Using Fast simulation to understand Angular Resolutions

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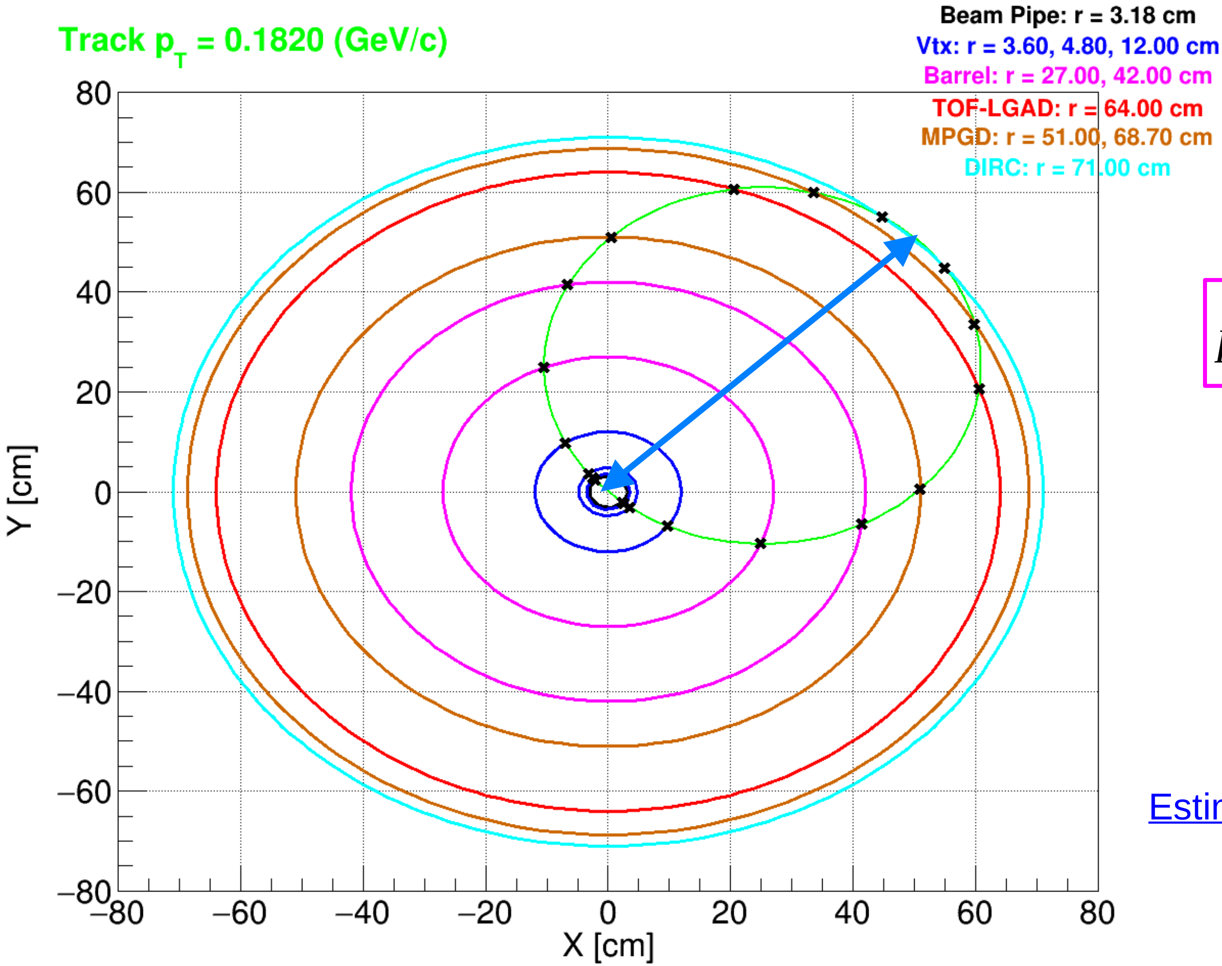
$$p_T(GeV/c) = 0.3\,B[T]\,R_{track}[m]$$



Extrapolation of track at DIRC R = 71 cm

At η = 0

$$R_{track} = \frac{R_{OutMPGD}}{2} = 0.3435\,m$$

$$p_{Tmin} = 0.3 * 1.7 * 0.3435 = 0.1752\,GeV/c$$

Fast Simulation based on **Global fit** and the **Kalman filter**

[Estimation of Theta/Phi resolutions at DIRC (71 cm)]

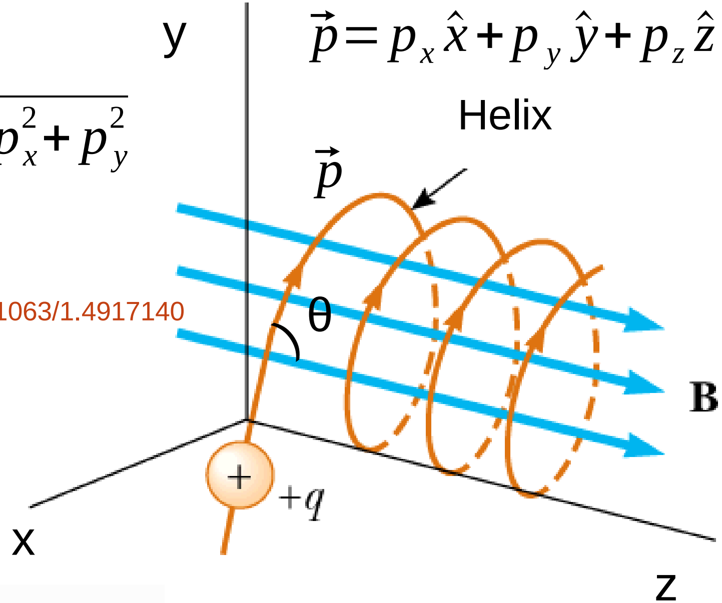
Charged Particle in Magetic Field (Lorentz Force)

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$p_T = \sqrt{p_x^2 + p_y^2}$$

If B is uniform the trajectory is helix (easier) apart from the deviations from the multiple scattering at each detector plane

https://doi.org/10.1063/1.4917140



RK propagator also used in Genfit (fun4All) B-field map

The Runge-Kutta-Nyström method from above can be adapted to handle second order differential equations, as is needed for the equations of motion in question,

Ref: ACTS

$$\frac{d^2\vec{r}}{ds^2} = \frac{q}{p}\left(\frac{d\vec{r}}{ds} \times \vec{B}(\vec{r})\right) = f(s, \vec{r}, \vec{T}), \qquad \vec{T} \equiv \frac{d\vec{r}}{ds},$$

Full Simulation: RK Propagator

Fast Simulation: Helix method

If B depends on **r (B map required)** the trajectory is predicted analytically solving RK method (RK track propagator)

Fast Simulation (Kalman and Global fit)

Fast Simulation (Kalman)

Track Parameters $\left(l_0,\ l_1\ ,\ \phi,\ \theta,\ 1/p\right)$

Parameter Covariance

$$\begin{array}{c|ccccc} & l_0 & l_1 & \phi & \theta & 1/p \\ \hline l_0 & \sigma_{l0}^2 & \sigma_{l0l1} & \sigma_{l0\phi} & \sigma_{l0\theta} & \sigma_{l0.1/p} \\ l_1 & \sigma_{l1l0} & \sigma_{l1}^2 & \sigma_{l1\phi} & \sigma_{l1\theta} & \sigma_{l1.1/p} \\ \phi & \sigma_{\phi l0} & \sigma_{\phi l1} & \sigma_{\phi}^2 & \sigma_{\phi\theta} & \sigma_{\phi.1/p} \\ \theta & \sigma_{\theta l0} & \sigma_{\theta l1} & \sigma_{\theta\phi} & \sigma_{\theta}^2 & \sigma_{\theta.1/p} \\ 1/p & \sigma_{1/p.l0} & \sigma_{1/p.l1} & \sigma_{1/p.\phi} & \sigma_{1/p.\theta} & \sigma_{1/p}^2 \end{array}$$

