Lattice QCD and Searches for Violations of Fundamental symmetries

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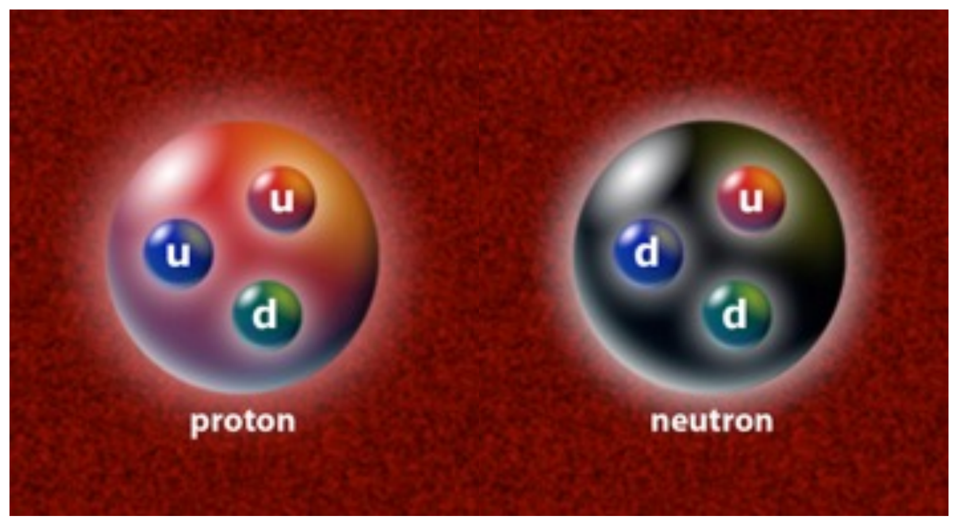

RBRC 25th Anniversary Celebration
BNL Physics Department, Jun 22, 2023

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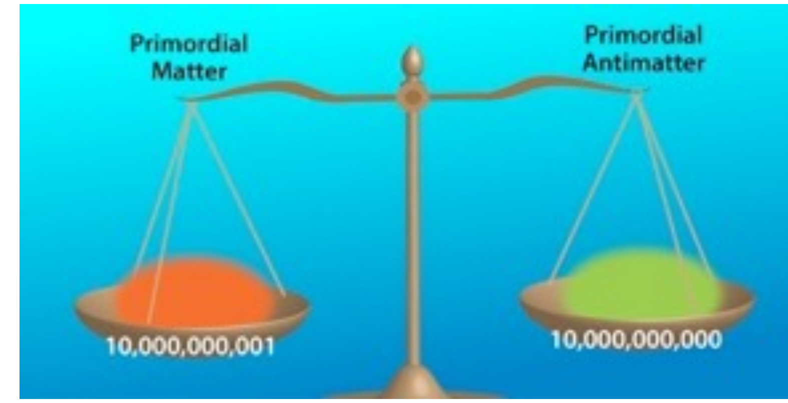
HOKUSAI GREAT WAVE

Baryogenesis and Broken Symmetries

Why does Universe have More Matter than Antimatter?

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \cdot 10^{-10}$$



[A.Sakharov (1967)] :

Three necessary conditions:

Baryon number-changing interactions

proton decay, neutron oscillations

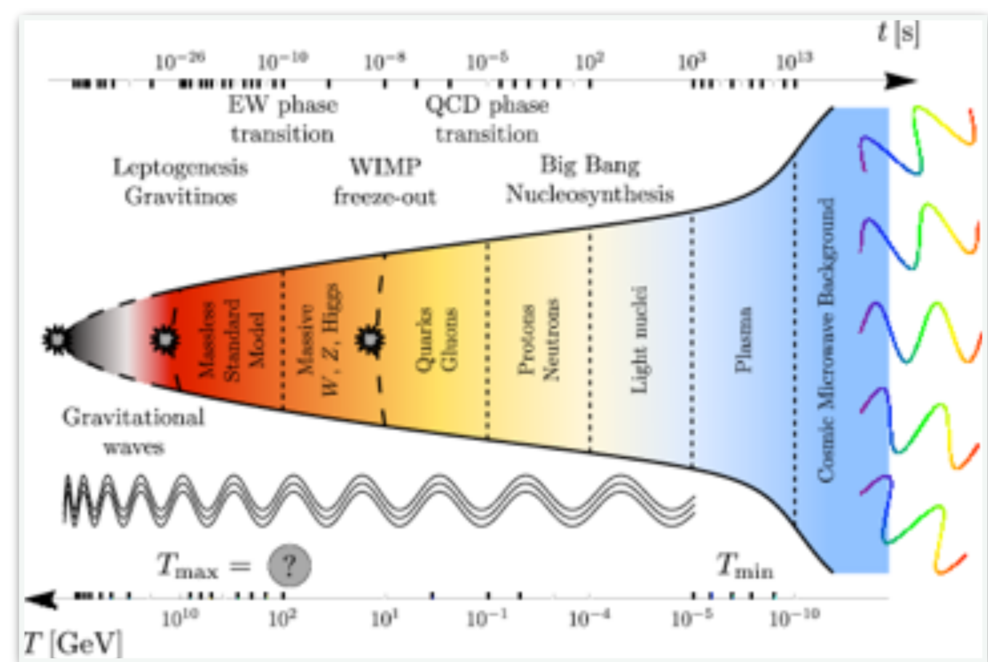
(alternatively: leptogenesis + sphalerons)

neutrinoless beta-decays

Violations of C- and CP-symmetries

(electric dipole moments of p, n, e⁻, nuclei, atoms)

Interactions out of equlibrium



Baryogenesis and Broken Symmetries

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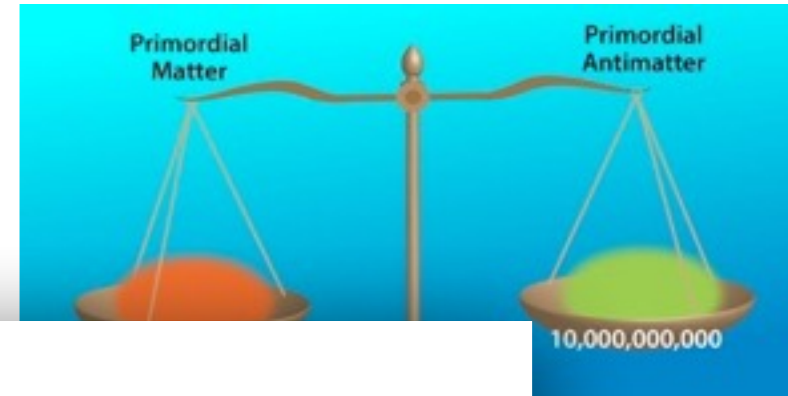


Violation of *CP* invariance, *C* asymmetry, and baryon asymmetry of the universe

A. D. Sakharov

(Submitted 23 September 1966)
Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32–35 (1967) [JETP Lett. **5**, 24–27 (1967).
Also S7, pp. 85–88]

Usp. Fiz. Nauk **161**, 61–64 (May 1991)

Из эффекта С. Окубо
при большой температуре
для Вселенной сшита шуба
по ее кривой фигуре

Literal translation: *Out of S. Okubo's effect*
At high temperature
A fur coat is sewed for the Universe
Shaped for its crooked figure.

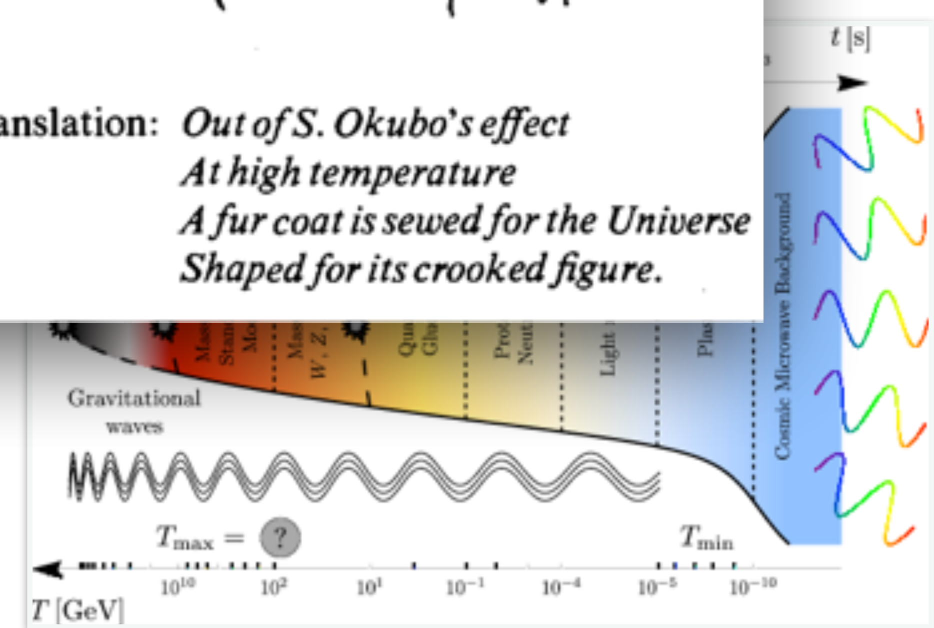
...ernatively:
...togenesis
...phalerons)

...utrinoless
...ta-decays

symmetries

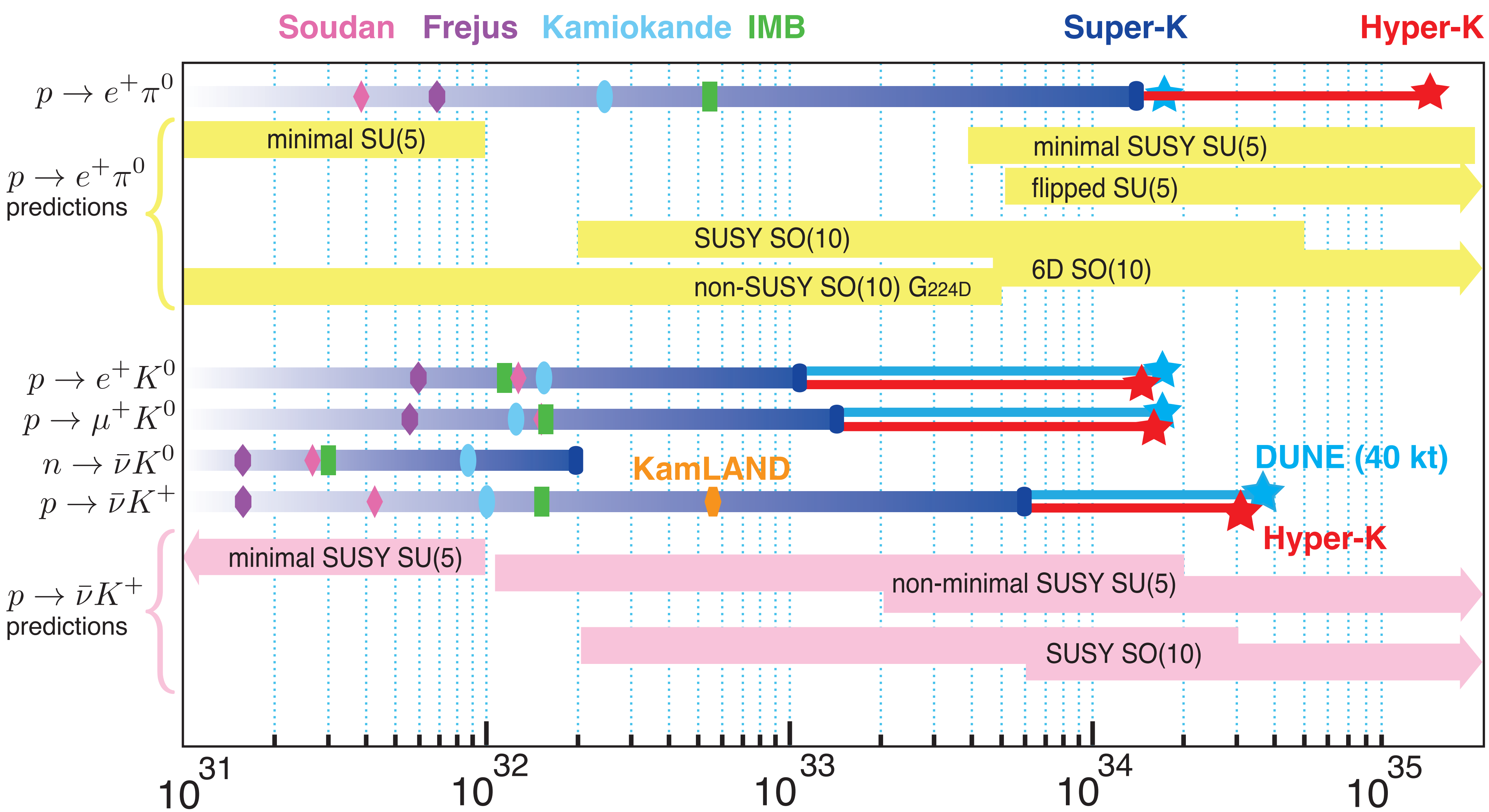
(electric dipole moments of p, n, e⁻, nuclei, atoms)

interactions out of equlibrium



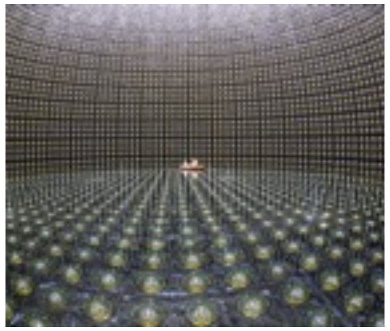
Does the Proton Decay?

- *Missing piece of Grand-Unified Theories*
- *Limit on nuclear matter stability*



[LBNF and DUNE CDR, R.Acciarri et al (2015)]

Soudan



Super Kamiokande

- Expected x10 improvement on lifetime limit from Hyper-K , DUNE
- Better sensitivity to $p \to \bar{\nu}K^+$ that affects supersymmetric GUT models

Proton Decays and Grand Unification

Proton lifetime in a Grand Unified Theory

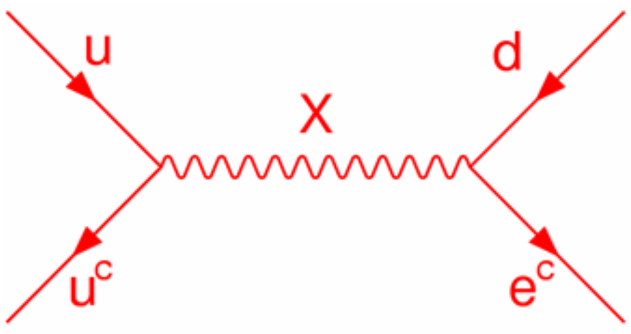
$$\frac{\tau_p}{Br(p\to\pi\bar\ell)} \approx \frac{1.4\cdot 10^{33}\text{ years}}{|c_I|^2\cdot\left|\langle\pi|\mathcal{O}_{\text{decay}}|p\rangle\right|^2}\cdot\left(\frac{\Lambda_{\text{GUT}}}{10^{15}\text{ GeV}}\right)^4$$

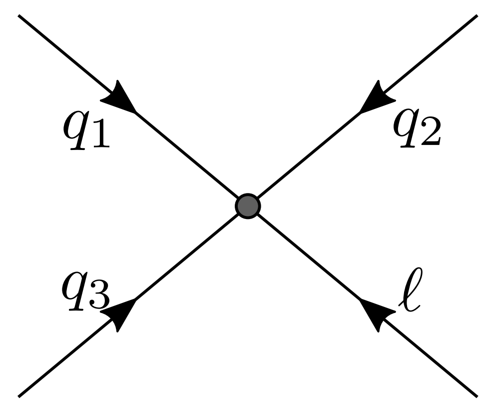
$\approx$ GUT boson mass

O(1) coupling at Λ_{GUT}

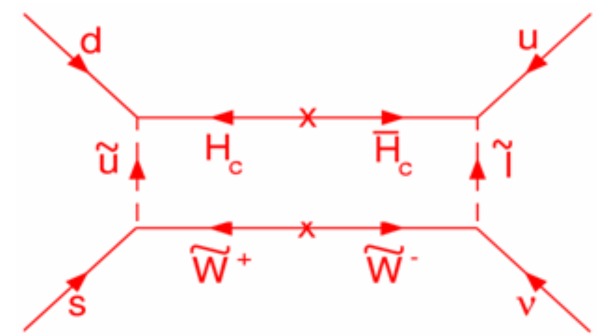
decay amplitude at nuclear scale

- ordinary GUT





- supersymmetric GUT



- Effective 4-quark interaction

$$\mathcal{L}_{\text{eff}} = \sum_I C_I \mathcal{O}_I + \text{h.c.}$$

$$\mathcal{O}_I = \epsilon^{abc}(\bar q_1^{aC} P_{\chi_I} q_2^b)(\bar\ell^C P_{\chi'_I} q_3^c) = \bar\ell^C_\alpha\, \mathcal{O}^{3q}_{I,\alpha}$$

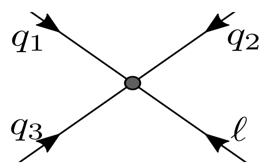
- Decay matrix elements $(W_{0,1})_I$ [S.Aoki et al, PRD62:014506 (2000)]

$$\langle\bar\ell(q)\Pi(p)|\mathcal{O}^{\chi'}|N(k)\rangle = \bar v^C_{\ell\alpha}(q)\, P_{\chi'}\left[W_0(-q^2) - \frac{i\not q}{m_N}W_1(-q^2)\right]u_N(k)$$

Protons Stable due to Topology?

