P3 2023

2023 Photocathode Physics for Photoinjectors Workshop

Brookhaven National Laboratory

3-5 October 2023

Theory Session

Beam Transport Parameter Sensitivities Using Adjoint Methods for 2D Axisymmetric Systems in Static Fields with MICHELLE

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Introduction

ICOPS 2023
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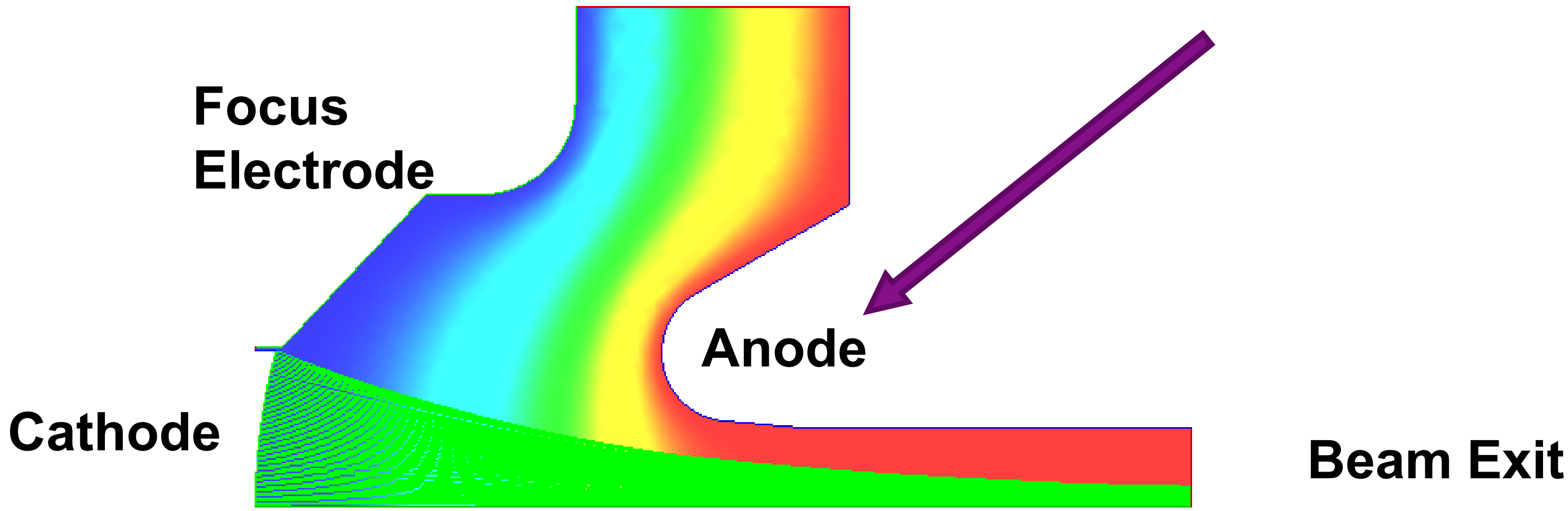
Determining Manufacturing Parameter Sensitivity Functions for Charged Particle Beam Electron Source (electron gun)

- Goal
  - Establish tolerances associated with a variety of manufacturing assembly processes
  - Tolerance sensitivities to include…
    - Alignment between parts
      - Clocking errors, shifts, tilts
    - Material properties, static field errors
    - Magnet location, orientation, and uniformity
  - Goal ➔ increase manufacturing yield (difficult with small scale devices)
- Enabler
  - Process embedded in, or linked to, a "gun" code (e.g., MICHELLE)

Adjoint Method Background: Sensitivity Function

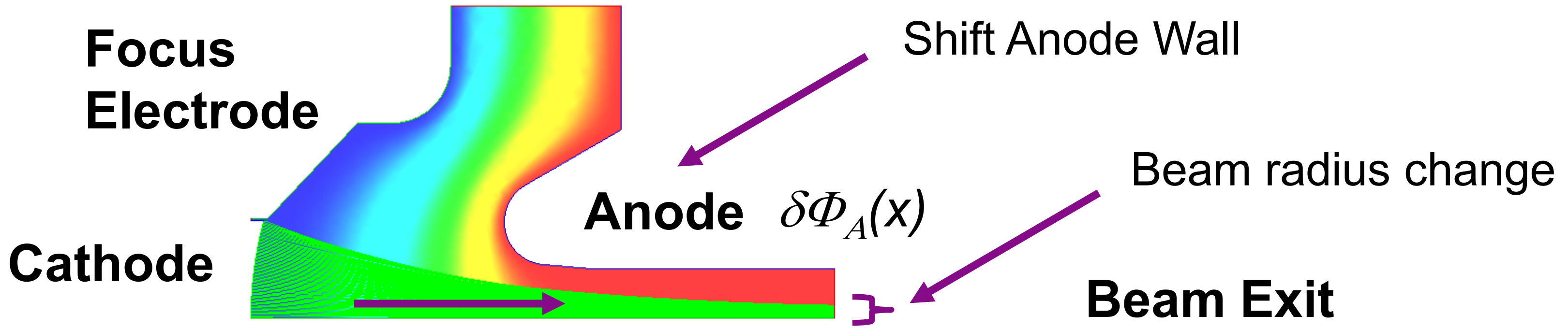
- Basic question: How do small changes in position or potential of anode affect the properties of the beam leaving the gun?



