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On radiatively-induced CP violation in the "real" two Higgs doublet model

Heather Logan

Carleton University

Ottawa, Canada

with Carlos H. de Lima (TRIUMF)

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Theory seminar (remote)

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Introduction

CP violation discovered 1964 (Cronin & Fitch) at the AGS at Brookhaven

Neutral kaons: $K_S \to \pi\pi$ CP even ($c\tau \simeq 2.7$ cm); $K_L \to 3\pi$ CP odd ($c\tau \simeq 15$ m); but K_L also decays to $\pi\pi$ about 0.3% of the time!

Explained in the SM by 3-generation CKM matrix (Kobayashi & Maskawa 1973); quantitatively established by the 1st-generation B-factories during the '00s.

CPV is one of the key ingredients needed to dynamically give rise to baryon asymmetry of the universe (Sakharov 1967) – not enough CP violation in the SM to achieve observed asymmetry $\to$ BSM sources?

Most BSM sources of CPV are severely constrained by limits on electric dipole moments (EDMs)

Introduction

In the SM Lagrangian there are very few “opportunities” for CP violation: need operators that are not self-Hermitian.

- The quark mixing matrix V_{CKM}: 2×2 not enough (phases can all be rotated away by field redefinitions); in 3×3 one physical CPV phase remains $\rightarrow$ original motivation for 3 quark generations

- $G^{\mu\nu}\tilde{G}_{\mu\nu}$ operator (strong interaction): Strong CP problem – coefficient of this operator constrained by neutron EDM to be $< 10^{-10}$. Very fine tuned! $\rightarrow$ most popular solution is Peccei-Quinn axion.

- Massive neutrinos (technically BSM): 3×3 lepton mixing matrix (PMNS) has its own CPV phase; also possibility for two additional Majorana phases.

Introduction

Beyond the SM, any term in the Lagrangian that is not self-Hermitian is a new possible source of CP violation.

$$\mathcal{L} \supset \left\{ C_i \mathcal{O}_i + C_i^* \mathcal{O}_i^\dagger \right\}$$

$+$ opportunity to explain baryon asymmetry of the universe!

$-$ generally strongly constrained by EDMs $\rightarrow$ fine-tuning

$\rightarrow$ Consider the two Higgs doublet model (2HDM)

Introduction

2HDM:

- Add a second Higgs doublet to the SM (Φ_1, Φ_2).

- Write down most general gauge-invariant Lagrangian.

- Immediately screw up flavour and CP.
  You don't know what you've got 'til it's gone :(

Have to do model-building (impose additional symmetries) to avoid experimentally-excluded levels of flavour- and CP-violation.

Introduction

The most general gauge-invariant scalar potential for the 2HDM:

$$\begin{aligned} V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \\ &+ \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left|\Phi_1^\dagger\Phi_2\right|^2 \\ &+ \left\{\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\} \end{aligned}$$

(10 parameters, 4 of them complex)

Yukawa Lagrangian: two copies of that of the SM:

$$\begin{aligned} \mathcal{L}_{Yuk} &= -Y_{ij}^{d1}\bar{Q}_{Li}\Phi_1 d_{Rj} - Y_{ij}^{u1}\bar{Q}_{Li}\tilde{\Phi}_1 u_{Rj} - Y_{ij}^{\ell 1}\bar{L}_{Li}\Phi_1 e_{Rj} + \text{h.c.} \\ &\quad -Y_{ij}^{d2}\bar{Q}_{Li}\Phi_2 d_{Rj} - Y_{ij}^{u2}\bar{Q}_{Li}\tilde{\Phi}_2 u_{Rj} - Y_{ij}^{\ell 2}\bar{L}_{Li}\Phi_2 e_{Rj} + \text{h.c.} \end{aligned}$$

Rotating to the fermion mass basis diagonalizes only the combinations $(Y^{u1}v_1 + Y^{u2}v_2)$, etc.; orthogonal combinations are not diagonal, source of FCNC and additional CPV.

Introduction

Sidestep the FCNC problem by imposing Natural Flavour Conservation (Glashow & Weinberg, 1977): Arrange for fermions of each different electric charge to couple to exactly one Higgs doublet.

Easy to impose using a Z_2 symmetry: $\Phi_1 \to -\Phi_1$, $\Phi_2 \to \Phi_2$

	u_R	d_R	e_R
Type I	$+$	$+$	$+$
Type II	$+$	$-$	$-$
Type X	$+$	$+$	$-$
Type Y	$+$	$-$	$+$

Also eliminates λ_6, λ_7, and m_{12}^2; can then absorb phase of λ_5 into unphysical rephasing of fields. No CPV in scalar potential!

Exact Z_2: trade m_{11}^2 and m_{22}^2 for Higgs vevs after EWSB; upper bound on all scalar masses $\sim \mathcal{O}(700$ GeV). Types II, X, and Y *fully excluded* by global fit including LHC data (Chowdhury & Eberhardt, 2017)

Introduction

Allow soft breaking of the Z_2 to reinstate m_{12}^2 and allow for a decoupling limit: "Complex 2HDM"

$$\begin{aligned} V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\ &+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 \\ &+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.} \right\}. \end{aligned}$$

Rephasing of scalar fields $\Phi_1 \to e^{i\theta}\Phi_1$ etc. $\to$ only 1 physically-meaningful CPV phase in scalar potential: Phase$\left[(m_{12}^{2*})^2 \lambda_5\right]$.
Usual approach: choose m_{12}^2 real; then Im(λ_5) contains the CPV.

Constrained by electron EDM: $|d_e| < 4.1 \times 10^{-30} e\,\text{cm}$ (JILA 2022)
Full 2-loop calculation in C2HDM (depends on Type):

$$\left(\frac{1\ \text{TeV}}{M}\right)^2 \text{Im}(\lambda_5) \times f(\sin^2\beta, \cos^2\beta) \lesssim 0.5 - 1\%$$

Altmannshofer, Gori, Hamer, & Patel, 2020

Introduction

Complex 2HDM is by now rather fine-tuned to avoid eEDM constraint.

