

# Neutrino Oscillations and Theory Biases

Peter B. Denton

BNL Summer Student Lecture

June 22, 2023



**Brookhaven™**  
National Laboratory



# About Me

1. Grew up in Michigan
2. Bachelors in physics and math from Rice, '10
3. PhD from Vanderbilt, '16
4. Year at Fermilab working with Stephen Parke, '15-'16
5. Postdoc at the Niels Bohr International Academy, '16-'18
6. Faculty at Brookhaven, '18-present

# About Me

1. Grew up in Michigan
2. Bachelors in physics and math from Rice, '10
3. PhD from Vanderbilt, '16
4. Year at Fermilab working with Stephen Parke, '15-'16
5. Postdoc at the Niels Bohr International Academy, '16-'18
6. Faculty at Brookhaven, '18-present

## Research interests

- ▶ Neutrino oscillations
- ▶ New physics in neutrinos
- ▶ Astroparticle physics
- ▶ Black holes
- ▶ Dark matter

## Other interests

- ▶ Ultimate frisbee
- ▶ Hiking
- ▶ Piano
- ▶ Photography

Stop by 2-16 anytime

# Key points

- ▶ Measuring neutrinos requires the biggest detectors
- ▶ Quantum mechanical neutrino oscillations occur on human scales
- ▶ Neutrinos unexpectedly have mass
- ▶ Neutrinos continue to **surprise**

## Neutrino masses: only left handed neutrinos?

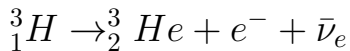
- ▶ Neutrinos: fermions only feel the weak (left) interaction
- ▶ Measure right handed fermions through electric charge
- ▶ Right handed neutrinos won't scatter off anything
- ▶ They don't exist?
- ▶ Neutrinos are massless?

This was the standard assumption until 1998!

Let's do a direct kinematic search

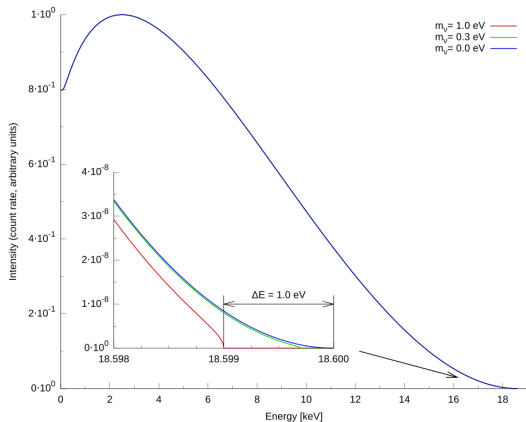


KATRIN 2006

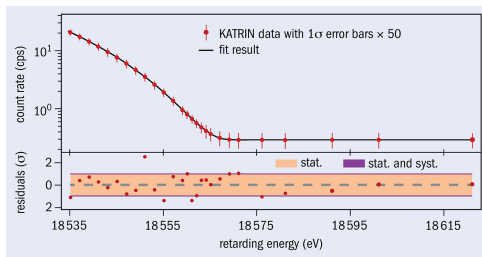


For massless neutrinos, what is the maximum electron energy?

# Neutrino masses: kinematic end point is hard



# Neutrino masses: kinematic end point is hard



KATRIN 2018

$$m_\nu \lesssim 1 \text{ eV}$$

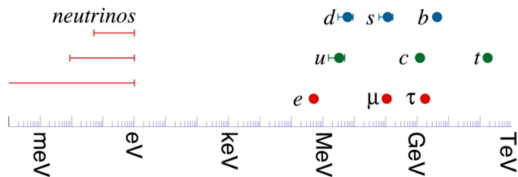


# Neutrino masses: small numbers?

- ▶ Other fermions get their mass from the Higgs field

See H. Davoudiasl's lecture on Tuesday, June 14

- ▶ “Expect” Yukawa couplings:  $y \sim 1$
- ▶ Top quark:  $y_t \sim 1$ , but electron:  $y_e \sim 10^{-6}$
- ▶ Neutrinos:  $y_\nu < 10^{-12}$  or nothing if no right handed neutrinos
- ▶ Weird?

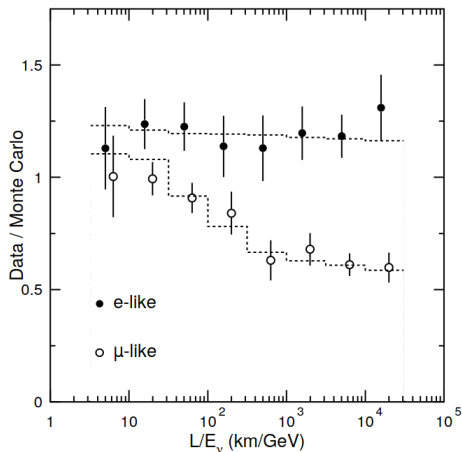


# Big surprise of 1998

- ▶ Electroweak understood, mediators ( $\gamma, W, Z$ ) found
- ▶ Strong understood, mediators (gluon) found
- ▶ All fermions detected except tau neutrino (2000), but no surprises expected
- ▶ Higgs boson still to be found
- ▶ Standard Model looks to be in great shape

# Atmospheric neutrinos disappear

Cosmic rays hit the atmosphere, produce  $\pi^+$ ,  $\mu$ , and  $\nu_\mu$



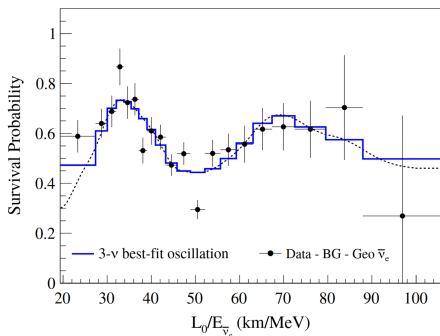
SuperKamiokande [hep-ex/9807003](https://arxiv.org/abs/hep-ex/9807003)

# Neutrinos really oscillate

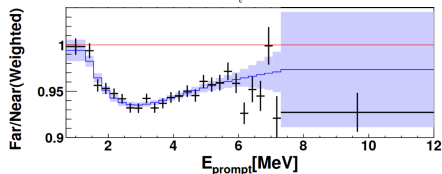
1. Neutrinos experience time  $\Rightarrow$  must have mass
2. Neutrino oscillate  $\Rightarrow$  must mix & masses must be different

# Neutrinos really oscillate

1. Neutrinos experience time  $\Rightarrow$  must have mass
2. Neutrinos oscillate  $\Rightarrow$  must mix & masses must be different



KamLAND [1303.4667](#)



Daya Bay [1809.02261](#)

## Two-flavor neutrino oscillation probability

Only one mixing angle, one  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ , no complex phase

## Two-flavor neutrino oscillation probability

Only one mixing angle, one  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ , no complex phase

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

Same flavor: disappearance

## Two-flavor neutrino oscillation probability

Only one mixing angle, one  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ , no complex phase

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

Same flavor: disappearance

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

New flavor: appearance



## Two-flavor neutrino oscillation probability

Only one mixing angle, one  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ , no complex phase

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

Same flavor: disappearance

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

New flavor: appearance

Can easily confirm unitarity:

$$\sum_{\beta} P(\nu_\alpha \rightarrow \nu_\beta) = 1$$

## Two-flavor neutrino oscillation probability

Only one mixing angle, one  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ , no complex phase

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

Same flavor: disappearance

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)$$

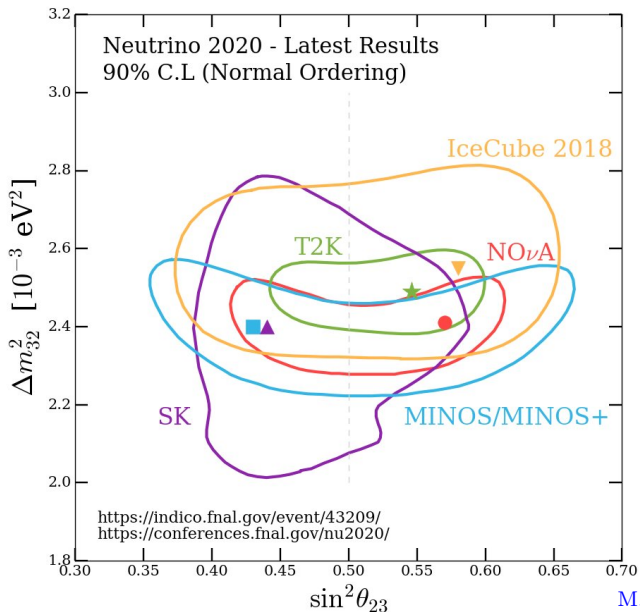
New flavor: appearance

Can easily confirm unitarity:

$$\sum_{\beta} P(\nu_\alpha \rightarrow \nu_\beta) = 1$$

What angle leads to maximal oscillations?

# Atmospheric parameters



M. Ross-Lonergan

## Maximal mixing: atmospheric neutrinos

Mixing for atmospheric angles seems to be maximal  $\theta_{23} \sim 45^\circ$

	$\theta_{23}$	$\theta_{13}$	$\theta_{12}$	$\delta$
Quarks	$2.4^\circ$	$0.20^\circ$	$13^\circ$	$69^\circ$
Leptons	$\sim 45^\circ$	X	X	unknown

Was an expectation that mixing angles should be small

Other atmospheric experiments had hints for oscillations, didn't frame it since "mixing angles should be small"

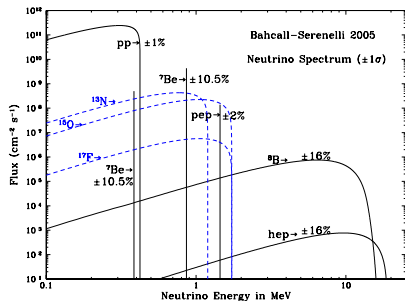
# Solar neutrinos

# Solar neutrinos

**Problem:** Too few neutrinos from the sun

## Problem: Too few neutrinos from the sun

### 1. John Bahcall predicted the solar neutrino flux



$${}^8\text{B flux} \propto T^{24}$$

J. Bahcall et al. [nucl-th/9601044](https://arxiv.org/abs/nuc1-th/9601044)

# Solar neutrinos

## 2. 1960s: Ray Davis's Homestake experiment used chlorine

$$E_{\nu, \text{tr}} = 0.8 \text{ MeV}$$

Homestake *ApJ.* 496 (1998) 505-526



# Solar neutrinos

## 2. 1960s: Ray Davis's Homestake experiment used chlorine

$$E_{\nu, \text{tr}} = 0.8 \text{ MeV}$$

Homestake [ApJ. 496 \(1998\) 505-526](#)

Measured  $\nu_e$  flux:  $2.56 \pm 0.23$

Predicted: 9.3, 6.4, 7.6

# Solar neutrinos

## 2. 1960s: Ray Davis's Homestake experiment used chlorine

$$E_{\nu, \text{tr}} = 0.8 \text{ MeV}$$

Homestake [ApJ. 496 \(1998\) 505-526](#)

Measured  $\nu_e$  flux:  $2.56 \pm 0.23$

Predicted: 9.3, 6.4, 7.6

Perhaps  $^8\text{B}$  is way lower than expected?

# Solar neutrinos

## 2. 1960s: Ray Davis's Homestake experiment used chlorine

$$E_{\nu, \text{tr}} = 0.8 \text{ MeV}$$

Homestake [ApJ. 496 \(1998\) 505-526](#)

Measured  $\nu_e$  flux:  $2.56 \pm 0.23$

Predicted: 9.3, 6.4, 7.6

Perhaps  $^8\text{B}$  is way lower than expected?

Perhaps Homestake is wrong?

# Solar neutrinos

## 2. 1960s: Ray Davis's Homestake experiment used chlorine

$$E_{\nu, \text{tr}} = 0.8 \text{ MeV}$$

Homestake [ApJ. 496 \(1998\) 505-526](#)

Measured  $\nu_e$  flux:  $2.56 \pm 0.23$

Predicted: 9.3, 6.4, 7.6

Perhaps  $^8\text{B}$  is way lower than expected?

Perhaps Homestake is wrong?

Perhaps both are wrong

# Solar neutrinos

## 3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

SNO [nucl-ex/0204008](#)

# Solar neutrinos

## 3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

## 4. Measured CC, ES, and NC processes for neutrinos from the Sun

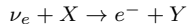
# Solar neutrinos

## 3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

## 4. Measured CC, ES, and NC processes for neutrinos from the Sun

- ▶ Charged current (CC) is  $\nu_e$  only



Electron neutrino disappearance

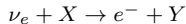
# Solar neutrinos

## 3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

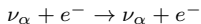
## 4. Measured CC, ES, and NC processes for neutrinos from the Sun

- ▶ Charged current (CC) is  $\nu_e$  only



Electron neutrino disappearance

- ▶ Elastic scattering (ES) is mostly  $\nu_e$  but also  $\nu_\mu$  and  $\nu_\tau$





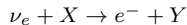
# Solar neutrinos

## 3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

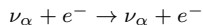
## 4. Measured CC, ES, and NC processes for neutrinos from the Sun

- ▶ Charged current (CC) is  $\nu_e$  only

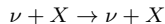


Electron neutrino disappearance

- ▶ Elastic scattering (ES) is mostly  $\nu_e$  but also  $\nu_\mu$  and  $\nu_\tau$



- ▶ Neutral current (NC) does not depend on the flavor



Total neutrino flux

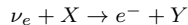
# Solar neutrinos

## 3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

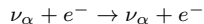
## 4. Measured CC, ES, and NC processes for neutrinos from the Sun

- ▶ Charged current (CC) is  $\nu_e$  only

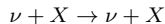


Electron neutrino disappearance

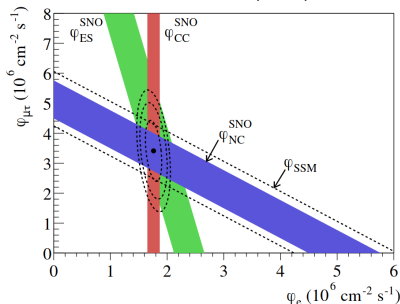
- ▶ Elastic scattering (ES) is mostly  $\nu_e$  but also  $\nu_\mu$  and  $\nu_\tau$



- ▶ Neutral current (NC) does not depend on the flavor



Total neutrino flux



SNO [nucl-ex/0204008](#)

# Solar neutrinos: matter effect

Presence of a dense electron field modifies oscillations

L. Wolfenstein [PRD 17 \(1978\)](#)

S. Mikheev, A. Smirnov [Nuovo Cim. C9 \(1986\) 17-26](#)

# Solar neutrinos: matter effect

Presence of a dense electron field modifies oscillations

L. Wolfenstein [PRD 17 \(1978\)](#)

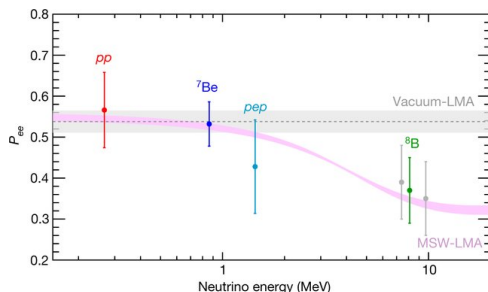
S. Mikheev, A. Smirnov [Nuovo Cim. C9 \(1986\) 17-26](#)

Low energy: no matter effect

High energy: large matter effect

$$P_{ee} \simeq 1 - \frac{1}{2} \sin^2 2\theta_{12}$$

$$P_{ee} \simeq \sin^2 \theta_{12}$$



Borexino

## What mixing angle fits this data?

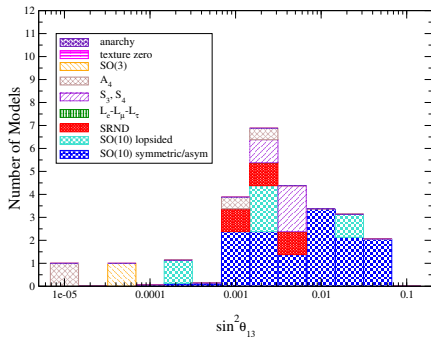
	$\theta_{23}$	$\theta_{13}$	$\theta_{12}$	$\delta$
Quarks	$2.4^\circ$	$0.20^\circ$	$13^\circ$	$69^\circ$
Leptons	$\sim 45^\circ$	X	$33^\circ$	unknown

Two large angles  
 Surely  $\theta_{13}$  will be small?!

