NC STATE UNIVERSITY

Center for Frontiers in Nuclear Science

Global analysis of polarized DIS & SIDIS data with improved small-x helicity evolution

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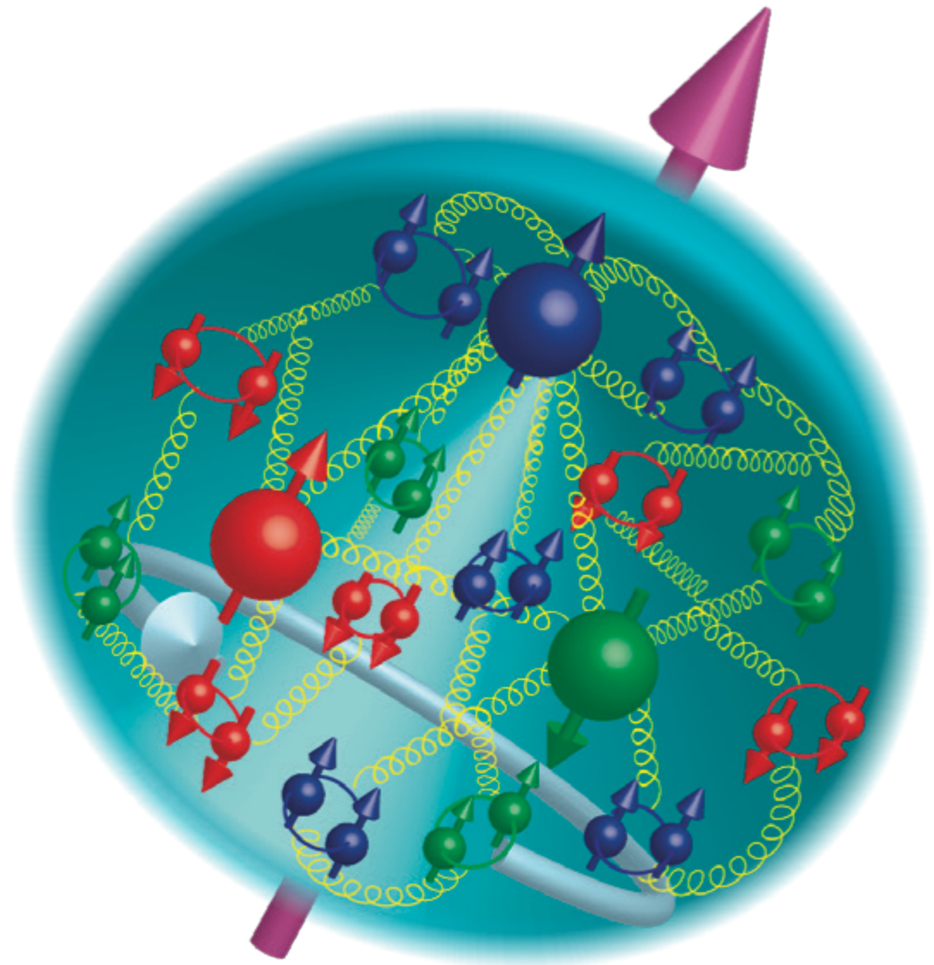
Based on D. Adamiak, N. Baldonado, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato, M. D. Sievert, A. Tarasov, and Y. Tawabutr (arXiv:2308.07461)

CFNS PostDoc Meeting, September 15, 2023

The proton's spin puzzle



The fundamental properties of hadrons, and in particular its spin, are defined by the complex dynamics of quarks and gluons which form a strongly bonded many-body parton system

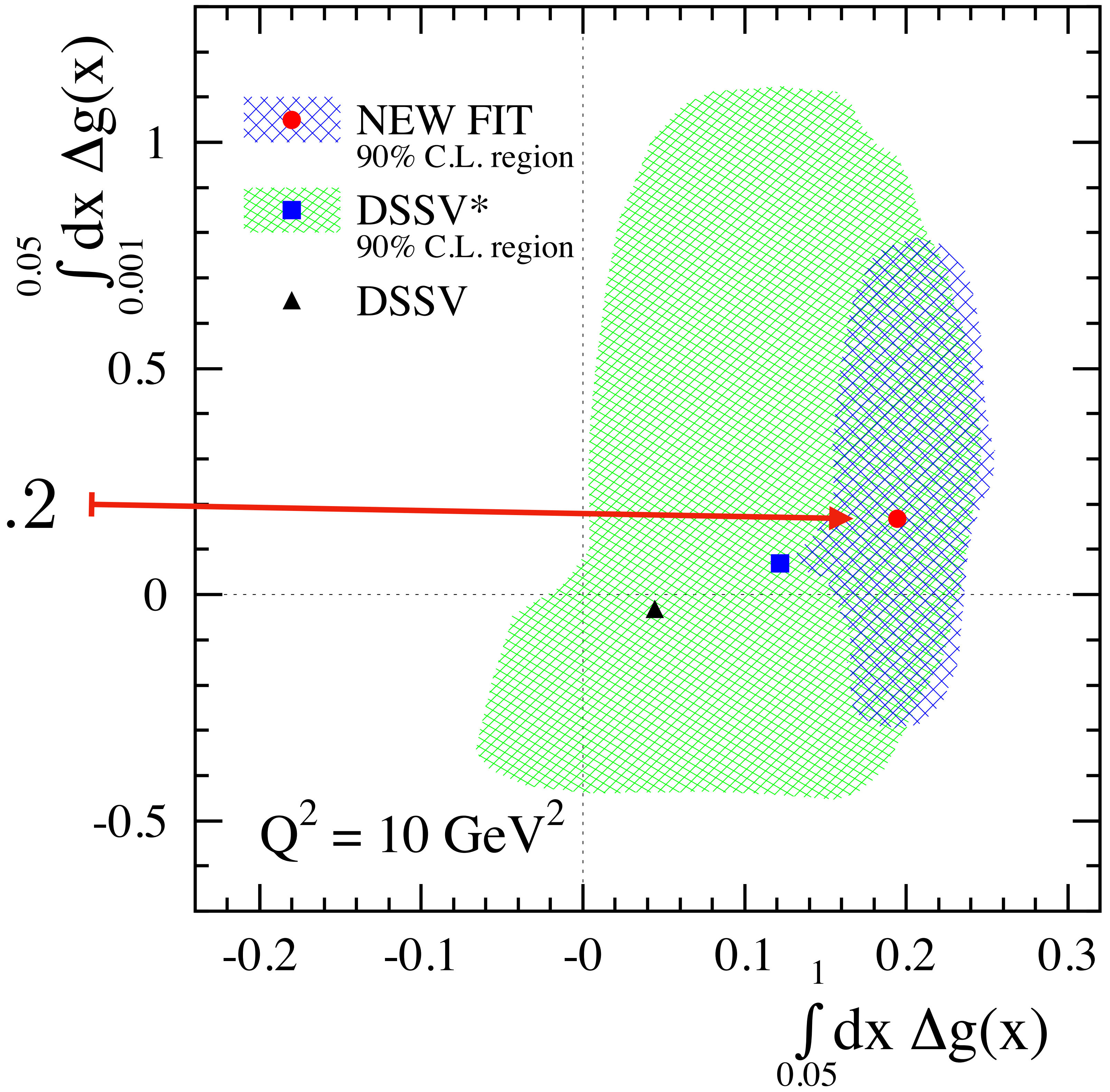
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Proton spin | Quark helicity (spin) | Gluon helicity | Orbital angular momentum

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton's spin: $\Delta\Sigma \approx 0.32$, which is much smaller than predicted by the quark model $\Delta\Sigma \approx 0.6$ - spin puzzle

The sum of quark and gluon helicities come short of 1/2 especially if one takes into account the error bars.