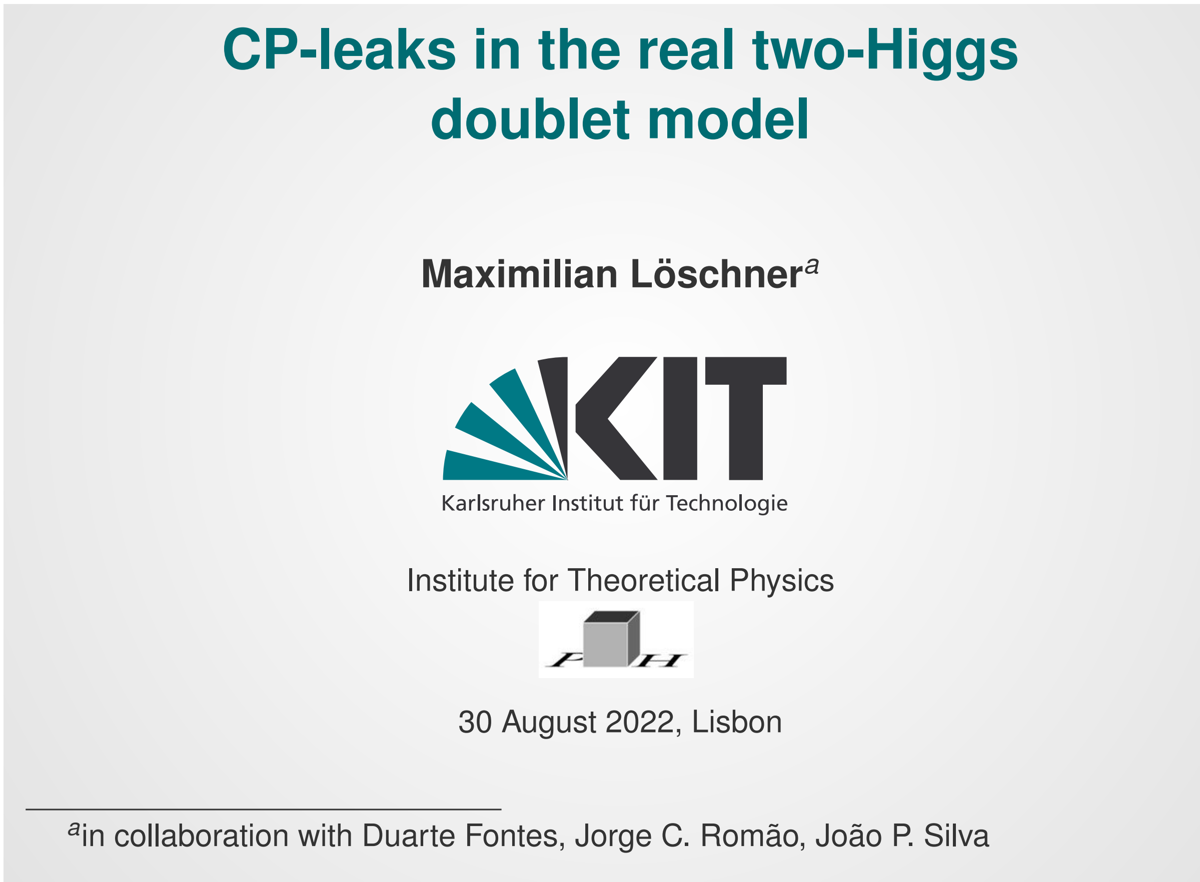
$\rightarrow$ Most 2HDM studies bypass this issue entirely by considering only the real 2HDM: impose CP conservation on the scalar potential.

Then CP-odd A^0 is a mass eigenstate; no mixing with CP-even H^0, h^0: tidy.

But... is this consistent?

Motivation

We got interested in this question after Carlos went to Lisbon Workshop on Multi-Higgs Models 2022.



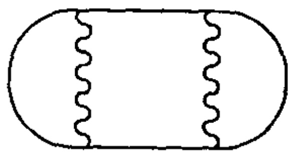
D. Fontes, M. Löschner, J.C. Romão, & J.P. Silva, 2103.05002 (EPJC)

Motivation

Fontes et al.'s argument:

- We know there is CP violation in the CKM matrix.

- CKM CPV can be transmitted to other operators via loop diagrams – e.g., contribution to Weinberg operator $f^{abc}\tilde{G}^a_{\alpha\beta}G^b_{\beta\mu}G^c_{\mu\alpha}$ in the SM has been computed at 3 loops (Pospelov 1994)



- No apparent reason why similar diagrams shouldn't generate imaginary parts for the operators multiplying m_{12}^2 and λ_5

- CKM phase is hard-breaking of CP, so no apparent reason why those generated imaginary parts shouldn't be divergent
$\rightarrow$ need complex 2HDM from the beginning to have the necessary imaginary counterterms!

Motivation

Fontes et al.'s calculation:

Computed leading $(1/\epsilon)^3$-divergent piece of A^0 tadpole diagram at 3 loops. (Most divergent piece $\rightarrow$ 3-loop counterterm)

- Minimum number of loops required to get the Jarlskog invariant (4 powers of CKM matrix) $\sim \mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i})$ [more on this later]

- At the very limit of modern Feynman-diagram computational technology

- Individual contributions are nonzero

- After summing over all 3 generations of up- and down-quark masses, the result is ZERO!?!

This talk $\rightarrow$ (1) Why is it zero? (2) Can we dig deeper?

Outline

- Jarlskog invariant and how to get it

- 3-loop A^0 tadpole: why the cancellation

- Symmetries of the 2HDM and the role of λ_5

- Loop diagrams in the unbroken phase and preliminary results

- Conclusions

The Jarlskog invariant

Reparameterization-invariant measure of the CP violation in the CKM matrix, introduced by Cecilia Jarlskog in 1985

$$J = \left|\mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i})\right|, \qquad (\alpha \neq \beta, i \neq j)$$

- unaffected by moving phases around in V
- related to the area of the unitarity triangles in B-physics

Before EWSB, all the CPV in the SM CKM sector can be considered to live in the 3×3 Yukawa matrices Y_u, Y_d. Define the Hermitian combinations:

$$\begin{aligned} H_u &= \frac{v^2}{2} Y_u Y_u^\dagger = U_{u_L} M_U^2 U_{u_L}^\dagger \\ H_d &= \frac{v^2}{2} Y_d Y_d^\dagger = U_{d_L} M_D^2 U_{d_L}^\dagger \end{aligned}$$

(CKM matrix is $V \equiv U_{u_L}^\dagger U_{d_L}$)

The Jarlskog invariant

Can then define another Jarlskog quantity, (Botella & Silva, 1995)

$$\begin{aligned}\bar{J} &= \mathrm{Im}\left\{\mathrm{Tr}\left(H_u H_d H_u^2 H_d^2\right)\right\} \\ &= \mathrm{Im}\left\{\mathrm{Tr}\left(V^\dagger M_U^2 V M_D^2 V^\dagger M_U^4 V M_D^4\right)\right\} \\ &= T(M_U^2)\, B(M_D^2)\, J,\end{aligned}$$