Symmetric matrix = 5(5+1)/2 = 15 independent entries

Two Independent Algorithms

Extrapolate to the Vertex

$\sigma_{l0} \rightarrow \sigma(DCA_{xy})$ $\quad \sigma_{l1} \rightarrow \sigma(DCA_z)$

$\sigma_\theta \quad \sigma_\phi$ $\quad \frac{\sigma_{1/p}}{(1/p)} = \frac{1/p^2 * \sigma_p}{(1/p)} = \frac{\sigma_p}{p}$

https://indico.bnl.gov/event/17750/contributions/71187/attachments/44843/75637/EPIC_Tracking_Meeting_Shyam1Dec2022.pdf

Three Options (Kalman):

1. **Outward-->Inward fitting**
2. **Inward--> Outward fitting**
3. Combined estimate (Weighted average)

Fast Simulation (Global fit)-generates intermediate distributions (DCA_{xy}, Δp_T, $\Delta\phi$, $\Delta\theta$)-See backup

Assuming uniform magnetic field (helix), ignoring energy loss, assuming Gaussian multiple scattering

Provide the **best parameters (global)** for the track based on simultaneous chi2 minimization considering all hit points

Global fit developed by Shyam with the help of Annalisa, F.Colamaria, Domenico Elia, and G.E. Bruno

Tracking Performances (Kalman and Global fit)

Spatial Resolution (SR): Uncertainity associated with pixel size ($\sigma_{r\varphi}$)
Multiple Scattering (MS): Uncertainity associated with material thickness (x/X_0)

$$\sigma_{d_0} = \sqrt{\sigma_{d_0 SR}^2 + \sigma_{d_0 MS}^2}$$

$$\frac{\sigma_{p_T}}{p_T} = \sqrt{\left(\frac{\sigma_{p_T}}{p_T}\right)_{SR}^2 + \left(\frac{\sigma_{p_T}}{p_T}\right)_{MS}^2}$$

arXiv:1805.12014 [physics.ins-det]

$$\Delta d_0|_{res} \approx \frac{3\sigma_{r\phi}}{\sqrt{N+5}}\sqrt{1+\frac{8r_0}{L_0}+\frac{28r_0^2}{L_0^2}+\frac{40r_0^3}{L_0^3}+\frac{20r_0^4}{L_0^4}}$$

$$\frac{\sigma_{p_T}}{p_T}(SR) \propto \sigma_{r\phi}\, p$$

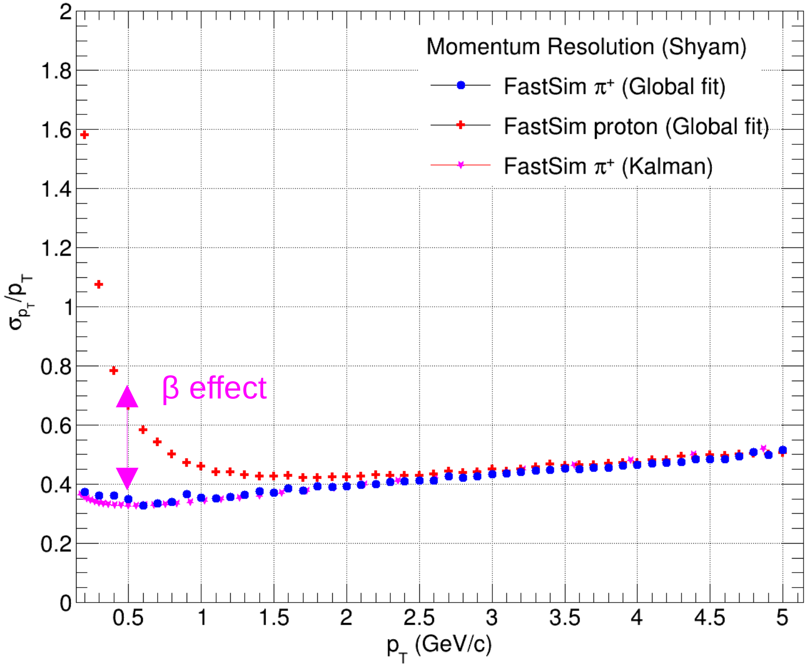
Curvature

$$\frac{\sigma_{p_T}}{p_T}(MS) \propto \frac{1}{\beta p} \qquad p = \frac{Const}{\beta}$$

Curvature

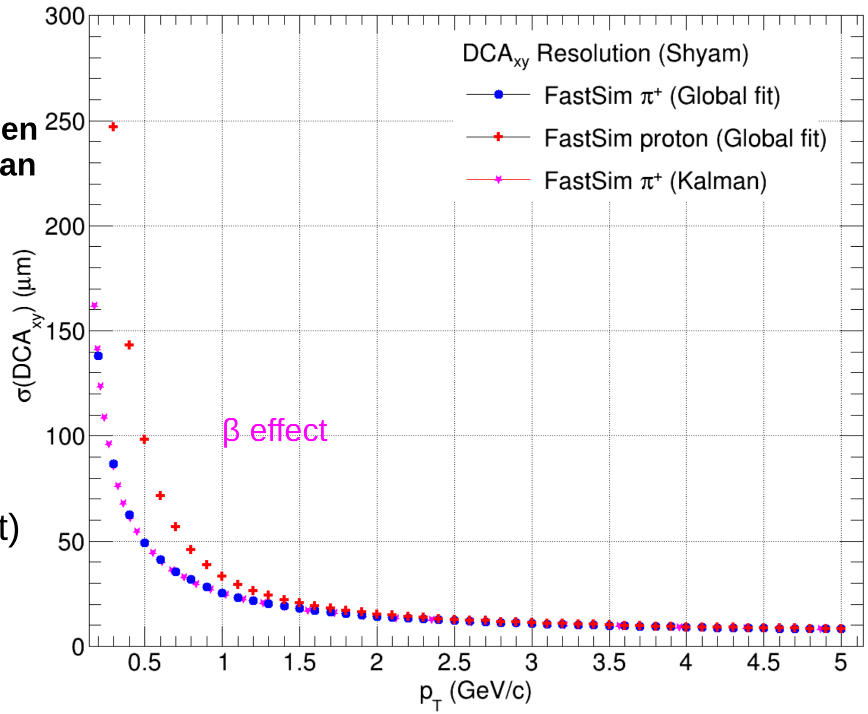
Momentum and mass Hypothesis

$$\Delta d_0|_{m.s.} \approx \frac{0.0136\,\mathrm{GeV/c}}{\beta p_T} r_0 \sqrt{\frac{d}{X_0 \sin\theta}}\sqrt{1+\frac{1}{2}\left(\frac{r_0}{L_0}\right)+\frac{N}{4}\left(\frac{r_0}{L_0}\right)^2}$$