$\Delta G \approx 0.2$

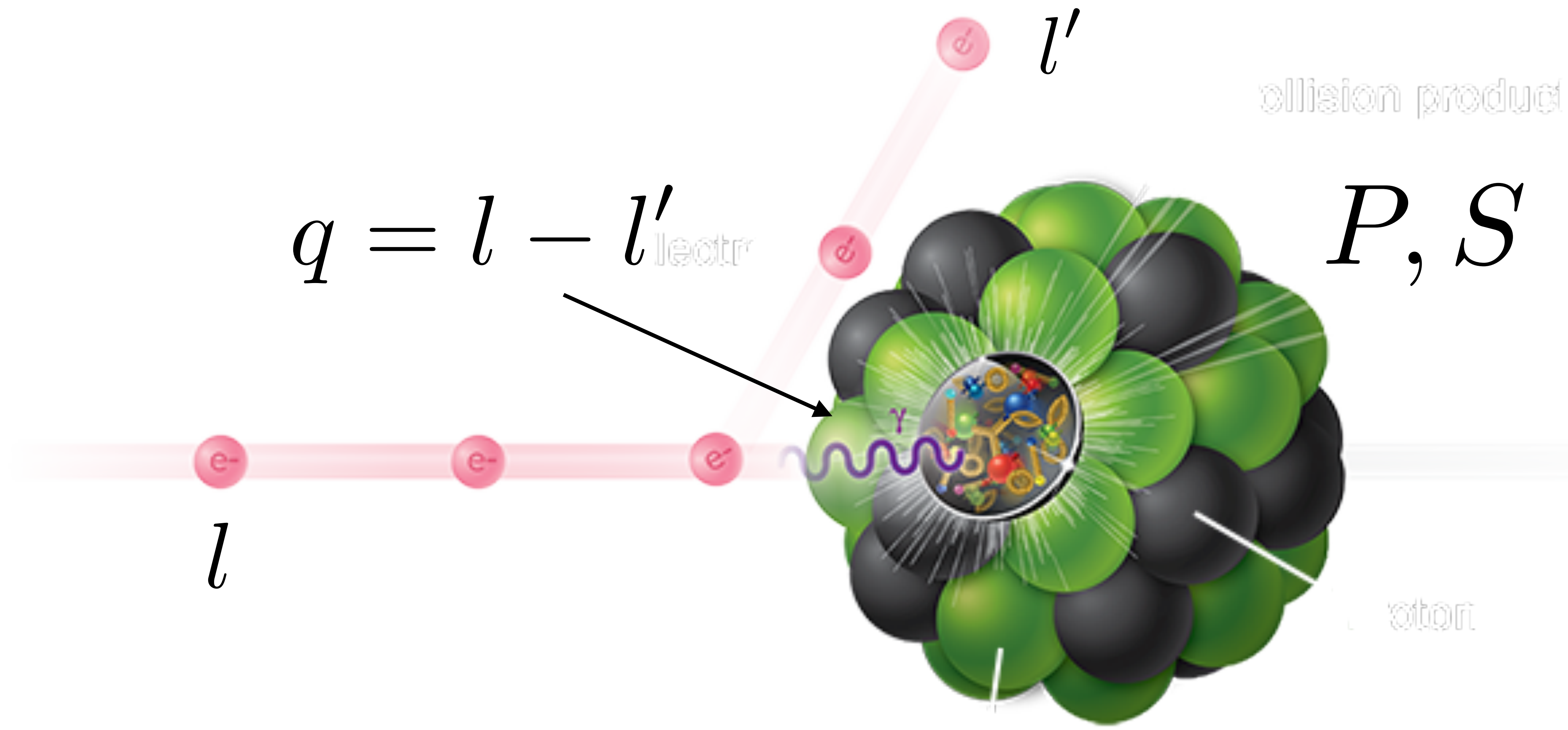


D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

Deep inelastic scattering

The helicity structure of the proton which can be measured in the polarized DIS

$$e(l) + N(P,S) \to e(l') + X$$



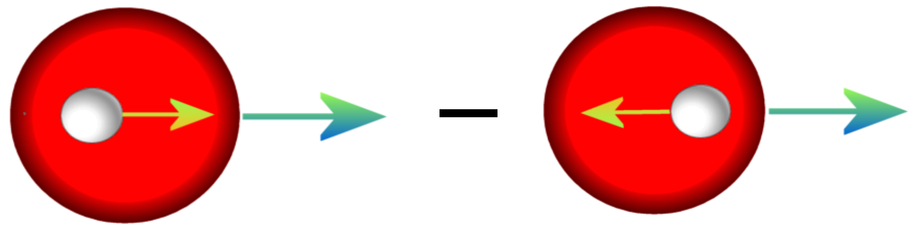
The process is characterized by its virtuality $Q^2 = -q^2$ and Bjorken variable $x_B = Q^2/(2P\cdot q)$.

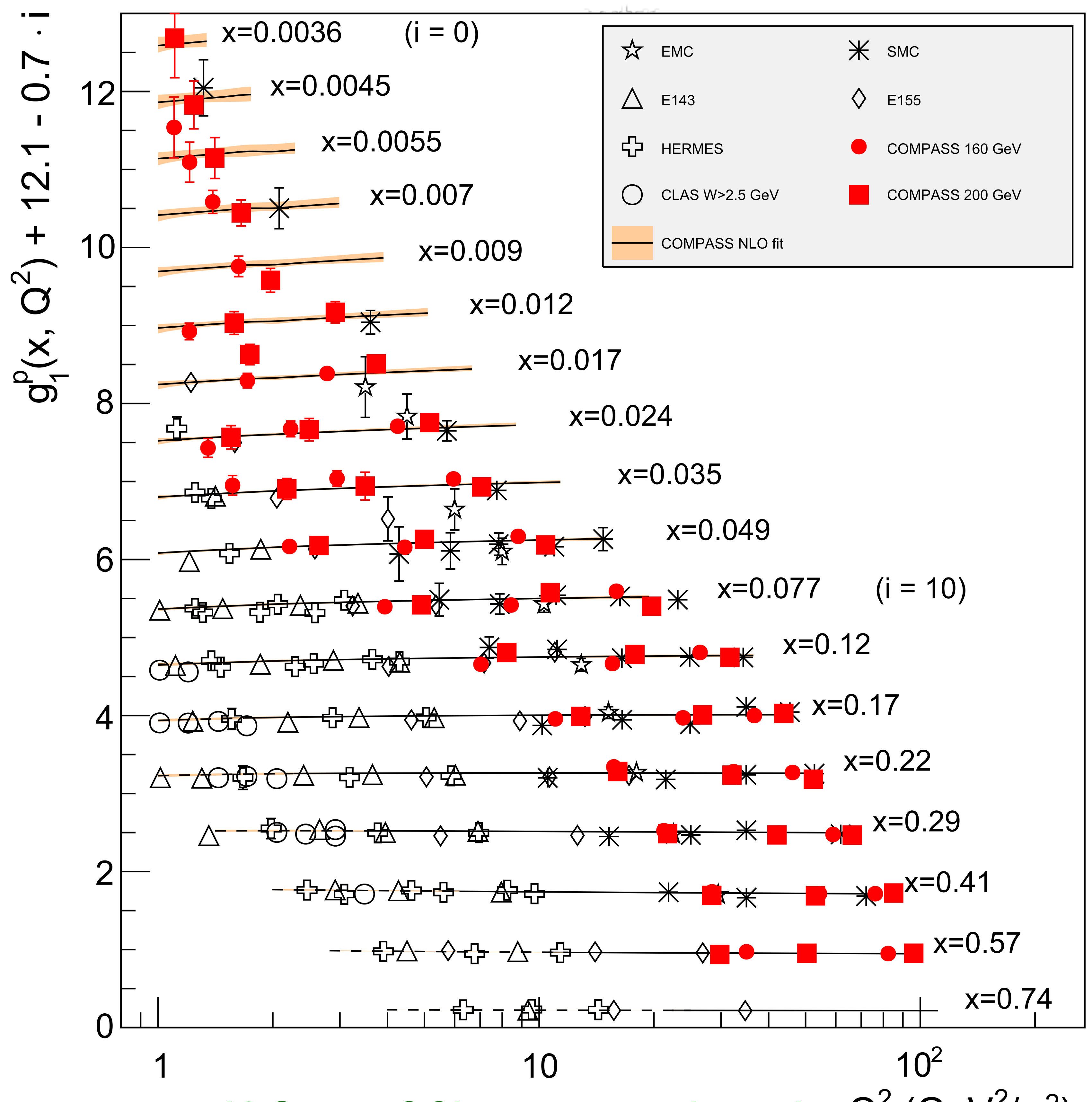
A key observable to study the proton helicity structure is the polarized structure function $g_1(x_B, Q^2)$:

$$\frac{1}{2}\left[\frac{d^2\sigma^{\leftrightarrows}}{dx_B dQ^2} - \frac{d^2\sigma^{\rightrightarrows}}{dx_B dQ^2}\right] \simeq \frac{4\pi\alpha^2}{Q^4} y(2-y) g_1(x_B, Q^2)$$

