Aspects of higher dimensional quantum Hall effect: Effective actions, entanglement entropy

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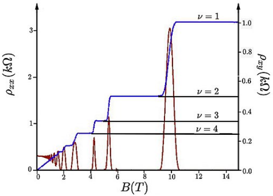
June 15, 2023

- (2+1) dim systems of (nonrelativistic) electrons in strong magnetic field
- Hall conductivity is quantized



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\sigma_H &= \frac{\nu}{2\pi} \frac{e^2}{\hbar}
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$\nu = 1, 2, \cdots$ for IQHE and $\nu = 1/3, 1/5, \cdots$ for FQHE.

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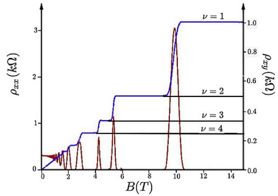

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- Framework for interesting ideas
  - topological field theories (Chern-Simons effective actions)
  - bulk-edge dynamics
  - non-commutative geometries, fuzzy spaces

Charged particle moving on 2d plane (or S^2) in strong external magnetic field (Landau problem)

- Landau levels, separated by energy gap ($\sim B$)
- Each Landau level is degenerate
- Lowest Landau level (LLL) :

$$\psi_n \sim z^n e^{-|z|^2/2}$$

$$z = x + iy$$

Many-body problem $\Longrightarrow$ quantum Hall droplets

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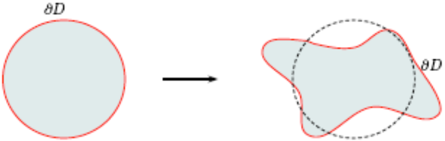

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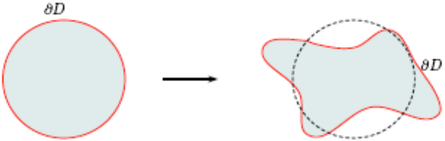

Edge dynamics is collectively described by 1d chiral boson ϕ (WEN, STONE,..)

$$S_{\text{edge}} = \int_{\partial D} \Big(\partial_t \phi + u\, \partial_\theta \phi\Big) \partial_\theta \phi, \qquad u \sim \frac{\partial V}{\partial r^2}\Big]_{\text{boundary}}$$

In the presence of electromagnetic fluctuations

- The bulk dynamics is described by an effective action

$$S_{\text{bulk}} = S_{CS} = \frac{\nu}{4\pi} \int_D \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

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- The bulk effective action S_{CS} captures the response of the system to electromagnetic fluctuations.

$$J^\mu = \frac{\delta S_{CS}}{\delta A_\mu} = \frac{\nu}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$

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$$S_{eff} = \frac{1}{4\pi} \int \left[[A + (s + \tfrac{1}{2})\omega]\, d[A + (s + \tfrac{1}{2})\omega] - \frac{1}{12}\omega d\omega \right] + \cdots$$

$$\omega = \text{spin connection} \qquad s = 0 \to LLL\ ,\ s = 1 \to \text{1st LL}, \cdots$$

$$T^{ij} = \frac{2}{\sqrt{g}} \frac{\delta S_{eff}}{\delta g_{ij}} = \frac{n_H}{2} (\epsilon^{il} \dot{g}^{lj} + \epsilon^{jl} \dot{g}^{li})$$

n_H = Hall viscosity

KLEVTSOV ET AL; BRADLYN, READ; CAN, LASKIN, WIEGMANN

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- Generalization to arbitrary even (spatial) dimensions

  QHE on $\mathbb{CP}^k$ (KARABALI AND NAIR, 2002...)

  - higher dimensionality
  - possibility of having both abelian and nonabelian magnetic fields

$\mathbb{CP}^k$: 2k dim space, locally parametrized by $z_i, i = 1, \cdots, k$

- Fubini-Study metric

$$ds^2 = \frac{dz \cdot d\bar{z}}{(1 + z \cdot \bar{z})} - \frac{\bar{z} \cdot dz \, z \cdot d\bar{z}}{(1 + z \cdot \bar{z})^2}$$

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- There are degenerate Landau levels, separated by energy gap.
- Each Landau level forms an irreducible $SU(k+1)$ representation, whose degeneracy and energy is easy to calculate.

