NNLO QCD corrections for the production of a **heavy-quark pair** in association with a **massive boson**

Chiara Savoini

University of Zurich

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in collaboration with *L.Buonocore, S.Catani, S.Devoto, M.Grazzini, S.Kallweit, J.Mazzitelli, L.Rottoli*

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Universität Zürich[UZH]

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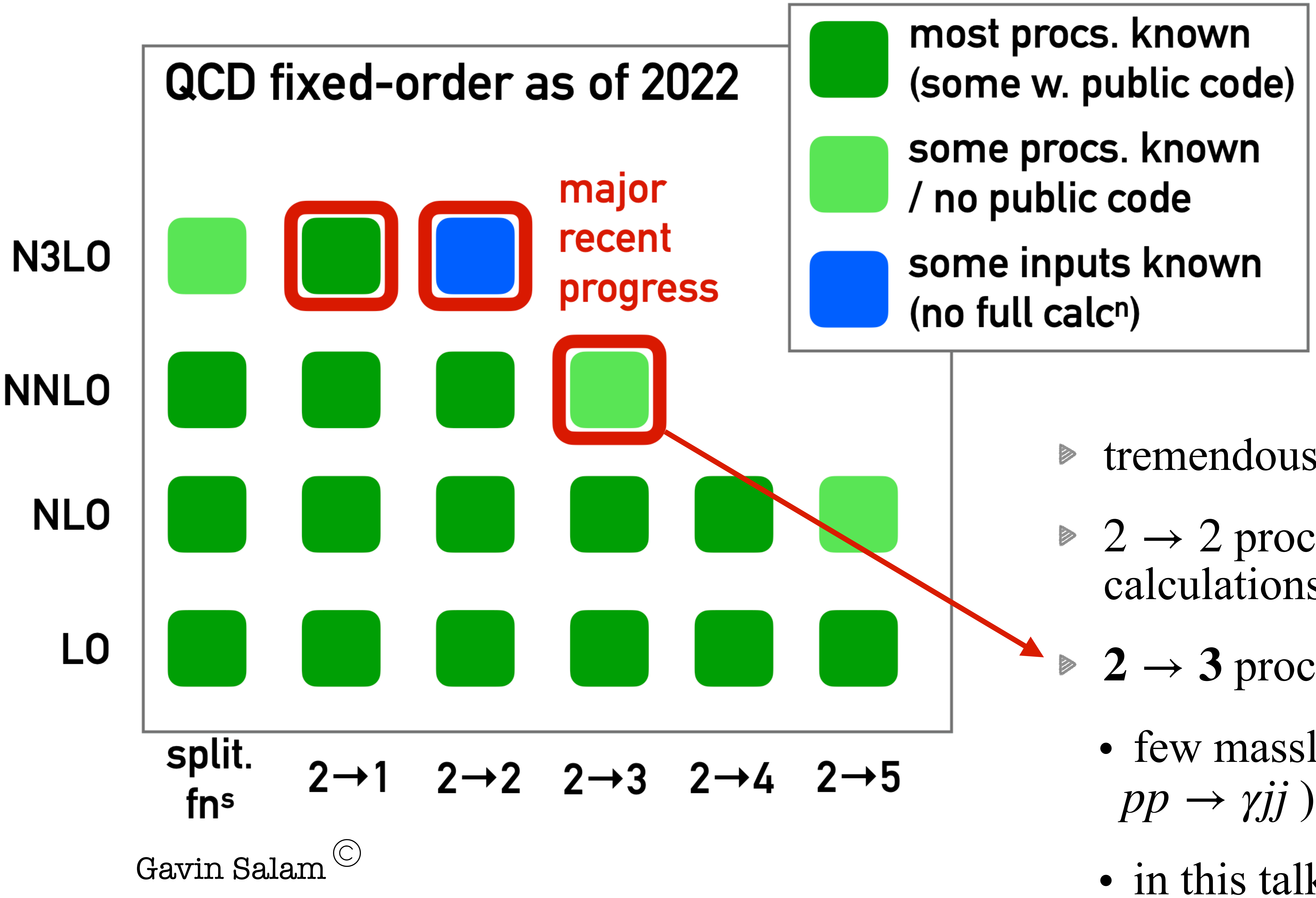
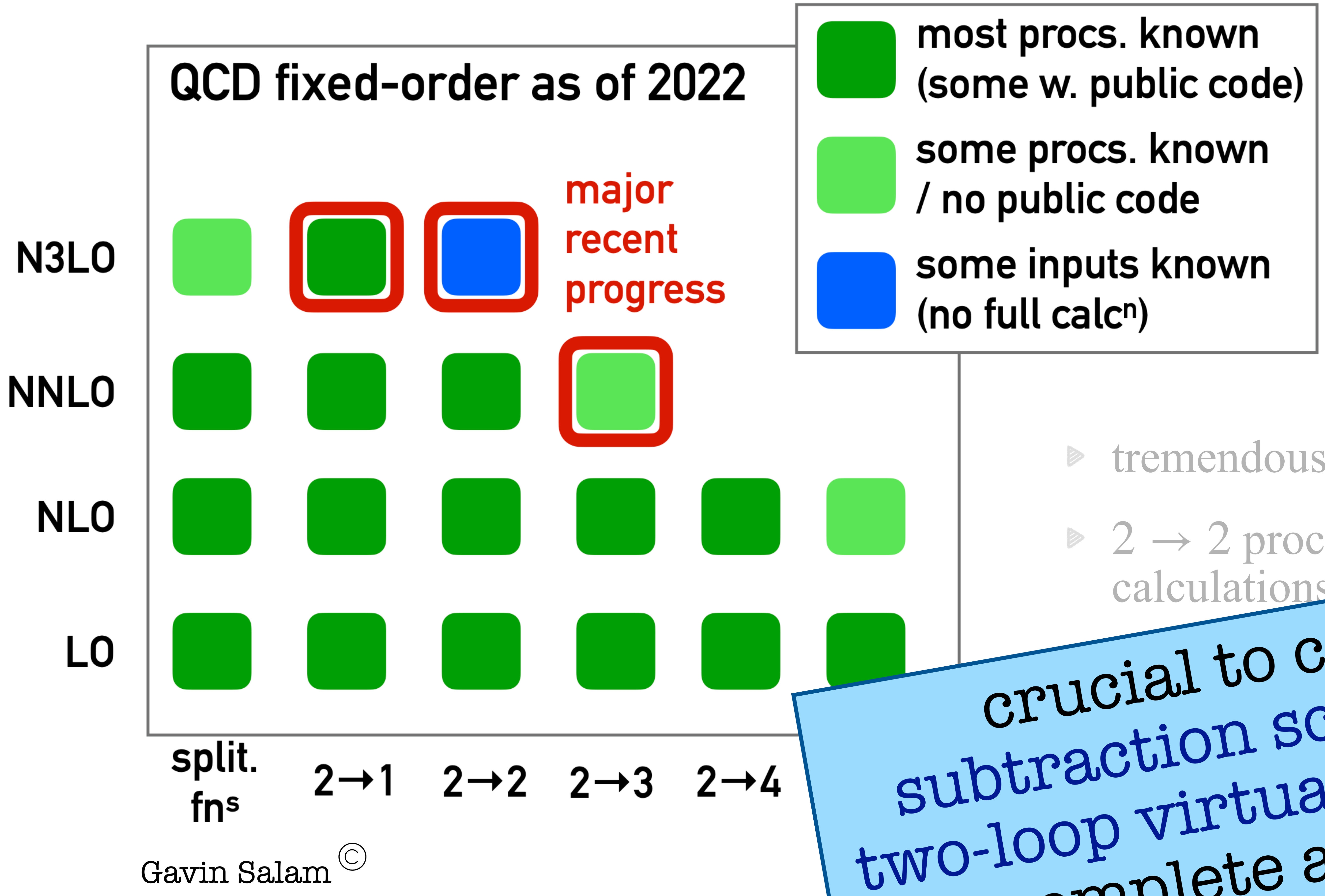
Introduction



- tremendous progress in the past ~10 years!
- 2 → 2 processes at NNLO are under control (independent calculations)
- **2 → 3** processes at NNLO represent the **current frontier**
  - few massless computations ($pp \to \gamma\gamma\gamma$, $pp \to \gamma\gamma j$, $pp \to jjj$, $pp \to \gamma jj$)
  - in this talk we will focus on **2 → 3** processes with **external massive legs**

$t\bar{t}H$ $Wb\bar{b}$ $t\bar{t}W$

Introduction



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- 2 → 2 processes at NNLO are … endent calculations)
- … ntier
- … $p \to jjj$,
- … will focus on **2** → **3** processes with **external massive legs**

crucial to construct an NNLO subtraction scheme and to have the two-loop virtual amplitudes in order to complete an NNLO calculation

The framework: q_T-subtraction

[Catani, Grazzini (2007)]

