Probing new physics through entanglement

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University of Edinburgh

THE UNIVERSITY *of* EDINBURGH

BNL Theory Seminar - 2023

Based on

Quantum SMEFT tomography: top quark pair production at the LHC

RA, Eric Madge, Fabio Maltoni and Luca Mantani

hep-ph/2203.05619

Phys. Rev. D 106 (2022) 5, 055007

Probing new physics through entanglement in diboson productions

RA, Eric Madge, Fabio Maltoni and Luca Mantani

hep-ph/2307.09675

Accepted in JHEP

"I would not call [entanglement] one but rather the characteristic trait of quantum mechanics"

- E. Schrodinger

Fresh new!

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

ATLAS EXPERIMENT

Submitted to: Nature

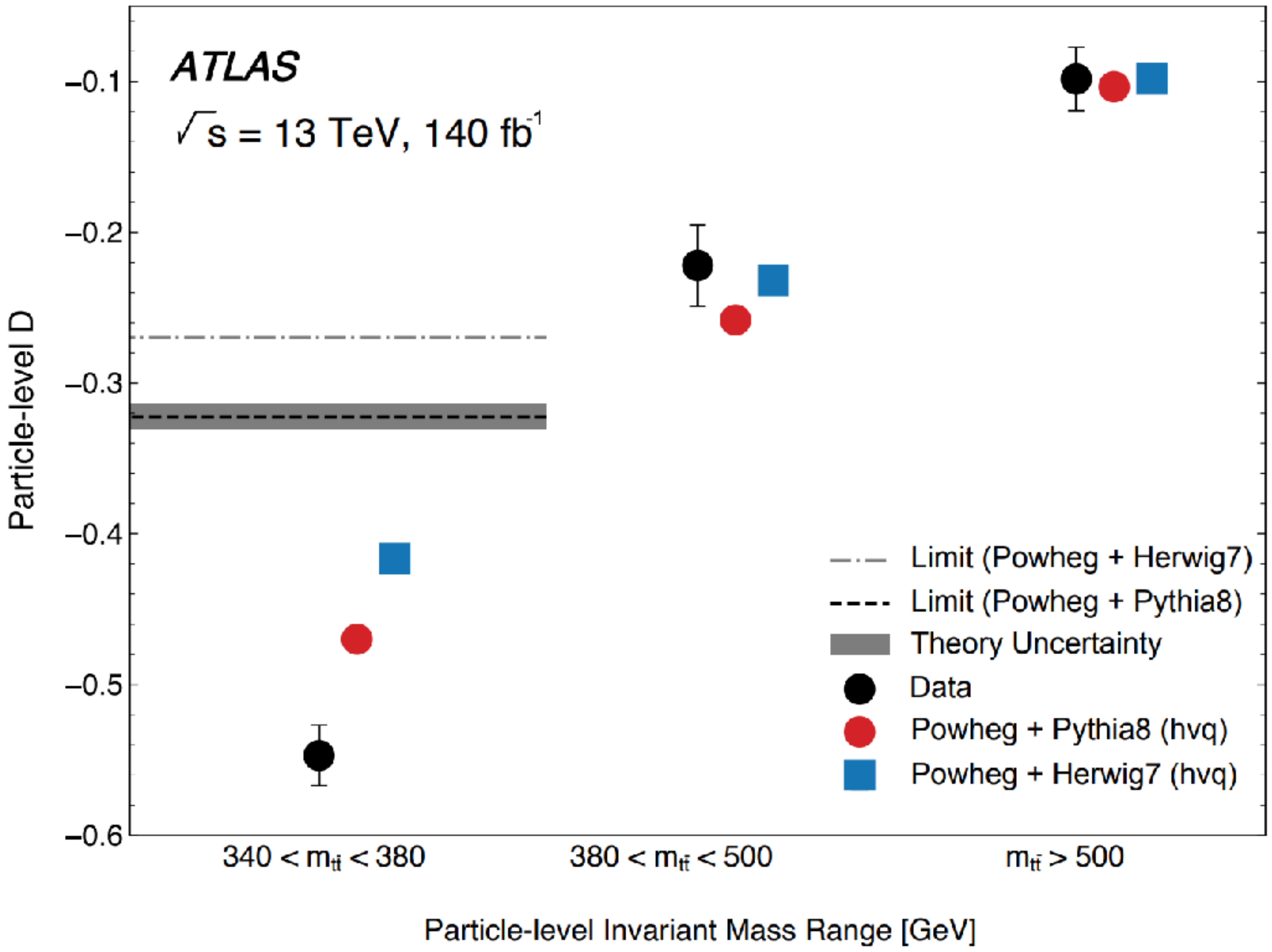
CERN-EP-2023-230
November 20, 2023

Observation of quantum entanglement in top-quark pairs using the ATLAS detector

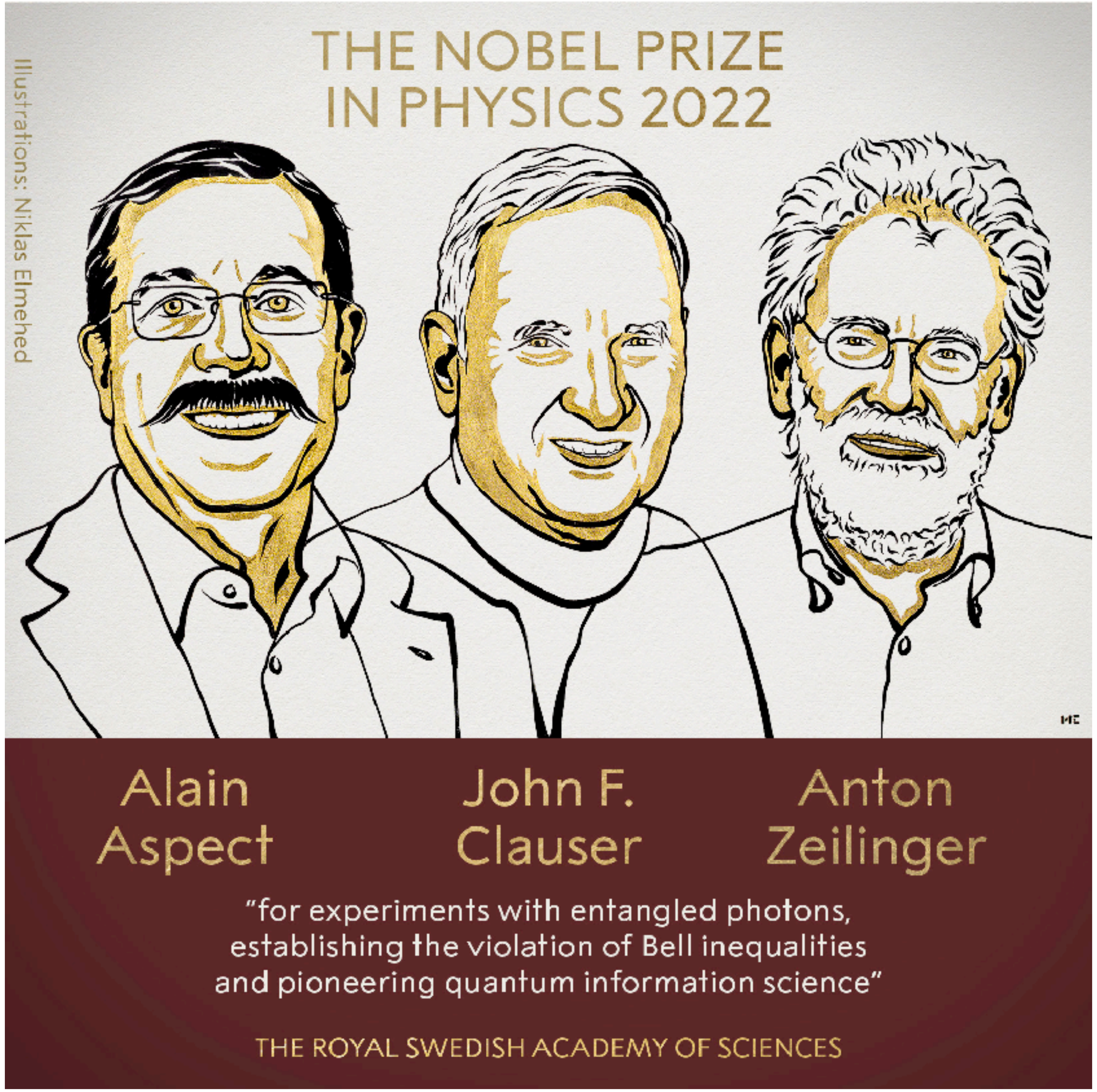
The ATLAS Collaboration

We report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision data set with a center-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 fb^{-1} recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top–antitop quark production threshold, where the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.547 \pm 0.002$ (stat.) $\pm$ 0.021 (syst.) for $340 < m_{t\bar{t}} < 380$ GeV. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes both the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement to date.

Fresh new!



D < -1/3 ⟷ *entangled*


THE NOBEL PRIZE
IN PHYSICS 2022
Illustrations: Niklas Elmehed
Alain
Aspect
John F.
Clauser
Anton
Zeilinger
"for experiments with entangled photons,
establishing the violation of Bell inequalities
and pioneering quantum information science"
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Fastly growing sub-field (since ’21)

Bipartite

Top-quark

[Afik and de Nova, ’21]
[Fabbrichesi, Floreanini, Panizzo, ’21]
[Severi, Degli, Maltoni, Sioli, ’21]
[Aoude, Madge, Maltoni, Mantani, ’22]
[Afik and de Nova, ’22]
[Aguilar-Saavedra, Casas, ’22]
[Fabbrichesi, Floreanini, Gabrielli, ’22]
[Severi, Vryinidou, ’22]
[Dong, Gonçalves, Kong, Navarro ’23]
[Aguilar-Saavedra, Casas, ’23]
[Han, Low, Wu ’23]
…

Diboson

[Barr, ’21]
[Barr, Caban, Rembielisnki, ’22]
[Aguilar-Saavedra et al ’22]
[Ashby-Pickering, Barr Wierchucka ’22]
[Fabbrichesi et al ’23]
[Aoude, Madge, Maltoni, Mantani, ’23]
[Aguilar-Saavedra ’23]
[Morales ’23]
[Altomonte, Barr ’23]
[Fabbri, Howarth, Maurin, ’23]
[Bernal, Caban, Rembielinski ’23]
[Bi, Qing-Hong Cao, Cheng, Zhang ’23]
…

Tripartite+

[Bernal, ’23]
[Sakurai, Spannowsky, ’23]

+more

$B^0 \to J/\psi K^*$

[Fabbrichesi et al ’23]
…

+ 2 specialised workshops

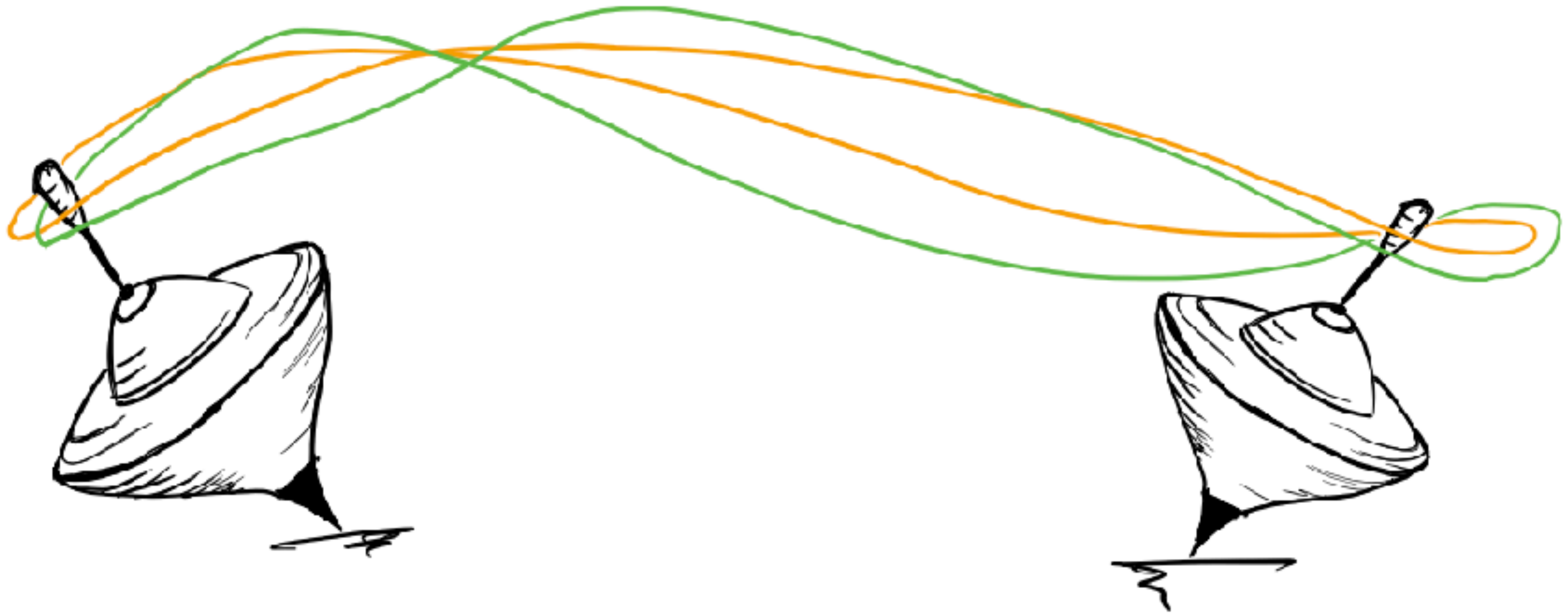
Foundational tests of Quantum Mechanics at the LHC

Mar 20-23, 2023

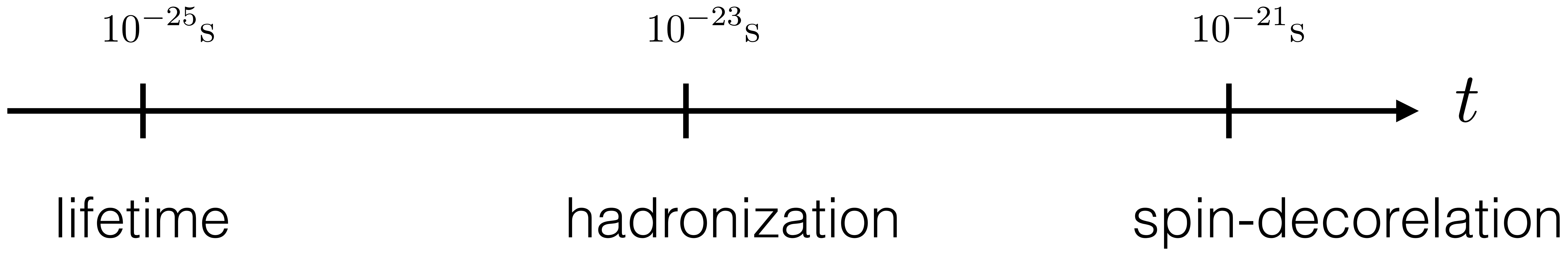
Quantum Observables for Collider Physics

Nov 6-10, 2023

Top-pair entanglement



Why top quarks?



- allows to efficiently reconstruct the spin from decay products

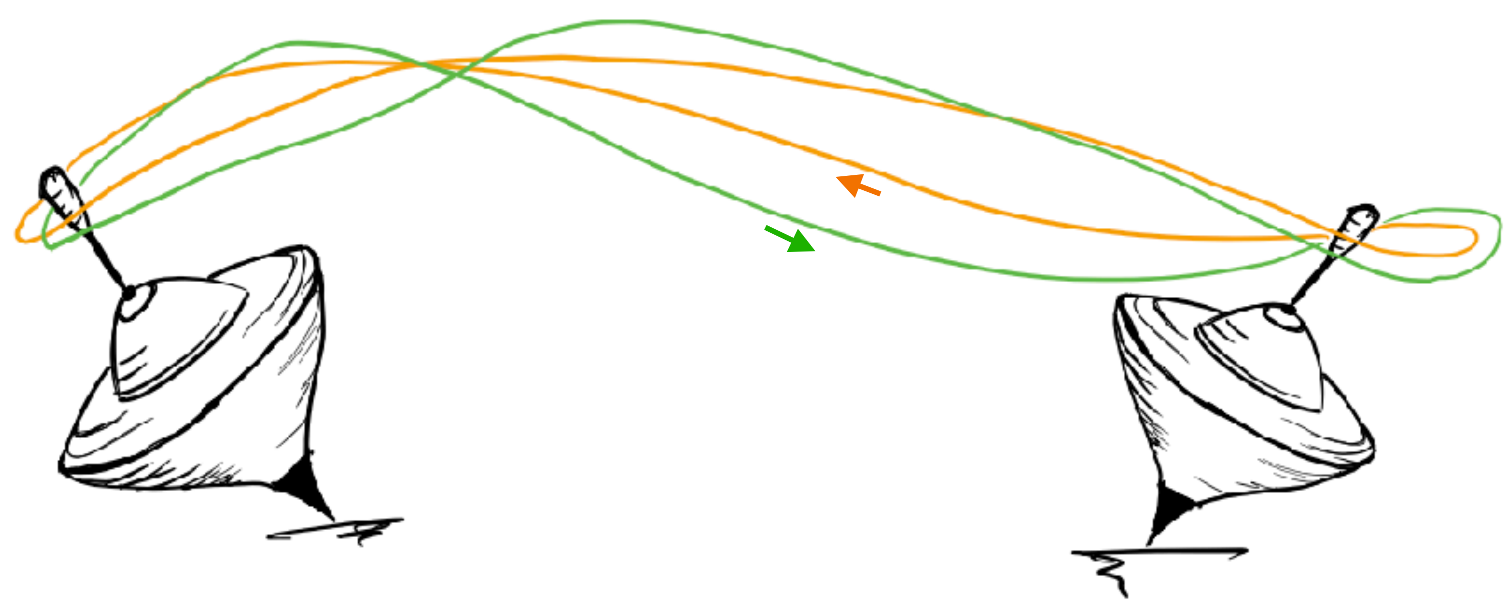
- top spin correlations vastly studied:

  D0 and CDF at Tevatron and ATLAS and CMS at LHC

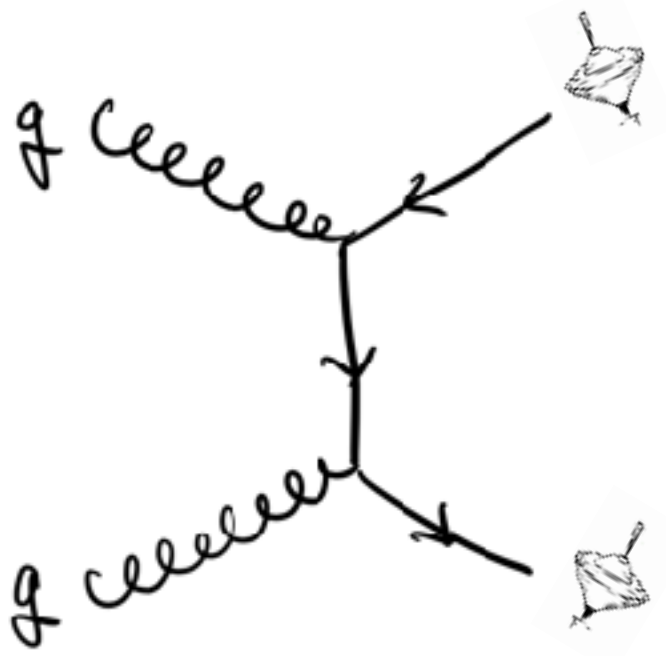
- Observation from ATLAS

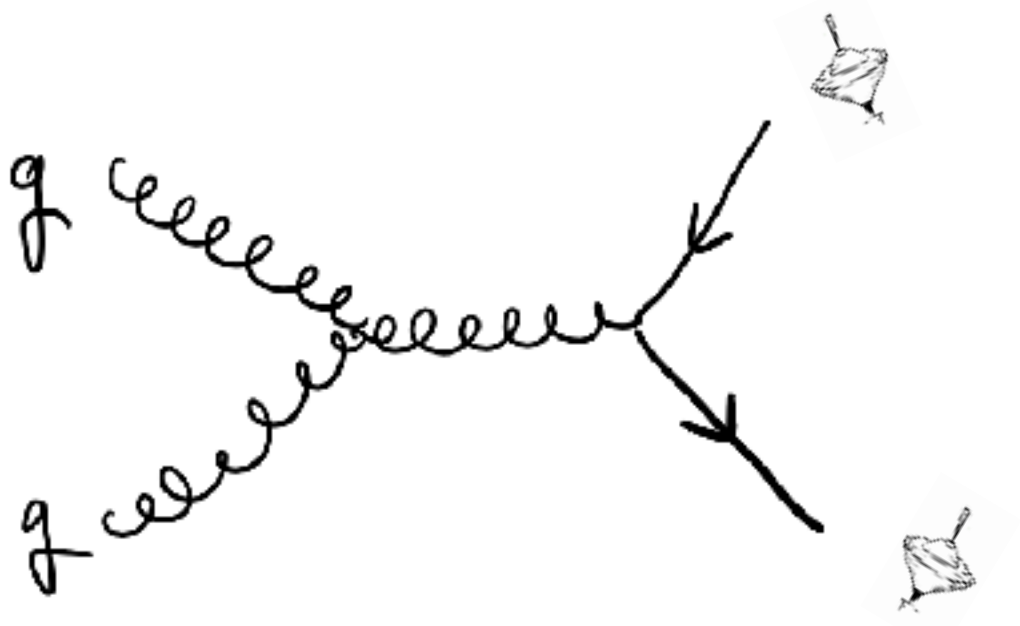
Entanglement in spin-space

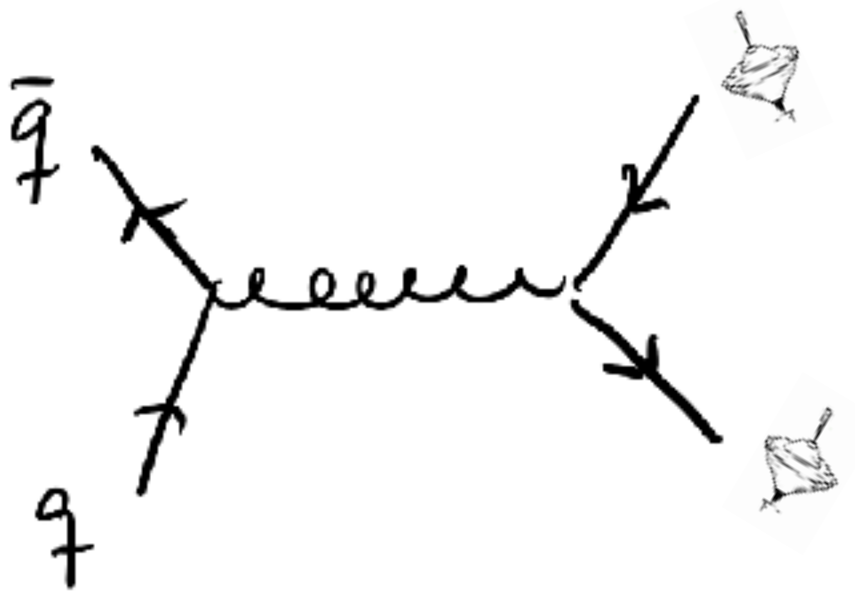
We want the entanglement in the top pair spin