- $\mathbb{CP}^k = SU(k+1)/U(k)$. We can use $(k+1)\times(k+1)$-matrix $g \in SU(k+1)$ as a coordinate, where
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- $\hat{R}_a,\ \hat{R}_{k^2+2k} \to$ gauge transformations $\quad(\,U(k)\,)$
- $\hat{R}_{+i}, \hat{R}_{-i} \to$ covariant derivatives $\quad(i = 1, \cdots, k)$
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  $[\hat{R}_{+i}, \hat{R}_{-j}] \sim f_{ija}\hat{R}_a, \quad a \in U(k)$
- How Ψ transforms under gauge transformations depends on choice of background fields

- Choose “uniform” $U(1)$ or $U(k)$ background magnetic fields.

$$U(1): \quad \bar{a} \sim in\mathrm{Tr}(t_{k^2+2k} g^{-1} dg) \quad \Rightarrow \quad \bar{F} = d\bar{a} = n\,\Omega, \quad \Omega = \text{Kahler } 2-\text{form}$$

$$SU(k): \quad \bar{A}^a \sim \mathrm{Tr}(t^a g^{-1} dg) \quad \Rightarrow \quad \bar{F}^a \sim \bar{R}^a \sim f^{aij} e^i \wedge e^j$$

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- Wavefunctions are written in terms of Wigner $\mathcal{D}$-functions

$$\Psi^J_{l,\alpha} \sim \mathcal{D}^J_{l,\alpha}(g) = \langle\, l \,|\, \hat{g} \,|\, \alpha \,\rangle$$

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$$\hat{R}^{k^2+2k}\, \Psi^J_{l,\alpha} = -\frac{nk}{\sqrt{2k(k+1)}}\, \Psi^J_{l,\alpha}, \qquad \hat{R}^a\, \Psi^J_{l,\alpha} = (T^a)_{\alpha\beta}\, \Psi^J_{l,\beta}$$

- Wavefunctions for each Landau level form an $SU(k+1)$ representation J

$$\Psi^J_{l;\alpha} \sim \langle l \mid \hat{g} \mid \underbrace{\alpha}\ \rangle$$

fixed $U(1)_R$ charge $\sim$ n and some finite $SU(k)_R$ repr. $\tilde{J}$

$l = 1, \cdots \dim J \Longrightarrow$ counts degeneracy within a Landau level

$\alpha =$ internal index $= 1, \cdots, N' = \dim\tilde{J}$

- Hamiltonian

$$H = \frac{1}{4mr^2} \sum_{i=1}^{k} (\hat{R}_{+i}\hat{R}_{-i} + \hat{R}_{-i}\hat{R}_{+i})$$
$$= \frac{1}{2mr^2} [C_2^{SU(k+1)}(J) - C_2^{SU(k)}(\tilde{J}) - \frac{n^2 k}{2(k+1)}]$$

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- Lowest Landau level: $\hat{R}_{-i}\Psi = 0$ Holomorphicity condition

  ($|\,\alpha\,\rangle$ is lowest weight state)

For a $U(1)$ magnetic field the LLL wavefunctions can be written in terms of complex coordinates as

$$\begin{aligned}\Psi_{i_1 i_2 \cdots i_k} &= \sqrt{N}\left[\frac{n!}{i_1! i_2! \ldots i_k!(n-s)!}\right]^{\frac{1}{2}} \frac{z_1^{i_1} z_2^{i_2} \cdots z_k^{i_k}}{(1+\bar{z}\cdot z)^{\frac{n}{2}}} \;, \\ s &= i_1 + i_2 + \cdots + i_k \;, \quad 0 \leq i_i \leq n \;, \quad 0 \leq s \leq n\end{aligned}$$

They form an $SU(k+1)$ representation of dimension

$$N = \mathrm{dim} J = \frac{(n+k)!}{n!\, k!}$$

They are degenerate with energy

$$E = \frac{1}{2mr^2} \frac{nk}{2}$$

- QHE on a compact space $M \Longrightarrow$ LLL defines an N-dim Hilbert space

  In the presence of confining potential $\Longrightarrow$ incompressible QH droplet