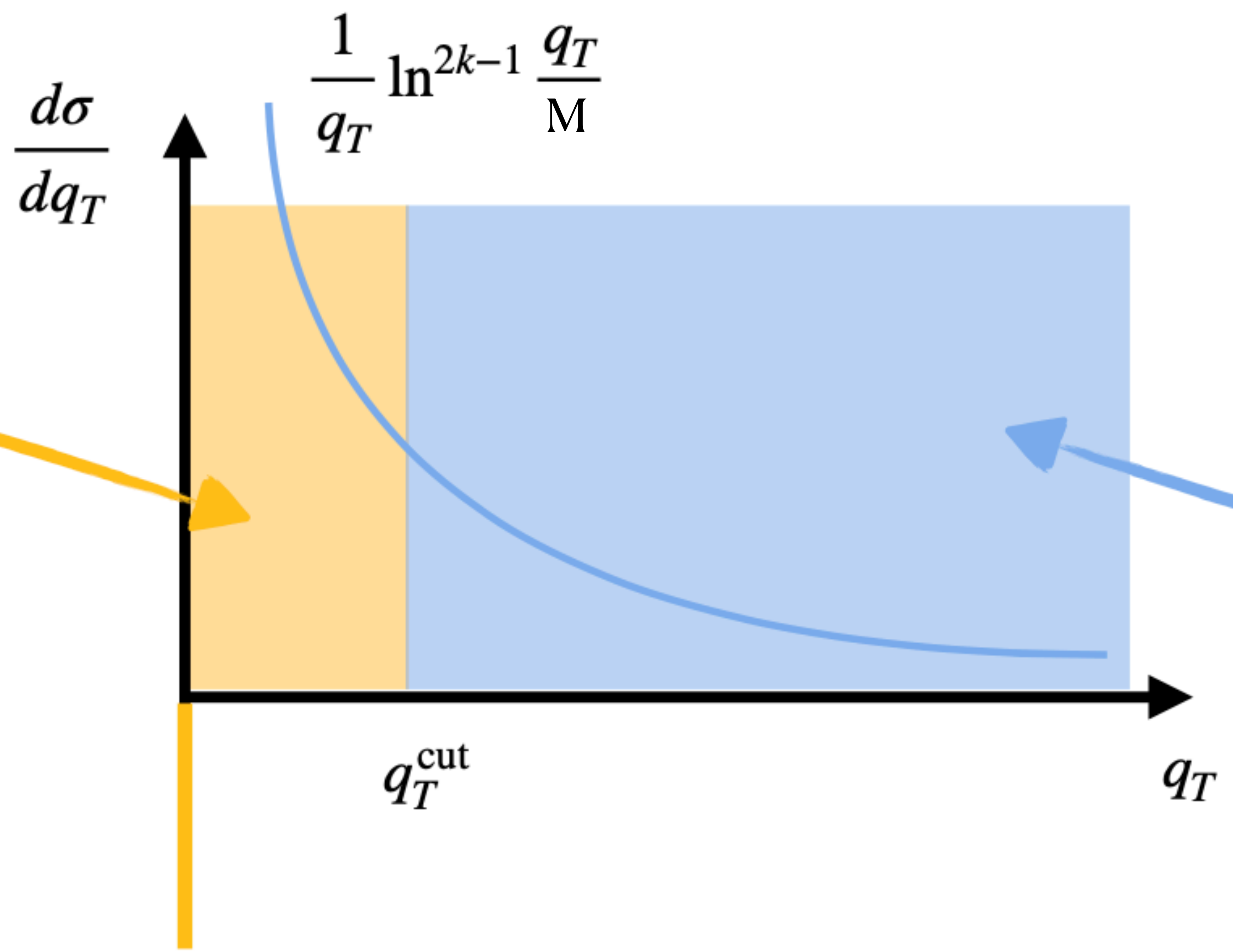
- cross section for the production of a triggered final state at N^{k}LO (in our case the triggered final state is $Q\bar{Q}F$)

crucial to keep the mass of the heavy quark m_Q

all emissions are unresolved

we can exploit the **QCD factorisation** of the matrix elements in the singular soft and/or collinear limits

ingredients from q_T - resummation



1 emission is always **resolved**

the complexity of the calculation is reduced by 1 order

logarithmic IR sensitivity to the cut

q_T and M are the transverse momentum and the invariant mass of the $Q\bar{Q}F$ system

$$d\sigma_{N^kLO} = \mathscr{H}_{N^kLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}]_{q_t > q_t^{\text{cut}}} + \mathcal{O}((q_t^{\text{cut}})^p)$$

The framework: q_T -subtraction [Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \mathscr{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_t > q_t^{\mathrm{cut}}} + \mathcal{O}((q_t^{\mathrm{cut}})^p)$$

☑ the required matrix elements can be computed with **automated tools** like OpenLoops2

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]

☑ the remaining NLO-type singularities can be removed by applying a **local subtraction** method

[Catani, Seymour (1998)] [Catani, Dittmaier, Seymour, Trocsanyi (2002)]

☑ automatised implementation in the **MATRIX** framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH [Grazzini, Kallweit, Wiesemann (2017)]

The framework: q_T-subtraction [Catani, Grazzini (2007)]

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- ☑ non trivial ingredient: **two-loop soft function** for an arbitrary kinematics of the heavy quarks

  [Catani, Devoto, Grazzini, Mazzitelli (2023)] [Devoto, Mazzitelli (in preparation)]

- ☑ all ingredients are known except for the **two-loop virtual amplitudes** contributing to the the hard-collinear coefficient

$$\mathcal{H}_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}^{(2)}(z_1, z_2)$$

where $$H^{(2)} = \left.\frac{2\Re(\mathcal{M}^{(2)}_{fin}(\mu_{IR},\mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=\mu_{IR}=M}$$

UV renormalised and IR subtracted amplitude at scale μ_{IR} (overall normalisation $(4\pi)^\epsilon e^{-\gamma_E \epsilon}$)

Remark: analogous definition for the hard-collinear coefficient at NLO $$H^{(1)} = \left.\frac{2\Re(\mathcal{M}^{(1)}_{fin}(\mu_{IR},\mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=\mu_{IR}=M}$$

The framework: q_T-subtraction [Catani, Grazzini (2007)]

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→ main bottleneck:
$2 \to 3$ two-loop amplitudes with internal and external massive legs are currently out of reach!

The framework: q_T-subtraction

[Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \mathcal{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_t > q_t^{\rm cut}} + \mathcal{O}((q_t^{\rm cut})^p)$$

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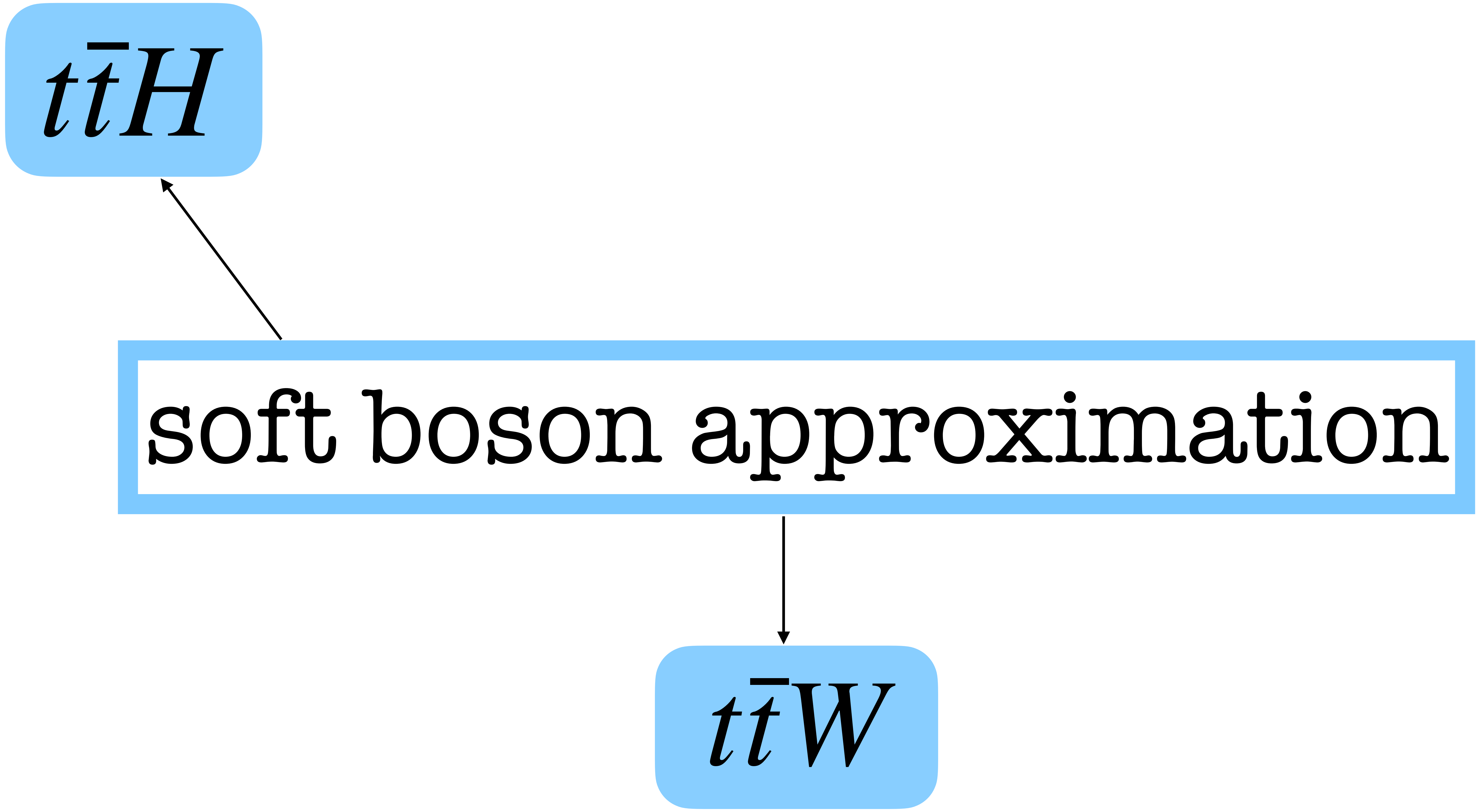
where $H^{(2)} = \frac{2\Re(\mathcal{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{\cdots}\Big|$

crucial to find one (or more) reasonable approximation

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]



[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (2023)]

Soft Higgs boson approximation

bottleneck: the two-loop amplitudes are at the frontier of the current techniques

solution: development of a soft boson approximation

- the main idea is to find an analogous formula to the well known factorisation in the case of **soft gluons**

$$\lim_{k\to 0} \mathscr{M}^{bare}(\{p_i\}, k) = J(k)\, \mathscr{M}^{bare}(\{p_i\})$$

see e.g. [Catani, Grazzini (2000)]

$$J(k) = g_s \mu^{\epsilon}(J^{(0)}(k) + g_s^2 J^{(1)}(k) + \ldots)$$

purely non abelian

- for a **soft scalar Higgs** radiated off a heavy quark with momentum p_j , we have that

soft insertion rules, only external legs matter!

$$\lim_{k\to 0} \mathscr{M}^{bare}(\{p_i\}, k) = J^{(0)}(k)\, \mathscr{M}^{bare}(\{p_i\})$$

bare mass of the heavy quark

$$J^{(0)}(k) = \sum_j \frac{m_{j,0}}{v} \frac{m_{j,0}}{p_j \cdot k}$$

- the naïve factorisation formula does NOT hold at the level of renormalised amplitudes!