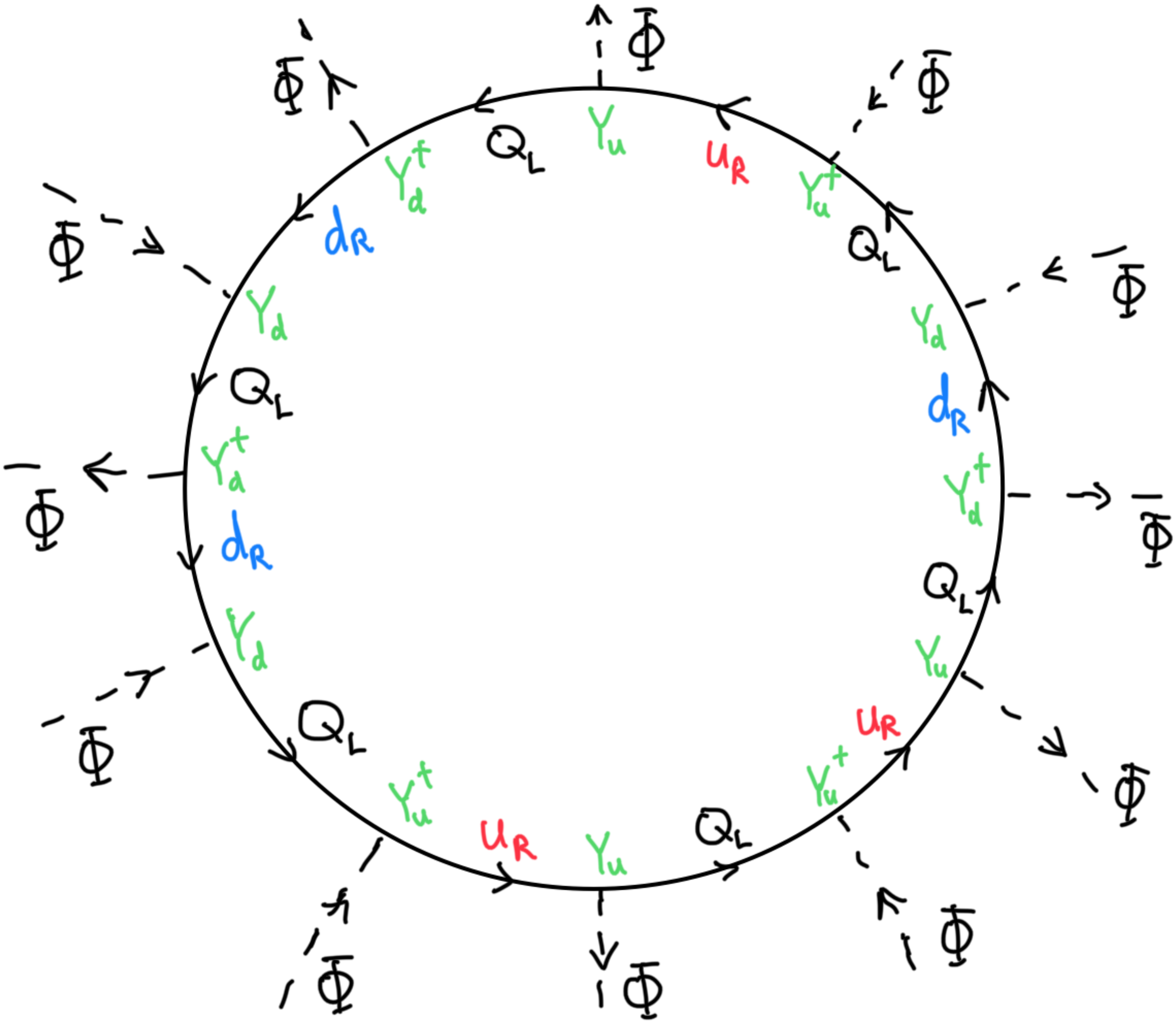
where

$$\begin{aligned}T(M_U^2) &= (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2), \\ B(M_D^2) &= (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2).\end{aligned}$$

Things to notice:

- H_u, H_d are Hermitian: $\mathrm{Tr}(H_u H_d H_u H_d)$ would be real because of cyclic property of the trace. Need a different exponent on the 1st and 2nd H_u's, and likewise H_d's, to get an imaginary part.

- J always comes with (at least) 6 powers of up-quark masses and 6 powers of down-quark masses (i.e., 12 Yukawa insertions).



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Y_u^†
Q_L
Y_d
d_R
Y_d^†
Q_L
Y_d
d_R
Y_d^†

The Jarlskog invariant

We want to generate operators in the 2HDM that contain J.

In the unbroken phase, getting a 4-scalar operator requires connecting 8 of the 12 scalar legs to each other $\rightarrow$ at least a 5-loop diagram.

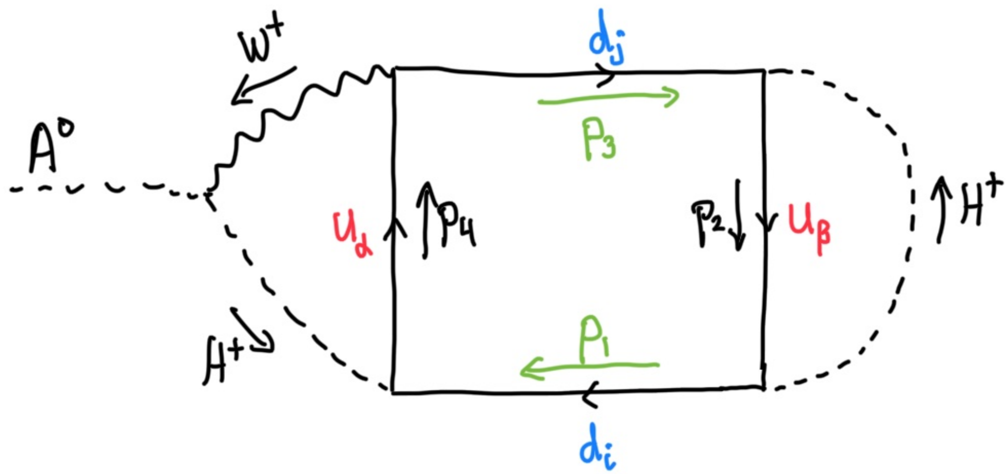
In the broken phase we should be able to replace some of the scalar lines with vevs and reduce the loop order.

But we still need (e.g.) $m_t^4 m_c^2 m_b^4 m_s^2$. This will be the key to understanding the cancellation of the 3-loop A^0 tadpole.

3-loop A^0 tadpole

The 3-loop A^0 tadpole winds up vanishing because there "aren't enough powers of quark masses in the numerator".
(A fuzzy statement which needs clarification.)

Consider one of the contributing diagrams: (note $p_1 = p_3$)



Multiplying out the couplings and quark propagator numerators gives a bunch of terms with mass and CKM structures like

$$\sum_{\alpha,\beta}\sum_{i,j} m_\alpha^2 V_{\alpha i} m_i^2 V_{\beta i}^* V_{\beta j} m_j^2 V_{\alpha j}^* \times \cdots$$

But swapping i and j gives the complex conjugate: Im(...) $= 0$!