- K states are filled, $N-K$ unoccupied

  Density matrix for ground state droplet : $\hat{\rho}_0$

$$\hat{\rho}_0 = \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & \ddots & & & & \\ & & & & 1 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & \ddots \\ & & & & & & & & 0 \end{bmatrix} \begin{array}{l} \updownarrow K \\ \updownarrow N-K \end{array}$$

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- Under time evolution: $\hat{\rho}_0 \to \hat{\rho} = \hat{U}\,\hat{\rho}_0\,\hat{U}^\dagger$

  $\hat{U} = N \times N$ unitary matrix ; "collective" variable describing excitations within the LLL

The action for $\hat{U}$ is

$$S_0 = \int dt \, \mathrm{Tr} \left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \; - \; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} \right]$$

which leads to the evolution equation for density matrix

$$i \frac{d\hat{\rho}}{dt} = [\hat{V}, \hat{\rho}]$$

S_0 : universal matrix action

No explicit dependence on properties of space on which QHE is defined, abelian or nonabelian nature of fermions, etc.

S_0 : action of a noncommutative field theory

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$$= N \int d\mu\, dt\; \left[i(\rho_0 * U^\dagger * \partial_t U) \;-\; (\rho_0 * U^\dagger * V * U)\right]$$

$$\underbrace{\hat{\rho}_0, \hat{U}, \hat{V}}_{(N\times N)\text{ matrices}} \quad\Longrightarrow\quad \underbrace{\rho_0(\vec{x}), U(\vec{x}, t), V(\vec{x})}_{\text{symbols}}$$

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- symbol: $O(\vec{x}, t) = \frac{1}{N} \sum_{m,l} \Psi_m(\vec{x}) \hat{O}_{ml}(t) \Psi_l^*(\vec{x})$

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$(N \times N)$ matrices — symbols

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S_0 = exact bosonic action describing the dynamics of LLL fermions

SAKITA, 1993: 2 dim. context

DAS, DHAR, MANDAL, WADIA, 1992

Edge effective action for $\nu = 1$

Large N limit ($n \to \infty$) $\Longrightarrow$ WZW-like chiral edge action

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- $\left([\hat{X},\ \hat{Y}]\right)_{symbol} \to \frac{i}{n}(\Omega^{-1})^{ij}\,\partial_i X(\vec{x},t)\,\partial_j Y(\vec{x},t) + \cdots$

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  ρ_0 = constant over the volume occupied by droplet
- $S_0\ \to$ edge effective action

$$S_0 \sim \int_{\partial D} \left(\partial_t \phi + u\ \mathcal{L}\phi\right)\mathcal{L}\phi$$

$(2k-1)$ (space) dim chiral action defined on droplet boundary

$$\mathcal{L}\phi = (\Omega^{-1})^{ij}\hat{r}_j \partial_i \phi, \qquad \mathcal{L} = \begin{cases} \text{derivative along boundary of droplet} \\ \to \partial_\theta \text{ in 2 dim.} \end{cases}$$

B. Nonabelian background magnetic field $U(k)$

- Wavefunction is a nontrivial representation of $SU(k) : dim(\tilde{J}) = N'$.
- Symbol = $(N' \times N')$ matrix valued function $\longrightarrow$ action in terms of $G \in U(N')$

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- Symbol = $(N' \times N')$ matrix valued function $\longrightarrow$ action in terms of $G \in U(N')$
- The effective edge action is a gauged WZW action in $(2k-1, 1)$ dimensions.

$$
\begin{aligned}
S_0 =& \frac{1}{4\pi} \int_{\partial D} \mathrm{tr} \left[\left(G^\dagger \dot{G} + u\, G^\dagger \mathcal{L} G \right) G^\dagger \mathcal{L} G \right] \\
&+ \frac{1}{4\pi} \int_D \mathrm{tr} \left[-d \left(i\bar{A} dG G^\dagger + i\bar{A} G^\dagger dG \right) + \frac{1}{3} \left(G^\dagger dG \right)^3 \right] \wedge \left(\frac{\Omega}{2\pi} \right)^{k-1} \frac{1}{(k-1)!} \\
\equiv& S_{\mathrm{WZW}}(A^L = A^R = \bar{A})
\end{aligned}
$$