Soft Higgs boson approximation

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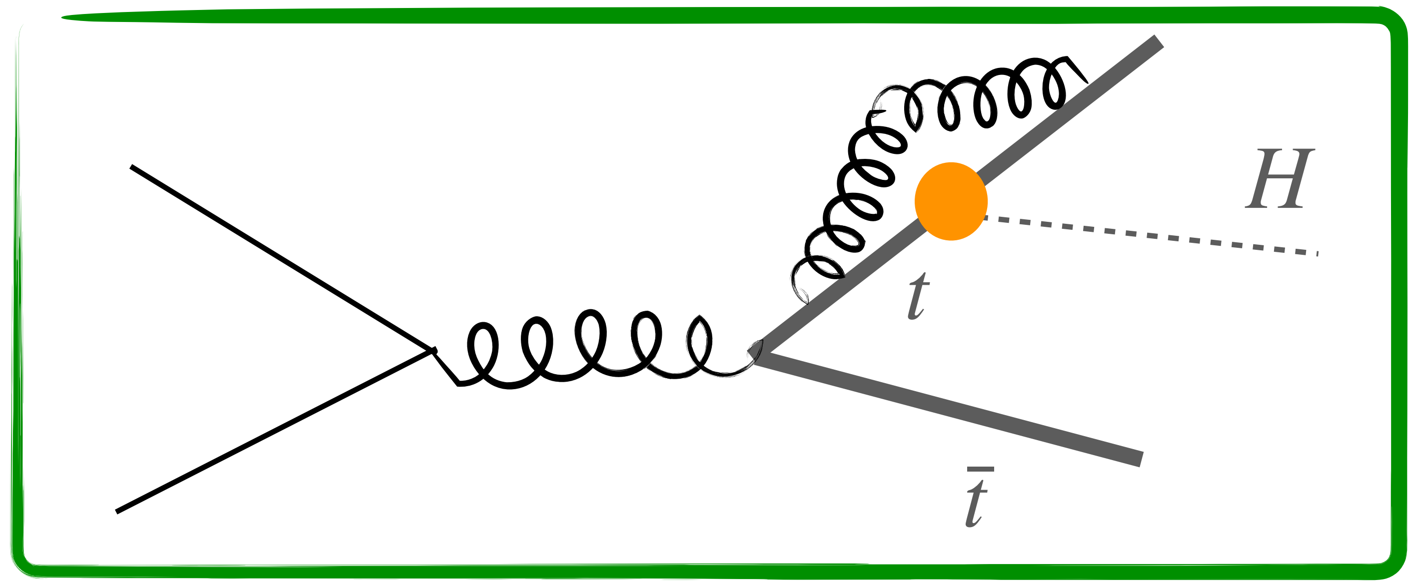
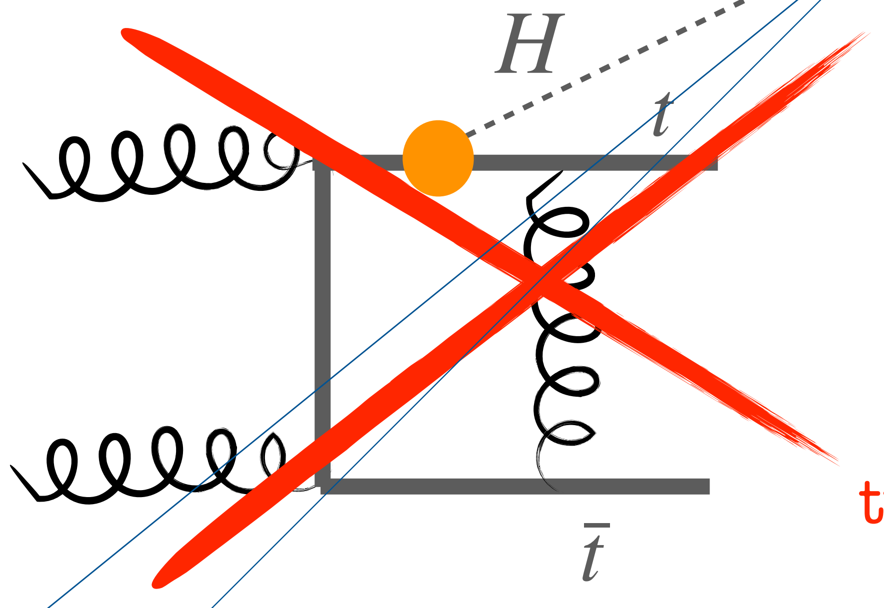
solution: development of a soft boson approximation

- already at one-loop, diagrams that are not captured by the naïve factorisation formula can give an **additional leading contribution** in the soft Higgs limit

one-particle diagrams





two-particle diagrams

- each 2P diagram contributes to the leading behaviour of the matrix element in the soft Higgs limit, if also the loop momentum is soft
- by considering all possible insertions of the Higgs boson on the top quark line, no additional contributions arise wrt the naïve factorisation formula

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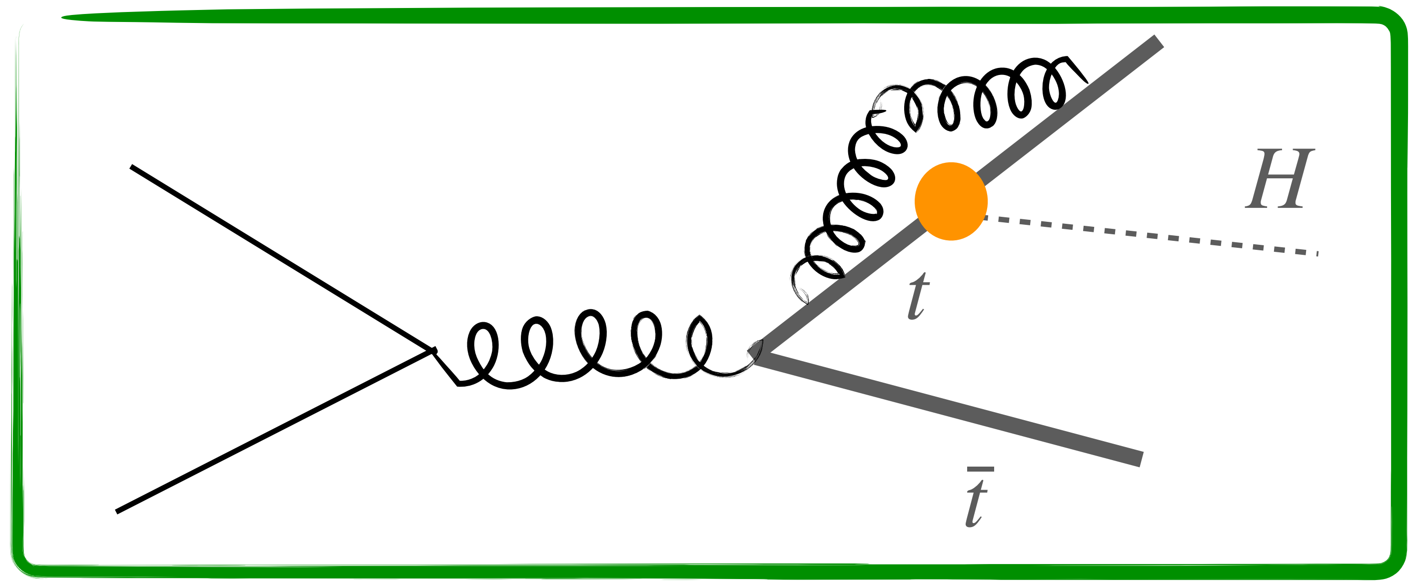

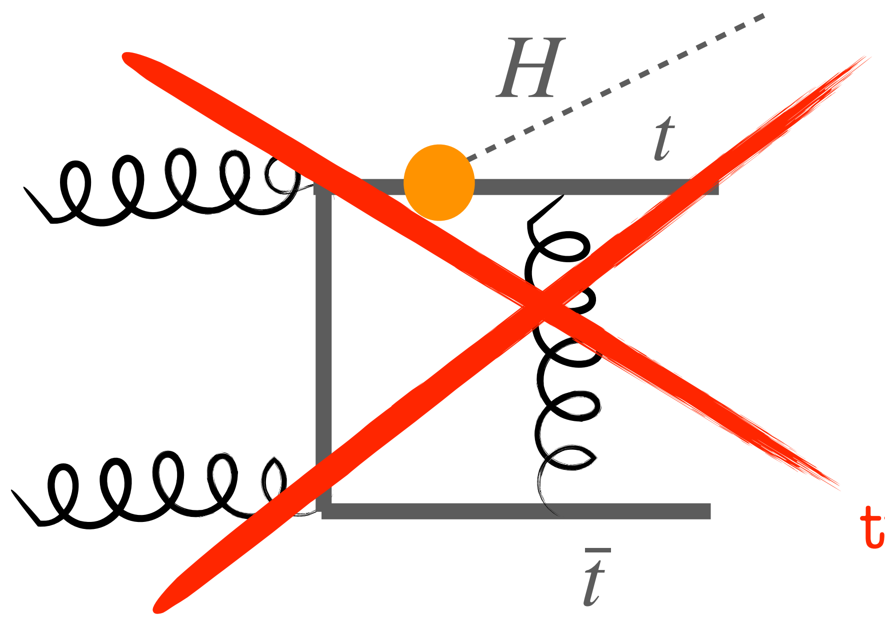

two-particle diagrams

- they give an additional contribution to the the naïve factorisation formula
- in other words, the renormalisation of the heavy-quark mass and wave function induces a modification of the Higgs coupling to the heavy quark

$$\lim_{k\to 0} \mathscr{M}(\{p_i\},k) = F(\alpha_s(\mu_R); m/\mu_R)\, J^{(0)}(k)\, \mathscr{M}(\{p_i\})$$

overall normalisation, finite, gauge-independent and perturbatively computable

renormalised mass

$$J^{(0)}(k) = \sum_j \frac{m}{v}\frac{m}{p_j \cdot k}$$

Soft Higgs boson approximation

- **master formula** in the soft Higgs limit ($k \to 0$, $m_H \ll m_t$)

$$\lim_{k\to 0} \mathcal{M}_{t\bar{t}H}(\{p_i\}, k) = F(\alpha_s(\mu_R); m_t/\mu_R)\, J^{(0)}(k) \mathcal{M}_{t\bar{t}}(\{p_i\})$$

[Bärnreuther, Czakon, Fiedler (2013)]

soft limit of the scalar form factor for the heavy quark [Bernreuther et al. (2005)] [Blümlein et al. (2017)]

$$F(\alpha_s(\mu_R); m_t/\mu_R) = 1 + \frac{\alpha_s(\mu_R)}{2\pi}(-3C_F) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C_F(n_L+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m_t^2}\right) + \mathcal{O}(\alpha_s^3)$$

up to two-loop order

we assume that all heavy quarks involved in the process have the same mass

- **NEW**: ongoing check of the soft factorisation formula at **three-loop order**, based on
  - three-loop on-shell renormalisation constants Z_m and Z_2 [Melnikov, Ritbergen (2000)]
  - decoupling relations at $\mathcal{O}(\alpha_s^3)$ [Chetyrkin, Kniehl, Steinhauser (1997)]
  - three-loop massive form factors [Fael, Lange, Schönwald, Steinhauser (2022, 2023)]

