On the gauge-invariance in SCET beyond leading power

Patrick Hager (MITP, JGU Mainz)

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Patrick Hager (MITP, JGU Mainz)

Based on

[2306.12412]

in collaboration with

Philipp Böer (MITP, JGU Mainz)

Soft-collinear Effective Theory

- SCET has proven itself as a powerful framework for factorisation and resummation in QCD.
- Leading-power SCET: Soft-collinear interactions are eikonal and can be decoupled.
- Next-to-leading power is receiving more attention.
- At NLP new interesting features appear and new problems arise:
  - Endpoint divergences, violation of KSZ, soft-quark emissions, ...
  - Factorisation more complicated: Ideas of "refactorisation".

A curious observation

[Bodwin, Ee, Kang, Wang 2302.05856]

- Consider the sub-subleading manifestly gauge-invariant Lagrangian

$$\mathcal{L}_{\xi q}^{(2)} = \overline{q} W_c^\dagger \big(i n_- D + i \not{D}_\perp \frac{1}{i n_+ D} i \not{D}_\perp \big) \frac{\not{n}_+}{2} \xi + \big[\overline{q} \overleftarrow{D}_s^\mu \big] x_{\perp\mu} W_c^\dagger i \not{D}_\perp \xi + \text{h.c.}$$

- Gauge-invariance implemented through the collinear Wilson line

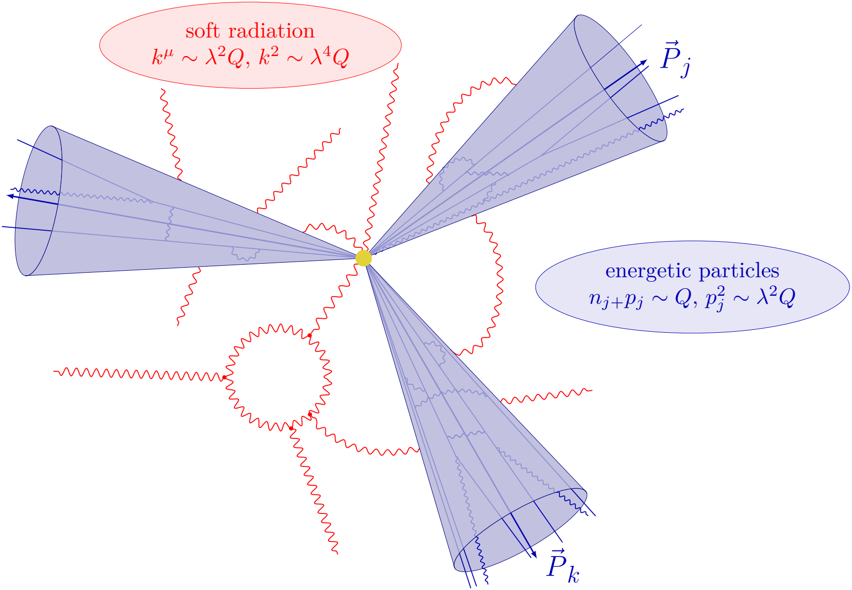
$$W_c(x) = \mathrm{P} \exp\Big(i g_s \int_{-\infty}^{0} ds\, n_+ A_c(x + s n_+) \Big)$$

- Pick out any individual term to compute the "radiative jet function"

$$\int d^4x \, \langle q_s'(\ell) \overline{q}_c'(p_2) q_c(p_1) | \, \mathcal{T}\big(J^{A0}(0), \mathcal{L}_{\xi q}^{(2)}(x) \big) \, |0\rangle$$

- Observation: Result is not gauge invariant. What is going wrong?

Kinematics


soft radiation
$k^\mu \sim \lambda^2 Q$, $k^2 \sim \lambda^4 Q$
$\vec{P}_j$
$\vec{P}_i$
energetic particles
$n_{j+}p_j \sim Q$, $p_j^2 \sim \lambda^2 Q$
$\vec{P}_k$

How to construct SCET

- Power-counting parameter $\lambda = |\frac{p_\perp}{n_+ p}| \ll 1$.
  [Bauer et al. hep-ph/0011336, hep-ph/0109045, hep-ph/0202088;]

- Introduce the mode split $\psi \sim \xi + \eta + q$, $A_\mu = A_{c\mu} + A_{s\mu}$
- Power counting:

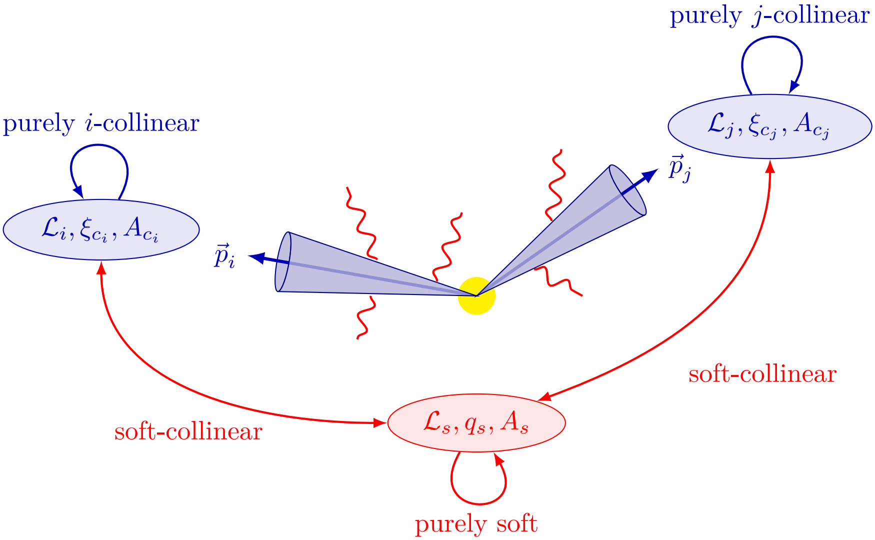
$$\xi \sim \lambda\,, \quad \eta \sim \lambda^2\,, \quad q \sim \lambda^3$$
$$(n_+ A_c, A_{c\perp}, n_- A_c) \sim (1, \lambda, \lambda^2)\,, \quad A_s \sim \lambda^2$$

- Control large field component $n_+ A_c$ through the Wilson line W_c.

- Multipole-expand the soft fields in soft-collinear products
  [Beneke, Chapovsky, Diehl, Feldmann hep-ph/0206152; Beneke, Feldmann hep-ph/0211358]

$$\phi_s(x) = \phi_s(x_-) + (x - x_-) \cdot \partial \phi_s + \dots$$

Intuitive Picture



Leading-power SCET

$$\mathcal{L}^{(0)} = \bar{\xi}\Big(in_-D + i\not{D}_{c\perp}\frac{1}{in_+D_c}i\not{D}_{c\perp}\Big)\frac{\not{n}_+}{2}\xi + \bar{q}i\not{D}_s q$$

- Soft-collinear interactions are mediated only through $n_-A_s \subset in_-D$.
- This eikonal interaction can be decoupled using soft Wilson lines.
- Gauge-invariance is manifest in each term, no subtleties.
- Beyond LP: two new types of terms
  - Non-eikonal coupling to gluon and soft quark
  - Multipole corrections to existing terms

More technical: the subleading SCET construction

- Start from the QCD Lagrangian in (collinear) light-cone gauge.
- Insert the soft and collinear modes

$$\mathcal{L}^{(1)}_{\xi q} = \overline{q} i \partial\!\!\!/_{\perp} \xi + \overline{q} i A\!\!\!/_{c\perp} \xi + \text{h.c.}$$

- Enforce "momentum-conservation":
  - Lagrangian contains terms of the form $\phi_s^n(x)\phi_c(x)$
  - To remove these terms: employ soft (background) equations of motion
  - This is a background field construction, collinear fluctuation

More technical: the subleading SCET construction

- Result: Lagrangian in collinear light-cone gauge, without unphysical terms

$$\mathcal{L}_{\xi q}^{(1)} = \bar{q}\not{A}_{c\perp}\xi + \text{h.c.}$$

$$\mathcal{L}_{\xi q}^{(2)} = \bar{q}n_- A_c \frac{\not{n}_+}{2}\xi + \bar{q}\not{A}_{c\perp}\frac{1}{in_+\partial}(i\not{\partial}_\perp + \not{A}_{c\perp})\frac{\not{n}_+}{2}\xi + [\bar{q}\overleftarrow{D}_s^\mu]x_{\perp\mu}\not{A}_{c\perp}\xi + \text{h.c.}$$