	$\theta_{23}$	$\theta_{13}$	$\theta_{12}$	$\delta$
Quarks	$2.4^\circ$	$0.20^\circ$	$13^\circ$	$69^\circ$
Leptons	$\sim 45^\circ$	X	$33^\circ$	unknown

Two large angles  
Surely  $\theta_{13}$  will be small?!

Models that Predict All 3 Angles

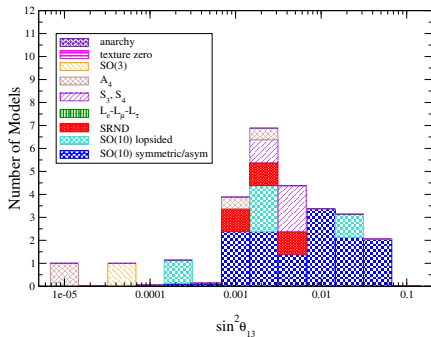


C. Albright, M-C. Chen [hep-ph/0608137](https://arxiv.org/abs/hep-ph/0608137)

	$\theta_{23}$	$\theta_{13}$	$\theta_{12}$	$\delta$
Quarks	$2.4^\circ$	$0.20^\circ$	$13^\circ$	$69^\circ$
Leptons	$\sim 45^\circ$	$8.5^\circ$	$33^\circ$	unknown

Two large angles  
Surely  $\theta_{13}$  will be small?!

Models that Predict All 3 Angles



True value:  
 $\sin^2 \theta_{13} = 0.02, \theta_{13} = 8.5^\circ$   
Quite large!

C. Albright, M-C. Chen [hep-ph/0608137](https://arxiv.org/abs/hep-ph/0608137)

Complex phase:  $\delta$

CP violation  $\Rightarrow$  particles and antiparticles act differently



## Complex phase: $\delta$

CP violation  $\Rightarrow$  particles and antiparticles act differently

To see CPV in oscillations need:

- ▶ Three nonzero mixing angles
- ▶ Neutrino flavor appearance

This is because CP violation implies T violation

## Complex phase: $\delta$

CP violation  $\Rightarrow$  particles and antiparticles act differently

To see CPV in oscillations need:

- ▶ Three nonzero mixing angles
- ▶ Neutrino flavor appearance

This is because CP violation implies T violation

In vacuum at first maximum:

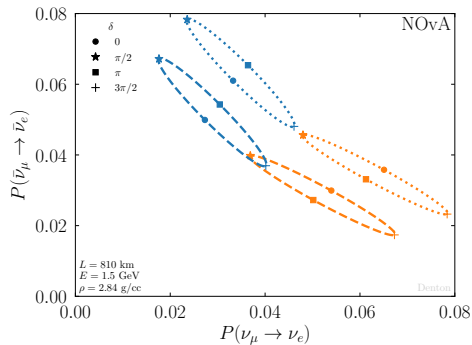
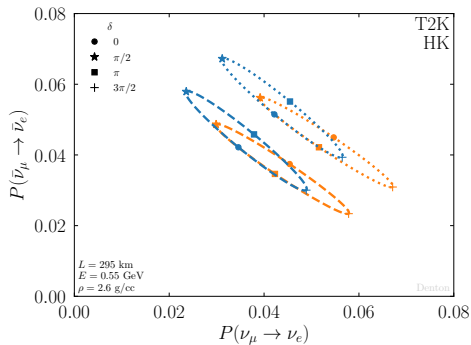
$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m_{21}^2}{\Delta m_{32}^2}$$

$$J \equiv s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)

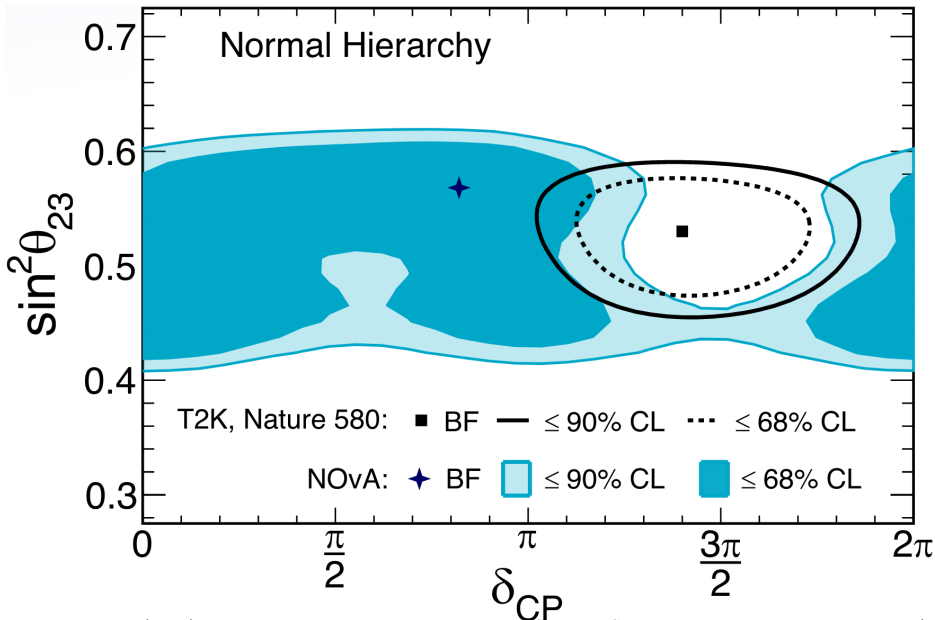
Matter effects are easily accounted for: [PBD](#), S. Parke [1902.07185](#)

# Complex phase: $\delta$ : how is it measured?



# Complex phase: $\delta$ : the data

A. Himmel [10.5281/zenodo.3959581](https://zenodo.org/record/3959581)



# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

2. Neutrinos can also get a Majorana mass term since they have no charge

# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

2. Neutrinos can also get a Majorana mass term since they have no charge
3. Two mass terms is fine, rich phenomenology

# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

2. Neutrinos can also get a Majorana mass term since they have no charge
3. Two mass terms is fine, rich phenomenology
4. Diagonalize from bare masses to physical masses:

$$m_\nu \simeq \frac{m_D^2}{m_M}$$



# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

2. Neutrinos can also get a Majorana mass term since they have no charge
3. Two mass terms is fine, rich phenomenology
4. Diagonalize from bare masses to physical masses:

$$m_\nu \simeq \frac{m_D^2}{m_M}$$

5. Suppose  $m_D \sim 100$  GeV, seems  $m_\nu \sim 0.01$  eV:

What Majorana mass works?

# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

2. Neutrinos can also get a Majorana mass term since they have no charge
3. Two mass terms is fine, rich phenomenology
4. Diagonalize from bare masses to physical masses:

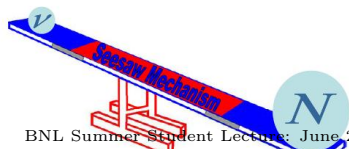
$$m_\nu \simeq \frac{m_D^2}{m_M}$$

5. Suppose  $m_D \sim 100$  GeV, seems  $m_\nu \sim 0.01$  eV:

What Majorana mass works?

6.  $\Rightarrow m_M \sim 10^{15}$  GeV at the unification scale!

Seesaw!



# Neutrino mass generation

1. Neutrinos could well get a Dirac mass term from the Higgs like other fermions

With three new right handed neutrinos

2. Neutrinos can also get a Majorana mass term since they have no charge
3. Two mass terms is fine, rich phenomenology
4. Diagonalize from bare masses to physical masses:

$$m_\nu \simeq \frac{m_D^2}{m_M}$$

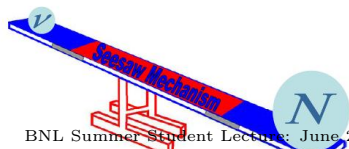
5. Suppose  $m_D \sim 100$  GeV, seems  $m_\nu \sim 0.01$  eV:

What Majorana mass works?

6.  $\Rightarrow m_M \sim 10^{15}$  GeV at the unification scale!

Seesaw!

We don't know if/how  
this works though



# Neutrino oscillation status: today

## Current status

- ▶ Discovery of neutrino oscillations added 7+ parameters

## Current status

- ▶ Discovery of neutrino oscillations added 7+ parameters
- ▶ Oscillations can probe 6 of them:

1.  $\Delta m_{21}^2$ : solar & reactor: **good**

Only have one good measurement of this

2.  $\Delta m_{31}^2$ : atmospheric, accelerator, & reactor:  
**know the magnitude, not the sign**

3.  $\theta_{12}$ : solar & reactor: **good**

Only have one good measurement of this

4.  $\theta_{13}$ : reactor: **good**

5.  $\theta_{23}$ : atmospheric & accelerator:  
okay, **don't know if  $> 45^\circ$  or  $< 45^\circ$**

6.  $\delta$ : accelerator: **unknown**

## Current status

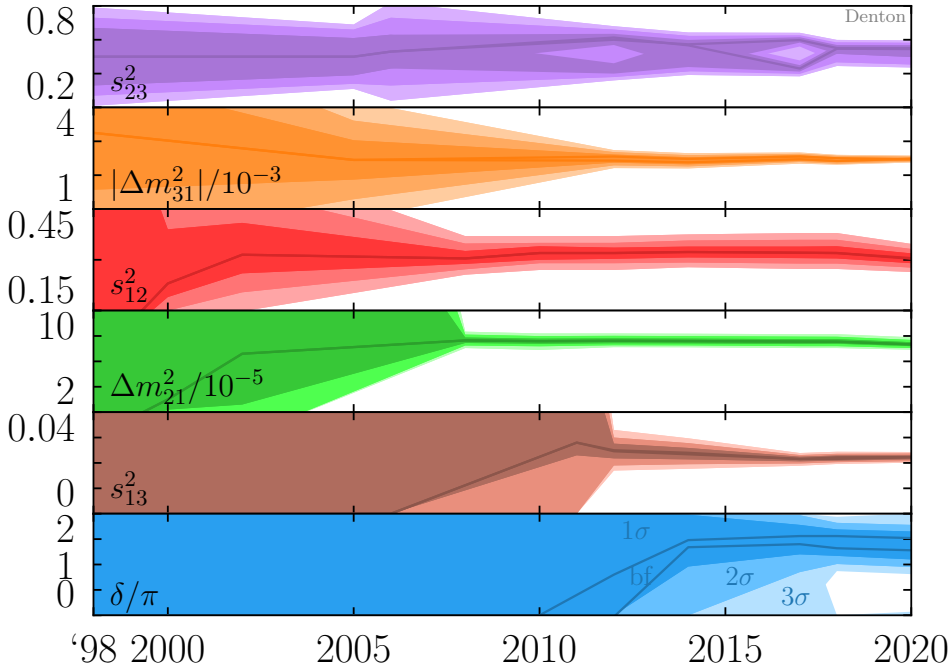
- ▶ Discovery of neutrino oscillations added 7+ parameters
- ▶ Oscillations can probe 6 of them:
  1.  $\Delta m_{21}^2$ : solar & reactor: **good**  
Only have one good measurement of this
  2.  $\Delta m_{31}^2$ : atmospheric, accelerator, & reactor:  
**know the magnitude, not the sign**
  3.  $\theta_{12}$ : solar & reactor: **good**  
Only have one good measurement of this
  4.  $\theta_{13}$ : reactor: **good**
  5.  $\theta_{23}$ : atmospheric & accelerator:  
okay, **don't know if  $> 45^\circ$  or  $< 45^\circ$**
  6.  $\delta$ : accelerator: **unknown**
- ▶ Seventh parameter is the absolute mass scale:  
Will be determined in cosmology  
Possibly KATRIN, and neutrinoless double beta decay searches

## Current status

- ▶ Discovery of neutrino oscillations added 7+ parameters
- ▶ Oscillations can probe 6 of them:
  1.  $\Delta m_{21}^2$ : solar & reactor: **good**  
Only have one good measurement of this
  2.  $\Delta m_{31}^2$ : atmospheric, accelerator, & reactor:  
**know the magnitude, not the sign**
  3.  $\theta_{12}$ : solar & reactor: **good**  
Only have one good measurement of this
  4.  $\theta_{13}$ : reactor: **good**
  5.  $\theta_{23}$ : atmospheric & accelerator:  
okay, **don't know if  $> 45^\circ$  or  $< 45^\circ$**
  6.  $\delta$ : accelerator: **unknown**
- ▶ Seventh parameter is the absolute mass scale:  
Will be determined in cosmology  
Possibly KATRIN, and neutrinoless double beta decay searches

Four remaining known unknowns in particle physics: all neutrinos!





# Precision is coming to neutrino physics

Discussion time!