Soft W-boson approximation

bottleneck: the two-loop amplitudes are at the frontier of the current techniques

solution: development of a soft boson approximation

- goal: compute NNLO QCD corrections for $t\bar{t}W$
- the idea is to follow a similar approach used in the case of $t\bar{t}H$: develop a **soft factorisation formula** also in the case of a W boson (only coupling to massless quarks, masses break the factorisation…)
- for a **soft gauge W boson** radiated off a massless quark with momentum p_j, we find that

$$\lim_{k\to 0} \mathcal{M}(\{p_i\}, k) = \frac{g_W}{\sqrt{2}} \sum_j \left(\sigma_j \frac{p_j \cdot \epsilon^*(k)}{p_j \cdot k} \mathcal{M}_{j_L}(\{p_i\}) \right)$$

valid at all perturbative orders

$$\sigma_j = \begin{cases} +1 & \text{incoming } \bar{q}\,, \text{ outgoing } q \\ -1 & \text{incoming } q\,, \text{ outgoing } \bar{q} \end{cases}$$

amplitude where the massless quark with momentum p_j is LEFT-HANDED

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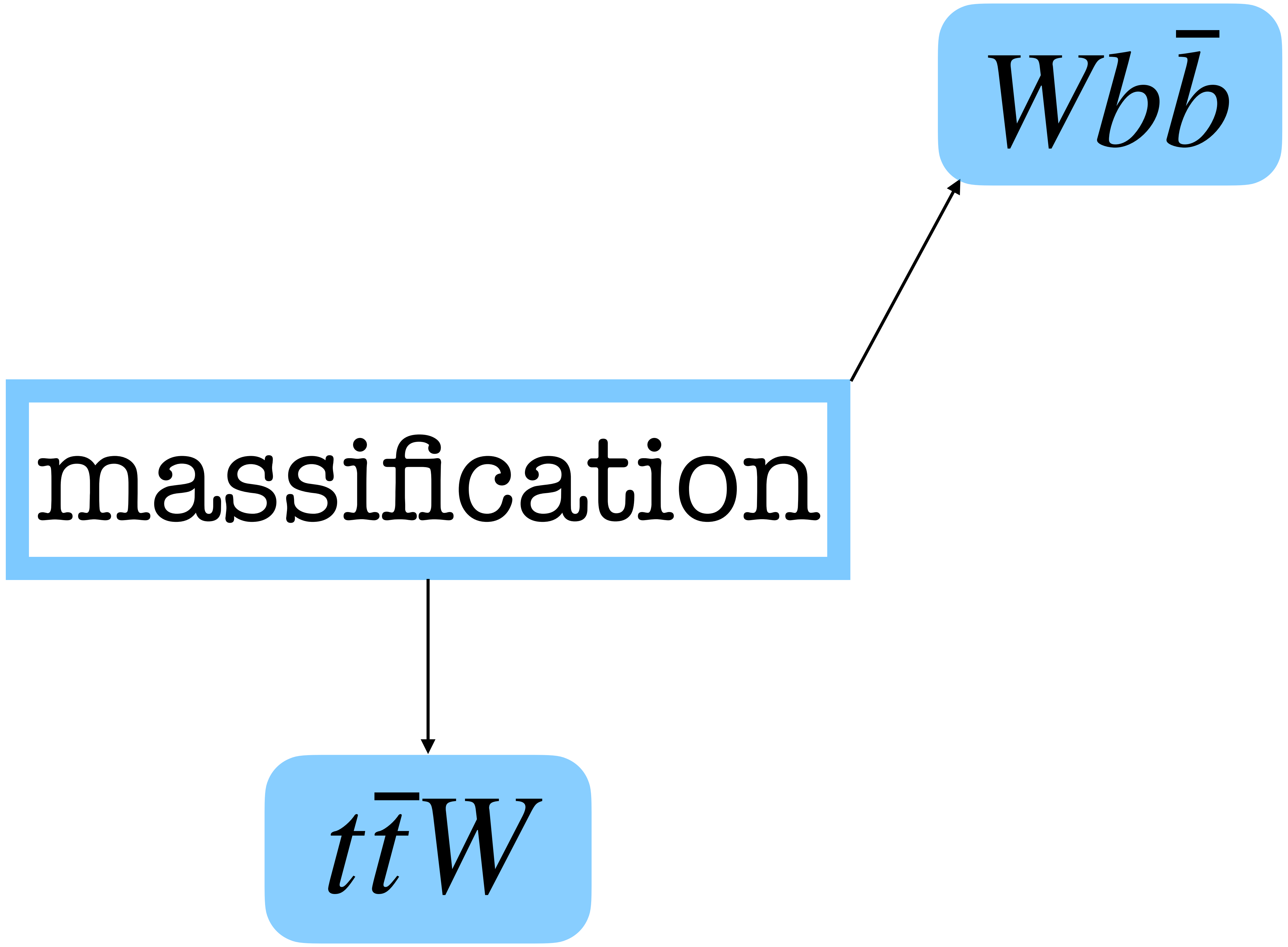
valid at all perturbative orders

main differences between W boson and Higgs :
- vectorial vs scalar current
- massless vs massive emitters
- no renormalisation effects
- selection of the polarisation state of the emitter

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, **CS** (2022)]



[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (2023)]

Massification

[Moch, Mitov (2007)]
[Becher, Melnikov (2007)]

- idea: exploit the recently computed leading-colour **massless two-loop 5-point amplitudes** for $q\bar{q}' \to WQ\bar{Q}$ production
  [Abreu at al. (2021)] [Badger at al. (2021)]
- and apply the **massification** technique to reconstruct the corresponding massive amplitudes up to power corrections in the mass m_Q
- massification relies on the **factorisation** properties of massless QCD amplitudes (into jet, hard and soft functions)

$$|\mathcal{M}_p\rangle = \mathcal{J}_0^{[p]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) \mathcal{S}_0^{[p]}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) |\mathcal{H}_p\rangle$$

- when the mass is introduced, some of the collinear singularities are screened

  $1/\epsilon$ poles are traded into $\log m_Q$

- in the limit $m_Q \ll Q$, the massive amplitude "shares" essential properties with the corresponding massless amplitude

  ↓ change in the regularisation scheme

- factorisation of massive QCD amplitudes (up to $\mathcal{O}(m_Q/Q)$)

$$|\mathcal{M}_p\rangle = \mathcal{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) \mathcal{S}_0^{[p]}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) |\mathcal{H}_p\rangle$$

ASSUMPTION: being the process-dependent soft and hard functions insensitive to collinear dynamics, they are assumed not to change, up to power corrections in the mass

Massification

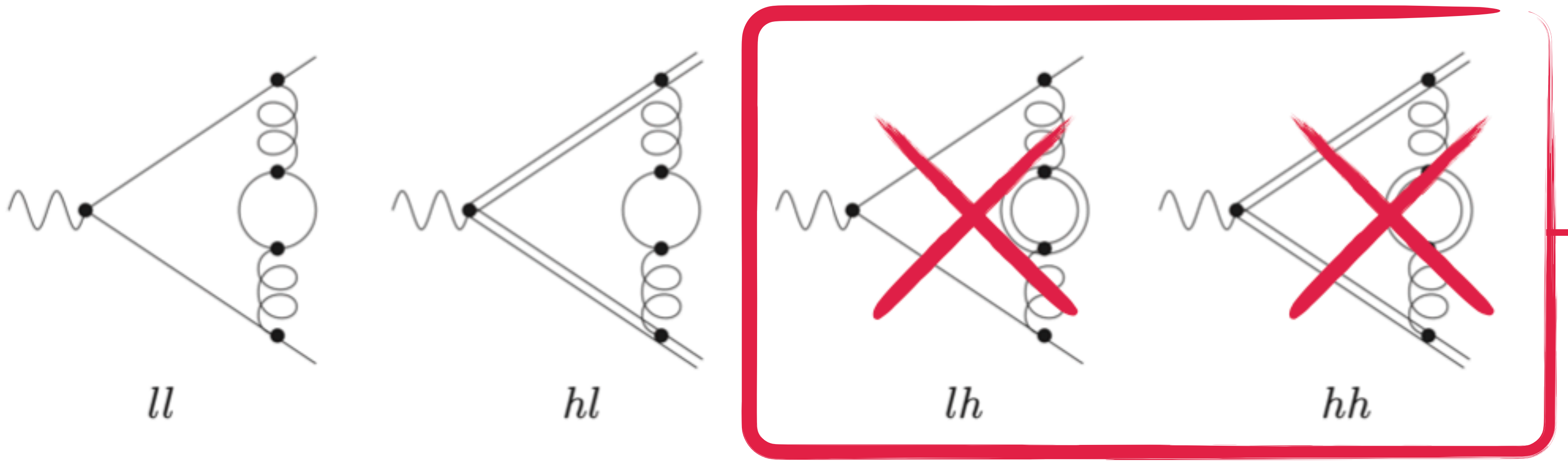
- the **master formula** is

$$\mathcal{M}^{[p],(m)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)=\prod_{i\in\{\text{all legs}\}}\left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)\right)^{\frac{1}{2}}\times\mathcal{M}^{[p],(m=0)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)$$

universal, perturbatively computable, ratio of massive and massless form factors

$$Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s,\varepsilon\right)=\mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s,\varepsilon\right)\left(\mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2},0,\alpha_s,\varepsilon\right)\right)^{-1}$$