Why NO proton decay seen ?

- ***more complicated*** GUT scenario ?
- ***other*** BNV mechanism ?
- ***small decay amplitude*** due to nucleon structure ?

"quark pudding" estimate: $\langle\text{vac}|$  $|N\rangle$

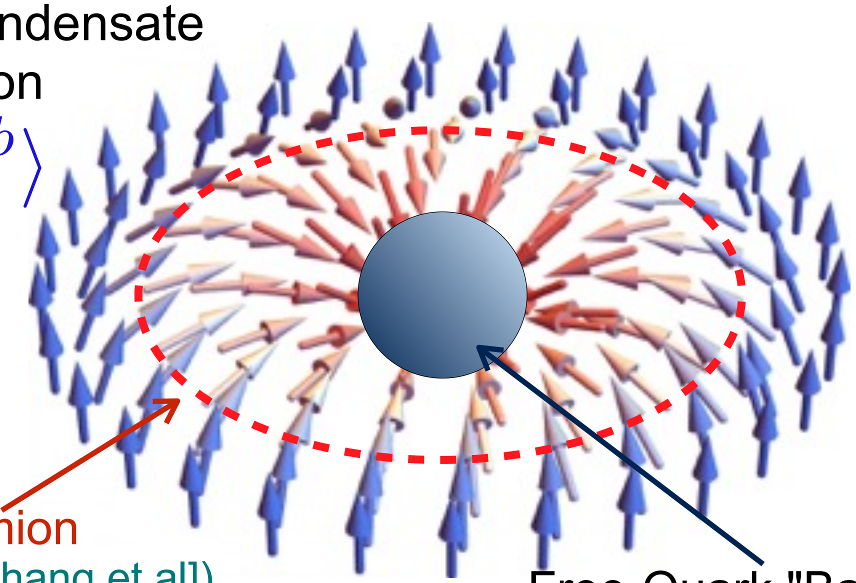
$$\langle\text{vac}|\mathcal{O}^{3q}|N\rangle \sim \rho_q^{3/2}\sqrt{V_N} \sim \frac{1}{V_N} \approx 0.004\,\text{GeV}^3$$

$$\langle\Pi|\mathcal{O}^{3q}|N\rangle \sim \langle\text{vac}|\mathcal{O}^{3q}|N\rangle/f_\pi \approx 0.03\,\text{GeV}^2$$

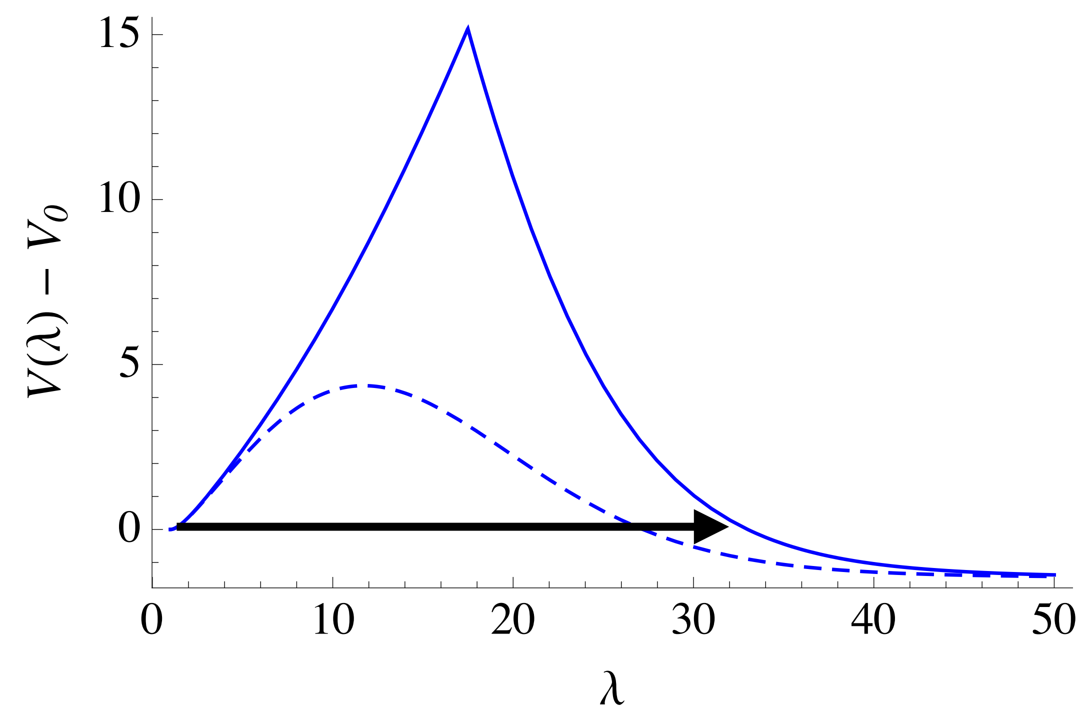
However, if the proton is a "Chiral Bag" [A.Martin, G.Stavenga '12]

- proton decay ≡ quantum tunneling of skyrmion over topological barrier

decay rate sensitive to R_{Bag}, quark masses; may be suppressed ~ $O(10^{-4})$ – $O(10^{-12})$



"Chiral Bag" Model

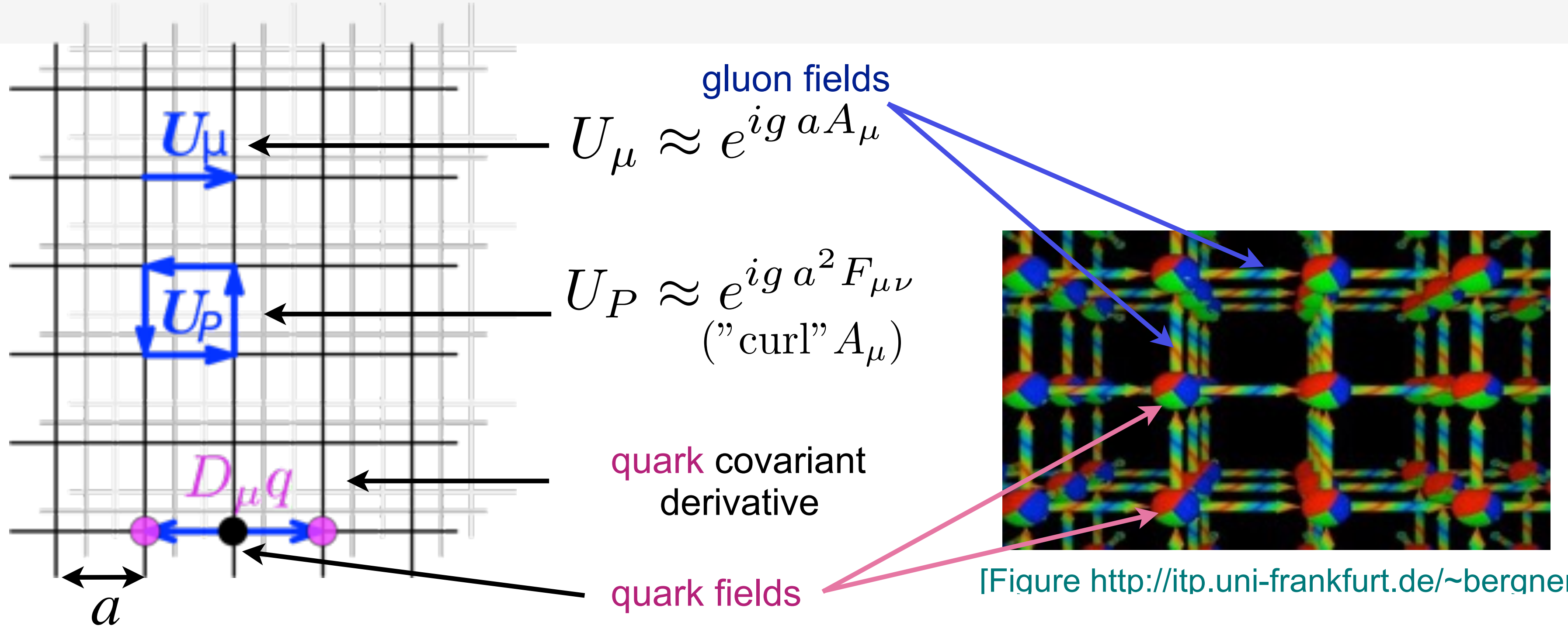


Uncertainty can be addressed only by a realistic ab initio QCD calculation

Hadron Correlators in Lattice QCD

Quarks and gluons
on a Lattice

- 4D Euclidean space
- discretized action
- controllable extrapolations
  $a \to 0$, $m_{quark} \to$ physical

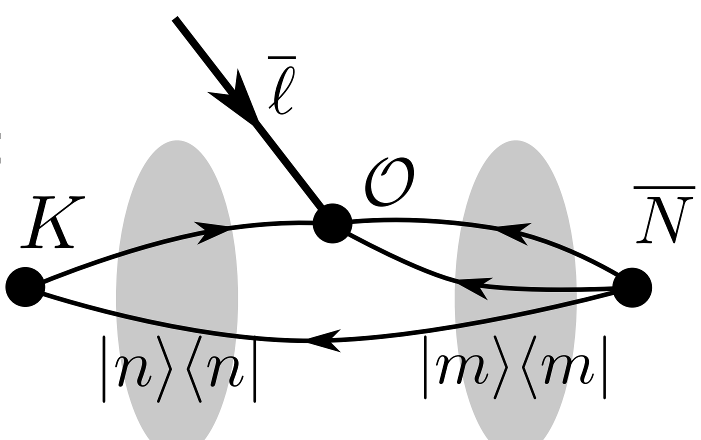


Observables from correlators of proton=***(ud)u*** , 3-quark decay operators , etc

$$\langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}\Big(Glue\Big) \int \mathcal{D}\Big(Quarks\Big)\; e^{-S_{Glue} - \bar{q}\left(\not{D}+m\right)q} \;\big[q_x \bar{q}_y \ldots\big]$$

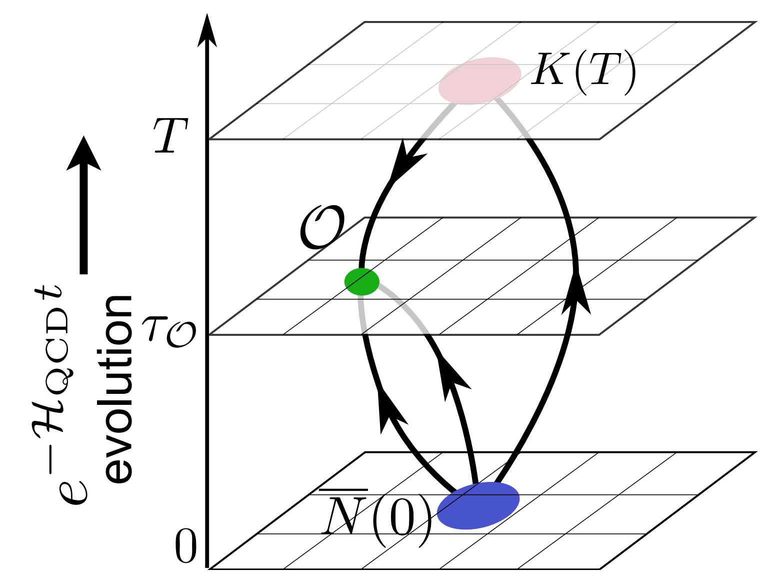
Nucleon-Meson Matrix Elements:

$$C_{\rm 3pt}^{K\mathcal{O}N} = \langle K(T)\mathcal{O}(\tau)\overline{N}(0)\rangle =$$



$$= \sum_{m(K),n(N)} Z_m e^{-E_m(T-\tau)} \langle m(K)|\mathcal{O}|n(N)\rangle e^{-E_n\tau} Z_n^*$$

(Ground state $n,m = 0$)