# Backups

# Schrödinger equation

Neutrinos propagate in eigenstates of the Hamiltonian

$$i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$$

# Schrödinger equation

Neutrinos propagate in eigenstates of the Hamiltonian

$$i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$$

In the absence of any interactions  $H_{\text{vac}} |\nu_i\rangle = E_i |\nu_i\rangle$ .

$$|\nu_i(L)\rangle = e^{-iE_i L} |\nu_i(0)\rangle \rightarrow e^{-im_i^2 L/2E} |\nu_i(0)\rangle$$

$i$  indicates mass eigenstate

## Schrödinger equation

Neutrinos propagate in eigenstates of the Hamiltonian

$$i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$$

In the absence of any interactions  $H_{\text{vac}} |\nu_i\rangle = E_i |\nu_i\rangle$ .

$$|\nu_i(L)\rangle = e^{-iE_i L} |\nu_i(0)\rangle \rightarrow e^{-im_i^2 L/2E} |\nu_i(0)\rangle$$

*i* indicates mass eigenstate

We don't produce neutrinos in eigenstates of the Hamiltonian in vacuum, e.g. mass eigenstates

$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle \quad \alpha \in \{e, \mu, \tau\}$$

## Schrödinger equation

Neutrinos propagate in eigenstates of the Hamiltonian

$$i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$$

In the absence of any interactions  $H_{\text{vac}} |\nu_i\rangle = E_i |\nu_i\rangle$ .

$$|\nu_i(L)\rangle = e^{-iE_i L} |\nu_i(0)\rangle \rightarrow e^{-im_i^2 L/2E} |\nu_i(0)\rangle$$

$i$  indicates mass eigenstate

We don't produce neutrinos in eigenstates of the Hamiltonian in vacuum, e.g. mass eigenstates

$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle \quad \alpha \in \{e, \mu, \tau\}$$

$U$  is a unitary  $3 \times 3$  matrix which has four degrees of freedom

Unitarity  $\Rightarrow$  9 dofs, rephasing  $\Rightarrow 9 - 5 = 4$



# The probability

The physical observable is the probability:  $P(\nu_\alpha \rightarrow \nu_\beta; L, E)$

# The probability

The physical observable is the probability:  $P(\nu_\alpha \rightarrow \nu_\beta; L, E)$

First calculate the transition amplitude:

1. Project from  $\nu_\alpha$  to  $\nu_i$  via  $U^*$
2. Propagate mass state  $\nu_i$
3. Project from  $\nu_i$  to  $\nu_\beta$  via  $U^T$
4. Sum over all intermediate states\*

\*coherency must apply

# The probability

The physical observable is the probability:  $P(\nu_\alpha \rightarrow \nu_\beta; L, E)$

First calculate the transition amplitude:

1. Project from  $\nu_\alpha$  to  $\nu_i$  via  $U^*$
2. Propagate mass state  $\nu_i$
3. Project from  $\nu_i$  to  $\nu_\beta$  via  $U^T$
4. Sum over all intermediate states\*

\*coherency must apply

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta; L, E) = \sum_{i=1}^3 U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i}$$

$$P = |\mathcal{A}|^2$$

# The probability

The physical observable is the probability:  $P(\nu_\alpha \rightarrow \nu_\beta; L, E)$

First calculate the transition amplitude:

1. Project from  $\nu_\alpha$  to  $\nu_i$  via  $U^*$
2. Propagate mass state  $\nu_i$
3. Project from  $\nu_i$  to  $\nu_\beta$  via  $U^T$
4. Sum over all intermediate states\*

\*coherency must apply

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta; L, E) = \sum_{i=1}^3 U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i}$$
$$P = |\mathcal{A}|^2$$

Discrete symmetries:

$$T : L \rightarrow -L, \quad CP : \nu \leftrightarrow \bar{\nu} \Leftrightarrow U_{\alpha i} \rightarrow U_{\alpha i}^* \Leftrightarrow E \rightarrow -E$$

Assume CPT is conserved:  $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$

# The probability

The physical observable is the probability:  $P(\nu_\alpha \rightarrow \nu_\beta; L, E)$

First calculate the transition amplitude:

1. Project from  $\nu_\alpha$  to  $\nu_i$  via  $U^*$
2. Propagate mass state  $\nu_i$
3. Project from  $\nu_i$  to  $\nu_\beta$  via  $U^T$
4. Sum over all intermediate states\*

\*coherency must apply

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta; L, E) = \sum_{i=1}^3 U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i}$$
$$P = |\mathcal{A}|^2$$

Discrete symmetries:

$$T : L \rightarrow -L, \quad CP : \nu \leftrightarrow \bar{\nu} \Leftrightarrow U_{\alpha i} \rightarrow U_{\alpha i}^* \Leftrightarrow E \rightarrow -E$$

Assume CPT is conserved:  $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$

Assume that  $E$  and direction don't change during propagation

# Coherent propagation

# Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap

## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT



## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region

## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties

## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this

## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this
  - ▶ All approaches give same answer

## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this
  - ▶ All approaches give same answer
- ▶ Nearly all cases of oscillations are known to be coherent

## Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this
  - ▶ All approaches give same answer
- ▶ Nearly all cases of oscillations are known to be coherent
- ▶ Exceptions:

# Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this
  - ▶ All approaches give same answer
- ▶ Nearly all cases of oscillations are known to be coherent
- ▶ Exceptions:
  - ▶ Solar neutrinos: decohere from sun to Earth

# Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this
  - ▶ All approaches give same answer
- ▶ Nearly all cases of oscillations are known to be coherent
- ▶ Exceptions:
  - ▶ Solar neutrinos: decohere from sun to Earth
  - ▶ Astrophysical neutrinos: (galactic or extragalactic) decohere



# Coherent propagation

- ▶ Neutrino oscillations requires all 3 wavefunctions to overlap
- ▶ Properly calculating this requires QFT
  - ▶ Need to integrate over production region
  - ▶ Need to account for detection uncertainties
- ▶ Literature is somewhat inconsistent in how to do this
  - ▶ All approaches give same answer
- ▶ Nearly all cases of oscillations are known to be coherent
- ▶ Exceptions:
  - ▶ Solar neutrinos: decohere from sun to Earth
  - ▶ Astrophysical neutrinos: (galactic or extragalactic) decohere
- ▶ Decohered probabilities are easy!

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^3 P_{\alpha i} P_{i i} P_{i \beta} = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Everything is at the probability level not the amplitude level  
This is the same expression as oscillation averaged probabilities

## Three flavor

Three angles, three  $\Delta m^2$  (two are close), one complex phase

## Three flavor

Three angles, three  $\Delta m^2$  (two are close), one complex phase

It is less easy to show that:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) = & 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) \\ & - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \\ & - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right) \end{aligned}$$

Many different ways to write these probabilities

## Three flavor: appearance

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) = & -4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) \\ & -4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \\ & -4\Re[U_{\alpha 2}U_{\beta 2}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right) \\ & +8\Im[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) \end{aligned}$$

## Three flavor: appearance

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) = & -4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) \\ & -4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \\ & -4\Re[U_{\alpha 2}U_{\beta 2}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right) \\ & +8\Im[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) \end{aligned}$$

Final coefficient:

$$8\Im[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \equiv 8J = 8s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

This is the same for all appearance channels (up to sign)

C. Jarlskog [PRL 55 \(1985\)](#)

$$s_{ij} = \sin \theta_{ij}, \quad c_{ij} = \cos \theta_{ij}$$

# More on probabilities

## More on probabilities

1. The Jarlskog term is the only term  $\propto \sin \delta$

## More on probabilities

1. The Jarlskog term is the only term  $\propto \sin \delta$
2. The Jarlskog term is the only term  $\propto L^3$

all others  $\propto L^2$

$\Rightarrow \delta$  is hard to measure



## More on probabilities

1. The Jarlskog term is the only term  $\propto \sin \delta$
2. The Jarlskog term is the only term  $\propto L^3$

all others  $\propto L^2$

$\Rightarrow \delta$  is hard to measure

3.  $\nu \rightarrow \bar{\nu} \Rightarrow U \rightarrow U^*$  which is  $\delta \rightarrow -\delta$

## More on probabilities

1. The Jarlskog term is the only term  $\propto \sin \delta$
2. The Jarlskog term is the only term  $\propto L^3$

all others  $\propto L^2$

$\Rightarrow \delta$  is hard to measure

3.  $\nu \rightarrow \bar{\nu} \Rightarrow U \rightarrow U^*$  which is  $\delta \rightarrow -\delta$
4.  $\delta \rightarrow -\delta \Rightarrow L \rightarrow -L$  or  $E \rightarrow -E$

Care is required because of the matter effect

## More on probabilities

1. The Jarlskog term is the only term  $\propto \sin \delta$
2. The Jarlskog term is the only term  $\propto L^3$

all others  $\propto L^2$

$\Rightarrow \delta$  is hard to measure

3.  $\nu \rightarrow \bar{\nu} \Rightarrow U \rightarrow U^*$  which is  $\delta \rightarrow -\delta$
4.  $\delta \rightarrow -\delta \Rightarrow L \rightarrow -L$  or  $E \rightarrow -E$

Care is required because of the matter effect

5. This follows from CPT. CP:  $\delta \rightarrow -\delta$  and  $T$  is  $L \rightarrow -L$

Matter effect causes apparent CPT violation

## Matter effect: constant

Call Schrödinger equation's eigenvalues  $m_i^2$  and eigenvectors  $U_i$ .

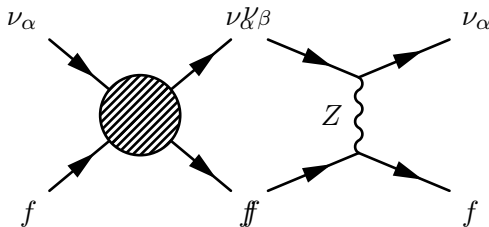
$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^3 U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i} \quad P = |\mathcal{A}|^2$$

## Matter effect: constant

Call Schrödinger equation's eigenvalues  $m_i^2$  and eigenvectors  $U_i$ .

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^3 U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i} \quad P = |\mathcal{A}|^2$$

In matter  $\nu$ 's propagate in a new basis that depends on  $a \propto N_e E_\nu$ .



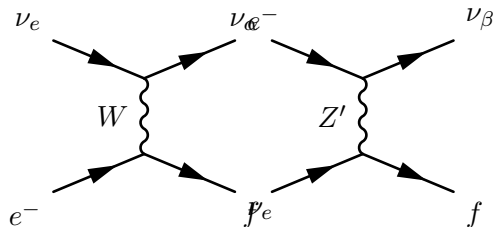
L. Wolfenstein [PRD 17 \(1978\)](#)

## Matter effect: constant

Call Schrödinger equation's eigenvalues  $m_i^2$  and eigenvectors  $U_i$ .

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^3 U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i} \quad P = |\mathcal{A}|^2$$

In matter  $\nu$ 's propagate in a new basis that depends on  $a \propto N_e E_\nu$ .



L. Wolfenstein [PRD 17 \(1978\)](#)

Eigenvalues:  $m_i^2 \rightarrow \widehat{m}_i^2(a)$

Eigenvectors are given by  $\theta_{ij} \rightarrow \widehat{\theta}_{ij}(a) \quad \Leftarrow \quad \text{Unitarity}$

## Hamiltonian dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

## Hamiltonian dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$



## Hamiltonian dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

For more on parameterizations see: [PBD](#), R. Pestes [2006.09384](#)

# Hamiltonian dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

For more on parameterizations see: [PBD](#), R. Pestes [2006.09384](#)

Find eigenvalues and eigenvectors:

$$H_{\text{flav}} = \frac{1}{2E} \widehat{U} \begin{pmatrix} 0 & & \\ & \widehat{\Delta m}_{21}^2 & \\ & & \widehat{\Delta m}_{31}^2 \end{pmatrix} \widehat{U}^\dagger$$

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

K. Kimura, A. Takamura, H. Yokomakura [hep-ph/0205295](#)

[PBD](#), S. Parke, X. Zhang [1907.02534](#)

# Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential  
Solution = MSW effect

S. Mikheev, A. Smirnov *Nuovo Cim. C9* (1986) 17-26

## Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential

Solution = MSW effect

S. Mikheev, A. Smirnov *Nuovo Cim. C9* (1986) 17-26

- ▶ Potential changes slowly  $\Rightarrow$  neutrinos remain in same state

## Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential

Solution = MSW effect

S. Mikheev, A. Smirnov *Nuovo Cim. C9* (1986) 17-26

- ▶ Potential changes slowly  $\Rightarrow$  neutrinos remain in same state
- ▶ In center of the sun  $\nu_e \approx \nu_2$

## Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential  
Solution = MSW effect

S. Mikheev, A. Smirnov *Nuovo Cim. C9* (1986) 17-26

- ▶ Potential changes slowly  $\Rightarrow$  neutrinos remain in same state
- ▶ In center of the sun  $\nu_e \approx \nu_2$
- ▶ Neutrinos leave sun as  $\nu_2$

## Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential  
Solution = MSW effect

S. Mikheev, A. Smirnov *Nuovo Cim.* C9 (1986) 17-26

- ▶ Potential changes slowly  $\Rightarrow$  neutrinos remain in same state
- ▶ In center of the sun  $\nu_e \approx \nu_2$
- ▶ Neutrinos leave sun as  $\nu_2$
- ▶ Probability to detect  $\nu_e$  is simply:

$$P_{ee} = P_{e2}^{\odot} P_{22}^{\text{vac}} P_{2e}^{\text{det}} \approx 1 \times 1 \times |U_{e2}|^2 \approx \sin^2 \theta_{12}$$

Bonus question: do we see more solar neutrinos at day or night?

## Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential  
Solution = MSW effect

S. Mikheev, A. Smirnov *Nuovo Cim.* C9 (1986) 17-26

- ▶ Potential changes slowly  $\Rightarrow$  neutrinos remain in same state
- ▶ In center of the sun  $\nu_e \approx \nu_2$
- ▶ Neutrinos leave sun as  $\nu_2$
- ▶ Probability to detect  $\nu_e$  is simply:

$$P_{ee} = P_{e2}^{\odot} P_{22}^{\text{vac}} P_{2e}^{\text{det}} \approx 1 \times 1 \times |U_{e2}|^2 \approx \sin^2 \theta_{12}$$

Bonus question: do we see more solar neutrinos at day or night?

Neutrinos in SNe experience MSW effect too,  
but they also experience neutrino-neutrino interactions

Propagation in SNe is much more involved



## Dirac mass term

If  $\nu_R$  (or  $\bar{\nu}_L$ ) existed it would be a gauge singlet

## Dirac mass term

If  $\nu_R$  (or  $\bar{\nu}_L$ ) existed it would be a gauge singlet

Charged leptons get their mass:

$$\mathcal{L} \supset y_e \bar{\ell}_L \phi e_R$$

## Dirac mass term

If  $\nu_R$  (or  $\bar{\nu}_L$ ) existed it would be a gauge singlet

Charged leptons get their mass:

$$\mathcal{L} \supset y_e \bar{\ell}_L \phi e_R$$

Neutrino oscillations likely indicate that  $\nu_R$  (and  $\bar{\nu}_L$ ) exist:

$$\mathcal{L} \supset y_{\nu_e} \bar{\ell}_L \phi \nu_{e,R}$$

## Dirac mass term

If  $\nu_R$  (or  $\bar{\nu}_L$ ) existed it would be a gauge singlet

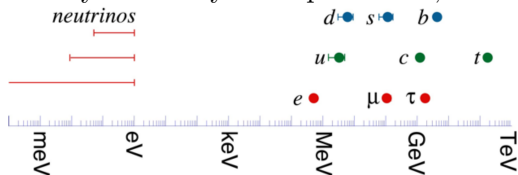
Charged leptons get their mass:

$$\mathcal{L} \supset y_e \bar{\ell}_L \phi e_R$$

Neutrino oscillations likely indicate that  $\nu_R$  (and  $\bar{\nu}_L$ ) exist:

$$\mathcal{L} \supset y_{\nu_e} \bar{\ell}_L \phi \nu_{e,R}$$

Perfectly valid way to acquire mass, but ...