- in the case of $WQ\bar{Q}$, the function $Z_{[q]}^{(m_Q|0)}$ is related to $\gamma^* q\bar{q}$ form factor



excluded, their description requires process-dependent information

Massification

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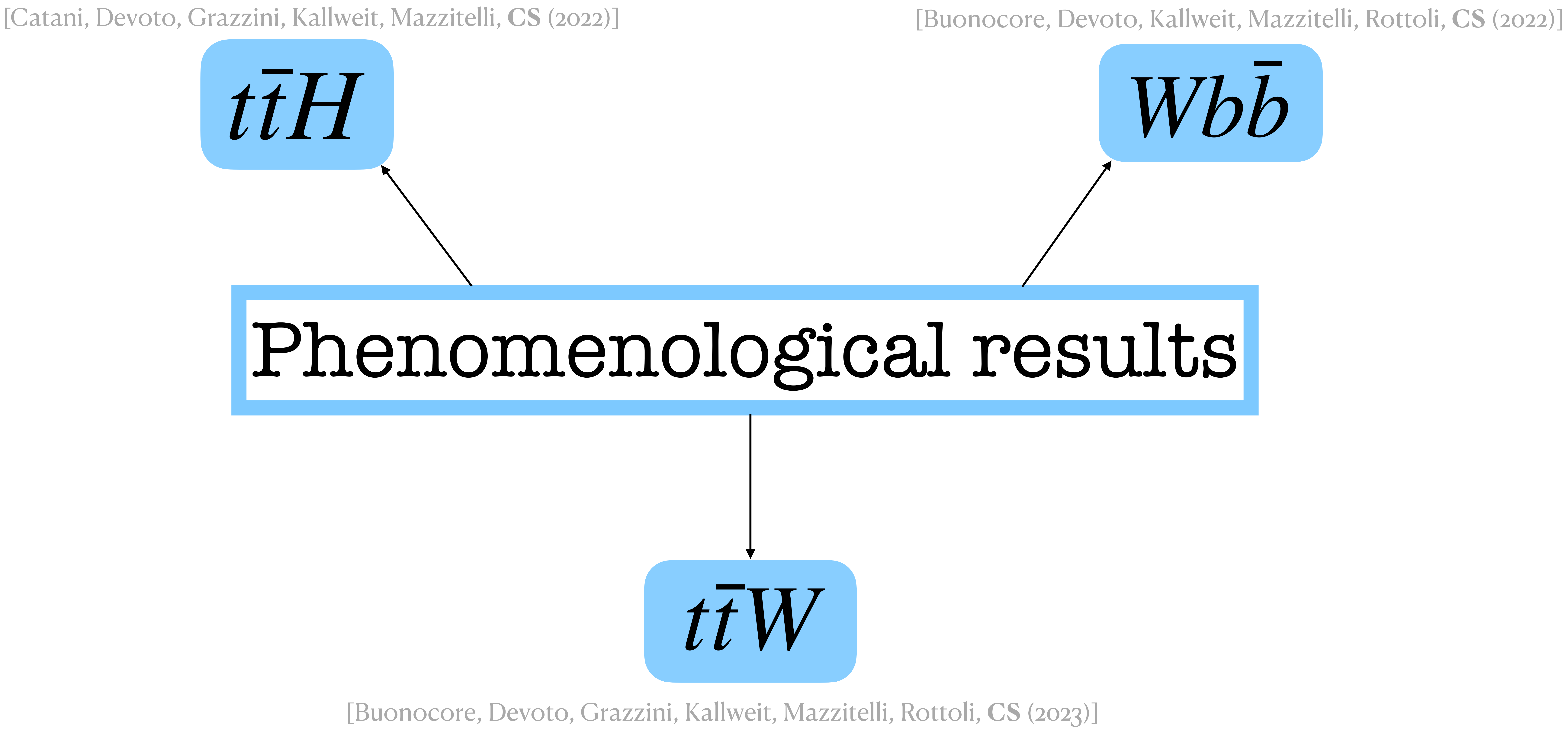
take-home message:

- the massification procedure predicts the correct ϵ poles, logarithms of the mass and mass independent terms of the massive amplitude
- power corrections in the mass and heavy-quark loop contributions cannot be retrieved


[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, CS (2022)]
$t\bar{t}H$
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$Wb\bar{b}$
Phenomenological results
$t\bar{t}W$
[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, CS (2023)]

Results

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

- direct probe of the **top Yukawa coupling**
- HL-LHC projection: $\mathcal{O}(2\%)$ [CERN Yellow Report (2019)]
- current theoretical predictions: $\mathcal{O}(10\%)$ [LHC cross section WG (2016)]
- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \to 3$ (2 masses) amplitudes
- prescription: **soft Higgs boson approximation**

all ingredients are computed exactly
except the two-loop contribution

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[Catani, de Florian, Ferrera, Grazzini (2015)]

- we construct a **mapping** to project a $t\bar{t}H$ event onto a $t\bar{t}$ one
- we test the **quality** of the approximation at Born and one-loop level: the observed deviation at NLO is used to estimate the uncertainty at NNLO

	$\sqrt{s} = 13\,\mathrm{TeV}$		$\sqrt{s} = 100\,\mathrm{TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0

$\sim 30\,\%$ (gg, 13 TeV)

$\sim 5\,\%$ ($q\bar{q}$, 13 TeV)

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- we test the **quality** of the approximation at Born and one-loop level: the observed deviation at NLO is used to estimate the uncertainty at NNLO
- at NNLO, the hard contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$
- it is clear that the quality of the final result depends on the size of the contribution we are approximating

FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO} , $\pm 15\,\%$ on $\Delta\sigma_{NNLO}$

$t\bar{t}H$

Results

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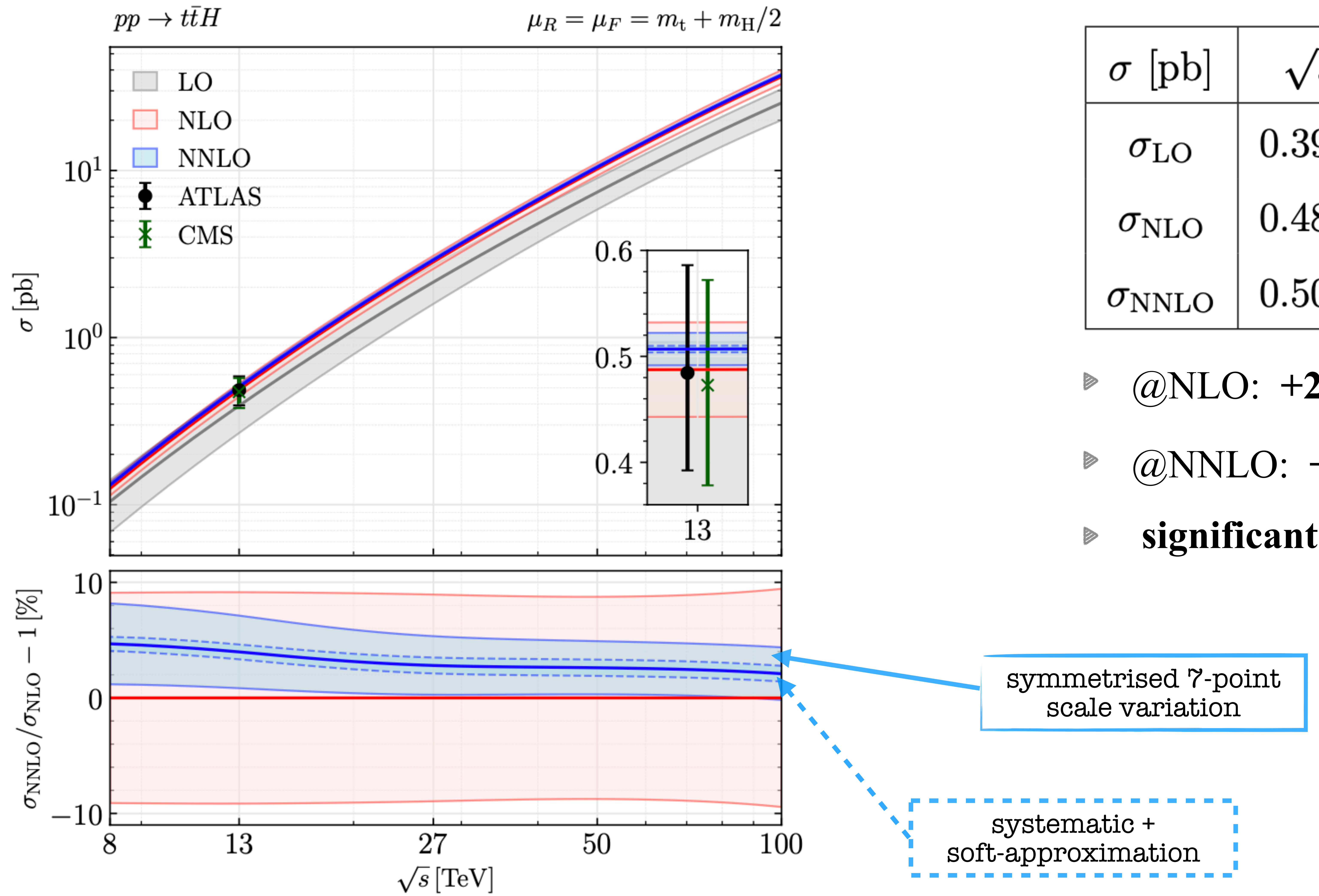

σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

- @NLO: **+25 (+44)%** at $\sqrt{s} = 13\,(100)\,TeV$
- @NNLO: **+4 (+2)%** at $\sqrt{s} = 13\,(100)\,TeV$
- **significant reduction** of the perturbative uncertainties

Note that a sensible comparison with data should eventually be done by including NLO EW corrections

$Wb\bar{b}$

Results

[CMS: arXiv 1608.07561]

setup: NNLO NNPDF31 4F , $\sqrt{s} = 8\,TeV$, $\mu_R = \mu_F = E_T(l\nu) + p_T(b_1) + p_T(b_2)$

$p_{T,l} > 30\,GeV \;\; |\eta_l| < 2.1$, $p_{T,b} > 25\,GeV \;\; |\eta_l| < 2.4$, $p_{T,j} > 25\,GeV \;\; |\eta_l| < 2.4$

- irreducible background to VH, single top production, BSM searches
- test of perturbative QCD: 4FS vs 5FS, **modelling of flavoured jets**
- large NLO QCD corrections
- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \rightarrow 3$ (2 masses) amplitudes
- prescription: **massification technique**

all ingredients are computed exactly except the two-loop contribution

$Wb\bar{b}$

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$p_{T,l} > 30\,GeV \;\; |\eta_l| < 2.1$, $p_{T,b} > 25\,GeV \;\; |\eta_l| < 2.4$, $p_{T,j} > 25\,GeV \;\; |\eta_l| < 2.4$

- irreducible background to *VH*, single top production, BSM searches
- test of perturbative QCD: 4FS vs 5FS, **modelling of flavoured jets**
- large NLO QCD corrections
- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \to 3$ (2 masses) amplitudes
- prescription: **massification technique**

all ingredients are computed exactly except the two-loop contribution

- we construct a **mapping** to project the massive bottom momenta to the massless ones (preserve the four momentum of the $b\bar{b}$ pair)
- we rely on the leading-colour two-loop massless amplitudes for $W + 4$ partons [Abreu at al. (2021)] [Badger at al. (2021)]
- **reliability** of the procedure:
  - the discrepancy between the exact and massified virtual contribution at NLO is only **3%** of the NLO correction
  - the part of the two-loop virtual amplitude computed in LCA contributes at the **2%** level of the full NNLO correction