In the parton model it can be related to the polarized parton distribution function (PDF) which represent parton dynamics of the proton:

$$g_1(x_B, Q^2) = \frac{1}{2}\sum_f e_f^2 \left(\Delta q_f(x_B, Q^2) + \Delta\bar{q}_f(x_B, Q^2)\right)$$

$\Delta q_f(x_B) =$ 



C.Adolph et al. (COMPASS), PLB 753 (2016)

First moment of the structure function

The helicity can be extracted from the first moment of the g_1 structure function

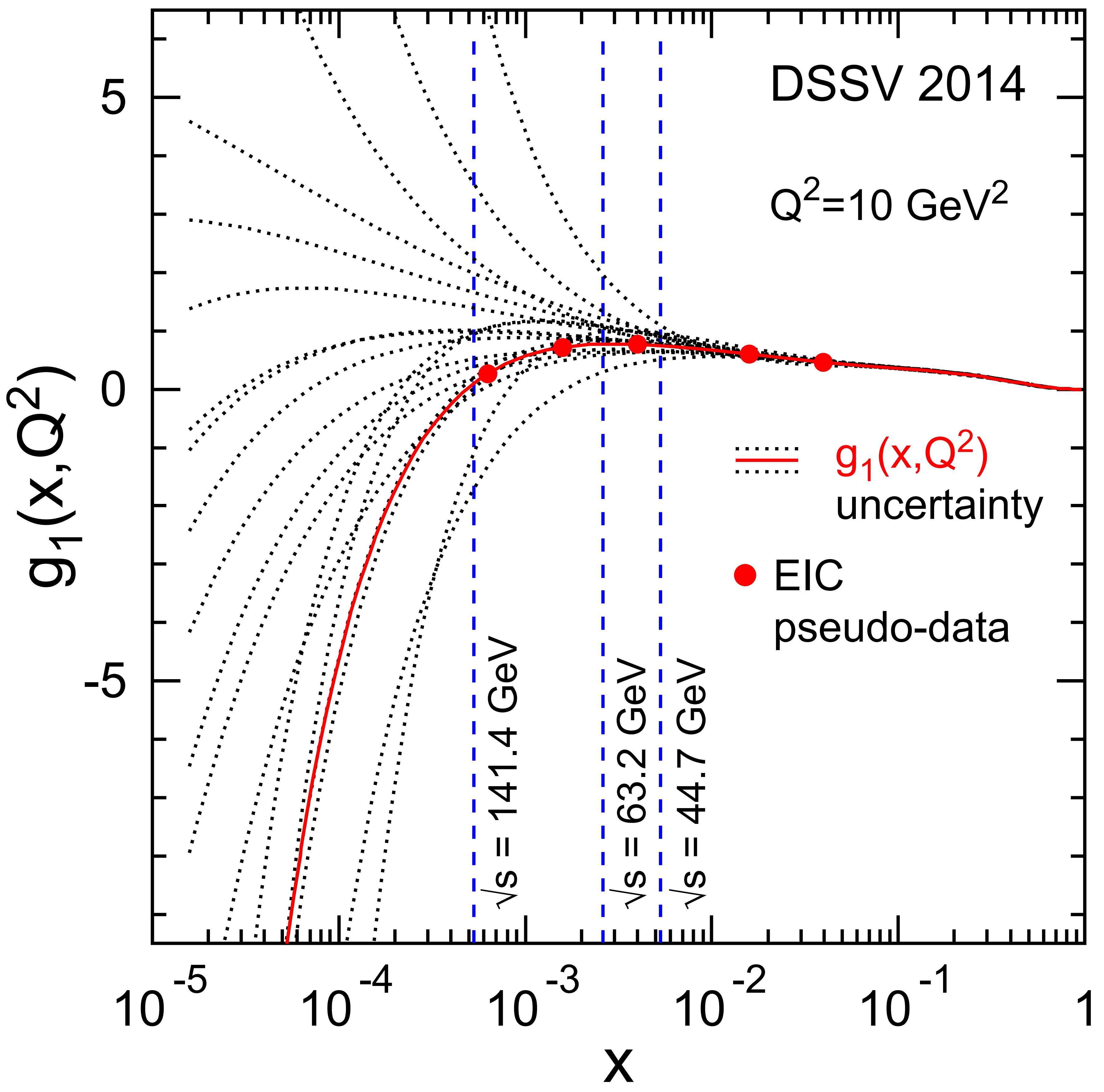
$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\boxed{\Sigma(Q^2)}\right)\left(1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

quark helicity

In terms of quark PDFs the helicity can be defined as

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B\, \left(\Delta q_f(x_B, Q^2) + \Delta\bar{q}_f(x_B, Q^2)\right)$$

- Large uncertainties in the region of small x
- “Hidden” spin might be in that region



D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)
Aschenauer et al., arXiv:1708.01527