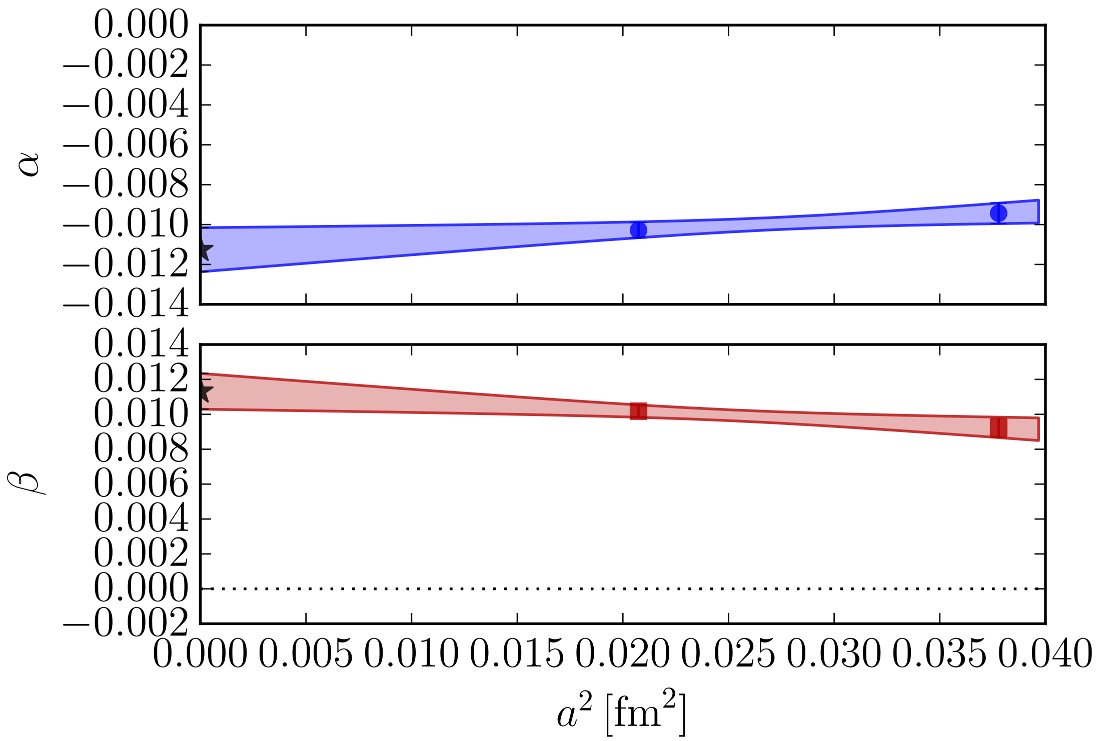


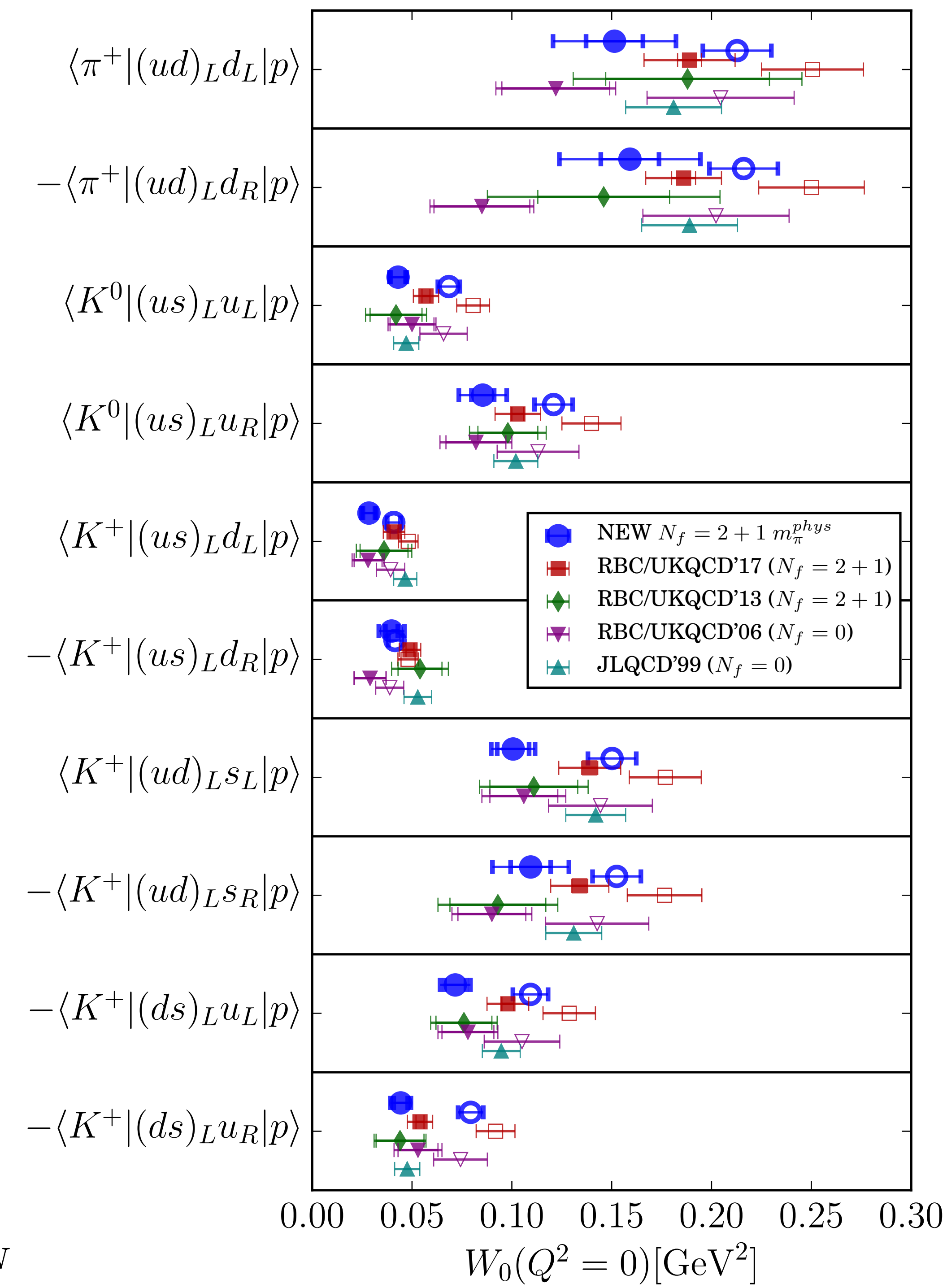
Quark lines = $(\not{D}+m)^{-1}\cdot\psi$

Proton Decay Amplitudes with Physical Quarks

Lattice calculations with chirally-symmetric quarks:

- Previously: at $m_\pi \gtrsim 300$ MeV [S.Aoki et al (2000), Y.Aoki et al (2006), (2013), (2017)]
- Physical quarks + continuum limit [J.Yoo, **S.S.** PRD'22]
- (NEXT: $p \to \pi\pi,\ p \to \pi K$)





Amplitudes $p \to \pi\bar{\ell}, K\bar{\ell}$

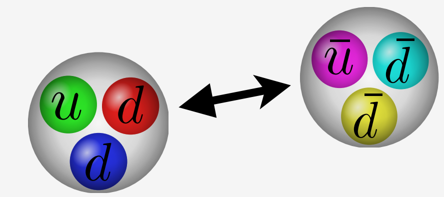
Proton decay constants ($p \to 3\ell$, also $p \to K\bar{\ell}$, $\pi\bar{\ell}$ from LO ChPT)

$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC}d^b)_R u^c_L|N\rangle = \alpha\, P_L U_N$$

$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC}d^b)_L u^c_L|N\rangle = \beta\, P_L U_N$$

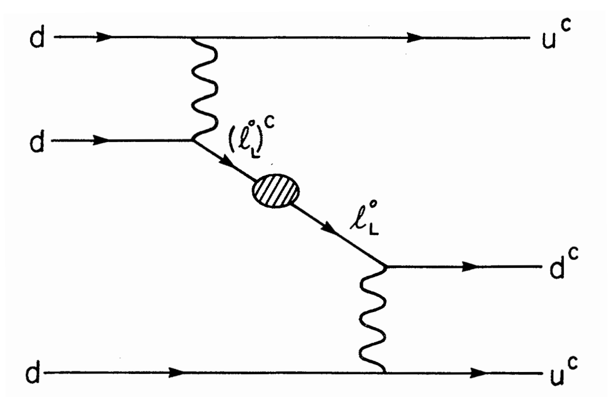
NO SUPPRESSION at physical quark masses $\Longrightarrow$ Protons sensitive to BNV from GUT

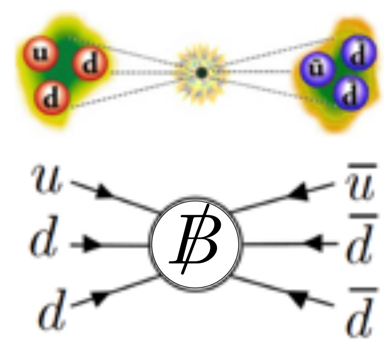
Δ(N_{Baryon})=2 Violation : n↔n̄ Oscillations

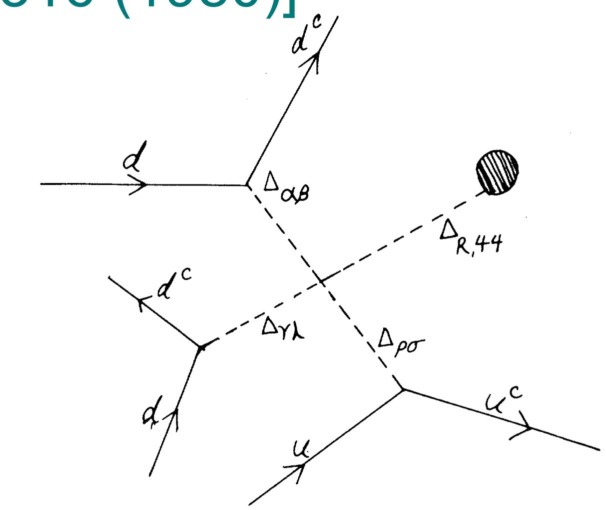


- GUT + massive Majorana lepton
  [T.K.Kuo, S.T.Love, PRL45:93 (1980)]

- partial unification and (B–L) viol.
  [R.N.Mohapatra, R.E.Marshak, PRL44:1316 (1980)]







Effective *ΔB*=2 interaction

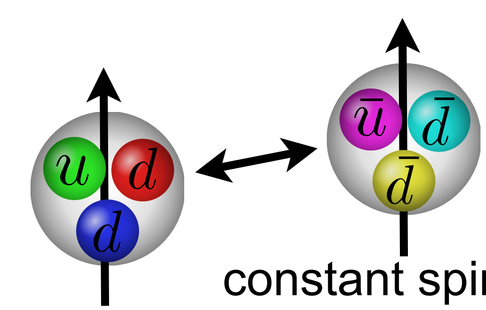
$$\mathcal{L}_{\rm eff} = \sum_i \left[c_i \mathcal{O}_i^{6q} + \text{h.c.}\right]$$

- Neutron no longer an eigenstate;
  n↔n̄ oscillation time in vacuum $\tau_{n\bar{n}} = (2\delta m)^{-1}$

$$\delta m = -\langle\bar{n}| \int d^4x\, \mathcal{L}_{\rm eff}\, |n\rangle = -\sum_i \frac{c_i}{M_X^5} \langle\bar{n}|\mathcal{O}_i^{6q}|n\rangle$$

BSM scale suppression
$M_X \gtrsim$ *(200–300)* TeV

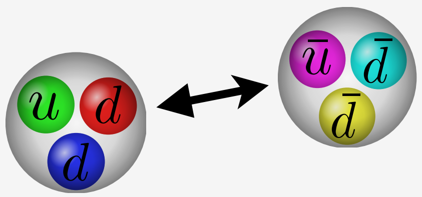
N-N̄ amplitude



constant spin :
flip mag.moment

- n↔n̄ oscillation in nuclei : suppressed by interaction $\Delta M \sim O(100\,\text{MeV})$

$n \leftrightarrow \bar{n}$ Oscillations: Experimental Status



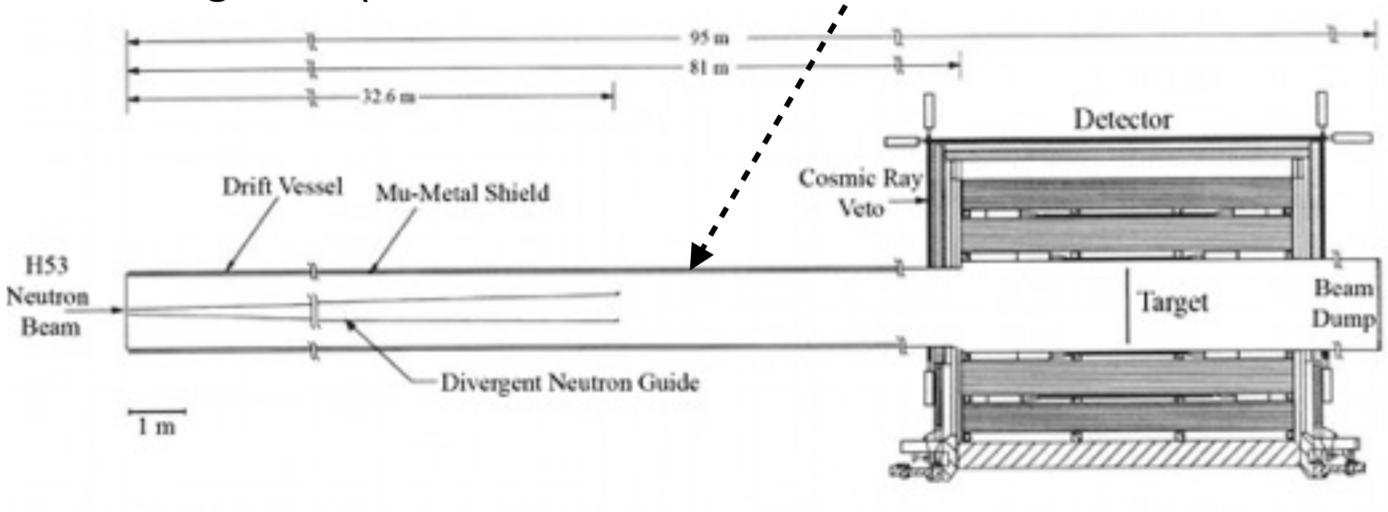
- "Quasi-free" reactor neutrons
  - ILL Grenoble high-flux reactor [M.Baldo-Ceolin et al, 1994)]

$$\tau_{n\bar{n}} \gtrsim 10^8\ s$$

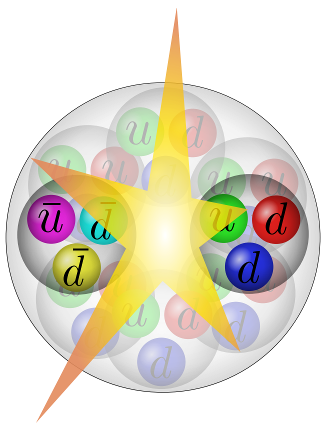
$$\delta m \lesssim 6 \cdot 10^{-24}\ \text{eV}$$

screening of mag.field for flight ~ $(\Delta M)^{-1} > 1\text{s}$

$$B < (2\mu_n t)^{-1} = 5\ \text{nT} = 10^{-4} B_\oplus$$



- In nuclei :
  - $\tau(^{56}Fe) \gtrsim 0.72 \cdot 10^{32}$ yr
    $\Longrightarrow \tau_{N\bar{N}} \gtrsim 1.4 \cdot 10^8$ s [Soudan]
  - $\tau(^{16}O) \gtrsim 1.77 \cdot 10^{32}$ yr
    $\Longrightarrow \tau_{N\bar{N}} \gtrsim 3.3 \cdot 10^8$ s [Super-K]
  - $\tau(^{2}H) \gtrsim 0.54 \cdot 10^{32}$ yr
    $\Longrightarrow \tau_{N\bar{N}} \gtrsim 1.96 \cdot 10^8$ s [SNO]

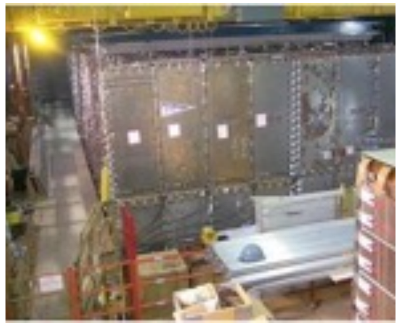


Nuclear decays from (ΔB=2) transitions: suppressed by nuclear medium:

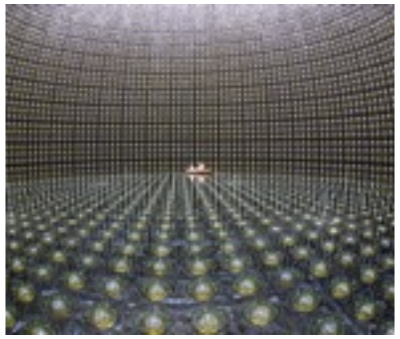
$$T_d = R \tau_{n\bar{n}}^2$$

$$R \sim 10^{23}\ s^{-1}$$

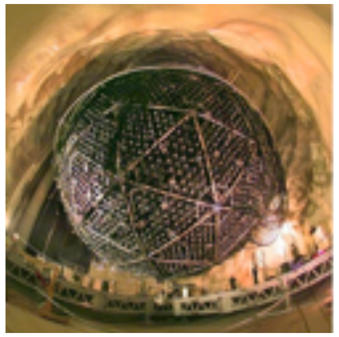
nuclear model uncertainty ~ 10-15% for ^{16}O [E.Friedman, A.Gal (2008)]



Soudan



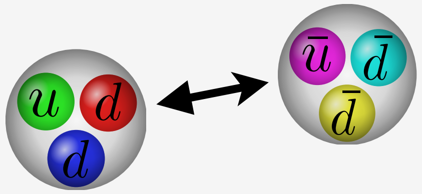
Super Kamiokande



SNO

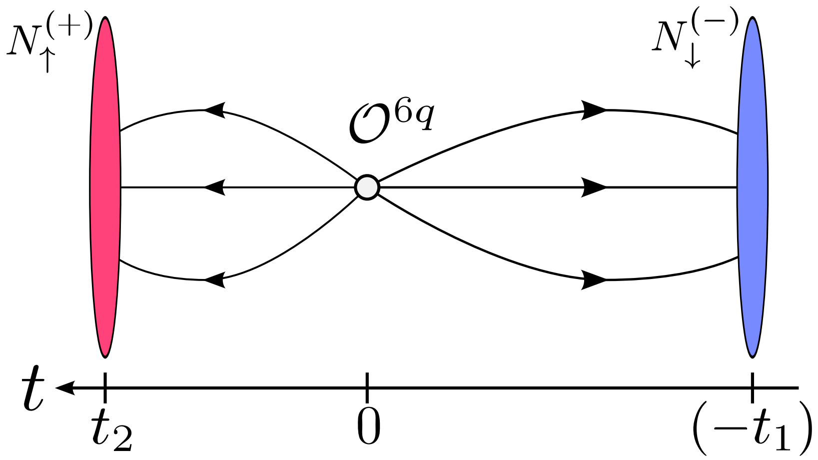
Sensitivity is limited by atmospheric neutrinos