Neutrino Yukawa couplings  $\lesssim 10^{-12}$

But electron Yukawa coupling  $\sim 10^{-6}$

# Majorana mass term

Not disallowed for neutrinos, so maybe it's there

$$\mathcal{L} \supset m \bar{\nu}_L \nu_L^c$$

Consequences:

# Majorana mass term

Not disallowed for neutrinos, so maybe it's there

$$\mathcal{L} \supset m \bar{\nu}_L \nu_L^c$$

Consequences:

1. Connects a  $\nu_L$  with a  $(\bar{\nu})_R$

# Majorana mass term

Not disallowed for neutrinos, so maybe it's there

$$\mathcal{L} \supset m \bar{\nu}_L \nu_L^c$$

Consequences:

1. Connects a  $\nu_L$  with a  $(\bar{\nu})_R$
2. Violates  $L$  number by two units

Leads to the neutrinoless double beta decay process

# Majorana mass term

Not disallowed for neutrinos, so maybe it's there

$$\mathcal{L} \supset m \bar{\nu}_L \nu_L^c$$

Consequences:

1. Connects a  $\nu_L$  with a  $(\bar{\nu})_R$
2. Violates  $L$  number by two units

Leads to the neutrinoless double beta decay process

3. Thus  $\nu \rightarrow \bar{\nu}$  mixing is possible

This is why charged leptons and quarks can't have Majorana mass terms



# Majorana mass term

Not disallowed for neutrinos, so maybe it's there

$$\mathcal{L} \supset m \bar{\nu}_L \nu_L^c$$

Consequences:

1. Connects a  $\nu_L$  with a  $(\bar{\nu})_R$
2. Violates  $L$  number by two units

Leads to the neutrinoless double beta decay process

3. Thus  $\nu \rightarrow \bar{\nu}$  mixing is possible

This is why charged leptons and quarks can't have Majorana mass terms

4. Only two total states:  $\nu_L$  and  $(\bar{\nu})_R$

If no Majorana term then probably four states:  $\nu_L$ ,  $\nu_R$ ,  $(\bar{\nu})_R$ , and  $(\bar{\nu})_L$

# Majorana mass term

Not disallowed for neutrinos, so maybe it's there

$$\mathcal{L} \supset m \bar{\nu}_L \nu_L^c$$

Consequences:

1. Connects a  $\nu_L$  with a  $(\bar{\nu})_R$
2. Violates  $L$  number by two units

Leads to the neutrinoless double beta decay process

3. Thus  $\nu \rightarrow \bar{\nu}$  mixing is possible

This is why charged leptons and quarks can't have Majorana mass terms

4. Only two total states:  $\nu_L$  and  $(\bar{\nu})_R$

If no Majorana term then probably four states:  $\nu_L$ ,  $\nu_R$ ,  $(\bar{\nu})_R$ , and  $(\bar{\nu})_L$

5. Difference is only relevant phenomenologically for  $p_\nu \sim m_\nu$

Cosmic neutrino background

Internal leg in neutrinoless double beta decay diagram

# Seesaw

Majorana mass term does not forbid Dirac mass term  
Many different seesaw realizations

Ingredients:



# Seesaw

Majorana mass term does not forbid Dirac mass term  
Many different seesaw realizations

Ingredients:

1. 3 left handed neutrinos  $\nu$  in a SU(2) doublet



# Seesaw

Majorana mass term does not forbid Dirac mass term  
Many different seesaw realizations

Ingredients:

1. 3 left handed neutrinos  $\nu$  in a SU(2) doublet
2. 3 right handed neutrinos  $N$  with Majorana mass term



# Seesaw

Majorana mass term does not forbid Dirac mass term  
Many different seesaw realizations

Ingredients:

1. 3 left handed neutrinos  $\nu$  in a SU(2) doublet
2. 3 right handed neutrinos  $N$  with Majorana mass term
3. Write down a Dirac mass term from the Higgs for  $\nu$  and  $N$



# Seesaw

Majorana mass term does not forbid Dirac mass term  
Many different seesaw realizations

Ingredients:

1. 3 left handed neutrinos  $\nu$  in a SU(2) doublet
2. 3 right handed neutrinos  $N$  with Majorana mass term
3. Write down a Dirac mass term from the Higgs for  $\nu$  and  $N$
4. After electroweak symmetry breaking

$$\mathcal{L} \supset -m_D \bar{\nu}_L N_R - \frac{1}{2} M_R \overline{(N^c)}_L N_R$$



# Seesaw

Majorana mass term does not forbid Dirac mass term  
Many different seesaw realizations

Ingredients:

1. 3 left handed neutrinos  $\nu$  in a SU(2) doublet
2. 3 right handed neutrinos  $N$  with Majorana mass term
3. Write down a Dirac mass term from the Higgs for  $\nu$  and  $N$
4. After electroweak symmetry breaking

$$\mathcal{L} \supset -m_D \bar{\nu}_L N_R - \frac{1}{2} M_R \overline{(N^c)}_L N_R$$

5. Diagonalize the mass matrix between bare and mass bases

$$\mathbb{N}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathbb{N}^* = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}, \quad \begin{pmatrix} \nu \\ N^c \end{pmatrix}_L = \mathbb{N} \begin{pmatrix} \nu_m \\ N_m^c \end{pmatrix}$$



Majorana mass term does not forbid Dirac mass term  
 Many different seesaw realizations

Ingredients:

1. 3 left handed neutrinos  $\nu$  in a SU(2) doublet
2. 3 right handed neutrinos  $N$  with Majorana mass term
3. Write down a Dirac mass term from the Higgs for  $\nu$  and  $N$
4. After electroweak symmetry breaking

$$\mathcal{L} \supset -m_D \bar{\nu}_L N_R - \frac{1}{2} M_R \overline{(N^c)}_L N_R$$

5. Diagonalize the mass matrix between bare and mass bases

$$\mathbb{N}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathbb{N}^* = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}, \quad \begin{pmatrix} \nu \\ N^c \end{pmatrix}_L = \mathbb{N} \begin{pmatrix} \nu_m \\ N_m^c \end{pmatrix}$$

6. Physical mass terms for  $M_R \gg m_D$ :

$$m_\nu \approx -\frac{m_D^2}{M_R}, \quad M_N \approx M_R$$

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$

SuperK, IMB, IceCube

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK
- ▶ Reactor  $\nu_e$  disappearance:

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK
- ▶ Reactor  $\nu_e$  disappearance:
  - ▶ LBL  $\rightarrow \sin 2\theta_{12}$  and  $|\Delta m_{21}^2|$   
KamLAND

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK
- ▶ Reactor  $\nu_e$  disappearance:
  - ▶ LBL  $\rightarrow \sin 2\theta_{12}$  and  $|\Delta m_{21}^2|$   
KamLAND
  - ▶ Future LBL  $\rightarrow \pm \Delta m_{31}^2$   
JUNO

# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK
- ▶ Reactor  $\nu_e$  disappearance:
  - ▶ LBL  $\rightarrow \sin 2\theta_{12}$  and  $|\Delta m_{21}^2|$   
KamLAND
  - ▶ Future LBL  $\rightarrow \pm \Delta m_{31}^2$   
JUNO
  - ▶ MBL  $\rightarrow \theta_{13}$ ,  $|\Delta m_{31}^2|$   
Daya Bay, RENO, Double Chooz



# Experiment to Oscillation Parameters

Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK
- ▶ Reactor  $\nu_e$  disappearance:
  - ▶ LBL  $\rightarrow \sin 2\theta_{12}$  and  $|\Delta m_{21}^2|$   
KamLAND
  - ▶ Future LBL  $\rightarrow \pm \Delta m_{31}^2$   
JUNO
  - ▶ MBL  $\rightarrow \theta_{13}$ ,  $|\Delta m_{31}^2|$   
Daya Bay, RENO, Double Chooz
- ▶ Accelerator LBL  $\nu_e$  appearance:  $\pm \Delta m_{31}^2$ ,  $\pm \cos 2\theta_{23}$ ,  $\theta_{13}$ ,  $\delta$   
T2K, NOvA, T2HK, DUNE

# Experiment to Oscillation Parameters

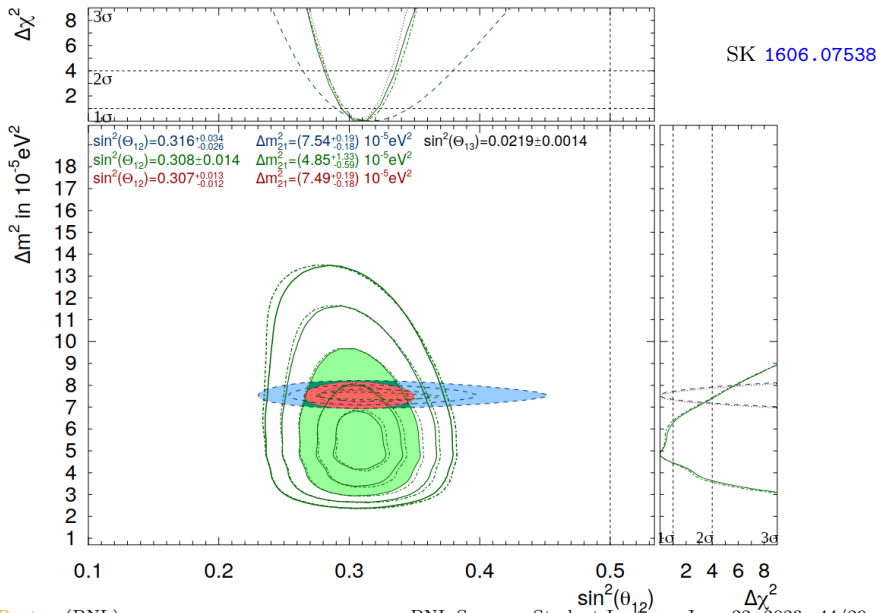
Six oscillation parameters:  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ,  $\delta$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$

- ▶ Atmospheric  $\nu_\mu$  disappearance  $\rightarrow \sin 2\theta_{23}$ ,  $|\Delta m_{31}^2|$   
SuperK, IMB, IceCube
- ▶ Solar  $\nu_e$  disappearance  $\rightarrow \pm \cos 2\theta_{12}$ ,  $\pm \Delta m_{21}^2$   
SNO, Borexino, SuperK
- ▶ Reactor  $\nu_e$  disappearance:
  - ▶ LBL  $\rightarrow \sin 2\theta_{12}$  and  $|\Delta m_{21}^2|$   
KamLAND
  - ▶ Future LBL  $\rightarrow \pm \Delta m_{31}^2$   
JUNO
  - ▶ MBL  $\rightarrow \theta_{13}$ ,  $|\Delta m_{31}^2|$   
Daya Bay, RENO, Double Chooz
- ▶ Accelerator LBL  $\nu_e$  appearance:  $\pm \Delta m_{31}^2$ ,  $\pm \cos 2\theta_{23}$ ,  $\theta_{13}$ ,  $\delta$   
T2K, NOvA, T2HK, DUNE

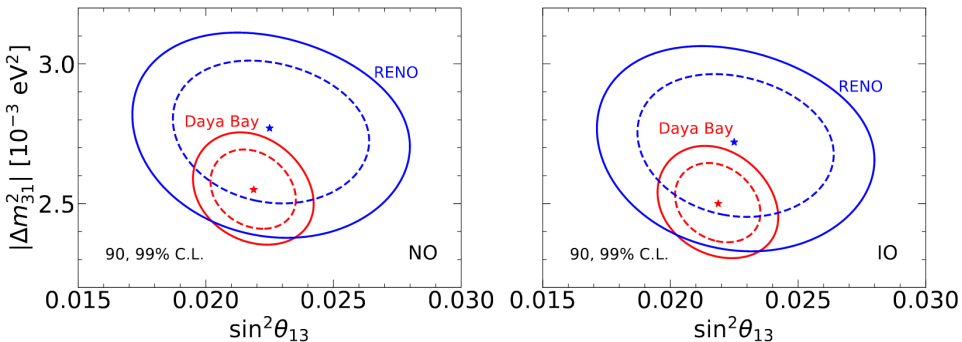
7th parameter: absolute mass scale

Cosmology, KATRIN,  $0\nu\beta\beta$

# Solar parameters: SK, SNO, KamLAND



# Reactor parameters



P. F. de Salas, et al. [2006.11237](#)

# Remaining oscillation unknowns

# Remaining oscillation unknowns

1. **Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$

# Remaining oscillation unknowns

1. **Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$ 
  - ▶ Matter effect

DUNE's strategy

# Remaining oscillation unknowns

1. **Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$

▶ Matter effect

DUNE's strategy

▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$

3% difference

JUNO's strategy



# Remaining oscillation unknowns

1. **Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$

- ▶ Matter effect

DUNE's strategy

- ▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$

3% difference

JUNO's strategy

- ▶ Slight ( $\sim 3\sigma$ ) preference for normal ordering ( $\Delta m_{31}^2 > 0$ )

# Remaining oscillation unknowns

1. **Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$

- ▶ Matter effect

DUNE's strategy

- ▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$

3% difference

JUNO's strategy

- ▶ Slight ( $\sim 3\sigma$ ) preference for normal ordering ( $\Delta m_{31}^2 > 0$ )
- ▶ Terminology: there are two orderings and three hierarchies

Quasi-degenerate is  $m_1 \sim m_2 \sim m_3$

# Remaining oscillation unknowns

- 1. Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$ 
  - ▶ Matter effect  
DUNE's strategy
  - ▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$   
3% difference  
JUNO's strategy
  - ▶ Slight ( $\sim 3\sigma$ ) preference for normal ordering ( $\Delta m_{31}^2 > 0$ )
  - ▶ Terminology: there are two orderings and three hierarchies  
Quasi-degenerate is  $m_1 \sim m_2 \sim m_3$
- 2.  $\theta_{23}$  octant:** whether  $\nu_3$  is more (upper) or less (lower)  $\nu_\mu$

# Remaining oscillation unknowns

- 1. Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$ 
  - ▶ Matter effect  
DUNE's strategy
  - ▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$   
3% difference  
JUNO's strategy
  - ▶ Slight ( $\sim 3\sigma$ ) preference for normal ordering ( $\Delta m_{31}^2 > 0$ )
  - ▶ Terminology: there are two orderings and three hierarchies  
Quasi-degenerate is  $m_1 \sim m_2 \sim m_3$
- 2.  $\theta_{23}$  octant:** whether  $\nu_3$  is more (upper) or less (lower)  $\nu_\mu$ 
  - ▶ Long-baseline appearance is good for this

# Remaining oscillation unknowns

- 1. Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$ 
  - ▶ Matter effect  
DUNE's strategy
  - ▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$   
3% difference  
JUNO's strategy
  - ▶ Slight ( $\sim 3\sigma$ ) preference for normal ordering ( $\Delta m_{31}^2 > 0$ )
  - ▶ Terminology: there are two orderings and three hierarchies  
Quasi-degenerate is  $m_1 \sim m_2 \sim m_3$
- 2.  $\theta_{23}$  octant:** whether  $\nu_3$  is more (upper) or less (lower)  $\nu_\mu$ 
  - ▶ Long-baseline appearance is good for this
- 3. Complex phase  $\delta$ :** governs if/how much CP is violated  
Relationship between  $\delta$  and CP violation: [PBD](#), R. Pestes [2006.09384](#)

# Remaining oscillation unknowns

- 1. Atmospheric mass ordering:** the sign of  $\Delta m_{31}^2$  &  $\Delta m_{32}^2$ 
  - ▶ Matter effect  
DUNE's strategy
  - ▶ Differentiate  $\Delta m_{31}^2$  and  $\Delta m_{32}^2$   
3% difference  
JUNO's strategy
  - ▶ Slight ( $\sim 3\sigma$ ) preference for normal ordering ( $\Delta m_{31}^2 > 0$ )
  - ▶ Terminology: there are two orderings and three hierarchies  
Quasi-degenerate is  $m_1 \sim m_2 \sim m_3$
- 2.  $\theta_{23}$  octant:** whether  $\nu_3$  is more (upper) or less (lower)  $\nu_\mu$ 
  - ▶ Long-baseline appearance is good for this
- 3. Complex phase  $\delta$ :** governs if/how much CP is violated  
Relationship between  $\delta$  and CP violation: [PBD](#), R. Pestes [2006.09384](#)
  - ▶ Long-baseline appearance is good for this