$\mathcal{L} = (\Omega^{-1})^{ij} \hat{r}_j D_i$ = covariant derivative along the boundary of droplet

- In the presence of gauge fluctuations one starts with a gauged matrix action.

$$\partial_t \to \hat{D}_t = \partial_t + i\hat{\mathcal{A}}$$

$$S = \int dt \,\mathrm{Tr}\left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \; - \; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} - \underbrace{\hat{\rho}_0 \; \hat{U}^\dagger \hat{\mathcal{A}} \hat{U}}\right]$$

gauge interactions

In terms of bosonic fields

$$S = N \int dt \, d\mu \; \mathrm{tr} \; \left[i\rho_0 * U^\dagger * \partial_t U \; - \; \rho_0 * U^\dagger * (V + \mathcal{A}) * U \right]$$

QUESTION: How is $\mathcal{A}$ related to the gauge fields coupled to the original fermions?

- S is invariant under

$$\begin{aligned} \delta U &= -i\lambda * U \\ \delta\mathcal{A}(\vec{x},t) &= \partial_t\lambda(\vec{x},t) - i\left(\lambda * (V+\mathcal{A}) - (V+\mathcal{A}) * \lambda\right) \end{aligned} \quad (1)$$

- Since S describes gauge interactions it has to be invariant under usual gauge transformations

$$\delta A_\mu = \partial_\mu \Lambda + i[\bar{A}_\mu + A_\mu, \Lambda], \qquad \delta\bar{A}_\mu = 0 \quad (2)$$

Background ($\bar{A}_\mu$)   Perturbation (A_μ)

The strategy is to choose

$$\begin{aligned} \mathcal{A} &= \text{function}(A_\mu, \bar{A}_\mu, V) \\ \lambda &= \text{function}(\Lambda, A_\mu, \bar{A}_\mu) \end{aligned}$$

such that the gauge transformation (2) induces $\delta\mathcal{A}$ in (1) (generalized Seiberg-Witten map) (KARABALI, 2005)

- In the large N limit the result is $S = S_{\text{edge}} + S_{\text{bulk}}$

$S_{\text{edge}} \sim$	$S_{\text{WZW}}\left(A^L = A + \bar{A}\,, A^R = \bar{A}\right)$	=	Chirally gauged WZW action in $2k$ dim
$S_{\text{bulk}} \sim$	$S_{CS}^{2k+1}(\tilde{A}) + \cdots$	=	$(2k+1)$ dim CS action

$\tilde{A} = (A_0 + V, \bar{a}_i + \bar{A}_i + A_i)$ = background + fluctuations

- Gauge Invariance $\Longrightarrow$ Anomaly Cancellation

$$\delta S_{\text{edge}} \neq 0, \quad \delta S_{\text{bulk}} \neq 0$$

$$\delta S_{\text{edge}} + \delta S_{\text{bulk}} = 0$$

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- So we can use

$$\frac{\delta S_{eff}}{\delta A_0} = J_0 = \text{Dolbeault index density}$$

and integrate up to get S_{eff}. (KARABALI AND NAIR, 2016)

- $\mathbb{CP}^1 = SU(2)/U(1)$; s-th LL

$$S_{3d} = \frac{1}{4\pi}\int\left\{\left(A + (s+\frac{1}{2})\omega\right)d\left(A + (s+\frac{1}{2})\omega\right) - \frac{1}{12}\omega\, d\omega\right\}$$

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- We have general results for arbitrary dimensions, higher Landau levels and nonabelian magnetic fields (KARABALI AND NAIR, 2016)
- $\mathbb{CP}^2 = SU(3)/U(2)$; LLL, Abelian gauge field

$$\begin{aligned} S_{5d}^{(LLL)} &= \frac{1}{(2\pi)^2} \int \left\{ \frac{1}{3!} \left(A + \omega^0\right) \left(dA + d\omega^0\right)^2 \right. \\ &\quad \left. - \frac{1}{12} \left(A + \omega^0\right) \left[(d\omega^0)^2 + \frac{1}{2} \mathrm{Tr}(\tilde{R} \wedge \tilde{R}) \right] \right\} \end{aligned}$$

$\omega^0 \sim U(1)$ part of spin connection; $\tilde{R} \sim SU(2)$ nonabelian part of the curvature.

