The role of exotic operators in determining the finite-volume spectrum from Lattice QCD and its consequences

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Introduction

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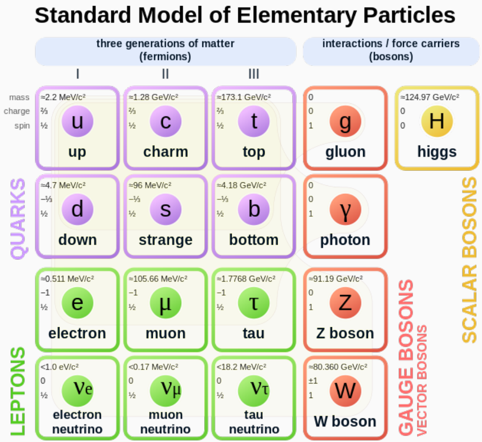
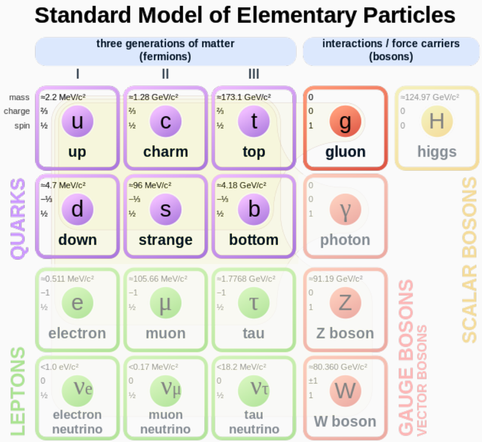
Standard Model



- Electromagnetic
- Weak
- Strong
- (No Gravity)

Figure from Wikipedia

Strong Force



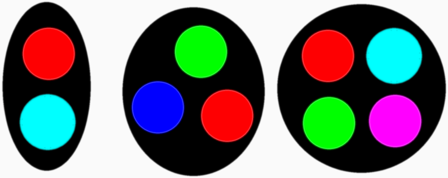
- Electromagnetic
- Weak
- Strong
- (No Gravity)

Figure from Wikipedia (modified)

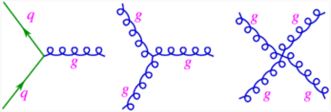
Quantum Chromodynamics

- Quarks (q)/antiquarks ($\overline{q}$) are fermions
- there are three color charges (RGB)
- quark confinement: no particles can have color

mesons: $q\overline{q}$
baryons: qqq
tetraquarks: $q\overline{q}q\overline{q}$
and more!

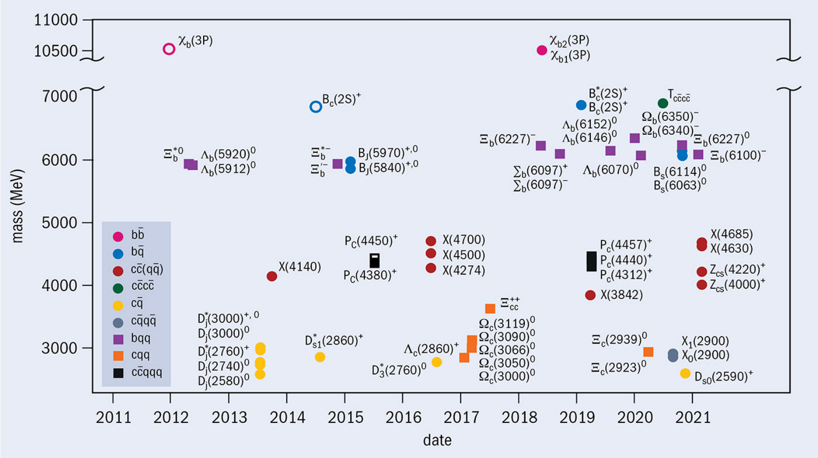


- gluons carry the color charge between quarks
- the basic QCD interactions are:



How many hadronic particles are there?

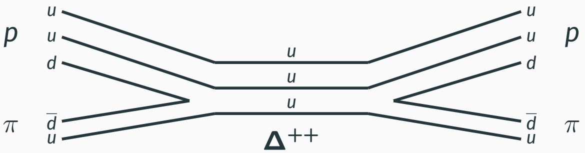
Here's 59 new ones discovered at LHC:



Resonances

Resonances can occur during scattering and affect the resulting scattering amplitudes

Δ resonance example:



These resonances are difficult to study because

- Exist for 10^{-23} seconds or less
- Extremely difficult to detect directly in experiment
- Form in low energy ranges
- Perturbative theories do not work

Lattice QCD can compute the effects specific to a resonance.

