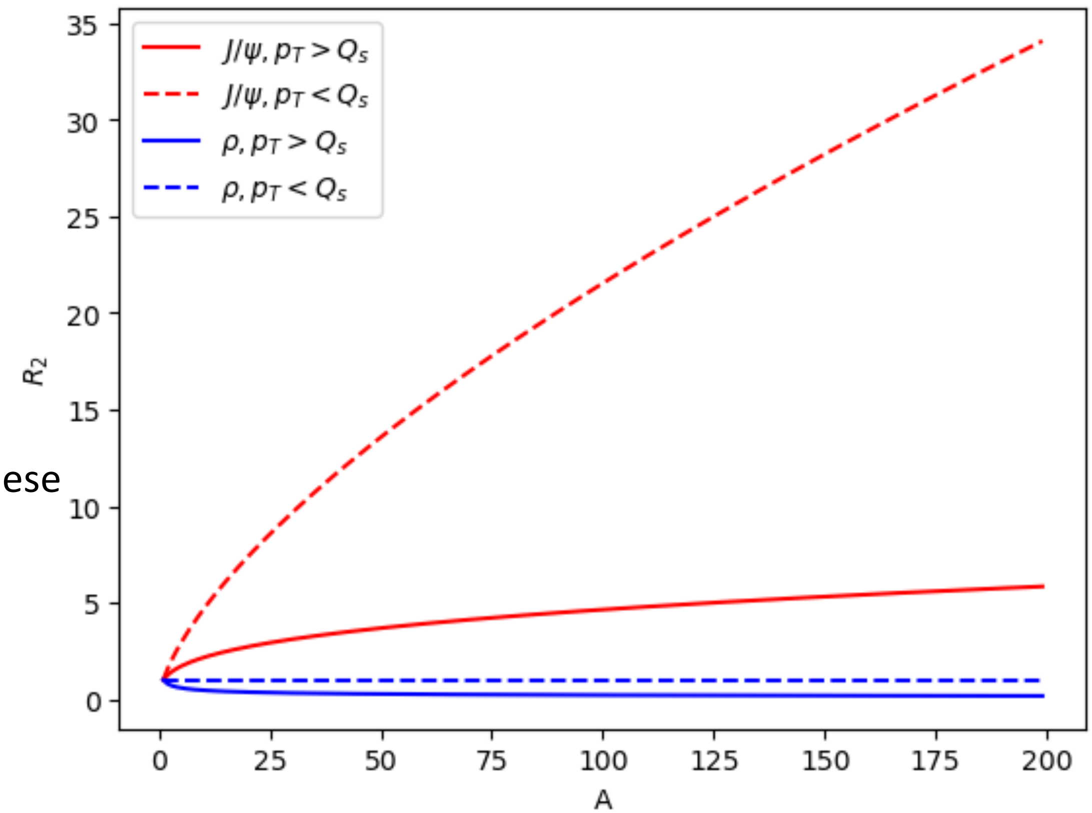
- $R_2(J/\psi) = \frac{R_1(\gamma^* A)}{R_1(\gamma^* p)} = \begin{cases} A^{\frac{1}{3}} \approx 6 & , \quad p_T > Q_s \\ A^{\frac{2}{3}} \approx 34 & , \quad p_T < Q_s \end{cases}$

- $R_2(\rho) = \begin{cases} A^{-\frac{1}{3}} \approx 0.17 & , \quad p_T > Q_s \\ A^0 \approx 0 & , \quad p_T < Q_s \end{cases}$

Double ratio for Au+p and Au+Au for ρ meson production

- Possible signal of saturation? (at Q^2=0)
- Note that the rho meson is nonperturbatively large, such that one can question our predictions for UPCs in pA.
- At EIC we would be able to study these processes at large Q^2, in the perturbative regime.