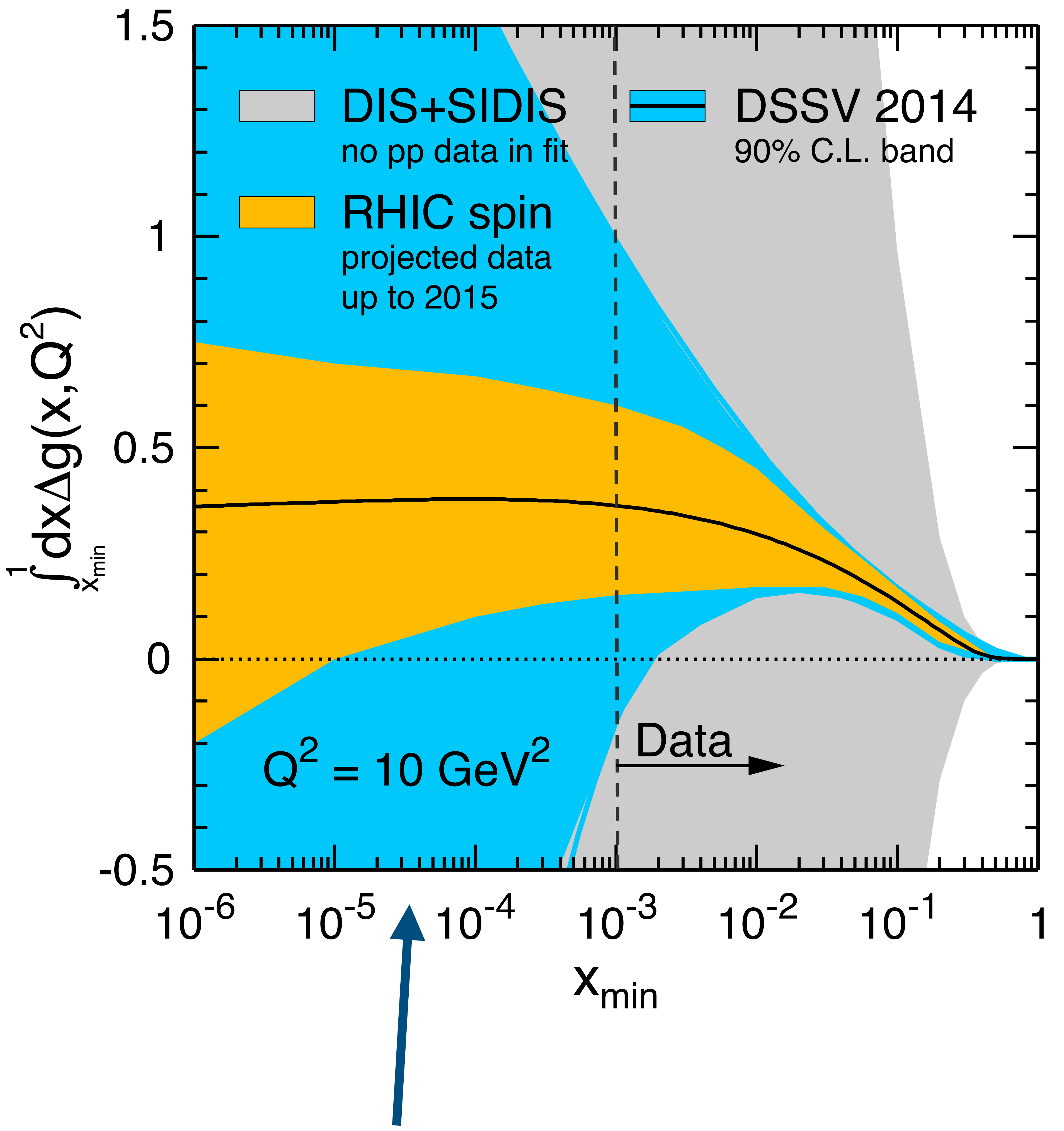
Extractions based on DGLAP

D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)
Aschenauer et al., arXiv:1708.01527

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma\\ \Delta\Gamma\end{pmatrix}=\begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0\\ -\frac{1}{2N_f}\Delta P_{\Sigma\Sigma}(a_s) & 0\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta\Gamma\end{pmatrix}$$

$$\Delta\Gamma(Q^2)\equiv a_s(Q^2)\Delta G(Q^2)$$

- The standard way to address the proton spin puzzle is by extracting the hPDFs from experimental data using collinear factorization along with the spin-dependent DGLAP evolution equations
- Since the DGLAP equations evolve PDFs in Q^2, they cannot truly predict the x dependence of PDFs
- The x dependence is greatly affected by the functional form of the PDF parametrization at the initial momentum scale Q_0^2, which gives the initial conditions for the DGLAP evolution