Compatiblity between Global fit and Kalman

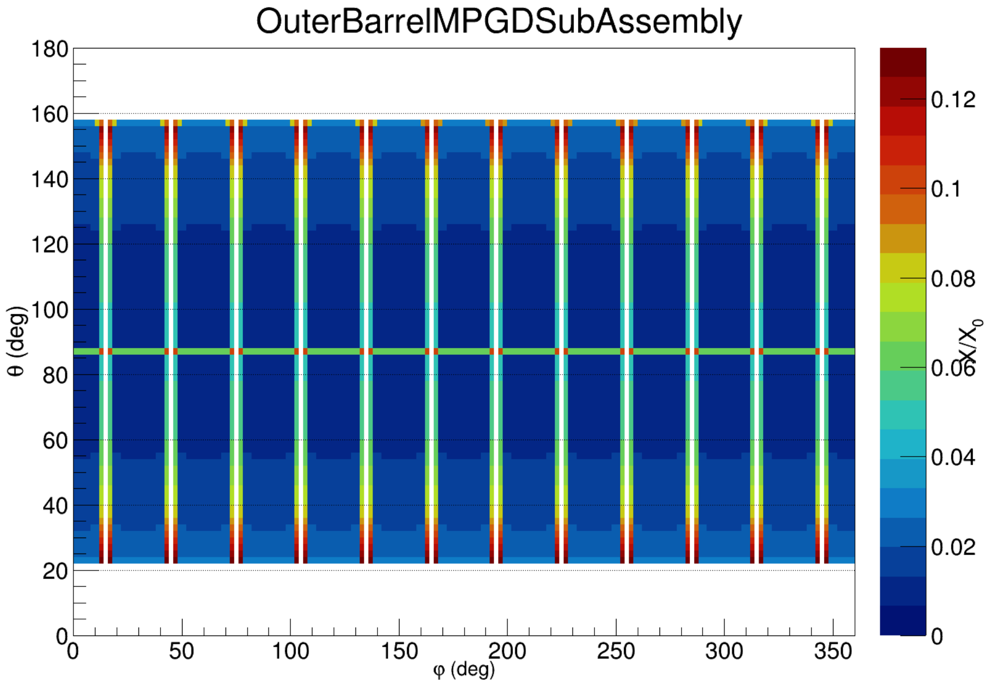
20k pion-simulations for each p_T (global fit)



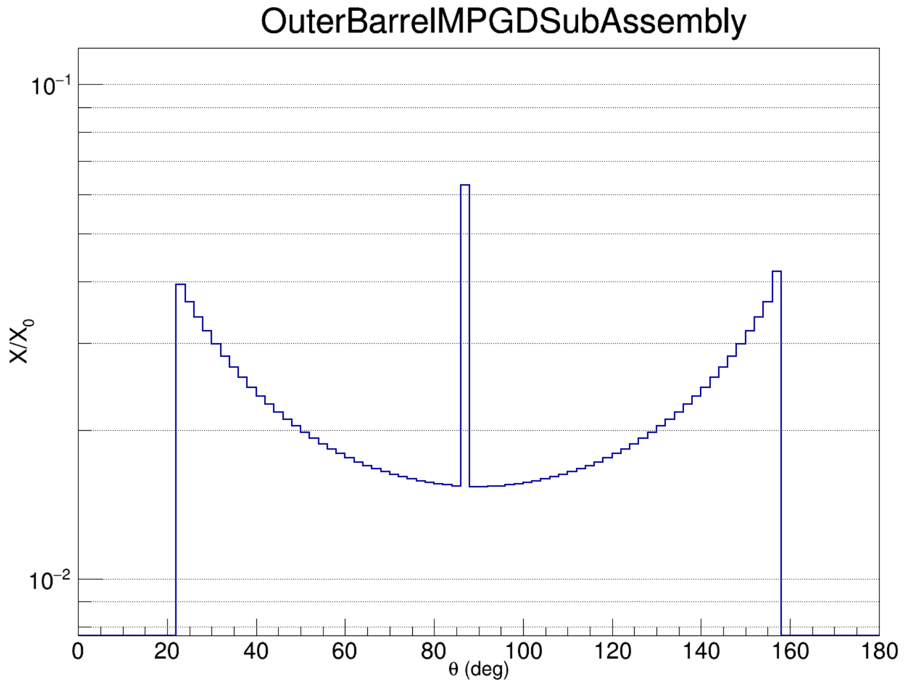
Material Budget (Outer MPGD)

In fast simulation used x/X_0 = 0.0153859

2D Map

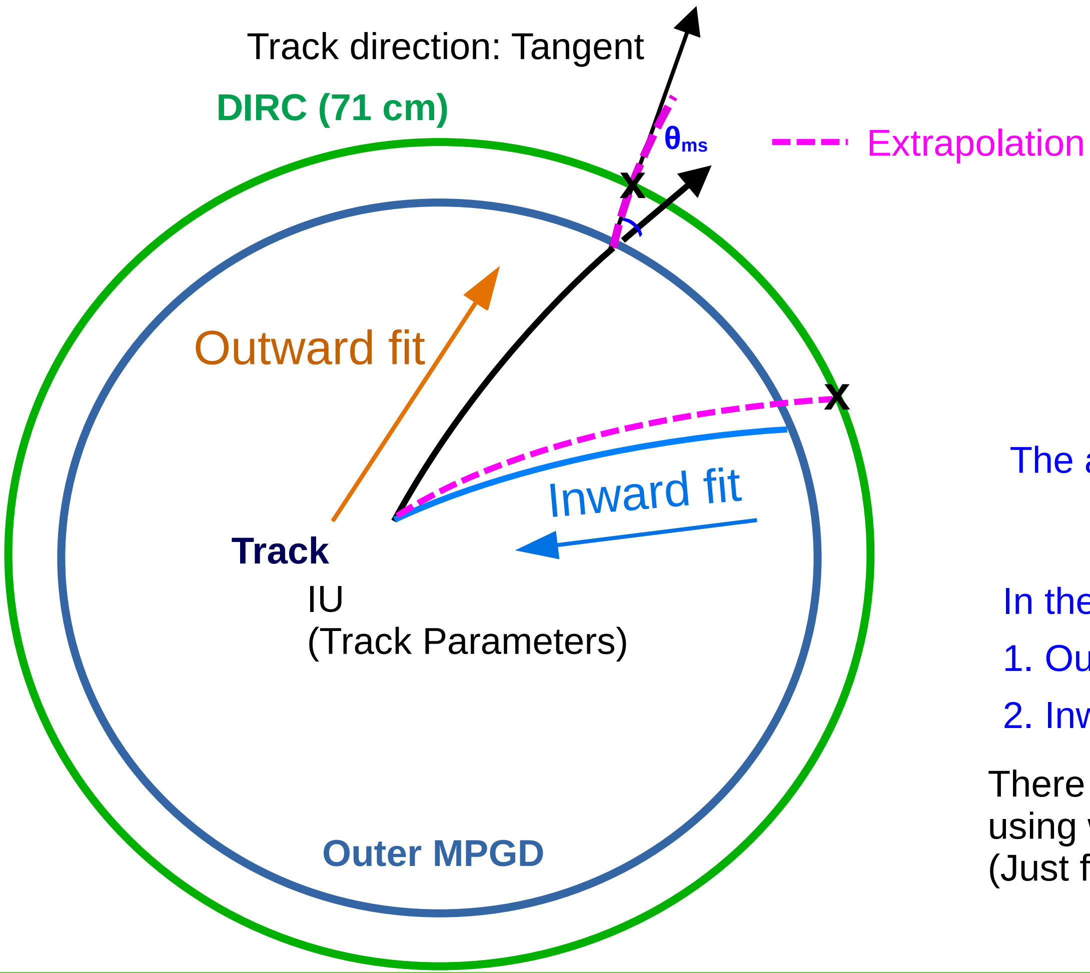


1D Map

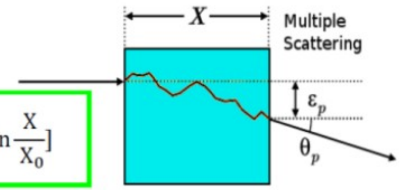


Fast Simulation Including Multiple Scattering of MPGD Layer (Pion+/Proton)