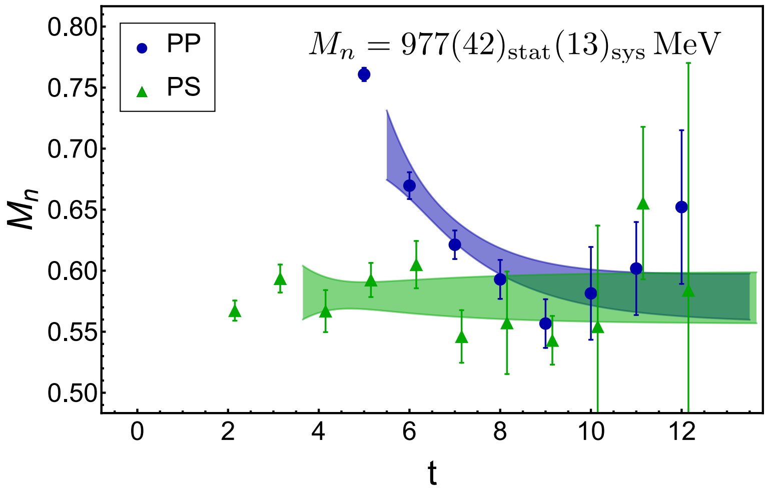
n↔n̄ Amplitudes from Lattice QCD

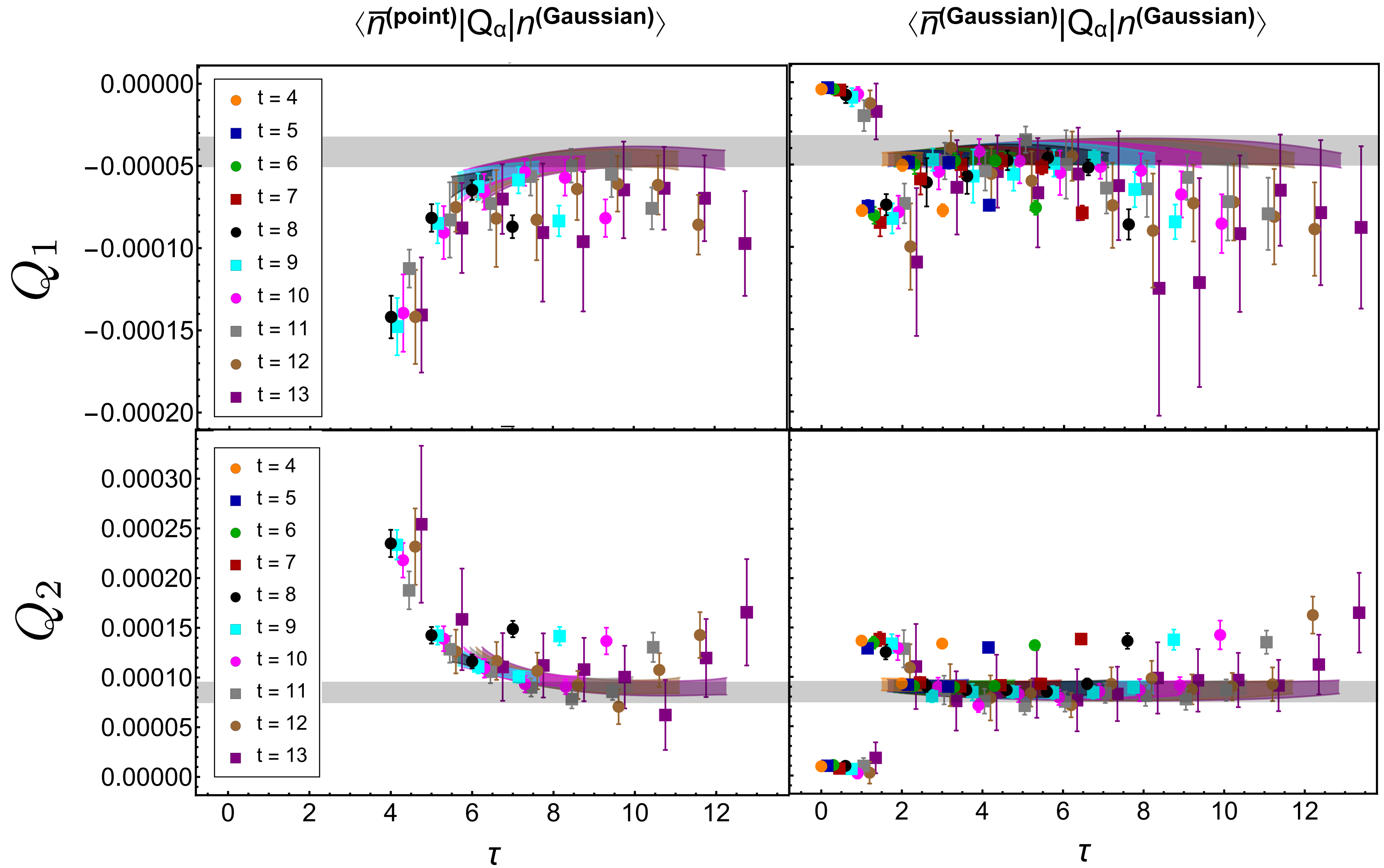


Lattice calculations at the physical point

[E.Rinaldi, S.S., M.Wagman; PRL'19; PRD'19]



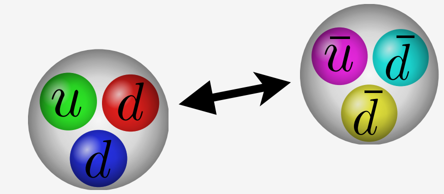




Control of systematic uncertainties

- Chiral-symmetric fermions with physical pion masses
- Variational analysis of ground/excited states

n↔n̄ Amplitudes from Lattice QCD



Lattice calculations at the physical point

[E.Rinaldi, S.S., M.Wagman; PRL'19; PRD'19]

	$\mathcal{O}^{\overline{MS}}(2\text{ GeV})$	Bag "A"	$\frac{\text{LQCD}}{\text{Bag "A"}}$	Bag "B"	$\frac{\text{LQCD}}{\text{Bag "B"}}$
$[(RRR)_3]$	0	0	–	0	–
$[(RRR)_1]$	45.4(5.6)	8.190	5.5	6.660	6.8
$[R_1(LL)_0]$	44.0(4.1)	7.230	6.1	6.090	7.2
$[(RR)_1L_0]$	-66.6(7.7)	-9.540	7.0	-8.160	8.1
$[(RR)_2L_1]^{(1)}$	-2.12(26)	1.260	-1.7	-0.666	3.2
$[(RR)_2L_1]^{(2)}$	0.531(64)	-0.314	-1.7	0.167	3.2
$[(RR)_2L_1]^{(3)}$	-1.06(13)	0.630	-1.7	-0.330	3.2
	$[10^{-5}\text{ GeV}^{-6}]$	$[10^{-5}\text{ GeV}^{-6}]$		$[10^{-5}\text{ GeV}^{-6}]$	

(comparison to MIT Bag model calculations [S.Rao, R.Shrock, PLB116:238 (1982)])

x(5-10) larger N-Nbar oscillation than previously expected

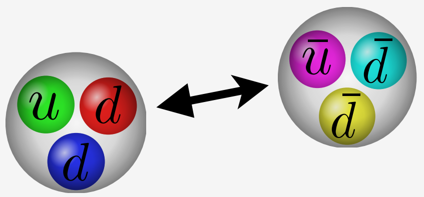
$\Longrightarrow$ ***Stronger constraints on BNV models;***

$\Longrightarrow$ ***Great motivation for new n↔n̄ experiments***

(Next steps:

- "crossed" 2-neutron annihilation amplitudes $\langle \text{vac}|O^{6q}|nn\rangle$
- Nuclear medium effects)

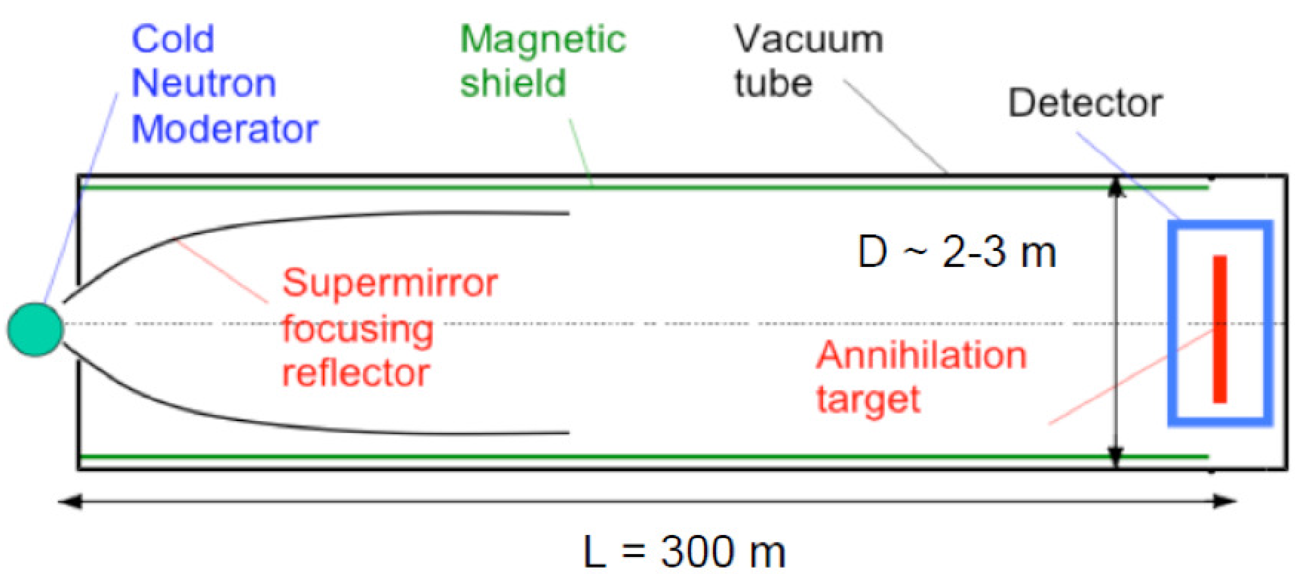
$n \leftrightarrow \bar{n}$ Oscillations: Experimental Outlook



Maximize Probability of oscillation $\sim N_n \; (T_{\mathrm{free}})^2$

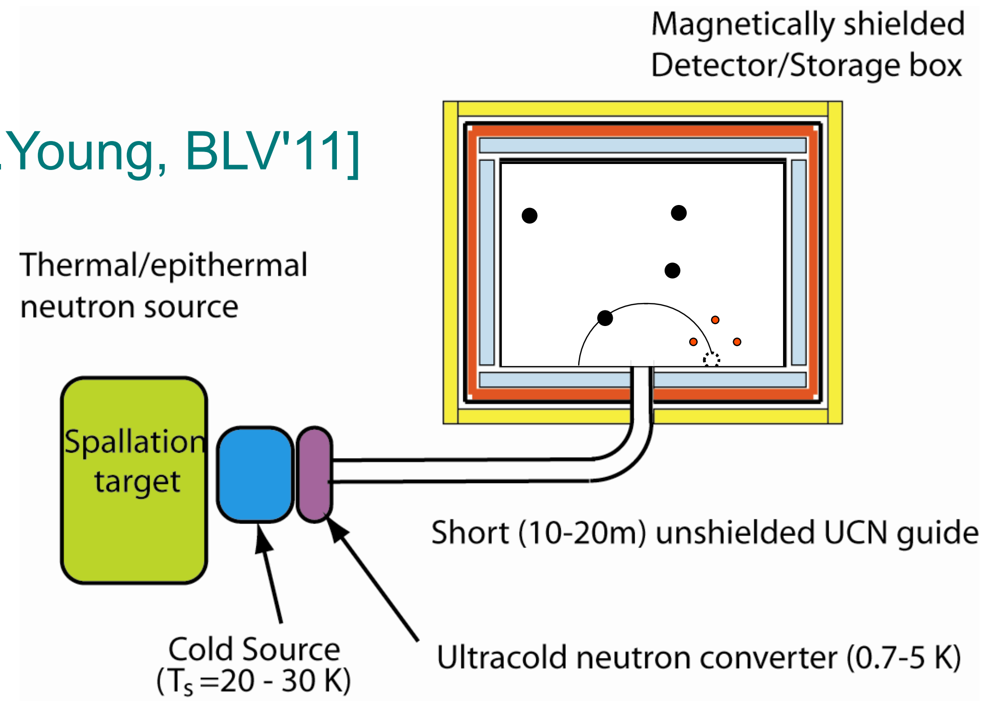
- Shielded beam (similar to ILL):
  Expected sensitivity x**10^2-10^3** ILL
  $\tau_{n\text{-}\bar{n}} \gtrsim 10^9\text{-}10^{10}$ s
  - ✦ Spallation sources: x12 flux @ESS
  - ✦ Elliptic focussing mirror
  - ✦ Better magnetic shielding (B < 1 nT)

[Phillips et al, arXiv:1410.1100]



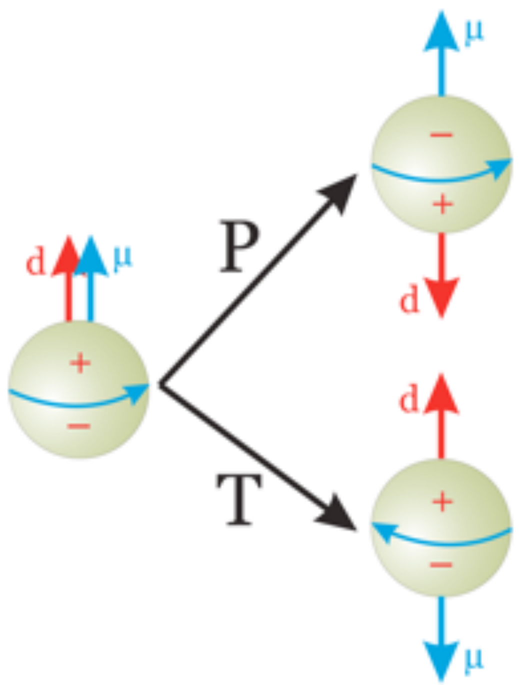
- stored ultra-cold neutrons
  $\tau_{n\text{-}\bar{n}} \gtrsim 2.2 \cdot 10^8$ s

[A.Young, BLV'11]



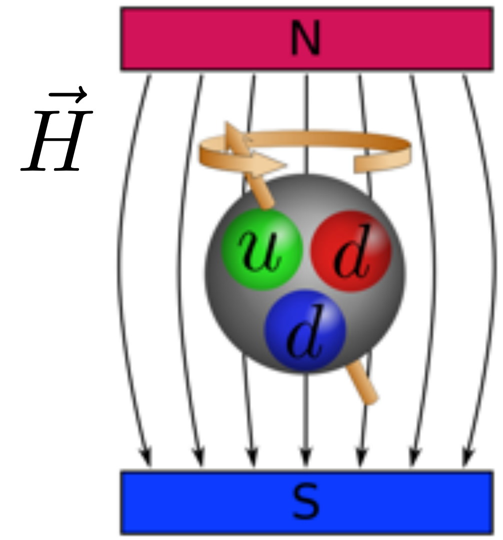
- Further improvements
  - ✦ Larger vessels
  - ✦ Better magnetic shielding (B < 1 nT)
  - ✦ Parabolic floor concentrators
  - ✦ Multiple coherent reflections

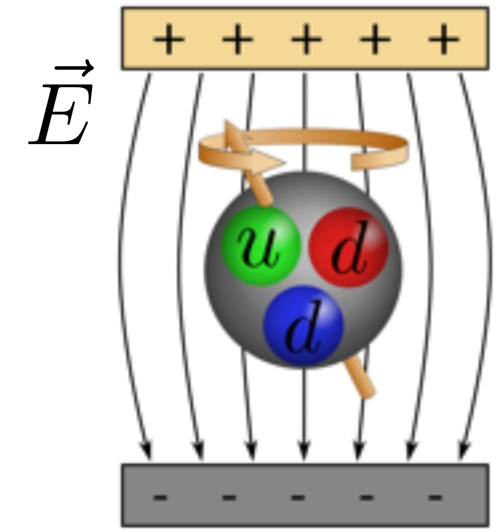
CP Violation & Neutron Electric Dipole Moment



$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$





Magnetic dipole moment $\vec{\mu}_n = \mu_n \vec{S}$

Electric dipole moment $\vec{d}_n = d_n \vec{S}$

EDMs are the most sensitive probes of CPv:

- Signals for beyond SM physics

  (SM = 10^{-5} of the current exp.bound)
- Prerequisite for Baryogenesis
- Strong ~~CP~~ problem : θ_{QCD}-induced EDM?

$$\langle N_{p'}|J^\mu|\bar{N}_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

Dirac — Pauli (anom.magnetic) — Electric dipole

Experimental Outlook

Current nEDM limits:

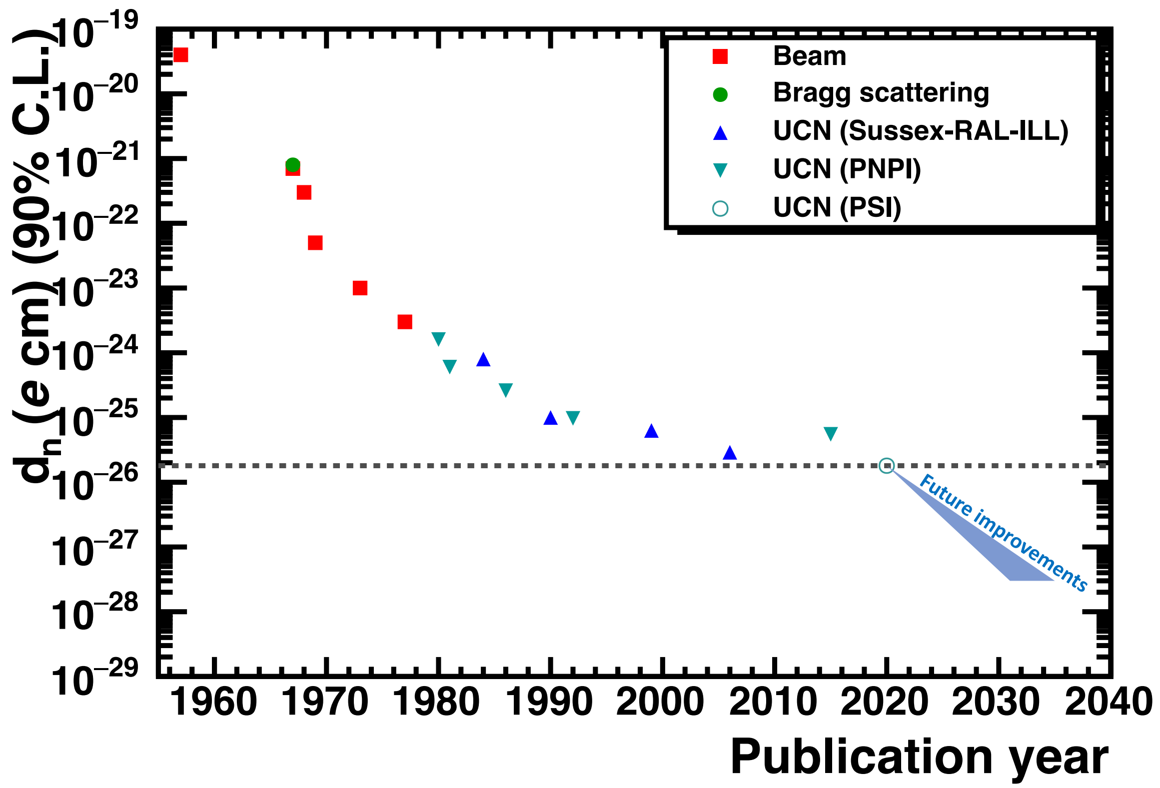
- $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$ (stored UC neutrons)
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26}\, e \cdot \mathrm{cm}$ (^{199}Hg)
  [Graner et al, PRL116:161601(2016)]

Future nEDM sensitivity :

- 1–2 years : next best limit?
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

	10^{-28} e cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*

[Snowmass EDM workshop report, arXiv:2203.08103]



Electric Dipole Moments: Window to New Physics

- New CP violating interactions induce CPv at quark-gluon level
  [Engel, Ramsey-Musolf, van Kolck, Prog.Part.Nucl.Phys. 71:21 (2013)]

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

suppressed by new-physics energy scale

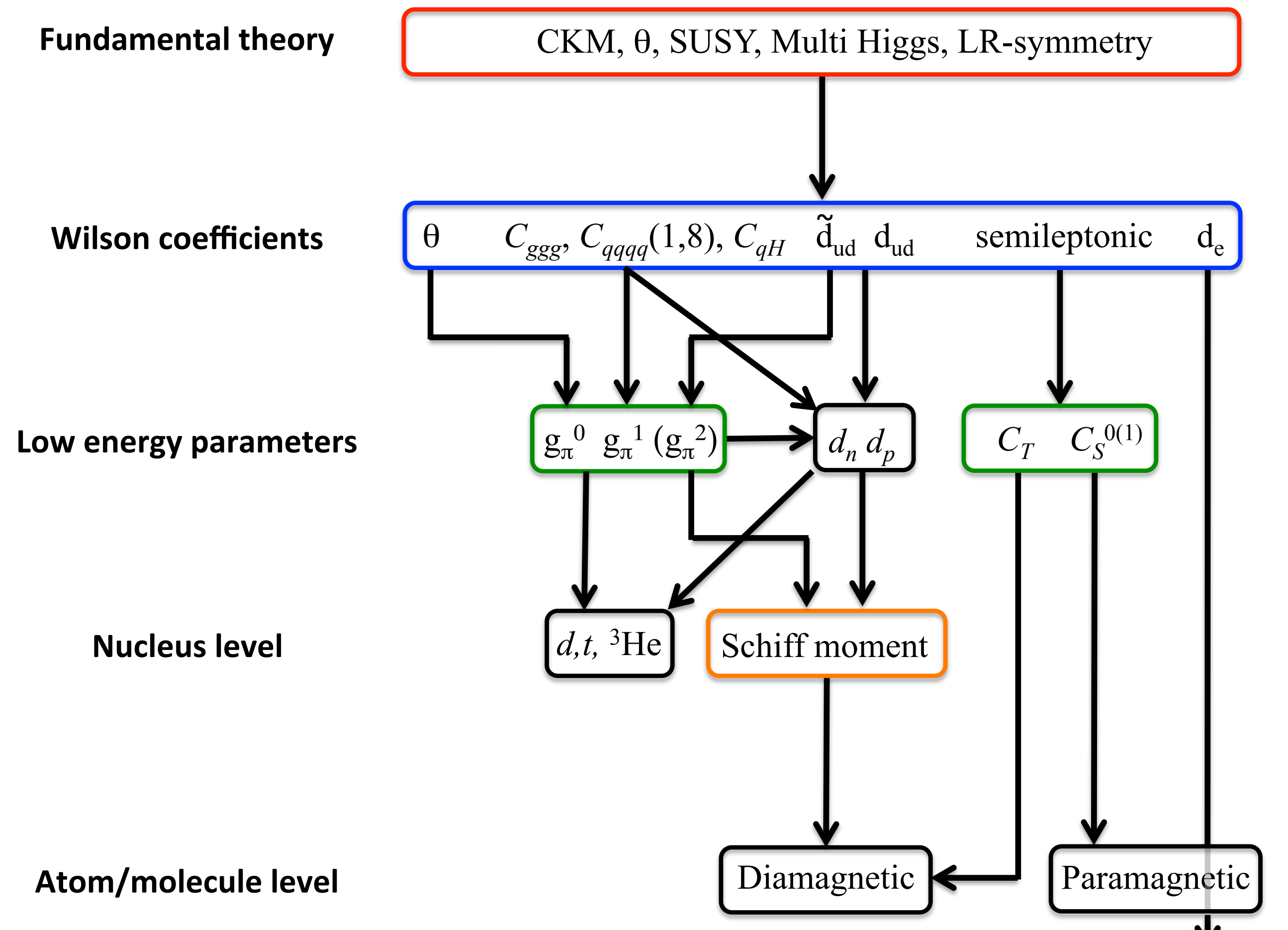
$d{=}4$: θ_{QCD}-term : *mix with $d{>}4$ isoscalar CPv*

$d{=}5$: quark EDM / chromo-EDM

$d{=}6$: 4-fermion CPv, 3-gluon (Weinberg)

....

$$d_{n,p} = d^{\theta}_{n,p}\theta_{\rm QCD} + d^{cEDM}_{n,p} c_{cEDM} + \dots$$



[Courtesy of Tim Chupp; Rev.Mod.Phys 91:015001 (2019)]

$$c_i \iff d_{n,p} \quad ?$$

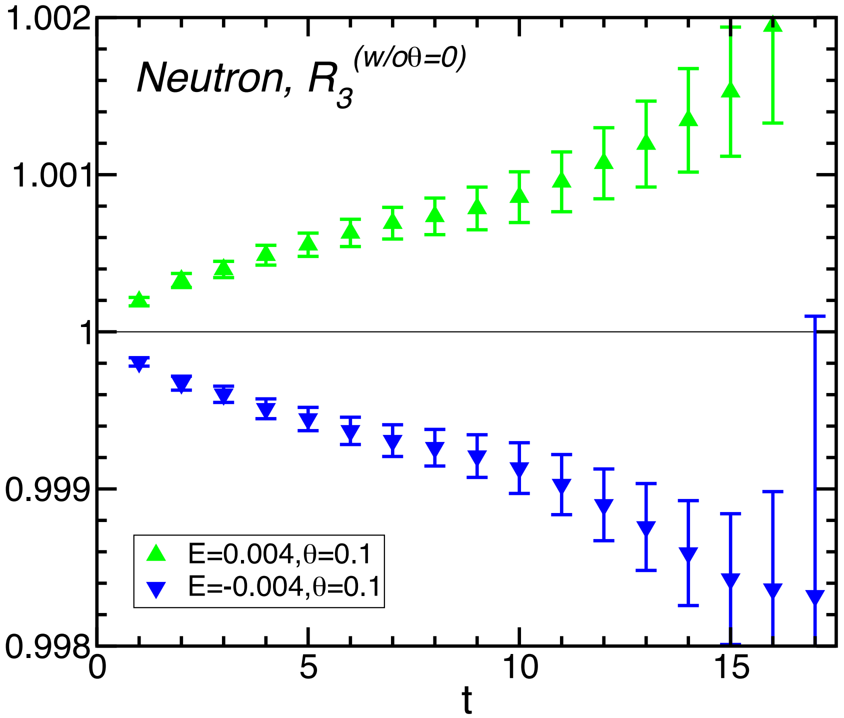
- Need Lattice QCD to relate
  Quark-gluon CPv interactions $\Longrightarrow$ nucleon EDMs , CPv πNN couplings

Determination of Nucleon EDM

- Compute ***Energy Shift*** in uniform electric field
  [S.Aoki et al '89 ; E.Shintani et al '06;
  E.Shintani et al, PRD75, 034507(2007)]

$$\langle N(t)\bar{N}(0)\rangle_{\theta,\vec{E}} \sim e^{-(E\pm\vec{d}_N\cdot\vec{E})t}$$

[Shintani et al (2015)]

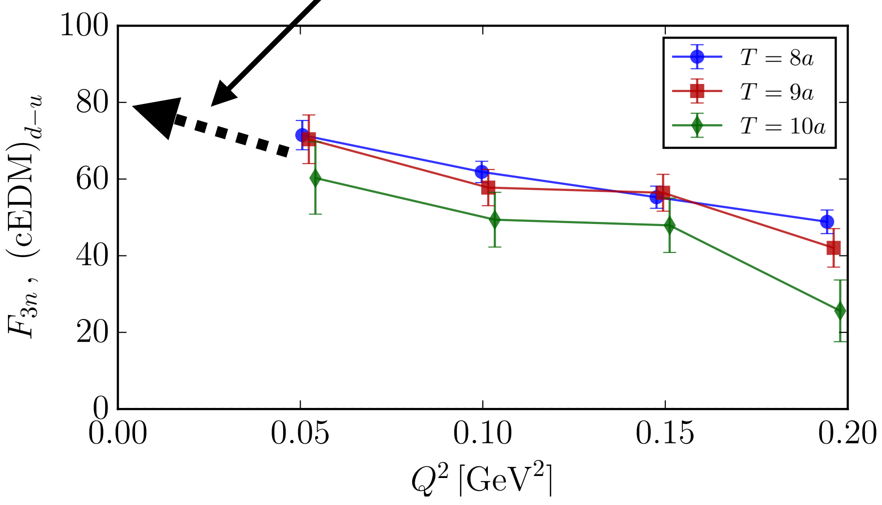


- Compute ***CPv Form-Factor*** F_3: $d_N = F_3(Q^2\to 0) / (2m_N)$
  [(everybody else, almost)]

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

- pre-2017 : spurious $\mu_n \leftrightarrow d_n$ mixing
- Dragos et al(2019) $d_n/\theta = -0.0015(7)\ e\cdot\text{fm}$
- Alexandrou et al(2020) $d_n/\theta = 0.0009(24)\ e\cdot\text{fm}$
- Bhattacharya et al (2021) $|d_n/\theta| \lesssim 0.01\ e\cdot\text{fm}$
- Liang et al (2023) $d_n/\theta = -0.0015(1)(3)\ e\cdot\text{fm}$

Need extrapolation to forward-limit $F_3(Q^2\to 0)$



Nucleon "Parity Mixing"

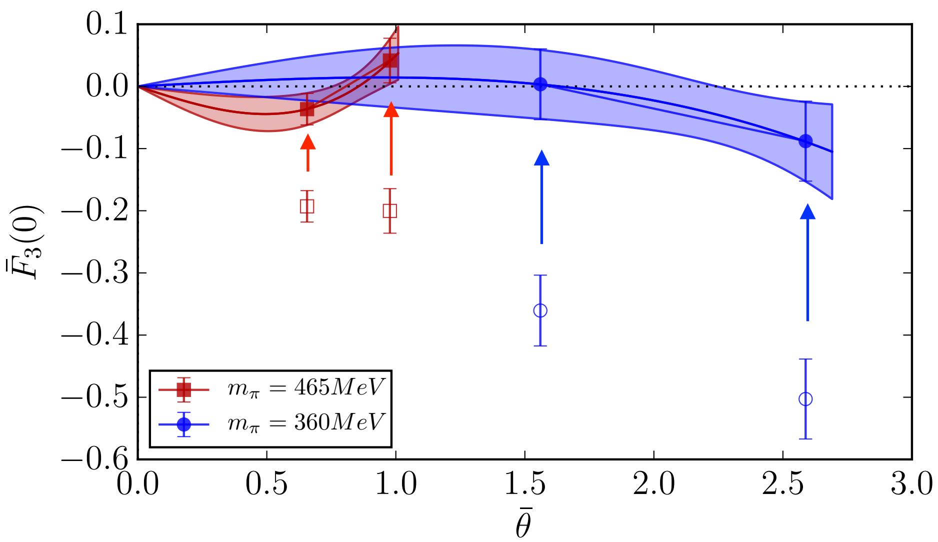
CPv interaction induces a chiral phase in nucleon spinor ;
lattice calculations of EDM have to account for that
[M.Abramczyk, S.Aoki, S.N.S, et al (2017) arXiv:1701.07792]

$$\langle \mathrm{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

Value of α-mixing is critical for correct determination of EDM:

$$F_3^{\mathrm{lat}}(Q^2) \approx \frac{m}{q_3} \underbrace{\langle N_\uparrow(0)|\bar{q}\gamma_4 q|N_\uparrow(-q_3)\rangle_{\not{CP}}}_{\text{CPv matrix element}} - \underbrace{\alpha_5 G_E(Q^2)}_{\text{Sachs form factor subtraction}}$$

$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\mathrm{true}} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$



Pre-2017 lattice results for θ_{QCD}-*n*EDM: ***original*** *and* ***corrected***

		m_π [MeV]	m_N [GeV]	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	−0.555(74)	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	−0.375(48)	−0.130(76)
	n	360	1.138(13)	−0.248(29)	0.020(58)