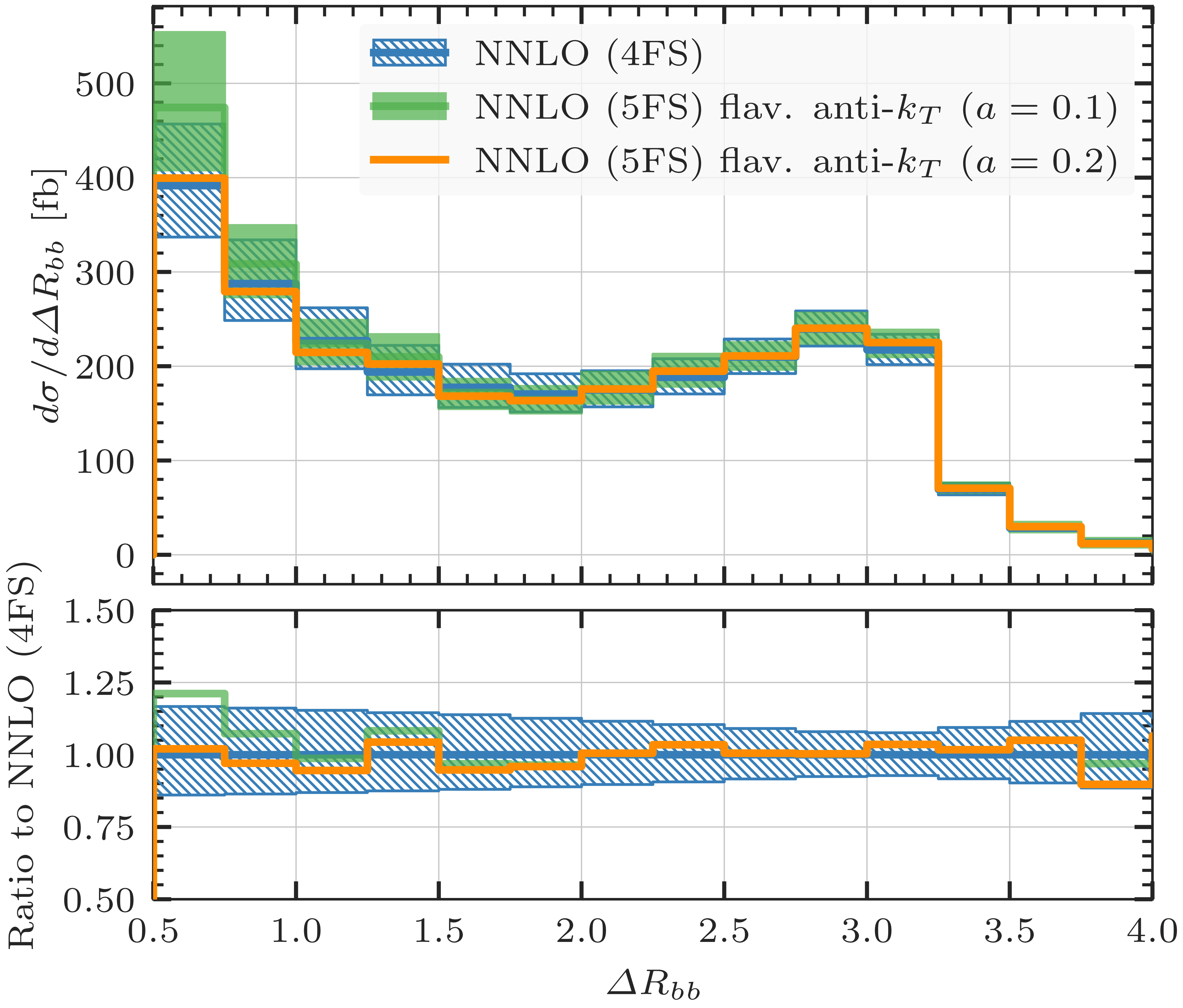
$Wb\bar{b}$

Results

[CMS: arXiv 1608.07561]

setup: NNLO NNPDF31 4F, $\sqrt{s} = 8\,TeV$, $\mu_R = \mu_F = E_T(l\nu) + p_T(b_1) + p_T(b_2)$

$p_{T,l} > 30\,GeV$ $|\eta_l| < 2.1$, $p_{T,b} > 25\,GeV$ $|\eta_l| < 2.4$, $p_{T,j} > 25\,GeV$ $|\eta_l| < 2.4$



order	σ^{4FS} [fb]	$\sigma^{5FS}_{a=0.05}$ [fb]	$\sigma^{5FS}_{a=0.1}$ [fb]	$\sigma^{5FS}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$649.9(1.6)^{+12.6\%}_{-11.0\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

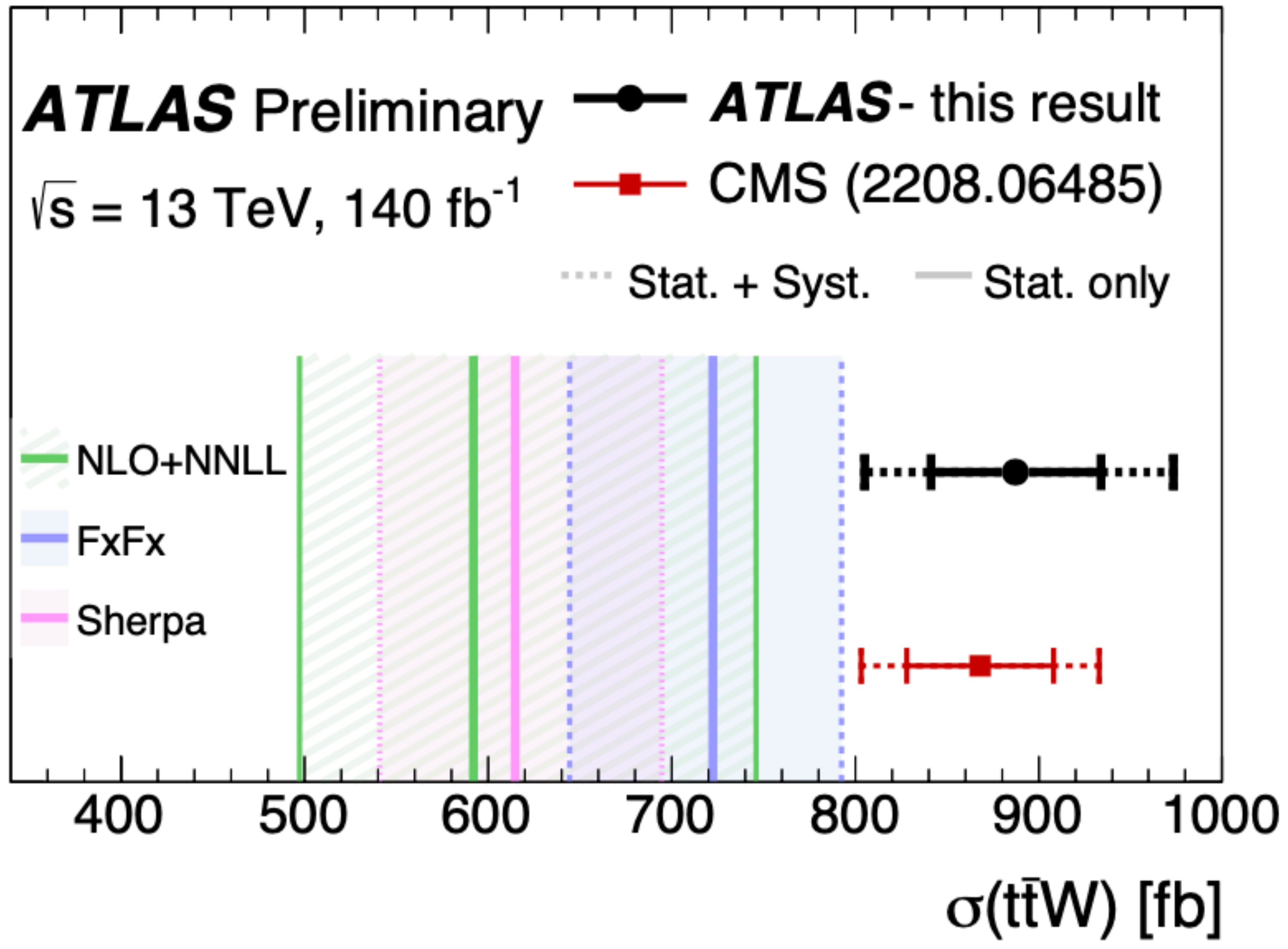
- comparison against the 5F massless computation [Poncelet et al. (2022)]
  - overall **good agreement** within the scale uncertainties
  - the uncertainties due to variation of $m_b \in [4.2, 4.92]\,GeV$ are at **2%** level (smaller than the ones due to the variation of a, ~**7%**)
- large positive NNLO corrections: **+40%**
- still **large** perturbative **uncertainties**

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385GeV$, $m_t = 173.2GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$

- relevant background for SM processes ($t\bar{t}H$, $t\bar{t}t\bar{t}$)
- multi-lepton signature relevant for BSM sources
- "**special**": large NLO QCD and EW corrections
- well known **tension** between theory and experiments (excess at **1-2σ level**) [ATLAS-CONF-2023-019] [CMS: arXiv 2208.06485]
- current **NLO QCD + EW** predictions, supplemented with **multi-jet merging** are affected by relatively large uncertainties [Frederix, Tsinikos (2021)]
- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \to 3$ (2 masses) amplitudes



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- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \to 3$ (2 masses) amplitudes