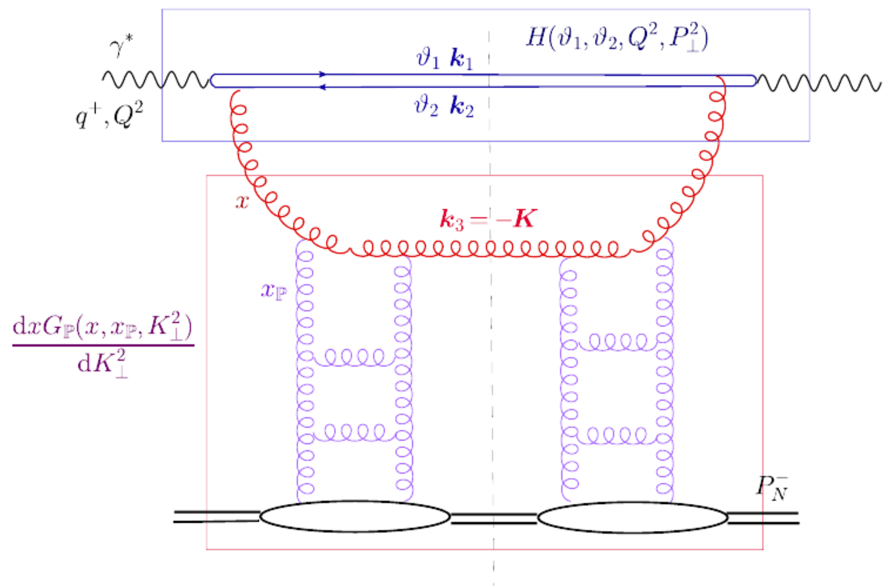
Huachen (Brian) Sun,
Zhoudunming Tu,
YK, in preparation



TMD factorisation for diffractive 2+1 jets

(E.I., A.H. Mueller, D.N. Triantafyllopoulos, Phys.Rev.Lett. 128 (2022) 20)

- At high $P_\perp \gg Q_s$, collinear factorisation emerges from the dipole picture
  - the gluon can alternatively be seen as a part of the Pomeron
  - essential condition: the gluon is relatively soft $\vartheta_3 \sim \frac{Q_s^2}{Q^2} \ll 1$



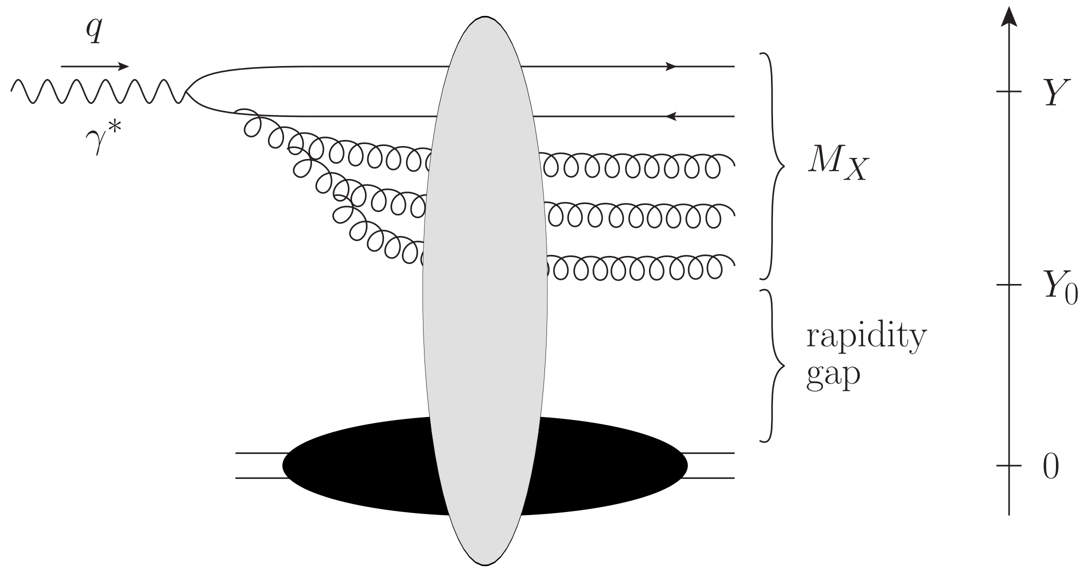
3-jet production in DIS at EIC and in UPCs.