3-loop A^0 tadpole

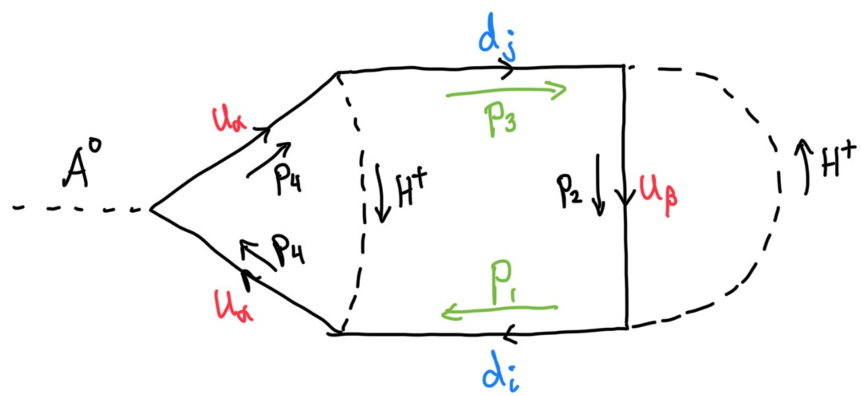
We need (e.g.) $m_\alpha^2 m_i^2 m_\beta^4 m_j^4$. We can get these additional powers of quark masses by expanding the denominators of the propagators:

$$\frac{1}{p^2 - m^2} = \frac{1}{p^2}\left(1 + \frac{m^2}{p^2} + \frac{m^4}{p^4} + \cdots\right)$$

But since $p_1 = p_3$, this procedure always gives pairs of terms that are complex conjugates of each other!

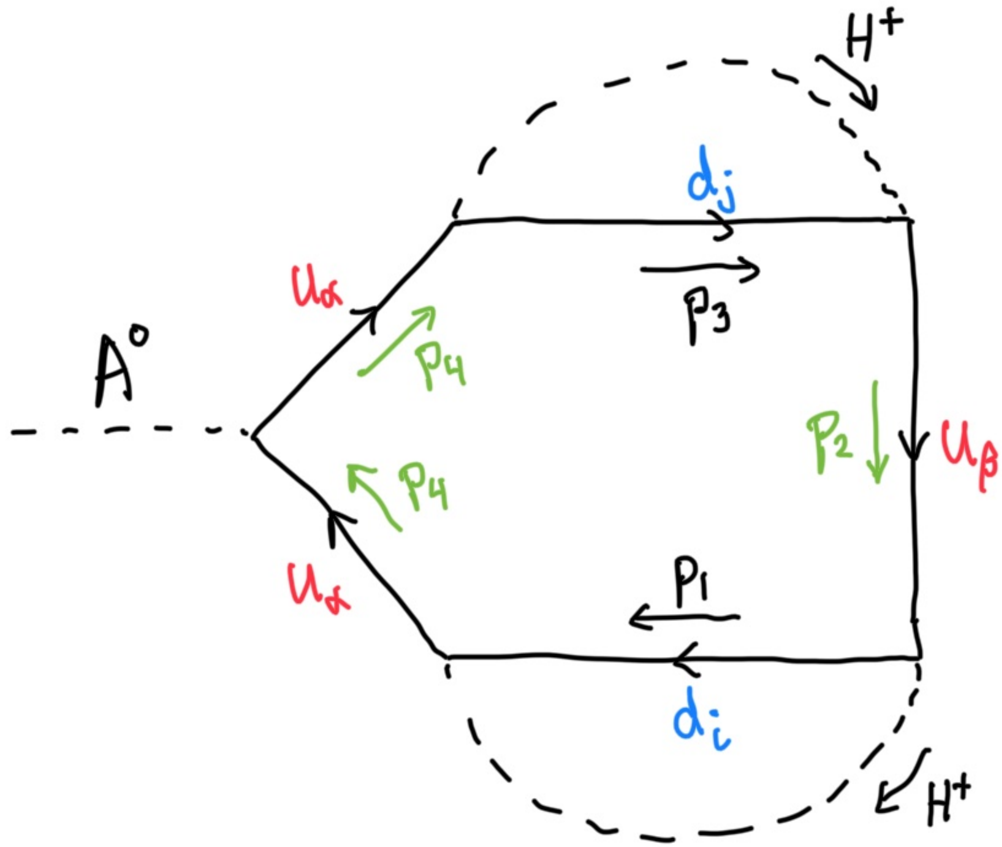
$$\sum_{\alpha,\beta}\sum_{i,j} m_\alpha^2 V_{\alpha i} m_i^2 V_{\beta i}^* V_{\beta j} m_j^2 V_{\alpha j}^* \times \frac{1}{p_1^2 p_3^2}\left[\frac{m_i^2}{p_1^2} + \frac{m_j^2}{p_3^2}\right] \times \cdots$$

Can show diagram-by-diagram that the same argument kills off all the diagrams with $p_1 = p_3$.



3-loop A^0 tadpole

The other class of diagrams have $p_2 = p_4$:



The argument is a little more subtle here, because there are two u_α propagators.

3-loop A^0 tadpole

But the numerator algebra around the A^0 vertex gives:

$$\frac{\not p_4 + m_\alpha}{p_4^2 - m_\alpha^2} \; y_\alpha \gamma^5 \; \frac{\not p_4 + m_\alpha}{p_4^2 - m_\alpha^2} = -\frac{(p_4^2 - m_\alpha^2)}{(p_4^2 - m_\alpha^2)^2} \; y_\alpha \gamma^5$$

i.e., due to the magic of the pseudoscalar coupling, one of the propagators disappears.

Again we get terms of the form (e.g.)

$$\sum_{\alpha,\beta} \sum_{i,j} m_\alpha^2 V_{\alpha i} m_i^2 V_{\beta i}^* m_\beta^2 V_{\beta j} m_j^2 V_{\alpha j}^* \times \cdots$$

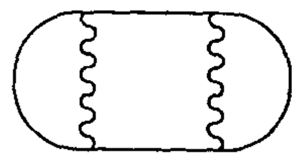
Recognizing that we need (e.g.) $m_\alpha^2 m_\beta^4$, we again need to expand the denominators of the propagators; but this always yields pairs of terms that are symmetric under $\{p_4, m_\alpha\} \leftrightarrow \{p_2, m_\beta\}$

Then, since $p_2 = p_4$, this procedure always gives pairs of terms which are simply complex conjugates of each other.

3-loop A^0 tadpole

In this way we can demonstrate that not only will the leading $(1/\epsilon)^3$ divergence vanish, but the entire tadpole diagram (including finite parts) must be zero.
(Terms with higher powers of quark masses also cancel pairwise.)

But there is no theorem here, because the SM 3-loop contribution to the (dimension-6) Weinberg operator $f^{abc}\tilde{G}^a_{\alpha\beta}G^b_{\beta\mu}G^c_{\mu\alpha}$ has been computed and shown to be nonzero (Pospelov 1994)



Same quark and charged boson topology; but now 3 gluons attached → different momentum structure inside and outside.