1. Fit the track with Kalman fit (Outward-->Inward) and estimate parameter at the Innermost Update (IU)
2. Extrapolate the track at DIRC layer (71 cm) including the multiple scattering effect of Outer MPGD layer (1.5% X_0)
3. Extract the uncertainties in Theta/Phi Resolutions



Multiple Scattering (M.S.)



$$\theta_p = 13.6 \frac{\text{MeV}z}{\beta c p} \sqrt{\frac{X}{X_0}} [1 + 0.038 \ln \frac{X}{X_0}]$$

$$\epsilon_p \approx \frac{1}{\sqrt{3}} \theta_p X$$

Ref: “Particle Detectors”, C. Grupen and B. A. Shwartz

At low p: m.s. dominated by $\propto \frac{1}{p}$

Issue: Extrapolation at very large distance (71 cm)

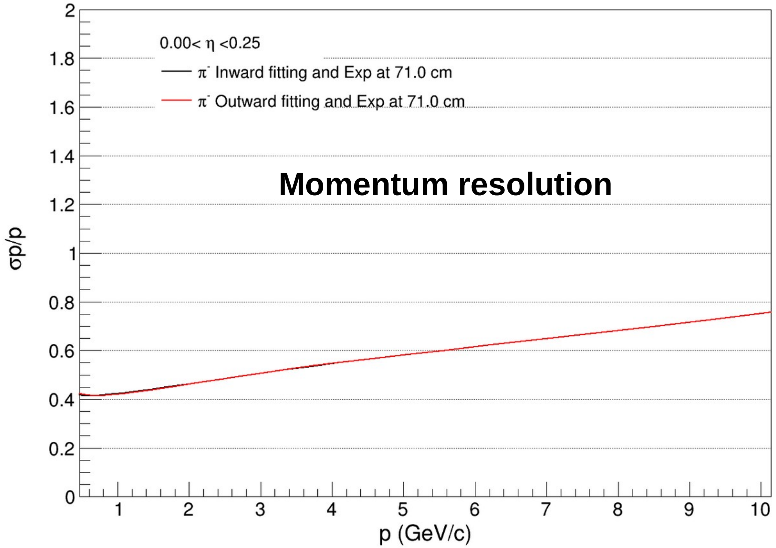
The algorithm is modified for Inward-->Outward track fitting to avoid extrapolation to large distance

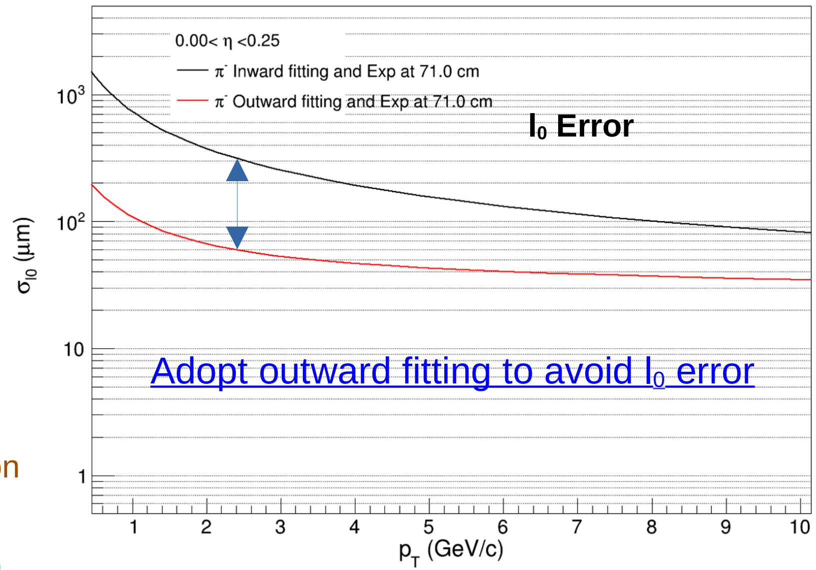
In the algorithm, its done in two steps

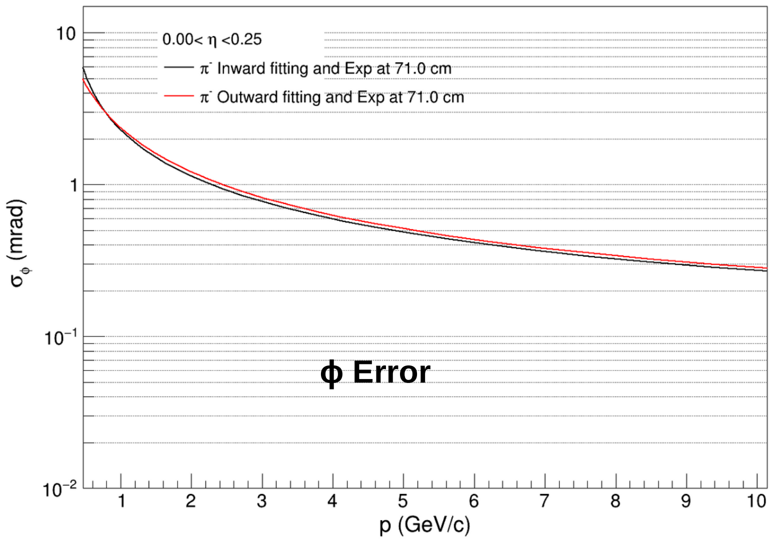
1. Outward-->Inward
2. Inward-->Outward

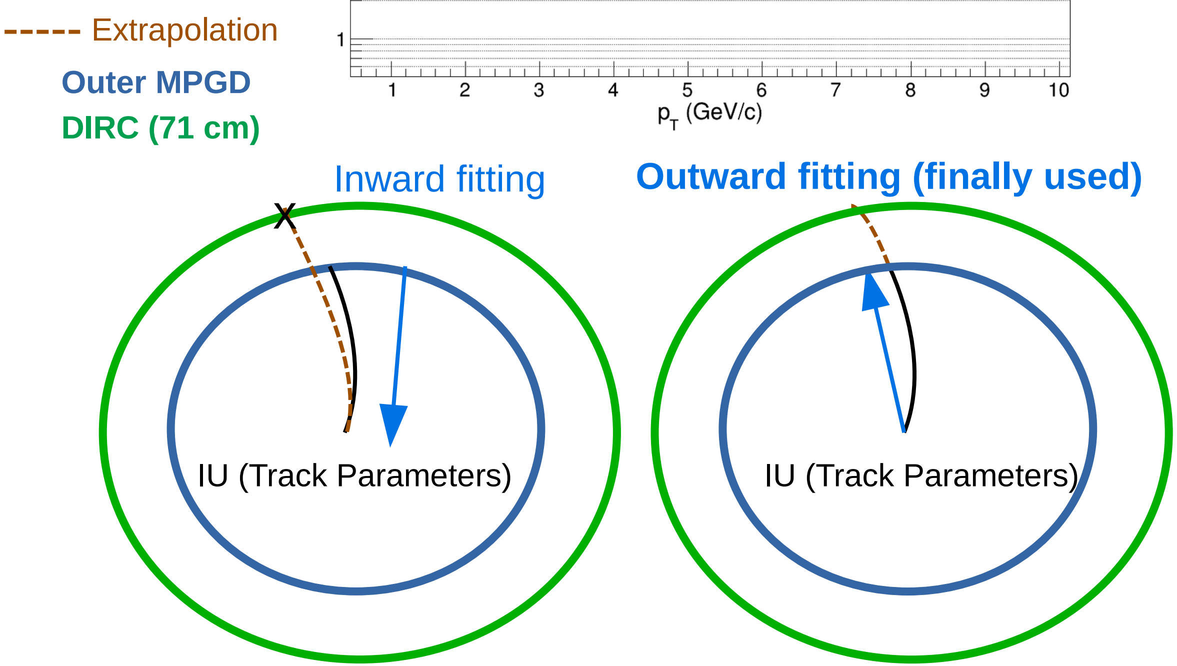
There is also a possibility to use the combined estimate using weighted average of Inward and Outward method (Just for information)