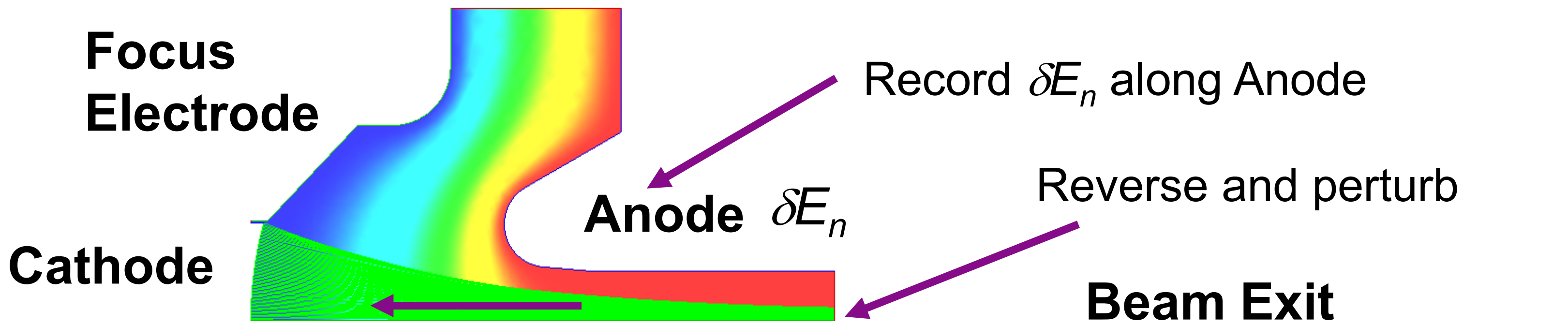
- Conventional solution: Trial and error. Do many simulations with different anode potentials or positions to understand sensitivities. Also leads to selecting the best (optimized) solution based on some performance metric.

Adjoint Method Background:
Based on Concept of Reciprocity

- Problem #1: If there is a shift in voltage by $\delta\Phi_A(x)$ due to wall displacement, you get a change in beam radius



- Problem #2: If you perturb the electron coordinates at the beam exit and reverse the beam, you can calculate a change in normal E-field on the Anode
  - Defines the sensitivity of the beam to wall displacements. *δE_n is the Sensitivity Function*



Adjoint Method Background:
Process - What the optics code contributes

- Exploits the symplectic property of Hamilton's equations
- Code (MICHELLE) solves the following equations:
  1. Integrates equations of motion (Hamilton's equations) for N particles j=1,N

$$\frac{d\mathbf{x}_j}{dt} = \frac{\partial H}{\partial \mathbf{p}} \qquad \frac{d\mathbf{p}_j}{dt} = -\frac{\partial H}{\partial \mathbf{x}}$$

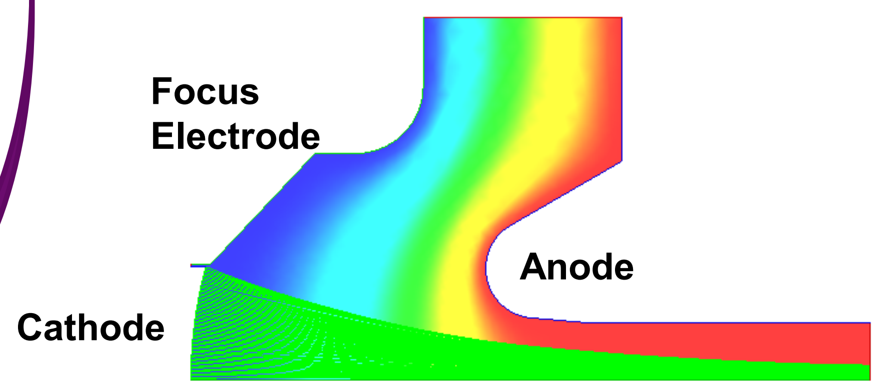
  2. Accumulates charge density

$$\rho(\mathbf{x}) = \sum_j I_j \int_0^{T_j} dt\, \delta(\mathbf{x} - \mathbf{x}_j(t))$$

