

Recent advances in flavor physics

Marzia Bordone



Brookhaven Forum 2023:
Advancing Searches for New Physics
05.10.2023

Outline:

1. The problem of flavour
2. Theoretical and experimental status
3. A glance into BSM physics

Motivation

Despite the SM successes,
there are open problems:

Motivation

Despite the SM successes,
there are open problems:

Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity

Motivation

Despite the SM successes,
there are open problems:

Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity

SM(EFT)

Λ_{EW}

Energy

Motivation

Despite the SM successes,
there are open problems:

Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity

UV theory

SM(EFT)

Λ_{UV}

Λ_{EW}

Energy

Motivation

Despite the SM successes,
there are open problems:

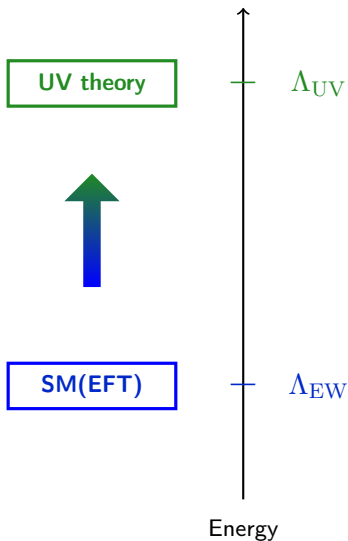
Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity



The (two) flavour problems

1. **The SM flavour problem:** The measured Yukawa pattern doesn't seem accidental

⇒ Is there any deeper reason for that?

2. **The NP flavour problem:** If we regard the SM as an EFT valid below a certain energy cutoff Λ , why don't we see any deviations in flavour changing processes?

⇒ Which is the flavour structure of BSM physics?

The SM flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} \text{light green circle} & \text{light green circle} & \text{dark green circle with } 0.003 \\ & \text{medium green circle} & \text{dark green circle with } 0.04 \\ & & 1 \end{pmatrix}$$

The SM flavour problem

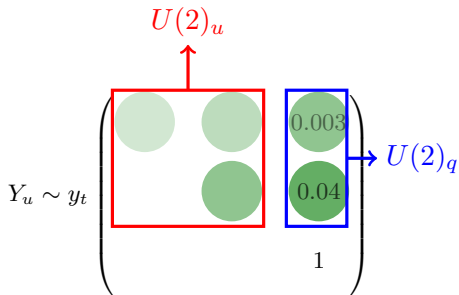
$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & 1 \end{pmatrix}$$

Exact $U(2)^n$ limit

The SM flavour problem

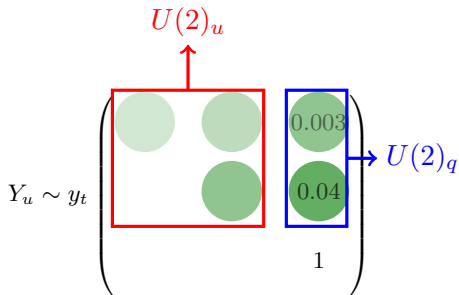
$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$



An approximate $U(2)^n$ is acting
on the light families!

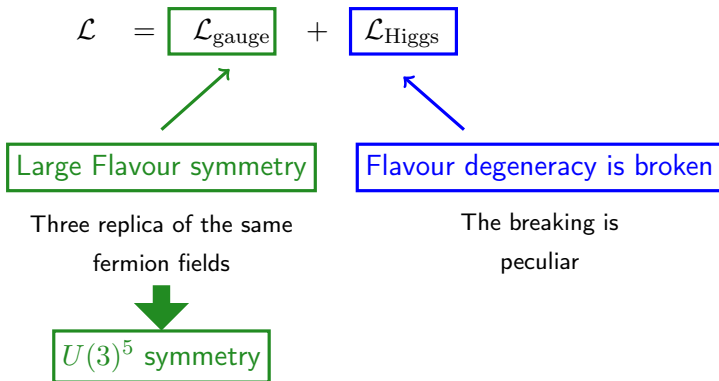
The SM flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$



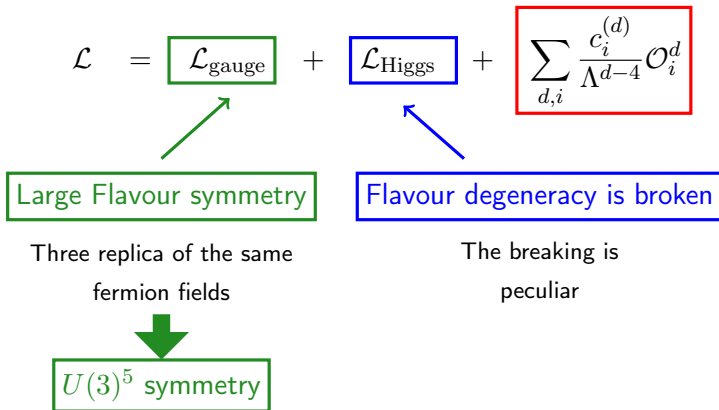
An approximate $U(2)^n$ is acting
on the light families!

The NP flavour problem



- In the SM: accidental $U(3)^5 \rightarrow \text{approx } U(2)^n$

The NP flavour problem

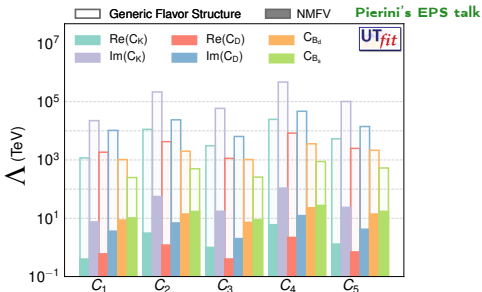
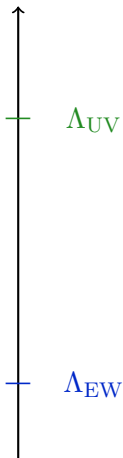


- In the SM: accidental $U(3)^5 \rightarrow$ approx $U(2)^n$
- **What happens when we switch on NP?**

The NP flavour problem

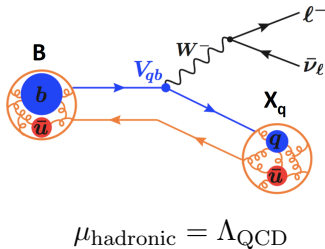
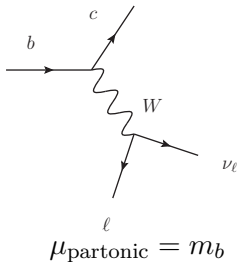
$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d$$

- What is the energy scale of NP?
- Why haven't observed any violation of accidental symmetries yet?



no breaking of the $U(2)^n$ flavour symmetry at low energies

Partonic vs Hadronic



**Fundamental challenge to match
partonic and hadronic descriptions**

What's the problem for BSM?