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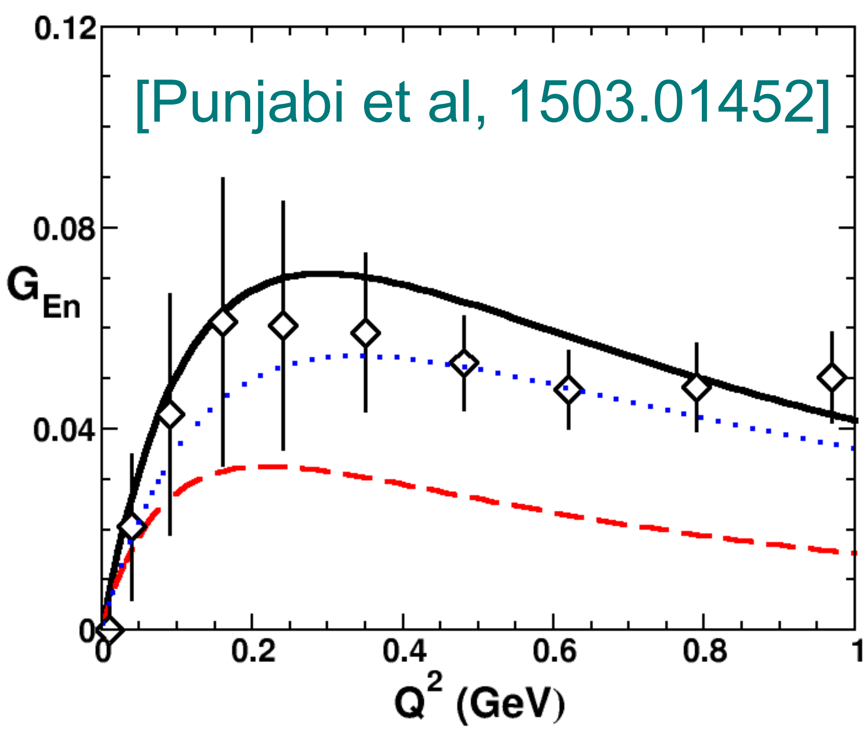
- Proton ($G_{Ep}(0)=1$) : Correction ~ α_5
- Neutron ($G_{En}(0)=0$) : No correction at $Q^2=0$
  However, $Q^2\to 0$ extrapolation may be skewed by neutron electric form factor ~$\alpha_5\, G_{En}(Q^2)$



Signal & Noise in θ_{QCD}-induced nEDM

Correlator for EDM $d_N \sim \langle Q \cdot (N J_\mu \bar{N}) \rangle$

(topological charge $Q \sim \int_{V_4} (G\tilde{G})$)

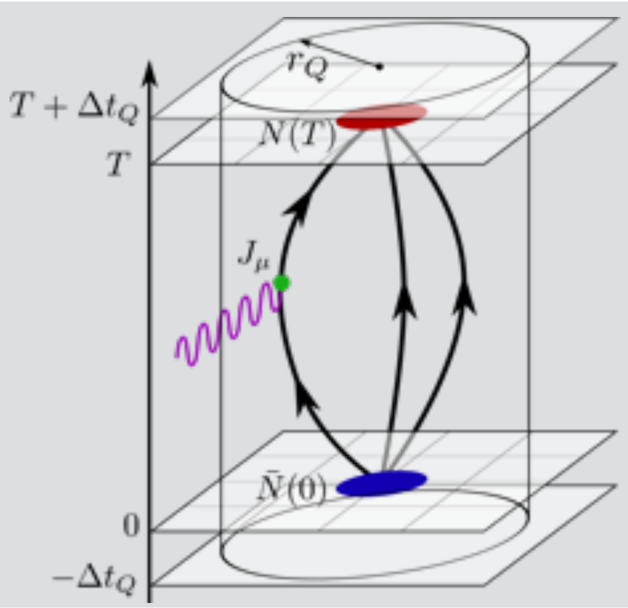
Stat. noise ~ *Variance of* Q $\sim (Volume)_{4d}$

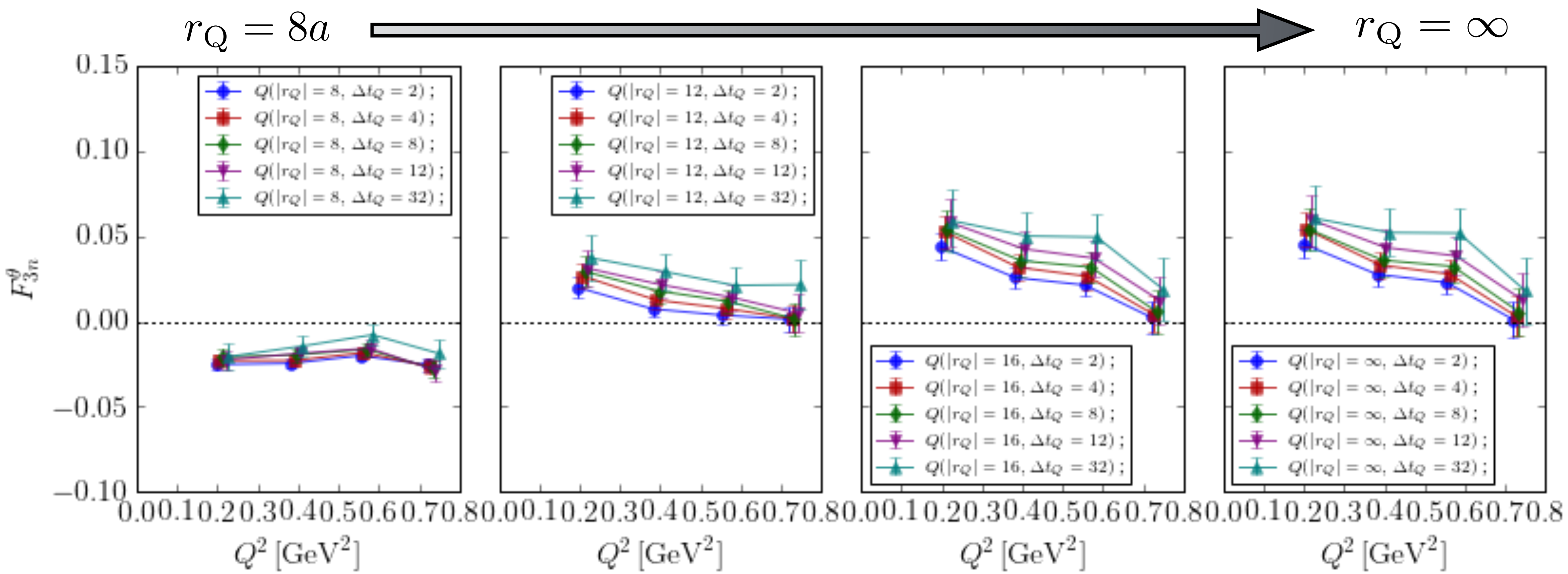
Q integral cutoff to reduce noise ("cluster decomposition")

[E.Shintani et al (2015) ; K.-F. Liu et al (2023) ; Dragos et al (2019)]

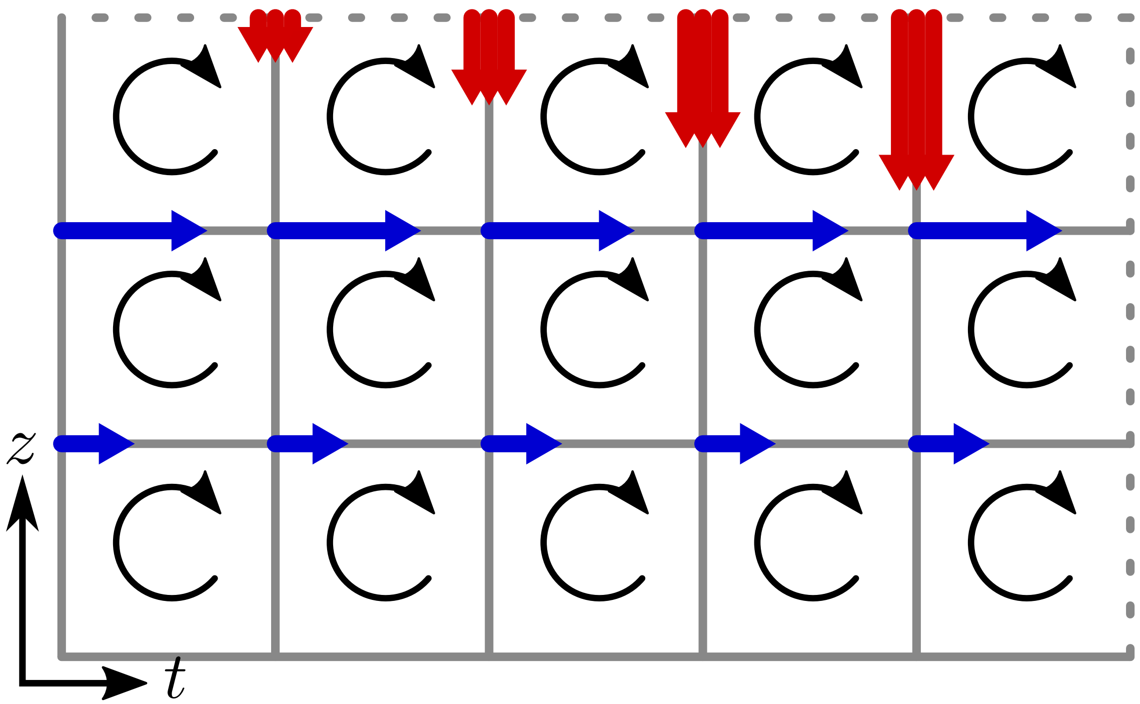
- in time around current, $|t_Q - t_J| < \Delta t$
- in space around nucleon $r < r_Q$

$\Longrightarrow$ *challenge: systematic bias?*





Alternative: Background Electric Field



Calculation of magnetic and electric moments, hadron polarizabilities [W.Detmold et al (2009)]

Electric field on a periodic lattice is "quantized"

$$\mathcal{E}_{\rm min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$$

$$\approx 0.037\,\mathrm{GeV}^2 = 187\,\mathrm{MV/fm}$$

for a (2.8 fm)^3 x (5.6 fm) lattice

Feynman-Hellman theorem : relate energy shift ...

$$m'_N = m_N - (d_N^\theta \theta)\,\vec\Sigma\cdot\vec{\mathcal{E}}$$

... to matrix element of <u>local topological charge density</u>

$$d_N^\theta \propto \left\langle N_\uparrow \right| \int d^3x\, G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu} \left| N_\uparrow \right\rangle_{\mathcal{E}_z}$$

polarized in ***spin*** and ***charge***

Advantages:

- sample $G\tilde{G}$ only on one time slice
  $\Longrightarrow$ noise reduction
- no need for $Q^2 \to 0$ momentum extrapolation

$\langle N_\uparrow|\, G\tilde{G}\, |N_\uparrow\rangle_{E=0}$ vanishes
⇩
sensitive to EDM directly
(no bias)

Topological Charge with Gradient Flow

[M.Luscher, JHEP08:071; 1006.4518]

Gradient flow: covariant *4D-diffusion* of quantum fields with "G.F." time t_{GF}:

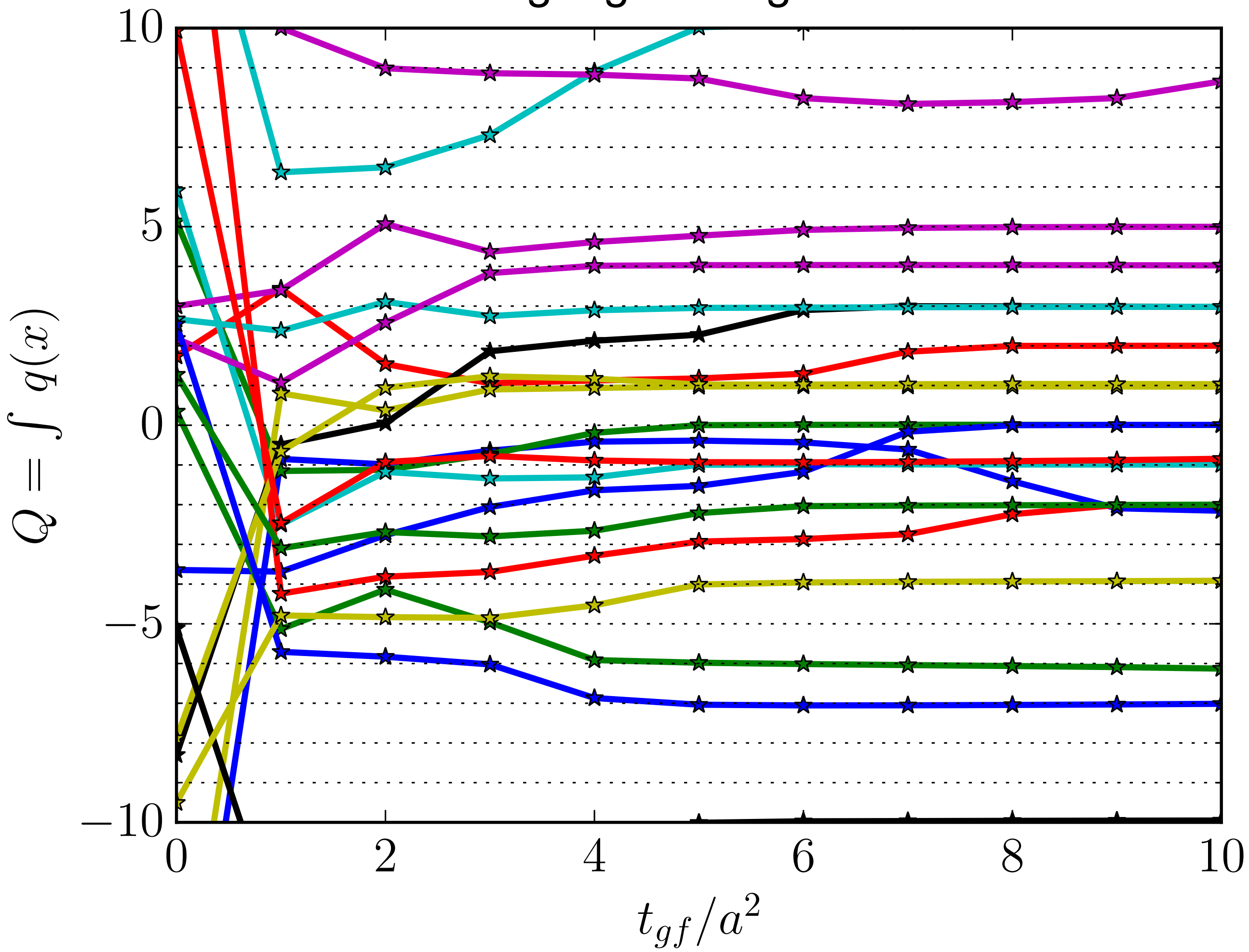
$$\frac{d}{dt_{\mathrm{GF}}} B_\mu(t_{\mathrm{GF}}) = D_\mu G_{\mu\nu}(t_{\mathrm{GF}})\,, \quad B_\mu(0) = A_\mu$$

Tree-level:

$$B_\mu(x, t_{\mathrm{GF}}) \propto \int d^4y \, \exp\Big[-\frac{(x-y)^2}{4t_{\mathrm{GF}}}\Big] A_\mu(y)$$

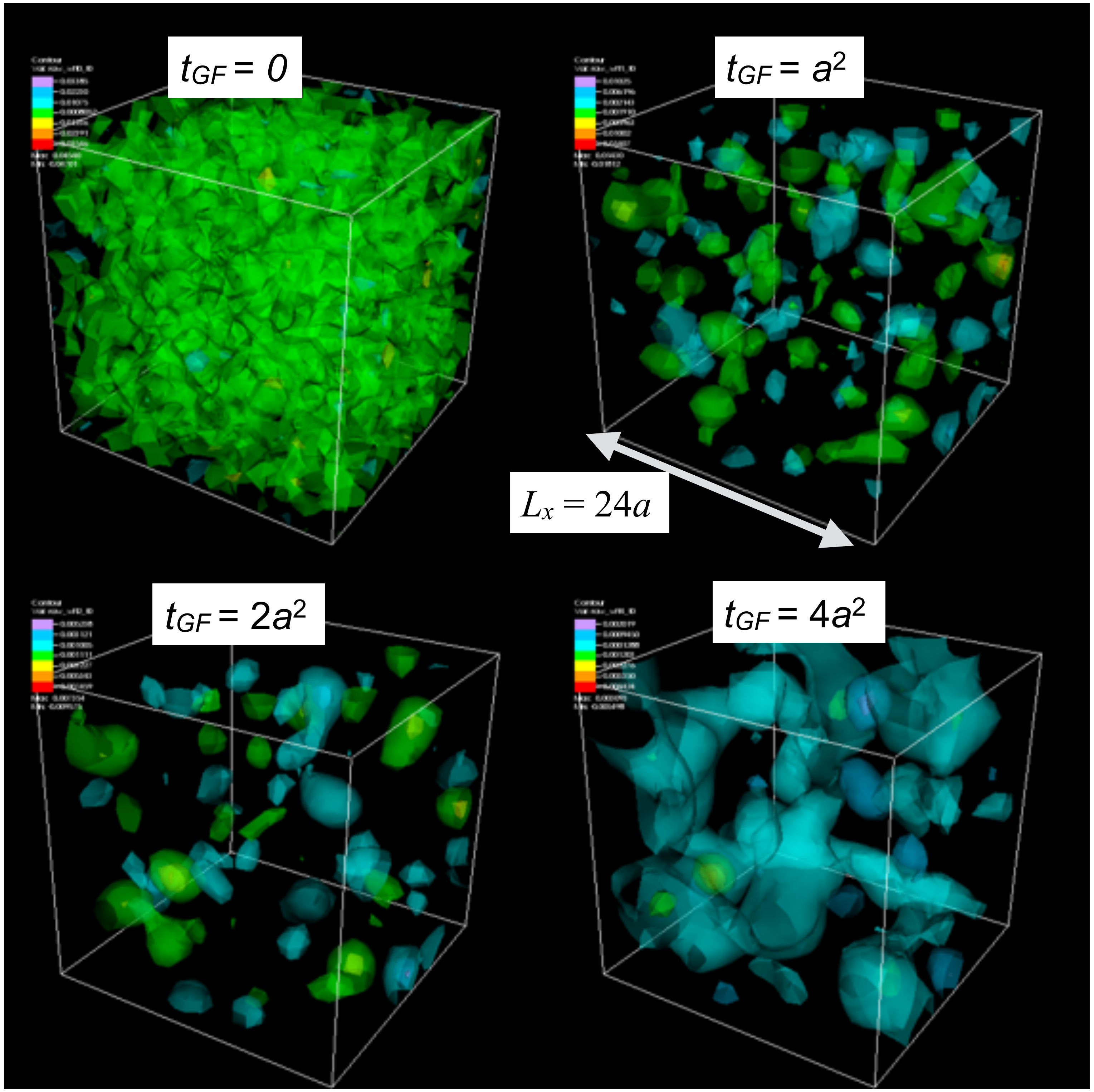
Gradient-flowed topological charge:

$$\tilde{Q}(t_{\mathrm{GF}}) = \int d^4x \, \frac{g^2}{32\pi^2} \Big[G_{\mu\nu}\widetilde{G}_{\mu\nu}\Big]\Big|_{t_{\mathrm{GF}}}$$