  3. Solves Poisson's Eq.

$$-\nabla^2 \Phi = 4\pi\rho$$

Repeats (iterates) until converged



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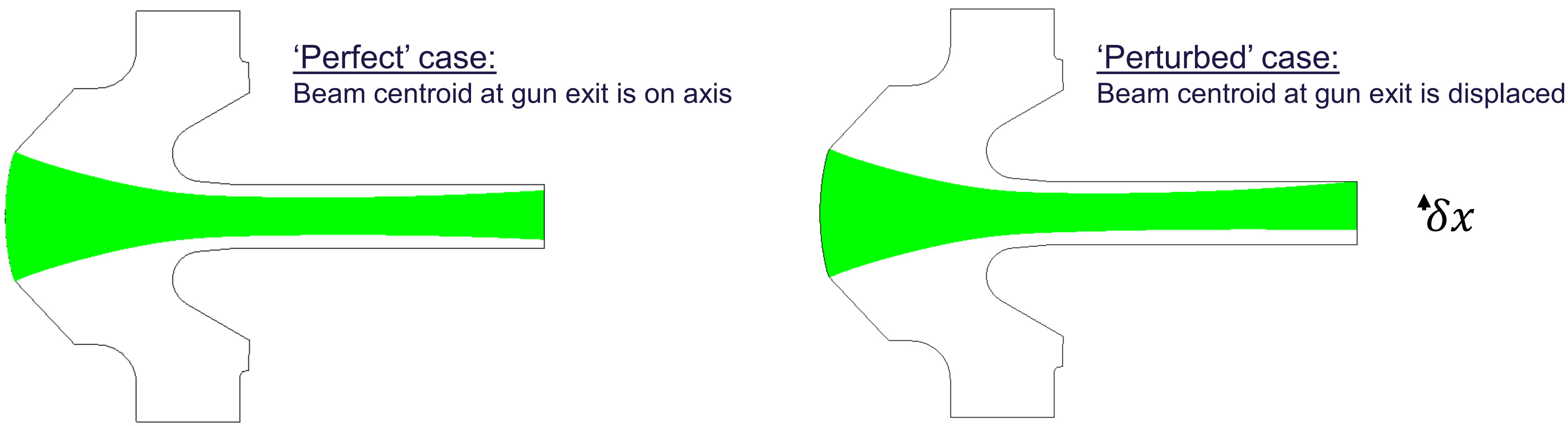
Adjoint Method Background:
Previous Example of the Adjoint Method (2017)

- Compute the displacement of the beam in a sheet beam gun due to a small change in anode potential or a small displacement of the anode

MICHELLE Simulations of Sheet Beam Gun

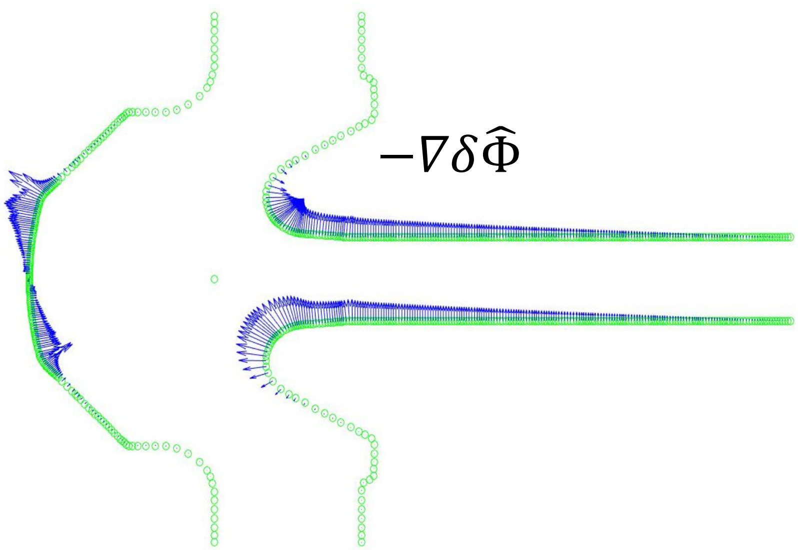


- The adjoint method gives us a way to compute the displacement of the beam due to an anode potential change or moving the anode
  - 1) With one extra run and 2) Without remeshing
  - ➔ Not changing the mesh is key to high accuracy of sensitivity prediction 🡐

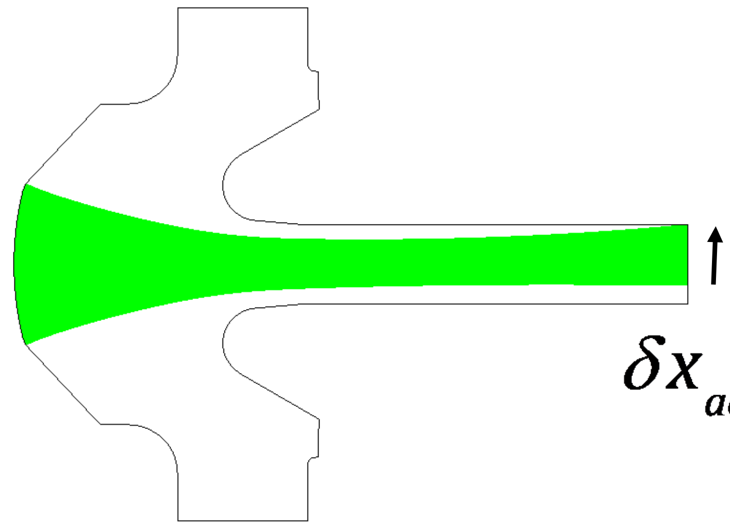
Adjoint Method Background:
Previous Example of the Adjoint Method (2017)