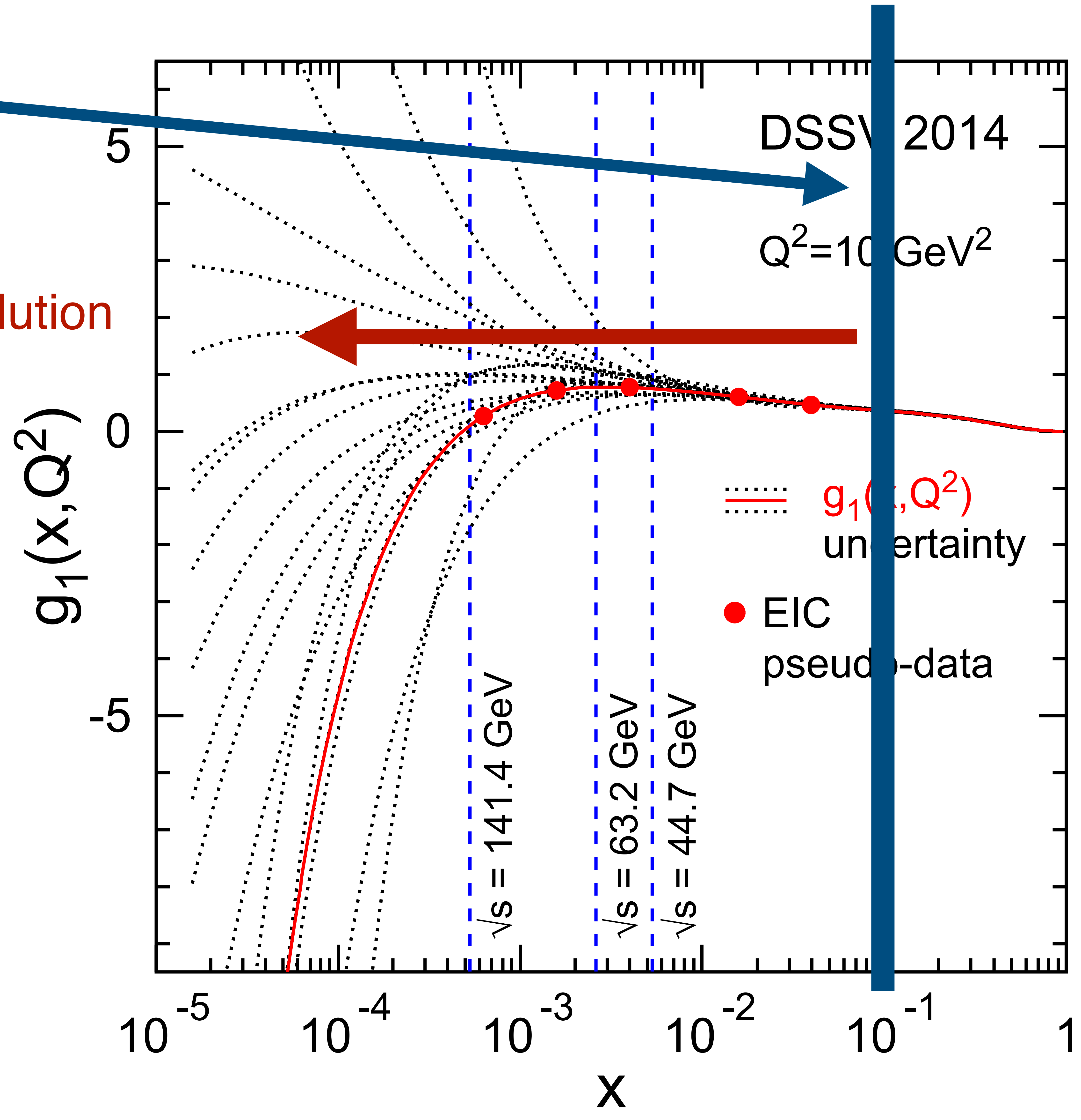


No data ⇒ no initial condition for DGLAP

DGLAP vs. small-x helicity evolution

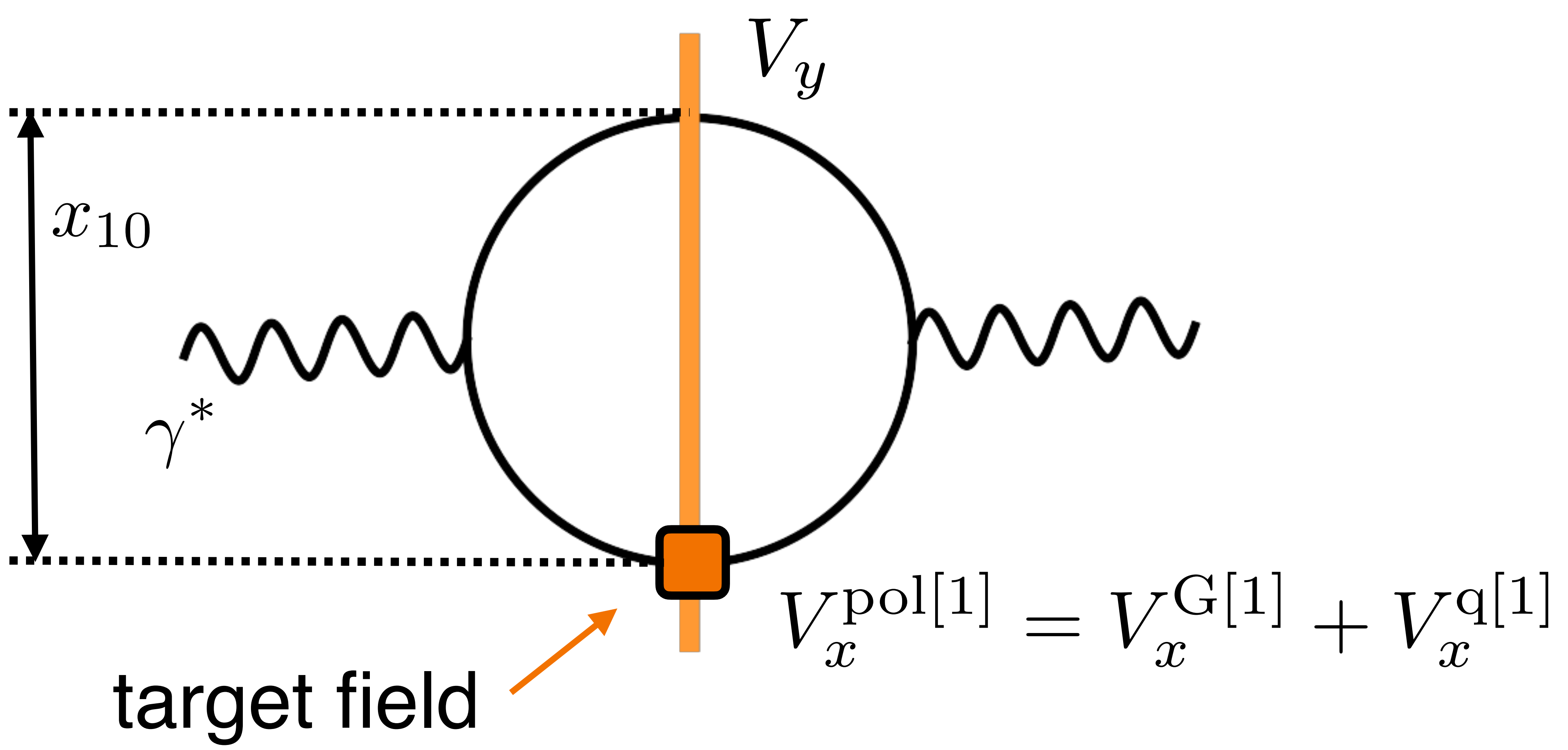
Initial condition at x_0

Small-x helicity evolution



- In the x region which has not yet been probed experimentally, DGLAP-based predictions typically acquire a broad uncertainty band due to extrapolation errors
- The benefit of small-x helicity evolution is it makes a genuine prediction for the hPDFs at small x given some initial conditions at a higher x_0

Small-x formalism for DIS



- Interaction of a virtual photon with a target is described via dipole amplitudes, e.g. Q_2, G_2 etc.

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \, \Delta q^+(x, Q^2)$$

- Quark distribution

quark amplitude ↓ gluon amplitude ↓

$$\Delta q^+(x, Q^2) \equiv \Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) = -\frac{N_c}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{\mathrm{d}z}{z} \int\limits_{1/zs}^{\min[1/zQ^2, 1/\Lambda^2]} \frac{\mathrm{d}x_{10}^2}{x_{10}^2} \left[Q_q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs) \right]$$

The dipole amplitudes depend on the transverse size of the dipole and center-of-mass energy squared

- Gluon distribution

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} \, G_2\left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x}\right)$$

KPS-CTT evolution

Kovchegov, Pitonyak, Sievert (2016-2019)

Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)

- How does the amplitude depend on z and x_{10}^2? Helicity dependent KPS-CTT evolution equations
- Sums up powers of $\alpha_s \ln 1/x$ and $\alpha_s \ln^2 1/x$
- Contains mixing between different types of operators (amplitudes)
- Consistent with small-x DGLAP evolution
- The equations are closed in the large-N_c and large-N_c & N_f limits.
- Large-N_c equations have been solved numerically (CKTT 2022) and analytically (J. Borden and Y. V. Kovchegov, 2023). The result is in agreement with the BER result:

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Global analysis

D. Adamiak, N. Baldonado, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato, M. D. Sievert, A. Tarasov, and Y. Tawabutr (arXiv:2308.07461)

- Perform, for the first time, a phenomenological analysis based on the KPS-CTT version of small-x helicity evolutions for gluon and flavor-singlet quark helicity distributions $\Delta q^{+}(x, Q^2)$
- Use 3 flavors (u, d, s)
- Base the analysis on the large-N_c & N_f limit
- To make the calculation more realistic we include running coupling corrections into the kernel of the evolution equations
- A system of six equations for six amplitude of the following form

$$
\begin{aligned}
Q_q(x_{10}^2, zs) = Q_q^{(0)}(x_{10}^2, zs) &+ \frac{N_c}{2\pi}\int_{1/x_{10}^2 s}^{z} \frac{\mathrm{d}z'}{z'} \int_{1/z's}^{x_{10}^2} \frac{\mathrm{d}x_{21}^2}{x_{21}^2}\, \alpha_s\!\left(\frac{1}{x_{21}^2}\right) \Big[2\,\widetilde{G}(x_{21}^2, z's) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) \\
&\qquad + Q_q(x_{21}^2, z's) - \overline{\Gamma}_q(x_{10}^2, x_{21}^2, z's) + 2\,\Gamma_2(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's)\Big] \\
&+ \frac{N_c}{4\pi}\int_{\Lambda^2/s}^{z} \frac{\mathrm{d}z'}{z'} \int_{1/z's}^{\min\left[x_{10}^2 z/z', 1/\Lambda^2\right]} \frac{\mathrm{d}x_{21}^2}{x_{21}^2}\, \alpha_s\!\left(\frac{1}{x_{21}^2}\right) \left[Q_q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's)\right],
\end{aligned}
$$

With initial conditions:

$$
\widetilde{G}^{(0)}(x_{10}^2, zs) = Q_q^{(0)}(x_{10}^2, zs) = \frac{\alpha_s^2 C_F}{2N_c}\pi\left[C_F \ln\frac{zs}{\Lambda^2} - 2\ln\left(zs x_{10}^2\right)\right], \qquad G_2^{(0)}(x_{10}^2, zs) = \frac{\alpha_s^2 C_F}{N_c}\pi \ln\frac{1}{x_{10}\Lambda},
$$

- implementation of the KPS-CTT evolution within the JAM Bayesian Monte Carlo framework

Flavor nonsinglet evolution at small x

- Measurements of the g_1 structure function in DIS off a nucleon are only sensitive to a specific linear combination of $\Delta q^+(x, Q^2)$
- The polarized SIDIS process, provides information on the individual flavor hPDFs $\Delta q^-(x, Q^2)$

$$\Delta q^-(x, Q^2) \equiv \Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2) = \frac{N_c}{2\pi^3} \int_{\Lambda^2/s}^{1} \frac{\mathrm{d}z}{z} \int_{1/zs}^{\min\left[1/zQ^2, 1/\Lambda^2\right]} \frac{\mathrm{d}x_{10}^2}{x_{10}^2} G_q^{\mathrm{NS}}(x_{10}^2, zs)$$

- Include both polarized DIS data and polarized SIDIS data
- Use large-N_c small-x helicity evolution equation to extract the flavor nonsinglet distribution $\Delta q^-(x, Q^2)$
- SIDIS cross section. Quarks and antiquarks have different fragmentation functions, so the cross section cannot be expressed in terms of $\Delta q^+(x, Q^2)$

$$g_1^h(x, z, Q^2) = \frac{1}{2} \sum_{q, \bar{q}} e_q^2 \, \Delta q(x, Q^2) \, D_1^{h/q}(z, Q^2)$$