B-physics

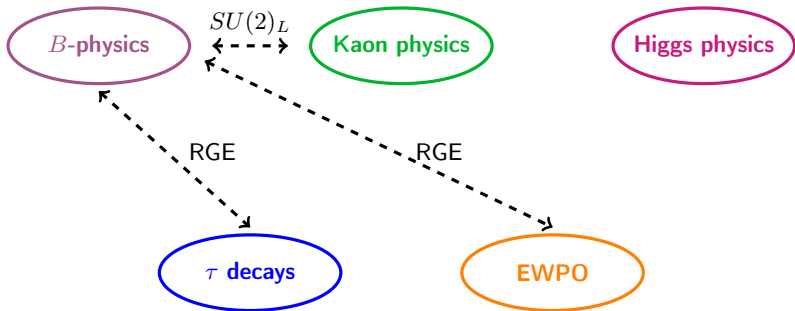
Kaon physics

Higgs physics

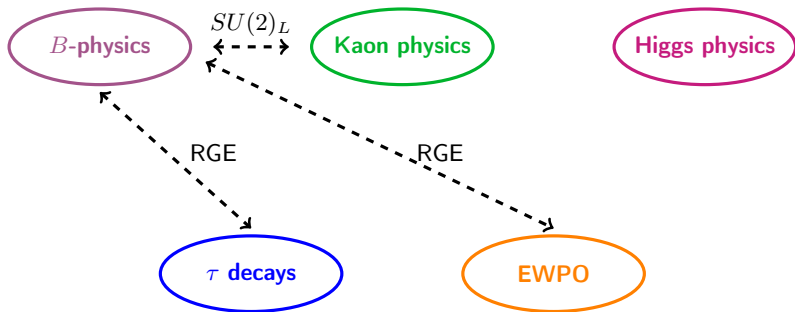
τ decays

EWPO

What's the problem for BSM?



What's the problem for BSM?



**How to satisfy all the constraints
at the same time?**

Theoretical and experimental status

Exclusive matrix elements

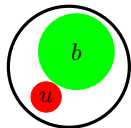
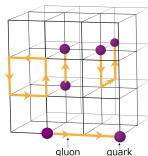
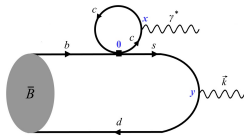
$$\langle H_q | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i$$

- Lattice QCD

- QCD SR, LCSR

- HQET (exploit $m_{b,c} \rightarrow \infty$ limit) + Data driven fits

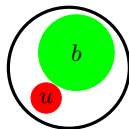
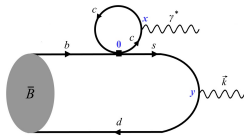
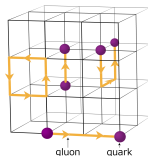
- Dispersive analysis



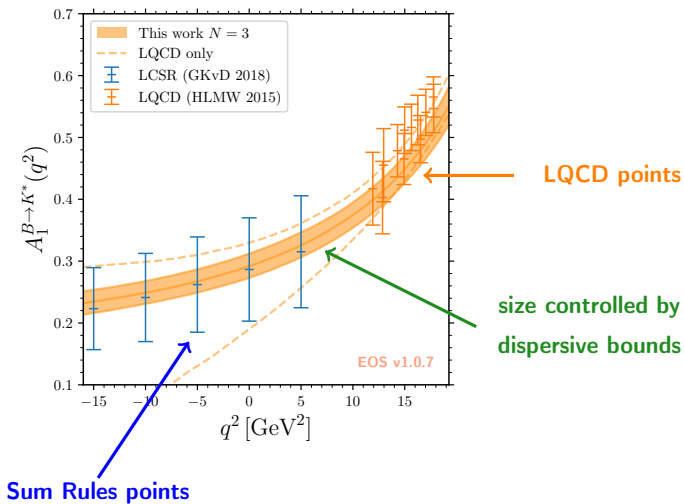
Exclusive matrix elements

$$\langle H_q | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i \quad \leftarrow \text{form factor}$$

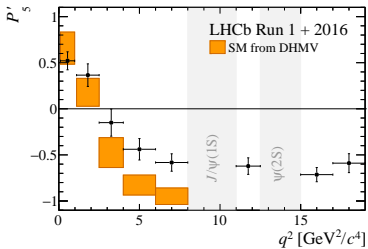
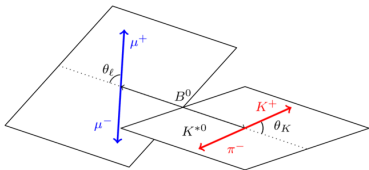
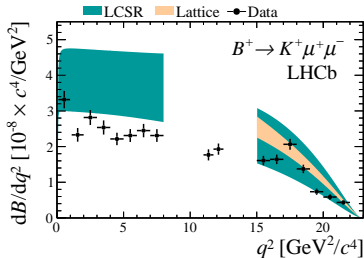
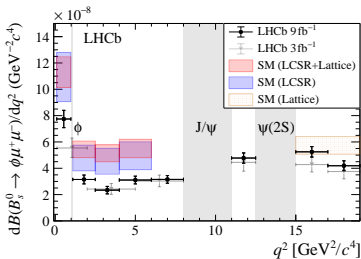
↙ ↘ scale Λ_{QCD}
↑ independent
 Lorentz structures



- Lattice QCD
- QCD SR, LCSR
- HQET (exploit $m_{b,c} \rightarrow \infty$ limit) + Data driven fits
- Dispersive analysis



“Anomalies” in $b \rightarrow sl^+l^-$ transitions

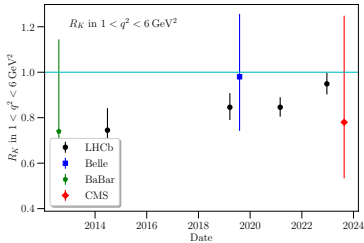


Lepton Flavour Universality violation

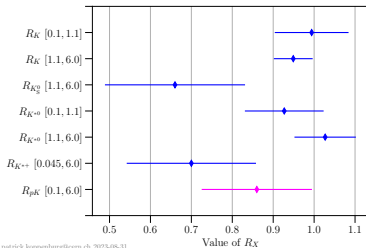
$$R_X = \frac{\mathcal{B}(H_b \rightarrow X\mu^+\mu^-)}{\mathcal{B}(H_b \rightarrow Xe^+e^-)}$$

- Test of Lepton Flavour Universality, which is one of the building principles of the SM
- With ratios, we reduce hadronic uncertainties at large extent
- For $q^2 \gg m_\ell^2 \rightarrow R_X = 1$
- Leading theoretical uncertainty coming from QED effects $\sim 1\%$