- Add another loop (and thus more Yukawa couplings)?
- Add some nontrivial external momentum flow?

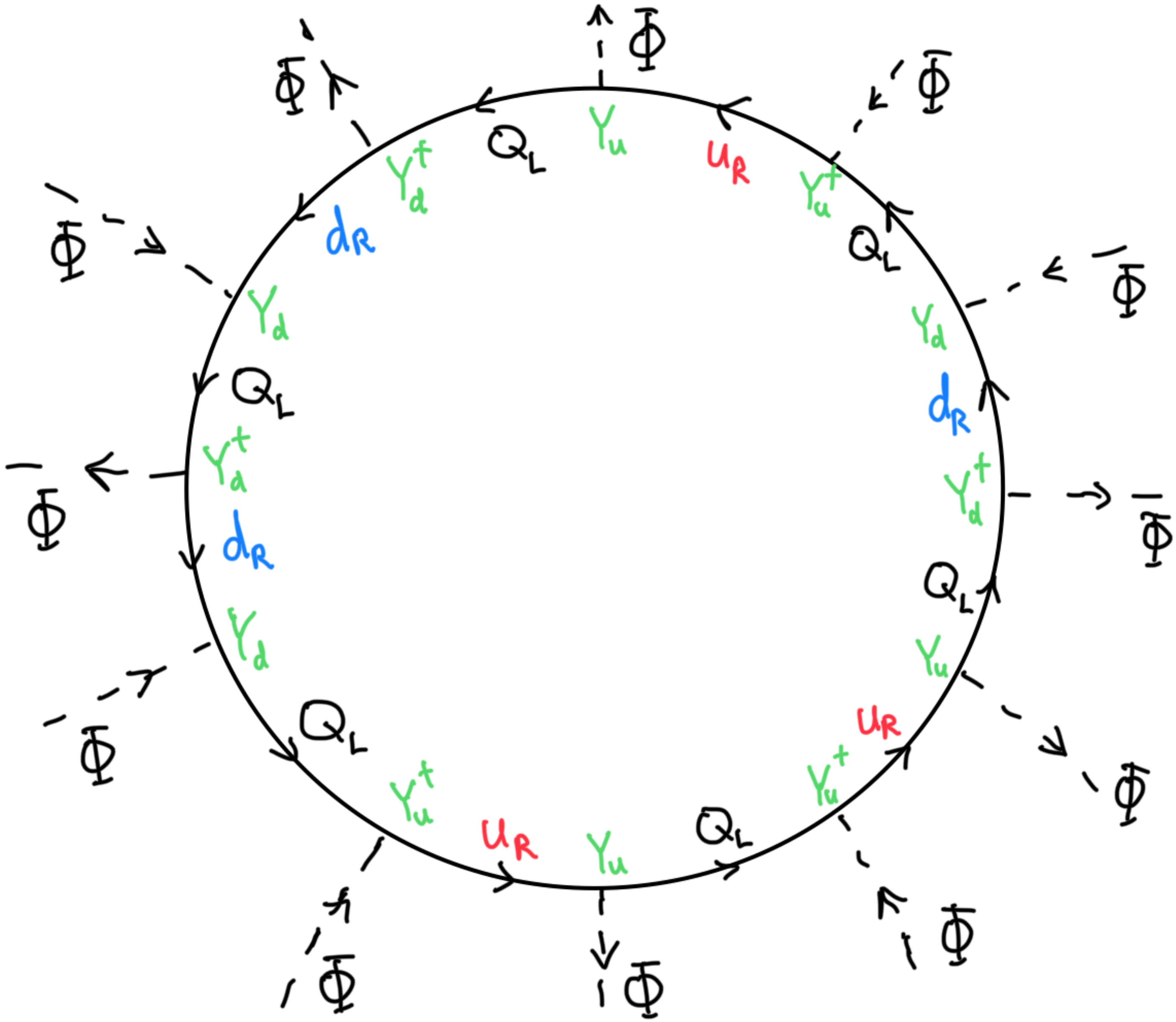
What do we need to add to get a CPV diagram?

Let's study the unbroken phase and think about what we need in order to generate an imaginary part for one of the non-Hermitian operators in the 2HDM scalar potential.

$$
\begin{aligned}
V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\
&+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 \\
&+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.} \right\}.
\end{aligned}
$$

$\Rightarrow$ interested in $\mathcal{O}_5 = \left(\Phi_1^\dagger \Phi_2\right)^2$ or $\mathcal{O}_{12} = \Phi_1^\dagger \Phi_2$.

Any CPV diagram must involve the Jarlskog invariant...



12 Yukawa insertions $\Rightarrow$ 12 scalar "legs"

What do we need to add to get a CPV diagram?

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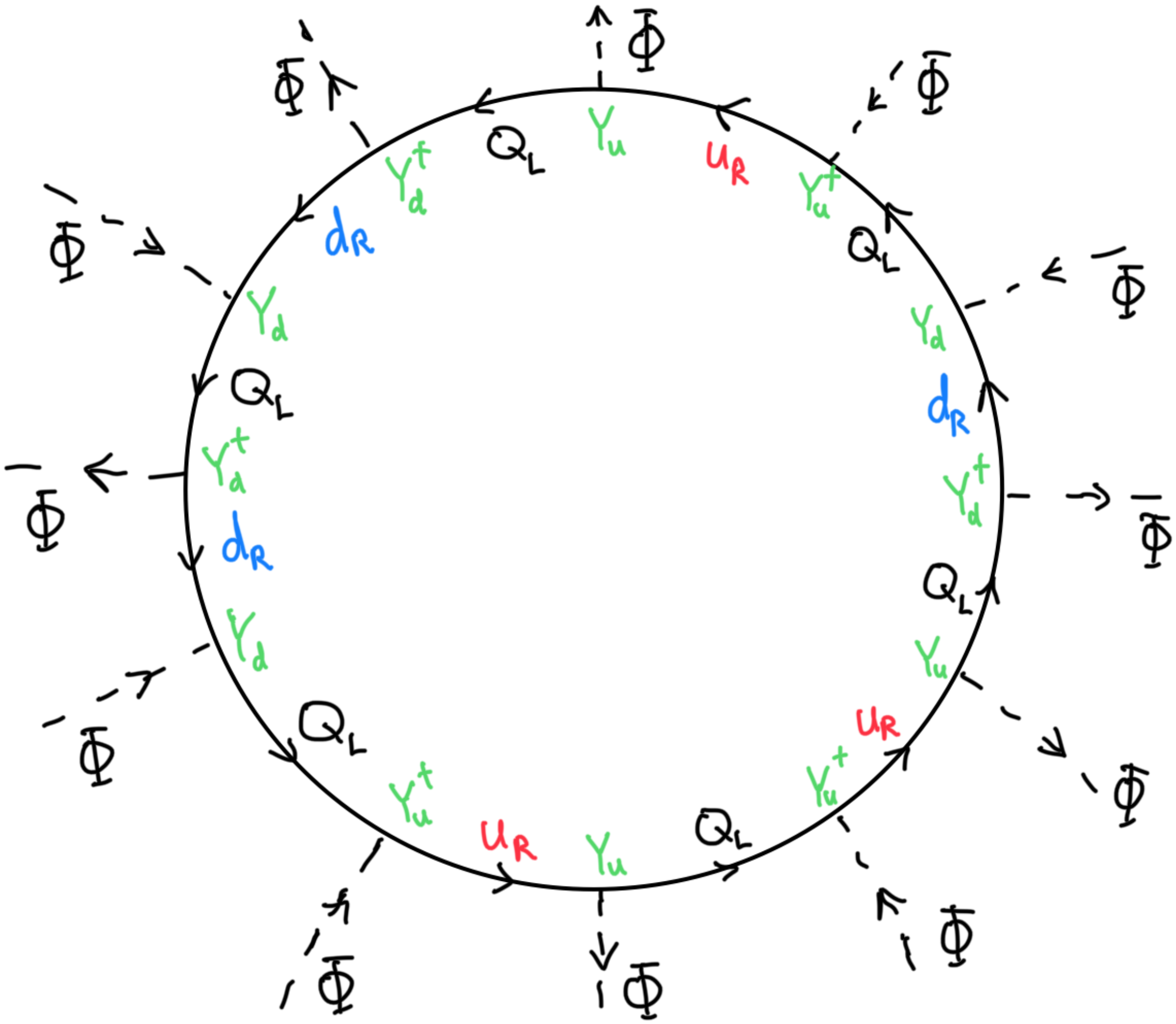
&+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 \\
&+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.} \right\}.
\end{aligned}
$$