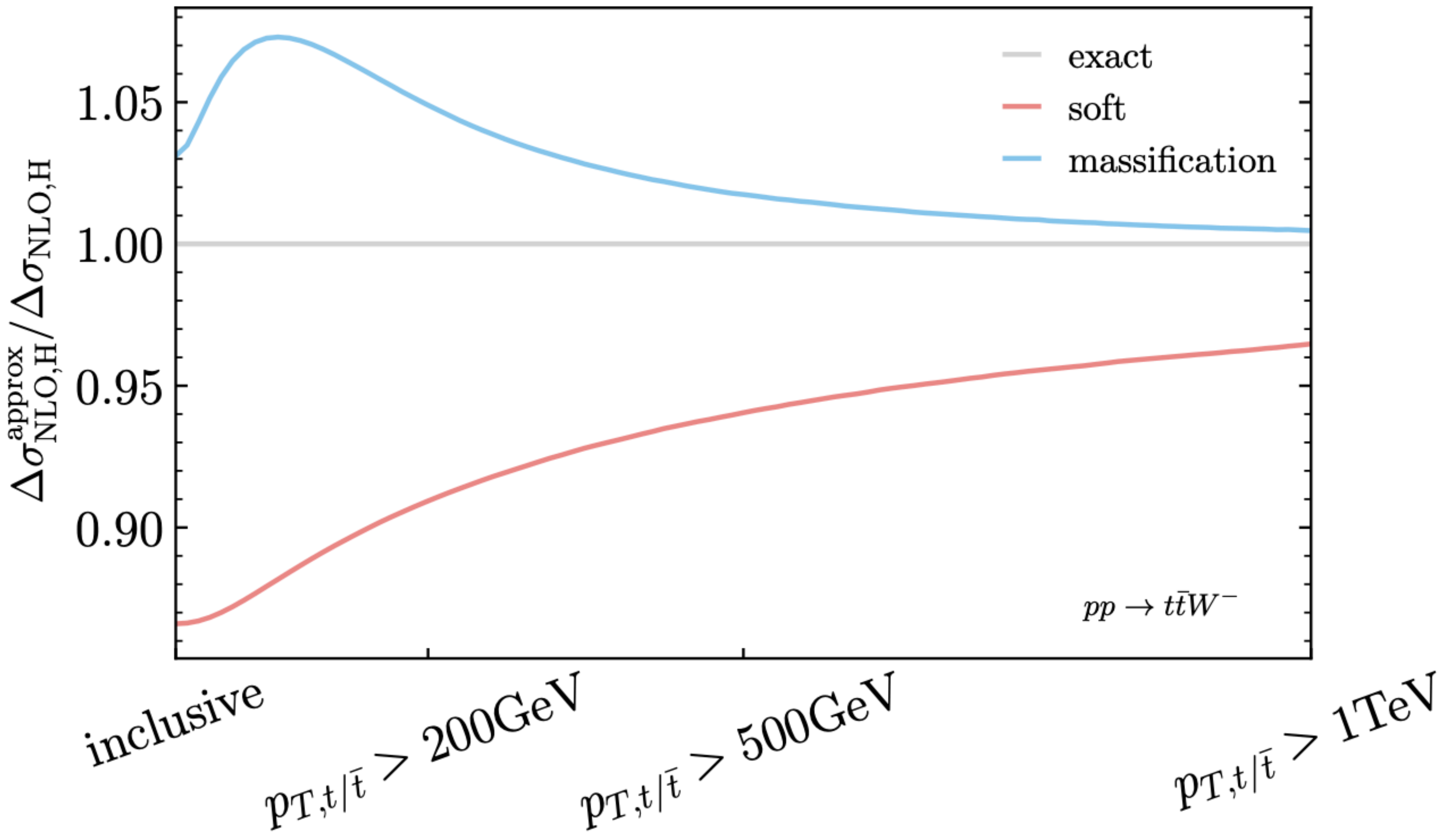
- good news! we have two rather different and complementary approximations of the exact two-loop virtual amplitudes
- **soft approximation**:
  - it works nicely in the case of $t\bar{t}H$, mainly due to the smallness of the approximated $H^{(2)}$ contribution
  - formally it is valid in the limit $E_W \to 0$, $m_W \ll m_t$ (which is not true for a physical W boson …)
- **massification**:
  - it works nicely in the case of $Wb\bar{b}$, mainly due to the smallness of the bottom mass (negligible power corrections)
  - formally it is valid in the limit $m_t \ll Q_{t\bar{t}W}$ (which is not true …)

how do these approximations perform for $Wt\bar{t}$?

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$

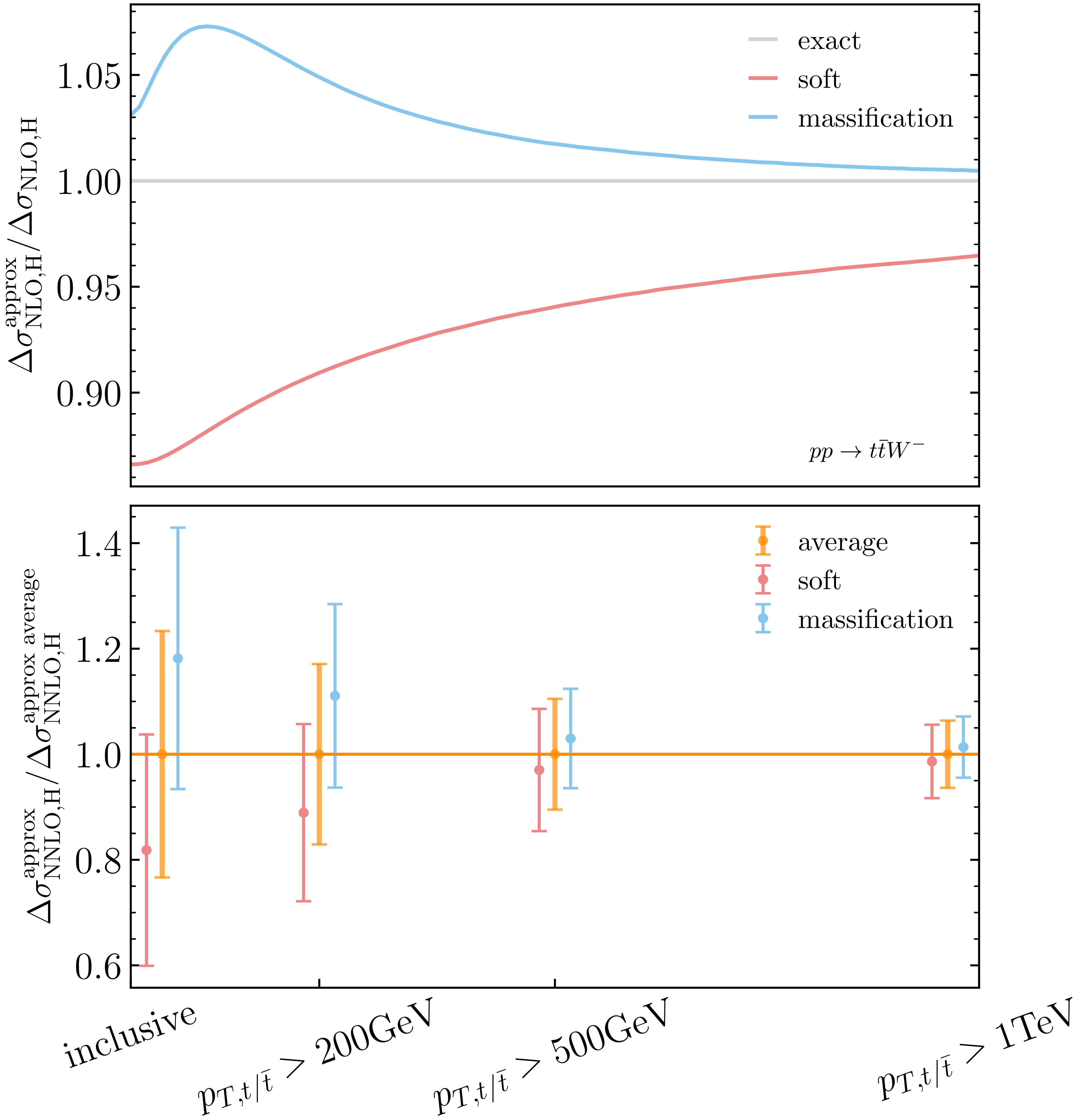


- validation at NLO:
  - both approaches provide a **good quantitative approximation** of the exact virtual coefficient (discrepancy of 5-15%)
  - the soft approximation tends to **undershoot** the exact result while the massification **overshoots** it
  - clear **asymptotic behaviour** towards the exact result for high $p_{T,t}$ where both approximations are expected to perform better (faster convergence of the massification)

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385GeV$, $m_t = 173.2GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- based on the validation at NLO, we define **our best prediction** at NNLO as the **average** of the two approximated results
- **systematic uncertainties** (on each approximation) are estimated as the maximum between what we obtain by varying the subtraction scale $1/2 \leq \mu_{IR}/Q \leq 2$ and twice the NLO deviation
- to be conservative, we linearly combine the uncertainties on the two approximations
- the two-loop contribution turns out to be **6-7%** of the NNLO cross section (both for $t\bar{t}W^+$ and $t\bar{t}W^-$)

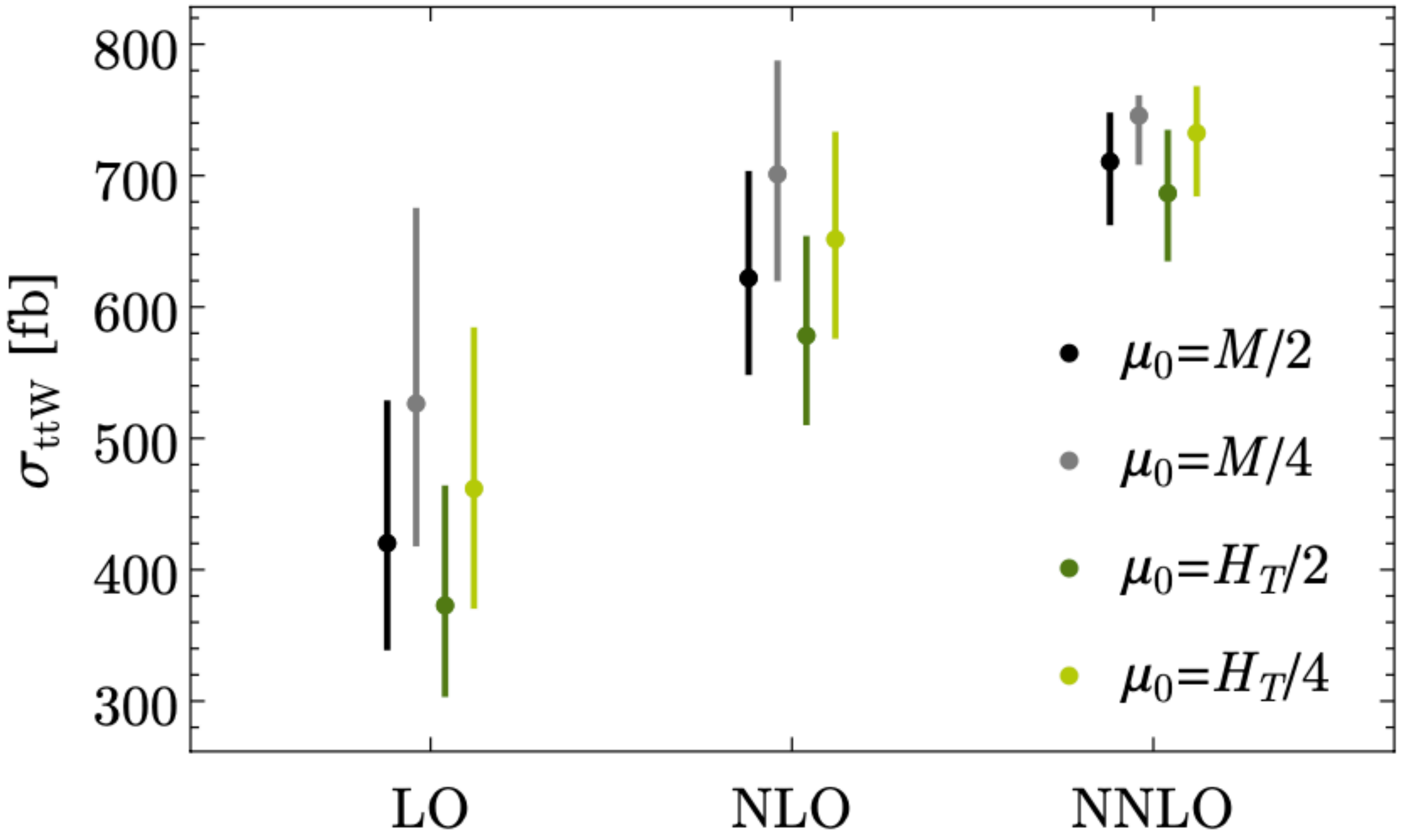
FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO} , $\mathcal{O}(25\%)$ on $\Delta\sigma_{NNLO,H}$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$