MB, Isidori, Pattori, '16
Isidori, Lancerini, Nabeebaccus, Zwicky, '22

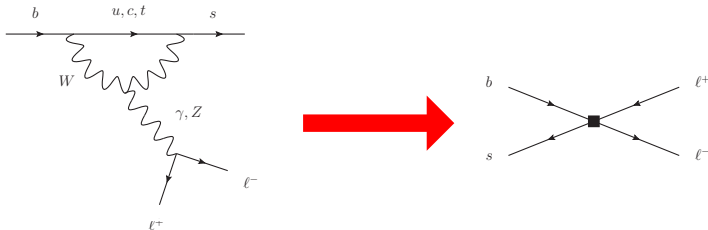


patrick.koppenburg@cern.ch 2023-08-31



patrick.koppenburg@cern.ch 2023-08-31

$b \rightarrow s \ell \ell$



$$\mathcal{H}_{\text{eff}} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* [-C_1 \mathcal{O}_1 - C_2 \mathcal{O}_2 + C_7 \mathcal{O}_7 + C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10}]$$

$$\mathcal{O}_1 = (\bar{s} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu \ell)$$

$$\mathcal{O}_2 = (\bar{s} \gamma^\mu T^a P_L b) (\bar{\ell} \gamma_\mu T^a \ell)$$

$$\mathcal{O}_9 = (\bar{s} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu \ell)$$

$$\mathcal{O}_{10} = (\bar{s} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu \gamma_5 \ell)$$

$$\mathcal{O}_7 = (\bar{s} \sigma^{\mu\nu} P_R b) F_{\mu\nu}$$

- Wilson coefficients are calculated at NNLO

Gorbahn, Haisch, '04, Bobeth, Gambino, Gorbahn, Haisch, '11

- The running to $\mu = m_b$ is known

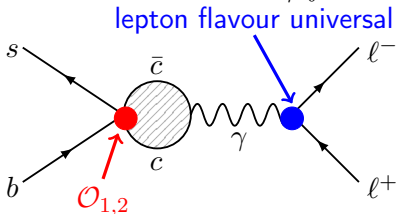
$$B \rightarrow K^{(*)} \ell^+ \ell^-$$

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local: $\mathcal{F}_\lambda^{(T)} = \langle K^{(*)}(k) | \bar{s} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$
 - form factors calculated on lattice and LCSR

HPQCD, '13, '22
 Bharucha, Straub, Zwicky, '15
 N. Gubernari, A. Kokulu, D. van Dyk, '18
 N. Gubernari, M. Reboud, D. van Dyk, J. Virto, '22

- Non-local: $\mathcal{H}_\lambda(q^2) = iP_\mu^\lambda \int d^4x e^{iqx} \langle K^{(*)}(k) | T \{ \mathcal{J}_{\text{em}}^\mu, C_i \mathcal{O}_i(0) \} | \bar{B}(k+q) \rangle$



$$C_9 \rightarrow C_9^{\text{eff}}(q^2) = C_9 + C_9^{\text{LD}}(q^2)$$

How do we parametrise these long-distance effects?

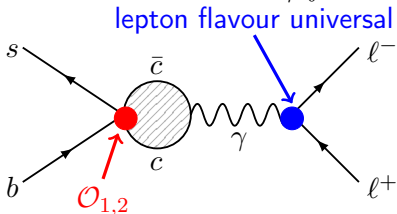
$$B \rightarrow K^{(*)} \ell^+ \ell^-$$

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- **Local:** $\mathcal{F}_\lambda^{(T)} = \langle K^{(*)}(k) | \bar{s} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$
 - form factors calculated on lattice and LCSR

HPQCD, '13, '22
 Bharucha, Straub, Zwicky, '15
 N. Gubernari, A. Kokulu, D. van Dyk, '18
 N. Gubernari, M. Reboud, D. van Dyk, J. Virto, '22

- Non-local: $\mathcal{H}_\lambda(q^2) = iP_\mu^\lambda \int d^4x e^{iqx} \langle K^{(*)}(k) | T \{ \mathcal{J}_{\text{em}}^\mu, C_i \mathcal{O}_i(0) \} | \bar{B}(k+q) \rangle$



$$C_9 \rightarrow C_9^{\text{eff}}(q^2) = C_9 + C_9^{\text{LD}}(q^2)$$

How do we parametrise these long-distance effects?

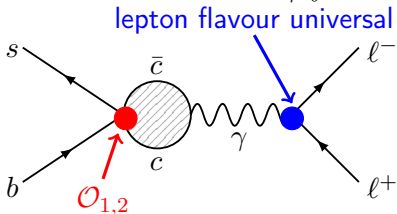
$$B \rightarrow K^{(*)} \ell^+ \ell^-$$

$$A_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- **Local:** $\mathcal{F}_\lambda^{(T)} = \langle K^{(*)}(k) | \bar{s} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$
 - form factors calculated on lattice and LCSR

HPQCD, '13, '22
 Bharucha, Straub, Zwicky, '15
 N. Gubernari, A. Kokulu, D. van Dyk, '18
 N. Gubernari, M. Reboud, D. van Dyk, J. Virto, '22

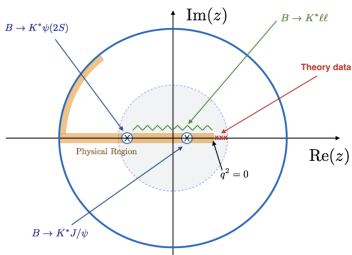
- **Non-local:** $\mathcal{H}_\lambda(q^2) = iP_\mu^\lambda \int d^4x e^{iqx} \langle K^{(*)}(k) | T \{ \mathcal{J}_{\text{em}}^\mu, C_i \mathcal{O}_i(0) \} | \bar{B}(k+q) \rangle$



$$C_9 \rightarrow C_9^{\text{eff}}(q^2) = C_9 + C_9^{\text{LD}}(q^2)$$

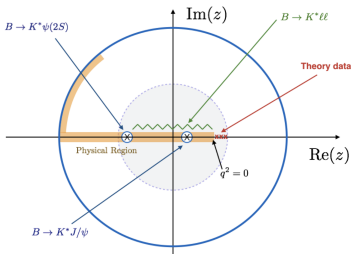
How do we parametrise these long-distance effects?

Charm-loop effects in $b \rightarrow sl^+l^-$



- Conformal transformation $q^2 \mapsto z(q^2)$, with $|z| < 1$
- $C_9^{\text{LD}} \propto \alpha_n z^n$ [1707.07305]
- Dispersive analysis allow to determine the truncation order of the series [2011.09813,2206.03797]
- Effects are **small**

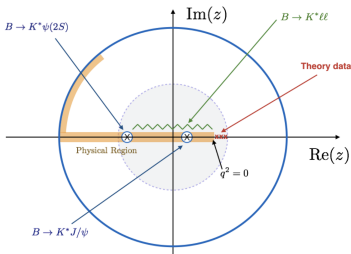
Charm-loop effects in $b \rightarrow sl^+l^-$



- Conformal transformation $q^2 \mapsto z(q^2)$, with $|z| < 1$
- $C_9^{\text{LD}} \propto \alpha_n z^n$ [1707.07305]
- Dispersive analysis allow to determine the truncation order of the series [2011.09813,2206.03797]
- Effects are **small**

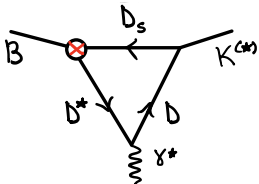
Is this all?

Charm-loop effects in $b \rightarrow sl^+l^-$



- Conformal transformation $q^2 \mapsto z(q^2)$, with $|z| < 1$
- $C_9^{\text{LD}} \propto \alpha_n z^n$ [1707.07305]
- Dispersive analysis allow to determine the truncation order of the series [2011.09813, 2206.03797]
- Effects are **small**

Is this all?