SM:







for instance: $$\frac{|t(\uparrow)\bar{t}(\uparrow)\rangle + |t(\downarrow)\bar{t}(\downarrow)\rangle}{\sqrt{2}}$$ would be a maximal entangled state

Spin production density matrix

[Afik and de Nova, 21']

The state-density matrix is obtained from the R-matrix

$$R^{I}_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^{*}_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

where $\mathcal{M}_{\alpha\beta} \equiv \langle t(k_1,\alpha)\bar{t}(k_2,\beta)|\mathcal{T}|a(p_1)b(p_2)\rangle$

$I = gg, q\bar{q}$

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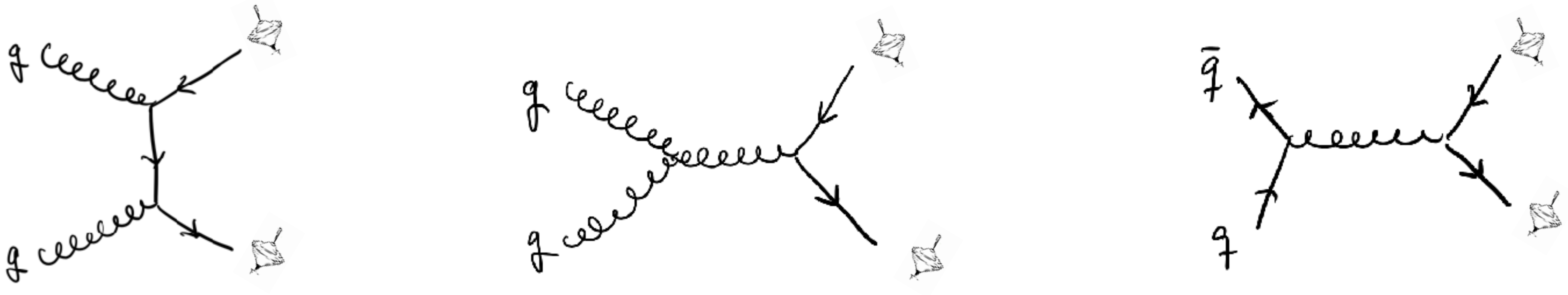

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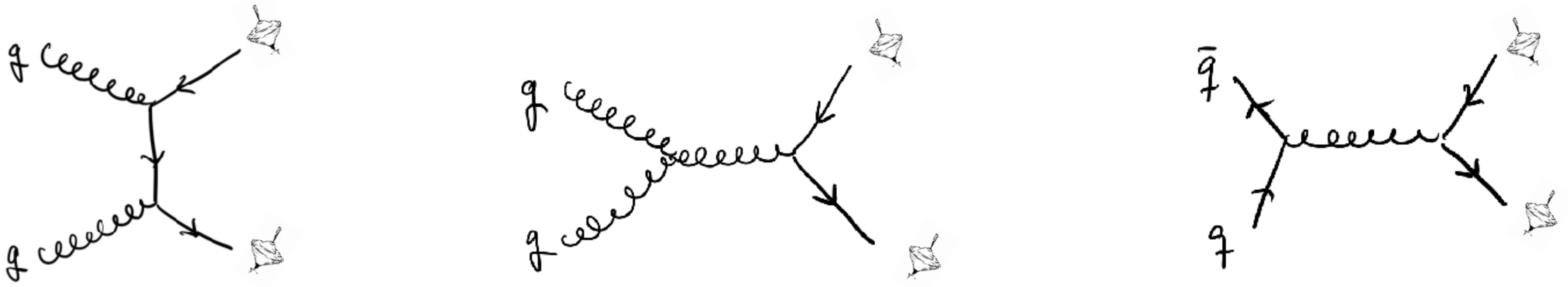

Mixed state of qq and gg initiated channels, weighted by the luminosity functions

$$R(\hat{s}, \boldsymbol{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s}, \boldsymbol{k})$$

Spin production density matrix

[Afik and de Nova, 21']

4x4 matrix in spin-space of the top pair.

Fano decomposition: (spanned by tensor prod. of Pauli and Identity)

$$R = \tilde{A}\,\mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^{+}\,\sigma^i \otimes \mathbb{1}_2 + \tilde{B}_i^{-}\,\mathbb{1}_2 \otimes \sigma^i + \tilde{C}_{ij}\,\sigma^i \otimes \sigma^j\,.$$

16-coefficients where the norm $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}\hat{s}} = \dfrac{\alpha_s^2\beta}{\hat{s}^2}\tilde{A}\left(\hat{s}, \boldsymbol{k}\right)$

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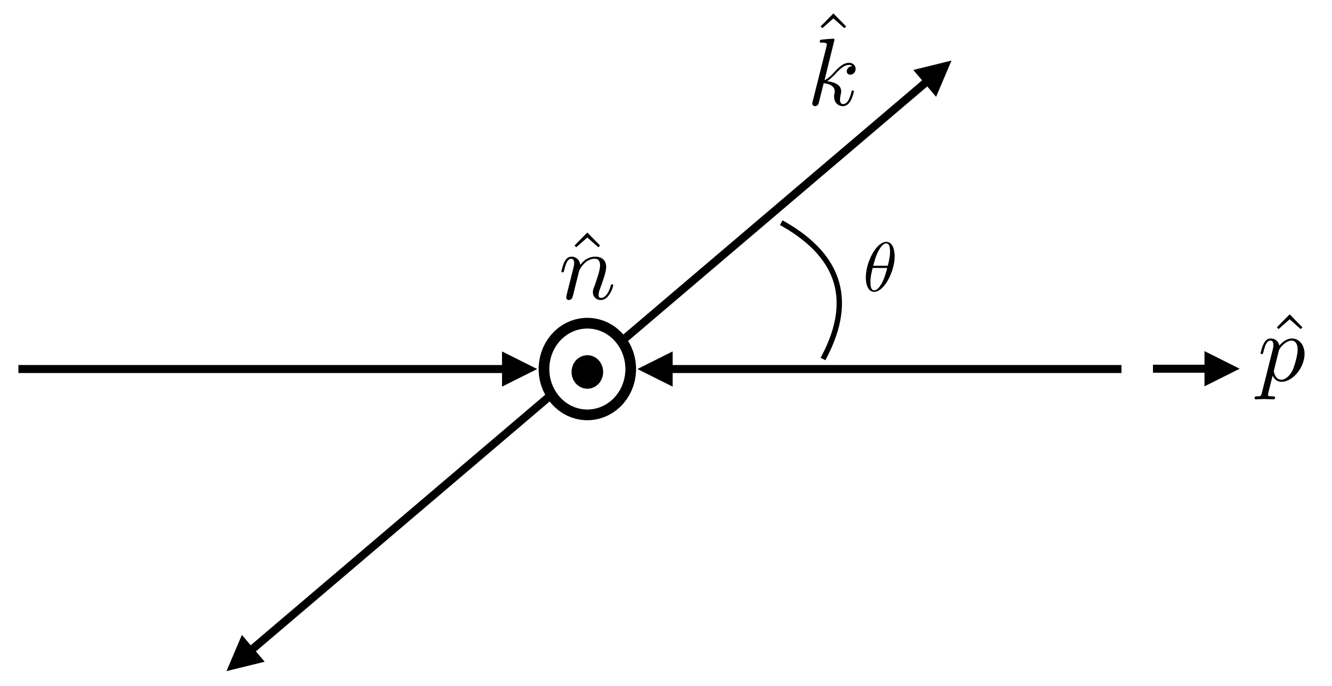
Normalize the state as $\rho = R/\mathrm{tr}(R)$

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^{+} \sigma^i \otimes \mathbb{1}_2 + B_i^{-} \mathbb{1}_2 \otimes \sigma^i + C_{ij}\, \sigma^i \otimes \sigma^j}{4} .$$

Density matrix and helicity-basis

Helicity basis:

$$\{\boldsymbol{k}, \boldsymbol{n}, \boldsymbol{r}\} : \ \boldsymbol{r} = \frac{(\boldsymbol{p} - z\boldsymbol{k})}{\sqrt{1-z^2}}, \quad \boldsymbol{n} = \boldsymbol{k} \times \boldsymbol{r},$$



To expand in this basis, e.g.

$$C_{nn} = \mathrm{tr}[C_{ij}\, \boldsymbol{n} \otimes \boldsymbol{n}\,]$$

Phase-space parametrized by: $\beta^2 = (1 - 4m_t^2/\hat{s})$ and $z = \cos\theta$

R-matrix coefficients at LO QCD

1. CP invariance ⟶ C_{ij} symmetric and $B_i^+ = B_i^-$
2. $C_{kn}, C_{rn}, B_n^\pm$ only generated at one-loop
3. $B_k^\pm \; B_r^\pm$ require P-odd ⟶ vanish for QCD

For gg-initiated SM

$$F_{gg} = \frac{7+9\beta^2 z^2}{192(1-\beta^2 z^2)^2}$$

$$\begin{aligned}
\tilde{A}^{gg,(0)} &= F_{gg}\big(1 + 2\beta^2(1-z^2) - \beta^4(z^4 - 2z^2 + 2)\big),\\
\tilde{C}_{nn}^{gg,(0)} &= -F_{gg}\big(1 - 2\beta^2 + \beta^4(z^4 - 2z^2 + 2)\big),\\
\tilde{C}_{kk}^{gg,(0)} &= -F_{gg}\big(1 - 2z^2(1-z^2)\beta^2 - (2 - 2z^2 + z^4)\beta^4\big),\\
\tilde{C}_{rr}^{gg,(0)} &= -F_{gg}\big(1 - (2 - 2z^2 + z^4)\beta^2(2-\beta^2)\big),\\
\tilde{C}_{rk}^{gg,(0)} &= F_{gg}\, 2z\,(1-z^2)^{3/2}\beta^2\sqrt{1-\beta^2},
\end{aligned}$$

Entanglement in bipartite systems

Given a bipartite system $\mathcal{H}_{\mathrm{ab}} = \mathcal{H}_{\mathrm{a}} \otimes \mathcal{H}_{\mathrm{b}}$

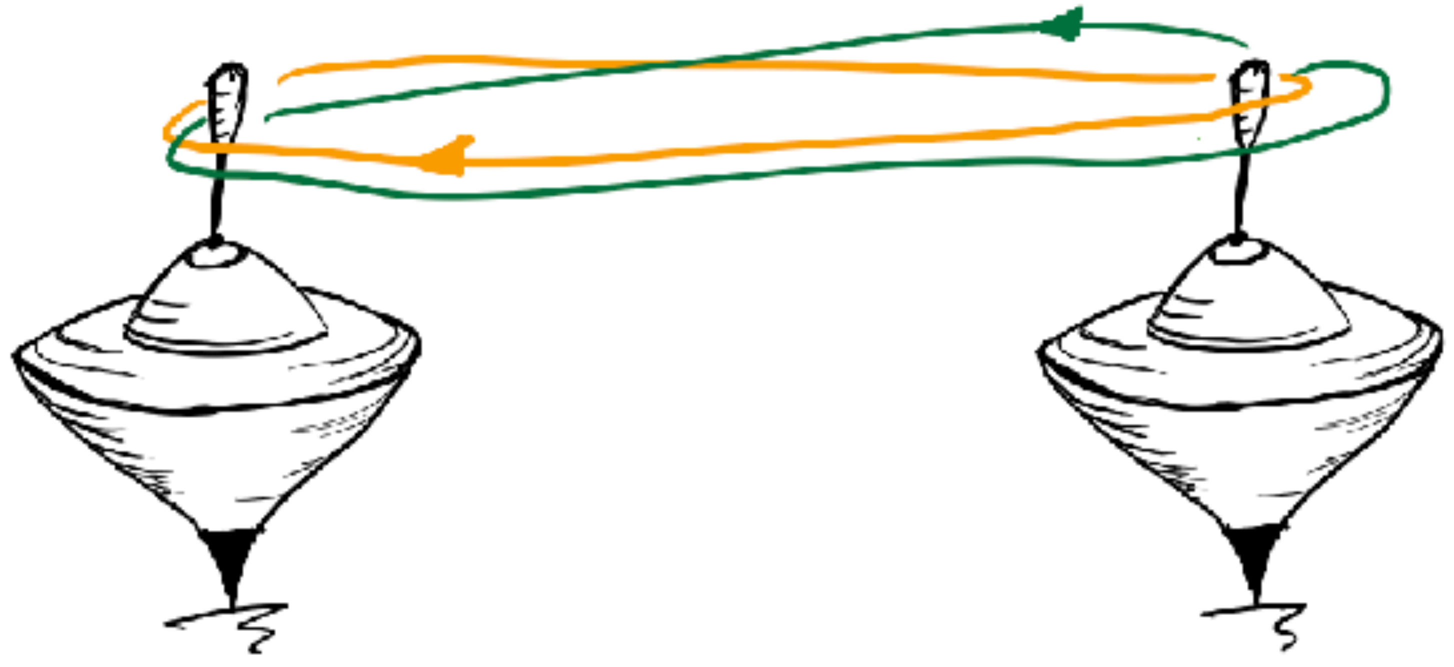
Can you write $|\Psi_{\mathrm{ab}}\rangle = |\Psi_{\mathrm{a}}\rangle \otimes |\Psi_{\mathrm{b}}\rangle$?

No? Then it is entangled.

Or more generally as product (mixed states): $\rho_{\mathrm{ab}} = \sum_k p_k \, \rho_{\mathrm{a}}^k \otimes \rho_{\mathrm{b}}^k$

Maximally entangled states (e.g Bell states):

$|\Phi^{\pm}\rangle = \dfrac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}}$ or $|\Psi^{\pm}\rangle = \dfrac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$



Entanglement in bipartite systems

Entanglement measures are more useful:

- Peres-Horodecki Criterion: $\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$

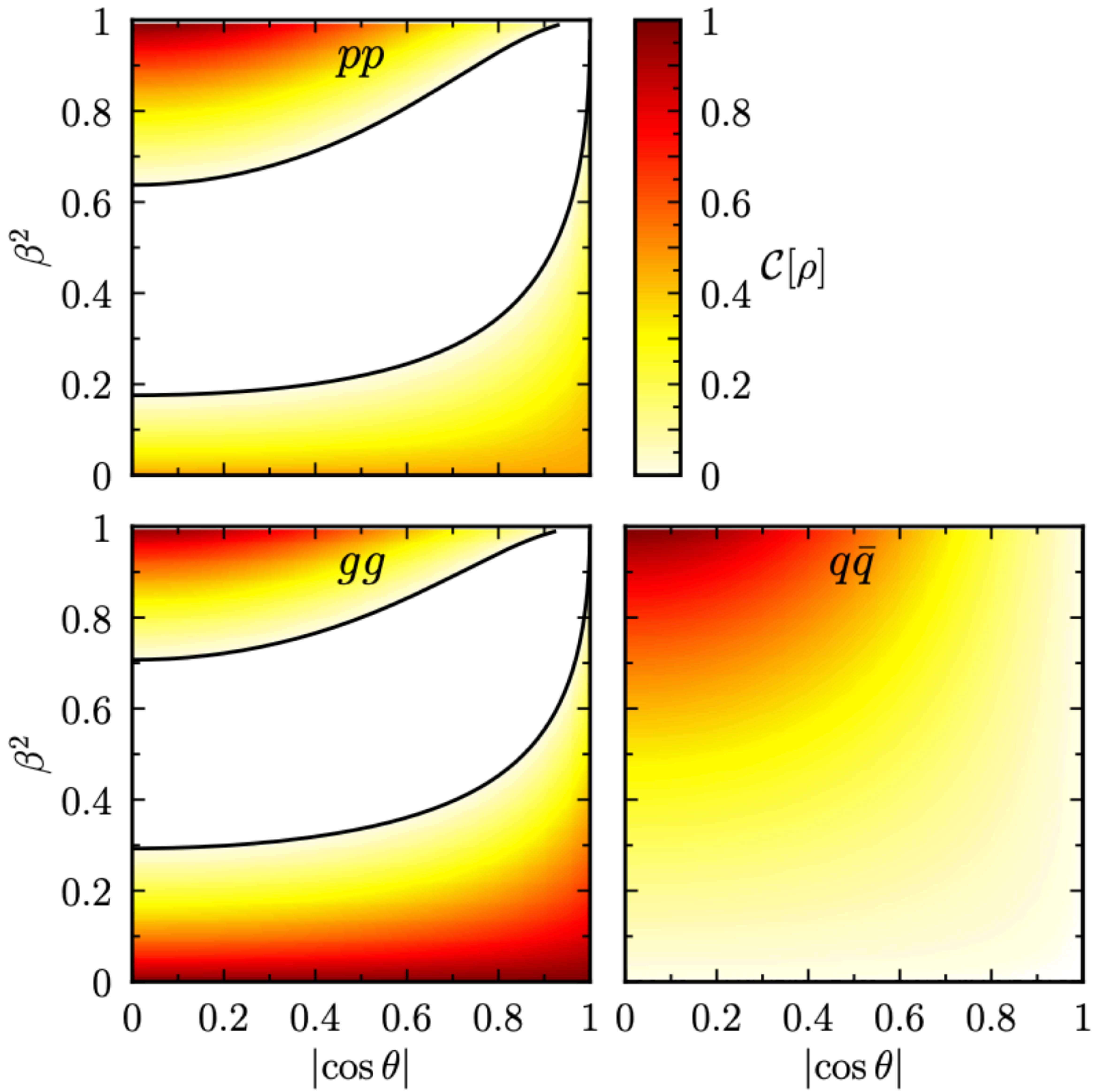
  (in the helicity-basis) (entangled)

- Concurrence: $C[\rho] = \max(\Delta/2, 0)$

  $C[\rho] = 1$ (maximally entangled)

What's the story for the SM?