- Comparison: predicted displacement/actual displacement

Vector plot of the 'sensitivity' or Green's function



'Direct' MICHELLE Simulation: Perturbed Anode Voltages



$$\delta x_{pred} = -\frac{q}{4\pi\lambda I}\int_S da\,\mathbf{n}\cdot\delta\Phi\nabla\delta\hat{\Phi}$$

$$\delta x_{pred} / \delta x_{actual} = 0.9969$$

Adjoint method predicted the deflection sensitivity to within 0.3%

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Application: 2D parallel plate sheet beam

- Manufacturing sensitivity to beam centering offset

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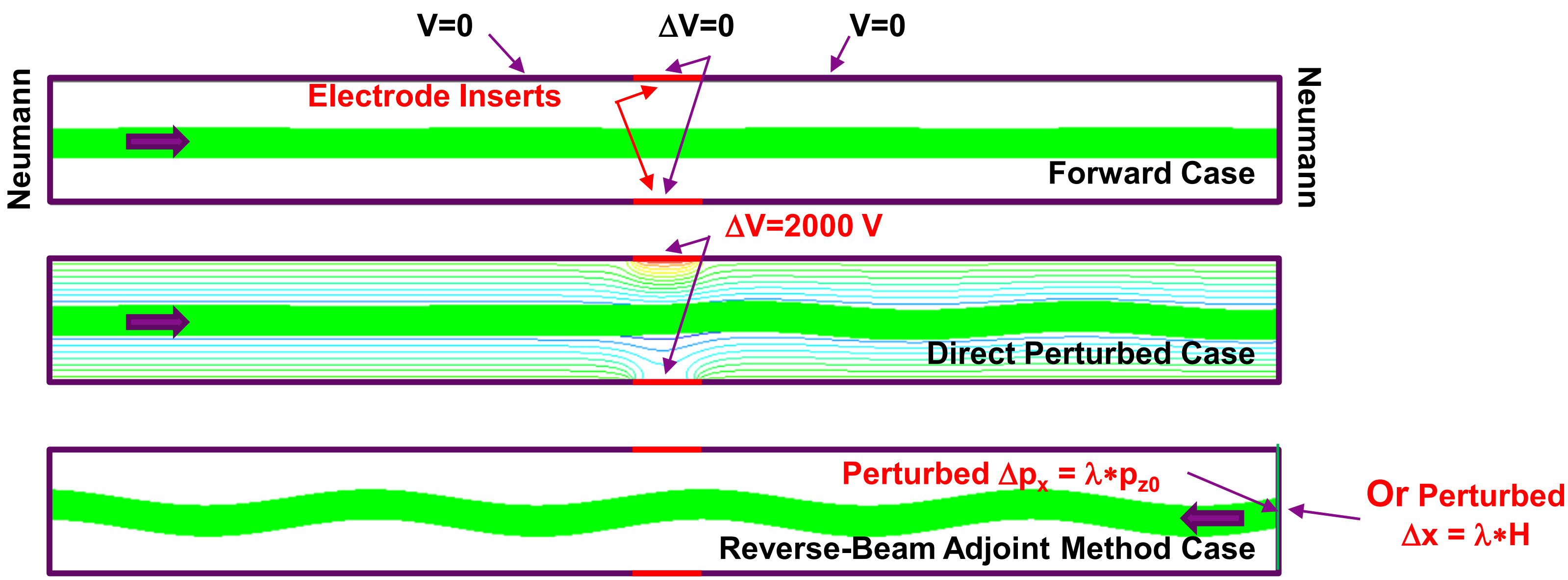
- Forward Case: Grounded Inserts top and bottom
- Direct Perturbation Case: Electrode inserts on top and bottom set to ΔV

  ΔV tested from 1 – 10,000 V
- Reverse-Beam Adjoint Method Case: Case launches beam in reverse direction with momentum perturbed by a constant value in the vertical transverse direction (1st) or a vertical position shift (2nd).