[2212.10516]

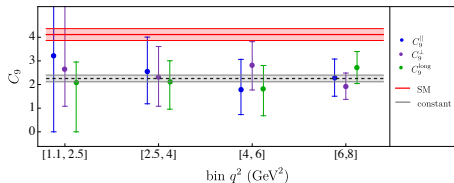
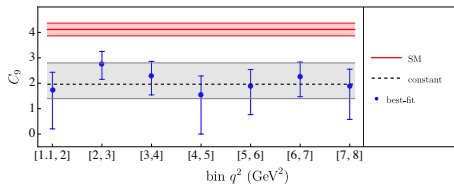
- Are these contributions included?
- Are they large that they can reconcile the tension in $B \rightarrow K^* \mu \mu$?

Charm loop effects in $B \rightarrow K^{(*)} \mu^+ \mu^-$

MB, Isidori, Maechler, Tinari, to appear

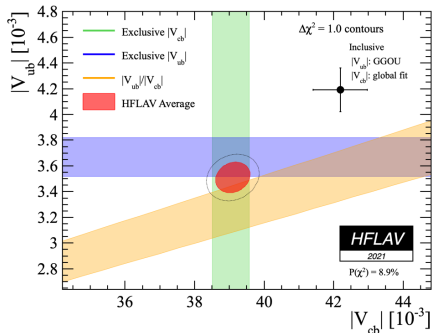
- Can we extract some hints of the shape of $C_9^{\text{LD}}(q^2)$ from data?
 - ⇒ NP yields a **constant** effect in the whole kinematic region
- Is the current sensitivity enough to claim anything?

$$C_9^{\text{eff}} = C_9 + \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{q^2}{(m_V^2 - q^2 - im_V\Gamma_V)} \frac{m_V\Gamma_V}{m_V^2 - q^2 - im_V\Gamma_V}$$



**No evidence
for q^2 dependence**

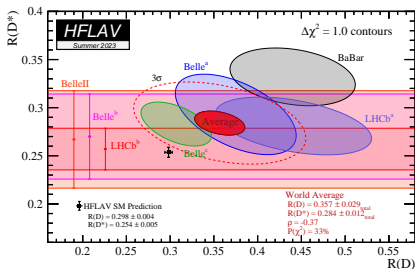
Inclusive vs. Exclusive determination of V_{cb}



Major impact for

- Test of unitarity for the CKM
- $\epsilon_K \sim |V_{cb}|^4$
- $\mathcal{B}(B_s \rightarrow \mu\mu) \sim |V_{cb}|^2$
- $\mathcal{B}(B \rightarrow K\nu\bar{\nu}) \sim |V_{cb}|^2$

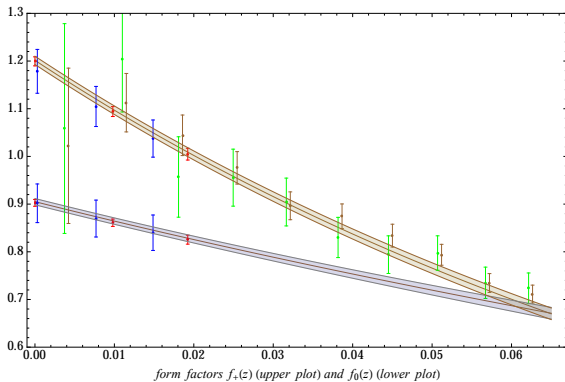
Lepton Flavour Universality Violation 2



$$R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \rightarrow D^{(*)} \ell \bar{\nu})}$$

- Combined tension of 3.3σ
- Other measurements like $R_{J/\psi}$, R_{Λ_c} , $R(X_c)$ are less significant and don't spoil the actual combination

- Belle+BaBar data and HPQCD+FNAL/MILC Lattice points



$$|V_{cb}| = (40.49 \pm 0.97) \times 10^{-3}$$

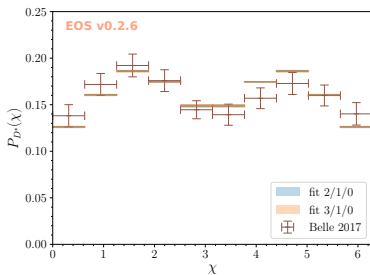
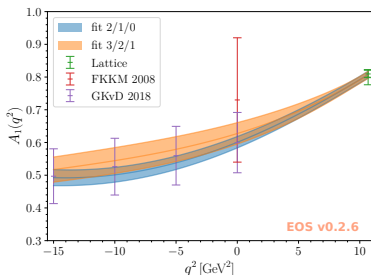
$B \rightarrow D^*$

- Until two years ago, only lattice points at zero-recoil were available
- Additional inputs were needed from LCSR and/or experimental data
- The use of Effective Theory like HQET was crucial to provide predictions

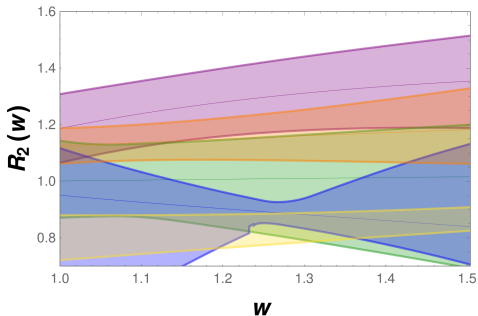
$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi} \right) \xi + \frac{\Lambda_{\text{QCD}}}{2m_b} \sum_j c_{ij} \xi_{\text{SSL}}^j + \frac{\Lambda_{\text{QCD}}}{2m_c} \sum_j d_{ij} \xi_{\text{SSL}}^j + \left(\frac{\Lambda_{\text{QCD}}}{2m_c} \right)^2 \sum_j g_{ij} \xi_{\text{SSL}}^j$$

$\Rightarrow 1/m, \alpha_s$ expansion parameters

$\Rightarrow \xi, \xi_{\text{SSL}}^i, \xi_{\text{SSL}}^i$ are the heavy quark form factors and are universal for all transitions
 $B^{(*)} \rightarrow D^{(*)}$



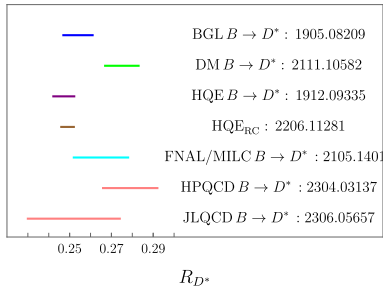
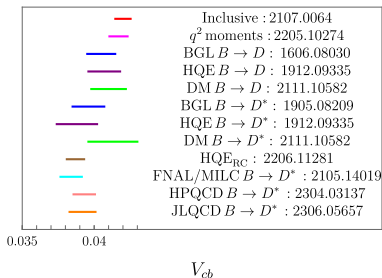
Lattice calculations at $q^2 \neq q_{\max}^2$