- "Unfix" light-cone gauge: introduce gauge-invariant building blocks

$$\chi = W_c^\dagger \xi\,, \quad \mathcal{A}_\perp^\mu = W_c^\dagger i D_\perp^\mu W_c - i\partial_\perp^\mu\,, \quad n_-\mathcal{A} = W_c^\dagger i n_- D W_c - i n_- D_s$$

and simply replace $\xi \to \chi$, $A_c \to \mathcal{A}$

- Yields the manifestly gauge-invariant "building-block" Lagrangian

$$\mathcal{L}_{\xi q}^{(1)} = \bar{q}\not{\mathcal{A}}_\perp\chi + \text{h.c.}$$

$$\mathcal{L}_{\xi q}^{(2)} = \bar{q}n_-\mathcal{A}\frac{\not{n}_+}{2}\chi + \bar{q}\not{\mathcal{A}}_\perp\frac{1}{in_+\partial}(i\not{\partial}_\perp + \not{\mathcal{A}}_\perp)\frac{\not{n}_+}{2}\chi + [\bar{q}\overleftarrow{D}_s^\mu]x_{\perp\mu}\not{\mathcal{A}}_\perp\chi + \text{h.c.}$$

The covariant Lagrangian

[Beneke, Chapovsky, Diehl, Feldmann hep-ph/0206152, Beneke, Feldmann hep-ph/0211358]

- Re-express the Lagrangian through the fundamental fields ξ, A_c

$$\mathcal{L}_{\xi q}^{(1)} = \overline{q} W_c^\dagger i\not{D}_\perp \xi - \overline{q} i\not{\partial}_\perp W_c^\dagger \xi + \text{h.c.}\,,$$

$$\mathcal{L}_{\xi q}^{(2)} = \overline{q} W_c^\dagger \big(in_- D + i\not{D}_\perp \frac{1}{in_+ D} i\not{D}_\perp \big) \frac{\not{n}_+}{2} \xi + \big[\overline{q} \overleftarrow{D}_s^\mu \big] x_{\perp\mu} W_c^\dagger i\not{D}_\perp \xi$$
$$- \Big(\overline{q} in_- D_s \frac{\not{n}_+}{2} + \big[\overline{q} \overleftarrow{D}_{s\perp}^\mu \big] x_{\perp\mu} i\not{\partial}_\perp \Big) W_c^\dagger \xi + \text{h.c.}$$

- The additional terms vanish either by total derivatives or through the soft-quark equations of motion.

- Gauge-invariance in the covariant Lagrangian is now subtle, as this Lagrangian contains unphysical interaction terms.

Gauge-invariance of the subleading Lagrangian

[Böer, PH, 2306.12412]

- Consider the first term in the Lagrangian

$$\overline{q} W_c^\dagger i n_- D \frac{\not{n}_+}{2} \xi = \overline{q} i n_- D_s \frac{\not{n}_+}{2} \xi + \overline{q} g_s n_- A_c \frac{\not{n}_+}{2} \xi + \overline{q} (W_c^\dagger - 1) i n_- D \frac{\not{n}_+}{2} \xi \,.$$

- All three pieces are required for invariance, but the first is unphysical.

- "Standard prescription": Drop unphysical terms from the Lagrangian in any computation.

- This procedure is not gauge-invariant.

- Does this matter?

A formal justification

- Dropping unphysical terms is again justified by the background field method
- "Standard prescription": Fluctuation ξ on top of soft background.

$$\mathcal{L}^{(2)}_{\xi q,\mathrm{unphys}} = -\Big(\overline{q} i n_- \overleftarrow{D}_s \frac{\not{n}_+}{2} + [\overline{q} i \overleftarrow{\not{D}}_{s\perp}] + m_q \overline{q}\Big)\xi\,,$$

- "Gauge-invariant prescription": Fluctuation $\chi = W_c^\dagger \xi$ instead

$$\mathcal{L}^{(2)}_{\xi q,\mathrm{unphys}} = -\Big(\overline{q} i n_- \overleftarrow{D}_s \frac{\not{n}_+}{2} + [\overline{q} i \overleftarrow{\not{D}}_{s\perp}] + m_q \overline{q}\Big) W_c^\dagger \xi\,,$$

  This results in the building block Lagrangian.

- For any sensible computation, both prescriptions agree, as they differ by soft-quark equations of motion.
- To avoid this problem entirely: Define radiative jet functions through a matching equation or use building blocks.

Conclusion

- Gauge-invariance in the covariant form of the SCET Lagrangian is subtle.
- Unphysical terms are required for manifest invariance but are dropped in computations.
- Insertions of individual pieces of the subleading Lagrangian are thus not necessarily gauge-invariant but can differ by soft equations of motion.
- Individual insertions are not physical, only the sum of all allowed vertices is.
- For any physical observable, this is therefore not an issue.
- Avoid this problem by defining the radiative jet function through a matching equation or by employing the building blocks.