[Afik and de Nova, 21']



White regions: zero-entanglement

Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$
- high-E: $\beta^2 \to 1, \cos\theta = 0$

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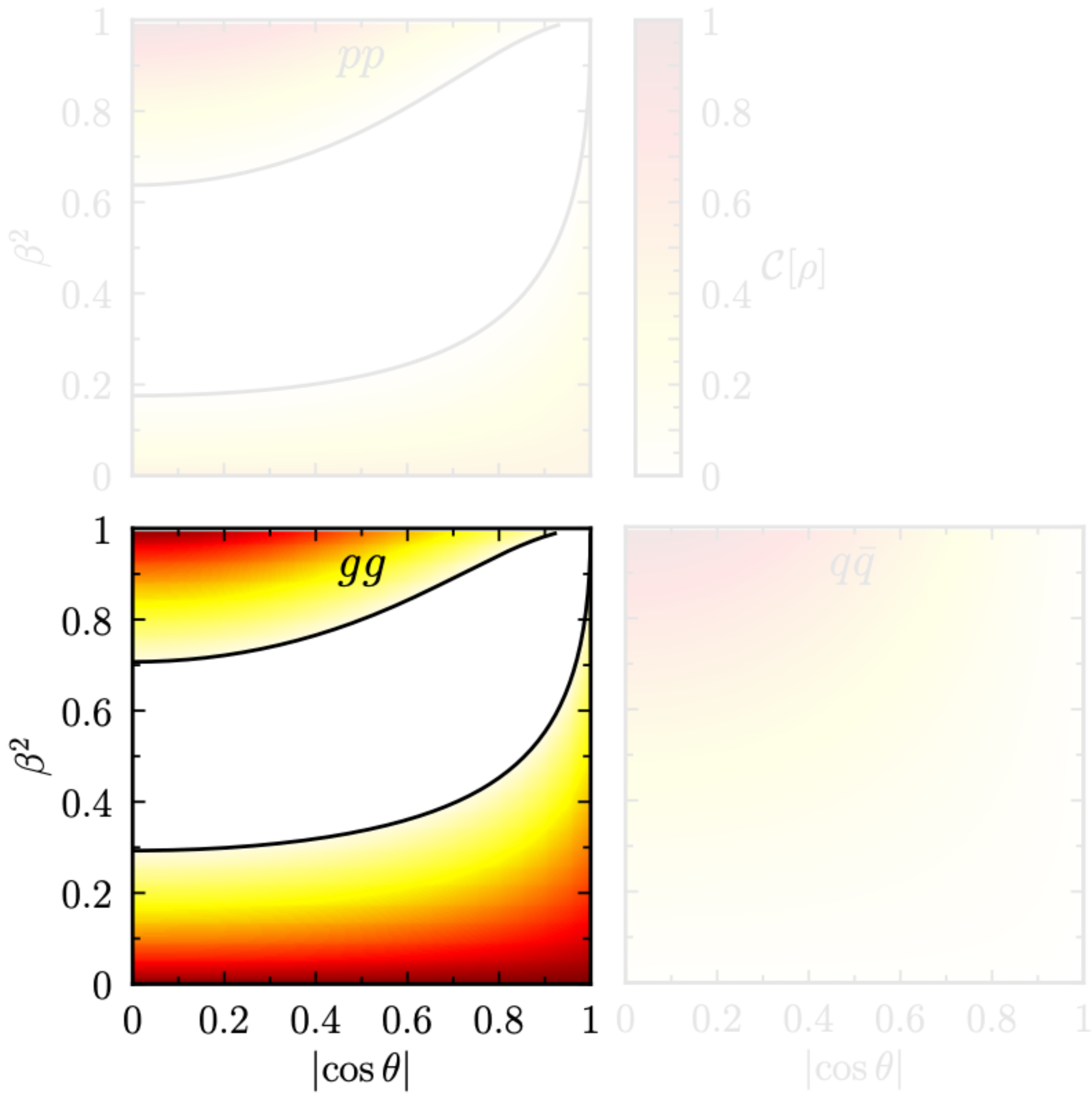


Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$

(singlet)

$$\rho_{gg}^{\text{SM}}(0, z) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

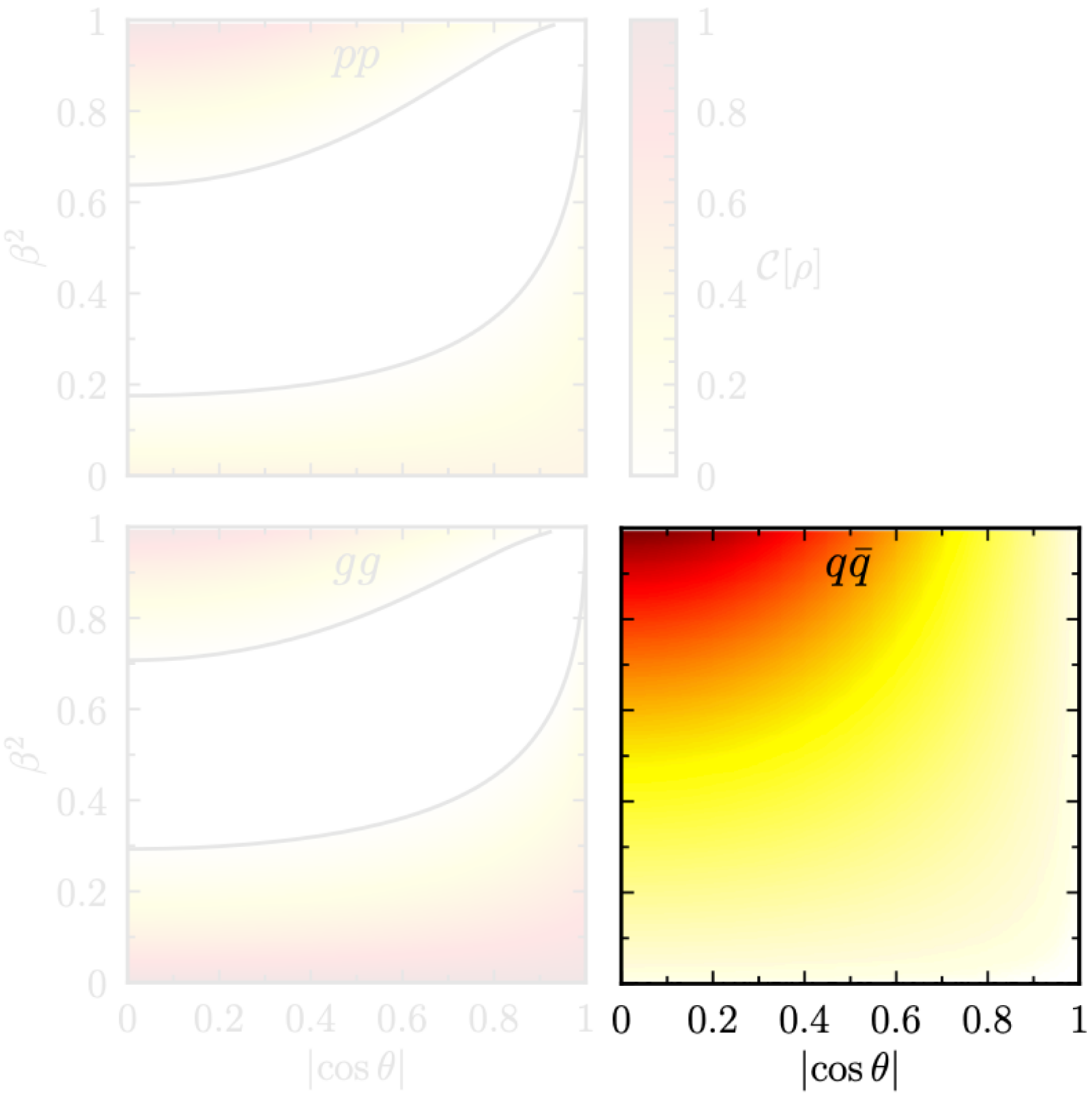
- high-E: $\beta^2 \to 1, \cos\theta = 0$

(triplet)

$$\rho_{gg}^{\text{SM}}(1, 0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}$$

What's the story for the SM?

[Afik and de Nova, 21']



Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$

  mixed but separable

- high-E: $\beta^2 \to 1, \cos\theta = 0$

  (triplet: same as gg)

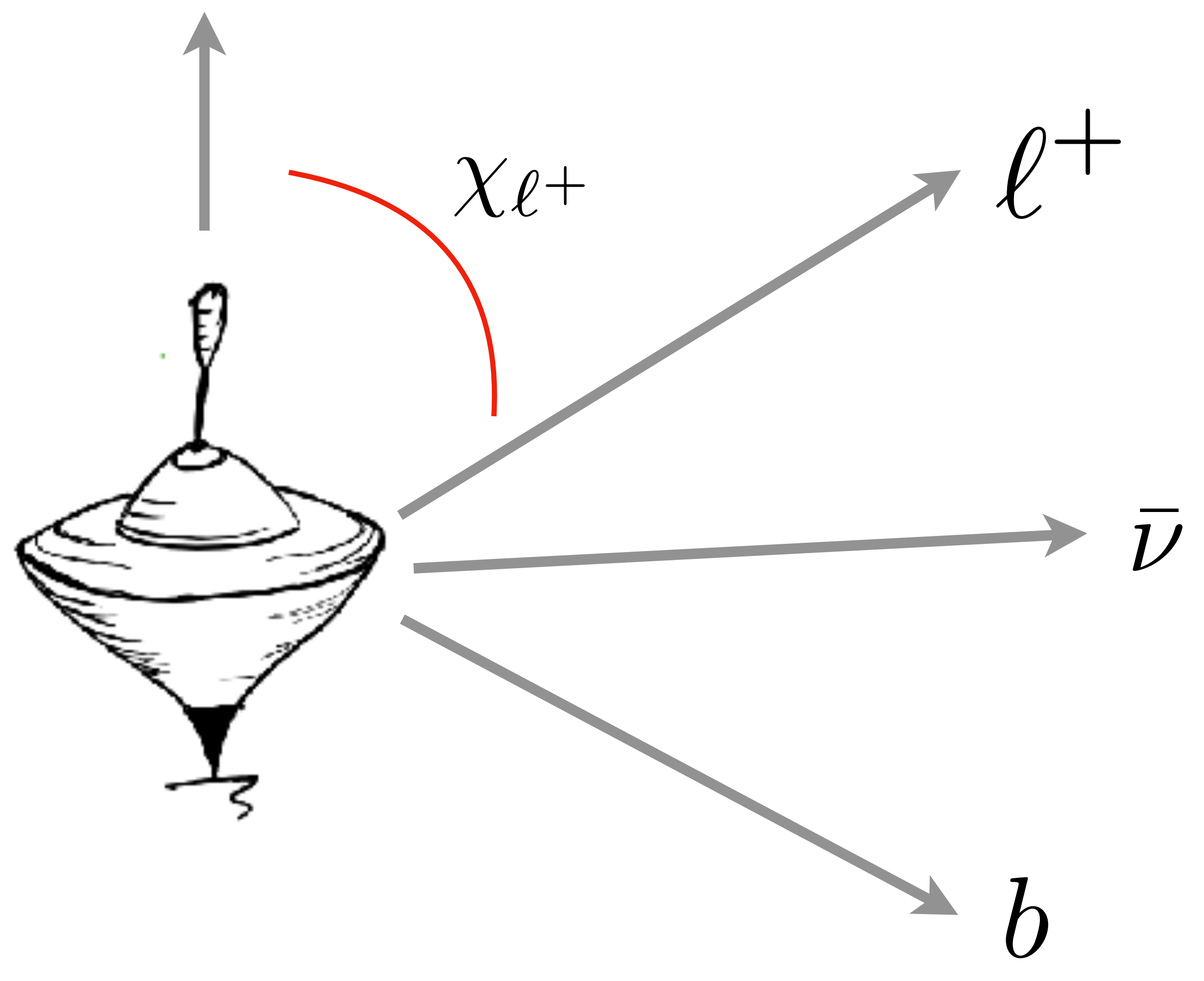
$$\rho^{\mathrm{SM}}_{q\bar{q}}(1,0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}.$$

Tomography and reconstruction of the state

[Mahlon, Parke 10']
[Baumgart, Tweede 12']
[Afik and de Nova, 21']

$\Gamma_t \sim 1\mathrm{GeV}$ top quickly decays ⟶ spin-info to decay products

$$t \to W^+ + b$$
$$(W^+ \to \ell^+ + \nu \text{ or } \bar{d} + u)$$



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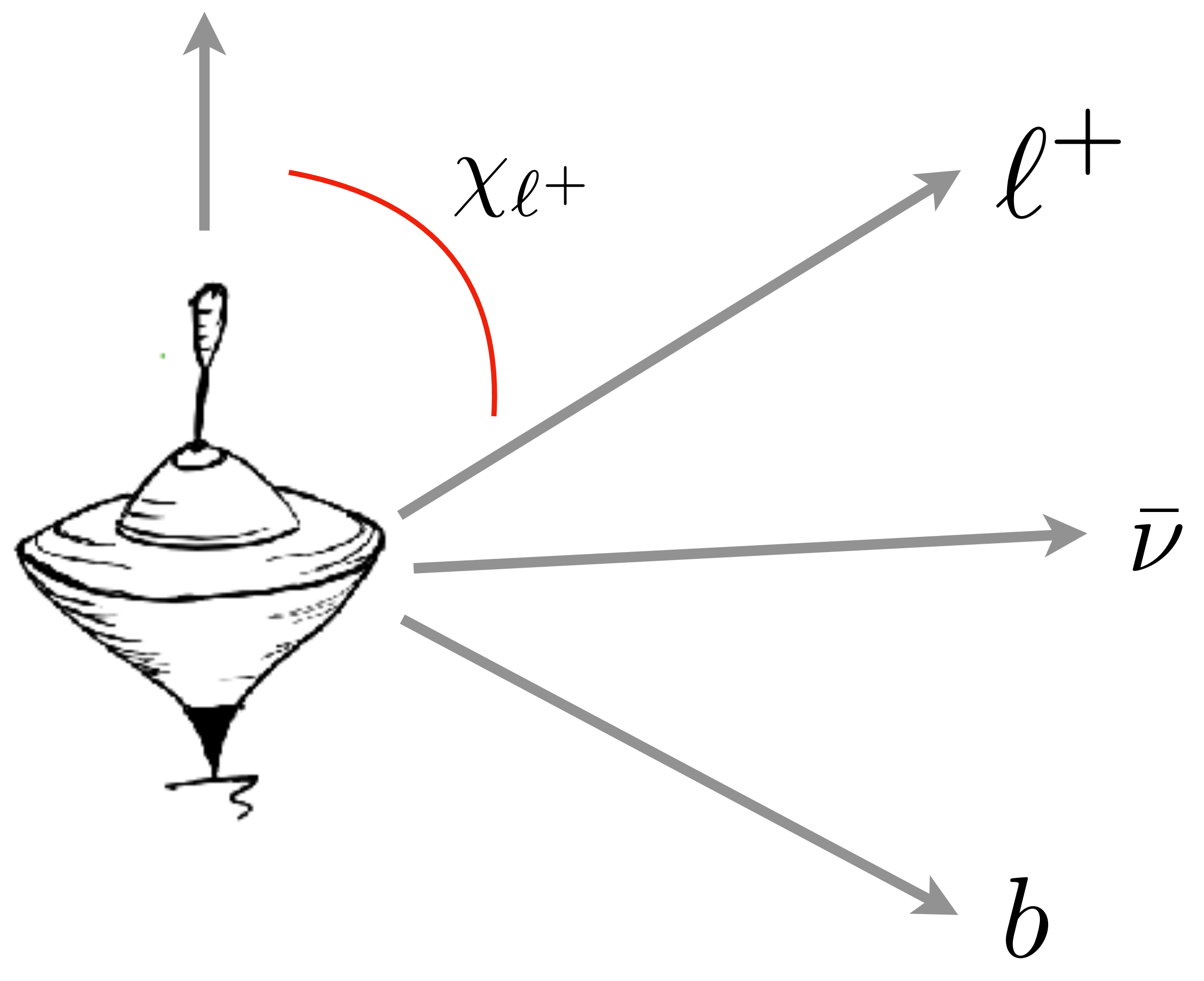
$$\frac{1}{\Gamma_t}\frac{d\Gamma}{d\cos\chi_i} = (1 + \alpha_i \cos\chi_i)/2 \qquad \alpha_i = \begin{cases} +1.0 & \ell^+ \text{ or } \bar{d}\text{-quark} \\ -0.31 & \bar{\nu} \text{ or } u\text{-quark} \\ -0.41 & b\text{-quark} \end{cases}$$



χ_i angle between i-th particle and top spin (top rest frame)

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When both top/antitop have lepton in the decay, the angles (fixed beam basis)

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_-}{(4\pi)^2}$$

$\hat{\mathbf{q}}_\pm$ lepton (antilepton) directions in the parent top (antitop) rest frame

[CMS, PRD 100, 072002]

Experimental detection

[Baumgart, Tweede 12']

[Afik and de Nova, 21']

[Aguilar-Saavedra, Casas, '22]

However, there is a direct experimental signature

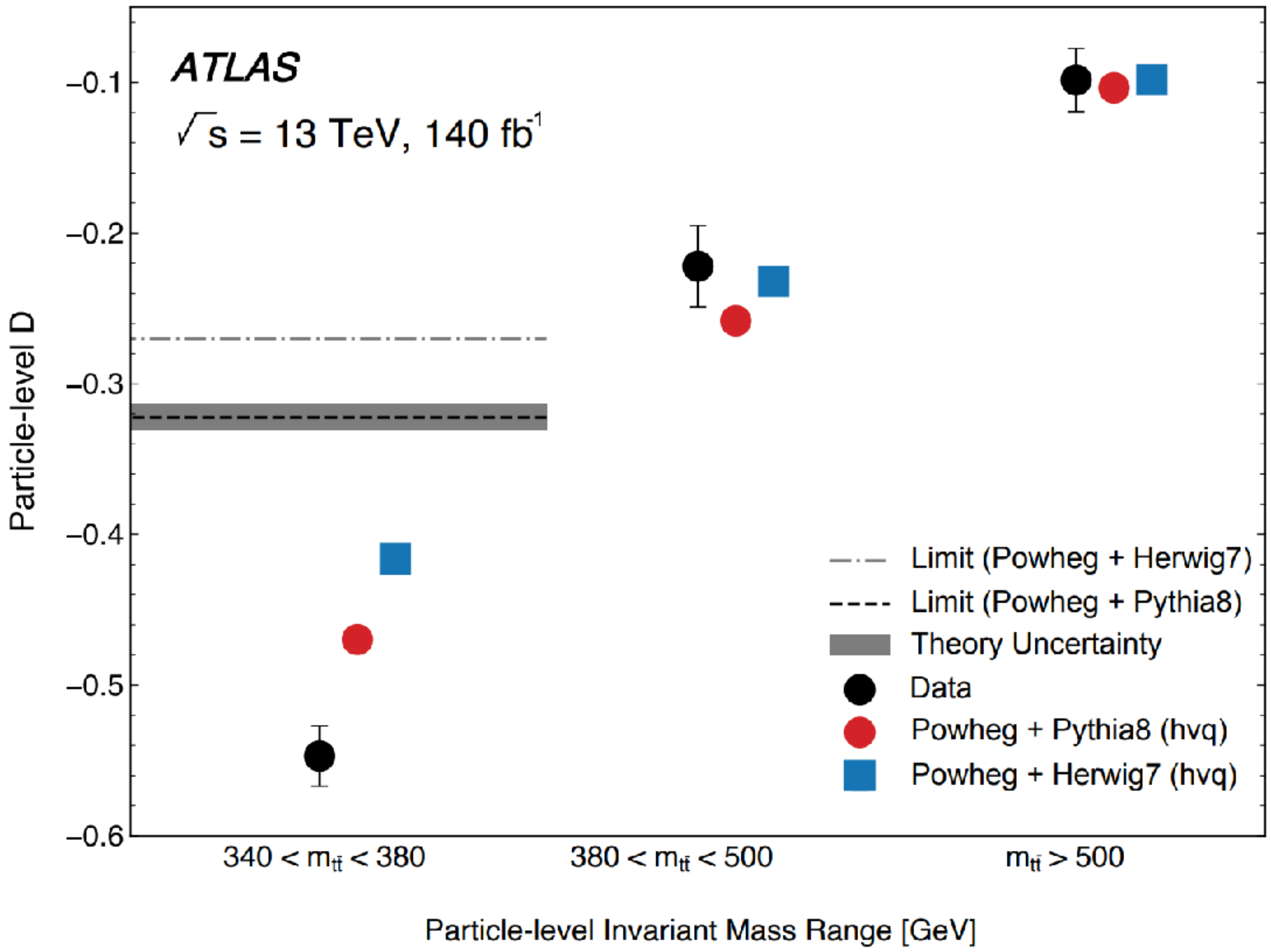
$$D = \frac{\mathrm{tr}[\mathbf{C}]}{3} \longrightarrow \frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

φ angle between leptons in each parent direction rest frames

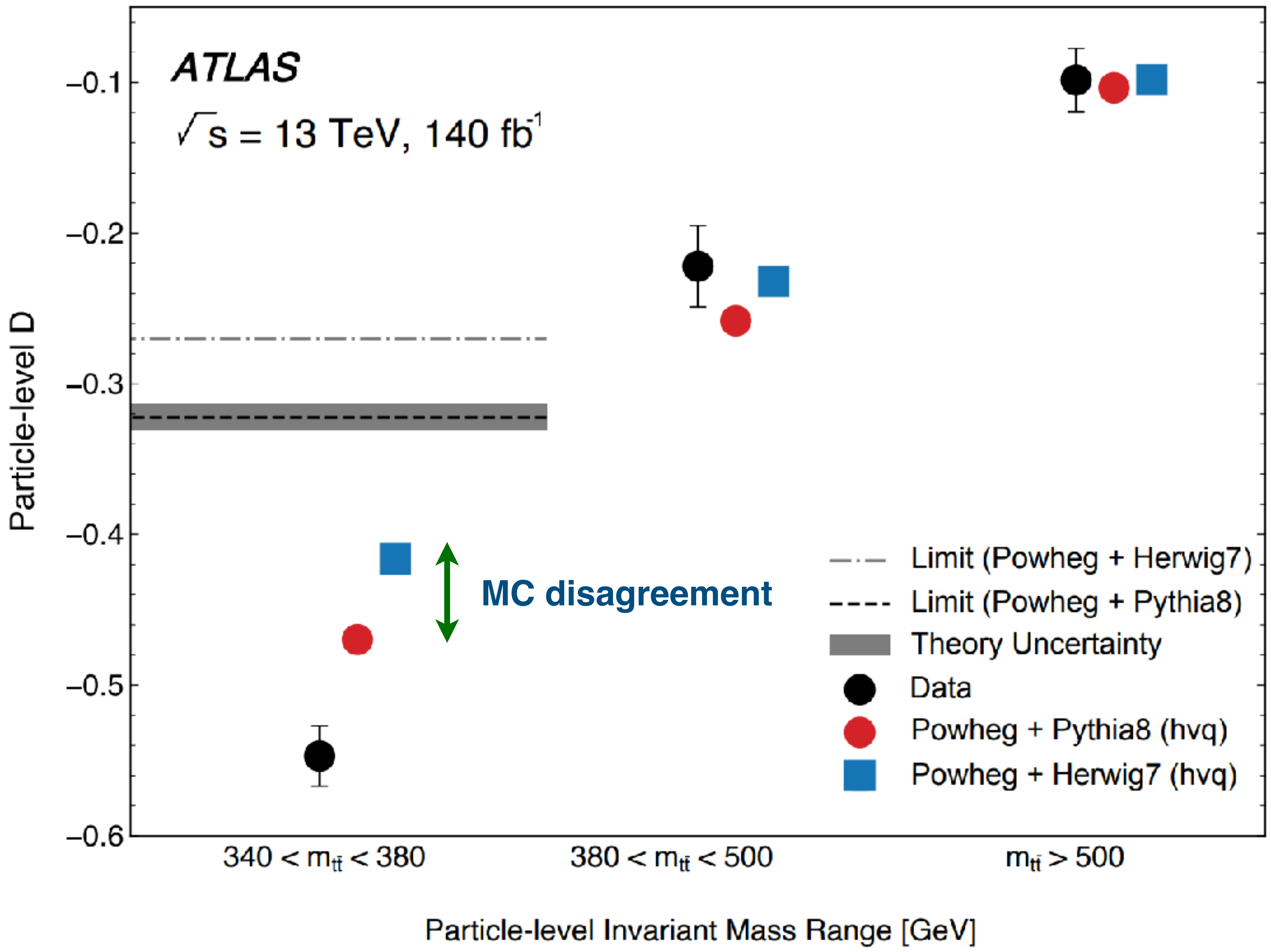
$$C[\rho] = \max(1 - 3D, 0)/2 \longrightarrow$$ entangled if $D < -1/3$