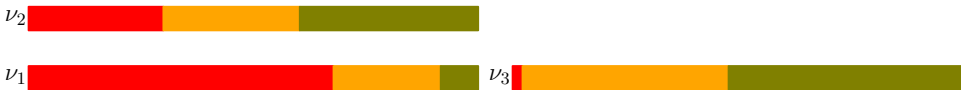
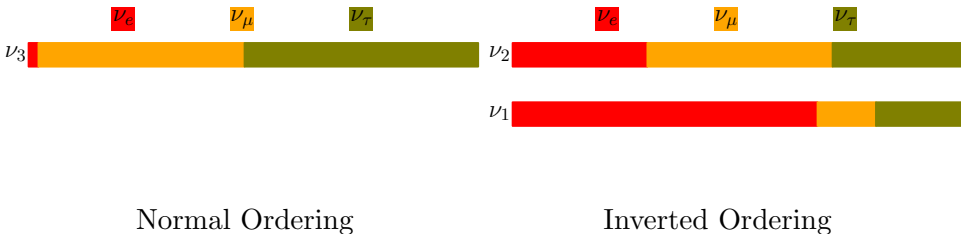
# Mass states in two orderings



Normal Ordering



# Mass states in two orderings

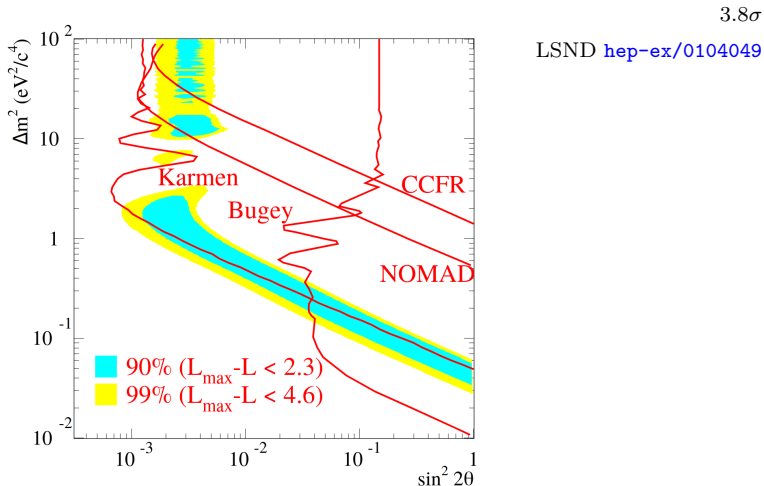




# LSND sees a $\sim 1$ eV sterile?

LSND at Los Alamos:

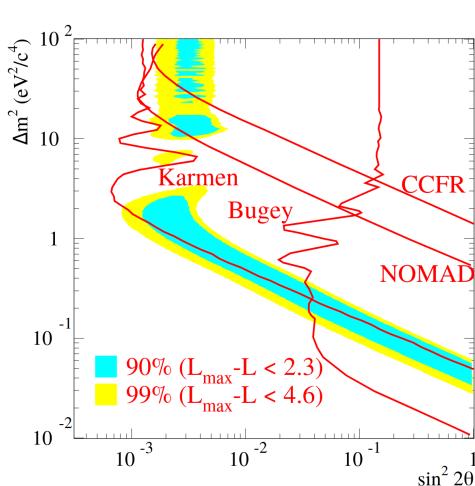
1.  $\bar{\nu}_\mu$  from  $\mu^+$  decay-at-rest
2. Saw an excess of  $\bar{\nu}_e$  events:  $87.9 \pm 22.4 \pm 6.0$



# LSND sees a $\sim 1$ eV sterile?

LSND at Los Alamos:

1.  $\bar{\nu}_\mu$  from  $\mu^+$  decay-at-rest
2. Saw an excess of  $\bar{\nu}_e$  events:  $87.9 \pm 22.4 \pm 6.0$



3.8 $\sigma$

LSND [hep-ex/0104049](https://arxiv.org/abs/hep-ex/0104049)

Could be a cut problem:  
J. Hill [hep-ex/9504009](https://arxiv.org/abs/hep-ex/9504009)

MiniBooNE also sees a  $\sim 1$  eV sterile?

Built at Fermilab to test LSND

# MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$

# MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$
2. Different production physics

# MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$
2. Different production physics
3. Different cross section

# MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$
2. Different production physics
3. Different cross section
4. Different backgrounds

# MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$
2. Different production physics
3. Different cross section
4. Different backgrounds
5. Different response to non-oscillation new physics



## MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$
2. Different production physics
3. Different cross section
4. Different backgrounds
5. Different response to non-oscillation new physics
6. Has an excess in both neutrino and anti-neutrino modes

## MiniBooNE also sees a $\sim 1$ eV sterile?

Built at Fermilab to test LSND

1. Higher energy and higher baseline: at the same  $L/E$
2. Different production physics
3. Different cross section
4. Different backgrounds
5. Different response to non-oscillation new physics
6. Has an excess in both neutrino and anti-neutrino modes
7. Excess is generally consistent with LSND under the oscillation hypothesis

# Latest MiniBooNE results

MiniBooNE 1805.12028

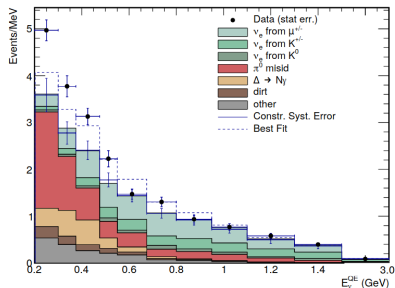


FIG. 1: The MiniBooNE neutrino mode  $E_{\nu}^{QE}$  distributions, corresponding to the total  $12.84 \times 10^{20}$  POT data, for  $\nu_e$  CCQE data (points with statistical errors) and background (histogram with systematic errors). The dashed curve shows the best fit to the neutrino-mode data assuming two-neutrino oscillations. The last bin is for the energy interval from 1500-3000 MeV.

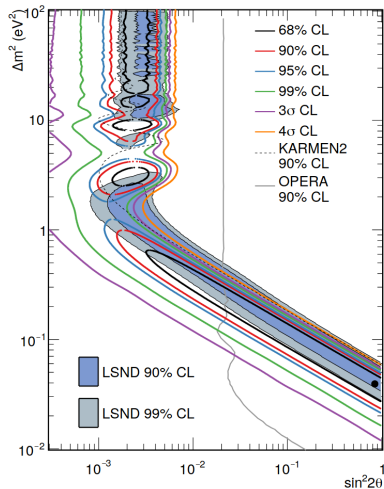


FIG. 3: MiniBooNE allowed regions in neutrino mode ( $12.84 \times 10^{20}$  POT) for events with  $200 < E_{\nu}^{QE} < 3000$  MeV within a two-neutrino oscillation model. The shaded areas show the 90% and 99% C.L. LSND  $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$  allowed regions. The black point shows the MiniBooNE best fit point. Also shown are 90% C.L. limits from the KARMEN [\[37\]](#) and OPERA [\[38\]](#) experiments.

# Gallium anomaly

# Gallium anomaly

1. GALLEX and SAGE were low energy solar experiments

GALLEX PLB 342 (1995) 440

SAGE PRL 77 (1996) 4708

# Gallium anomaly

1. GALLEX and SAGE were low energy solar experiments

GALLEX PLB 342 (1995) 440

SAGE PRL 77 (1996) 4708

2. Calibrated detectors with intense beta decay sources

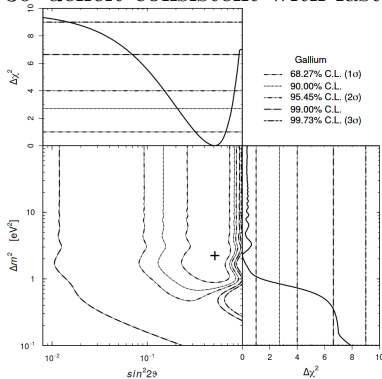
# Gallium anomaly

1. GALLEX and SAGE were low energy solar experiments

GALLEX PLB 342 (1995) 440

SAGE PRL 77 (1996) 4708

2. Calibrated detectors with intense beta decay sources
3.  $3\sigma$  deficit consistent with fast oscillations



C. Giunti, M. Laveder [1006.3244](#)

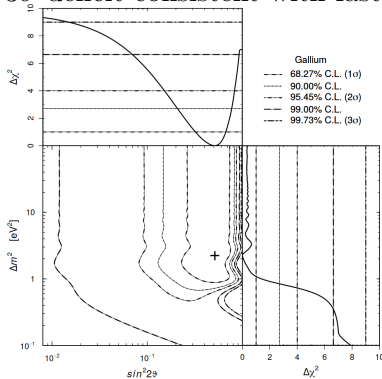
# Gallium anomaly

1. GALLEX and SAGE were low energy solar experiments

GALLEX PLB 342 (1995) 440

SAGE PRL 77 (1996) 4708

2. Calibrated detectors with intense beta decay sources
3.  $3\sigma$  deficit consistent with fast oscillations



C. Giunti, M. Laveder [1006.3244](#)

4. Using improved nuclear shell models:  $3.0\sigma \rightarrow 2.3\sigma$

J. Kostensalo, et al. [1906.10980](#)

BNL Summer Student Lecture: June 22, 2023 51/29



# Reactor anti-neutrino anomaly

# Reactor anti-neutrino anomaly

- ▶ Near detectors  $\mathcal{O}(100)$  m from cores, expect no oscillations

# Reactor anti-neutrino anomaly

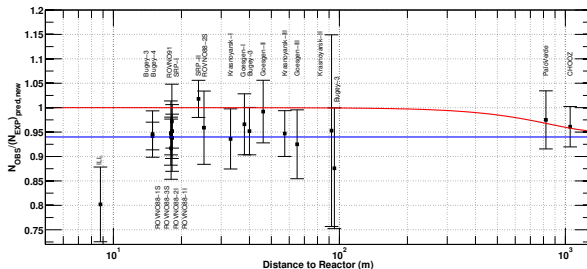
- ▶ Near detectors  $\mathcal{O}(100)$  m from cores, expect no oscillations
- ▶ New large frequency fast oscillations  $\Rightarrow$  overall flux deficit

Large frequency  $\Rightarrow$  large  $\Delta m_{41}^2$

# Reactor anti-neutrino anomaly

- ▶ Near detectors  $\mathcal{O}(100)$  m from cores, expect no oscillations
- ▶ New large frequency fast oscillations  $\Rightarrow$  overall flux deficit
- ▶ Have to compare to theory prediction

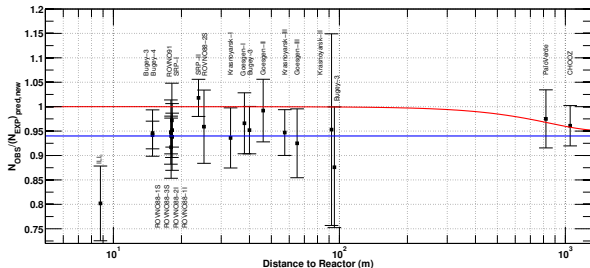
Large frequency  $\Rightarrow$  large  $\Delta m_{41}^2$



# Reactor anti-neutrino anomaly

- ▶ Near detectors  $\mathcal{O}(100)$  m from cores, expect no oscillations
- ▶ New large frequency fast oscillations  $\Rightarrow$  overall flux deficit
- ▶ Have to compare to theory prediction

Large frequency  $\Rightarrow$  large  $\Delta m_{41}^2$



- ▶ Deficit compared to theory  
 $\Rightarrow \Delta m_{41}^2 \gtrsim 1.5 \text{ eV}^2 \sin^2 2\theta_{14} \sim 0.14$

G. Mention, et al. [1101.2755](#)

# Reactor anti-neutrino anomaly: fuel evolution

# Reactor anti-neutrino anomaly: fuel evolution

- ▶ Reactor flux calculations involve thousands of isotopes

Many never directly observed

## Reactor anti-neutrino anomaly: fuel evolution

- ▶ Reactor flux calculations involve thousands of isotopes  
Many never directly observed
- ▶ The amount of isotopes in reactors varies in time

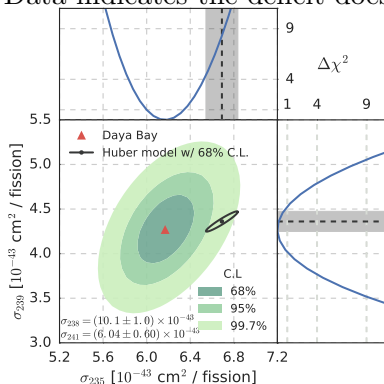


## Reactor anti-neutrino anomaly: fuel evolution

- ▶ Reactor flux calculations involve thousands of isotopes  
Many never directly observed
- ▶ The amount of isotopes in reactors varies in time
- ▶ If the deficit was due to neutrino physics it would be independent of the flux

# Reactor anti-neutrino anomaly: fuel evolution

- ▶ Reactor flux calculations involve thousands of isotopes  
Many never directly observed
- ▶ The amount of isotopes in reactors varies in time
- ▶ If the deficit was due to neutrino physics it would be independent of the flux
- ▶ Data indicates the deficit does evolve with flux



Daya Bay [1704.01082](https://arxiv.org/abs/1704.01082)

# Light sterile global picture

## Light sterile global picture

- ▶ Appears that there is  $\nu_\mu \rightarrow \nu_e$  with  $\Delta m_{41}^2 \sim 1 \text{ eV}^2$

LSND, MiniBooNE

## Light sterile global picture

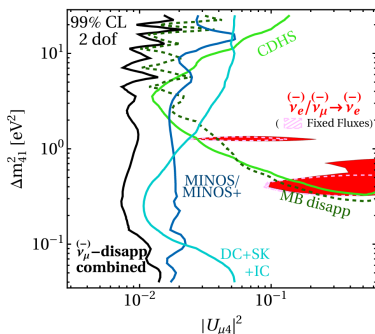
- ▶ Appears that there is  $\nu_\mu \rightarrow \nu_e$  with  $\Delta m_{41}^2 \sim 1 \text{ eV}^2$   
LSND, MiniBooNE
- ▶ Could be  $\nu_e \rightarrow \nu_e$  at comparable  $\Delta m_{41}^2$   
Gallium, reactor anti-neutrino anomaly

## Light sterile global picture

- ▶ Appears that there is  $\nu_\mu \rightarrow \nu_e$  with  $\Delta m_{41}^2 \sim 1 \text{ eV}^2$   
LSND, MiniBooNE
- ▶ Could be  $\nu_e \rightarrow \nu_e$  at comparable  $\Delta m_{41}^2$   
Gallium, reactor anti-neutrino anomaly
- ▶ Appearance also **needs**  $\nu_\mu \rightarrow \nu_\mu$