- FNAL/MILC '21
- HQE@1/ m_c^2
- Exp data (BGL)
- JLQCD '23
- HPQCD '23

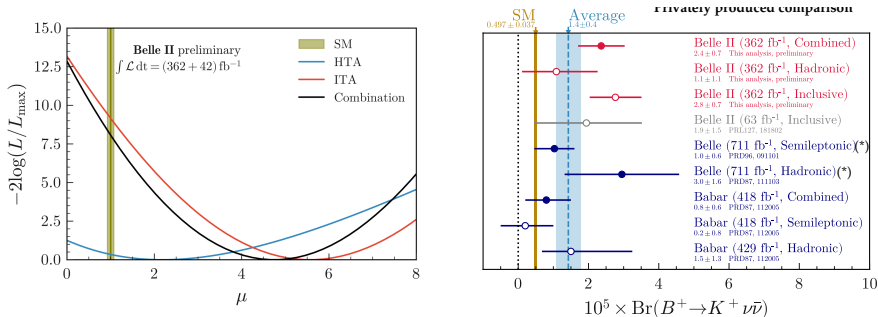
- Tensions between different lattice determinations, experimental data and non-lattice theory determination
- No consensus yet, ongoing checks
- New Belle analysis available

Pheno Status



- For what concerns V_{cb} , the inclusive determination is stable
- The lack of a consensus for $B \rightarrow D^*$ form factors results in a large spread in the values for V_{cb} and R_{D^*}
- The situation is rather unclear
- A lot of WIP between different theory communities

$B^+ \rightarrow K^+ \nu \bar{\nu}$ from Belle II



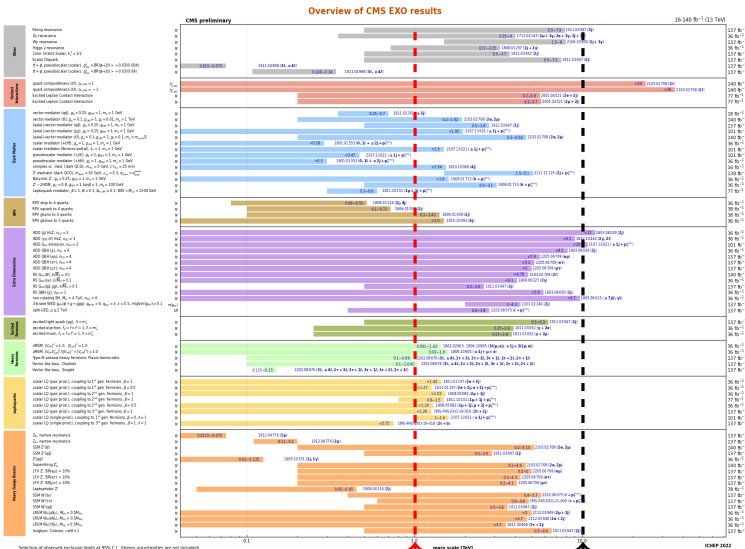
- First evidence of the $B^+ \rightarrow K^+ \nu \bar{\nu}$ process at 3.6σ with

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = (2.4 \pm 0.5(\text{stat})_{-0.4}^{+0.5}(\text{syst})) \times 10^{-5}$$

- Tension with the SM of $\sim 2.8\sigma$

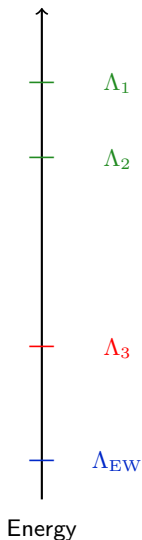
A glance into BSM physics

Status of high energy bounds



Flavour Non-Universal New Physics

Dvali, Shifman, '00
Panico, Pomarol, '16
MB, Cornella, Fuentes-Martin, Isidori '17
Allwicher, Isidori, Thomsen '20
Barbieri, Cornella, Isidori, '21
Davighi, Isidori '21



Basic idea:

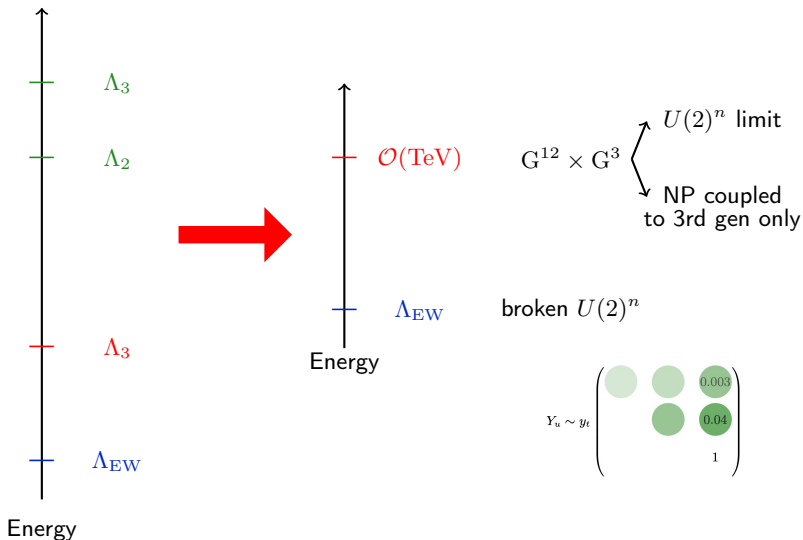
- 1st and 2nd have small masses and small couplings to NP because they are generated by dynamics at a heavier scale
- 3rd generation is linked to dynamics at lower scales and has stronger couplings

Flavour deconstruction:

fermion families interact with different gauge groups and flavour hierarchies emerge as accidental symmetries

Flavour Non-Universal New Physics

Dvali, Shifman, '00
 Panico, Pomarol, '16
MB, Cornella, Fuentes-Martin, Isidori '17
 Allwicher, Isidori, Thomsen '20
 Barbieri, Cornella, Isidori, '21
 Davighi, Isidori '21



What do we expect in the SMEFT?

$$\mathcal{L}_{\text{EFT}} \supset \frac{C_{bc\tau\tau}}{\Lambda^2} (\bar{b}_L^i \gamma_\nu c_L^j) (\bar{\nu}_\tau \gamma^\mu \tau_L)$$

From $U(2)^n \Rightarrow C_{bc\tau\tau} \sim V_{cb} \mathcal{O}(1)$

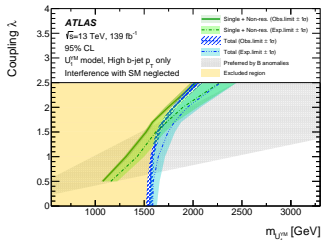
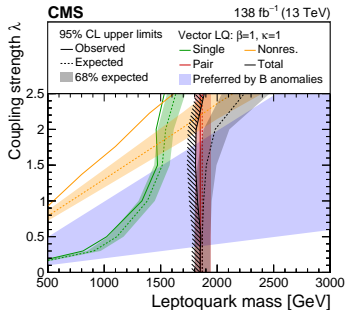
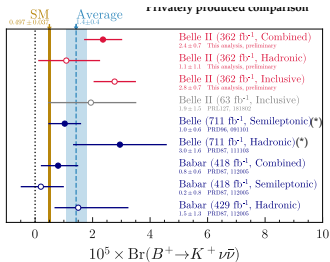
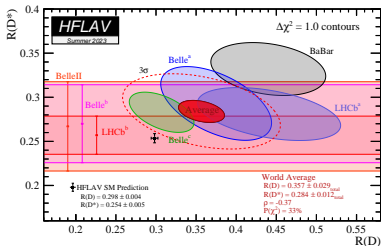
From $R_{D^{(*)}} \Rightarrow \Lambda \sim \mathcal{O}(\text{TeV})$

Using $SU(2)_L$ invariance, we have

$$\mathcal{L}_{\text{EFT}} \supset \frac{C_{bs\tau\tau}}{\Lambda^2} (\bar{b}_L^i \gamma_\nu s_L^j) (\bar{\nu}_\tau \gamma^\mu \nu_L)$$

From $U(2)^n \Rightarrow C_{bs\tau\tau} \sim V_{cb} \mathcal{O}(1)$

Belle II measurement of $B \rightarrow K \nu \bar{\nu}$
in agreement with $U(2)^n$



The present hints align well together, but it is too soon to claim victory...