Motivation for Lattice QCD

Things that make studying QCD difficult:

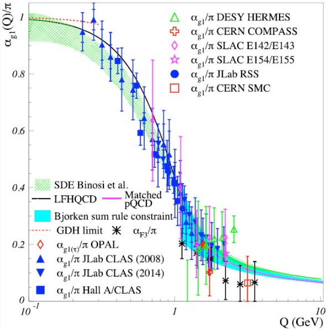
- quark confinement
- gluonic self interactions

Things that make a perturbative approach difficult:

- asymptotic freedom
- hot QCD background

Advantages of Lattice QCD:

- lattice QCD is exact and only limited by statistics



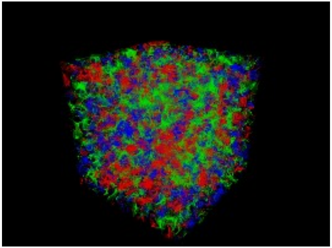
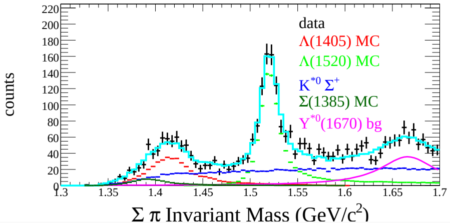
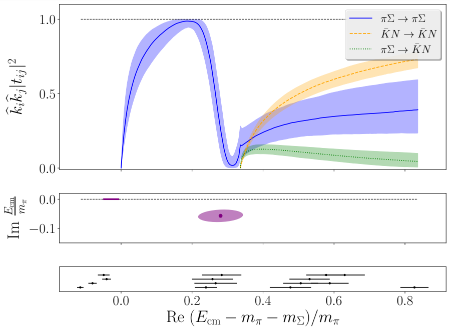


Fig 1: Deur, A. The QCD Running Coupling at All Scales and the Connection Between Hadron Masses and Λ_S. *Few-Body Syst* 59, 146 (2018).

Example: Λ(1405) Resonance

- Questions whether the **Λ**(1405) was actually two nearby resonances (**Λ**(1405) and **Λ**(1380))
- Difficult to experimentally discern [CLAS, 2013]
- Recent coupled-channel $\bar{K}N$-$\pi\boldsymbol{\Sigma}$ analysis in Lattice QCD distinctly shows two. [Bulava et al, 2023]

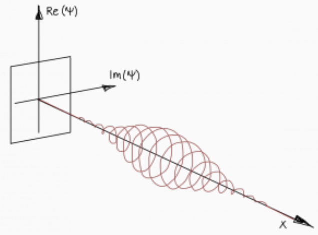




Methods

Quantum Mechanics

- All Standard Model particles and partons are described using quantum mechanics.
- Particles are steady state solutions to various wave equations.



Path Integral Formulation

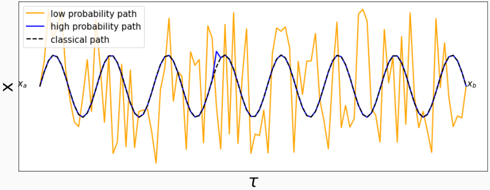
Classical mechanics → path of least action, $S = \int_{t_a}^{t_b} L\, dt$

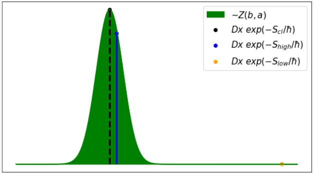
Quantum mechanics → all paths are possible

- physics is determined by the transition amplitude that gives a probability of getting from point a to b

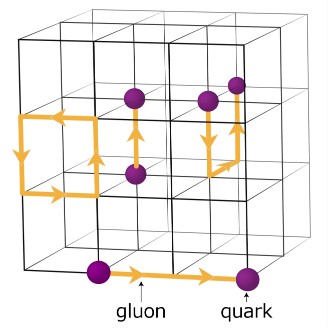
$$Z(b,a) = \int_a^b \mathcal{D}x\, e^{iS/\hbar} \xrightarrow{t \to -i\tau} \int_a^b \mathcal{D}x\, e^{-S/\hbar}$$