# Light sterile global picture

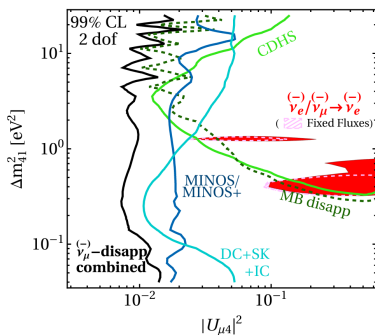
- ▶ Appears that there is  $\nu_\mu \rightarrow \nu_e$  with  $\Delta m_{41}^2 \sim 1 \text{ eV}^2$   
LSND, MiniBooNE
- ▶ Could be  $\nu_e \rightarrow \nu_e$  at comparable  $\Delta m_{41}^2$   
Gallium, reactor anti-neutrino anomaly
- ▶ Appearance also **needs**  $\nu_\mu \rightarrow \nu_\mu$
- ▶ Strong constraints: IceCube (atm) and MINOS+ (LBL acc)



M. Dentler, et al. [1803.10661](https://arxiv.org/abs/1803.10661)

## Light sterile global picture

- ▶ Appears that there is  $\nu_\mu \rightarrow \nu_e$  with  $\Delta m_{41}^2 \sim 1 \text{ eV}^2$   
LSND, MiniBooNE
- ▶ Could be  $\nu_e \rightarrow \nu_e$  at comparable  $\Delta m_{41}^2$   
Gallium, reactor anti-neutrino anomaly
- ▶ Appearance also **needs**  $\nu_\mu \rightarrow \nu_\mu$
- ▶ Strong constraints: IceCube (atm) and MINOS+ (LBL acc)



M. Dentler, et al. [1803.10661](https://arxiv.org/abs/1803.10661)

- ▶ Are also cosmological bounds



## Other anomalies

- ▶ ANITA

- ▶ Balloon looking for UHE earth-skimming tau neutrinos
- ▶ Neutrinos are readily absorbed at these energies
- ▶ Detected several neutrinos at  $30^\circ$  below the horizon
- ▶ Remains unexplained

ANITA [1803.05088](#)

## Other anomalies

- ▶ ANITA
  - ▶ Balloon looking for UHE earth-skimming tau neutrinos
  - ▶ Neutrinos are readily absorbed at these energies
  - ▶ Detected several neutrinos at  $30^\circ$  below the horizon
  - ▶ Remains unexplained ANITA [1803.05088](#)
- ▶ X-ray 3.5 keV line
  - ▶ An x-ray line has been seen in galaxies  
E. Bulbul, et al. [1402.2301](#) & A. Boyarsky, et al. [1402.4119](#)
  - ▶ Consistent with 7 keV sterile neutrino DM decaying
  - ▶ Separate analysis of our galaxy finds nothing  
C. Dessert, N. Rodd, B. Safdi [1812.06976](#)

## Other anomalies

- ▶ ANITA
  - ▶ Balloon looking for UHE earth-skimming tau neutrinos
  - ▶ Neutrinos are readily absorbed at these energies
  - ▶ Detected several neutrinos at  $30^\circ$  below the horizon
  - ▶ Remains unexplained ANITA [1803.05088](#)
- ▶ X-ray 3.5 keV line
  - ▶ An x-ray line has been seen in galaxies  
E. Bulbul, et al. [1402.2301](#) & A. Boyarsky, et al. [1402.4119](#)
  - ▶ Consistent with 7 keV sterile neutrino DM decaying
  - ▶ Separate analysis of our galaxy finds nothing  
C. Dessert, N. Rodd, B. Safdi [1812.06976](#)
- ▶ Track and cascade spectra at IceCube
  - ▶ IceCube measures tracks ( $\nu_\mu$ ) and cascades ( $\nu_e, \nu_\tau$ )
  - ▶ At  $> 3\sigma$  neutrino decay is preferred  
PBD, I. Tamborra [1805.05950](#)

# Other anomalies

- ▶ ANITA
  - ▶ Balloon looking for UHE earth-skimming tau neutrinos
  - ▶ Neutrinos are readily absorbed at these energies
  - ▶ Detected several neutrinos at  $30^\circ$  below the horizon
  - ▶ Remains unexplained ANITA [1803.05088](#)
- ▶ X-ray 3.5 keV line
  - ▶ An x-ray line has been seen in galaxies  
E. Bulbul, et al. [1402.2301](#) & A. Boyarsky, et al. [1402.4119](#)
  - ▶ Consistent with 7 keV sterile neutrino DM decaying
  - ▶ Separate analysis of our galaxy finds nothing  
C. Dessert, N. Rodd, B. Safdi [1812.06976](#)
- ▶ Track and cascade spectra at IceCube
  - ▶ IceCube measures tracks ( $\nu_\mu$ ) and cascades ( $\nu_e, \nu_\tau$ )
  - ▶ At  $> 3\sigma$  neutrino decay is preferred
- ▶ NOvA and T2K slightly disagree PBD, I. Tamborra [1805.05950](#)
  - ▶ Flavor changing CP violating non-standard interactions
  - ▶ Model preference is slight  $\sim 2\sigma$
  - ▶ Testable at IceCube and COHERENT  
PBD, J. Gehrlein, R. Pestes [2008.01110](#)

# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

1. Any new neutrino  $m_\nu \lesssim M_Z/2$  must be right handed

# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

1. Any new neutrino  $m_\nu \lesssim M_Z/2$  must be right handed
2. A fourth neutrino could affect oscillations directly

# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

1. Any new neutrino  $m_\nu \lesssim M_Z/2$  must be right handed
2. A fourth neutrino could affect oscillations directly
3. Add one more  $\Delta m_{41}^2$

Known to not be near  $\Delta m_{21}^2$  or  $\Delta m_{31}^2$



# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

1. Any new neutrino  $m_\nu \lesssim M_Z/2$  must be right handed
2. A fourth neutrino could affect oscillations directly
3. Add one more  $\Delta m_{41}^2$

Known to not be near  $\Delta m_{21}^2$  or  $\Delta m_{31}^2$

4. Add three new angles and two new complex phases

# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

1. Any new neutrino  $m_\nu \lesssim M_Z/2$  must be right handed
2. A fourth neutrino could affect oscillations directly
3. Add one more  $\Delta m_{41}^2$

Known to not be near  $\Delta m_{21}^2$  or  $\Delta m_{31}^2$

4. Add three new angles and two new complex phases
5. In most oscillation cases  $3 + 1$  is sufficient

# Sterile neutrino

Sterile neutrinos may well exist, but at  $\sim 1$  eV?

1. Any new neutrino  $m_\nu \lesssim M_Z/2$  must be right handed
2. A fourth neutrino could affect oscillations directly
3. Add one more  $\Delta m_{41}^2$

Known to not be near  $\Delta m_{21}^2$  or  $\Delta m_{31}^2$

4. Add three new angles and two new complex phases
5. In most oscillation cases  $3 + 1$  is sufficient
6. Note that the NC term in the matter effect matters now

## Steriles: 1 eV

### For:

1. LSND
2. MiniBooNE
3. Gallium
4. Reactor anti-neutrino

### Against:

1. MINOS+: long-baseline accelerator with both near and far detectors
2. IceCube atmospheric:  
via the matter effect

## Steriles: 1 eV

### For:

1. LSND
2. MiniBooNE
3. Gallium
4. Reactor anti-neutrino

### Against:

1. MINOS+: long-baseline accelerator with both near and far detectors
2. IceCube atmospheric:  
via the matter effect

Evidence for requires theory predictions or measuring energy  
Evidence against leverages near/far detectors or angles

## Steriles: 1 eV

### For:

1. LSND
2. MiniBooNE
3. Gallium
4. Reactor anti-neutrino

### Against:

1. MINOS+: long-baseline accelerator with both near and far detectors
2. IceCube atmospheric: via the matter effect

Evidence for requires theory predictions or measuring energy  
Evidence against leverages near/far detectors or angles

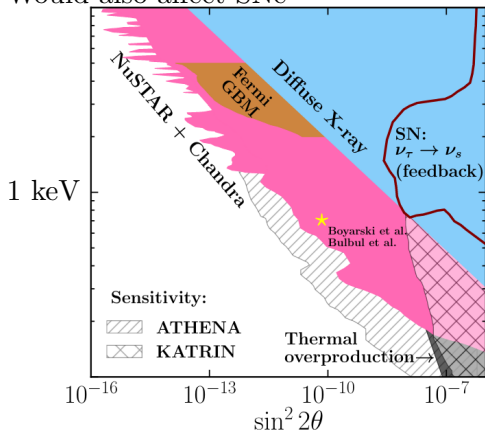
### Ongoing/upcoming probes:

1. MicroBooNE → Short baseline neutrino program (three detectors)
2. Short baseline reactor experiments: see wiggles directly!

NEOS, DANSS, PROSPECT

# Steriles: keV

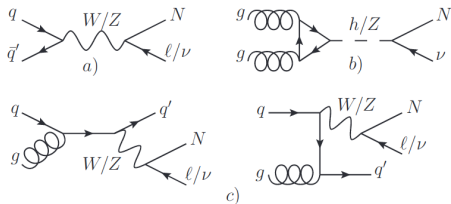
- ▶ keV sterile neutrinos can be DM
- ▶ Would be a bit high in temperature
- ▶ A possible hint of their existence at 7 keV
- ▶ Would also affect SNe



A. Suliga, I. Tamborra, M. Wu [2004.11389](#)

# Steriles: GeV+

If they are heavy they won't affect oscillations, just kinematics



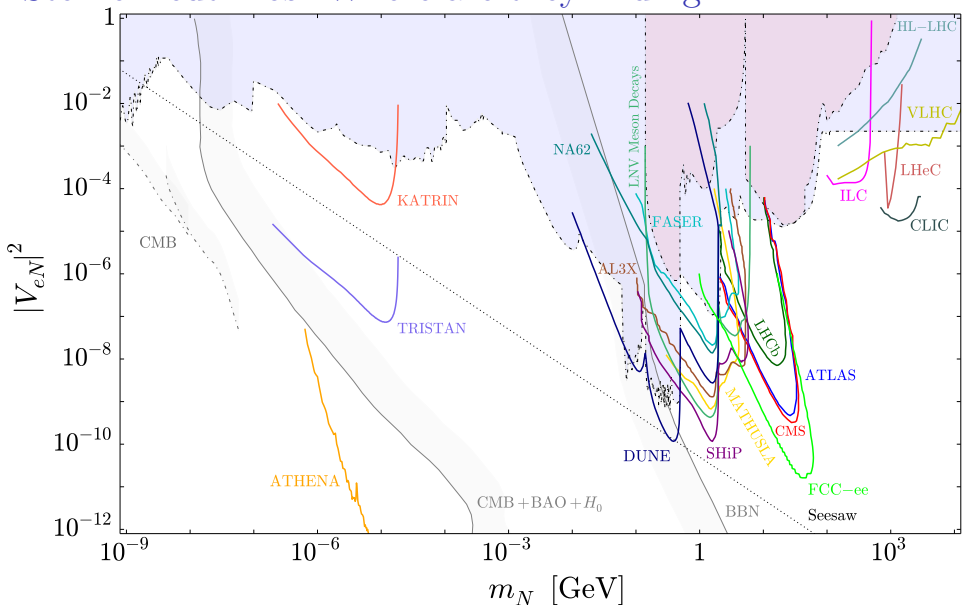
**Figure 7.** HNL production channels: a) Drell-Yan-type process; b) gluon fusion; c) quark-gluon fusion.

K. Bondarenko, et al. [1805.08567](#)

- ▶ Look in colliders, beam dumps
- ▶ Battle between energy and intensity



# Sterile Neutrinos: Where are they Hiding?



F. Deppisch CERN Neutrino Platform '19

BNL Summer Student Lecture: June 22, 2023 60/29

# Non-standard neutrino interactions

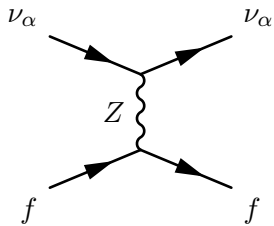
What if there was a new matter-effect like interaction?

L. Wolfenstein [PRD 17 \(1978\)](#)

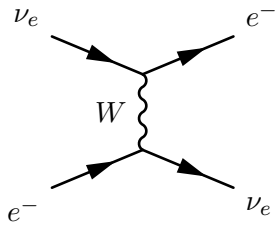
Recent overview: [PBD](#), et al. [1907.00991](#)

- ▶ Can affect propagation, production, detection
- ▶ Scales like the matter potential
- ▶ Can have own non-trivial flavor & CP violating structure
- ▶ Testable in scattering experiments, early universe, and SNe
- ▶ Leads to a degeneracy: mass ordering can't be determined

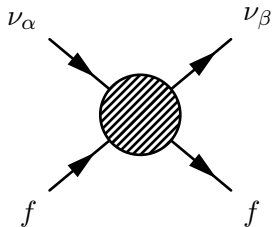
# Matter Effects in Feynman Diagrams



$$V_{\text{NC}} = \mp \frac{1}{2} \sqrt{2} G_F n_n$$

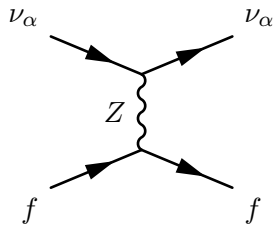


$$V_{\text{CC}} = \pm \sqrt{2} G_F n_e$$

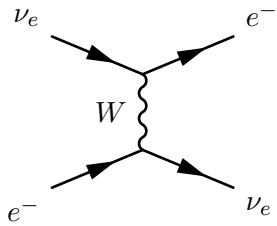


$$V_{\text{NSI}} = \pm \epsilon_{\alpha\beta}^{f,X} \sqrt{2} G_F n_f$$

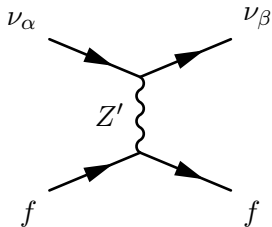
# Matter Effects in Feynman Diagrams



$$V_{\text{NC}} = \mp \frac{1}{2} \sqrt{2} G_F n_n$$



$$V_{\text{CC}} = \pm \sqrt{2} G_F n_e$$



$$V_{\text{NSI}} = \frac{n_f}{2} \frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$

## NSI at the Hamiltonian Level

$$H^{\text{vac}} = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger$$

$$H^{\text{mat,SM}} = \frac{a}{2E} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

## NSI at the Hamiltonian Level

$$H^{\text{vac}} = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger$$

$$H^{\text{mat,SM}} = \frac{a}{2E} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

$$H^{\text{mat,NSI}} = \frac{a}{2E} \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

$$H = H^{\text{vac}} + H^{\text{mat,SM}} + H^{\text{mat,NSI}}$$

# NSI at the Lagrangian Level

EFT Lagrangian:

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P f)$$

$$\text{with } \Lambda = \frac{1}{\sqrt{2\sqrt{2}\epsilon}G_F}.$$

# NSI at the Lagrangian Level

EFT Lagrangian:

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P f)$$

$$\text{with } \Lambda = \frac{1}{\sqrt{2}\sqrt{2}\epsilon G_F}.$$

Simplified model Lagrangian:

$$\mathcal{L}_{\text{NSI}} = g_\nu Z'_\mu \bar{\nu} \gamma^\mu \nu + g_f Z'_\mu \bar{f} \gamma^\mu f$$

which gives a potential

$$V_{\text{NSI}} \propto \frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$



# NSI at the Lagrangian Level

EFT Lagrangian:

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P f)$$

$$\text{with } \Lambda = \frac{1}{\sqrt{2}\sqrt{2}\epsilon G_F}.$$

Simplified model Lagrangian:

$$\mathcal{L}_{\text{NSI}} = g_\nu Z'_\mu \bar{\nu} \gamma^\mu \nu + g_f Z'_\mu \bar{f} \gamma^\mu f$$

which gives a potential

$$V_{\text{NSI}} \propto \frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$

Models with large NSIs consistent with CLFV:

- Y. Farzan, I. Shoemaker [1512.09147](#)   Y. Farzan, J. Heeck [1607.07616](#)  
D. Forero and W. Huang [1608.04719](#)   U. Dey, N. Nath, S. Sadhukhan [1804.05808](#)  
K. Babu, A. Friedland, P. Machado, I. Mocioiu [1705.01822](#)   Y. Farzan [1912.09408](#)  
PBD, Y. Farzan, I. Shoemaker [1804.03660](#)

# Neutrino Decay

Since neutrinos have different masses, they decay

- ▶ Loop suppressed
- ▶ Long lifetime:  $\tau \gtrsim 10^{35}$  years

Test this!