  $\lambda = \Delta p_x / p_{z0}$ (1st) and $\lambda = \Delta x/H$ (2nd) tested from 0.00001 to 0.16384



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Mean Displacement: *Hamiltonian Approach* (New)

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Case 1: Original Forward case (Forward)

Case 2: Reverse Beam – "(Y)" is perturbed by amount **λ**

Test Case 3: Perturbed Voltage – "(X)" Direct **ΔV** change

Adjoint Relation

$$\sum_j I_j \left\{\left(-\lambda \delta x^{(X)}\right)\right\}_L = -q\varepsilon_0 \int_B d^2x \left(\delta\phi^{(X)} \mathbf{n}\cdot\nabla\delta\phi^{(Y)}\right)$$

Test 1 Mean displacement

$$\delta y^{(Y)} = 0\,,$$

$$\delta x^{(Y)} = 0$$

$$\delta p_x^{(Y)} = \lambda \quad \text{a constant}$$

$$\delta p_y^{(Y)} = 0$$

<u>First Test:</u> In this case there is no requirement for correcting the canonical angular momentum

$$\sum_j I_j \left\{\left(-\lambda \delta x^{(X)}\right)\right\}_L = -q\varepsilon_0 \int_B d^2x \left(\delta\phi^{(X)} \mathbf{n}\cdot\nabla\delta\phi^{(Y)}\right)$$

$$-\lambda \sum_j I_j \left\{\left(\delta x^{(X)}\right)\right\}_L = -q\varepsilon_0 \Delta V \int_B d^2x \left(\delta E_{perp}\right)$$

ΔV runs

λ – Adjoint Perturbed Runs

$$\frac{1}{\Delta V}\sum_j I_j \left\{\left(\delta x^{(X)}\right)\right\}_L = \frac{q\varepsilon_0}{\lambda} \int_B d^2x \left(\delta E_{perp}\right)$$

Mean Displacement: 2D parallel plate sheet beam

- Manufacturing sensitivity to beam centering offset

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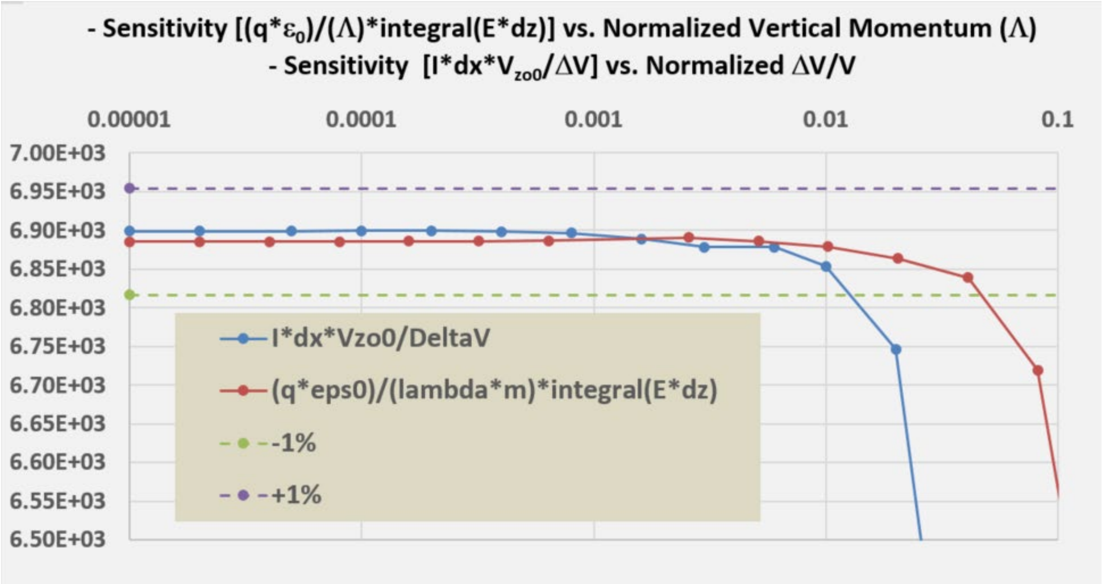
- Results of direct vs. adjoint methods agree to within **0.20%.**
- Verification: ***Hamiltonian Approach*** Excellent first successful Adjoint method to beam transport in a magnetic field.
- Results:
  - As the perturbed-case voltage values became small enough it easily entered the linear regime.
  - There is very a broad range of both Λ and ΔV where the results are all in a linear regime.