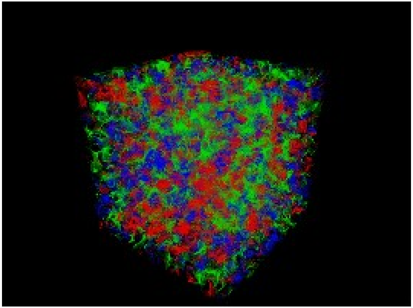
Ex: Simple harmonic oscillator

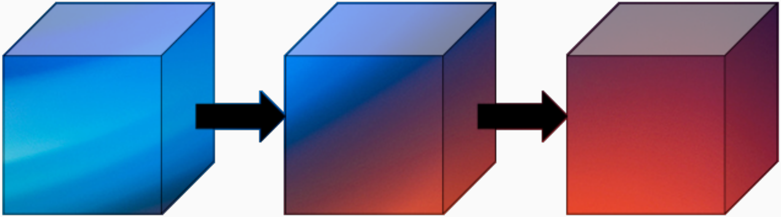




Lattice QCD







Computational Framework

1. Compute lattice configurations of fields

   quarks: $\psi^f, \bar{\psi}^f|_{f=u,d,s}$ gluons: $\mathcal{A}_\mu$

2. Create operators with the make-up and quantum numbers of the particles of interest

   $$\pi^+ = \bar{d}u$$

3. Construct matrices of two-point correlation functions within the channels of interest

   $$\langle 0|\pi\overline{\pi}|0\rangle, \langle 0|[N\pi][\overline{N\pi}]|0\rangle, \langle 0|\mathbf{\Delta}[\overline{N\pi}]|0\rangle ...$$

4. Use GEVP and fitting method to extract the steady state energies of the channel

   $$\langle 0|\pi\overline{\pi}|0\rangle = \sum_{n=0}^{\infty} Ae^{-E_n t}$$

5. Fit to those energies using Lüscher formalism to calculate phase shifts and matrix elements

Notes on Operator/Correlator Construction

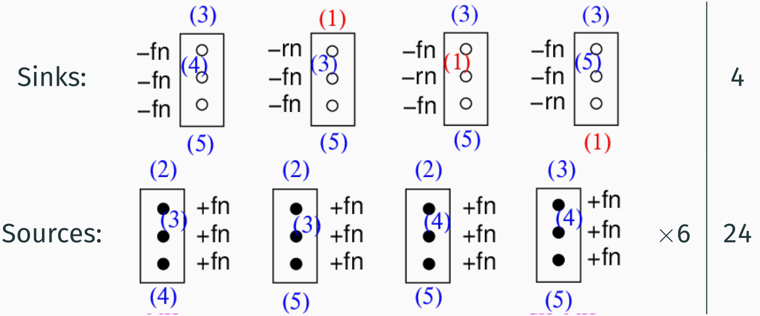
Operator Notes:

- Gluons $\rightarrow$ Stout smearing
- Quarks $\rightarrow$ LapH smearing

Correlator Notes:

- compute correlators including
  - mesons
  - baryons
  - tetraquarks
  - hexaquarks
- stochastic factorization $\rightarrow$ tensor contraction
  - split correlators into *sources* and *sinks*
  - multi-hadron correlators can be made out of the same contractions as single hadron correlators
  - efficient algorithm $\rightarrow$ produce many different correlators

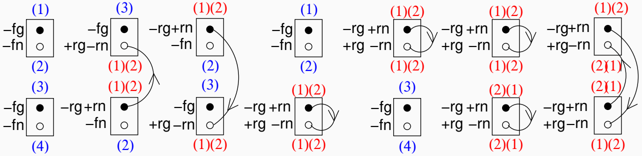
Computational costs of baryon correlators



Baryon sinks and sources can be used for B-B, B-MB, and MB-MB correlators.