- We divide the system into two regions, D and its complementary D^C, and define the reduced density matrix

$$\rho_D = \mathrm{Tr}_{D^C} |GS\rangle \langle GS|$$

where $|GS\rangle = \prod_m c_m^\dagger |0\rangle$.

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- The entanglement entropy is defined as

$$S = -\mathrm{Tr}\rho_D \log \rho_D$$

- We choose D to be the spherically symmetric region of $\mathbb{CP}^k$ satisfying $z \cdot \bar{z} \leq R^2$. For $\mathbb{CP}^1 \sim S^2$, D is a polar cap around the north pole with latitude angle θ. $R = \tan\theta/2$ via stereographic projection.

- The entanglement entropy can also be written as

$$S = -\mathrm{Tr}\rho_D \log \rho_D = -\sum_{m=1}^{N} [\lambda_m \log \lambda_m + (1-\lambda_m)\log(1-\lambda_m)]$$

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- λ's are eigenvalues of the two-point correlator (PESCHEL, 2003)

$$C(r, r') = \sum_{m=1}^{N} \Psi_m^*(z)\, \Psi_m(z') \;, \qquad z, z' \in D$$

$$\int_D C(r, r') \Psi_l^*(z') d\mu(z') = \lambda_l\, \Psi_l^*(z)$$

where

$$\lambda_l = \int_D |\Psi_l|^2 d\mu$$

- For 2d gapped systems

$$S = c\,L - \gamma + \mathcal{O}(1/L)$$

L: perimeter of boundary

c: non-universal constant

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- For integer QHE on $S^2 = \mathbb{CP}^1$ RODRIGUEZ AND SIERRA, 2009

For $\nu = 1$: $c = 0.204$

General results on Kähler manifolds CHARLES AND ESTIENNE, 2019

A. QHE on $\mathbb{CP}^k$ with $U(1)$ magnetic field

Entanglement Entropy for $\nu = 1$ on $\mathbb{CP}^k$ and Abelian magnetic field

A. QHE on $\mathbb{CP}^k$ with $U(1)$ magnetic field

The LLL wavefunctions are essentially the coherent states of $\mathbb{CP}^k$.

$$\begin{aligned} \Psi_{i_1 i_2 \cdots i_k} &= \sqrt{N}\left[\frac{n!}{i_1! i_2! \ldots i_k! (n-s)!}\right]^{\frac{1}{2}} \frac{z_1^{i_1} z_2^{i_2} \cdots z_k^{i_k}}{(1+\bar{z}\cdot z)^{\frac{n}{2}}} , \\ s &= i_1 + i_2 + \cdots + i_k , \quad 0 \leq i_i \leq n , \quad 0 \leq s \leq n \end{aligned}$$

They form an $SU(k+1)$ representation of dimension

$$N = \mathrm{dim} J = \frac{(n+k)!}{n!\, k!}$$

The volume element for $\mathbb{CP}^k$ is

$$d\mu = \frac{k!}{\pi^k} \frac{d^2 z_1 \cdots d^2 z_k}{(1+\bar{z}\cdot z)^{k+1}} \quad , \quad \int d\mu = 1$$

- The eigenvalues $\lambda = \int_D \Psi^* \Psi$ are given by

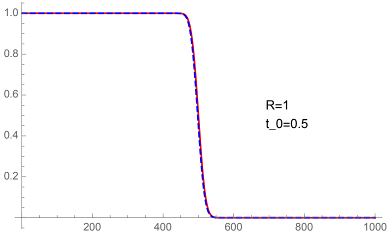
$$\lambda_{i_1 i_2 \cdots i_k} \equiv \lambda_s = \frac{(n+k)!}{(n-s)!(s+k-1)!} \int_0^{t_0} dt\, t^{s+k-1}\, (1-t)^{n-s}$$

where $t_0 = R^2/(1+R^2)$.