* things are actually more subtle due to mass window constraints

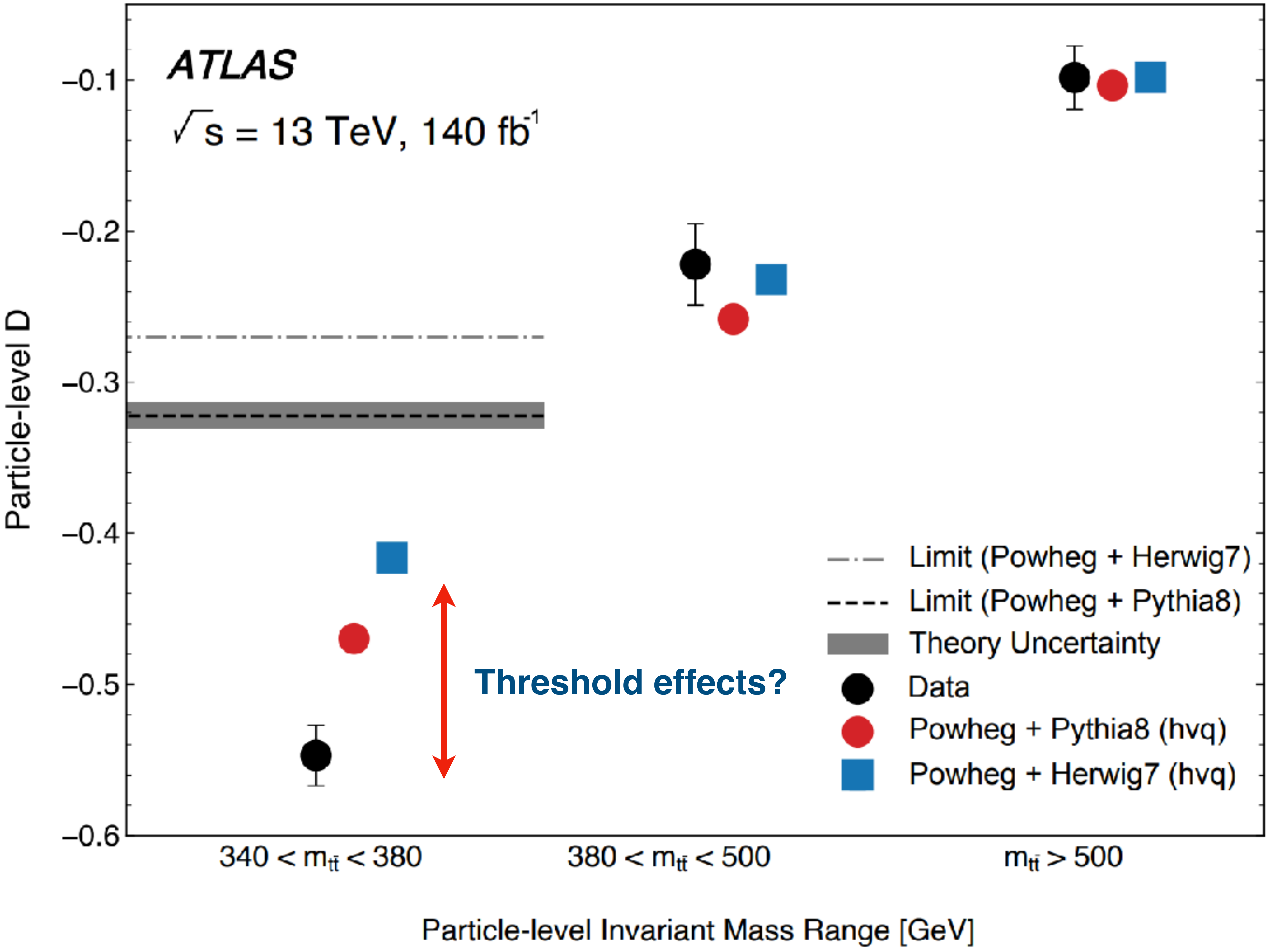
Experimental detection



Experimental detection

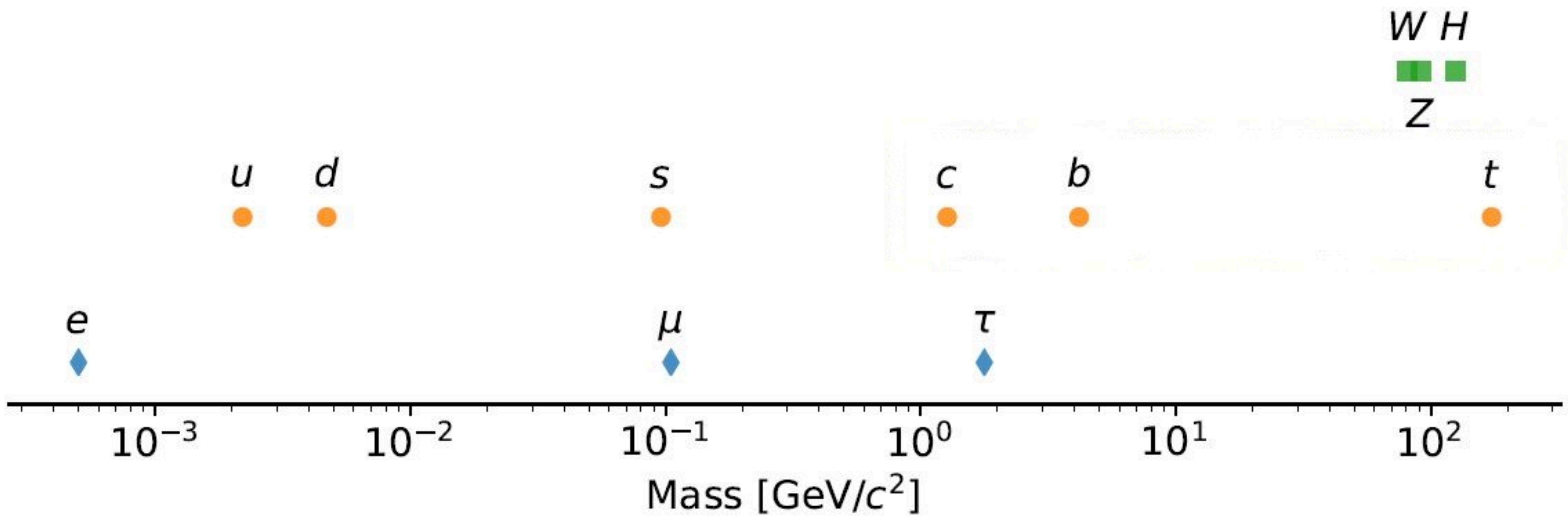


Experimental detection



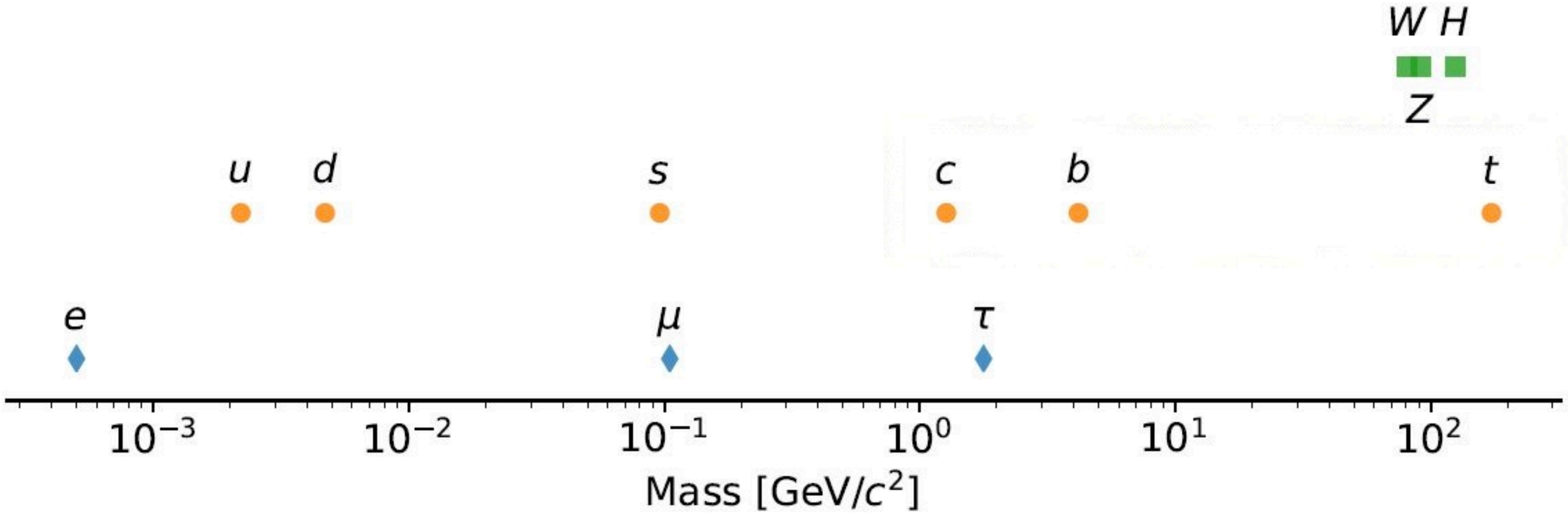
Too early for any fit or claim in this region!!

SM particles



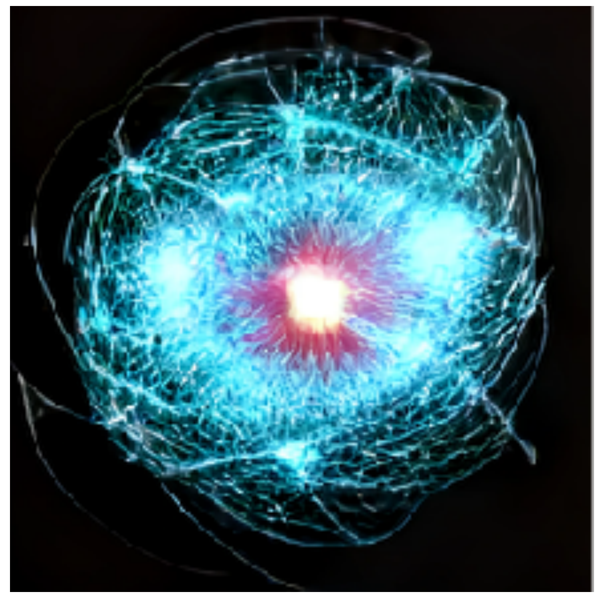
[Fig from CMS collaboration]

BSM particles?



[Fig from CMS collaboration]

New particle?

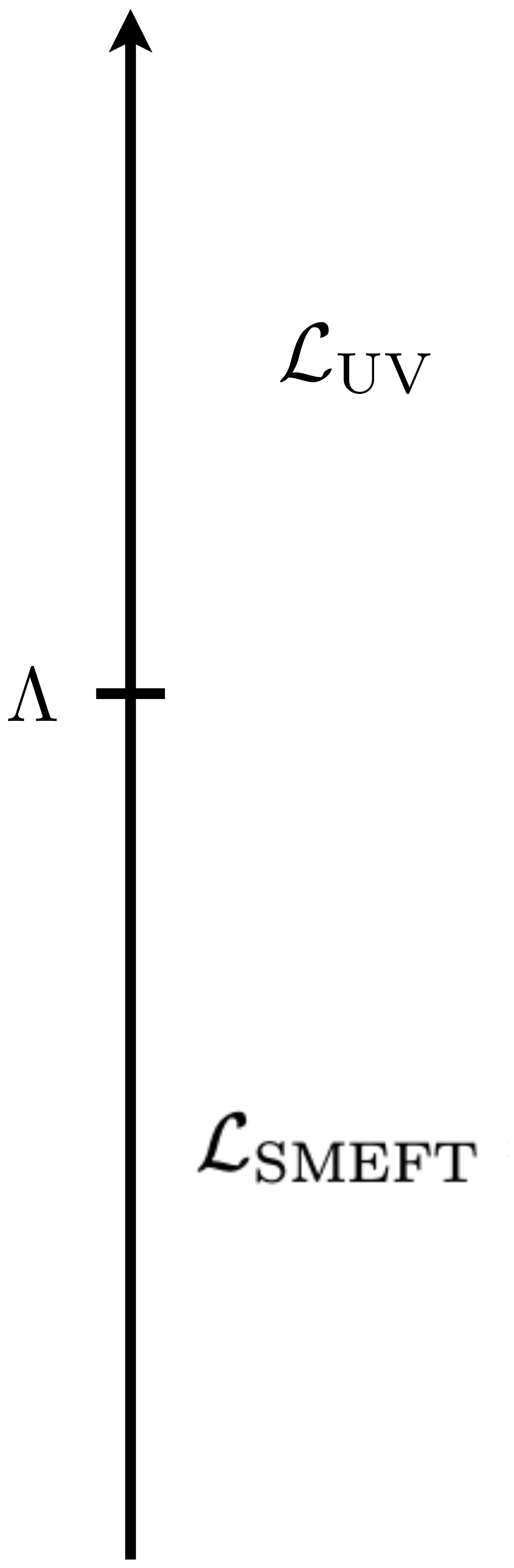


AI generated image:
prompt: new particle

[craiyon.com]

SMEFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}\sum_i c_i \mathcal{O}_i$$



LO-QCD in ttbar prod.

$$\mathcal{O}_G = g_s f^{ABC} G_\nu^{A,\mu} G_\rho^{B,\nu} G_\mu^{C,\rho}$$

$$\mathcal{O}_{\varphi G} = \left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) G_A^{\mu\nu} G_{\mu\nu}^A$$

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu} T^A\, t)\tilde{\varphi} G_{\mu\nu}^A + \text{h.c.}$$

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 10’]

[Aguilar-Saavedra et. al, 18’]

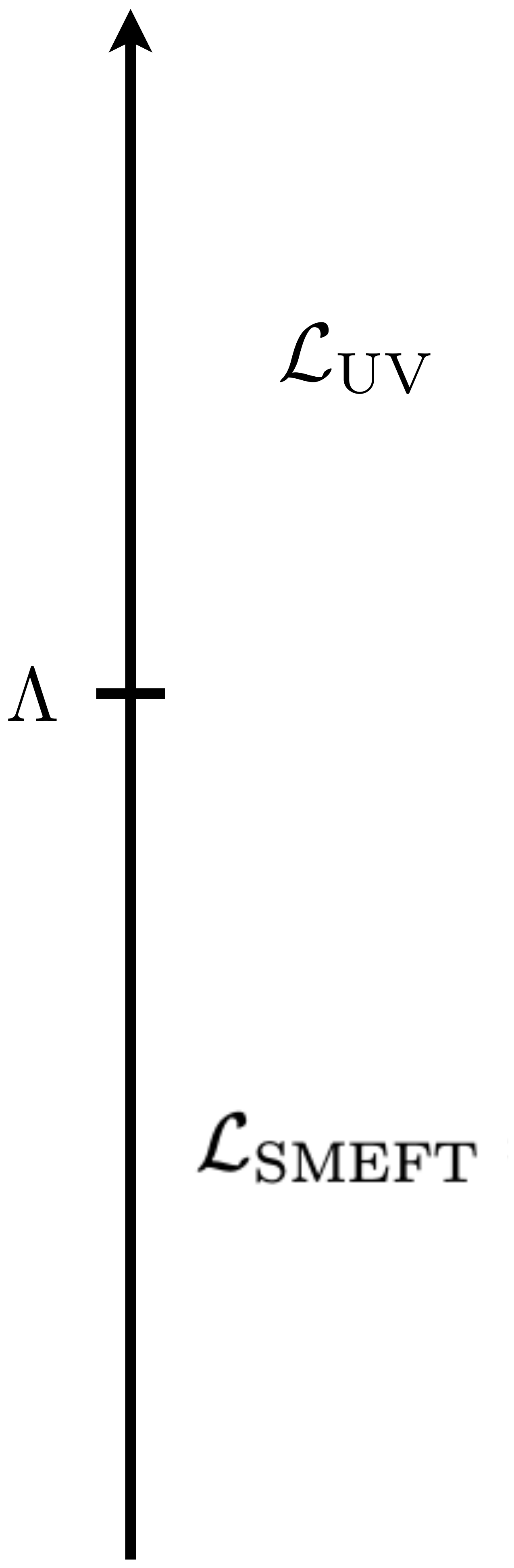
[SMEFTatNLO: Degrande et. al, 20’]

+4F operators

$$\mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{tu}^{(8)}, \mathcal{O}_{td}^{(8)}, \mathcal{O}_{Qu}^{(8)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{tq}^{(8)}$$

SMEFT

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2}\sum_i c_i \mathcal{O}_i$$



[Grzadkowski, Iskrzynski, Misiak, Rosiek, 10']

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[SMEFTatNLO: Degrande et. al, 20']

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+4F operators

$$\mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{tu}^{(8)}, \mathcal{O}_{td}^{(8)}, \mathcal{O}_{Qu}^{(8)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{tq}^{(8)}$$

Maximal points are affected by SMEFT?

Can SMEFT induce new regions?

SMEFT

Back to the R-matrix… $R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$

With dim-six contributions:

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}^{\text{SM}}_{\alpha\beta} + \frac{1}{\Lambda^2}\mathcal{M}^{(\text{d6})}_{\alpha\beta} \longrightarrow \rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

The Fano coefficients $X = X^{(0)} + \frac{1}{\Lambda^2}X^{(1)} + \frac{1}{\Lambda^4}X^{(2)}$ where

$$X = \tilde{A},\ \tilde{C}_{ij} \text{ and } \tilde{B}^{\pm}_i$$

$\mathcal{O}(\Lambda^{-4})$ from dim-6 sq.

SMEFT

Back to the R-matrix…

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

With dim-six contributions:

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}^{\text{SM}}_{\alpha\beta} + \frac{1}{\Lambda^2}\mathcal{M}^{(\text{d6})}_{\alpha\beta} \longrightarrow \rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

At $\mathcal{O}(\Lambda^{-2})$

$$\tilde{C}^{gg,(1)}_{nn} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{-7g_s^2 v m_t}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right]$$

SMEFT entanglement: gg-initiated

[Aoude, Madge, Maltoni, Mantani, 22']

only $\mathcal{O}_{tG}, \mathcal{O}_{G}, \mathcal{O}_{\varphi G}$ contributes

$$\rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

gg-initiated at threshold $\beta^2 = 0$

- linear interference exactly cancel, maximally entangled state unchanged
- quadratics vanish for $\mathcal{O}_{\varphi G}$ and decreases for $\mathcal{O}_{tG}, \mathcal{O}_{G}$

gg-initiated at high-E: $\beta^2 \to 1$: EFT not valid but $m_t^2 \ll \hat{s} \ll \Lambda^2$