- Actually: the "unintegrated" (TMD) version of collinear factorisation

High-Mass Diffraction

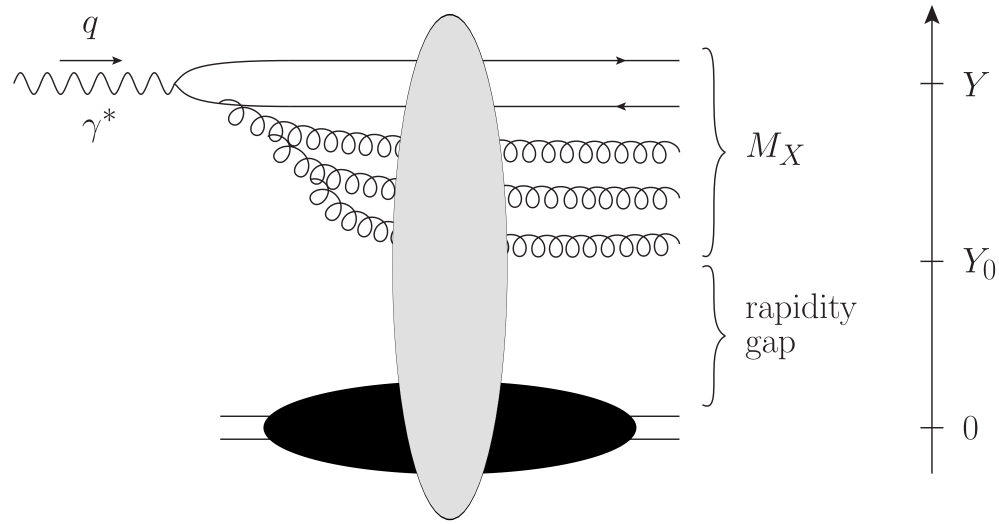
$M_X^2 >> Q^2$

What if M_X is large?



- If $\ln(M_X/Q)$ is large, need an evolution equation that re-sums all those gluon emissions contributing to M_X.
- One can ask questions like whether large M_X or small M_X are more probable?
- Also, what happens in the black disk limit? Theoretically only completely elastic and inelastic (no rapidity gaps) processes should remain, with 50% probability each.

Diffraction with fixed M_X



- Consider the differential cross section for diffractive dissociation with the fixed mass M_X.

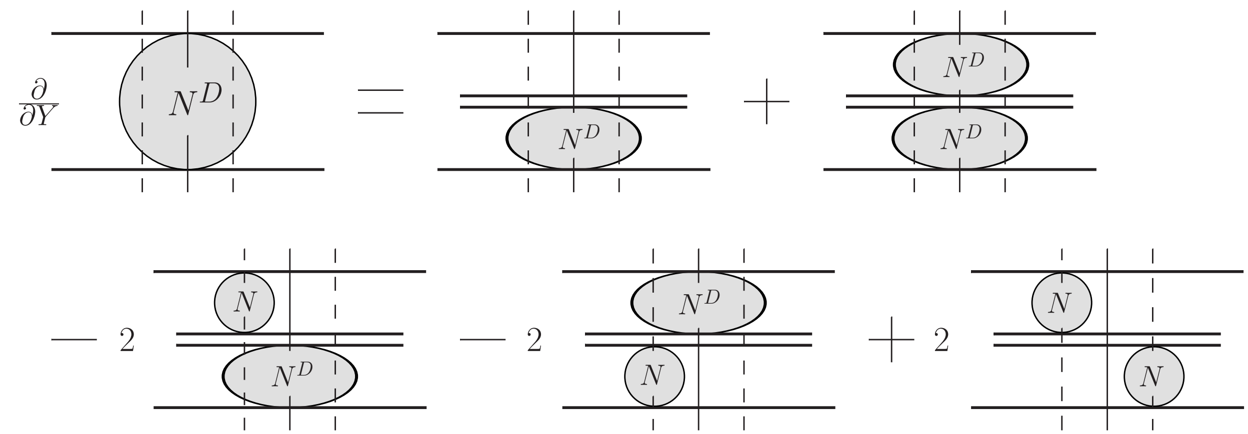
- It can be written as

$$M_X^2 \, \frac{d\sigma_{diff}^{\gamma^* A}}{dM_X^2} = -\int \frac{d^2 x_\perp}{4\,\pi} \, d^2 b_\perp \int_0^1 \frac{dz}{z\,(1-z)} \, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \, \frac{\partial S^D(\vec{x}_\perp, \vec{b}_\perp, Y, Y_0)}{\partial Y_0}$$

where S_D is the (exclusive) dipole S-matrix with the rapidity gap greater than Y_0.