- continuous "cooling":
  effective scale $\Lambda_{\mathrm{UV}} \to (t_{\mathrm{GF}})^{-1/2}$
- smoothing fields (reduce $|G_{\mu\nu}|$)
  remove $G_{\mu\nu}$ dislocations;
  dynamical separation of top. sectors [M.Luscher, JHEP08:071; 1006.4518]
- "diffusion" of topological charge density makes it nonlocal

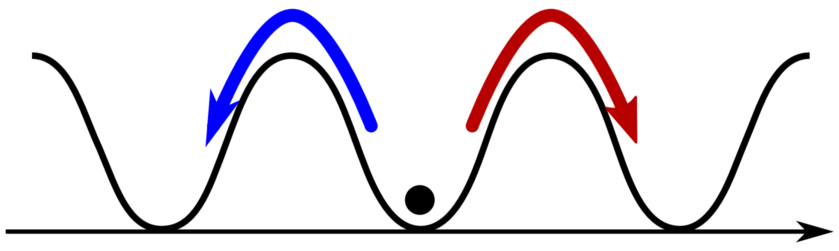
Gradient-Flowed Topological Charge Density



$24^3 \times 64$ lattice, $m\pi \approx 340\ MeV$

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$
$$\approx \frac{1}{16\pi^2}\frac{1}{a^4}\mathrm{Tr}\left[G^{\rm lat}_{\mu\nu}\widetilde{G}^{\rm lat}_{\mu\nu}\right]$$
$$\propto (\mathbf{E}\cdot\mathbf{H})_{\rm color}$$

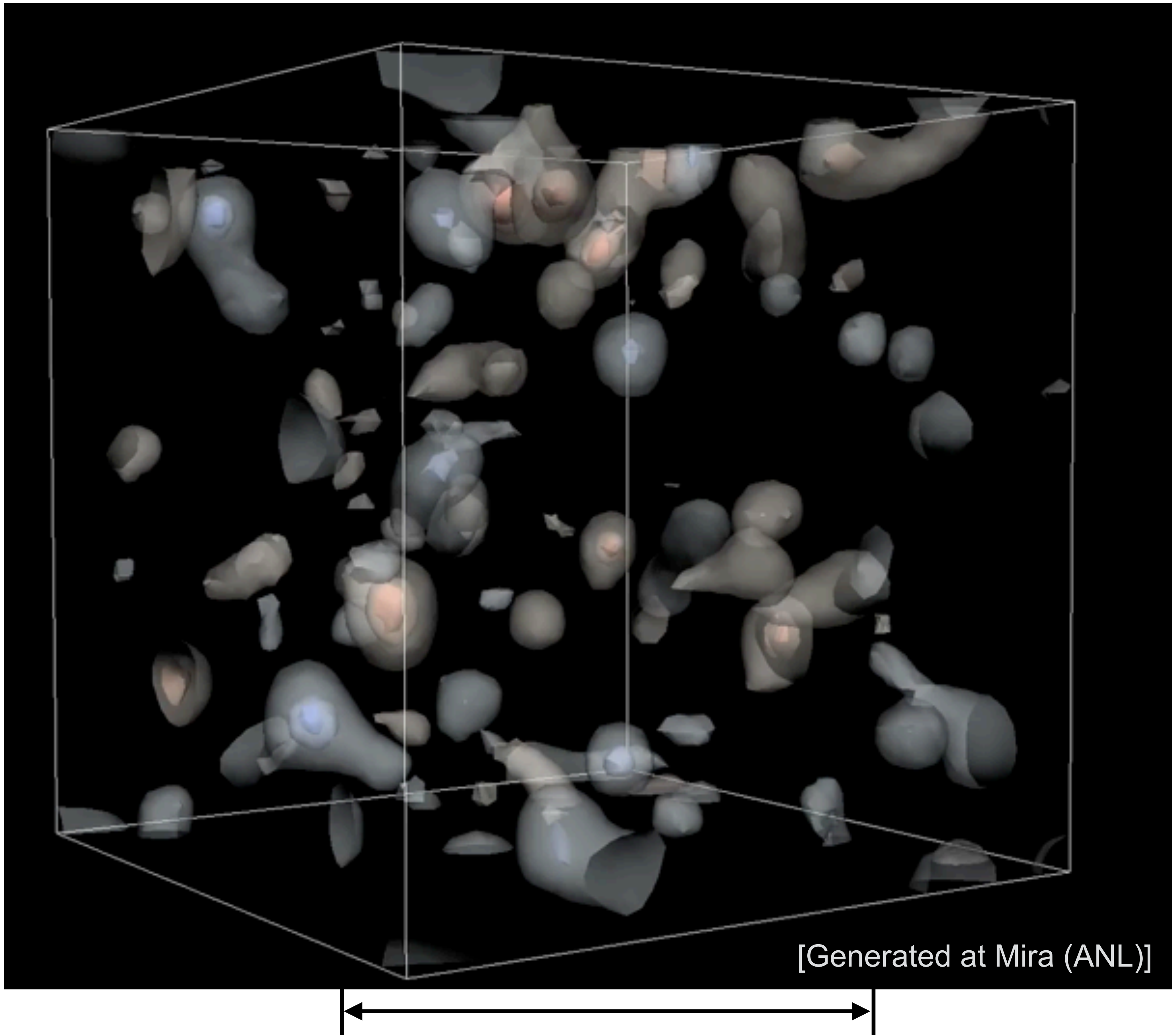
Instantons and Anti-Instantons : Quantum tunneling of gluon field between topological sectors



CPv-QCD Θ-Vacuum :

$$|vac\rangle_\theta = \sum_Q e^{i\theta Q}\,|Q\rangle$$

Tunneling Between Topology Sectors



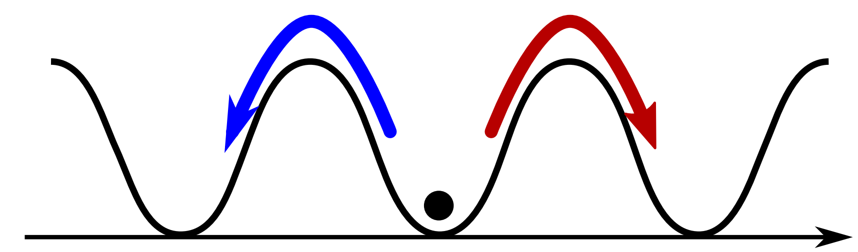
5 fm=5·10^{-15}m

6 s video =5 fm / c =1.7·10^{-23} s real time

[Lattice QCD at the physical point]

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}$$
$$\approx \frac{1}{16\pi^2}\frac{1}{a^4}\mathrm{Tr}\left[G^{\mathrm{lat}}_{\mu\nu}\widetilde{G}^{\mathrm{lat}}_{\mu\nu}\right]$$
$$\propto (\mathbf{E}\cdot\mathbf{H})_{\mathrm{color}}$$

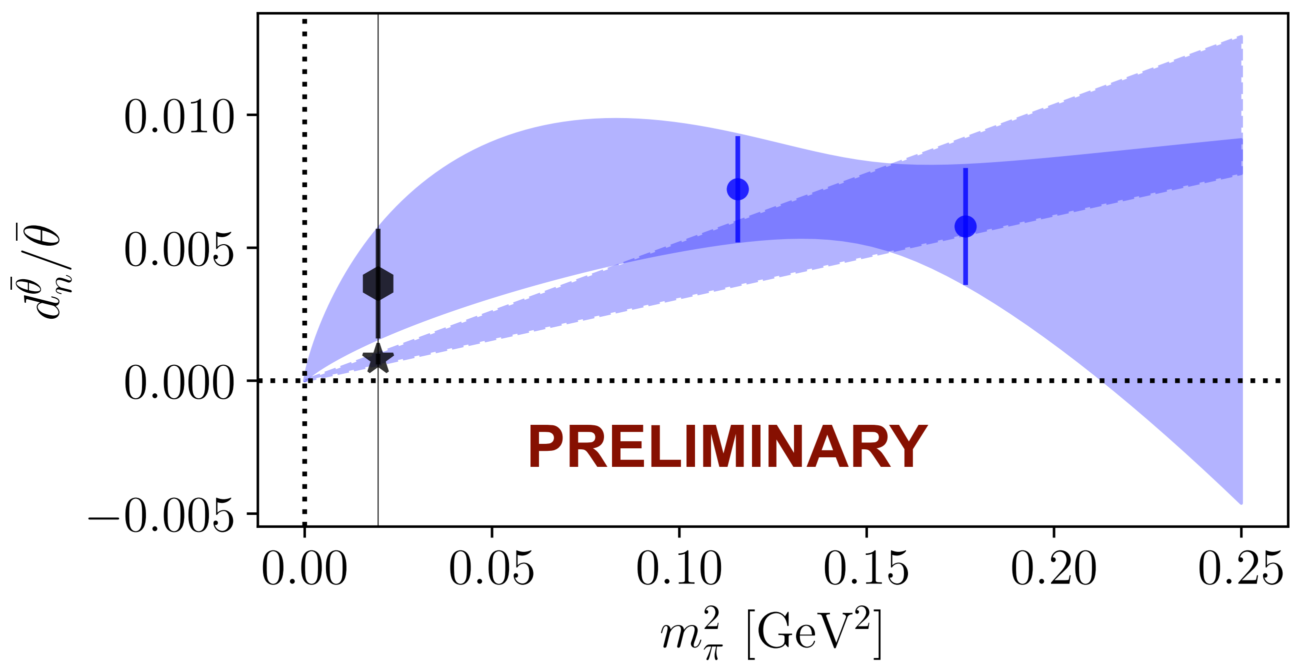
Instantons and Anti-Instantons :
Quantum tunneling of gluon field between topological sectors

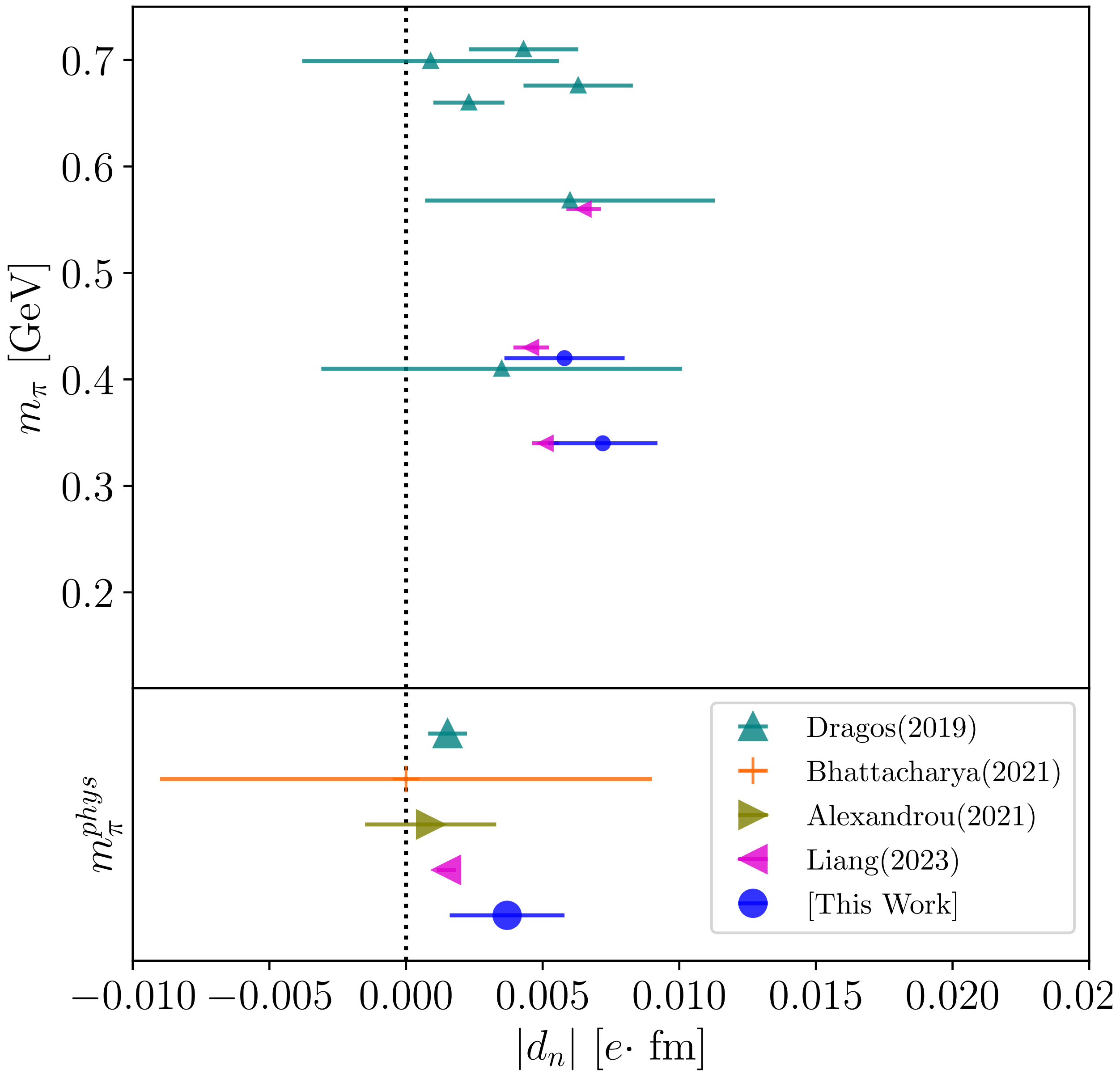


CPv-QCD Θ-Vacuum :

$$|vac\rangle_\theta = \sum_Q e^{i\theta Q}\,|Q\rangle$$

Extrapolation to the Physical Point





Chiral extrapolation
[Hockings, van Kolck (2005)]

$$d_n(m_\pi) = C_1 m_\pi^2 + C_2 m_\pi^2 \log \frac{m_\pi^2}{m_N^2}$$

(Only multiplicative O(a^2) corrections with chiral-symmetric lattice fermions)

Summary of neutron θ-EDM from Lattice QCD

Summary

- *Nucleon structure calculations on a lattice are critical to searches for symmetry violations, understanding the origin of nuclear matter*