- linear interference: sign dependent
- quadratics always decreases

SMEFT entanglement: qq-initiated

[Aoude, Madge, Maltoni, Mantani, 22']

only $\mathcal{O}_{tG}$ and 4F contributes

$$\rho = \frac{R^{\rm SM} + R^{\rm EFT}}{{\rm tr}(R^{\rm SM}) + {\rm tr}(R^{\rm EFT})}$$

qq-initiated at threshold $\beta^2 = 0$

- no contributions for linear and quad

qq-initiated at high-E: $m_t^2 \ll \hat{s} \ll \Lambda^2$

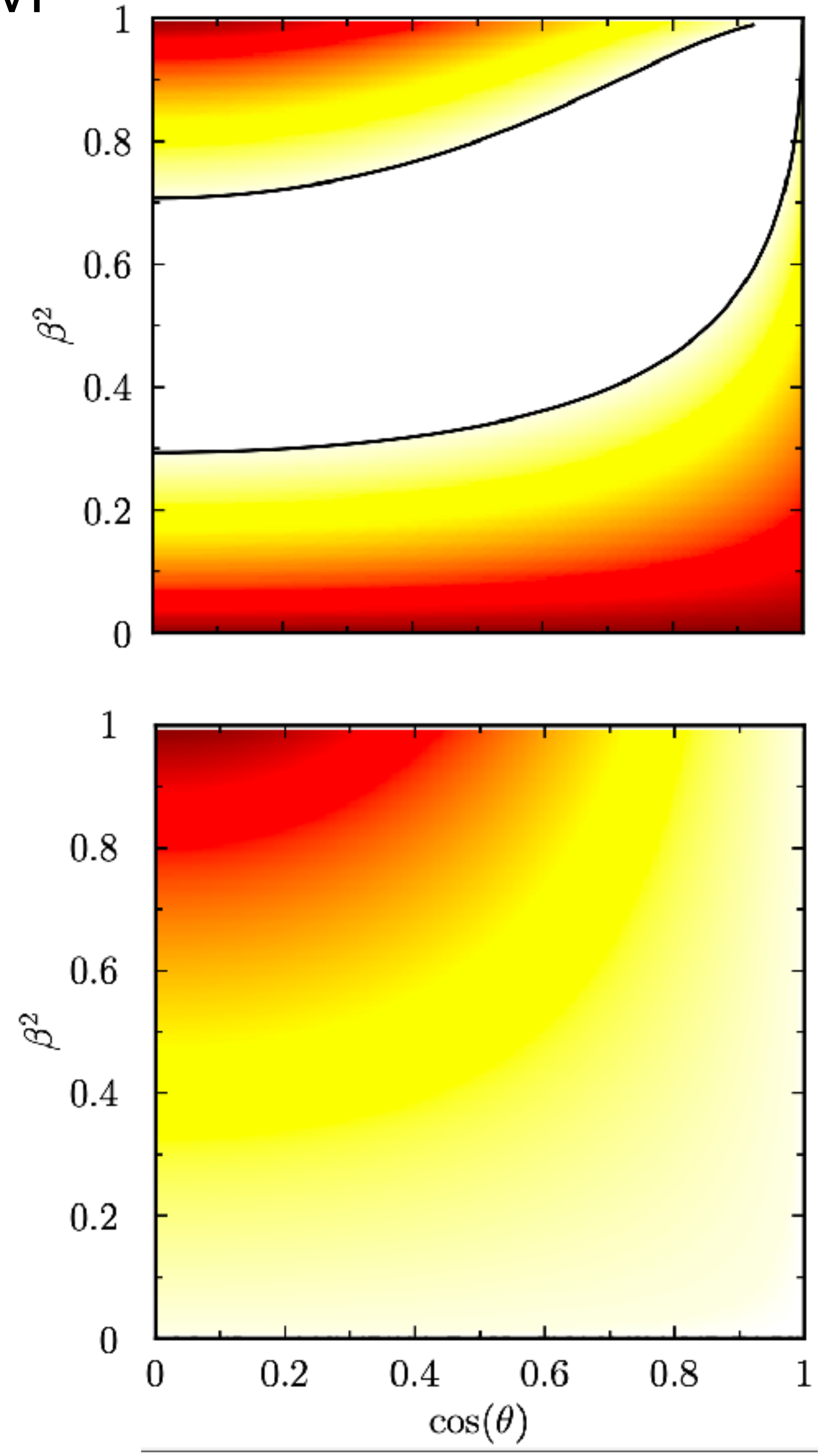
- sign dependent for linear and quadratics always decreases

everything gets more involved for pp

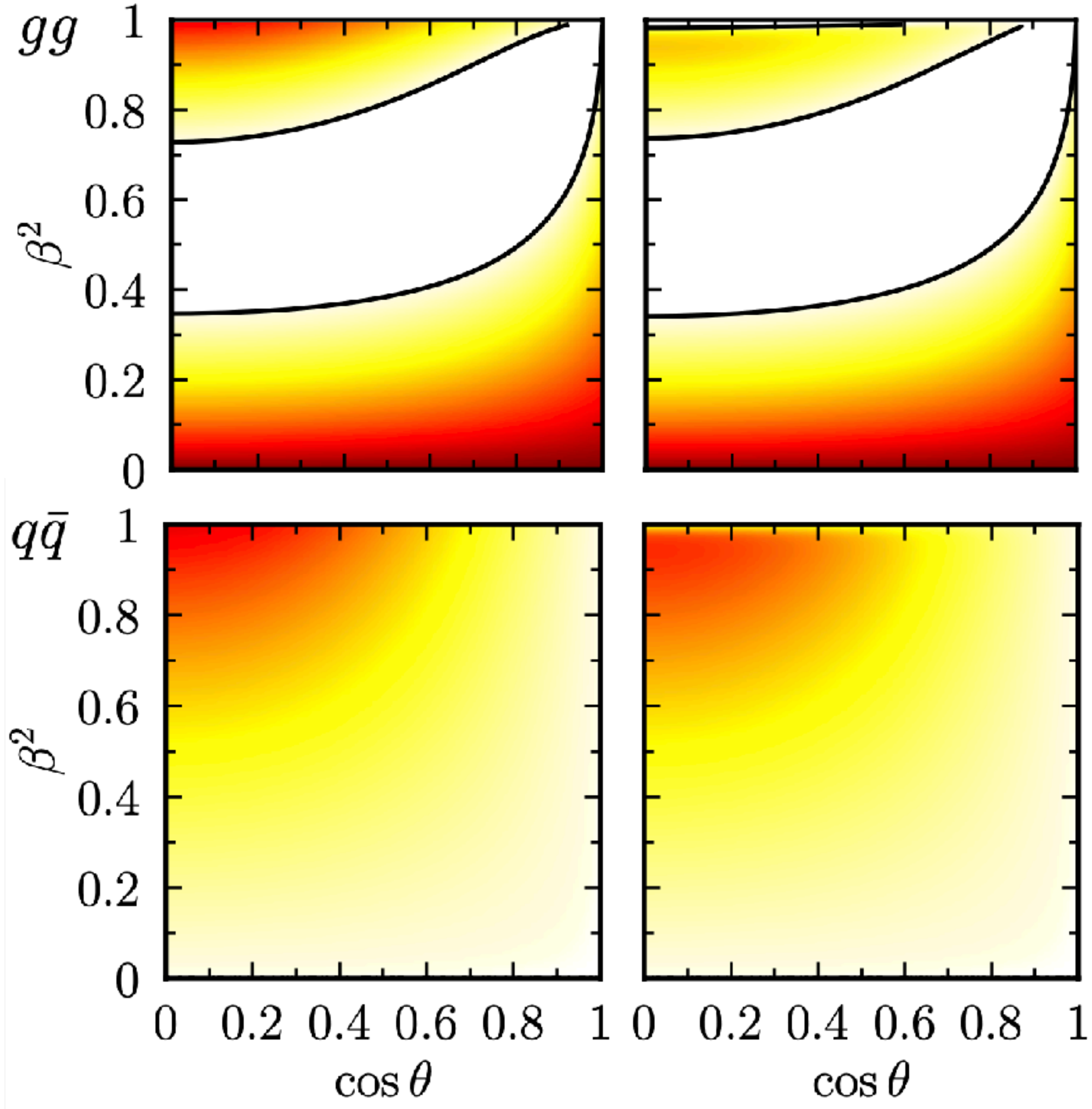
SMEFT entanglement

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu}T^A\, t)\tilde{\varphi}G^A_{\mu\nu} + \text{h.c.}$$

SM



linear quad



SMEFT entanglement marker

$$\rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

Δ_0 calculated with SM R's

$$\Delta_1 \equiv \Delta - \Delta_0$$

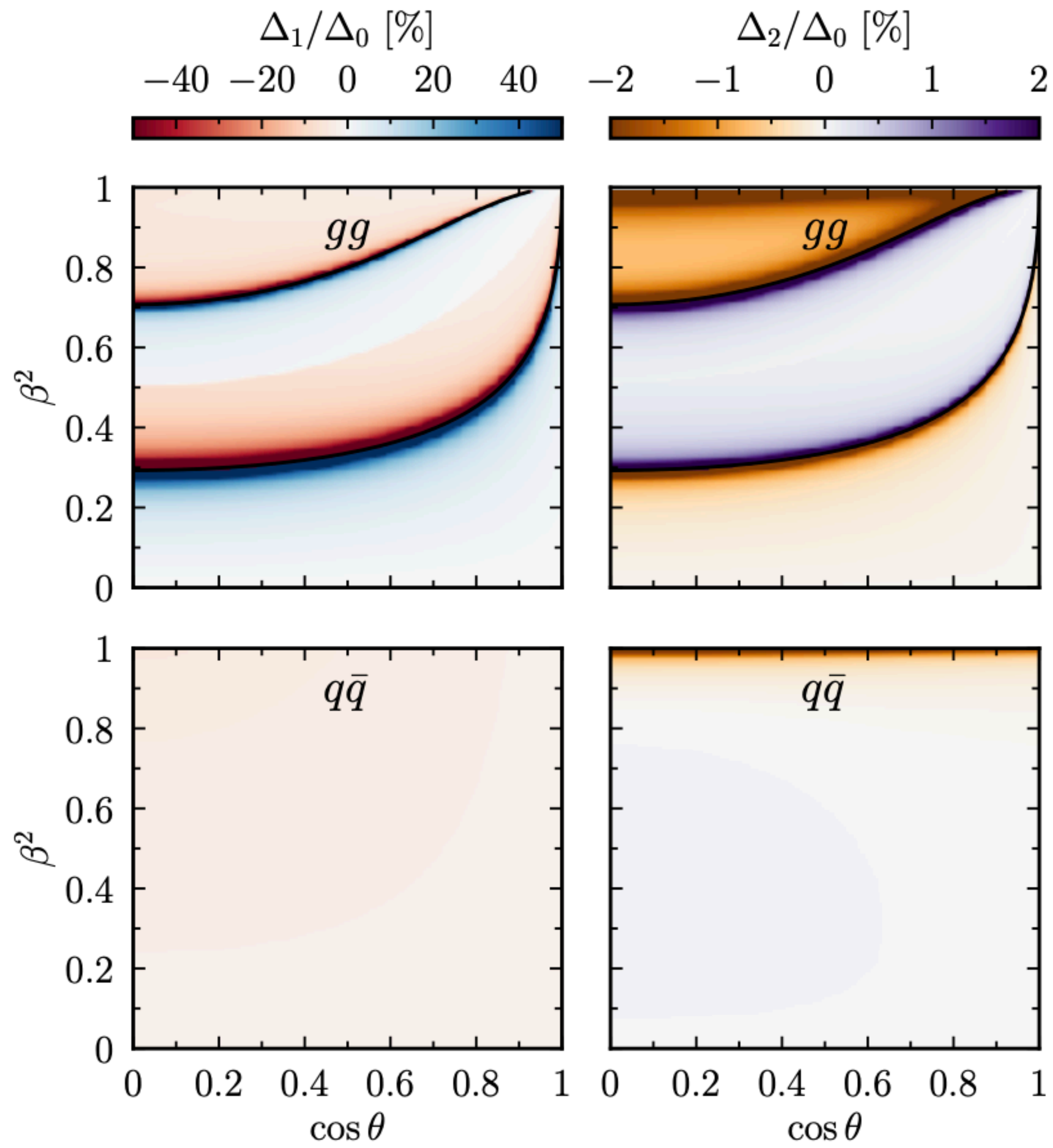
→ calculated with SMEFT R's up to $\mathcal{O}(\Lambda^{-2})$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

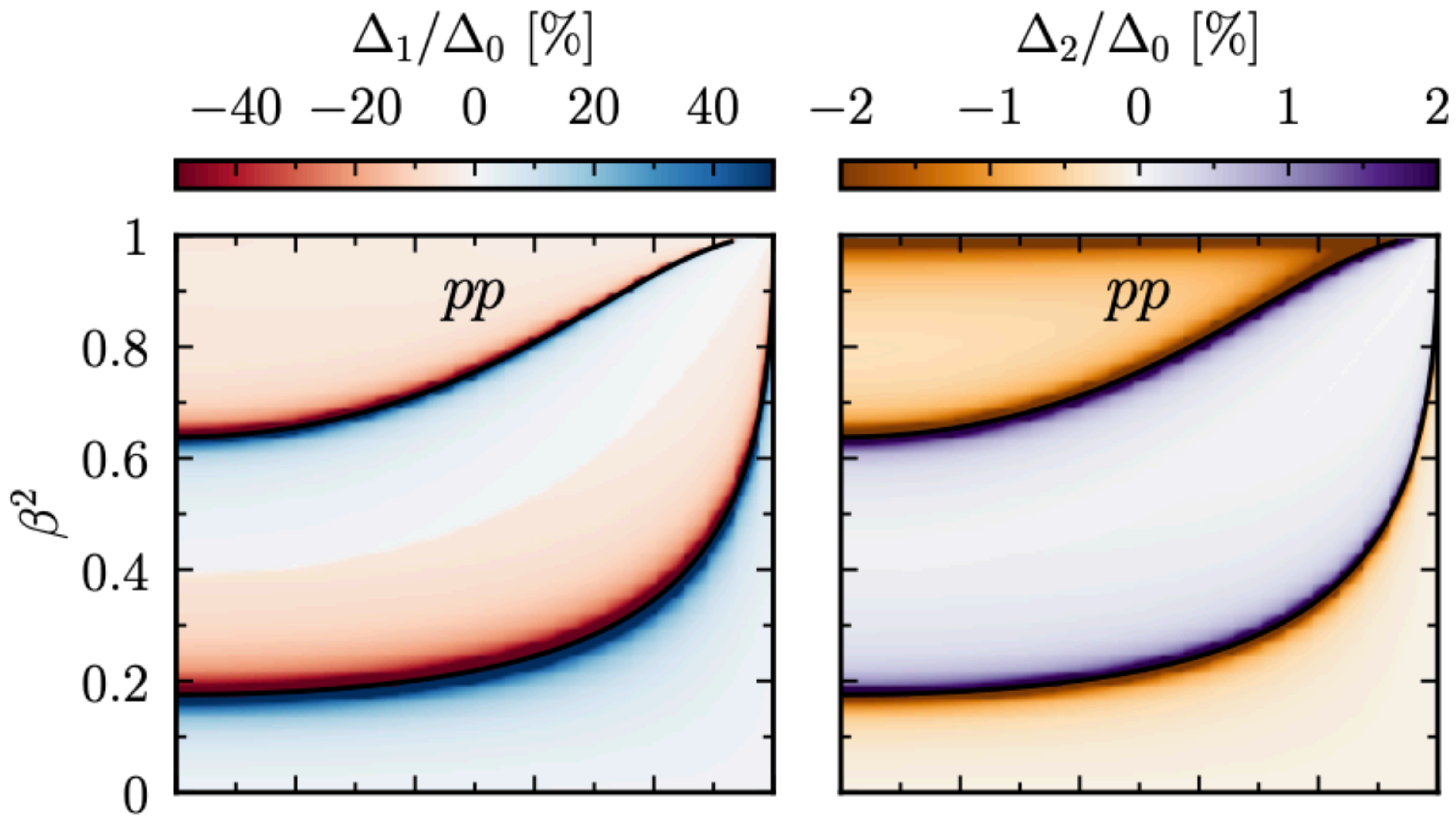
→ calculated with SMEFT R's up to $\mathcal{O}(\Lambda^{-4})$

SMEFT entanglement marker

separate channels



mixed state



SMEFT averaged concurrence

Average over the solid angle

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \,,$$

$\longrightarrow$

PHC implies

$$\delta \equiv -C_z + |2C_\perp| - 1 > 0$$

$$C[\rho] = \max(\delta/2, 0)$$

(fixed beam basis)

SMEFT averaged concurrence

Average over the solid angle

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega\, R(\hat{s}, \boldsymbol{k}) \,,$$

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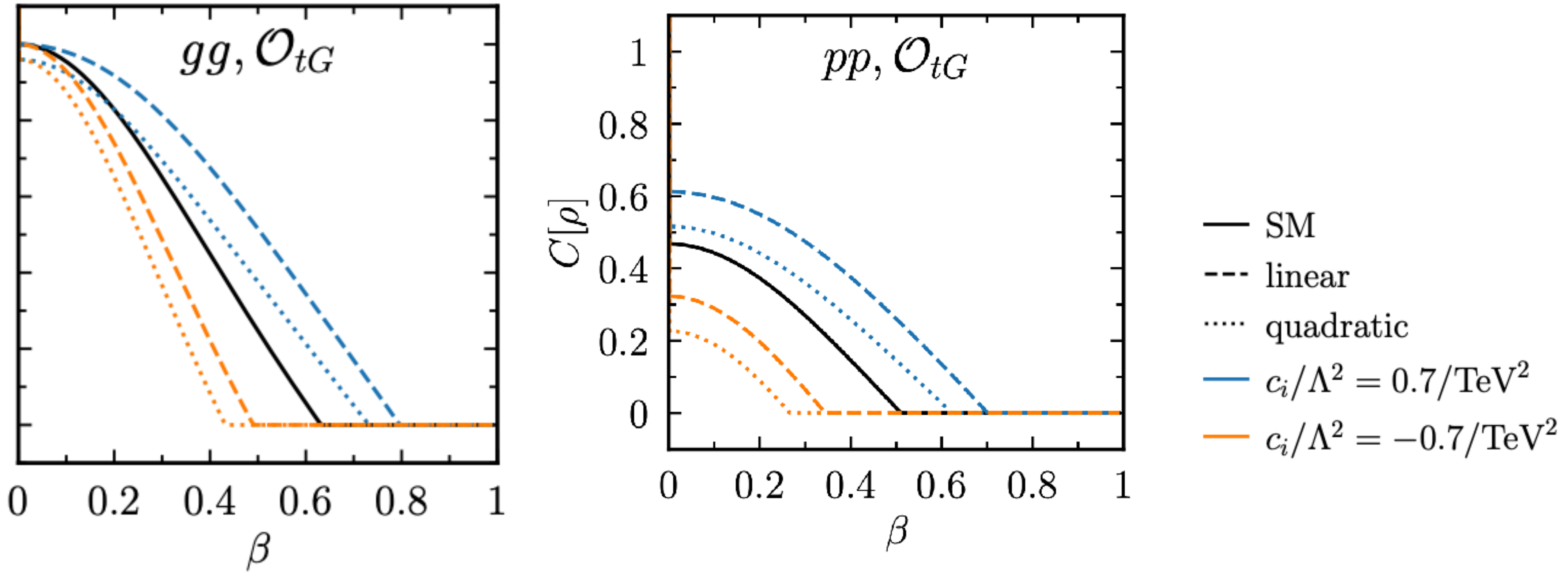

Tomography and reconstruction of the state

[Mahlon, Parke 10']
[Baumgart, Tweede 12']
[Afik and de Nova, 21']

$$t \to W^+ + b \quad (W^+ \to \ell^+ + \nu \text{ or } \bar{d} + u) \qquad \frac{1}{\Gamma_t}\frac{d\Gamma}{d\cos\chi_i} = (1 + \alpha_i \cos\chi_i)/2$$

In the SM:

$$\alpha_i = \begin{cases} +1.0 & \ell^+ \text{ or } \bar{d}\text{-quark} \\ -0.31 & \bar{\nu} \text{ or } u\text{-quark} \\ -0.41 & b\text{-quark} \end{cases}$$

Tomography and reconstruction of the state

[Mahlon, Parke 10']
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[Afik and de Nova, 21']

$t \to W^+ + b \quad (W^+ \to \ell^+ + \nu \text{ or } \bar{d} + u)$

$$\frac{1}{\Gamma_t}\frac{d\Gamma}{d\cos\chi_i} = (1 + \alpha_i \cos\chi_i)/2$$

In the SMEFT:

[Zhang, Willenbrock, 11']

$$\alpha_b = -\frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} + \frac{\mathrm{Re}C_{tW}v^2}{\Lambda^2 V_{tb}}\frac{8\sqrt{2}m_t m_W(m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2}$$

$$\alpha_v = \frac{m_t^6 - 12m_t^4 m_W^2 + 3m_t^2 m_W^4(3 + 8\ln(m_t/m_W)) + 2m_W^6}{m_t^6 - 3m_t^2 m_W^4 + 2m_W^6} - \frac{\mathrm{Re}C_{tW}v^2}{\Lambda^2 V_{tb}}\frac{12\sqrt{2}m_t m_W(m_t^6 - 6m_t^4 m_W^2 + 3m_t^2 m_W^4(1 + 4\ln(m_t/m_W)) + 2m_W^6)}{(m_t^2 + 2m_W^2)^2(m_t^2 - m_W^2)^2}$$

$$\alpha_{e^+} = 1$$

In the SM:

$$\alpha_i = \begin{cases} +1.0 & \ell^+ \text{ or } \bar{d}\text{-quark} \\ -0.31 & \bar{\nu} \text{ or } u\text{-quark} \\ -0.41 & b\text{-quark} \end{cases}$$

$\alpha_{\ell^+} = +1$ unchanged by dim-6 linear effects

* can be changed to dim-6 squared but very small

Diboson entanglement

Diboson entanglement

Massive spin-1 bosons (W,Z) have three polarisations ⟶ Qutrits!

Single vector boson

$$\rho = \frac{1}{3}\mathbb{I} + \sum_{i=1}^{8} a_i \lambda_i$$

a_i : 8 real parameters

Two vector bosons

$$\rho = \frac{1}{9}\,\mathbb{I}\otimes\mathbb{I} + \frac{1}{3}\sum_{i=1}^{8} a_i\,\lambda_i\otimes\mathbb{I} + \frac{1}{3}\sum_{j=1}^{8} b_j\,\mathbb{I}\otimes\lambda_j + \sum_{i=1}^{8}\sum_{j=1}^{8} c_{ij}\,\lambda_i\otimes\lambda_j$$

$a_i\,, b_j\,, c_{ij}$ 8+8+64 real parameters

λ_i are Gell-Mann matrices

Concurrence for qutrits

[Horodecki, Horodecki, Horodecki, Horodecki, 09']

[Ashby-Pickering, Barr, Wierzchuca, '22]

How is the concurrence for qutrits? Not as simple as in the qubit case

$$\mathcal{C}(\rho) = \inf \left[\sum_i p_i c(|\psi_i\rangle) \right] \qquad c(|\psi_i\rangle) = \sqrt{2(1 - \mathrm{tr}_A[(\mathrm{tr}_B|\psi_i\rangle\langle\psi_i|)^2])}$$

Infimum of all ensembles of the decomposition $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

… complicated optimisation problem

$$0 \leq \mathcal{C}(\rho) \leq \frac{2}{\sqrt{3}} \qquad \mathcal{C}(\rho) > 0 \longrightarrow \text{entangled}$$

Only calculable for pure states.