Flavor nonsinglet evolution at small x

- One can construct a numerical solution of the equations by discretizing integrals in our evolution equations

$$Q_q(x_{10}^2, zs) = Q_q^{(0)}(x_{10}^2, zs) + \frac{N_c}{2\pi} \int_{1/x_{10}^2 s}^{z} \frac{\mathrm{d}z'}{z'} \int_{1/z's}^{x_{10}^2} \frac{\mathrm{d}x_{21}^2}{x_{21}^2} \alpha_s\left(\frac{1}{x_{21}^2}\right) \Big[2\,\widetilde{G}(x_{21}^2, z's) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) + Q_q(x_{21}^2, z's) - \overline{\Gamma}_q(x_{10}^2, x_{21}^2, z's) + 2\,\Gamma_2(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's)\Big] + \frac{N_c}{4\pi} \int_{\Lambda^2/s}^{z} \frac{\mathrm{d}z'}{z'} \int_{1/z's}^{\min\left[x_{10}^2 z/z', 1/\Lambda^2\right]} \frac{\mathrm{d}x_{21}^2}{x_{21}^2} \alpha_s\left(\frac{1}{x_{21}^2}\right) \left[Q_q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's)\right],$$

- Initial conditions can are parametrized

$$Q_q^{(0)}(s_{10}, \eta) = a_q\,\eta + b_q\,s_{10} + c_q\,,$$ ← 3 flavors

$$\widetilde{G}^{(0)}(s_{10}, \eta) = \widetilde{a}\,\eta + \widetilde{b}\,s_{10} + \widetilde{c}\,,$$

$$G_2^{(0)}(s_{10}, \eta) = a_2\,\eta + b_2\,s_{10} + c_2\,.$$

← gluons

- 15 parameters for singlet + 9 parameters for non-singlet

$$G_q^{\mathrm{NS}\,(0)} = a_q^{\mathrm{NS}}\,\eta + b_q^{\mathrm{NS}}\,s_{10} + c_q^{\mathrm{NS}},$$ ← 3 flavors

Experimental data

Data set (A_1)	Target	N_{pts}	χ^2/N_{pts}
SLAC (E142) [137]	^{3}He	1	0.60
EMC [142]	p	5	0.20
SMC [143, 145]	p	6	1.29
	p	6	0.53
	d	6	0.67
	d	6	2.26
COMPASS [146]	p	5	1.02
COMPASS [147]	p	17	0.74
COMPASS [148]	d	5	0.88
HERMES [149]	n	2	0.73
Total		59	0.91

Data set ($A_\parallel$)	Target	N_{pts}	χ^2/N_{pts}
SLAC(E155) [140]	p	16	1.28
	d	16	1.62
SLAC (E143) [139]	p	9	0.56
	d	9	0.92
SLAC (E154) [138]	^{3}He	5	1.09
HERMES [150]	p	4	1.54
	d	4	0.98
Total		63	1.19

Dataset (A_1^h)	Target	Tagged Hadron	N_{pts}	χ^2/N_{pts}
SMC [144]	p	h^+	7	1.03
	p	h^-	7	1.45
	d	h^+	7	0.82
	d	h^-	7	1.49
HERMES [154]	p	π^+	2	2.39
	p	π^-	2	0.01
	p	h^+	2	0.79
	p	h^-	2	0.05
	d	π^+	2	0.47
	d	π^-	2	1.40
	d	h^+	2	2.84
	d	h^-	2	1.22
	d	K^+	2	1.81
	d	K^-	2	0.27
	d	$K^+ + K^-$	2	0.97
HERMES [155]	^{3}He	h^+	2	0.49
	^{3}He	h^-	2	0.29
COMPASS [152]	p	π^+	5	1.88
	p	π^-	5	1.10
	p	K^+	5	0.42
	p	K^-	5	0.31
COMPASS [153]	d	π^+	5	0.50
	d	π^-	5	0.78
	d	h^+	5	0.90
	d	h^-	5	0.86
	d	K^+	5	1.50
	d	K^-	5	0.78
Total			104	1.01

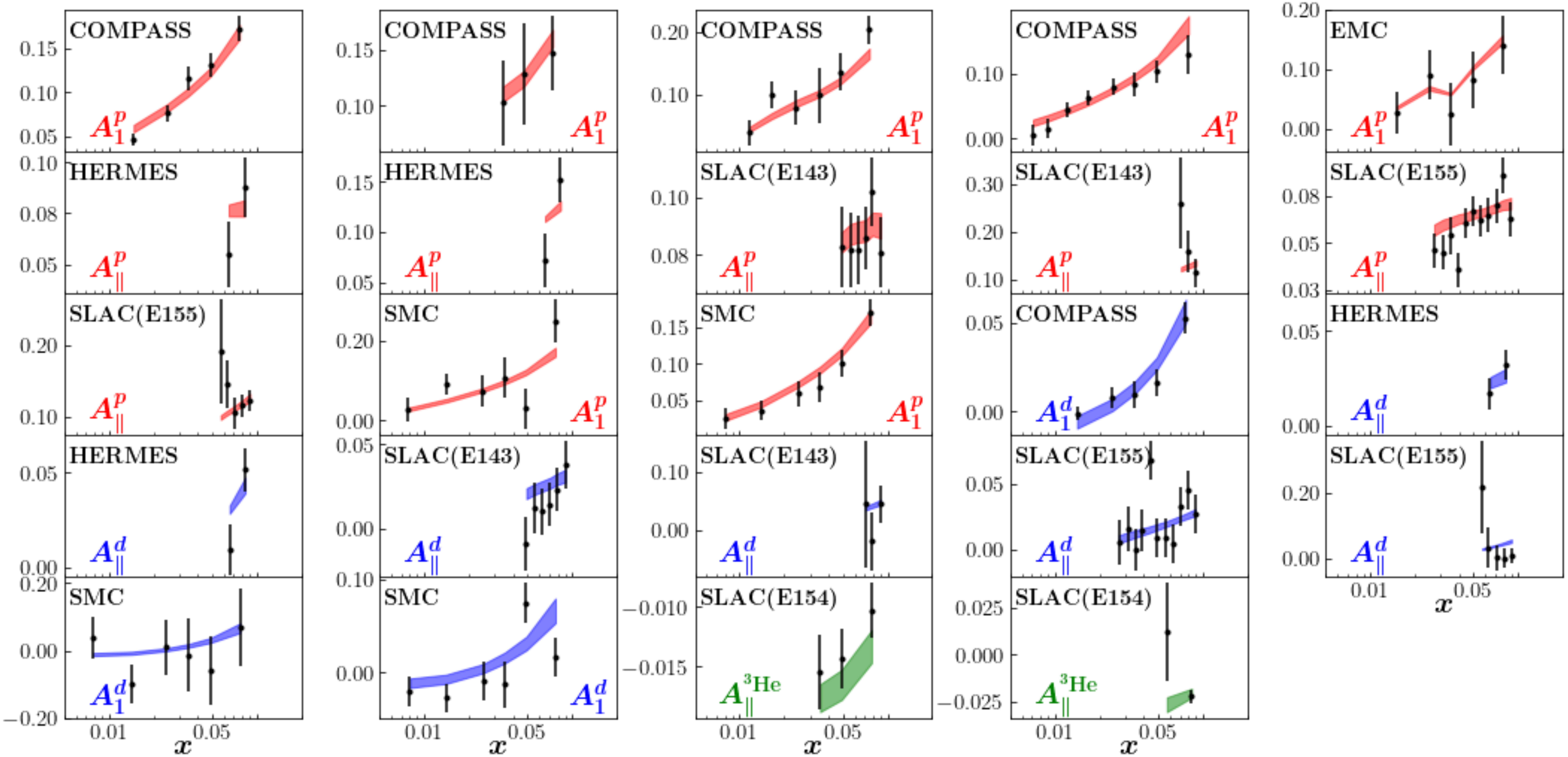
$$5 \times 10^{-3} < x < 0.1 \equiv x_0$$