Conclusions

- Flavour physics is a powerful test for new physics living at different energy scales
- At the current status, we haven't observed any clear sign of new physics
- No clear sign of new physics can hint to a peculiar structure for the flavour structure of NP and to flavour deconstruction
 - ⇒ Theoretical and Experimental efforts will shed light on puzzles in hadronic predictions, aiming to a deeper understanding of the SM
 - ⇒ From the phenomenological point of view, a few hints point to a strong link between new physics and the third generations, with possible new physics reach close to the current searches

Appendix

$B \rightarrow D^{(*)}$ form factors

- 7 (SM) + 3 (NP) form factors
- Lattice computation for $q^2 \neq q_{\max}^2$ only for $B \rightarrow D$
- Calculation usually give only a few points
- q^2 dependence must be inferred
- Conformal variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- $t_+ = (m_B + m_{D^{(*)}})^2$ pair production threshold
- $t_0 < t_+$ free parameter that can be used to minimise $|z_{\max}|$
- $|z| \ll 1$, in the $B \rightarrow D$ case $|z| < 0.06$

The HQE parametrisation 1

- Expansion of QCD Lagrangian in $1/m_{b,c} + \alpha_s$ corrections

[Caprini, Lellouch, Neubert, '97]

- In the limit $m_{b,c} \rightarrow \infty$: all $B \rightarrow D^{(*)}$ form factors are given by a **single** Isgur-Wise function

$$F_i \sim \xi$$

- at higher orders the form factors are still related \Rightarrow **reduction** of free parameters

$$F_i \sim \left(1 + \frac{\alpha_s}{\pi}\right) \xi + \frac{\Lambda_{\text{QCD}}}{2m_b} \xi_{\text{SL}}^i + \frac{\Lambda_{\text{QCD}}}{2m_c} \xi_{\text{SL}}^i$$

- at this order 1 leading and 3 subleading functions enter
- ξ^i are not predicted by HQE, they have to be determined using some other information

The HQE parametrisation 2

- Important point in the HQE expansion: $q^2 = q_{\max}^2$
- At this point Luke's Theorem applies: the subleading corrections vanish for some form factors
- The leading Isgur-Wise function is normalised: $\xi(q^2 = q_{\max}^2) = 1$
- **Problem:** contradiction with lattice data!
- $1/m_c^2$ corrections **have to be systematically included**
 - well motivated also since $\alpha_s/\pi \sim 1/m_b \sim 1/m_c^2$

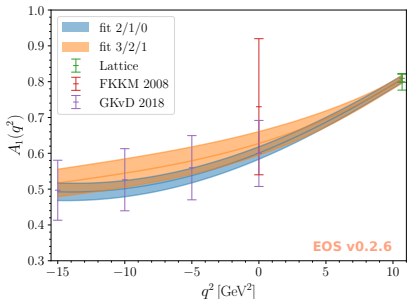
[Jung, Straub, '18,
MB, M.Jung, D.van Dyk, '19]

The HQE results

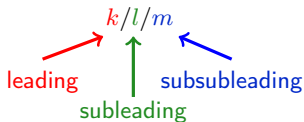
[MB, Jung, van Dyk, EPJC 80 (2020),
MB, Gubernari, Jung, van Dyk, EPJC 80 (2020)]

Data points:

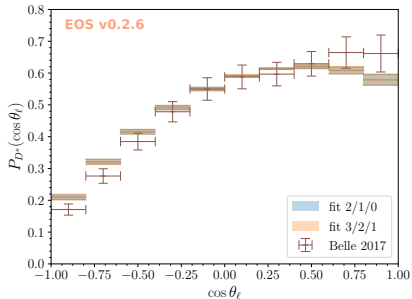
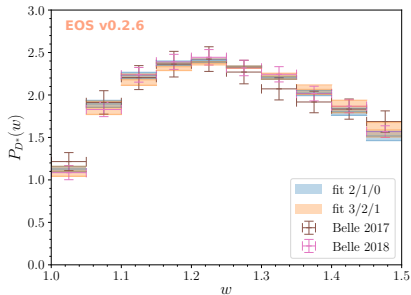
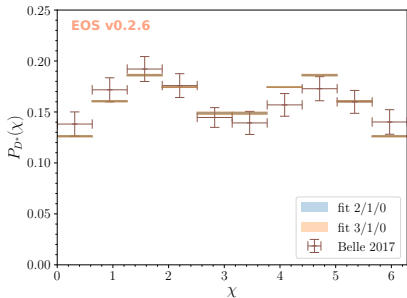
- theory inputs only (Lattice QCD, QCD Sum Rules, Light-cone Sum Rules, Dispersive Bounds)



- Expansion in z up to order



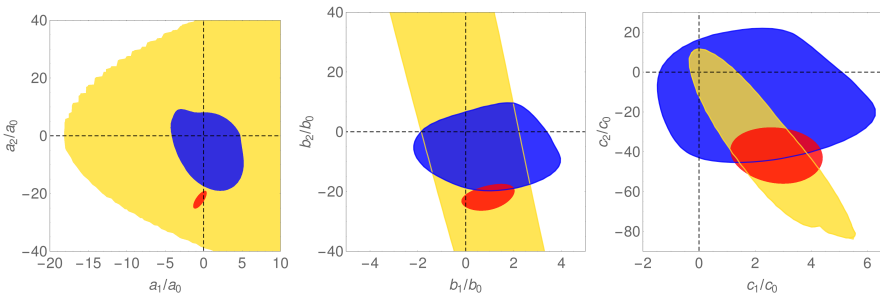
Comparison with kinematical distributions



good agreement with kinematical distributions

Fit stability

- BGL fit to Belle 2017 and 2018 data (yellow)
- HQE fit 2/1/0 (red)
- HQE fit 3/2/1 (blue)



- compatibility of HQE fit with data driven one
- 2/1/0 underestimates massively uncertainties