Typical Lagrangian for  $\nu_i \rightarrow \nu_j + \phi$  with  $m_i > m_j$

$$\mathcal{L} \supset \frac{g_{ij}}{2} \bar{\nu}_j \nu_i \phi + \frac{g'_{ij}}{2} \bar{\nu}_j i \gamma_5 \nu_i \phi$$

# Neutrino Decay Phenomenology

Neutrino decay is phenomenologically classified into:

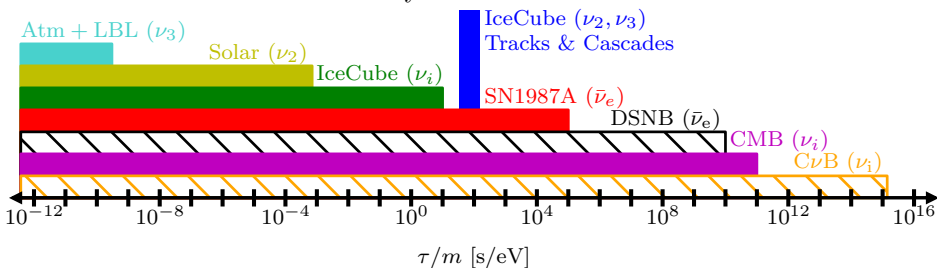
- ▶ **Invisible decay:**

- ▶ The decay products are sterile or too low energy to be detected
- ▶ Results in a *depletion* of the flux below the relevant energy

- ▶ **Visible decay:**

- ▶ Decay products are detected
- ▶ In addition to depletion, there is *regeneration*
- ▶ Regeneration happens at a lower energy than depletion

## Invisible $\nu$ Decay Constraints and Evidence



$\tau/m$  [s/eV]

M. Gonzalez-Garcia and M. Maltoni 0802.3699

J. Berryman, A. de Gouvea, D. Hernandez 1411.0308

G. Pagliaroli, et al. 1506.02624

PBD, I. Tamborra 1805.05950

Kamiokande-II, PRL 58 1490 (1987)

S. Ando hep-ph/0307169

S. Hannestad, G. Raffelt hep-ph/0509278

A. Long, C. Lunardini, E. Sabancila 1405.7654

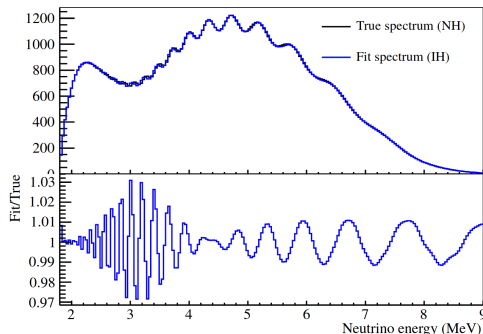
# Other new physics searches

1. Unitary violation
2. Decoherence
3. Lorentz invariance violation and CPT violation
4. Dark matter interactions
5. Neutrino magnetic moment
6. Combination of new physics scenarios
7.  $\vdots$

# Next generation oscillation experiments

JUNO: KamLAND 2.0, coming online in  $\sim 1$  year

1. Improved measurement of solar parameters  $\theta_{12}$ ,  $\Delta m_{21}^2$
2. Measurements of MBL reactor parameters  $\theta_{13}$ ,  $\Delta m_{31}^2$
3. Mass ordering measurement by  $\Delta m_{31}^2$  vs.  $\Delta m_{32}^2$  discrimination



JUNO 1508.07166

# Next generation oscillation experiments

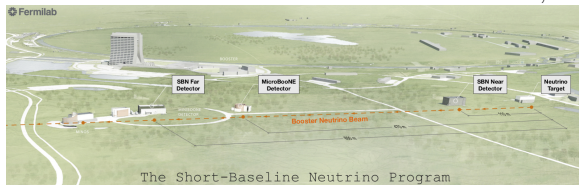
- ▶ Accelerator

- ▶ Short baseline neutrino program at Fermilab

MicroBooNE: taking data since 2015

Short baseline neutrino detector (SBND): near detector, coming online nowish

ICARUS: far detector, coming online nowish



P. Machado, O. Palamara, D. Schmitz [1903.04608](#)

# Next generation oscillation experiments

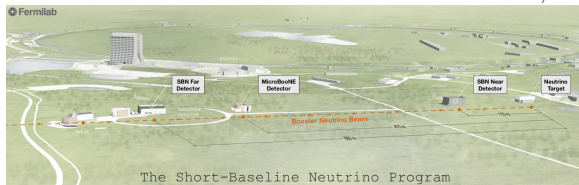
- ▶ Accelerator

- ▶ Short baseline neutrino program at Fermilab

MicroBooNE: taking data since 2015

Short baseline neutrino detector (SBND): near detector, coming online nowish

ICARUS: far detector, coming online nowish



P. Machado, O. Palamara, D. Schmitz [1903.04608](#)

- ▶ DUNE from Fermilab to SURF in South Dakota: 6+ years out

1300 km: longest long-baseline accelerator experiment

Broadband beam peaked at  $\sim 2.5$  GeV: highest energy accelerator experiment



# Next generation oscillation experiments

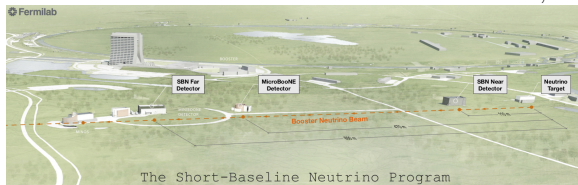
## ▶ Accelerator

### ▶ Short baseline neutrino program at Fermilab

MicroBooNE: taking data since 2015

Short baseline neutrino detector (SBND): near detector, coming online nowish

ICARUS: far detector, coming online nowish



P. Machado, O. Palamara, D. Schmitz [1903.04608](#)

## ▶ DUNE from Fermilab to SURF in South Dakota: 6+ years out

1300 km: longest long-baseline accelerator experiment

Broadband beam peaked at  $\sim 2.5$  GeV: highest energy accelerator experiment

## ▶ T2HK in Japan: Similar to T2K: 5+ years out

Increasing protons on target (POT)

New far detector, HyperK

# Next generation oscillation experiments

Hyper-KamiokaNDE: A new much larger SuperK-like detector under a different mountain

- ▶ Long-baseline program is called T2HK
- ▶ Will have additional solar neutrino physics
  - Less sensitive than SK due to less overburden and more backgrounds
- ▶ Atmospheric neutrinos
- ▶ Galactic supernova neutrinos
- ▶ Diffuse supernova neutrino background (DSNB)

Super-K was loaded with Gadolinium last year to reduce backgrounds to detect the DSNB

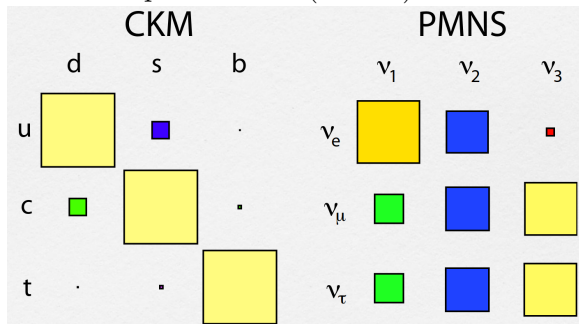
# Next generation oscillation experiments

## **Possible** future oscillation experiments

- ▶ T2HKK: Put one of the HK detectors in Korea
- ▶ ESSnuSB: Long baseline accelerator experiment in Sweden  
The above two are targeting the second oscillation appearance maximum
- ▶ INO: Magnetized atmospheric experiment in India
- ▶ Neutrino factory: muon storage ring
- ▶ ⋮

# Flavor models

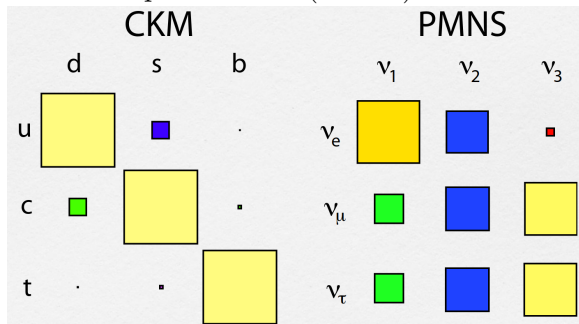
Quark matrix (CKM) is perturbative  
Lepton matrix (PMNS) isn't



Review: S. King [1510.02091](#)

# Flavor models

Quark matrix (CKM) is perturbative  
Lepton matrix (PMNS) isn't



Review: S. King [1510.02091](#)

Is there any structure?

## Flavor models

Popular early models: Bimaximal, tri-bimaximal, & golden ratio

All predicted  $U_{e3} = 0 \Rightarrow \theta_{13} = 0$

Now know  $\theta_{13} = 8.5^\circ$

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

## Flavor models

Popular early models: Bimaximal, tri-bimaximal, & golden ratio

All predicted  $U_{e3} = 0 \Rightarrow \theta_{13} = 0$

Now know  $\theta_{13} = 8.5^\circ$

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Need more degrees of freedom: sum rules

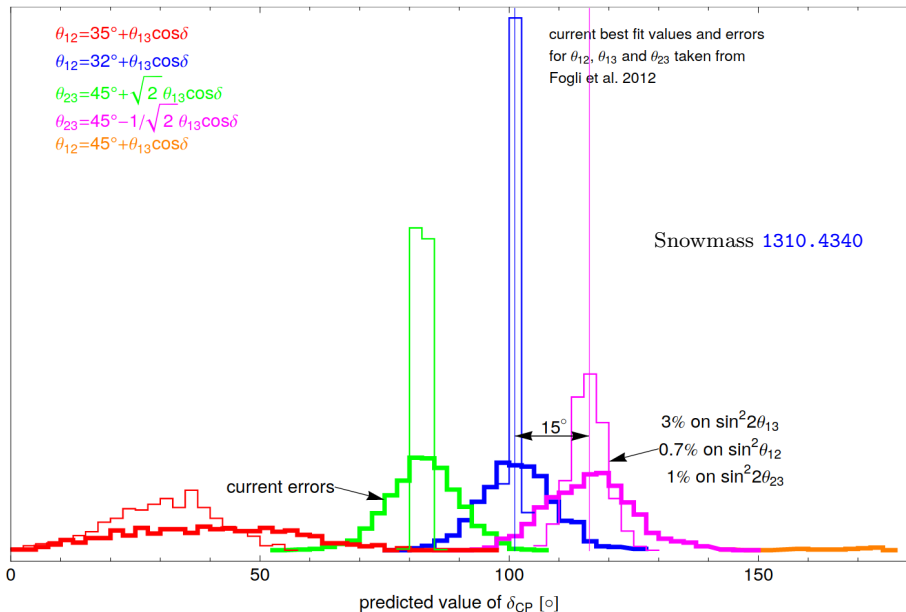
Perhaps:

$$U = \begin{pmatrix} c_\phi & s_\phi e^{-i\psi} & 0 \\ -s_\phi e^{i\psi} & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} U_{TBM}$$

which predicts:

$$\cos \delta \approx \frac{\theta_{12} - \sin^{-1} \frac{1}{\sqrt{3}}}{\theta_{13}}$$

# Flavor models





## Related topics that were skipped

- ▶ Absolute mass scale measurements
  - ▶ Cosmological/astrophysical measurements
  - ▶ Neutrino-less double beta decay
  - ▶ Tritium end point
- ▶ Supernova neutrinos
  - ▶ Galactic and diffuse background
  - ▶ Physics during propagation and inside SNe
- ▶ High energy astrophysical flux
  - ▶ IceCube (10 years ago) and its upgrade (soon)
  - ▶ KM3NeT/ARCA/ANTARES (construction ongoing)
  - ▶ Baikal GVD (construction ongoing)
  - ▶ ANITA (has performed several balloon flights)
  - ▶ GRAND, POEMMA, P-ONE, ARA, ARIANNA, RNO, PUEO, BEACON, TAROGE (none are funded ... yet!)
- ▶ Many other oscillation BSM scenarios
  - ▶ Decoherence
  - ▶ Lorentz invariance or CPT violation
  - ▶ Dark matter interactions
  - ▶ Unitarity violation
- ▶ Leptogenesis
- ▶ Early universe measurements of neutrino properties
- ▶ Neutrino cross sections
  - ▶ Coherent elastic  $\nu$  nucleus scattering (CEvNS) at COHERENT, ...
- ▶ Geoneutrinos

# Hamiltonian Dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

# Hamiltonian Dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$$s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}$$

PBD, R. Pestes [2006.09384](#)

# Hamiltonian Dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

$$a = 2\sqrt{2}G_F N_e E$$

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$$s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}$$

Find eigenvalues and eigenvectors:

PBD, R. Pestes [2006.09384](#)

$$H_{\text{flav}} = \frac{1}{2E} \widehat{U} \begin{pmatrix} 0 & & \\ & \widehat{\Delta m}_{21}^2 & \\ & & \widehat{\Delta m}_{31}^2 \end{pmatrix} \widehat{U}^\dagger$$

J. Kopp [physics/0610206](#)

Computationally works, but we can do better than a **black box**...

Analytic expression?