$$1.69\ \mathrm{GeV}^2 < Q^2 < 10.4\ \mathrm{GeV}^2$$

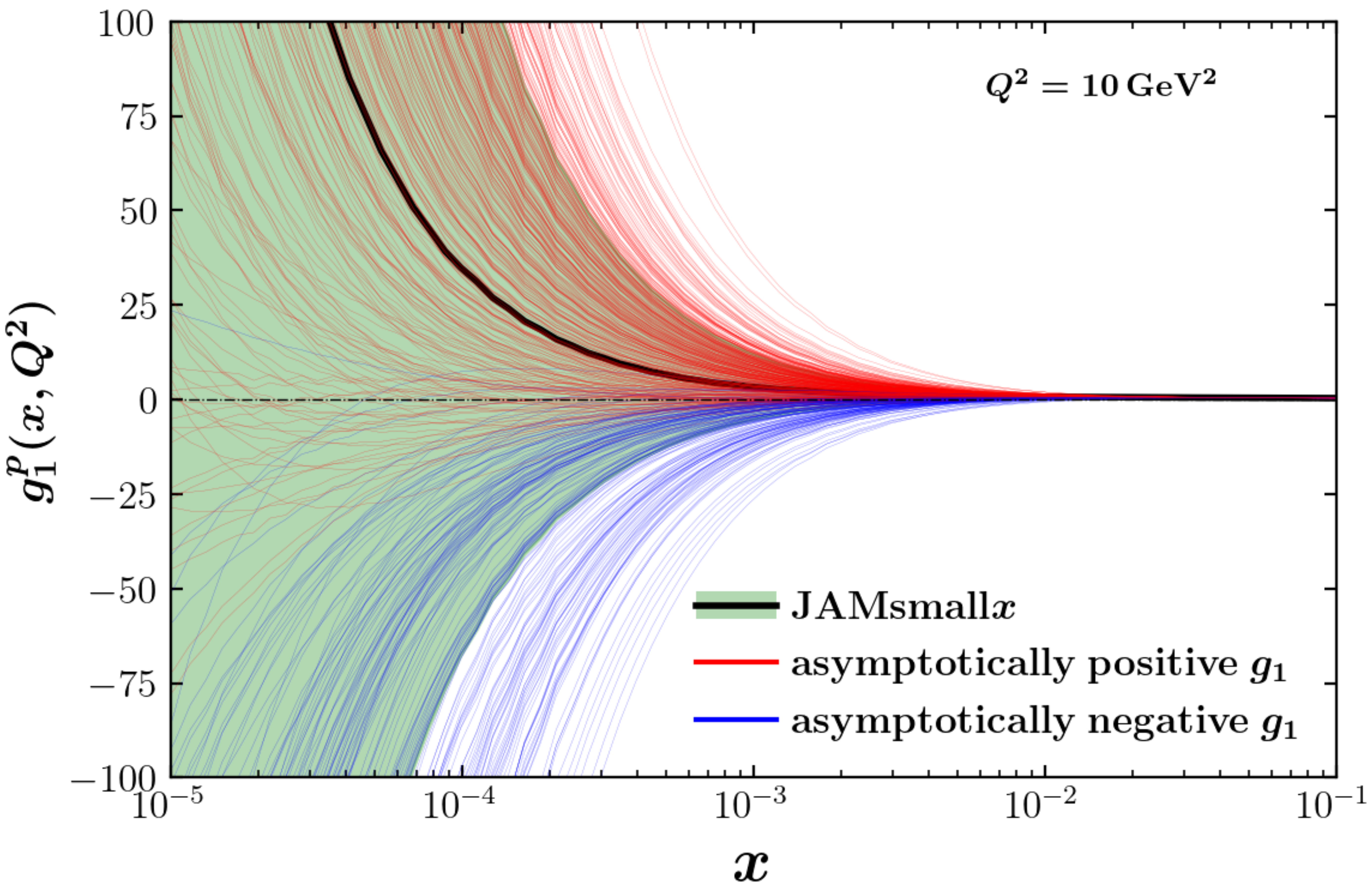
$$0.2 < z < 1.0$$

$$N_{\mathrm{pts}} = 226$$

Data versus theory

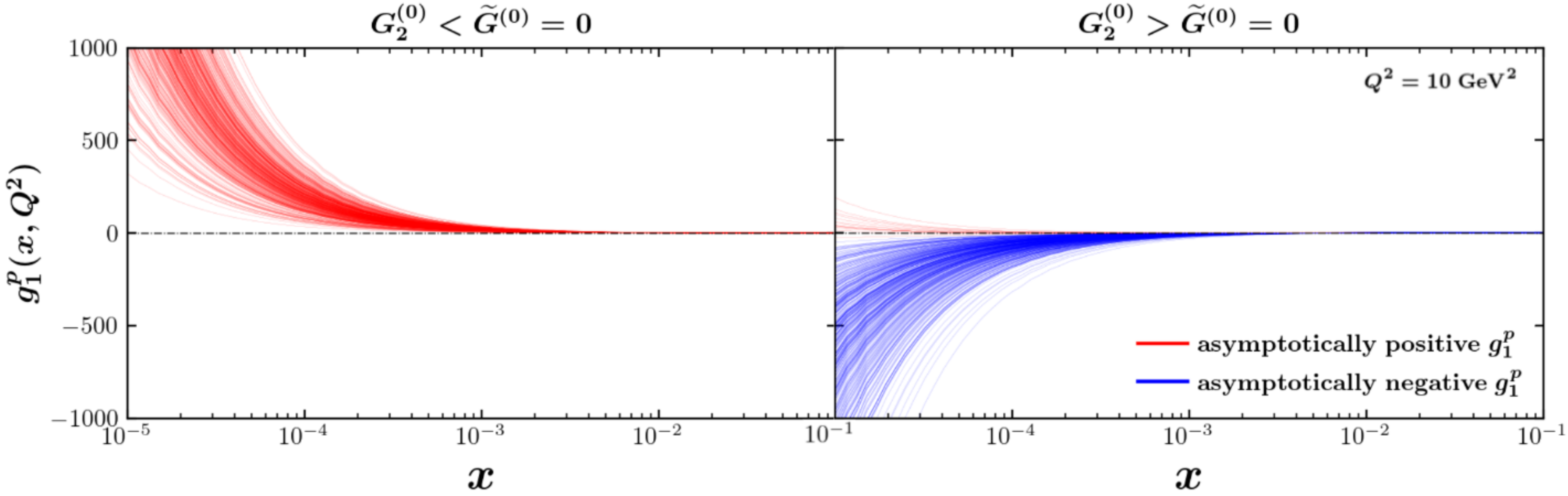


g_1 structure function (x dependence)



- 500 replicas
- Each replica represents an individual fit of the experimental data
- largely unconstrained at smaller x
- g_1 is well constrained in the region where there is experimental data
- Evolution equations guarantees that the small x behavior of g_1 must be exponential in $\ln(1/x)$