3/2/1 is our nominal fit

Phenomenological results

- V_{cb} extraction

$$V_{cb}^{\text{average}} = (41.1 \pm 0.5) \times 10^{-3}$$

compatibility of 1.8σ between inclusive and exclusive

- Universality ratios

$$R_{D^*} = 0.2472 \pm 0.0050 \quad R_{D_s^*} = 0.2472 \pm 0.0050$$

towards the combined 4σ discrepancy

- We observe no $SU(3)_F$ breaking
- Good compatibility with LHCb $\bar{B}_s \rightarrow D_s^{(*)}$ analysis in 2001.03225

Inclusive vs Exclusive determination of V_{cb}

Inclusive determination of V_{cb} :

$$V_{cb}^{\text{incl}} = (42.00 \pm 0.65) \times 10^{-3}$$

[P. Gambino, C. Schwanda, 1307.4551
A. Alberti, P. Gambino, K. J. Healey, S. Nandi, 1411.6560
P. Gambino, K. J. Healey, S. Turczyk, 1606.06174]

Exclusive determination of V_{cb} : depends on the data set used and the assumptions for the hadronic parameters

- $B \rightarrow D\ell\bar{\nu}$: $V_{cb}^{\text{excl}}|_{BD} = (40.49 \pm 0.97) \times 10^{-3}$

[P. Gambino, D. Bigi, 1606.08030, + ...]

- $B \rightarrow D^*\ell\bar{\nu}$: not a general consensus yet, but systematically lower $V_{cb}^{\text{excl}}|_{BD}$

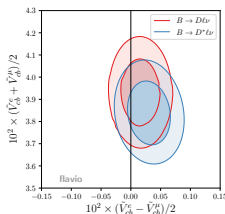
[P. Gambino, M. Jung, S. Schacht, '19
F. Bernlochner, Z. Ligeti, M. Papucci, D. Robinson, '17 + ...]

- $B_s \rightarrow D_s^{(*)}\ell\bar{\nu}$: new extraction by LHCb \Rightarrow still large uncertainties

[2001.03225]

**No evidence so far that
this tension is due to NP**

[M. Jung, D. Straub, 1801.01112]



HQET in a nutshell

- In HQET it is convenient to work with velocities instead of momenta
- Instead of q^2 we use the dimensionless variable $w = v_B \cdot v_{D^*}$
- When the $B(b)$ decays such that the $D^*(c)$ is at rest in the $B(b)$ frame

$$v_B = v_{D^*} \quad \Rightarrow \quad w = 1$$

- The brown muck doesn't realise that anything changed
- At zero recoil, the leading IW function is normalized

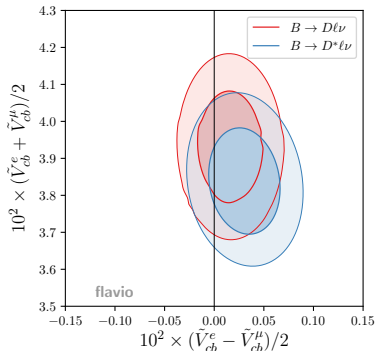
$$\xi(w = 1) = 1$$

- If we allow LFUV between μ and electrons

$$\tilde{V}_{cb}^\ell = V_{cb}(1 + C_{V_L}^\ell)$$

- Fitting data from Babar and Belle

$$\frac{\tilde{V}_{cb}^e}{\tilde{V}_{cb}^\mu} = 1.011 \pm 0.012$$



$$\frac{1}{2}(\tilde{V}_{cb}^e + \tilde{V}_{cb}^\mu) = (3.87 \pm 0.09)\%$$
$$\frac{1}{2}(\tilde{V}_{cb}^e - \tilde{V}_{cb}^\mu) = (0.022 \pm 0.023)\%$$

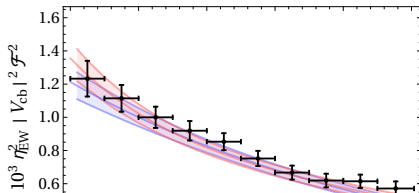
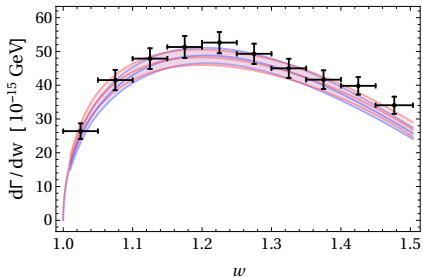
BGL vs CLN

- Both BGL and CLN parametrisation of form factors rely on using unitarity arguments.

[Boyd, Grinstein, Lebed, '95]

Caprini, Neubert, Lellouch, '98]

- CLN relies on HQET.
- Unfolded distributions from Belle allowed to repeat an independent fit.



BGL vs CLN parametrisations

CLN

[Caprini, Lellouch, Neubert, '97]

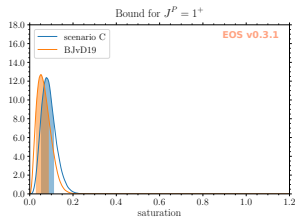
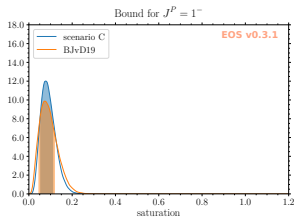
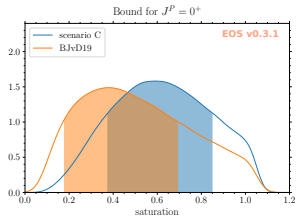
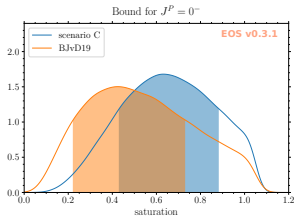
- Expansion of FFs using HQET
- $1/m_{b,c}$ corrections included
- Expansion of leading IW function up to 2nd order in $(w - 1)$

BGL

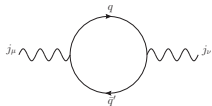
[Boyd, Grinstein, Lebed, '95]

- Based on analyticity of the form factors
- Expansion of FFs using the conformal variable z
- Large number of free parameters

Results: unitary bounds



Unitarity Bounds



$$= i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu(x), j_\nu^\dagger(0) \} | 0 \rangle = (g_{\mu\nu} - q_\mu q_\nu) \Pi(q^2)$$

- If $q^2 \ll m_b^2$ we can calculate $\Pi(q^2)$ via perturbative techniques $\Rightarrow \chi(0)$
- Dispersion relations link $\text{Im}(\Pi(q^2))$ to sum over matrix elements

$$\sum_i |F_i(0)|^2 < \chi(0)$$

[Boyd, Grinstein, Lebed, '95
Caprini, Lellouch, Neubert, '97]

- The sum runs over **all** possible states hadronic decays mediated by a current $\bar{c}\Gamma_\mu b$
 - The unitarity bounds are more effective the most states are included in the sum
 - The unitarity bounds introduce correlations between FFs of different decays
 - $B_s \rightarrow D_s^{(*)}$ decays are expected to be of the same order of $B_{u,d} \rightarrow D_{u,d}^{(*)}$ decays due to $SU(3)_F$ symmetry

Theory framework

$$\Gamma = \frac{1}{m_B} \text{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) \right\} | B(p) \rangle$$