Concurrence Bounds for qutrits

However, we can define lower and upper bounds $\mathcal{C}_{\rm LB} \leq \mathcal{C}(\rho) \leq \mathcal{C}_{\rm UB},$

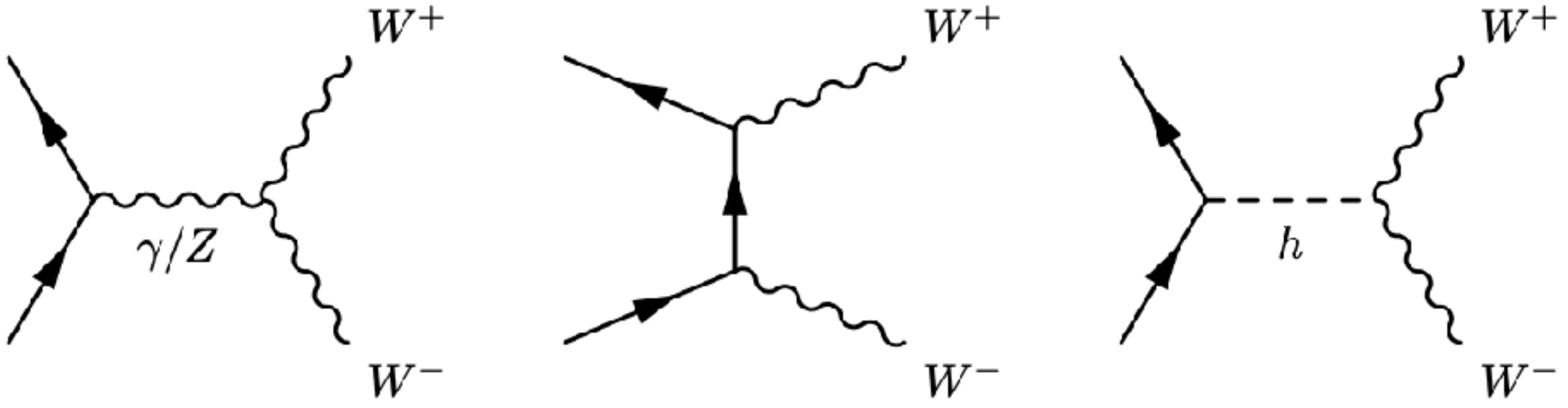
Lower: $$(\mathcal{C}(\rho))^2 \geq 2\max\left(0, \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[\rho_B^2\right]\right) \equiv \mathcal{C}_{\rm LB}^2 \,,$$

Upper: $$(\mathcal{C}(\rho))^2 \leq 2\min\,\left(1 - \mathrm{Tr}[\rho_A^2], 1 - \mathrm{Tr}[\rho_B^2]\right) \equiv \mathcal{C}_{\rm UB}^2 \,,$$

For a pure state: $P(\rho) = \mathrm{tr}\rho^2 = 1$ ⟶ $\mathcal{C}_{\rm LB}(\rho) = \mathcal{C}(\rho) = \mathcal{C}_{\rm UB}(\rho)$

EW boson production at colliders

WW:



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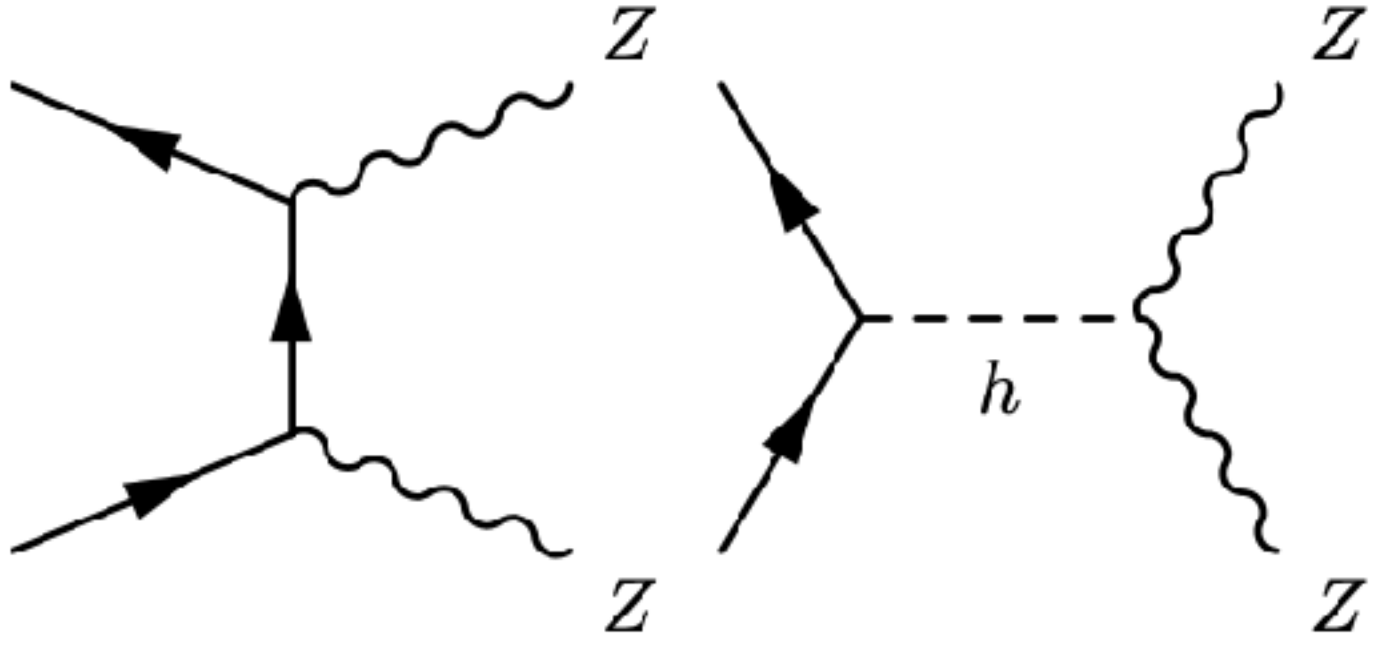

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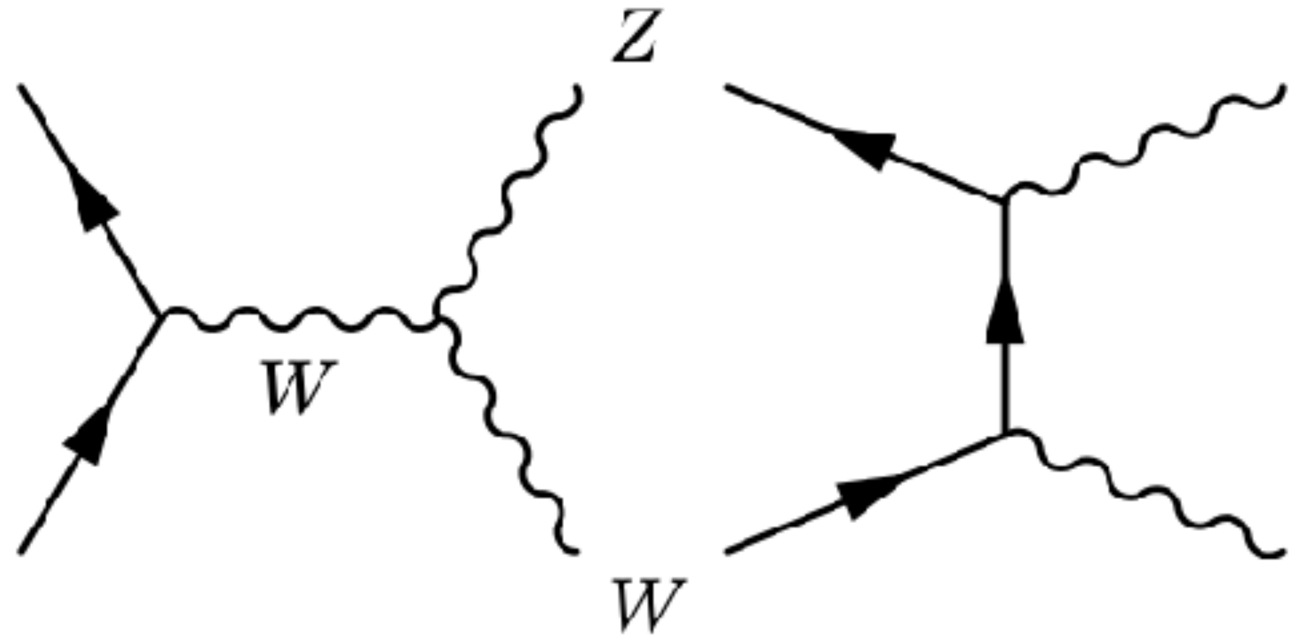

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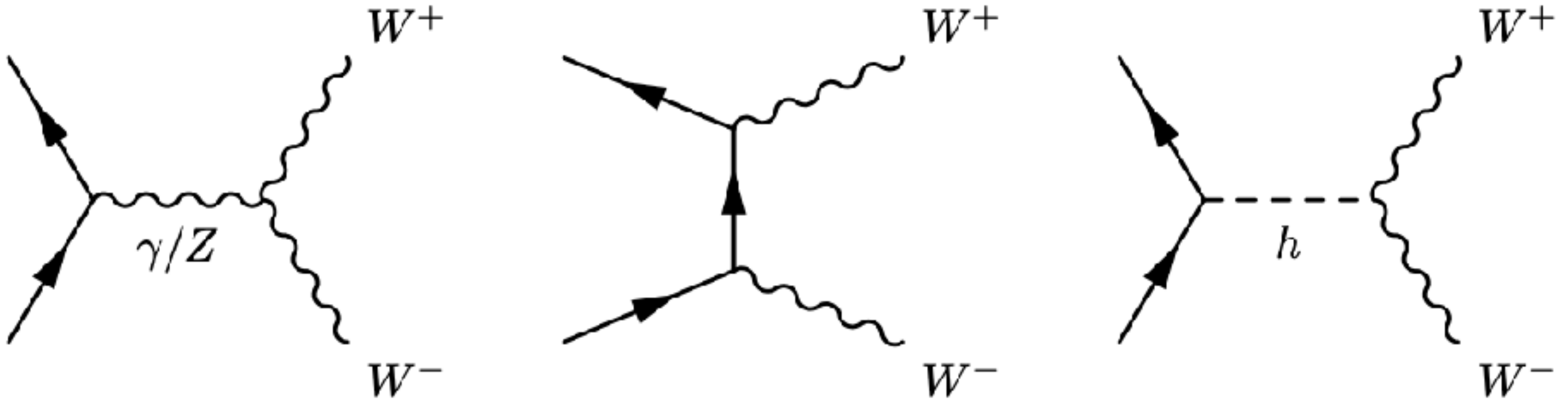
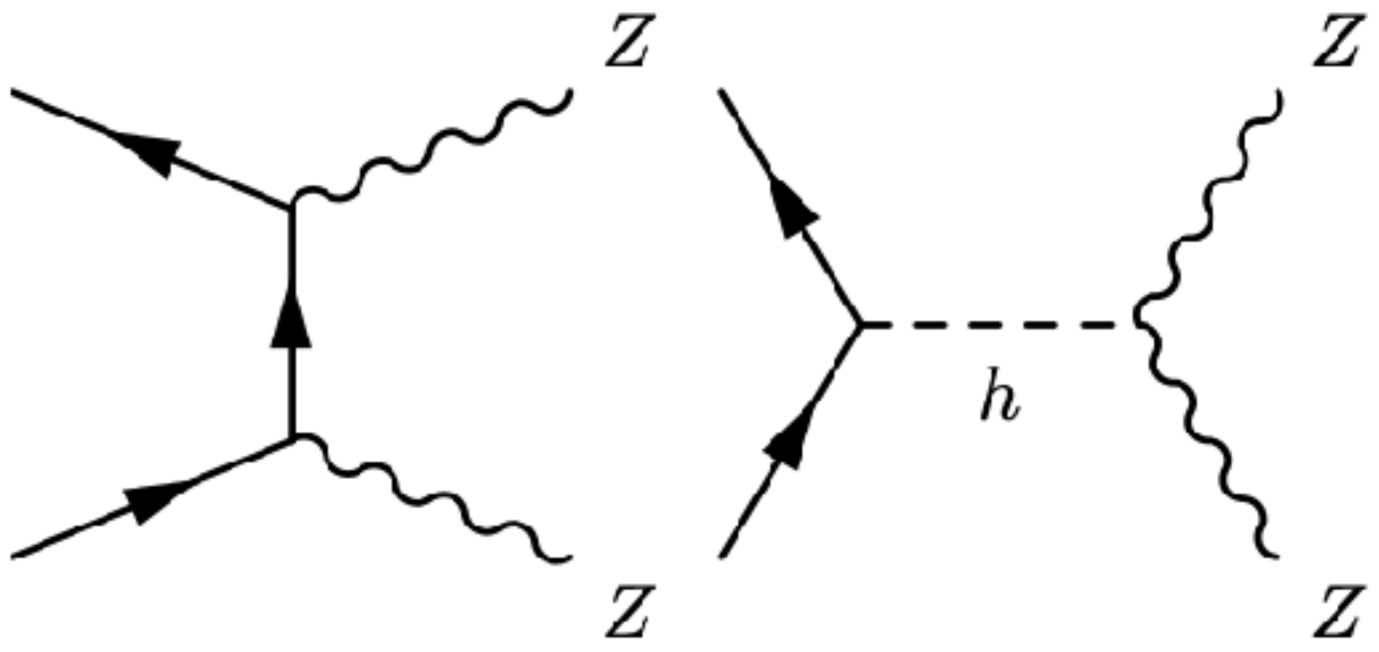
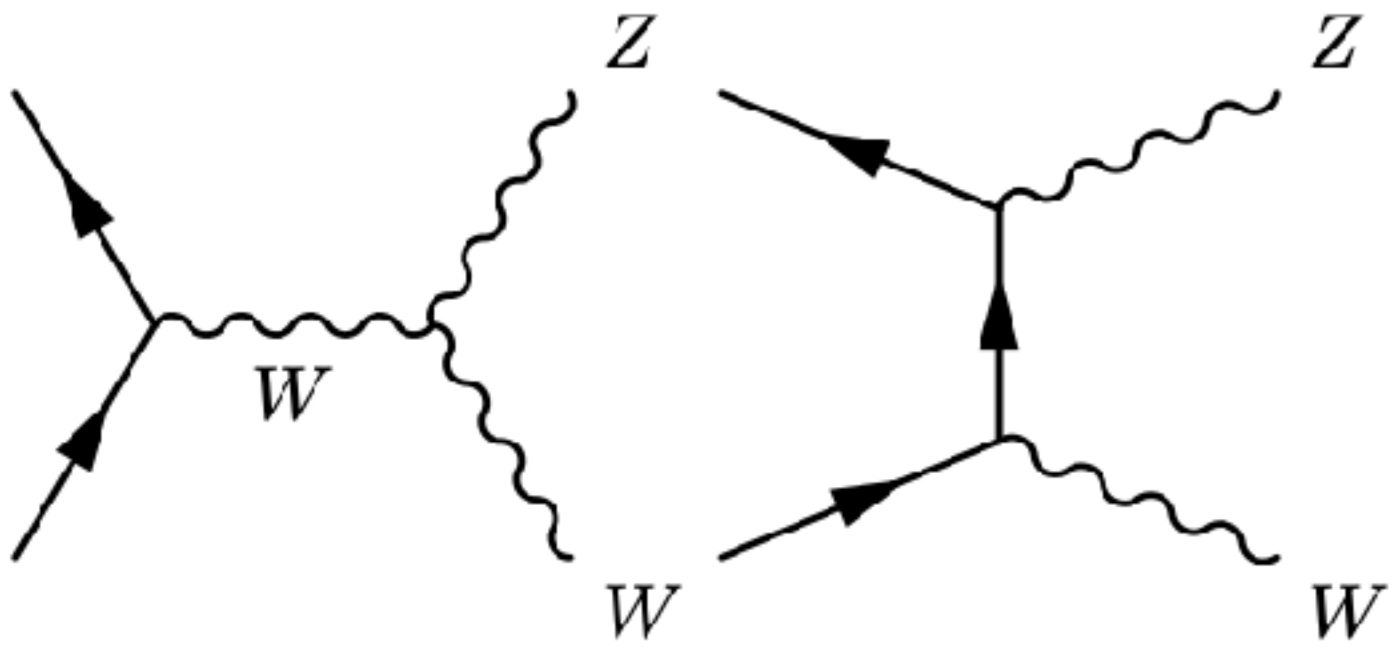


ZZ:



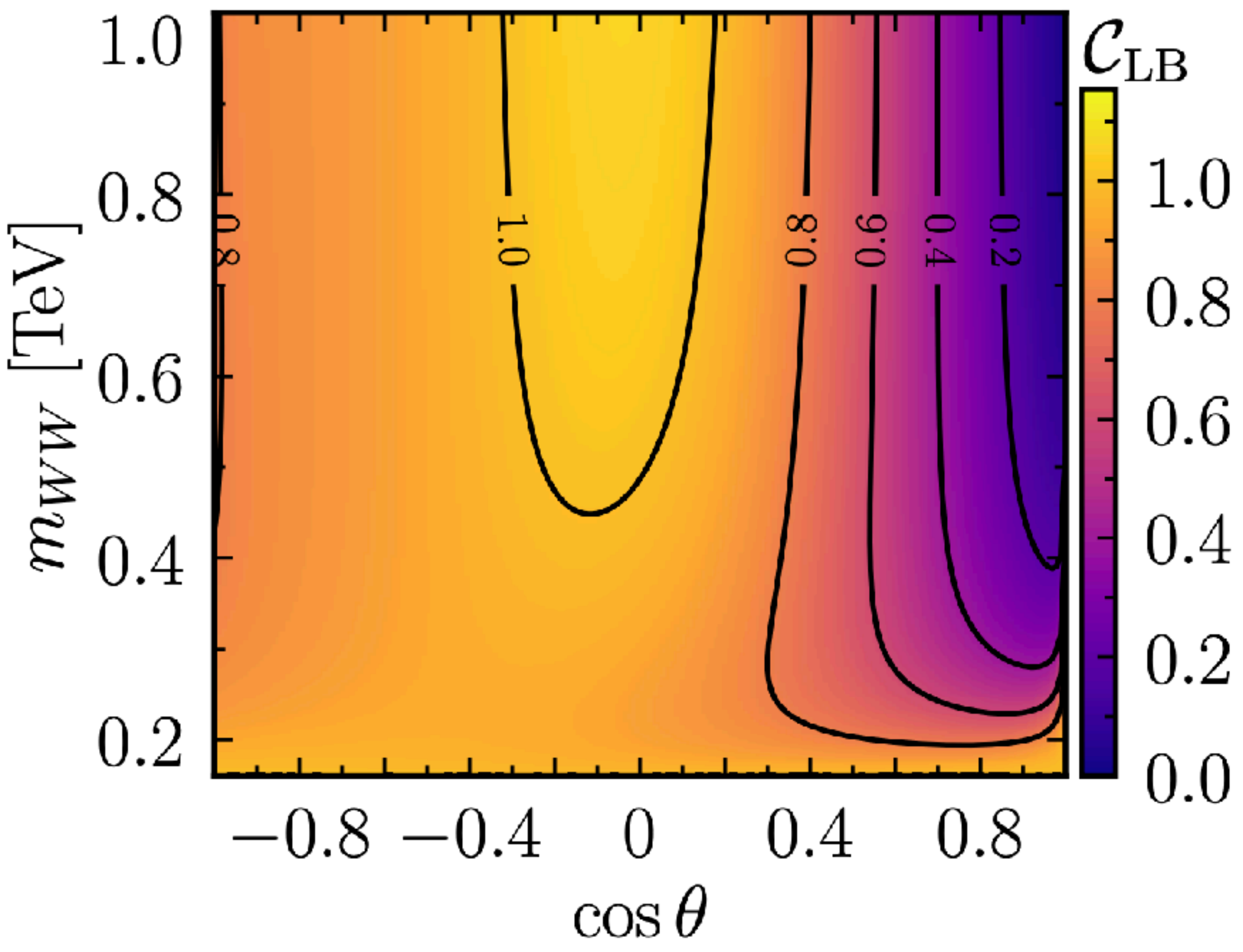
WZ:



What's the story for the Standard Model?