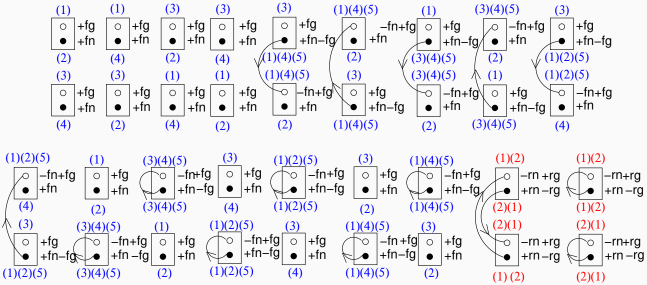
Diagrams provided by of Colin Morningstar

Computational costs of tetraquark correlators



Tetraquark sinks

24



Tetraquark sources

66

Diagrams provided by Colin Morningstar

GEVP

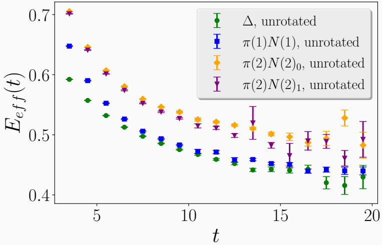
Correlation matrix elements in the same channel share the same FV energy levels

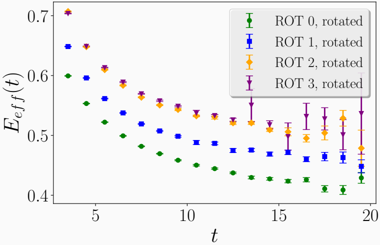
$$\langle 0|\mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)} e^{-E_n t}$$

Separate out by solving GEVP of $N \times N$ matrix and eigenvalues are

$$\lim_{t\to\infty} \lambda_n(t) \approx b_n e^{-E_n t}$$

Example ($N\pi$, $I = 3/2$, $H_g(0)$):





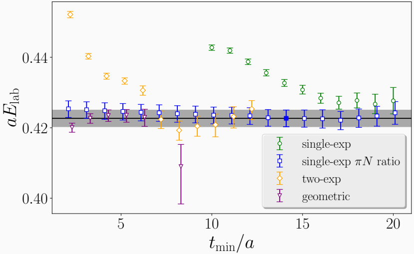
Finite-Volume Energy Spectrum

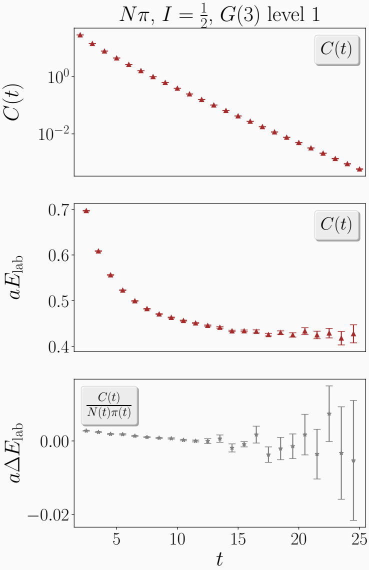
Fitting methods:

- single-exp: Ae^{-Et}
- double-exp: $Ae^{-Et}(1 + Re^{-D^2t})$
- geometric: $Ae^{-Et}/(1 - Re^{-Dt})$

Ratio:

$$R(t) = \frac{\lambda_n(t)}{C_1(t)C_2(t)}$$

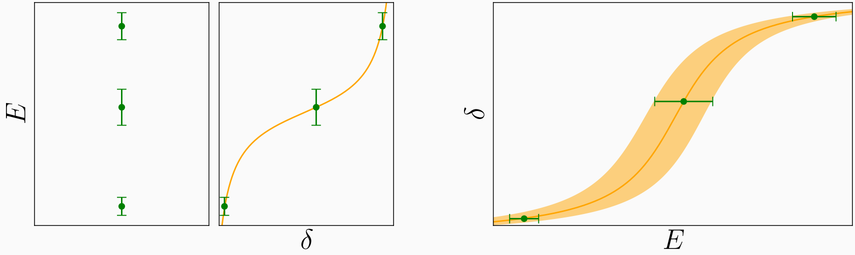




Connect finite-volume to infinite-volume via Lücsher:

$$\det[\widetilde{K}^{-1}(E_{\mathrm{cm}}) - B^{P}(E_{\mathrm{cm}})] = 0$$

- truncate higher waves
- $\widetilde{K}$ - related to the usual scattering K-matrix
- B^{P} (‘box matrix’) - finite volume irreps
- only works for 2-2 scattering



Results

$N\pi$ channel information

$$N\pi \to N\pi$$

Correlation Matrix Information:

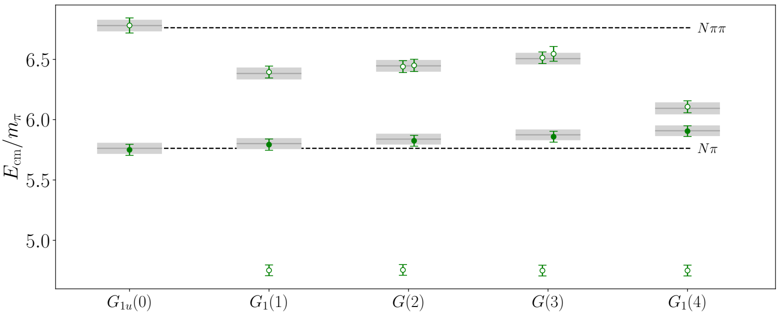
$a_{N\pi}^{I=1/2}$

- $I = 1/2$
- operators:
  - N
  - $N\pi$
- momenta: $\mathrm{d}^2 = 0, 1, 2, 3, 4$

$\boldsymbol{\Delta}(1232)$, $a_{N\pi}^{I=3/2}$

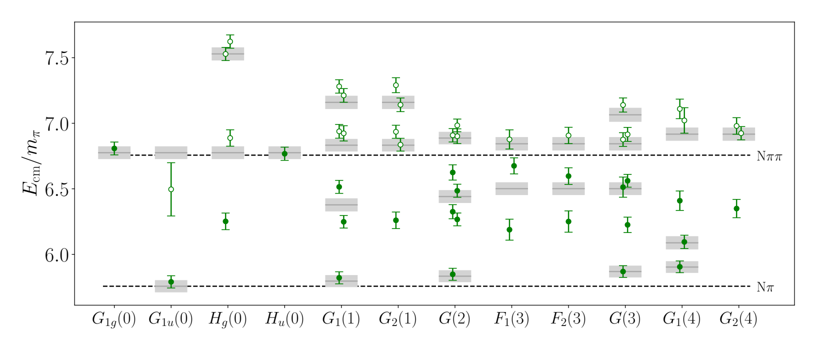
- $I = 3/2$
- operators:
  - $\boldsymbol{\Delta}$
  - $N\pi$
- momenta: $\mathrm{d}^2 = 0, 1, 2, 3, 4$

I=1/2 $N\pi$



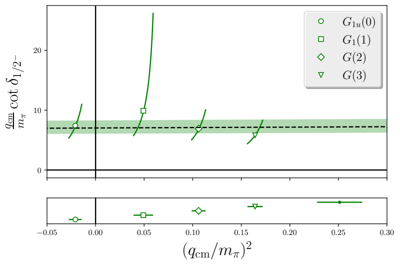
- Grey bands: noninteracting scattering levels (N, π correlators)
- Green dots: interacting levels ($N\pi$, N correlators)
- Filled green dots: levels used for constraining $a_{N\pi}^{I=1/2}$

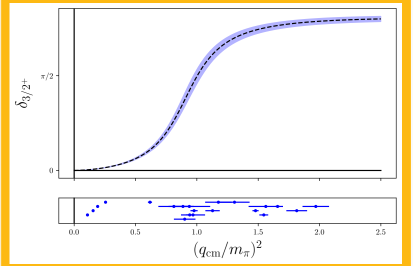
I=3/2 $N\pi$, $\Delta(1232)$

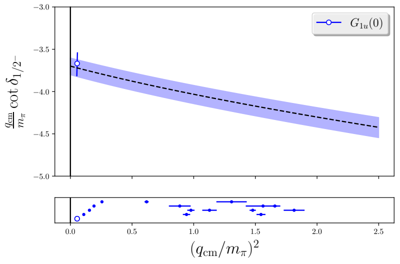


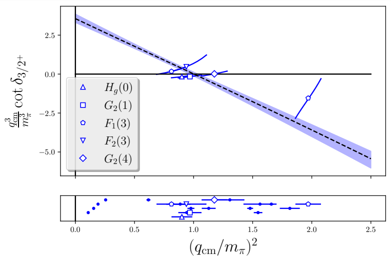
- Grey bands: noninteracting scattering levels (N, π correlators)
- Green dots: interacting levels ($N\pi$, Δ correlators)
- Filled green dots: levels used for calculating $a_{N\pi}^{I=3/2}$