Evolution equation for diffraction

$$S^D(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y, Y_0) = 1 - 2\,N(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y) + N^D(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y, Y_0)$$



- The "diffractive S-matrix" obeys the following evolution equation (YK, E. Levin, 2000):

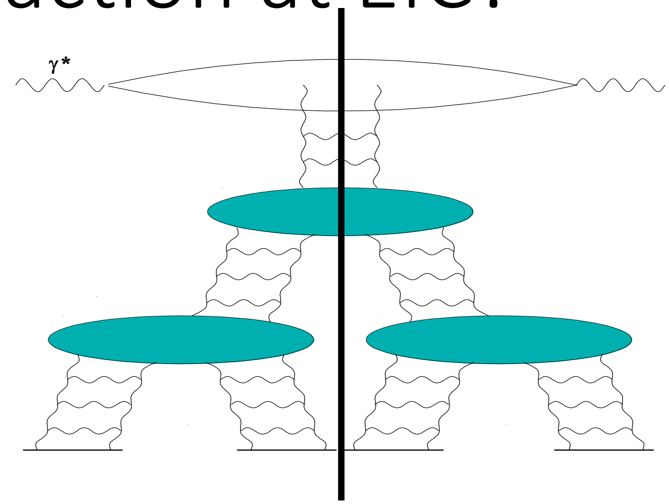
$$\partial_Y S^D(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y, Y_0) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \, \frac{x_{10}^2}{x_{20}^2 \, x_{21}^2} \left[S^D(\vec{x}_{1\perp}, \vec{x}_{2\perp}, Y, Y_0)\, S^D(\vec{x}_{2\perp}, \vec{x}_{0\perp}, Y, Y_0) - S^D(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y, Y_0) \right]$$

- The initial condition is given by

$$S^D(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y = Y_0, Y_0) = S^2(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y_0) = [1 - N(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y_0)]^2$$

with the dipole amplitude N found from the standard nonlinear dipole evolution.

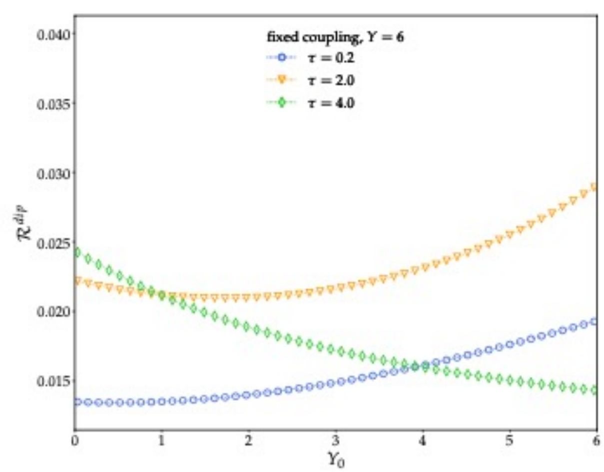
Can we measure high-mass diffraction at EIC?



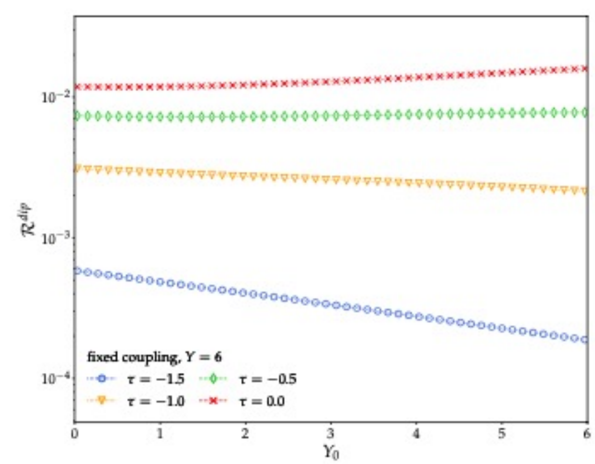
- Need large rapidity gap and large mass M_X. This is tough at the EIC.
- Perhaps A-dependence may help reveal evolution signatures?
- Dependence on M_X may also signal effects of nonlinear small-x evolution and saturation
  (see A.D. Le, A. H. Mueller, S. Munier, 2021, 2103.10088 [hep-ph]; A.D. Le, 2103.07724 [hep-ph]).

$$\tau = \ln \frac{2}{r_\perp Q_s(Y)}$$

outside saturation region:



inside saturation region:



Conclusions

- Low-mass diffraction: saturation signals may be found by studying the diffractive/total ratio and the eAu/ep double ratio versus M_X.
- Exclusive VM production: can find a saturation signal by studying $d\sigma/dt$ vs $|t|$.
- UPCs: A-dependence of the elastic VM production may tell us about nuclear effects and, possibly, saturation. (Is it perturbative?)
- High-mass diffraction: interesting, but can we do it at EIC? If yes, how? Studying the detailed structure of the final state with a rapidity gap may help us better understand and detect the onset of saturation.