- The entanglement entropy is

$$\begin{aligned} S &= \sum_{s=0}^{n} \overbrace{\frac{(s+k-1)!}{s!(k-1)!}}^{degeneracy} H_s \\ H_s &= [-\lambda_s \log \lambda_s - (1-\lambda_s) \log(1-\lambda_s)] \end{aligned}$$

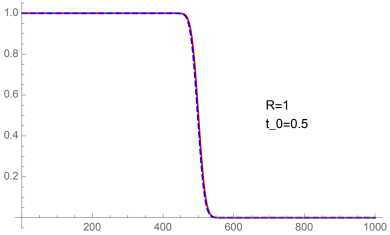
- For large n, this is amenable to an analytical semiclassical calculation for all $k \ll n$.



Graph of λ_s vs s

Transition ($\lambda = \frac{1}{2}$) at $s^* \sim n\, t_0$

$k = 1$, $k = 5$



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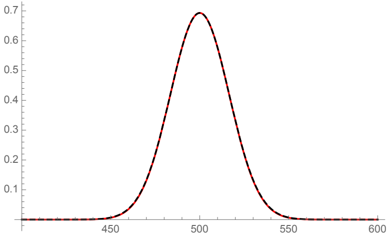
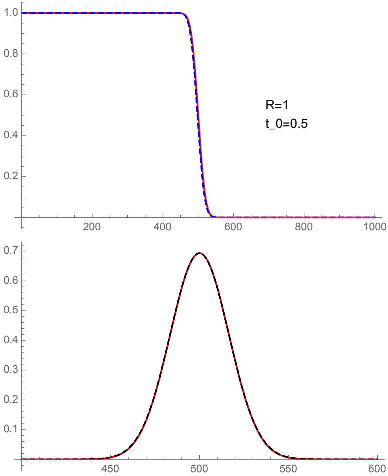


Graph of H_s vs s

—- exact

- - - - Gaussian approximation



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Only wavefunctions localized around the boundary of the entangling surface contribute to entropy.

From semiclassical analysis

$$S \sim n^{k-\frac{1}{2}} \frac{\pi (\log 2)^{3/2}}{2\, k!} \underbrace{2k \frac{R^{2k-1}}{(1+R^2)^k}}_{\textit{geometric area}} \sim c_k \text{ Area}$$

In agreement with $k = 1$ result by RODRIGUEZ AND SIERRA

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- Formula for entropy becomes universal if expressed in terms of a "phase space" area instead of a geometric area.

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- $V_{\text{phase space}} \to \frac{n^k}{k!} \int \Omega^k = \frac{n^k}{k!} \int d\mu$

$$A_{\text{phase space}} = \frac{n^{k-\frac{1}{2}}}{k!} A_{\text{geom}} = \frac{n^{k-\frac{1}{2}}}{k!}\, 2k\frac{R^{2k-1}}{(1+R^2)^k}$$

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- What about higher Landau levels?

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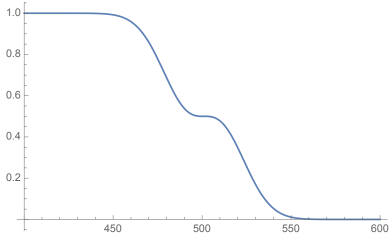

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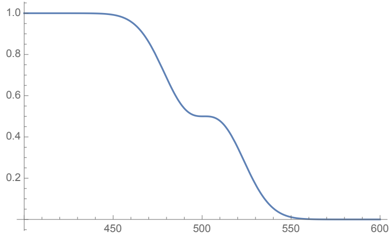
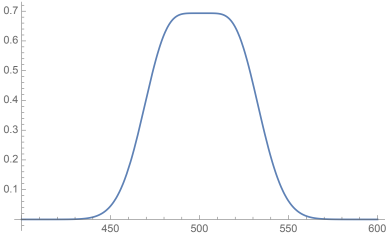


- Step-like pattern around the transition point.