Adjoint method predicted the deflection sensitivity to within 0.2%

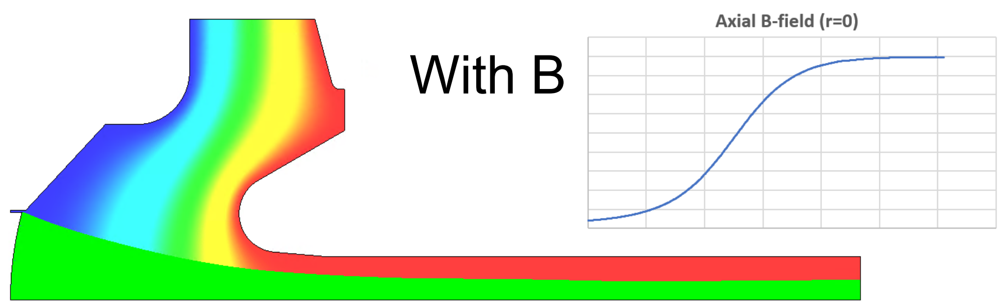
Adjoint Advances:

New

Now adding 2D axisymmetric optics – with and without a B-field

- Previous cases were 2D planar
- Extend the approach to 2D axisymmetric electron guns with and without magnetostatic fields
  - Geometry: 2D axisymmetric
  - Beam axial energy: 10 keV
  - Beam guide field: ~0.1 T (variable for beam capture)
  - **Manufacturing Sensitivity:**
    - Adjoint case mimics Anode wall displacement or voltage errors
    - Direct case: Apply AK-Gap voltage shifts to Anode electrodes

Without B



With B

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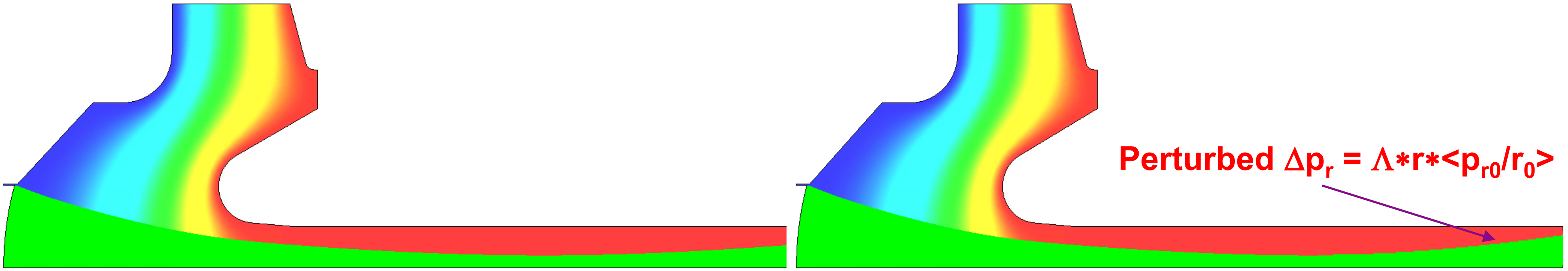
Application: 2D Axisymmetric Pierce Diode (No B)

- Manufacturing sensitivity to Anode (AK-Gap) offset

- Forward Case: Standard Electrostatic-only Electron Gun @ 10 KeV
- Direct Perturbation Case: Anode electrode ΔV applied

  ΔV tested from 0.1 – 30 V
- Reverse-Beam Adjoint Method Case: Case launches beam in reverse direction with momentum perturbed by a constant value in the radial direction.

  $\Lambda = \Delta p_r/(r^* \langle p_{r0}/r_0 \rangle)$ tested from 0.000001 to 0.01