Backup Slides

Black Disk Limit

- Start with basic scattering theory: the final and initial states are related by the S-matrix operator,

$$|\psi_f\rangle = \hat{S}\,|\psi_i\rangle$$

- Write it as
$$|\psi_f\rangle = |\psi_i\rangle + \left[\hat{S} - 1\right]\,|\psi_i\rangle$$

- The total cross section is

$$\sigma_{tot} \propto \left|\left[\hat{S} - 1\right]\,|\psi_i\rangle\right|^2 = 2 - S - S^*$$

  where the forward matrix element of the S-matrix operator is

$$S = \langle\psi_i|\,\hat{S}\,|\psi_i\rangle$$

  and we have used unitarity of the S-matrix

$$\hat{S}\,\hat{S}^\dagger = 1$$

Black Disk Limit

- Now, since $$|\psi_f\rangle = |\psi_i\rangle + \left[\hat{S} - 1\right]\, |\psi_i\rangle$$

  the elastic cross section is

$$\sigma_{el} \propto \left|\langle\psi_i|\, \left[\hat{S} - 1\right] |\psi_i\rangle\right|^2 = |1 - S|^2$$

- The inelastic cross section can be found via

$$\sigma_{tot} = \sigma_{inel} + \sigma_{el}$$

- In the end, for scattering with impact parameter b we write

$$\sigma_{tot} = 2 \int d^2b\, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2b\, |1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2b \left[1 - |S(b)|^2\right]$$

Unitarity Limit

- Unitarity implies that $$1 = \langle \psi_i | \, \hat{S} \, \hat{S}^\dagger \, | \psi_i \rangle = \sum_X \langle \psi_i | \, \hat{S} | X \rangle \, \langle X | \hat{S}^\dagger \, | \psi_i \rangle \geq |S|^2$$
- Therefore $|S| \leq 1$

  leading to the unitarity bound on the total cross section

$$\sigma_{tot} = 2 \int d^2 b \; [1 - \mathrm{Re}\, S(b)] \leq 4 \int d^2 b = 4\pi R^2$$

- Notice that when S=-1 the inelastic cross section is zero and

$$\sigma_{tot} = 4\pi R^2 = \sigma_{el}$$

$$\sigma_{tot} = 2 \int d^2 b \, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2 b \, |1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2 b \left[1 - |S(b)|^2 \right]$$

This limit is realized in low-energy scattering!

Black Disk Limit

- At high energy inelastic processes dominate over elastic. Imposing

$$\sigma_{inel} \geq \sigma_{el}$$

we get

$$\mathrm{Re}\, S \geq 0$$

- The bound on the total cross section is (aka the **black disk limit**)

$$\sigma_{tot} = 2 \int d^2b\, [1 - \mathrm{Re}\, S] \leq 2 \int d^2b = 2\pi R^2$$

- The inelastic and elastic cross sections at the black disk limit are

$$\sigma_{inel} = \sigma_{el} = \pi R^2$$

$$\sigma_{tot} = 2 \int d^2b\, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2b\, |1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2b \left[1 - |S(b)|^2\right]$$

Notation

- At high energies

$$\mathrm{Im}\, S \approx 0$$

while the dipole amplitude N is the imaginary part of the T-matrix (S=1+iT), such that

$$\mathrm{Re}\, S = 1 - N$$

- The cross sections are

$$\sigma_{tot} = 2 \int d^2b\, N(x_\perp, b_\perp)$$

$$\sigma_{el} = \int d^2b\, N^2(x_\perp, b_\perp)$$

$$\sigma_{inel} = \int d^2b \left[2\, N(x_\perp, b_\perp) - N^2(x_\perp, b_\perp)\right]$$

- We see that N=1 is the black disk limit. Hence $N \leq 1$.