$e^+e^- \to W^+W^-$

Lower bound:



No symmetry around $\theta = \pi/2$ as in ttbar

Entanglement is mostly present across the phase space

Zero entanglement at $\theta = 0$

High entanglement at central HE region

Relevant SMEFT Operators for diboson

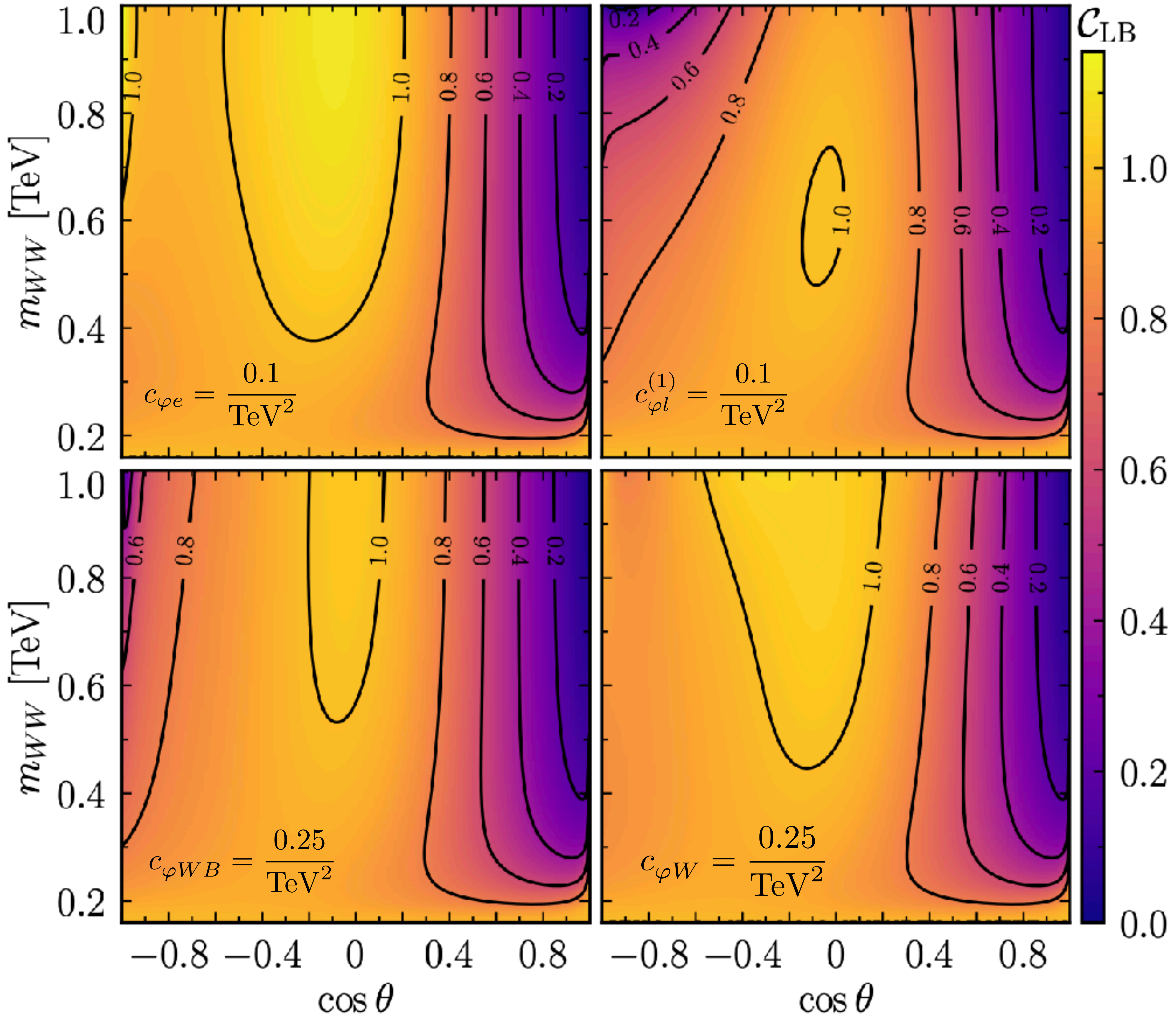
Operator	Coefficient	Definition	95 % CL bounds
		two-fermion operators	
$\mathcal{O}_{\varphi u}$	$c_{\varphi u}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{u}\gamma^\mu u\big)$	$[-0.17, 0.14]$
$\mathcal{O}_{\varphi d}$	$c_{\varphi d}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{d}\gamma^\mu d\big)$	$[-0.07, 0.09]$
$\mathcal{O}_{\varphi q}^{(1)}$	$c_{\varphi q}^{(1)}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{q}\,\gamma^\mu\, q\big)$	$[-0.06, 0.22]$
$\mathcal{O}_{\varphi q}^{(3)}$	$c_{\varphi q}^{(3)}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu\, \tau_I \varphi\big)\big(\bar{q}\,\gamma^\mu\, \tau^I q\big)$	$[-0.21, 0.05]$
$\mathcal{O}_{\varphi e}$	$c_{\varphi e}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{e}\gamma^\mu e\big)$	$[-0.21, 0.26]$
$\mathcal{O}_{\varphi l}^{(1)}$	$c_{\varphi l}^{(1)}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{l}\gamma^\mu l\big)$	$[-0.11, 0.13]$
$\mathcal{O}_{\varphi l}^{(3)}$	$c_{\varphi l}^{(3)}$	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi\big)\big(\bar{l}\gamma^\mu \tau^I l\big)$	$[-0.21, 0.05]$

		bosonic operators	
$\mathcal{O}_W$	c_W	$\varepsilon_{IJK} W^I_{\mu\nu} W^{J,\nu\rho} W^{K,\mu}_\rho,$	$[-0.18, 0.22]$
$\mathcal{O}_{\varphi W}$	$c_{\varphi W}$	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) W^{\mu\nu}_I W^I_{\mu\nu}$	$[-0.15, 0.30]$
$\mathcal{O}_{\varphi B}$	$c_{\varphi B}$	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) B_{\mu\nu} B^{\mu\nu}$	$[-0.11, 0.11]$
$\mathcal{O}_{\varphi WB}$	$c_{\varphi WB}$	$(\varphi^\dagger \tau_I \varphi) B^{\mu\nu} W^I_{\mu\nu}$	$[-0.17, 0.27]$
$\mathcal{O}_{\varphi D}$	$c_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^\dagger (\varphi^\dagger D_\mu \varphi)$	$[-0.52, 0.43]$
		four-fermion operator	
$\mathcal{O}_{ll}$	c_{ll}	$\big(\bar{l}\gamma_\mu l\big)\big(\bar{l}\gamma^\mu l\big)$	$[-0.16, 0.02]$

Bounds from [SMEFit Collaboration ’21]

SMEFT entanglement deviations

$e^+e^- \to W^+W^-$



$\mathcal{O}_{\varphi e}$ changes $Ze^+_R e^-_R$ vertex

- more ent. in central and backward HE

$\mathcal{O}^{(1)}_{\varphi l}$ changes $Ze^+_L e^-_L$ vertex

- less ent. in central and backward HE

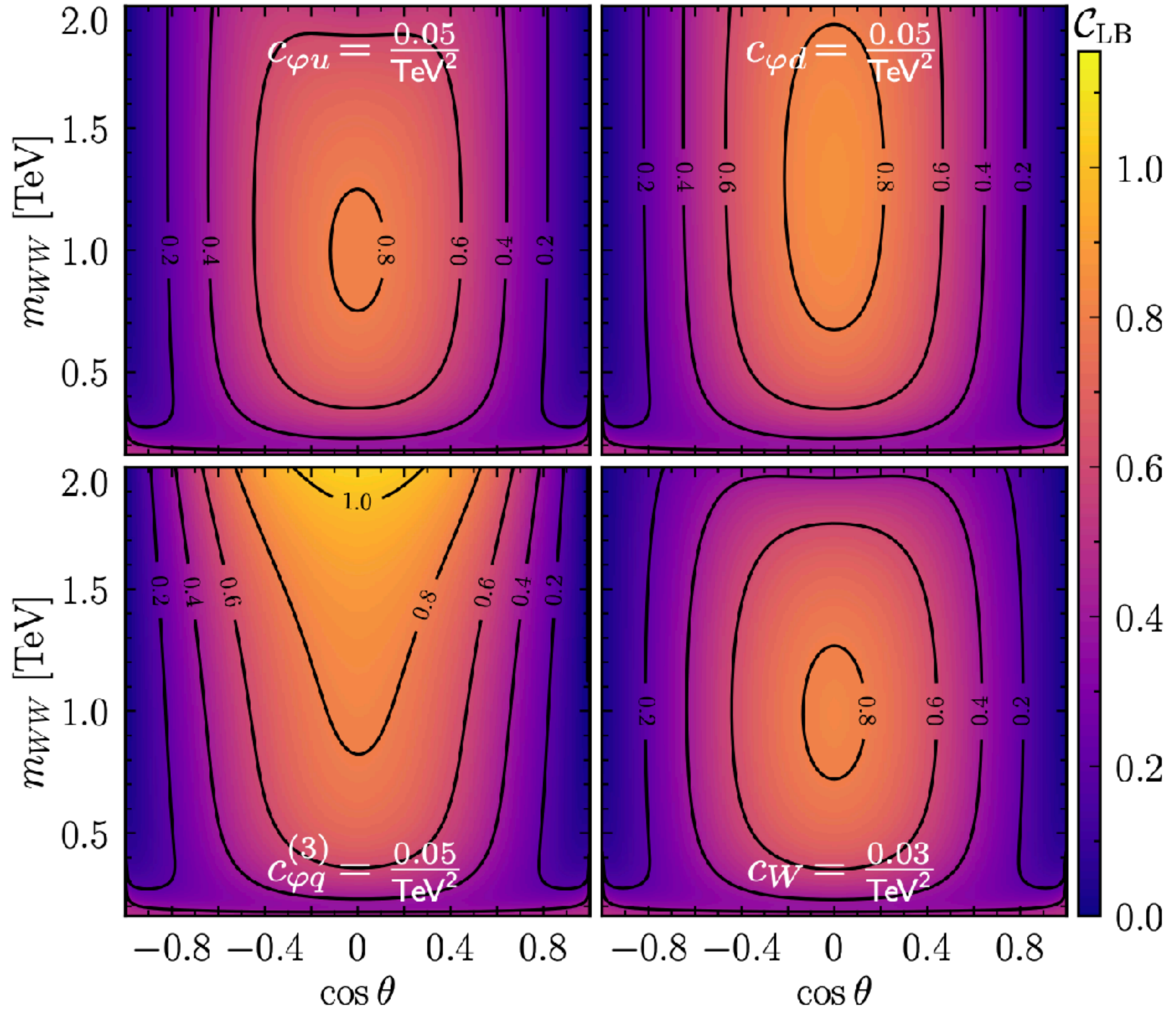
$\mathcal{O}_{\varphi WB}$ changes TGC coupling

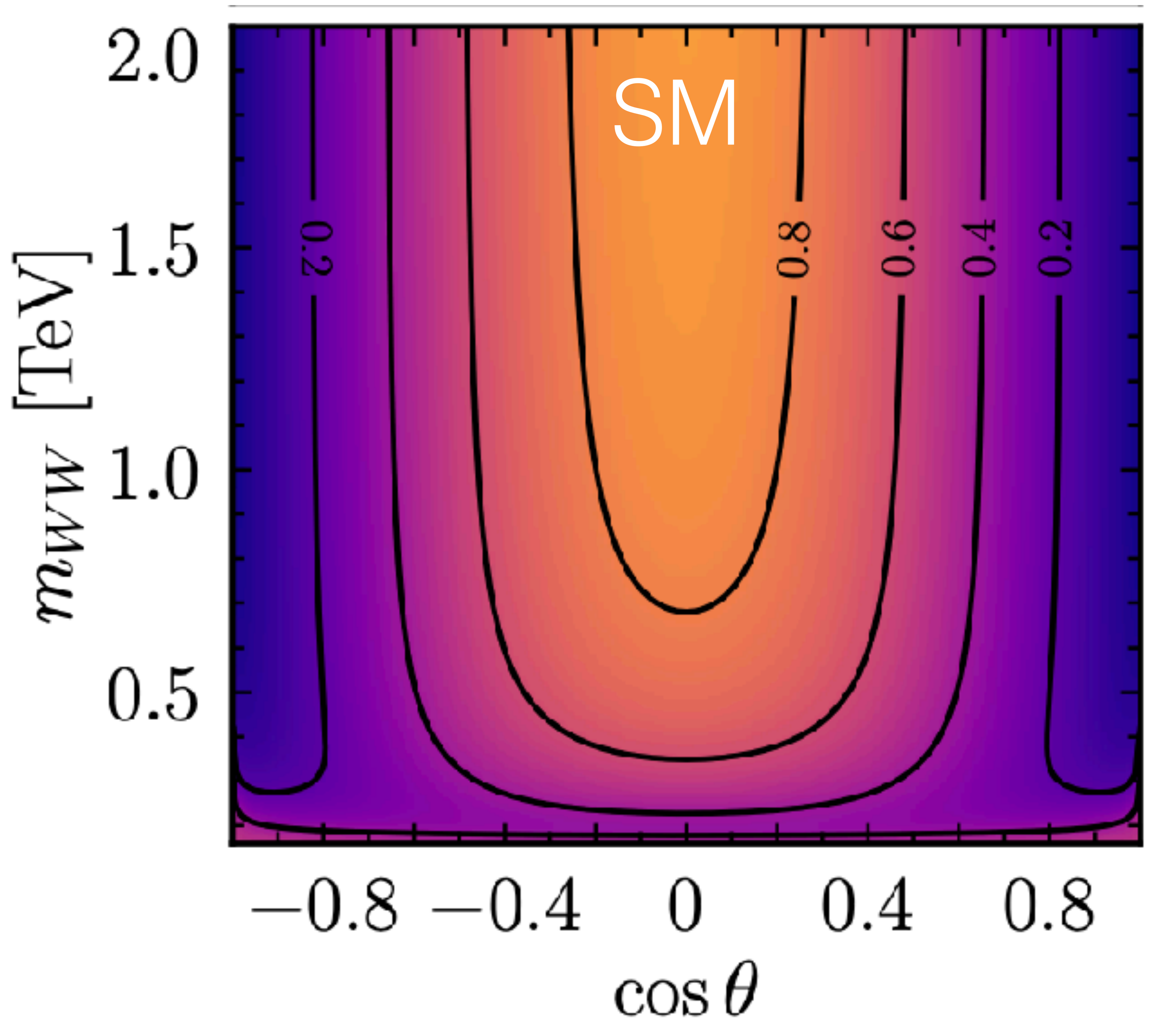
$\mathcal{O}_W$ new Lorentz structure

- small effect

SMEFT entanglement deviations

$pp \to W^+W^-$

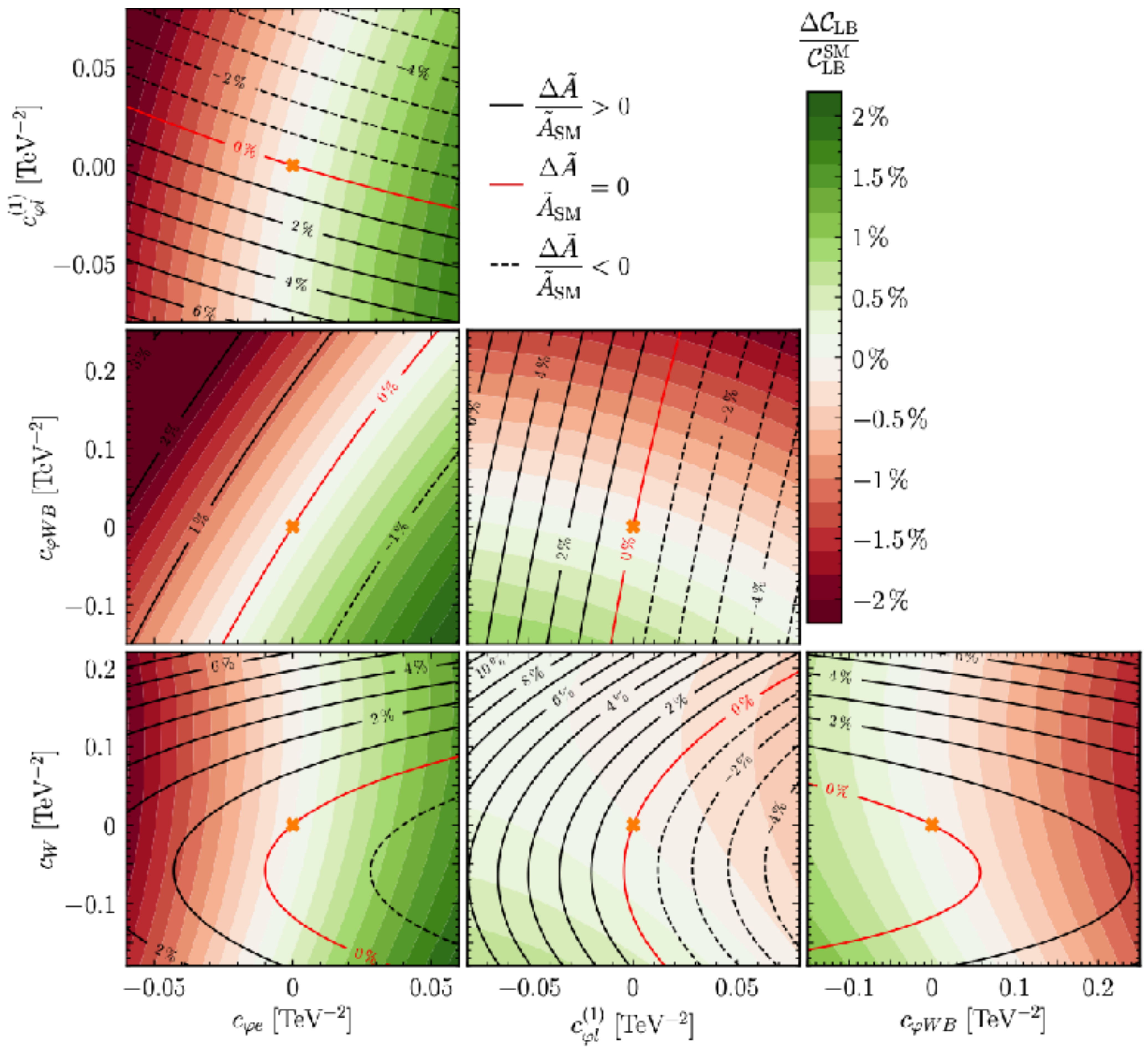




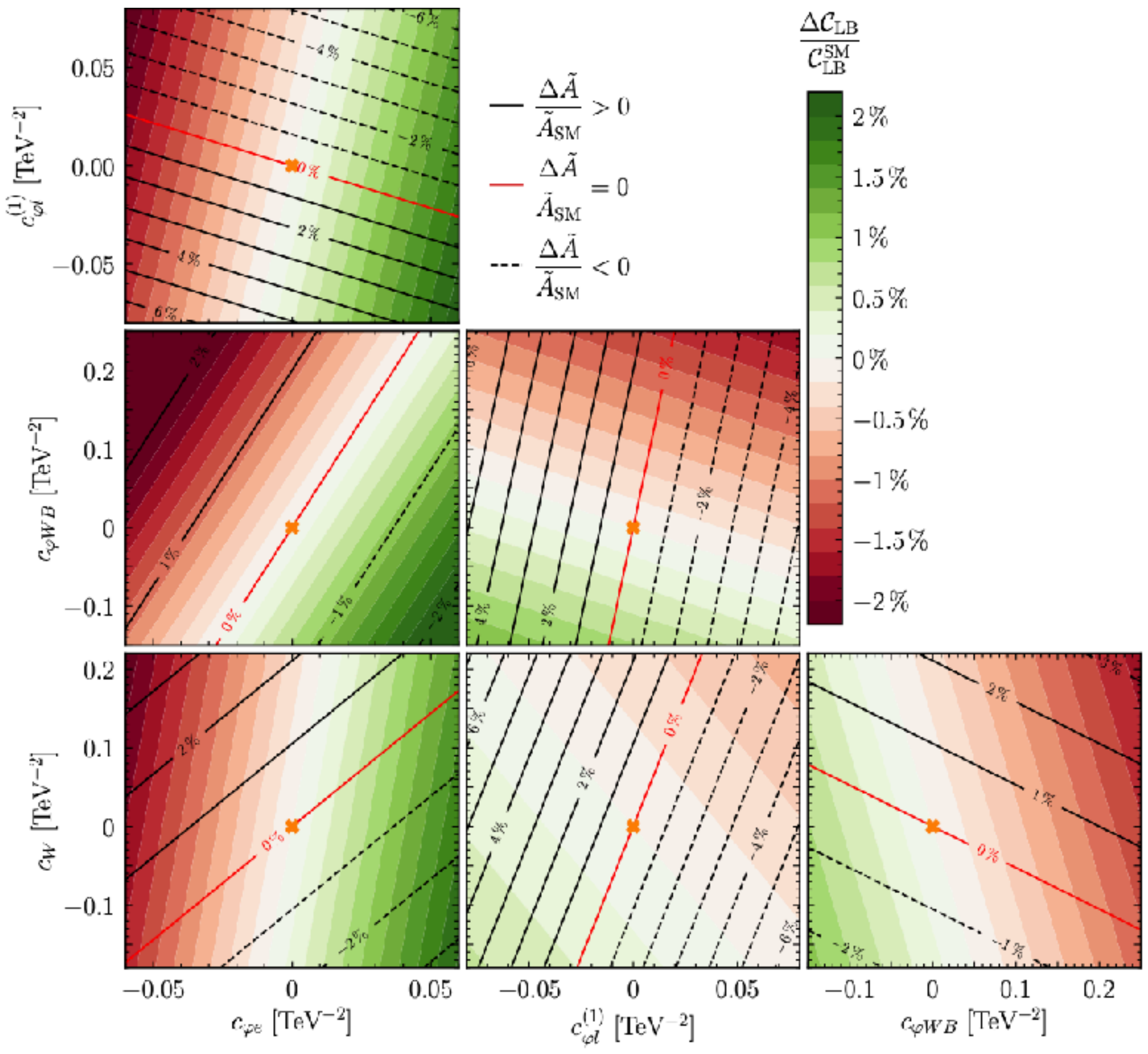
Similar balance changes for hadron collider.