Sign of g_1 structure function



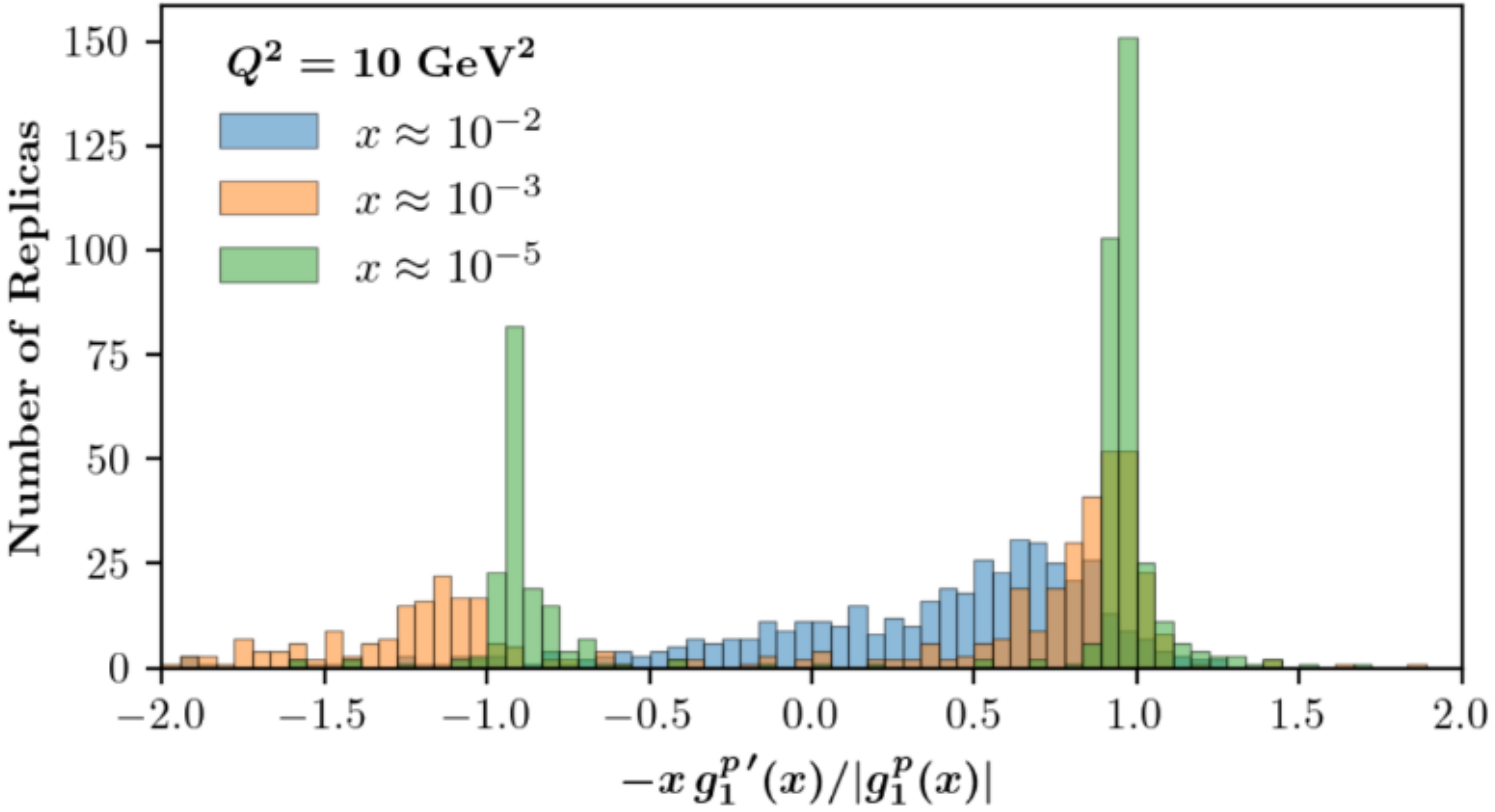
- The major difficulty in constraining g_1 is caused by the insensitivity of the data to the G_2 and $\tilde{G}$ amplitudes

Asymptotic behavior

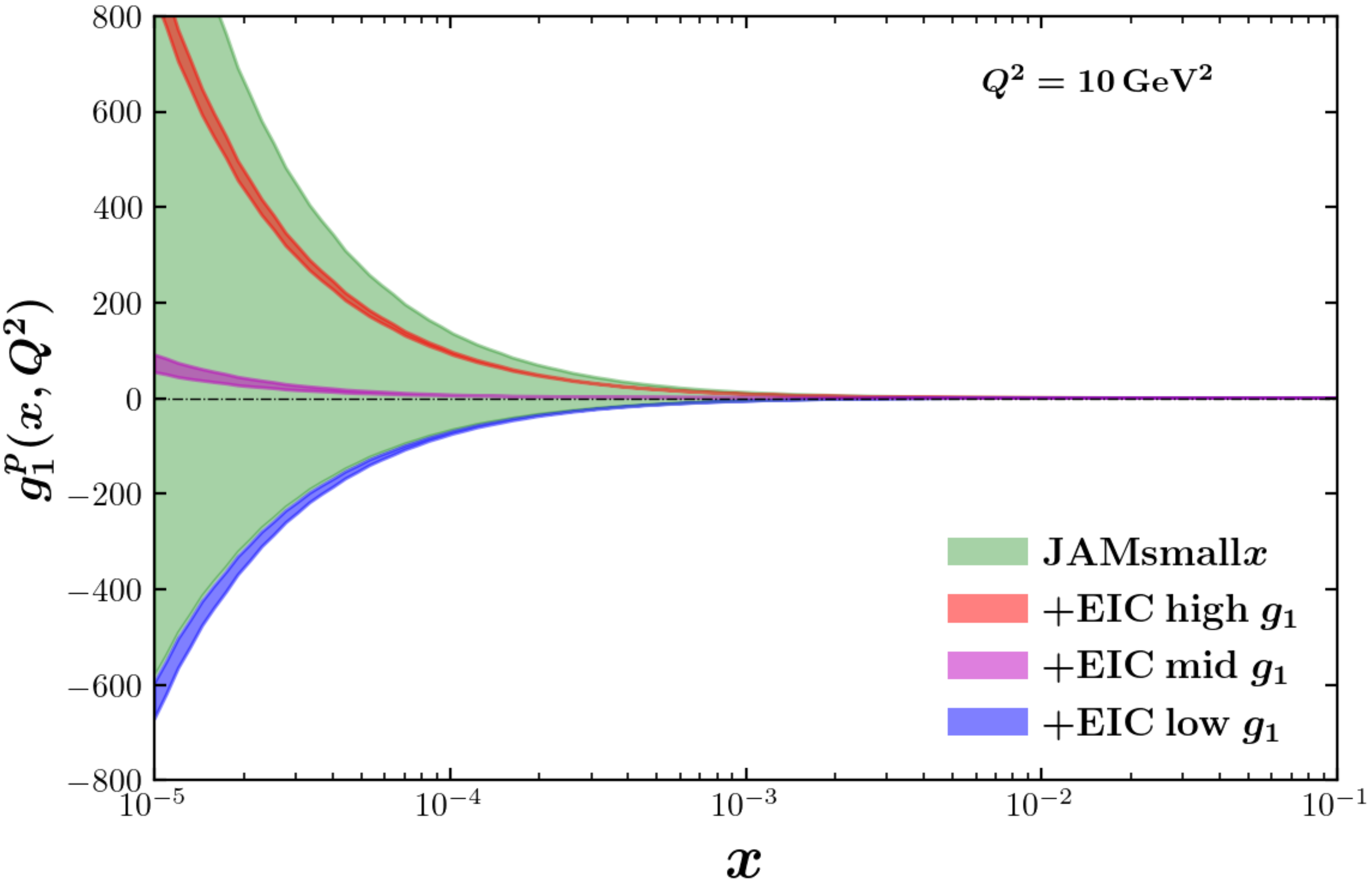
- Evolution equations that we use guaranty the asymptotic behavior

$$\lim_{x\to 0} g_1^p(x) \equiv g_1^{p\,(0)}\, x^{-\alpha_h(x)}$$

$$\alpha_h(x) \equiv \frac{1}{g_1^p(x)} \frac{\mathrm{d}\, g_1^p(x)}{\mathrm{d} \ln(1/x)}$$



Impact of EIC data



- In order to study the impact of lower x measurements on our ability to predict the behavior of g_1 and the hPDFs at even smaller x, we utilized EIC pseudodata for the kinematic region of $10^{-4} < x < 10^{-1}$ and $1.69\ GeV^2 < Q^2 < 50\ GeV^2$

Thank you for your attention!