  1st excited level wavefunctions have a node.

- The step-like plateau of λ causes the broadening of the entropy H_s around $\lambda = 1/2$. H_s cannot be approximated with a simple Gaussian.



- Previous semiclassical analysis does not work.

$$S^{(q=1)} = 1.65\, S^{(q=0)}$$

$\nu = 2$ CASE

What happens when both $q = 0$ and $q = 1$ Landau levels are full, namely $\nu = 2$?

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The two-point correlator now is given by

$$C(r, r') = \sum_{s=0}^{n} \Psi_s^{*0}(r)\Psi_s^0(r') + \sum_{s=0}^{n+2} \Psi_s^{*1}(r)\Psi_s^1(r')$$

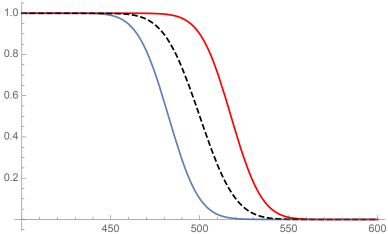
There are $2n + 4$ eigenvalues: λ_0^1 , $\tilde{\lambda}_s^{\pm}$, λ_{n+2}^1, $s = 0, \cdots, n$ and

$$\tilde{\lambda}_s^{\pm} = \frac{\lambda_s^0 + \lambda_{s+1}^1 \pm \sqrt{(\lambda_s^0 - \lambda_{s+1}^1)^2 + 4(\delta\lambda)^2_{s,s+1}}}{2}$$

where

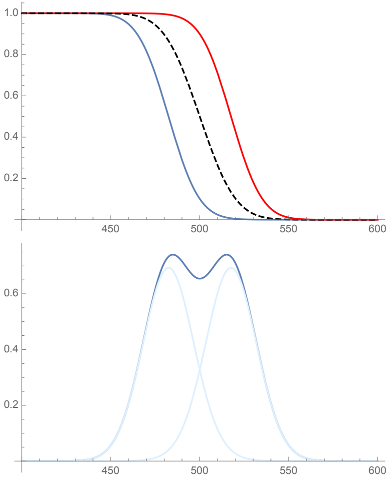
$$\delta\lambda_{s,s+1} = \int_D \Psi_s^{*(q=0)}(r)\ \Psi_{s+1}^{(q=1)}(r)\ d\mu$$

$\nu = 2$ CASE



$\color{red}{\tilde{\lambda}_s^+}$, $\color{blue}{\tilde{\lambda}_s^-}$

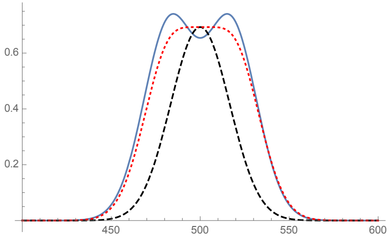
- - - for $\nu = 1$



$\tilde{\lambda}_s^+$, $\tilde{\lambda}_s^-$

- - - for $\nu = 1$

$\tilde{H}_s{}^+ + \tilde{H}_s{}^-$



- - - $H_s^{\nu=1}$

... $H_s^{q=1}$

—- $H_s^{\nu=2}$

$$S = \sum H_s$$

$$S^{(\nu=2)} > S^{(q=1)} > S^{(\nu=1)}$$

$$S^{(q=1)} = 1.65\, S^{(\nu=1)}$$

$$S^{(\nu=2)} = 1.76\, S^{(\nu=1)}$$

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- Entanglement entropy for higher dim QHE on $\mathbb{CP}^k$: For $\nu = 1$ there is a universal formula valid for any k, Abelian or non-Abelian background if area is expressed in terms of phase-space area.

- When the boundary of the entangling surface intersects the edge boundary there is additional log contribution in 2d, $S_{edge} \sim \frac{c}{6}\log(l)$.

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  What are the contributions from non-Abelian droplets?

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THANK YOU!