$\Rightarrow$ interested in $\mathcal{O}_5 = \left(\Phi_1^\dagger \Phi_2\right)^2$ or $\mathcal{O}_{12} = \Phi_1^\dagger \Phi_2$.

$\mathcal{O}_5$: close 8 legs, need at least 5-loop diagram. $\Leftarrow$ focus on this.

$\mathcal{O}_{12}$: close 10 legs, need at least 6-loop diagram.



Type I:

6 incoming Φ_2's

6 outgoing Φ_2's

Type II:

3 incoming Φ_1's

3 outgoing Φ_1's

3 incoming Φ_2's

3 outgoing Φ_2's

$\mathcal{O}_5$ is $(\Phi_1^\dagger \Phi_2)^2$:

need to convert e.g. two outgoing Φ_2's into Φ_1's!

Can do this by inserting a λ_5 vertex. Novel ingredient!

Symmetries of the 2HDM and the role of λ_5

Consider again the quark Yukawa couplings after imposing Natural Flavour Conservation:

$$\mathcal{L}_{Yuk} = -Y^d_{ij}\bar{Q}_{Li}\Phi_1 d_{Rj} - Y^u_{ij}\bar{Q}_{Li}\tilde{\Phi}_2 u_{Rj} + \text{h.c.}$$

(for Type II; replace Φ_1 with Φ_2 for Type I.)

We normally enforce this by imposing a Z_2 symmetry.

But we could equally well have achieved this form for the Yukawa couplings by imposing a global $U(1)$ symmetry, e.g.:

$$\Phi_1 \to e^{-i\theta}\Phi_1, \qquad \Phi_2 \to e^{i\theta}\Phi_2$$

with Q_L invariant and

$$u_R \to e^{i\theta}u_R, \qquad d_R \to e^{-i\theta}d_R \qquad \text{(Type I)}$$
$$u_R \to e^{i\theta}u_R, \qquad d_R \to e^{i\theta}d_R \qquad \text{(Type II)}$$

(For Type II, this is equivalent to the Peccei-Quinn $U(1)$.)

Symmetries of the 2HDM and the role of λ_5

Most general scalar potential:

$$
\begin{aligned}
V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\
&+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right)\left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left| \Phi_1^\dagger \Phi_2 \right|^2 \\
&+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \left[\lambda_6 \left(\Phi_1^\dagger \Phi_1 \right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right) \right] \left(\Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right\}.
\end{aligned}
$$

Imposing $U(1)_{PQ}$ kills off m_{12}^2, λ_6, λ_7, and λ_5!

$U(1)_{PQ}$ can't be exact or A^0 is massless (physical Goldstone boson of the spontaneous breaking of the extra $U(1)$).

Softly break $U(1)_{PQ}$: reinstate m_{12}^2. Complex, but its phase can be trivially rotated away using the $U(1)_{PQ}$.

Then the scalar potential has no possible CPV terms.
Protected by a softly-broken symmetry: radiative corrections cannot generate a *divergent* $\text{Im}(\lambda_5)$ (or even $\text{Re}(\lambda_5)$).
(Finite & calculable radiatively-generated $\text{Im}(\lambda_5)$ is ok.)

Symmetries of the 2HDM and the role of λ_5

Corollary 1: any diagrams in the softly-broken-Z_2 2HDM that could generate a divergent Im(λ_5) must know about $\lambda_5 \neq 0$, or they will be equivalent to the corresponding diagrams in the softly-broken-$U(1)_{PQ}$ 2HDM and the divergent parts will sum to zero.

$\rightarrow$ Require a λ_5 insertion in the diagrams!

Unbroken phase: convert two outgoing Φ_2's into Φ_1's. Minimum of 6 loops!

Broken phase: must show up via triple- or quartic-Higgs couplings that still depend on λ_5 after all other quartic couplings are re-expressed in terms of masses and mixing angles.

8 Lagrangian parameters:	7 + 1 physical parameters:
m_{11}^2, m_{22}^2, m_{12}^2, and 5 λ's	m_h, m_H, m_A, m_{H^+}, v, α, β, λ_5

Symmetries of the 2HDM and the role of λ_5

Corollary 2: If one wants a real 2HDM that is guaranteed in an obvious way to be safe from CPV "leaks" (and hence theoretically consistent), use the softly-broken-$U(1)_{PQ}$ 2HDM.

- Freedom of scalar masses and mixing angles is identical to that in softly-broken-Z_2 model. (Still fully viable phenomenologically.)

- One coupling degree of freedom is removed from triple- and quartic-scalar couplings: $U(1)_{PQ}$ model is more predictive (less general) than Z_2 version, but the differences are experimentally rather subtle.

λ_5 freedom shows up in $h^0 H^+ H^-$ coupling: $U(1)_{PQ}$ restricts the charged Higgs contribution to $h^0 \to \gamma\gamma$.

Analysis of 6-loop diagrams (preliminary)

- Work in the unbroken phase and aim for $\mathcal{O}_5 = (\Phi_1^\dagger \Phi_2)^2$

- Consider diagrams with a quark loop giving the 12-Yukawa-matrix Jarlskog structure, as well as a λ_5 insertion.