Forward Case | **Reverse-Beam Adjoint Method Case**

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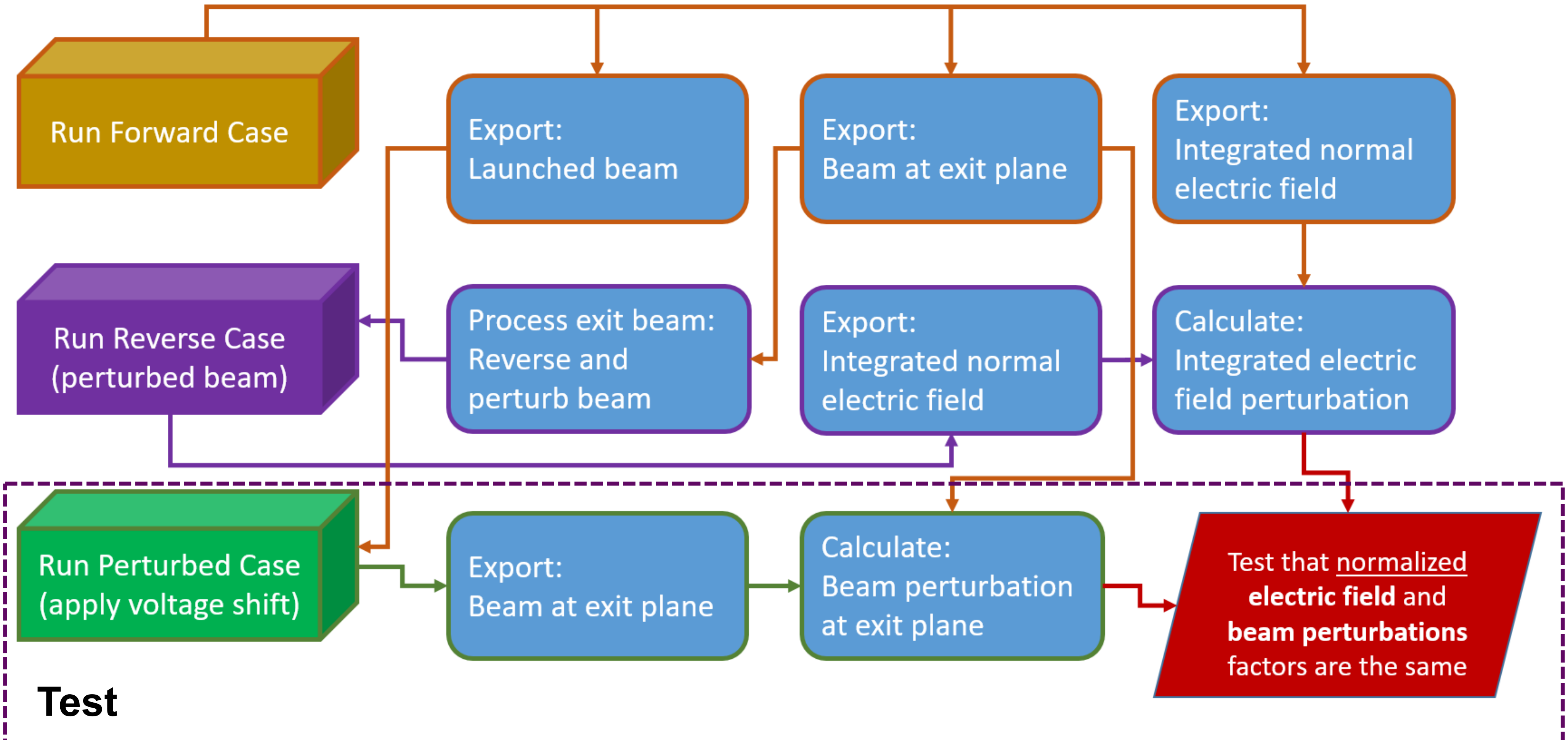
Application and Testing/Verification Process:
- Hamiltonian basis



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Mean Displacement: *Hamiltonian Approach* (No B)

Case 1: Original Forward case (Forward)
Case 2: Reverse Beam – “(Y)” is perturbed by amount **λ**
Test Case 3: Perturbed Voltage – “(X)” Direct **ΔV** change

Adjoint Relation

$$\sum_j I_j\left[\left(\delta p_{rj}^{(X)}\delta r_j^{(Y)} - \delta p_{rj}^{(Y)}\delta r_j^{(X)}\right)_{t=0}^{z=L}\right] = -q\varepsilon_0\int_B d^2x\left(\delta\phi^{(X)}\mathbf{n}\cdot\nabla\delta\phi^{(Y)} - \delta\phi^{(Y)}\mathbf{n}\cdot\nabla\delta\phi^{(X)}\right)$$

$$\sum_j I_j\left\{\left(-\lambda\delta r^{(X)}\right)\right\}_L = -q\varepsilon_0\int_B d^2x\left(\delta\phi^{(X)}\mathbf{n}\cdot\nabla\delta\phi^{(Y)}\right)$$

Transverse Momentum kick

$$\delta r^{(Y)} = 0$$

$$\delta p_r^{(Y)} = \lambda = \Lambda\, r^{(X)} \langle p_{r0}/r_0\rangle$$

$$\Lambda_k = \Lambda \langle p_{r0}/r_0\rangle$$

$$\Lambda_k\sum_j I\left\{r^{(X)}\delta r^{(X)}\right\}_L = -q\varepsilon_0\Delta V\int_B d^2x\left(\delta E_{perp}\right)$$

ΔV runs

λ – Adjoint Perturbed Runs

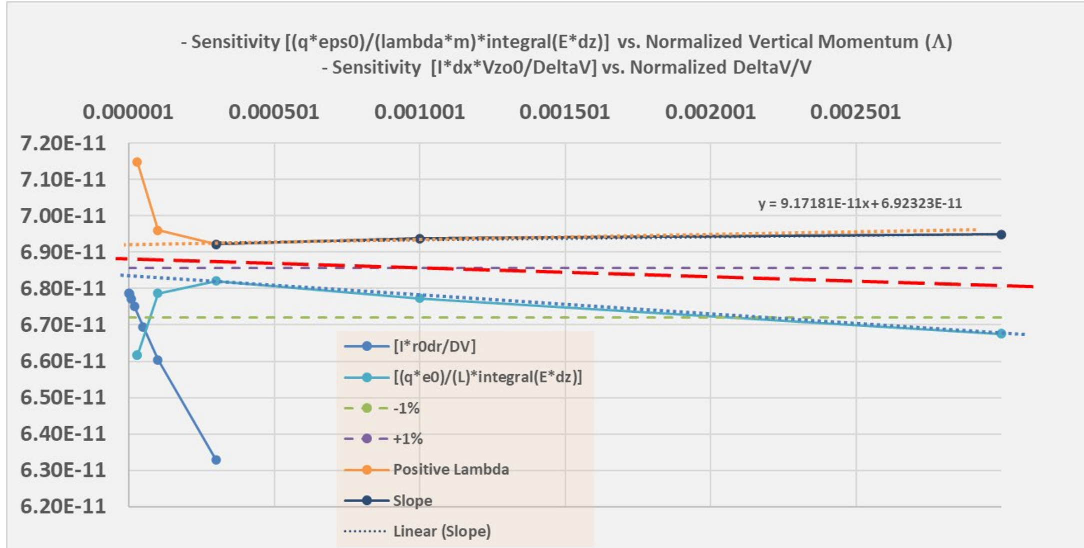
$$\frac{1}{\Delta V}\sum_j I\left\{r^{(X)}\delta r^{(X)}\right\}_L = \frac{q\varepsilon_0}{\Lambda_k}\int_B d^2x\left(\delta E_{perp}\right)$$