- Proton decays $p \to \pi/K$, $p \to$ leptons

  No topological suppression of nucleon decay found; confirm limits on GUTs

  NEXT: $p \to \rho \to \pi\pi$, $p \to K^* \to \pi K$ amplitudes

- Neutron-antineutron oscillation

  Amplitudes × (6 ... 8) larger than from pheno.models

  NEXT: $nn \to$ vacuum amplitudes, $n \to \bar{n}$ in nuclear medium

- Novel method to compute nEDM from local topological charge

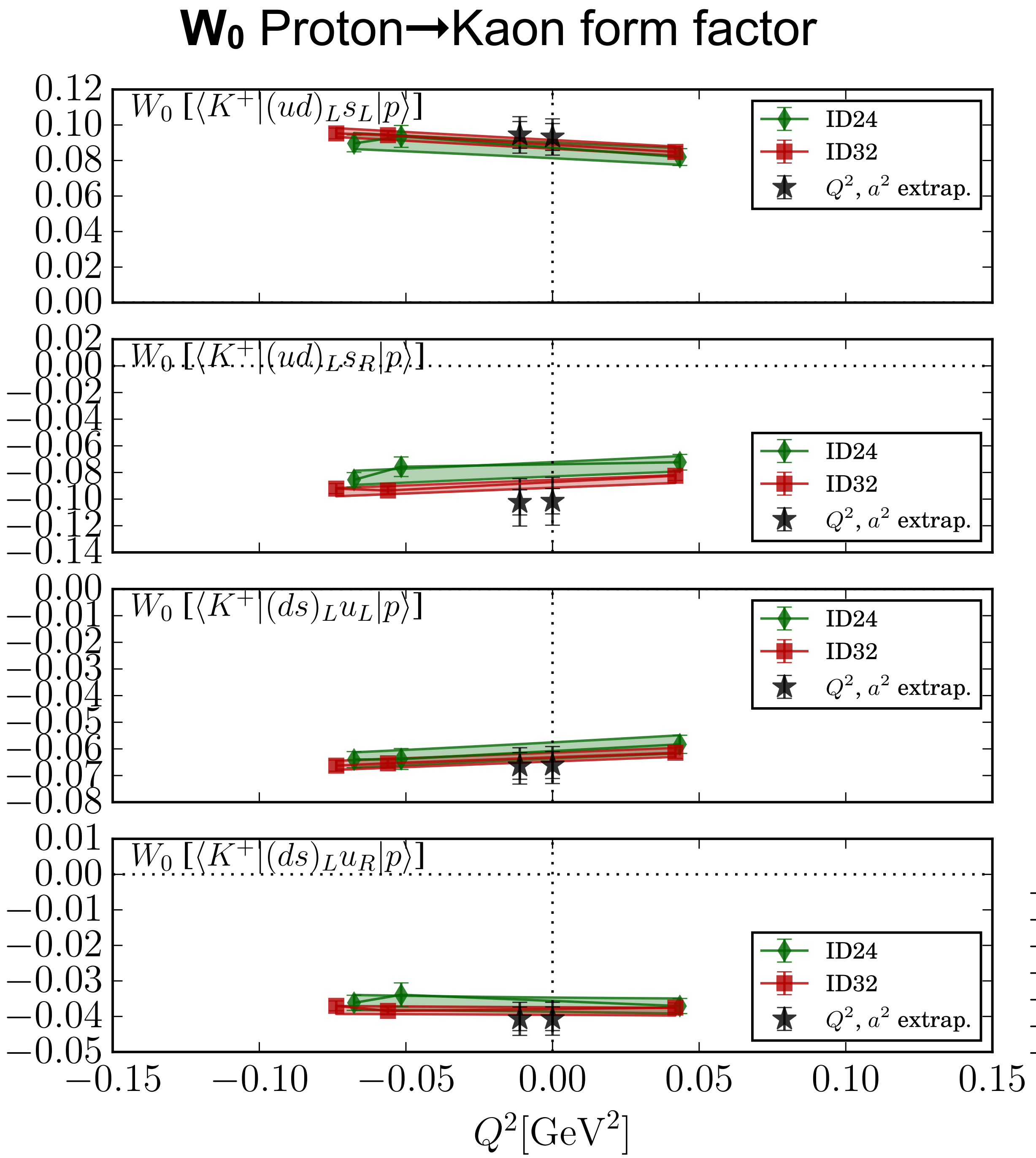
  Cross-check for electric-dipole form factor calculation

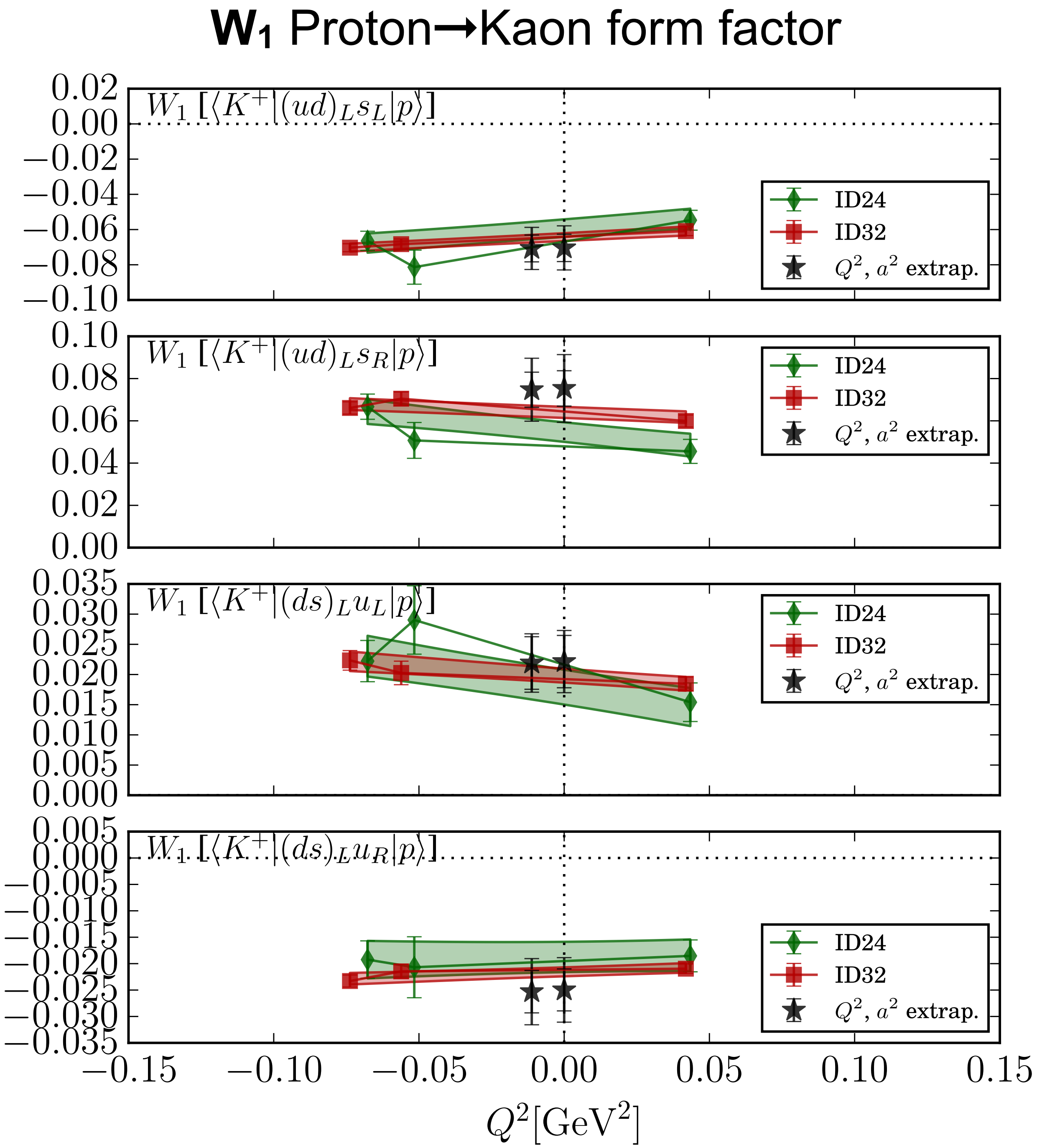
  Results consistent with earlier works

  Potential method of choice for physical-point calculations with large V_4

BACKUP

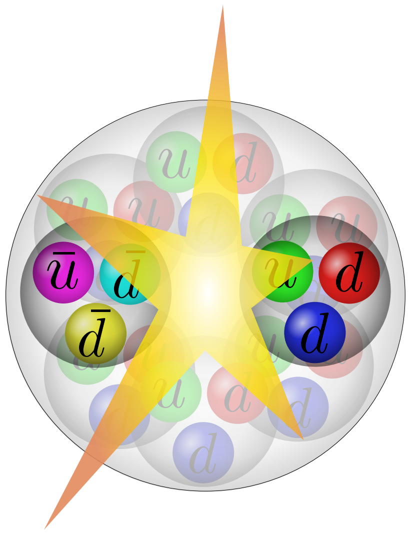
Proton Decay : Extrap. in Q^2 and Lattice Spacing





- linear momentum extrapolation $Q^2 \rightarrow m_e^2, m_\mu^2$ to the decay kinematics
- Continuum extrapolation $A(a^2) \sim (A_0 + A_2\, a^2)$; sys.error = $|A_0 - A_{[a=0.14\mathrm{fm}]}|$

Searches for N⇔N̄ in Nuclei



Nucleus decay from (*nn*) annihilation

$T_d = R\tau_{n\bar{n}}^2$

$R \sim 10^{23}\ s^{-1}$

nuclear model uncertainty
~ 10-15% for ^{16}O
[E.Friedman, A.Gal (2008)]

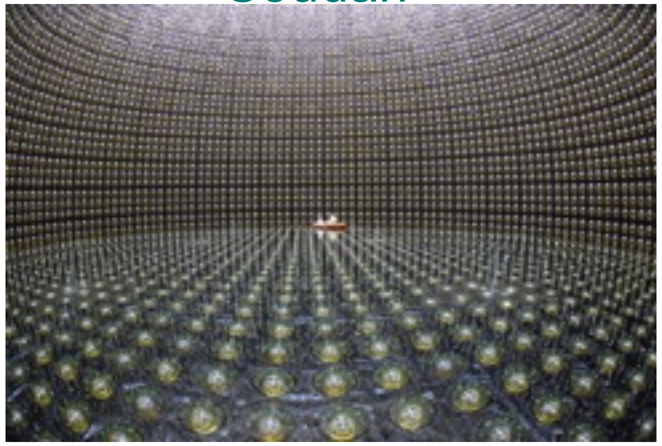
	oscillation in nuclei	oscillation in vacuum
^{56}Fe [Soudan 2]	$T_d(^{56}Fe) > 0.72 \cdot 10^{32}$ yr $\longrightarrow$	$\tau_{n\bar{n}} > 1.4 \cdot 10^8$ s
^{16}O [Super-K]	$T_d(^{16}O) > 1.77 \cdot 10^{32}$ yr $\longrightarrow$	$\tau_{n\bar{n}} > 3.3 \cdot 10^8$ s
^{2}H [SNO]	$T_d(^{2}H) > 0.54 \cdot 10^{32}$ yr $\longrightarrow$	$\tau_{n\bar{n}} > 1.96 \cdot 10^8$ s

Nuclear medium effect suppresses neutron/antineutron oscillation

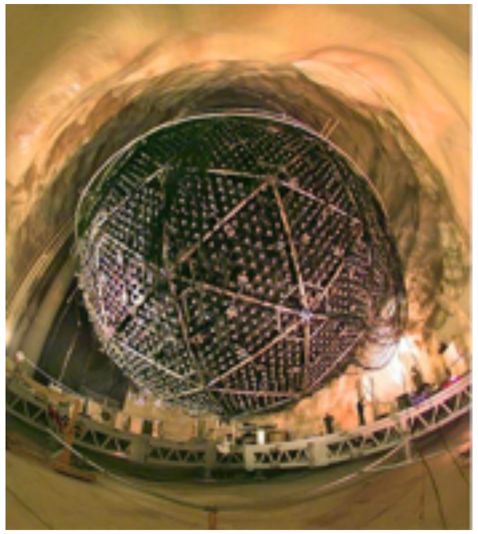
$$\Delta M \sim 100\ \mathrm{MeV}$$

Sensitivity is limited by atmospheric neutrinos

Soudan

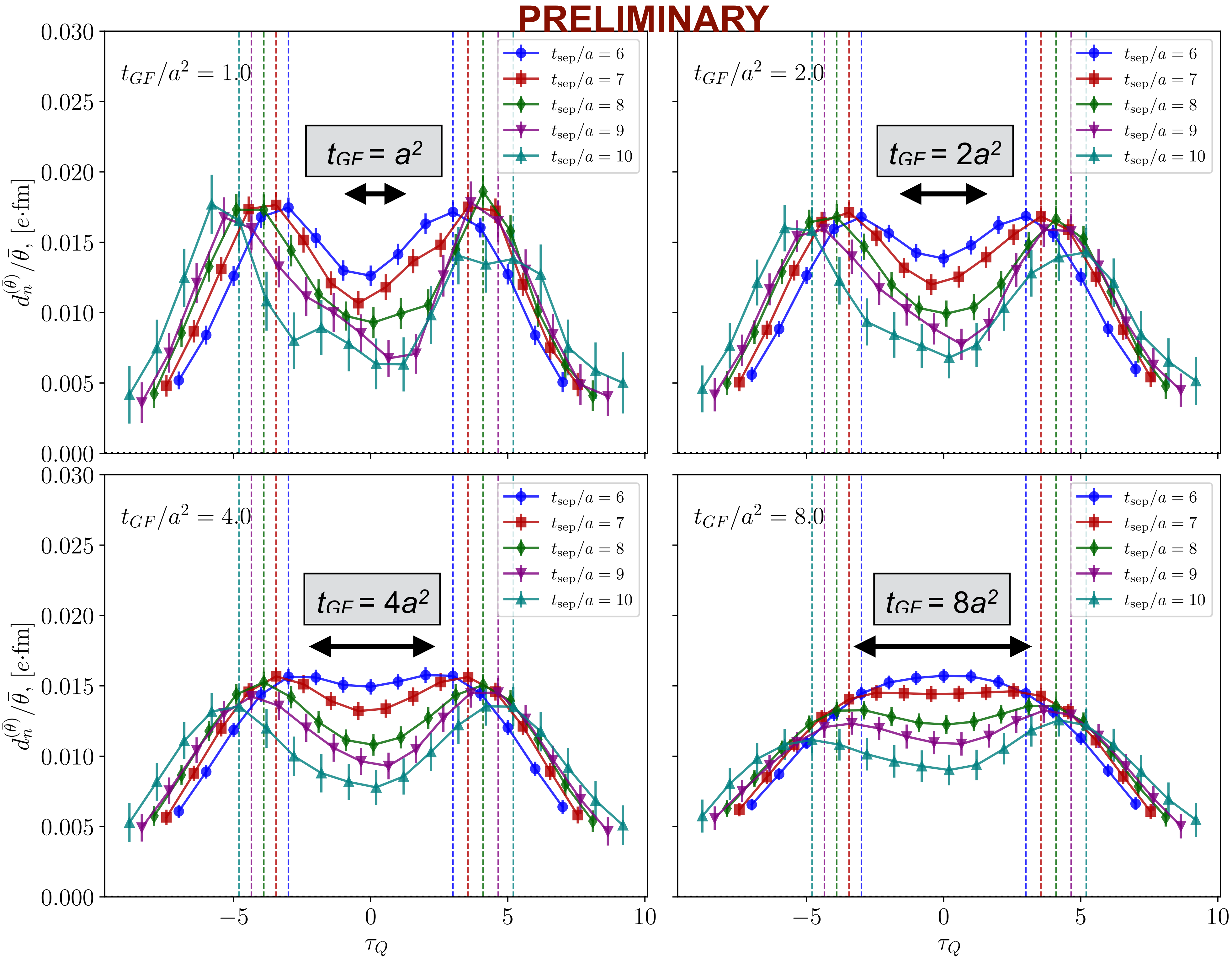


Super Kamiokande



SNO

Matrix Elements of GG (Low-mode Improved)



Two effects observed:

1. Convergence to ground state matrix el.
2. Diffusion of top.charge for $t_{sep} \lesssim 7a$

PRELIMINARY estimates
$2md_n = F_3(0) \approx 0.11 .. 0.13$
agree with form factor

Analysis of (τ_Q, t_{GF}) required to detangle

$$\langle N|G\widetilde{G}|N\rangle\,,$$
$$\langle N|G\widetilde{G}|N\rangle_{exc}\,,$$
$$\langle \text{vac}|G\widetilde{G}|N\bar{N}\rangle\,,$$
$$\ldots$$