- Look for the most divergent piece of the diagram: $\sim [\log(\Lambda)]^6$ in a cutoff scheme, or $(1/\epsilon)^6$ in Dimensional Regularization. (Only the most divergent piece is cancelled by the corresponding-loop-order counterterm – all less-divergent pieces will be canceled by lower-order counterterms, which in our case must be real.)

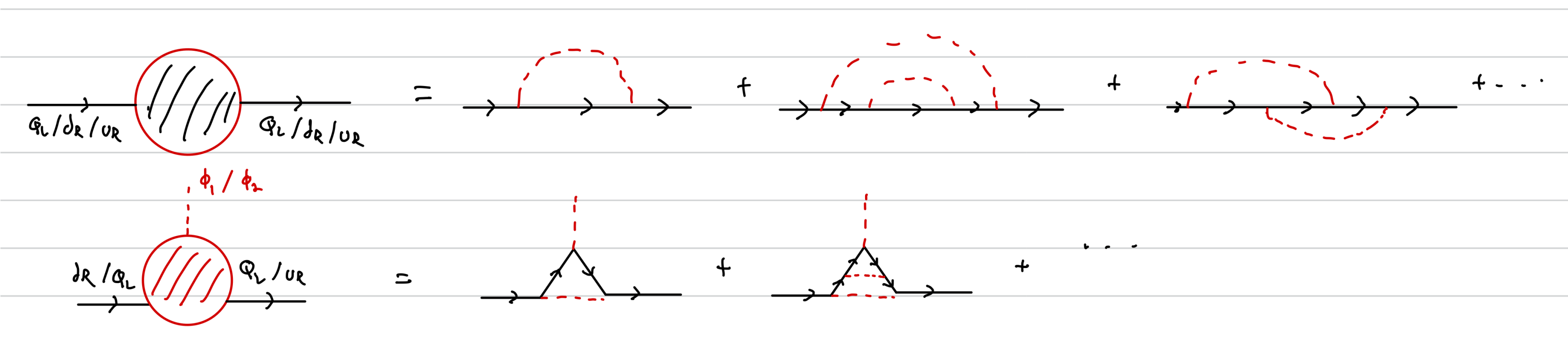
Can determine whether a given diagram has a $[\log(\Lambda)]^6$ divergence by shrinking one loop at a time – if it's possible to choose an order of shrinkings such that each one gives a $\log(\Lambda)$, then we have found a contribution to the most divergent piece.

Analysis of 6-loop diagrams (preliminary)

Let's start with Type I: only Φ_2 couples to the quarks.

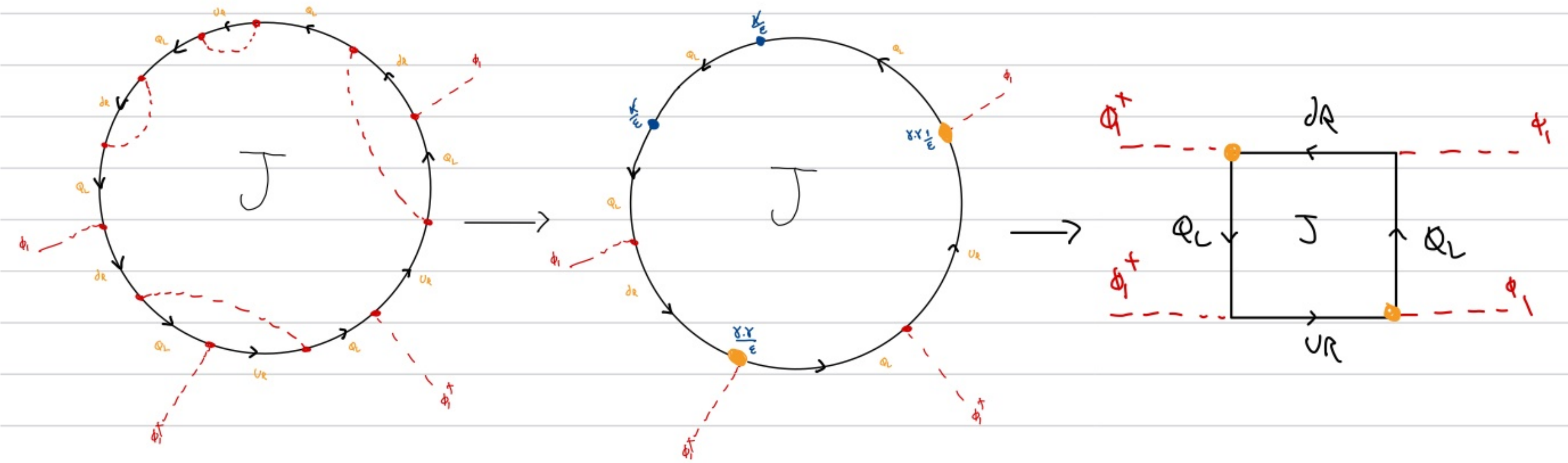
1. Close up 8 legs, leaving 4 external scalars

2. Integrate the internal sub-loops, starting with the ones that are divergent: only fermion self-energies and triangle corrections to Yukawa vertices give $\log(\Lambda)$ divergences



Analysis of 6-loop diagrams (preliminary)

3. Integrate the remaining fermion box last (otherwise the integral will have more than 4 fermion propagators and will be finite)