- we estimate the **perturbative uncertainties** (due to missing higher orders) on the basis of
  - 7-point scale variation
  - behaviour of the perturbative series
  - choice of different scales: $M/2$, $M/4$, $H_T/2$, $H_T/4$
  - breakdown of the corrections into partonic channels



first signs of convergence starting from NNLO. Lower scales are preferred

no new large contributions from channels opening up at NNLO. NNLO corrections are dominated by gq channel (which is NLO accurate)

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ -6.7\%}$
CMS [10]	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

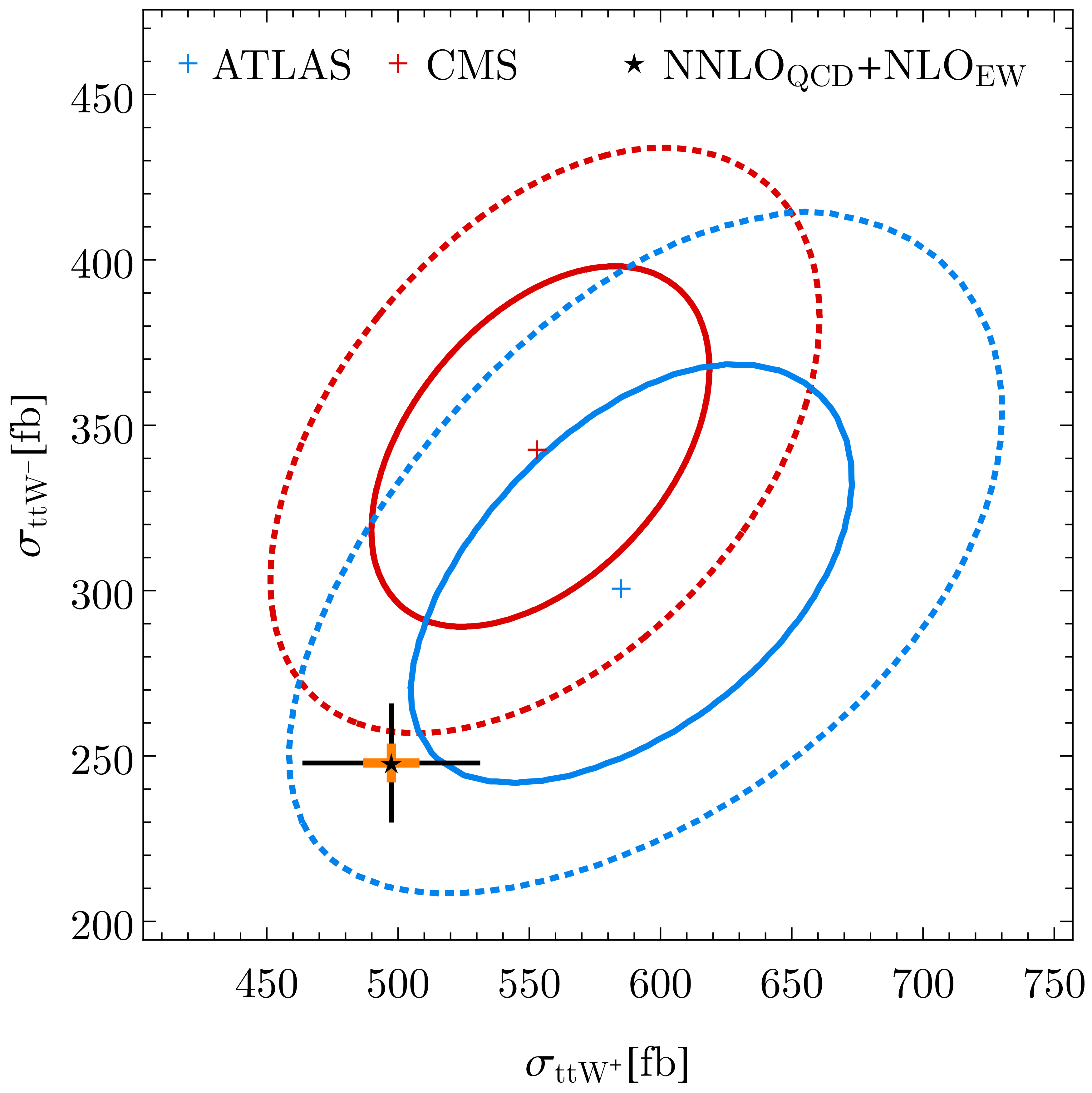
- @NLO QCD: large corrections (**+50%**)
- @NNLO QCD: moderate corrections (**+15%**)
- inclusion of **all subdominant LO and NLO** contributions ($\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s^2\alpha^2)$, $\mathcal{O}(\alpha_s\alpha^3)$, $\mathcal{O}(\alpha^4)$) labelled as NLO EW (**+5%**)
- the ratio $\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-)$ is slightly reduced (very stable perturbative behaviour)

our result is fully compatible with FxFx with smaller perturbative uncertainties !!

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385GeV$, $m_t = 173.2GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



[ATLAS-CONF-2023-019]

[CMS: arXiv 2208.06485]

- comparison against the most recent ATLAS and CMS data:
  - the **agreement is at the** 1σ **and** 2σ **level** respectively
  - reduction of the perturbative scale uncertainties
  - systematic uncertainties due the two-loop approximation are under control and much smaller than the scale uncertainties

take-home message:
two completely different approximations lead to compatible results for the missing two-loop virtual contribution!!

Summary & Outlook

summary:

- the current and expected precision of LHC data requires **NNLO QCD predictions**
- the actual frontier is represented by NNLO corrections for $2 \to 3$ processes with **several massive external legs**
- the IR divergencies are regularised within the q_T **-subtraction** framework: two-loop soft function for arbitrary kinematics
- the only missing ingredient is represented by the **two-loop amplitudes**:
  - first approximation based on a soft boson factorisation formula
  - second approximation based on the massification procedure of the corresponding massless amplitudes
- for all three processes considered ($t\bar{t}H$, $Wb\bar{b}$, $t\bar{t}W$), we have a **good control of the systematic uncertainties** associated to the approximation (much smaller than the perturbative uncertainties)

outlook:

- test the performance of the soft approximation in a fiducial setup and at the differential level
- match the $Wb\bar{b}$ fixed order calculation to parton shower
- explore other processes of the same class!

BACKUP SLIDES

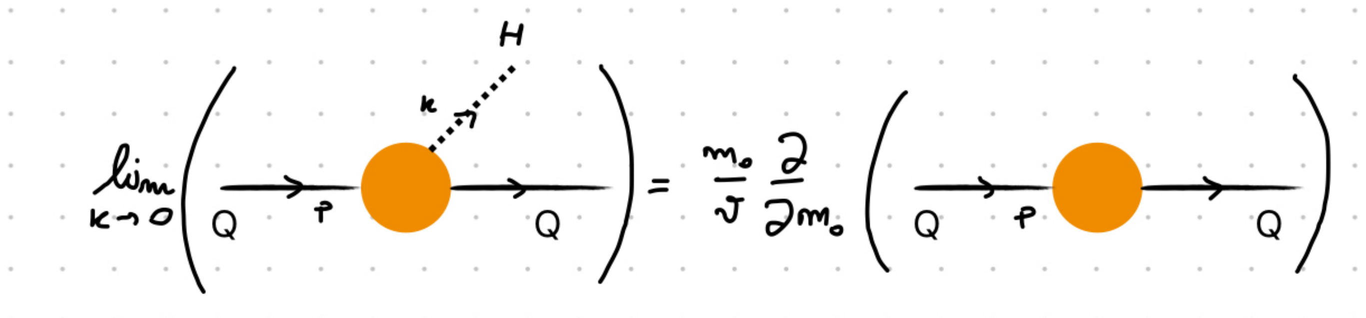
Soft Higgs approximation: more details

[Shifman, Vainshtein, Voloshin, Zakharov (1979)]

- the effective coupling can also be derived by exploiting Higgs **low-energy theorems** (LETs) [Kniehl, Spira (1995)]

$$\lim_{k\to 0} \mathcal{M}^{bare}_{Q\to QH}(p,k) = \frac{1}{v}\frac{\partial}{\partial \log m_0} \mathcal{M}^{bare}_{Q\to Q}(p) \Big|_{p^2=m^2}$$

heavy-quark self-energy



In the soft limit, the Higgs boson is not a dynamical d.o.f.

Its effect is to shift the mass of the heavy quark:

$$m_0 \to m_0\left(1+\frac{H}{v}\right)$$

$$\mathcal{M}^{bare}_{Q\to Q}(p) = \bar{Q}_0 \left\{ m_0[-1+\Sigma_S(p)] + \not{p}\,\Sigma_V(p) \right\} Q_0$$

[Broadhurst, Grafe, Gray, Schilcher (1990)]
[Broadhurst, Gray, Schilcher (1991)]

$$\Sigma_S(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^{\epsilon}}\right]^n \left(A_n(m_0^2/p^2) - B_n(m_0^2/p^2)\right)$$

$$\Sigma_V(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^{\epsilon}}\right]^n B_n(m_0^2/p^2)$$

- renormalisation of the quark mass and wave function $m_0\bar{Q}_0Q_0 = m\bar{Q}QZ_mZ_2$
- $\overline{MS}$ renormalisation of the strong coupling + decoupling of the heavy quark

[Chetyrkin, Kniehl, Steinhauser (1997)]