Fast Simulation [Inward vs Outward]







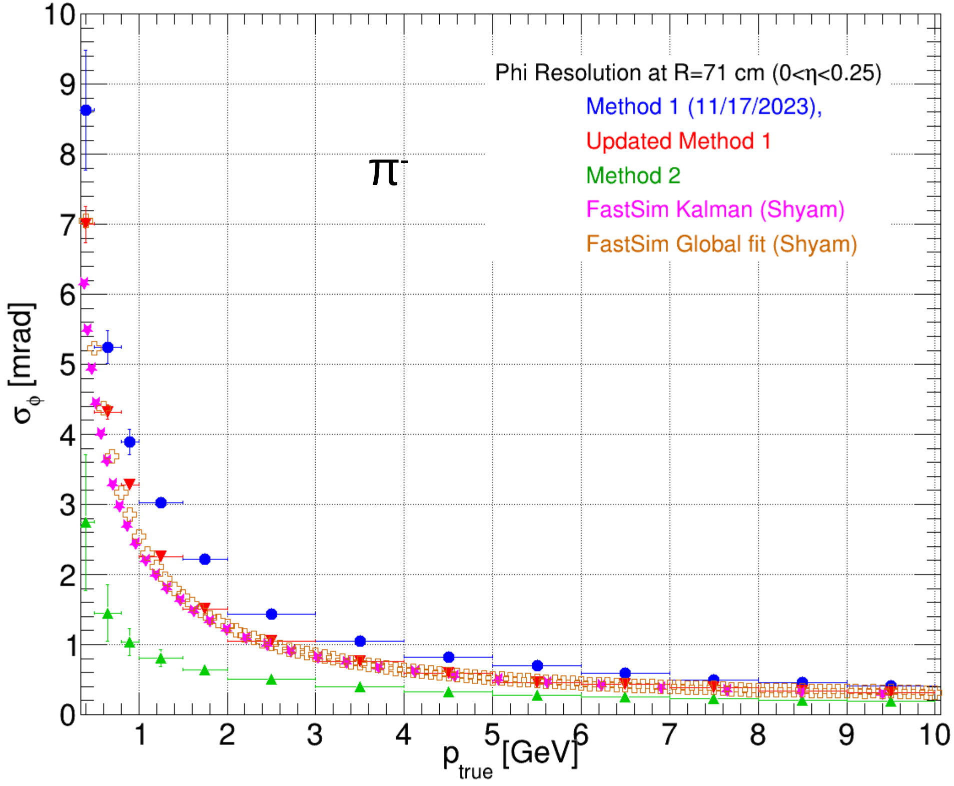


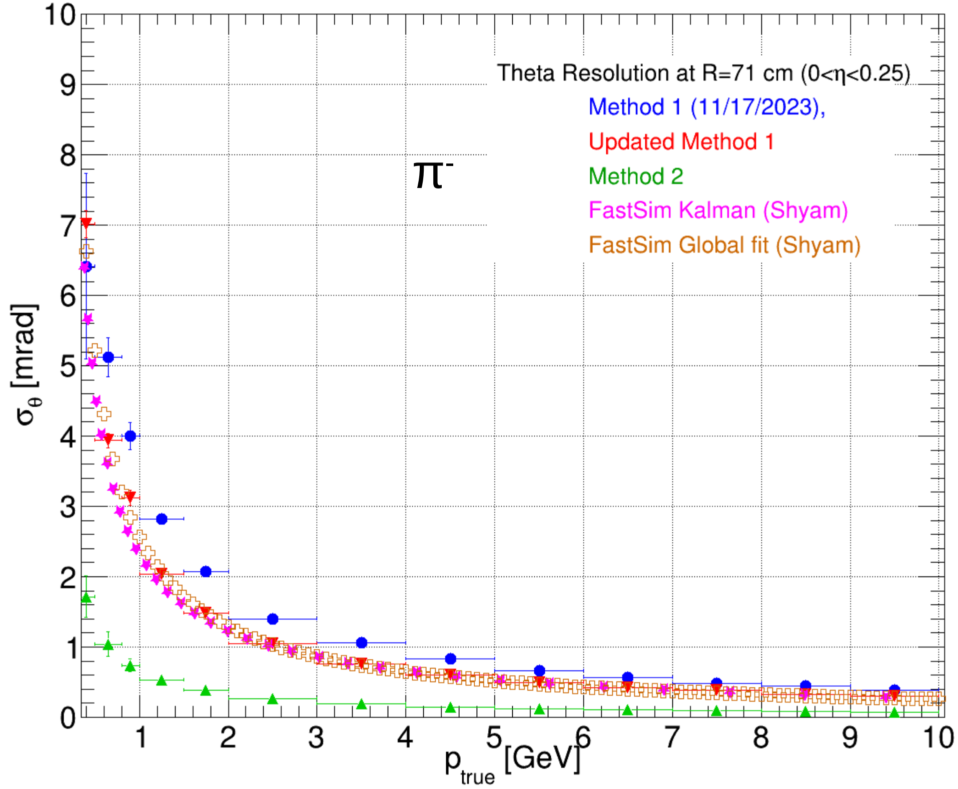
Theta/Phi Resolutions for Pion+

FastSim (Kalman) uses Inward to Outward fitting algorithm considering multiple scattering at the Outer MPGD layer

Global fit also take care of multiple scattering at Outer MPGD layer (parameters are global)

Updated Method 1 is closest to the two independent Fast Simulation method



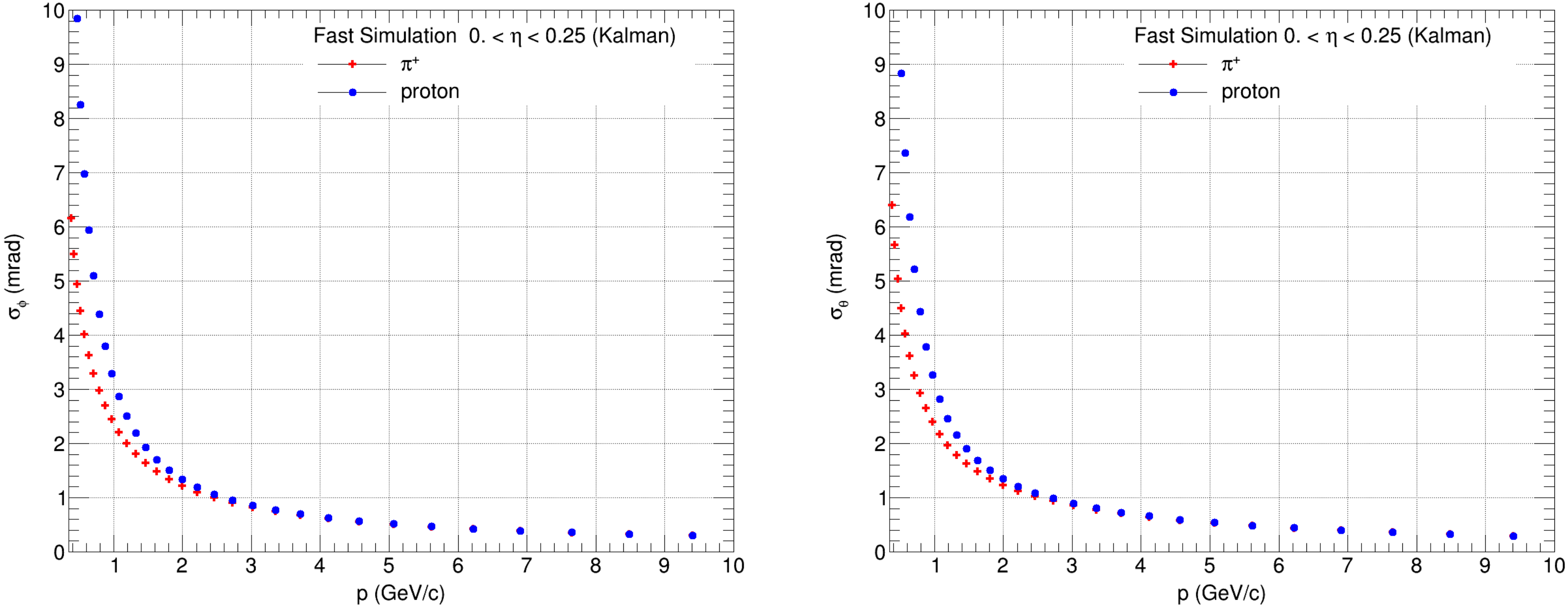


For Method 1, Method 2, and Updated Method 1

https://indico.bnl.gov/event/21559/contributions/84571/attachments/51631/88303/TrackingWG_12-14-2023.pdf