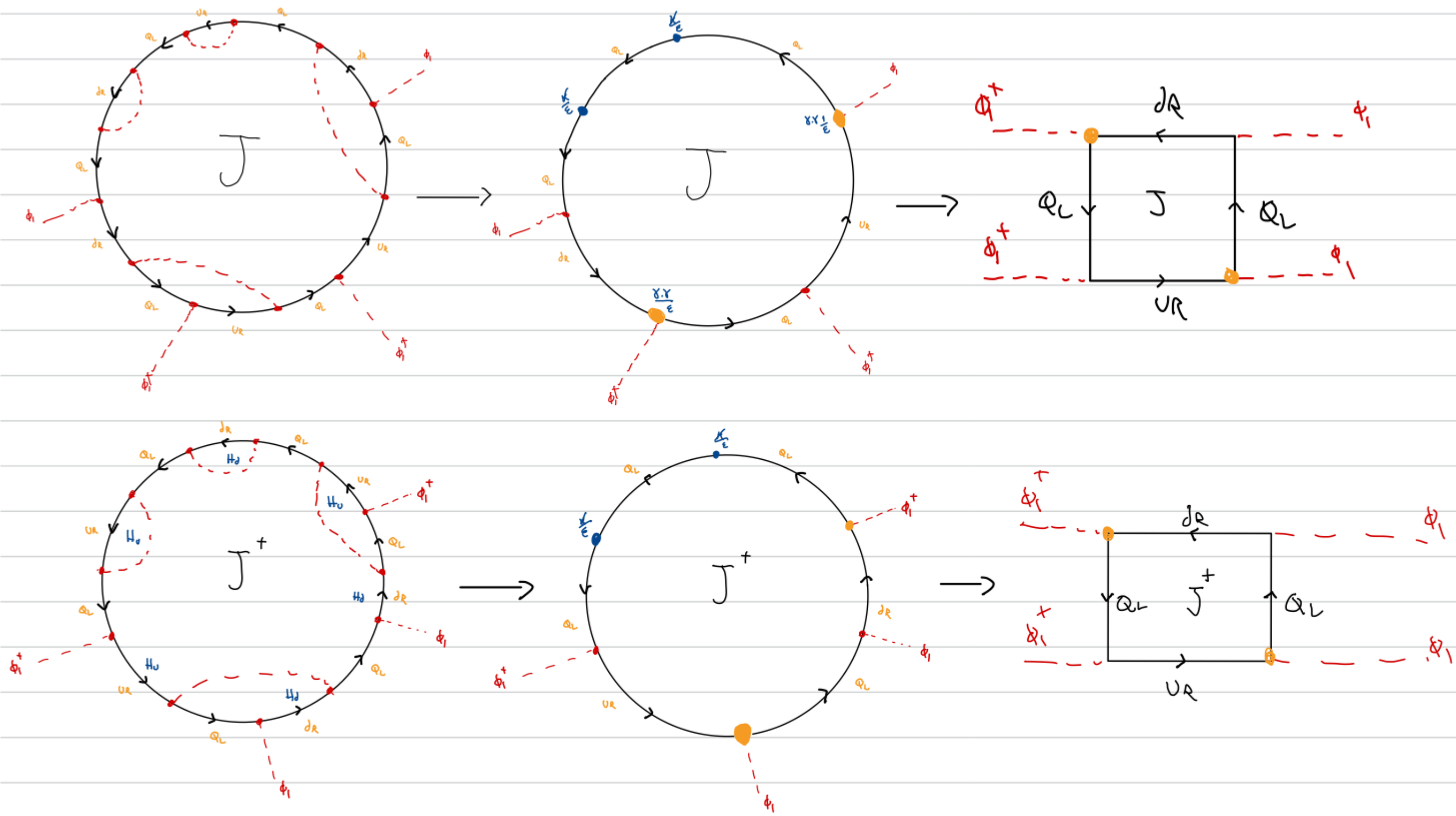


4. Attach two of the external scalars to a λ_5 vertex and integrate the remaining (log-divergent) 2-point scalar loop

Can show that we get $[\log(\Lambda)]^6$!
(Consistent with superficial degree of divergence = 0.)
But: the result of step 3 is always a Hermitian operator!!

Analysis of 6-loop diagrams (preliminary)

At step 3, for each diagram $\sim \text{Tr}(H_u H_d H_u^2 H_d^2)$, we also get a diagram $\sim \text{Tr}(H_d H_u H_d^2 H_u^2)$: the imaginary part of the most divergent piece cancels!!! (Verified using QGRAF + Mathematica.)



Most divergent piece doesn't depend on internal momenta, and hence can't distinguish different placements of the internal loops.

Analysis of 6-loop diagrams (preliminary)

For Type II, there are 3 classes of diagrams giving $\mathcal{O}_5$:

- λ_5 vertex attached to two external scalars $\to$ cancellation of the imaginary part of the $[\log(\Lambda)]^6$ divergence follows as for Type I.

- λ_5 vertex attached to four "internal" scalars $\to$ we are always stuck with a finite sub-loop before we can get to the stage of integrating the quark box – no $[\log(\Lambda)]^6$ divergence.

- Two m_{12}^2 insertions on "internal" scalar lines and no λ_5 vertex (5-loop) $\to$ finite sub-integrals; does not contribute to the most-divergent piece (as expected from $U(1)_{PQ}$ symmetry argument)

Analysis of 6-loop diagrams (preliminary)

Counter to our expectation, we have shown by a diagrammatic argument in the unbroken phase that the imaginary part of the most-divergent 6-loop contribution to $\mathcal{O}_5$ cancels!

This despite the necessary ingredients of 12 Yukawa insertions (to produce J) and the λ_5 insertion (to hard-break the U(1)$_{PQ}$) being present.

To complete the "leak-proofing" we need to show:

1) That there is likewise no imaginary most-divergent contribution to $\mathcal{O}_{12} = \Phi_1^\dagger \Phi_2$ at the lowest nontrivial order – same style of argument: most-divergent diagrams' imaginary parts cancel! (still working on finalizing this); and

2) That these results continue to hold at higher orders (we are still thinking about ways to address this).

Analysis of 6-loop diagrams (preliminary)

It would also be interesting to know whether the finite 6-loop contribution to $\mathcal{O}_5$ has an imaginary part – we have no reason to expect it to cancel.

(We think that once one can track the momentum flow through the diagrams, the diagrams and their conjugates will become distinguishable.)

From an EFT perspective, the 5-loop diagram before the external λ_5 attachment can in principle give rise to the C-odd offshell operator

$$(\Phi^\dagger(\partial^2\Phi))(\Phi^\dagger\Phi) - ((\partial^2\Phi^\dagger)\Phi)(\Phi^\dagger\Phi)$$

A deeper understanding?

Qing-Hong Cao, K. Cheng, & C. Xu, 2201.02989
conjecture that the CPV phase of the CKM matrix cannot "leak" into the 2HDM effective potential on geometrical grounds:

They represent the scalar potential in a vector space of scalar bilinears, in which charge conjugation (C) comprises a reflection across a particular plane. C is conserved when all quantities are symmetric under reflections across this plane. Yukawa bilinears can be represented as vectors in this space, but the phase that ultimately winds up in the CKM matrix cannot.

The weak point (in our opinion) is that the Yukawa couplings themselves are not captured by the field bilinear representation, and it is the Yukawa couplings that mix C and P in the scalar sector. This hints at the possible importance of C-violating versus P-violating CP violation to this puzzle.

We think that demonstrating finite CPV leakage would invalidate the conjecture.

Conclusions

On the face of it, there seems to be no convincing reason why the (hard!) CP violation in the CKM matrix should not divergently "leak" into the real 2HDM at high enough loop order.

Working in the unbroken phase, we identified two necessary ingredients for such hard leakage of CP violation:
- 12 Yukawa insertions (to produce J); and
- a λ_5 insertion (to hard-break the would-be $\mathrm{U}(1)_{PQ}$).

Yet even with those ingredients present (at 6 loops), we are able to show by a diagrammatic argument that the would-be most-divergent imaginary contribution to $\mathcal{O}_5 = (\Phi_1^\dagger \Phi_2)^2$ cancels!

CP violation is subtle and mysterious, and evidently has more to teach us about the symmetries of the 2HDM.

BACKUP SLIDES