Phase Shifts

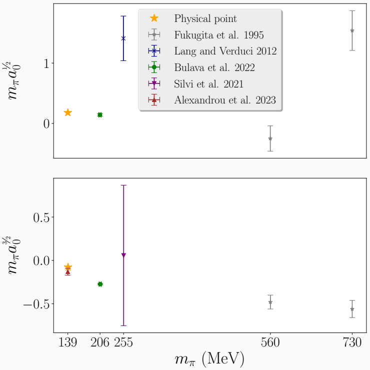


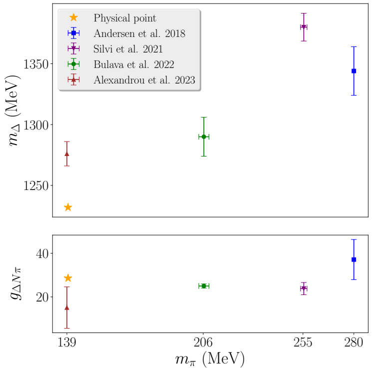






Phase Shifts



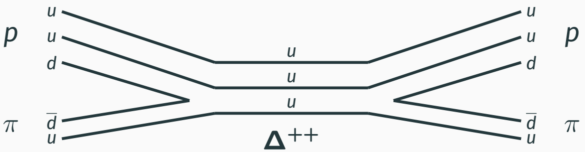


$\boldsymbol{\Delta}$ Operator

The correlation matrix used for $\boldsymbol{\Delta}$ channel included

$$\langle 0|[N\pi][\overline{N\pi}]|0\rangle,\ \langle 0|\boldsymbol{\Delta}[\overline{N\pi}]|0\rangle,\ \text{and}\ \langle 0|\boldsymbol{\Delta}\overline{\boldsymbol{\Delta}}|0\rangle$$

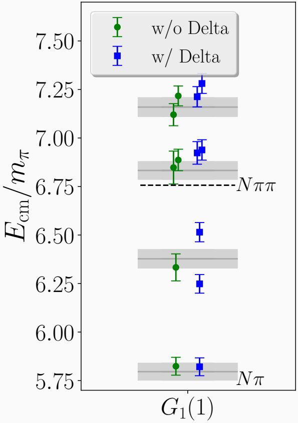
The $\boldsymbol{\Delta}$ is not a bound state at this pion mass. Why include it?



Still couples to energy states within the channel $\rightarrow$ increase precision and number of states we can retrieve.

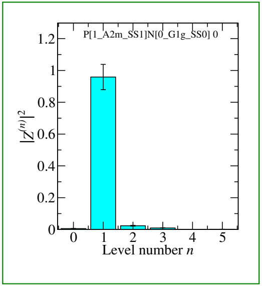
How important was the $\boldsymbol{\Delta}$ operator?

Δ Operator's Impact

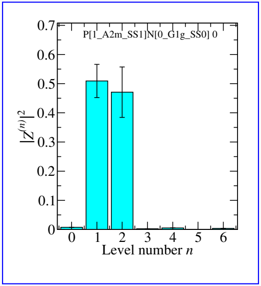


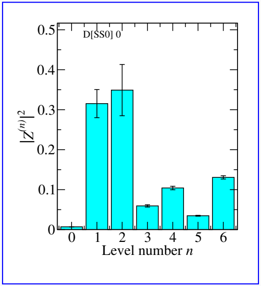
$$\langle 0|\mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)} e^{-E_n t}$$

$N\pi$



$N\pi$, Δ





Tetraquark Resonances

Two coupled-channel scattering channels investigated:

$$K\pi, K\eta \to K\pi, K\eta$$

- resonance: κ
- $I = 1/2$
- operators:
  - K
  - $K\pi$
  - $K\eta$ ($\eta = u\bar{u} + d\bar{d}$)
  - $K\phi$ ($\phi = s\bar{s}$)
  - $\bar{s}u\bar{s}s$ (diquark-antidiquark)
- momentums: $\mathrm{d}^2 = 0$

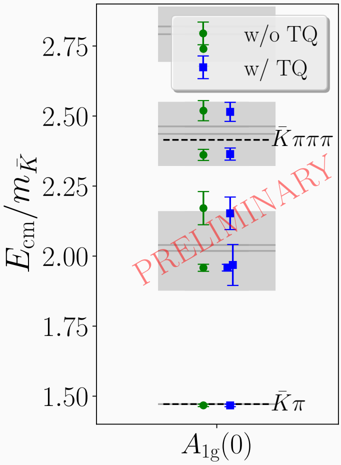
$$K\bar{K}, \pi\eta \to K\bar{K}, \pi\eta$$