Fast Simulation Including Multiple Scattering of MPGD Layer (Pion+/Proton)

Inward-->Outward fitting, then extrapolation at 71 cm considering M.S. at Outer MPGD layer



Mass hypothesis is changed to proton

Spatial Resolution and Multiple Scattering (Fast Simulation)

$$\Delta\phi|_{res.} = \frac{\sqrt{12}\sigma_{r\phi}}{L_0\sqrt{(N-1)(N+1)(N+2)(N+3)}}\sqrt{(16N^3+2N^2-3N)+\frac{60N^3 r_0}{L_0}+\frac{60N^3 r_0^2}{L_0^2}}$$

$$\approx \frac{\sigma_{r\phi}}{L_0}\frac{8\sqrt{3}}{\sqrt{N+5}}\sqrt{1+\frac{15}{4}\frac{r_0}{L_0}+\frac{15}{4}\frac{r_0^2}{L_0^2}}$$

$$\Delta\phi|_{m.s.} = \frac{1}{\beta p_T} f\left(\frac{d}{X_0\sin\theta}\right)\sqrt{\frac{N-3/4}{N-1}+\frac{N}{N-1}\left(\frac{r_0}{L_0}\right)+\frac{N^2}{N-1}\left(\frac{r_0}{L_0}\right)^2}$$

$$\approx \frac{0.0136\,\mathrm{GeV/c}}{\beta p_T}\sqrt{\frac{d}{X_0\sin\theta}}\sqrt{1+\left(\frac{r_0}{L_0}\right)+\left(\frac{r_0}{L_0}\right)^2}$$

$$\Delta\theta|_{res.} = \frac{\sigma_z\sin^2\theta}{L_0}\sqrt{\frac{12N}{(N+1)(N+2)}}$$

$$\approx \frac{2\,\sigma_z\sin^2\theta}{L_0}\sqrt{\frac{3}{N+3}}$$

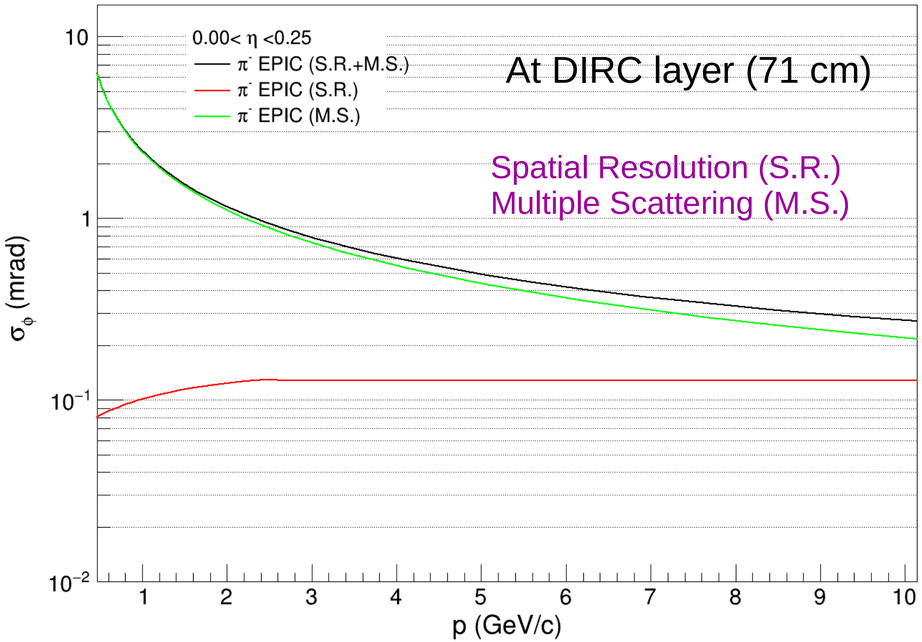
$$\Delta\theta|_{m.s.} = \frac{\sin\theta}{\beta\, p_T} f\left(\frac{d}{X_0\sin\theta}\right)$$

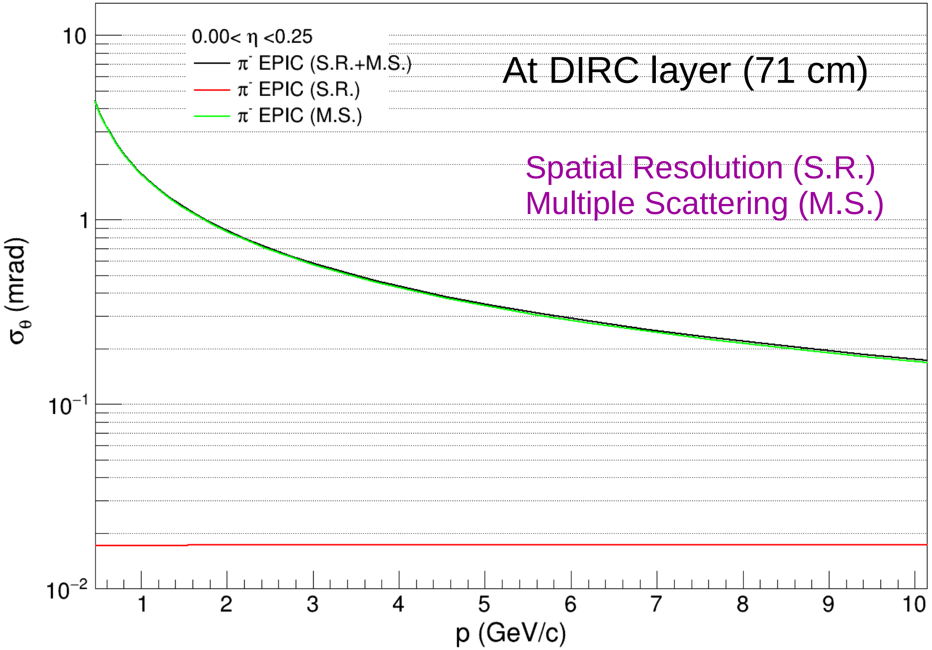
$$\approx \frac{0.0136\,\mathrm{GeV/c}\,\sin\theta}{\beta\, p_T}\sqrt{\frac{d}{X_0\sin\theta}}$$

Fomula for Theta/Phi resolution w.r.t. vertex

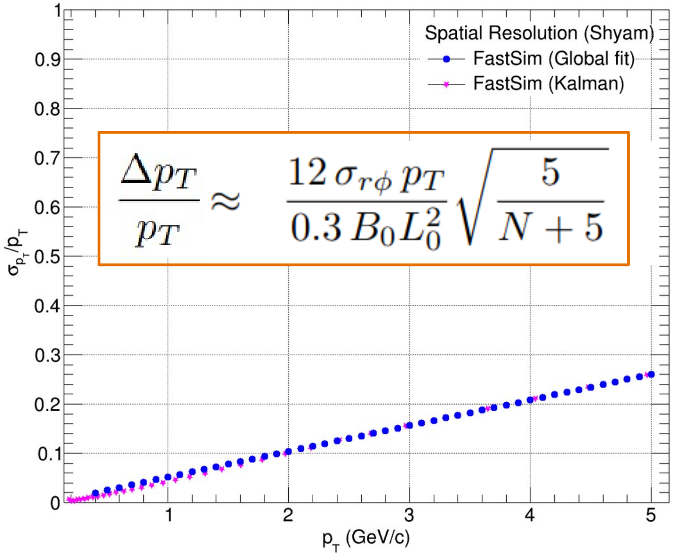
arXiv:1805.12014 [physics.ins-det]

Case of Outward-->Inward fitting and then extrapolation to 71.0 cm considering M.S.