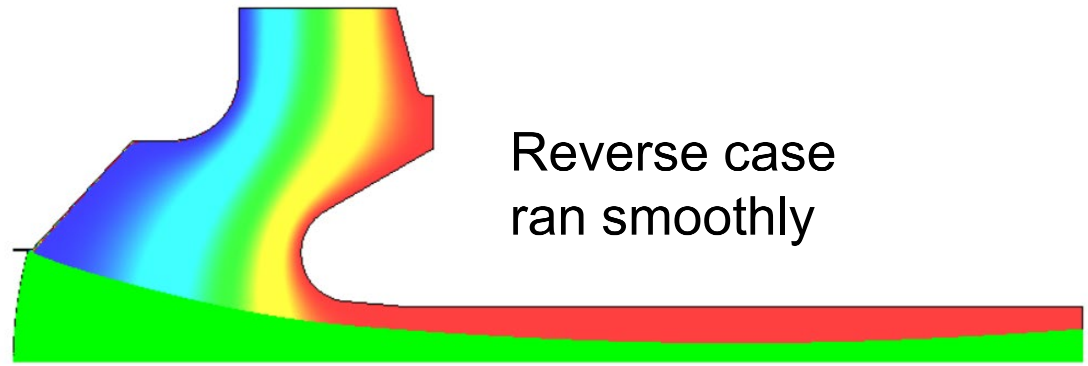
Mean Displacement: 2D Axisymmetric Electron Gun

- Manufacturing sensitivity to AK-Gap axial offset

No-B

- Results:
  - The test case where the direct perturbation of a voltage change worked as expected.
  - The reverse-beam case was oddly sensitive to changes.





Reverse case ran smoothly

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Transverse Momentum: *Hamiltonian Approach* (*With B*)

Case 1: Original Forward case (Forward)

Case 2: Reverse Beam – "(Y)" is perturbed by amount λ

Test Case 3: Perturbed Voltage – "(X)" Direct ΔV change

Adjoint Relation

$$\sum_j I_j \left\{ \left(\delta\mathbf{x}^{(Y)} \cdot \delta\mathbf{p}^{(X)} - \delta\mathbf{x}^{(X)} \cdot \left(\delta\mathbf{p}^{(Y)} + q\mathbf{B} \times \delta\mathbf{x}^{(Y)} \right) \right) \right\}_L = -q\varepsilon_0 \int_B d^2x \left(\delta\phi^{(X)} \mathbf{n} \cdot \nabla \delta\phi^{(Y)} \right)$$

Two Tests…

- Mean Displacement

$$\delta r^{(Y)} = \lambda\, r^{(X)} / r_0$$

$$\delta p_r^{(Y)} = 0$$

- Transverse Momentum kick

$$\delta r^{(Y)} = 0$$

$$\delta p_r^{(Y)} = \lambda = \Lambda\, r^{(X)} \langle p_{r0}/r_0 \rangle$$

First Test: In this case there is no requirement for correcting the canonical angular momentum

Second Test: In this case there is a required conservation of canonical angular momentum

Summary

- Manufacturing assembly sensitivities can be determined for charged particle 2D axisymmetric beams in electron gun sources
  - Without B-field
  - With B-Field 🡐 in progress

- The Hamiltonian basis for the Adjoint method enables fine sensitivities to be captured, including relativistic corrections for beam energies ~10 keV.

- ***The Hamiltonian method properly handles optics in the presence of B-fields.***

- We have demonstrated the capability of the Adjoint method applied to 2D electrons guns operating under space charge limited emission
    - AK-Gap voltage variation Predicted sensitivity agreement to within 0.2% of direct calculations

- Method has previously proven to apply successfully to Electron guns for…
  - Planar:
    - AK-Gap variation / AK-Voltage variation / Vertical offset misalignment
    - Planar: Beam transport with static guide B-fields