# Analytic Oscillation Probabilities in Matter

☑ Solar:  $P_{ee} \simeq \sin^2 \theta_{\odot}$

Approx: S. Mikheev, A. Smirnov [Nuovo Cim. C9 \(1986\) 17-26](#)

Exact: S. Parke [PRL 57 \(1986\) 2322](#)

☑ Long-baseline: All three flavors

Exact: H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

Approx: [PBD](#), H. Minakata, S. Parke, [1604.08167](#)

Review: G. Barenboim, [PBD](#), S. Parke, C. Ternes [1902.00517](#)

☑  $\nu_e$  disappearance (neutrino factory):

$$\Delta \widehat{m}_{ee}^2 = \widehat{m}_3^2 - (\widehat{m}_1^2 + \widehat{m}_2^2 - \Delta m_{21}^2 c_{12}^2)$$

[PBD](#), S. Parke, [1808.09453](#)

☐ Atmospheric

Get the eigen**values**

# Eigenvalues Analytically: The Exact Solution

Solve the cubic characteristic equation: eigenvalues

$$(\widehat{m^2}_i)^3 - A(\widehat{m^2}_i)^2 + B\widehat{m^2}_i - C = 0$$

$$A \equiv \sum_i \widehat{m^2}_i = \Delta m_{31}^2 + \Delta m_{21}^2 + a$$

$$B \equiv \sum_{i>j} \widehat{m^2}_i \widehat{m^2}_j = \Delta m_{31}^2 \Delta m_{21}^2 + a(\Delta m_{ee}^2 c_{13}^2 + \Delta m_{21}^2)$$

$$C \equiv \prod_i \widehat{m^2}_i = a \Delta m_{31}^2 \Delta m_{21}^2 c_{13}^2 c_{12}^2$$

G. Cardano *Ars Magna* 1545

V. Barger, et al. [PRD 22 \(1980\) 2718](#)

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

# Eigenvalues Analytically: The Exact Solution

Solve the cubic characteristic equation: eigen**values**

$$(\widehat{m^2}_i)^3 - A(\widehat{m^2}_i)^2 + B\widehat{m^2}_i - C = 0$$

$$A \equiv \sum_i \widehat{m^2}_i = \Delta m_{31}^2 + \Delta m_{21}^2 + a$$

$$B \equiv \sum_{i>j} \widehat{m^2}_i \widehat{m^2}_j = \Delta m_{31}^2 \Delta m_{21}^2 + a(\Delta m_{ee}^2 c_{13}^2 + \Delta m_{21}^2)$$

$$C \equiv \prod_i \widehat{m^2}_i = a \Delta m_{31}^2 \Delta m_{21}^2 c_{13}^2 c_{12}^2$$

G. Cardano *Ars Magna* 1545

V. Barger, et al. [PRD 22 \(1980\) 2718](#)

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

Then write down eigen**vectors** (mixing angles)

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

K. Kimura, A. Takamura, H. Yokomakura [hep-ph/0205295](#)

[PBD](#), S. Parke, X. Zhang [1907.02534](#)



# Eigenvalues Analytically: The Exact Solution

Solve the cubic characteristic equation: eigen**values**

$$(\widehat{m^2}_i)^3 - A(\widehat{m^2}_i)^2 + B\widehat{m^2}_i - C = 0$$

$$A \equiv \sum_i \widehat{m^2}_i = \Delta m_{31}^2 + \Delta m_{21}^2 + a$$

$$B \equiv \sum_{i>j} \widehat{m^2}_i \widehat{m^2}_j = \Delta m_{31}^2 \Delta m_{21}^2 + a(\Delta m_{ee}^2 c_{13}^2 + \Delta m_{21}^2)$$

$$C \equiv \prod_i \widehat{m^2}_i = a \Delta m_{31}^2 \Delta m_{21}^2 c_{13}^2 c_{12}^2$$

G. Cardano *Ars Magna* 1545

V. Barger, et al. [PRD 22 \(1980\) 2718](#)

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

Then write down eigen**vectors** (mixing angles)

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

K. Kimura, A. Takamura, H. Yokomakura [hep-ph/0205295](#)

[PBD](#), S. Parke, X. Zhang [1907.02534](#)

“Unfortunately, the algebra is rather impenetrable.”

V. Barger, et al.

# Eigenvalues Analytically: The Exact Solution

The cubic solution (in neutrino terms)

$$\widehat{m^2}_1 = \frac{A}{3} - \frac{1}{3}\sqrt{A^2 - 3BS} - \frac{\sqrt{3}}{3}\sqrt{A^2 - 3B}\sqrt{1 - S^2}$$

$$\widehat{m^2}_2 = \frac{A}{3} - \frac{1}{3}\sqrt{A^2 - 3BS} + \frac{\sqrt{3}}{3}\sqrt{A^2 - 3B}\sqrt{1 - S^2}$$

$$\widehat{m^2}_3 = \frac{A}{3} + \frac{2}{3}\sqrt{A^2 - 3BS}$$

$$A = \Delta m_{21}^2 + \Delta m_{31}^2 + a$$

$$B = \Delta m_{21}^2 \Delta m_{31}^2 + a \left[ c_{13}^2 \Delta m_{31}^2 + (c_{12}^2 c_{13}^2 + s_{13}^2) \Delta m_{21}^2 \right]$$

$$C = a \Delta m_{21}^2 \Delta m_{31}^2 c_{12}^2 c_{13}^2$$

$$S = \cos \left\{ \frac{1}{3} \cos^{-1} \left[ \frac{2A^3 - 9AB + 27C}{2(A^2 - 3B)^{3/2}} \right] \right\}$$

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)

Get the eigenvectors

# Values and Vectors

Probability amplitude:

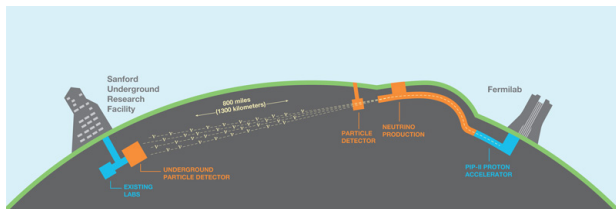
$$A_{\alpha\beta} = \sum_i \hat{U}_{\alpha i}^* e^{-im_i^2 L/2E} \hat{U}_{\beta i}$$

- ▶ **Eigenvalues** give the frequencies of the oscillations

Where should DUNE be?

- ▶ **Eigenvectors** give the amplitudes of the oscillations

How many events will DUNE see?



# Exact Neutrino Oscillations in Matter: Mixing Angles

$$s_{12}^2 = \frac{-\left[(\widehat{m^2_2})^2 - \alpha\widehat{m^2_2} + \beta\right] \Delta\widehat{m^2_{31}}}{\left[(\widehat{m^2_1})^2 - \alpha\widehat{m^2_1} + \beta\right] \Delta\widehat{m^2_{32}} - \left[(\widehat{m^2_2})^2 - \alpha\widehat{m^2_2} + \beta\right] \Delta\widehat{m^2_{31}}}$$
$$s_{13}^2 = \frac{(\widehat{m^2_3})^2 - \alpha\widehat{m^2_3} + \beta}{\Delta\widehat{m^2_{31}} \Delta\widehat{m^2_{32}}}$$
$$s_{23}^2 = \frac{s_{23}^2 E^2 + c_{23}^2 F^2 + 2c_{23}s_{23}c_\delta EF}{E^2 + F^2}$$
$$e^{-i\delta} = \frac{c_{23}s_{23} (e^{-i\delta} E^2 - e^{i\delta} F^2) + (c_{23}^2 - s_{23}^2) EF}{\sqrt{(s_{23}^2 E^2 + c_{23}^2 F^2 + 2EFc_{23}s_{23}c_\delta) (c_{23}^2 E^2 + s_{23}^2 F^2 - 2EFc_{23}s_{23}c_\delta)}}$$
$$\alpha = c_{13}^2 \Delta m_{31}^2 + (c_{12}^2 c_{13}^2 + s_{13}^2) \Delta m_{21}^2, \quad \beta = c_{12}^2 c_{13}^2 \Delta m_{21}^2 \Delta m_{31}^2$$
$$E = c_{13}s_{13} \left[ \left( \widehat{m^2_3} - \Delta m_{21}^2 \right) \Delta m_{31}^2 - s_{12}^2 \left( \widehat{m^2_3} - \Delta m_{31}^2 \right) \Delta m_{21}^2 \right]$$
$$F = c_{12}s_{12}c_{13} \left( \widehat{m^2_3} - \Delta m_{31}^2 \right) \Delta m_{21}^2$$

# New Physics

DUNE and T2HK will have unprecedented capabilities to test the three-neutrino oscillation picture

Extend DMP to new physics progress report:

Sterile

S. Parke, X. Zhang [1905.01356](#)

NSI

S. Agarwalla, et al. [2103.13431](#)

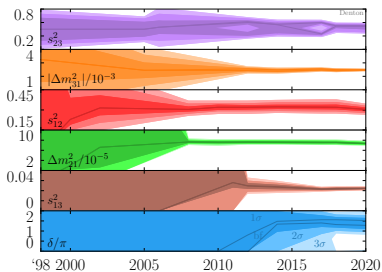
Neutrino decay

Decoherence

...

Given Rosetta, extensions should be considerably simpler

# References



SK [hep-ex/9807003](#)

M. Gonzalez-Garcia, et al. [hep-ph/0009350](#)

M. Maltoni, et al. [hep-ph/0207227](#)

SK [hep-ex/0501064](#)

SK [hep-ex/0604011](#)

T. Schwetz, M. Tortola, J. Valle [0808.2016](#)

M. Gonzalez-Garcia, M. Maltoni, J. Salvado [1001.4524](#)

T2K [1106.2822](#)

D. Forero, M. Tortola, J. Valle [1205.4018](#)

D. Forero, M. Tortola, J. Valle [1405.7540](#)

P. de Salas, et al. [1708.01186](#)

# CP violation in matter



# The CPV Term in Matter

The amount of CPV is

$$P_{\alpha\beta} - \bar{P}_{\alpha\beta} = \pm 16J \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \quad \alpha \neq \beta$$

where the Jarlskog is

$$J \equiv \Im[U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*] \quad \alpha \neq \beta, i \neq j$$

$$J = c_{12}s_{12}c_{13}^2s_{13}c_{23}s_{23} \sin \delta$$



C. Jarlskog [PRL 55 \(1985\)](#)

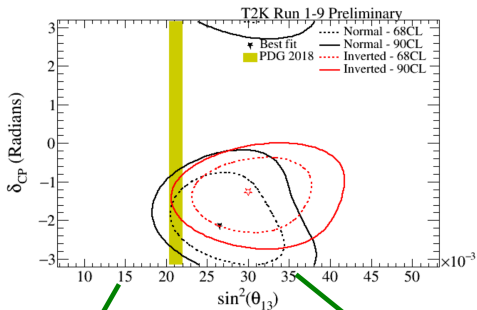
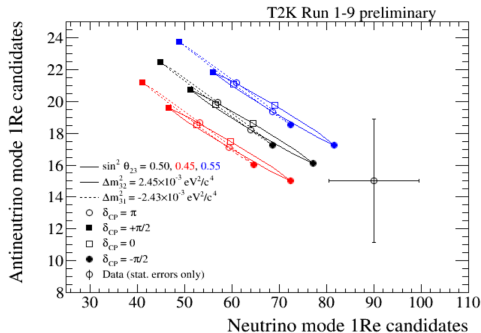
The exact term in matter is known to be

$$\frac{\hat{J}}{J} = \frac{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}{\widehat{\Delta m}_{21}^2 \widehat{\Delta m}_{31}^2 \widehat{\Delta m}_{32}^2}$$

V. Naumov [IJMP 1992](#)

P. Harrison, W. Scott [hep-ph/9912435](#)

# CPV Tension at T2K



$$J = c_{12} s_{12} c_{13}^2 s_{13} c_{23} s_{23} \sin \delta$$

## CPV in Matter

CPV in matter can be written sans  $\cos(\frac{1}{3} \cos^{-1}(\dots))$  term.

$$\frac{\widehat{J}}{J} = \frac{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}{\widehat{\Delta m}_{21}^2 \widehat{\Delta m}_{31}^2 \widehat{\Delta m}_{32}^2}$$

$$\left(\widehat{\Delta m}_{21}^2 \widehat{\Delta m}_{31}^2 \widehat{\Delta m}_{32}^2\right)^2 = (A^2 - 4B)(B^2 - 4AC) + (2AB - 27C)C$$

$$A \equiv \sum_j \widehat{m}_j^2 = \Delta m_{31}^2 + \Delta m_{21}^2 + a$$

$$B \equiv \sum_{j>k} \widehat{m}_j^2 \widehat{m}_k^2 = \Delta m_{31}^2 \Delta m_{21}^2 + a(\Delta m_{ee}^2 c_{13}^2 + \Delta m_{21}^2)$$

$$C \equiv \prod_j \widehat{m}_j^2 = a \Delta m_{31}^2 \Delta m_{21}^2 c_{13}^2 c_{12}^2$$

**This is the only oscillation quantity in matter that can be written exactly without  $\cos(\frac{1}{3} \cos^{-1}(\dots))$ !**

H. Yokomakura, K. Kimura, A. Takamura [hep-ph/0009141](https://arxiv.org/abs/hep-ph/0009141)

[1902.07185](https://arxiv.org/abs/1902.07185)

BNL Summer Student Lecture: June 22, 2023 90/29

## CPV Factorizes

Thus  $\hat{J}^{-2}$  is fourth order in matter potential:  
only two matter corrections are needed.

$$\frac{\hat{J}}{J} = \frac{1}{|1 - (a/\alpha_1)e^{i2\theta_1}| |1 - (a/\alpha_2)e^{i2\theta_2}|}$$

## CPV Factorizes

Thus  $\hat{J}^{-2}$  is fourth order in matter potential:  
only two matter corrections are needed.

$$\frac{\hat{J}}{J} = \frac{1}{|1 - (a/\alpha_1)e^{i2\theta_1}| |1 - (a/\alpha_2)e^{i2\theta_2}|}$$

CPV in matter can be well approximated:

$$\frac{\hat{J}}{J} \approx \frac{1}{|1 - (a/\Delta m_{ee}^2)e^{i2\theta_{13}}| |1 - (c_{13}^2 a/\Delta m_{21}^2)e^{i2\theta_{12}}|}$$

PBD, Parke [1902.07185](#)

See also X. Wang, S. Zhou [1901.10882](#)

Precise at the  $< 0.04\%$  level!

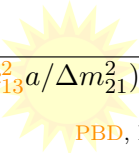

## CPV Factorizes

Thus  $\hat{J}^{-2}$  is fourth order in matter potential:  
only two matter corrections are needed.

$$\frac{\hat{J}}{J} = \frac{1}{|1 - (a/\alpha_1)e^{i2\theta_1}| |1 - (a/\alpha_2)e^{i2\theta_2}|}$$

CPV in matter can be well approximated:

$$\frac{\hat{J}}{J} \approx \frac{1}{|1 - (a/\Delta m_{ee}^2)e^{i2\theta_{13}}| |1 - (c_{13}^2 a/\Delta m_{21}^2)e^{i2\theta_{12}}|}$$

 PBD, Parke [1902.07185](#)

See also X. Wang, S. Zhou [1901.10882](#)

Precise at the  $< 0.04\%$  level!

## One caveat in support of $\delta$

If the goal is **CP violation** the Jarlskog should be used

however

If the goal is **measuring the parameters** one must use  $\delta$

Given  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ , and  $J$ , I can't determine the sign of  $\cos \delta$   
which is physical

e.g.  $P(\nu_\mu \rightarrow \nu_\mu)$  depends on  $\cos \delta$  a tiny bit

- ▶ As T2(H)K has almost no  $\cos \delta$  sensitivity, they should focus on  $J$
- ▶ NOvA/DUNE has some  $\cos \delta$  sensitivity, so both  $J$  and  $\delta$  should be reported