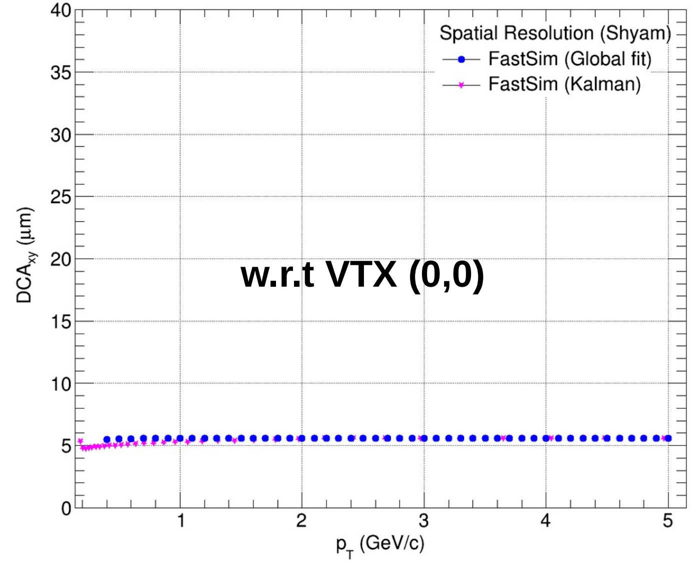
Fast Simulation (Global fit)-Spatial Resolution (η = 0)

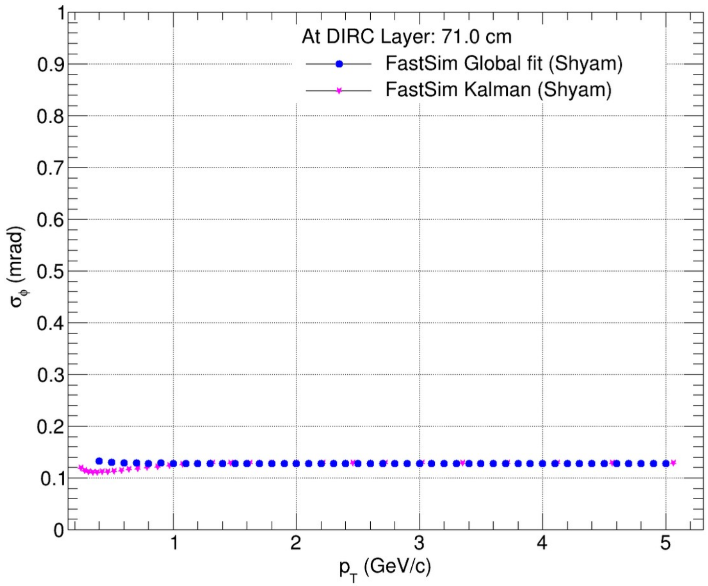


$$\frac{\Delta p_T}{p_T} \approx \frac{12\,\sigma_{r\phi}\,p_T}{0.3\,B_0 L_0^2}\sqrt{\frac{5}{N+5}}$$

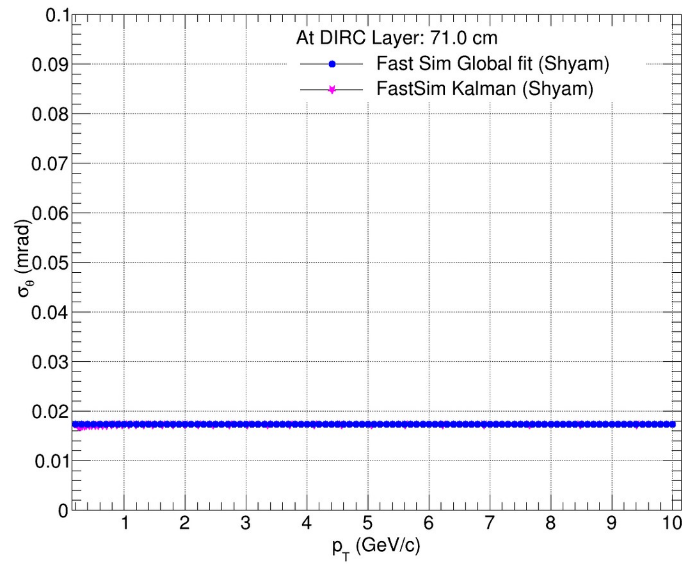
Considering only spatial resolution effect and ignoring the multiple scattering

arXiv:1805.12014 [physics.ins-det]





[Inward-->Outward fitting]