Theory framework

$$\Gamma = \frac{1}{m_B} \text{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) \right\} | B(p) \rangle$$



$$\sum_{n,i} \frac{1}{m_b^n} C_{n,i} \mathcal{O}_{n+3,i}$$

Theory framework

$$\Gamma = \frac{1}{m_B} \text{Im} \int d^4x \langle B(p) | T \{ \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) \} | B(p) \rangle$$



$$\sum_{n,i} \frac{1}{m_b^n} C_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p) | \mathcal{O}_{n+3,i} | B(p) \rangle$ are non perturbative
 - ⇒ They need to be determined with non-perturbative methods, e.g. Lattice QCD
 - ⇒ They can be extracted from data
 - ⇒ With large n , large number of operators

Theory framework

$$\Gamma = \frac{1}{m_B} \text{Im} \int d^4x \langle B(p) | T \{ \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) \} | B(p) \rangle$$



$$\sum_{n,i} \frac{1}{m_b^n} C_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p) | \mathcal{O}_{n+3,i} | B(p) \rangle$ are non perturbative
 - ⇒ They need to be determined with non-perturbative methods, e.g. Lattice QCD
 - ⇒ They can be extracted from data
 - ⇒ With large n , large number of operators



loss of predictivity

Theory framework

$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[1 + a_1 \left(\frac{\alpha_s}{\pi} \right) + a_2 \left(\frac{\alpha_s}{\pi} \right)^2 + a_3 \left(\frac{\alpha_s}{\pi} \right)^3 - \left(\frac{1}{2} - p_1 \left(\frac{\alpha_s}{\pi} \right) \right) \frac{\mu_\pi^2}{m_b^2} + \left(g_0 + g_1 \left(\frac{\alpha_s}{\pi} \right) \right) \frac{\mu_G^2(m_b)}{m_b^2} + d_0 \frac{\rho_D^3}{m_b^3} - g_0 \frac{\rho_{LS}^3}{m_b^3} + \dots \right]$$

$$\mu_\pi^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu \quad \mu_G^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$$

- Coefficients of the expansions are known
- Ellipses stands for higher orders

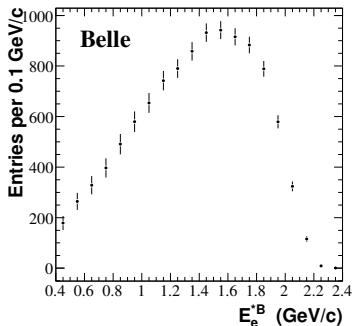
Theory framework

$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[1 + a_1 \left(\frac{\alpha_s}{\pi} \right) + a_2 \left(\frac{\alpha_s}{\pi} \right)^2 + a_3 \left(\frac{\alpha_s}{\pi} \right)^3 - \left(\frac{1}{2} - p_1 \left(\frac{\alpha_s}{\pi} \right) \right) \frac{\mu_\pi^2}{m_b^2} + \left(g_0 + g_1 \left(\frac{\alpha_s}{\pi} \right) \right) \frac{\mu_G^2(m_b)}{m_b^2} + d_0 \frac{\rho_D^3}{m_b^3} - g_0 \frac{\rho_{LS}^3}{m_b^3} + \dots \right]$$

$$\mu_\pi^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu \quad \mu_G^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$$

- Coefficients of the expansions are known
- Ellipses stands for higher orders

How do we constrain the OPE parameters?



- Lepton energy and hadronic invariant mass distributions can be used to extract non perturbative information
- Moments of the kinematic distributions

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell, \text{cut}}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell, \text{cut}}}}$$

$$R^* = \frac{\int_{E_\ell > E_{\ell, \text{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int dE_\ell \frac{d\Gamma}{dE_\ell}}$$

- Similar definition for hadronic mass moments

- The moments give access to the distribution, but not to the normalisation

- They admit an HQE as the rate

⇒ No $\mathcal{O}(\alpha_s^3)$ terms are known yet

An alternative for the inclusive determination

- q^2 moments

$$R^* = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \frac{d\Gamma}{dq^2}} \quad \langle (q^2)^n \rangle = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 (q^2)^n \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \frac{d\Gamma}{dq^2}}$$

- Exploits HQE to reduce numbers of higher dimensional operators [Fael, Mannel, Vos, '18]
- Preliminary result:

$$|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$$

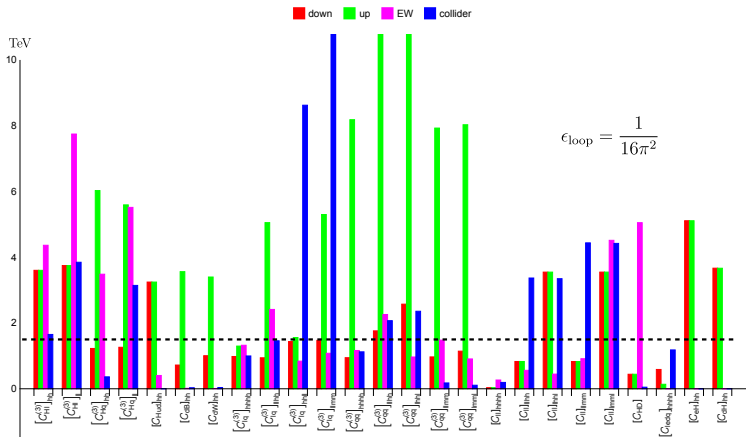
What's the issue with the previous determination?

- The q^2 moments require a measurement of the branching ratio with a cut in q^2 which is not available yet
- By extrapolating from the current available measurements, the branching ratio is lower than what used
- If the same branching ratios is used, the two methods give the **same** result

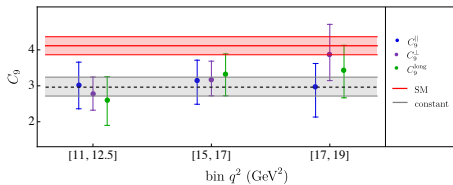
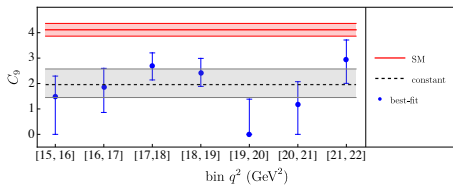
The results for inclusive V_{cb} are stable

SMEFT with Flavour 1

[Allwicher, Cornella, Isidori, Stefaneke, in preparation]



C_9 from $B \rightarrow K^{(*)} \mu^+ \mu^-$ data



Inclusive vs. Exclusive determination of V_{cb}

The inclusive determination is solid

- The traditional determination using data for the hadronic mass moments and lepton energy moments yields stable results up to $\mathcal{O}(\alpha_s^3)$ corrections in the width

[2011.13654]

MB, Capdevila, Gambino, '21

- New determination using q^2 moments yields very compatible results

[2205.10274]

- Only caveat: QED corrections for charged current decays are enhanced by the Coulomb factor (for neutral B mesons)

MB, Bigi, Gambino, Haisch, Piccione '23

⇒ The impact has to be checked for each measurement

The exclusive determination depends on the dataset and hadronic form factor used

- Work in progress on the theory side
- New experimental data are available and have to be still scrutinised