SMEFT entanglement deviations: Central region

$$m_{WW} = 500\,\mathrm{GeV} \quad \cos\theta = 0$$



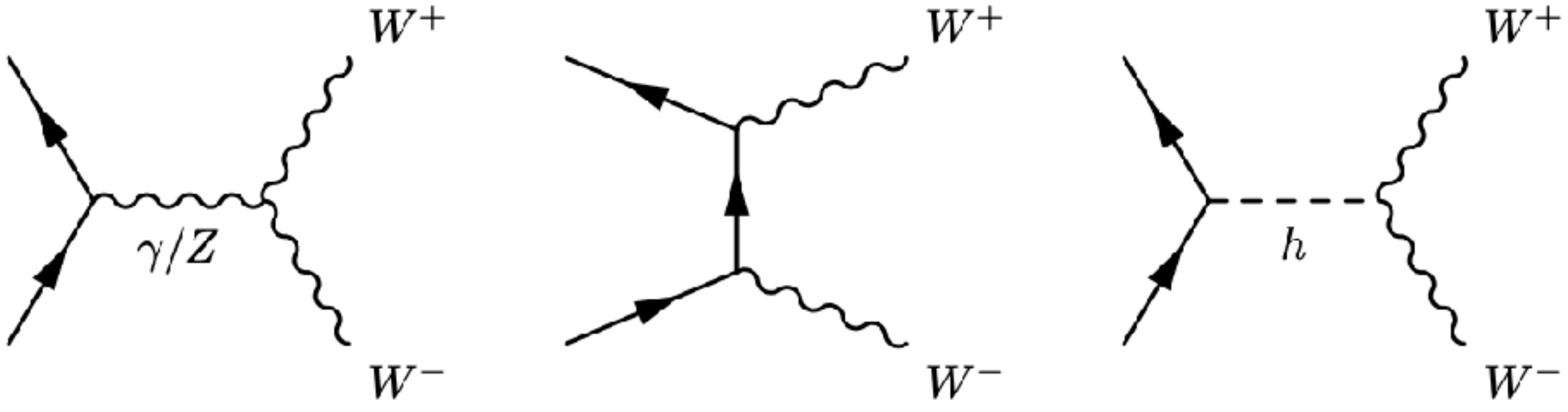
linear+squared dim-6



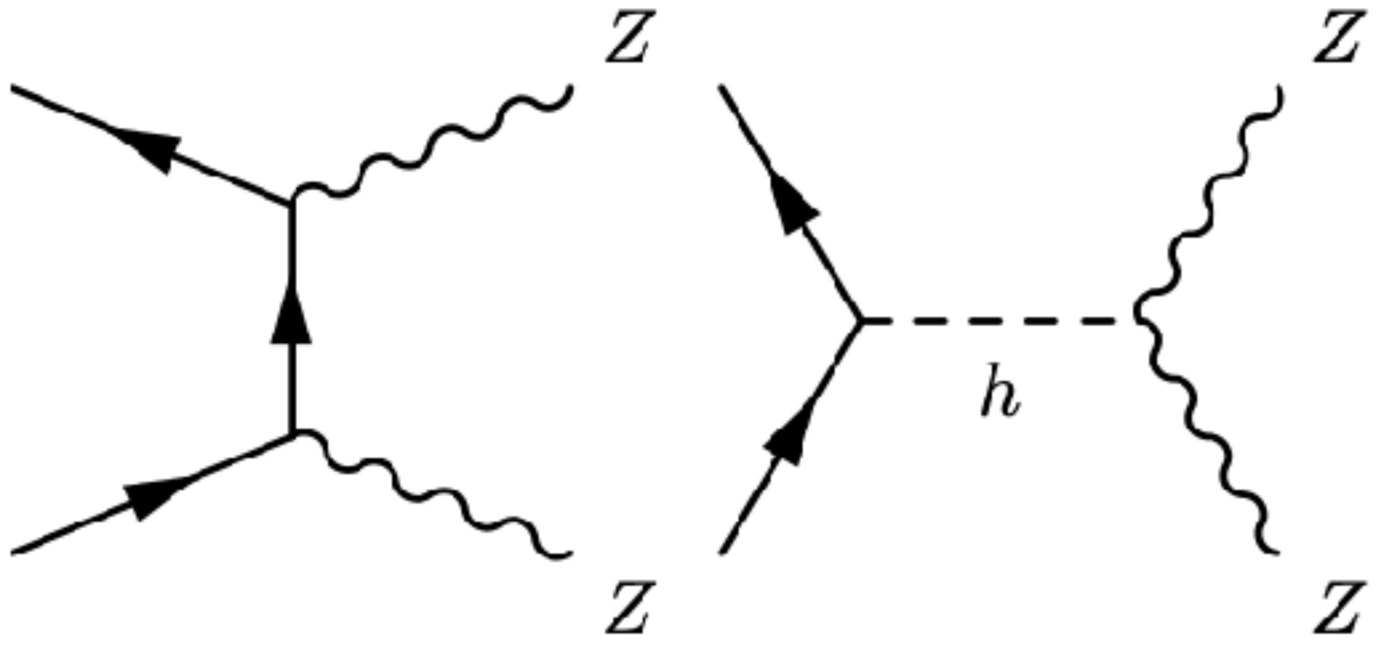
Linear dim-6

EW boson production at colliders

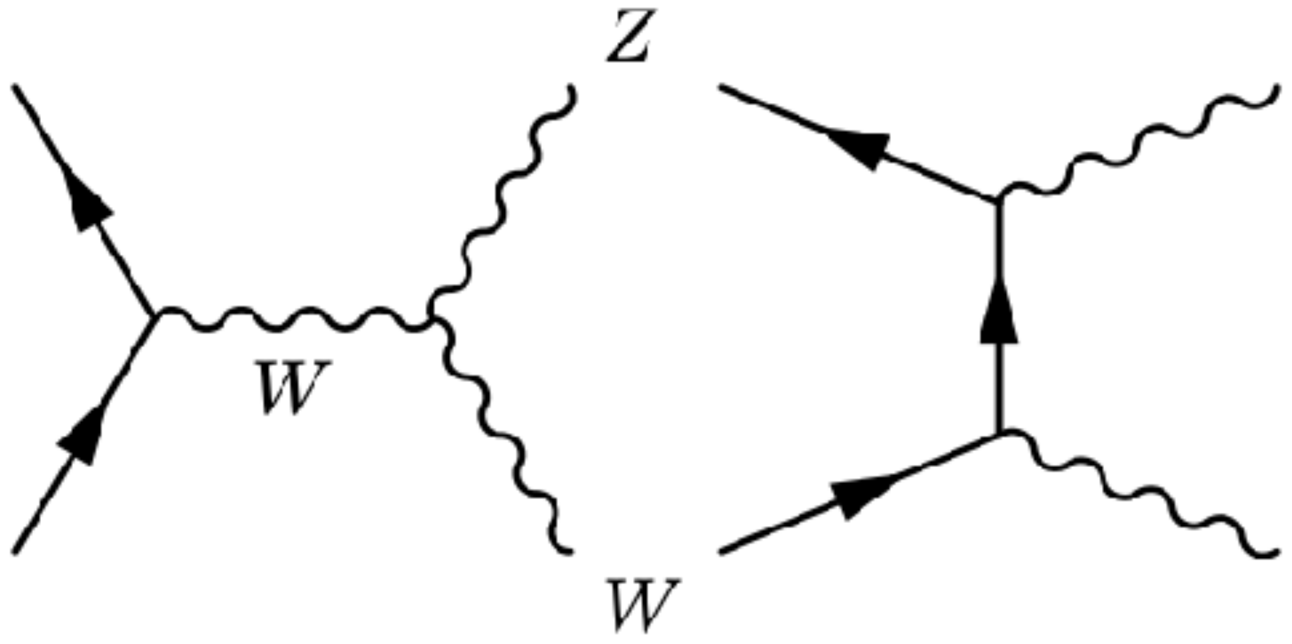
WW:



ZZ:

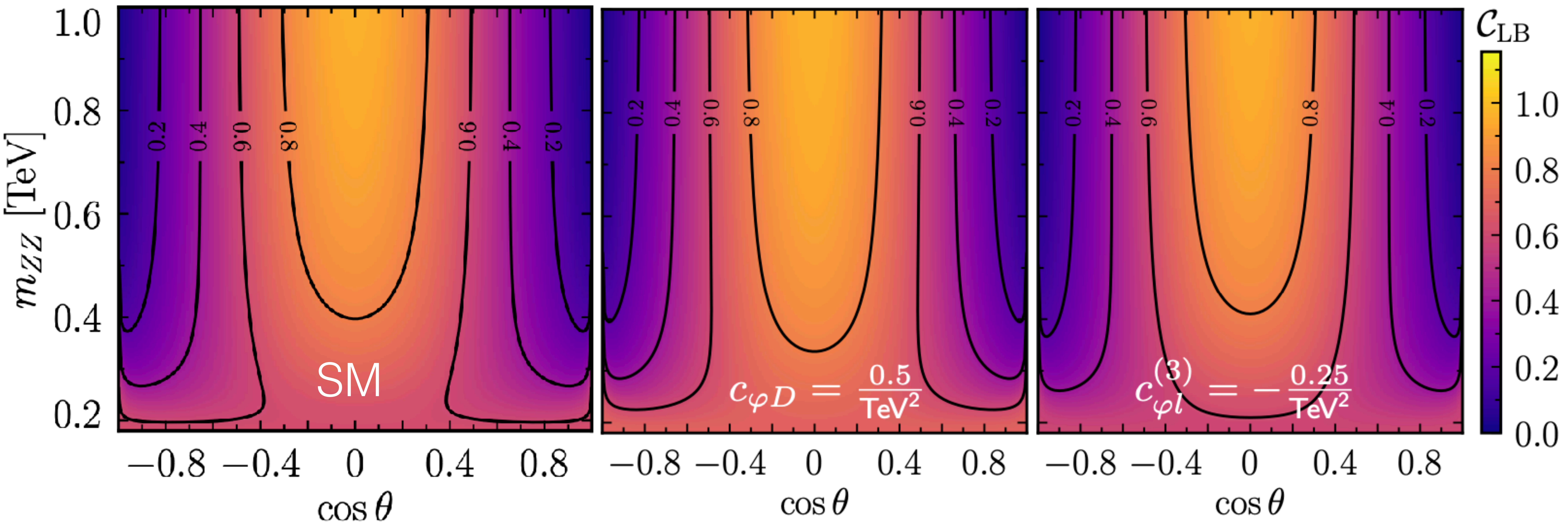


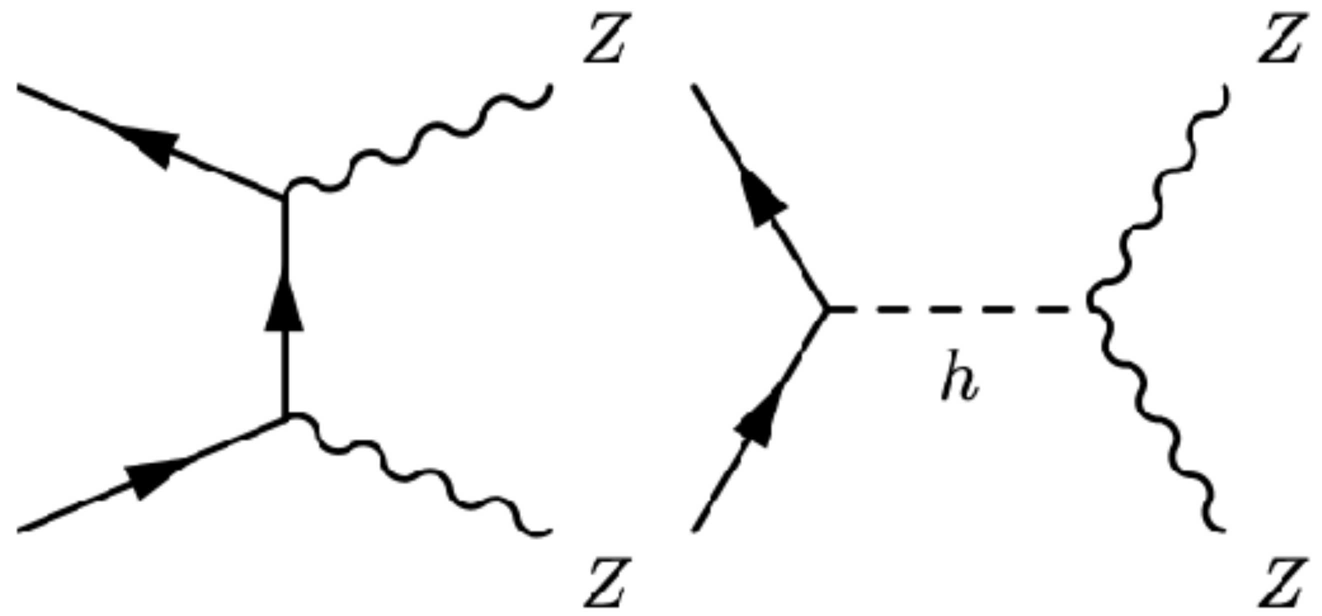
WZ:



EW boson production at colliders

$e^+e^- \to ZZ$





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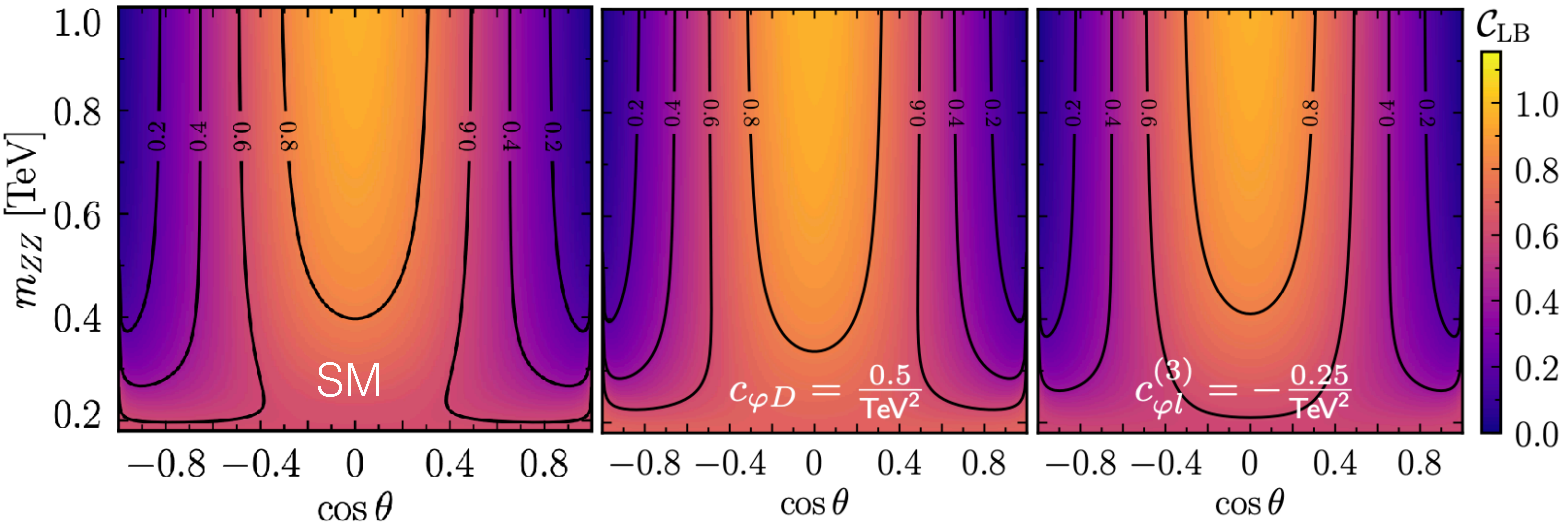

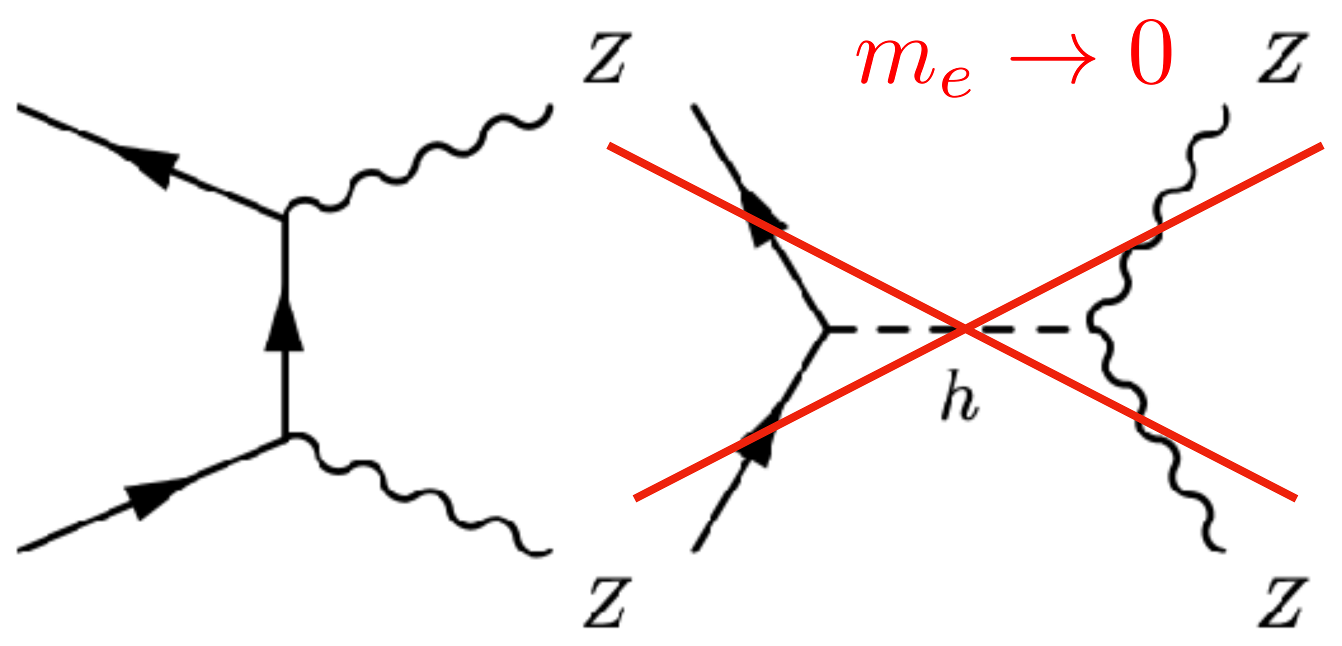

SMEFT just change the balance between RH and LH couplings

effects are reduced in $pp \to ZZ$

Perturbative Unitarity and Entanglement

The density matrix (and angular observables) are sensitive to new directions

$$e^+e^- \to W^+W^-$$

$(\lambda_1\lambda_2\|\alpha\,\beta)$	SM	EFT Λ^{-2} : c_{WWW}
$+-00$	$-2\sqrt{2}G_F m_Z^2 \sin\theta$	-
$+--+$	$2\sqrt{2}G_F m_W^2 \sin\theta$	-
$+-+-$	$-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3\theta \csc^4(\theta/2)$	-
$+-\pm\pm$	-	$3\cdot 2^{1/4}\sqrt{G_F}m_W \sin\theta\,(4m_W^2 x^2 - m_Z^2)$
$+-0\pm$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\pm 1+\cos\theta)\,x$
$+-\pm 0$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\mp 1+\cos\theta)\,x$
$-+00$	$2\sqrt{2}G_F(m_Z^2 - m_W^2)\sin\theta$	-
$-+\pm\pm$	-	$6\cdot 2^{1/4}\sqrt{G_F}m_W(m_Z^2 - m_W^2)\sin\theta$

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$+-\pm\pm$	-	$3\cdot 2^{1/4}\sqrt{G_F}m_W \sin\theta\,(4m_W^2x^2 - m_Z^2)$
$+-0\pm$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\pm 1+\cos\theta)\,x$
$+-\pm 0$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\mp 1+\cos\theta)\,x$
$-+00$	$2\sqrt{2}G_F(m_Z^2 - m_W^2)\sin\theta$	-
$-+\pm\pm$	-	$6\cdot 2^{1/4}\sqrt{G_F}m_W(m_Z^2 - m_W^2)\sin\theta$

No interference! ⟶ Cross-section $\tilde{A}(\mathcal{O}_W) \sim 0$

Perturbative Unitarity and Entanglement

$(\lambda_1\lambda_2\|\alpha\,\beta)$	SM	EFT Λ^{-2} : c_{WWW}
$+-00$	$-2\sqrt{2}G_F m_Z^2 \sin\theta$	-
$+--+$	$2\sqrt{2}G_F m_W^2 \sin\theta$	-
$+-+-$	$-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3\theta \csc^4(\theta/2)$	-
$+-\pm\pm$	-	$3\cdot 2^{1/4}\sqrt{G_F}m_W \sin\theta\,(4m_W^2 x^2 - m_Z^2)$
$+-0\pm$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\pm 1+\cos\theta)\,x$
$+-\pm 0$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\mp 1+\cos\theta)\,x$
$-+00$	$2\sqrt{2}G_F(m_Z^2 - m_W^2)\sin\theta$	-
$-+\pm\pm$	-	$6\cdot 2^{1/4}\sqrt{G_F}m_W(m_Z^2 - m_W^2)\sin\theta$

$$\rho = \begin{bmatrix} \mathcal{M}_{++}\mathcal{M}^*_{++} & \mathcal{M}_{++}\mathcal{M}^*_{+-} & \cdots \\ \mathcal{M}_{+-}\mathcal{M}^*_{++} & \mathcal{M}_{+-}\mathcal{M}^*_{+-} & \cdots \\ \vdots & & \ddots \end{bmatrix}$$

The spin-density matrix has different helicity products

$$\tilde{a}_1(\mathcal{O}_W) \simeq \tilde{b}_1(\mathcal{O}_W) \simeq \bar{c}_W\, 2^{5/4}\, x\ \cos^4(\theta/2)(\cos\theta + 3)\csc\theta\,,$$

Entanglement is sensitive to off-diagonal contractions

$$\tilde{c}_{13} \simeq 3\,\bar{c}_W \cdot 2^{3/4}\cos^2(\theta/2)(3\cos\theta + 1)\cot(\theta/2)\,x$$

Recovers the energy growth!

Conclusions

SM induces maximal entanglement points/regions in ttbar

Purely linear interference SMEFT effects vanish in these regions!

Quadratic interference decreases the entanglement at these points

Missing dim-8 linear interference and double-insertions at $\mathcal{O}(\Lambda^{-4})$

Conclusions

SM induces maximal entanglement points/regions in ttbar

Purely linear interference SMEFT effects vanish in these regions!

Quadratic interference decreases the entanglement at these points

Missing dim-8 linear interference and double-insertions at $\mathcal{O}(\Lambda^{-4})$

Diboson production: Qutrits   Entanglement measures are more subtle

$e^+e^- \to W^+W^-, pp \to W^+W^-$ and $pp \to WZ$ are sensitive to dim-6 modifications

while $e^+e^- \to ZZ$ and $pp \to ZZ$ are less (but potentially for dim-8)

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