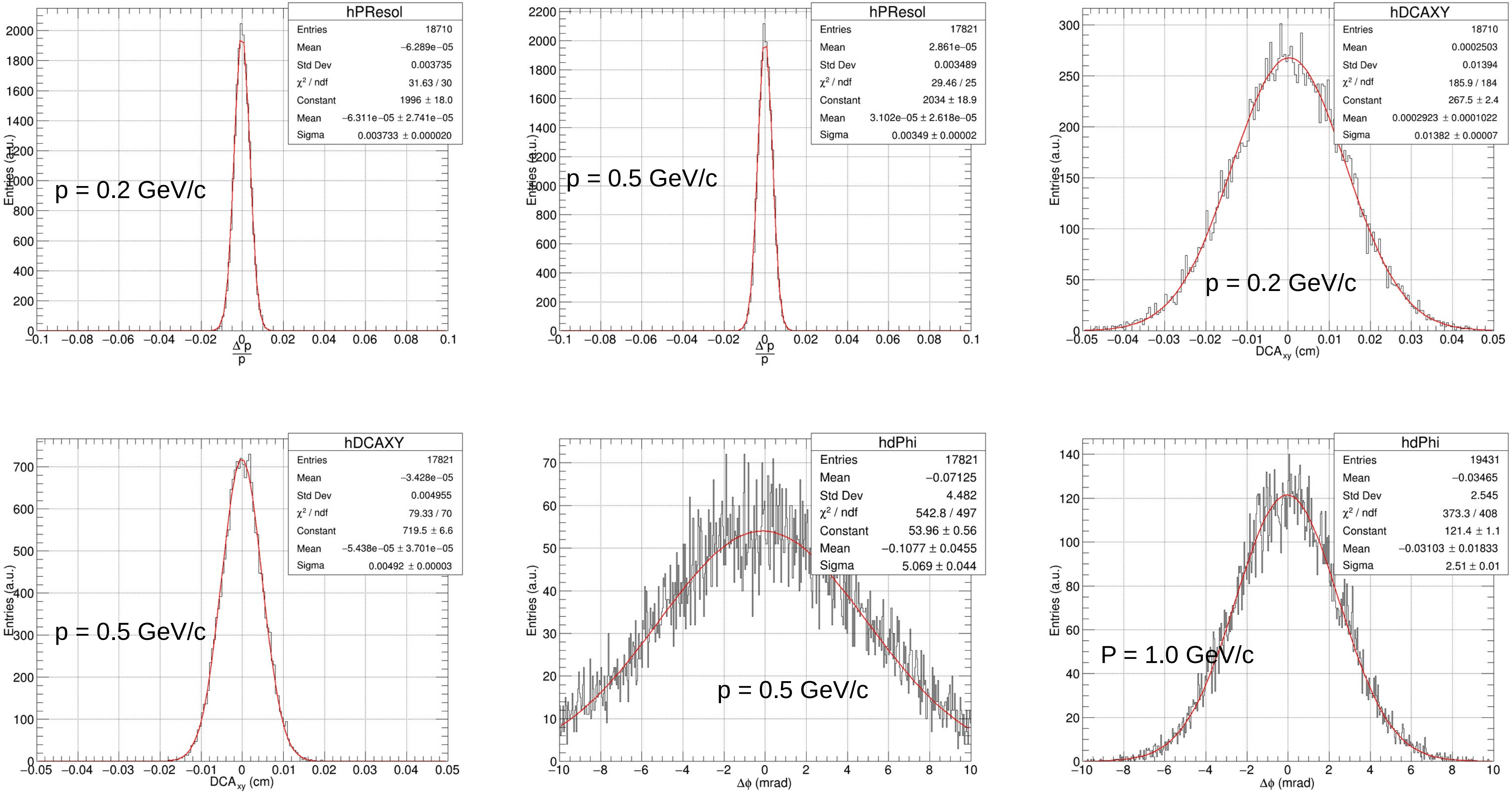
Summary

- Presented fast simulation studies for the theta/phi resolutions at DIRC layer (71 cm)
- Updated Method 1 gives the closest results to the two independent fast simulation methods
- Global fit and Kalman and can further used to study several other cases

Thank You !!

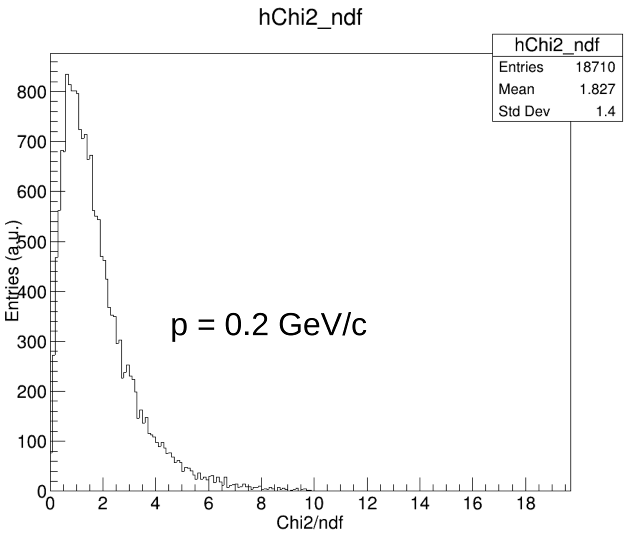
Intermediate distributions from Global fit (Pion)

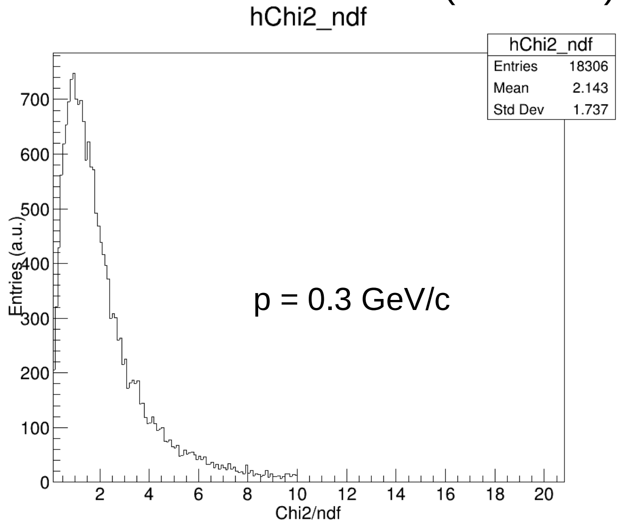
20k pion simulations for each momentum

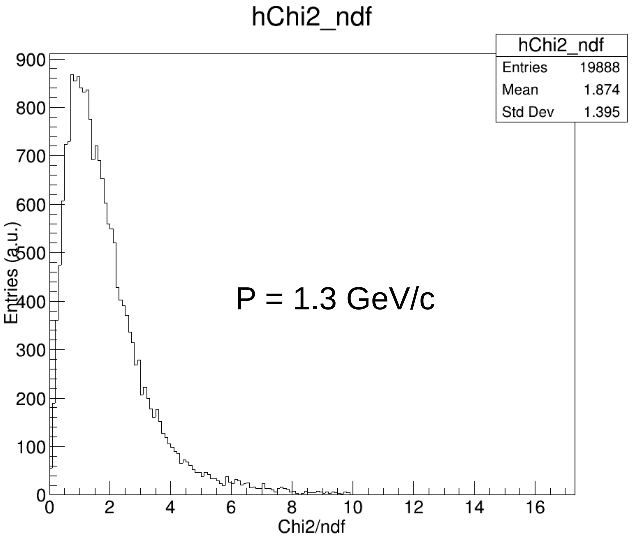


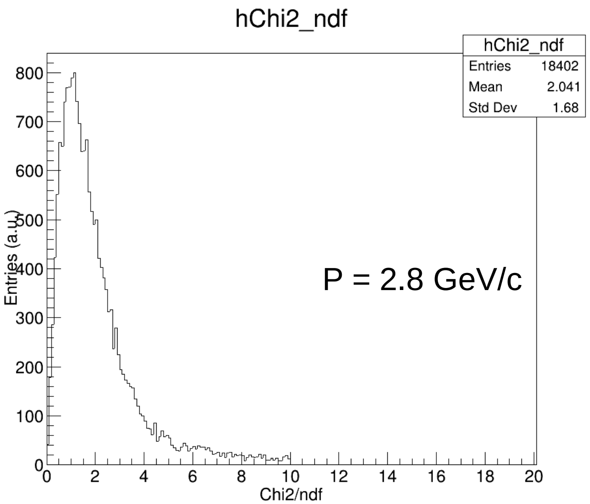
Distributions from Global fit (Pion)

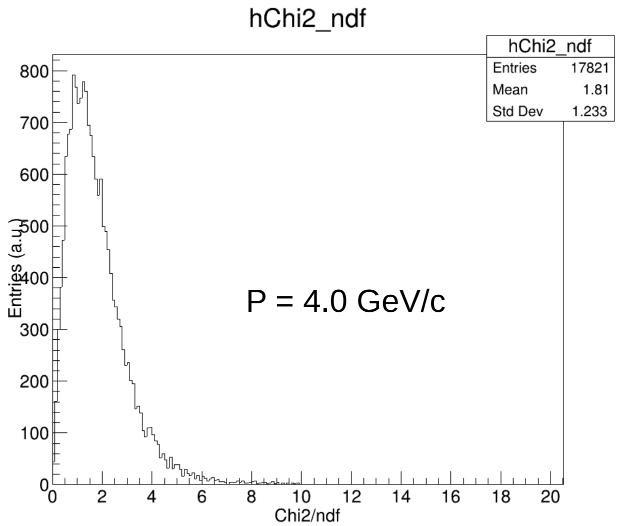
Chi2 distribution in Fast sim (Global fit)











- Chi2 is good in global fit

Different beta for different particle hypothesis (momentum dependent) and so carefully chosen
ACTS using Pion hypothesis for track reconstruction while transporting covariance (Important for Method 2)