- resonance: $a_0(980)$
- $I = 1$
- operators:
  - π
  - $K\bar{K}$
  - $\pi\eta$ ($\eta = u\bar{u} + d\bar{d}$)
  - $\pi\phi$ ($\phi = s\bar{s}$)
  - $\bar{u}u\bar{d}u$ (diquark-antidiquark)
- momentums: $\mathrm{d}^2 = 0$

Meson-Meson Spectrums

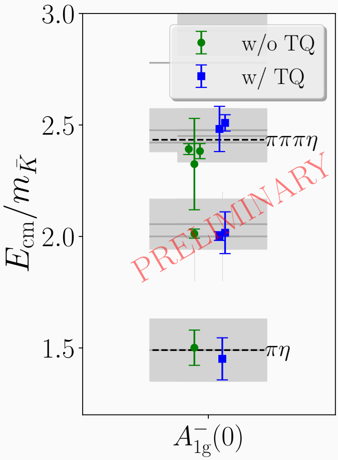
κ channel

TQ = $\bar{s}u\bar{s}s$



a_0 channel

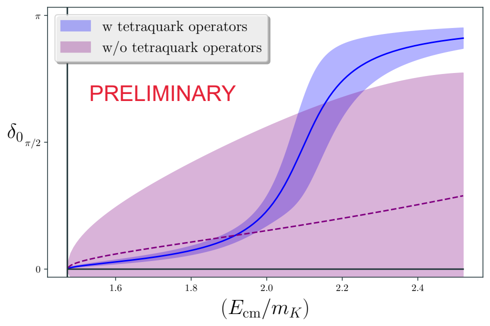
TQ = $\bar{u}u\bar{d}u$



Amplitudes

$K\pi$–$K\eta$ Spectrum (κ channel)

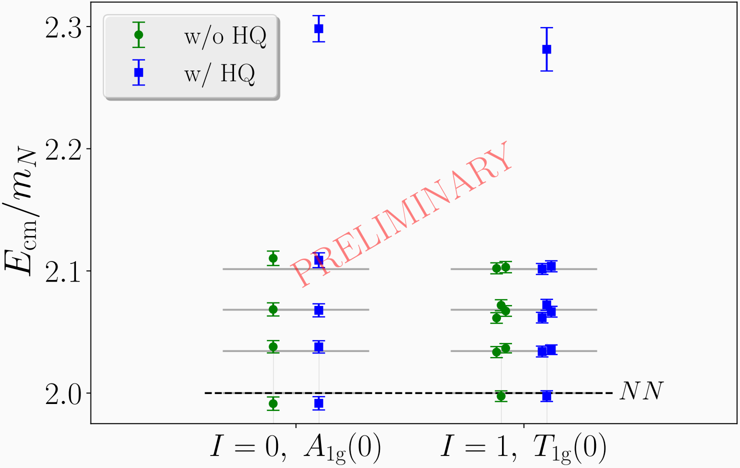
- Without tetraquark $\to$ no resonance (fit to 5 levels)
- With tetraquark $\to$ resonance at $\sim 2.1 m_K$ (fit to 5+TQ levels)



$K\bar{K}$–$\pi\eta$ Spectrum (a_0 channel)

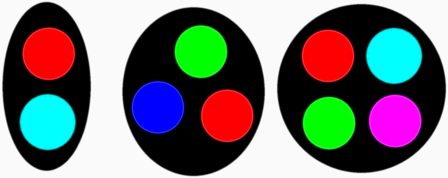
- Without tetraquark $\to$ no resonance (fit to 3 levels)
- With tetraquark $\to$ virtual bound state (fit to 2+TQ levels)

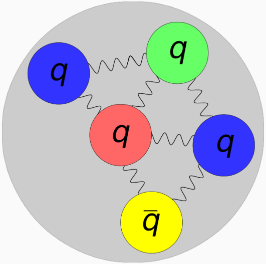
Not always: *NN* scattering with Hexaquarks (HQ)



Deuteron Dineutron

What's the limit?





Final Notes:

- As long as the operator has the quantum numbers of your channel, it can be included
- Not every operator will reveal a new state in the energy regime of interest... but it might.
- Solution? Run low statistics with all operators you can compute to check + prayers

Thanks for listening!