Frontiers of parton-shower accuracy

HET seminar BNL - 12/10/2023

Melissa van Beekveld

UNIVERSITY OF OXFORD

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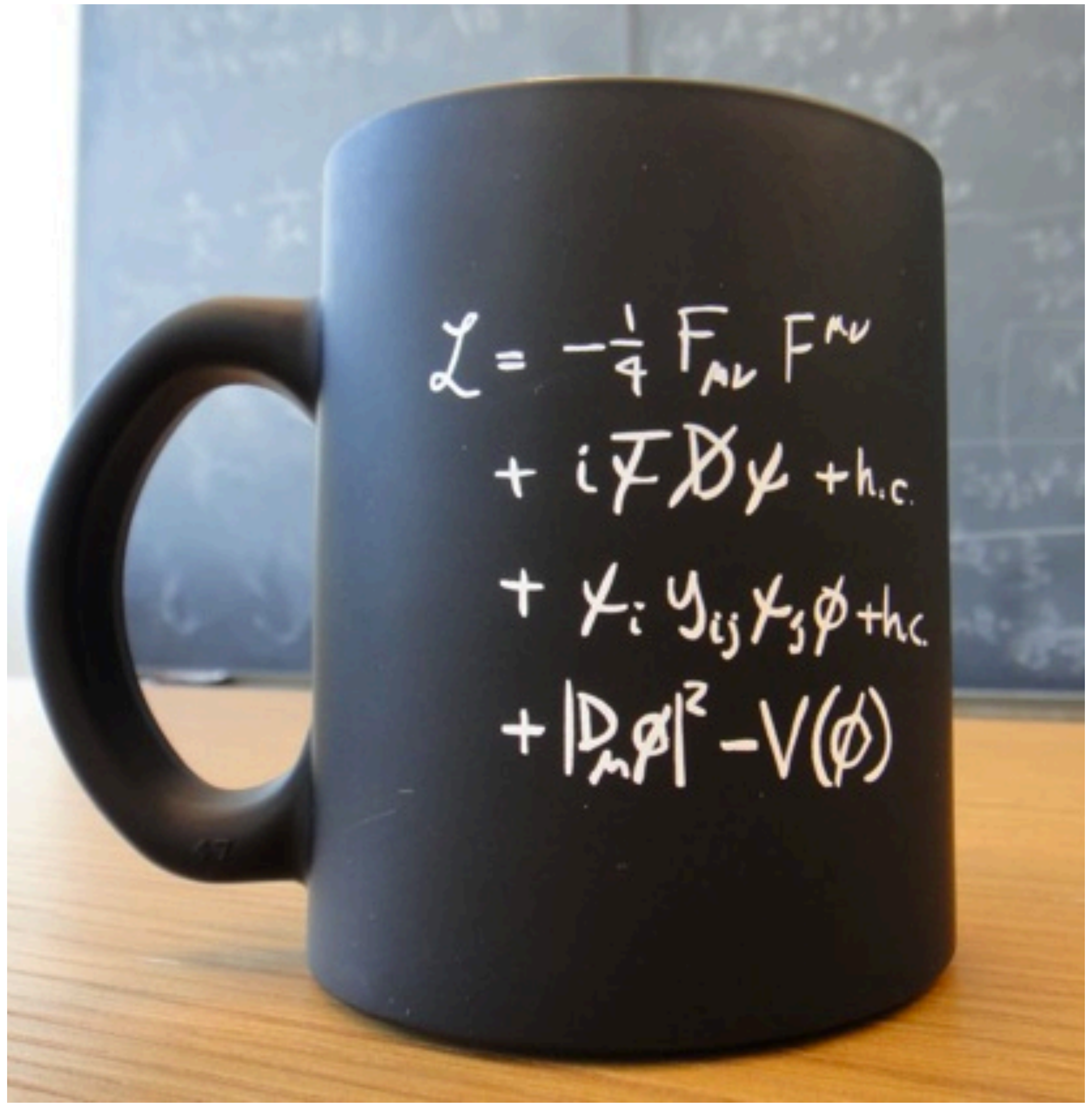
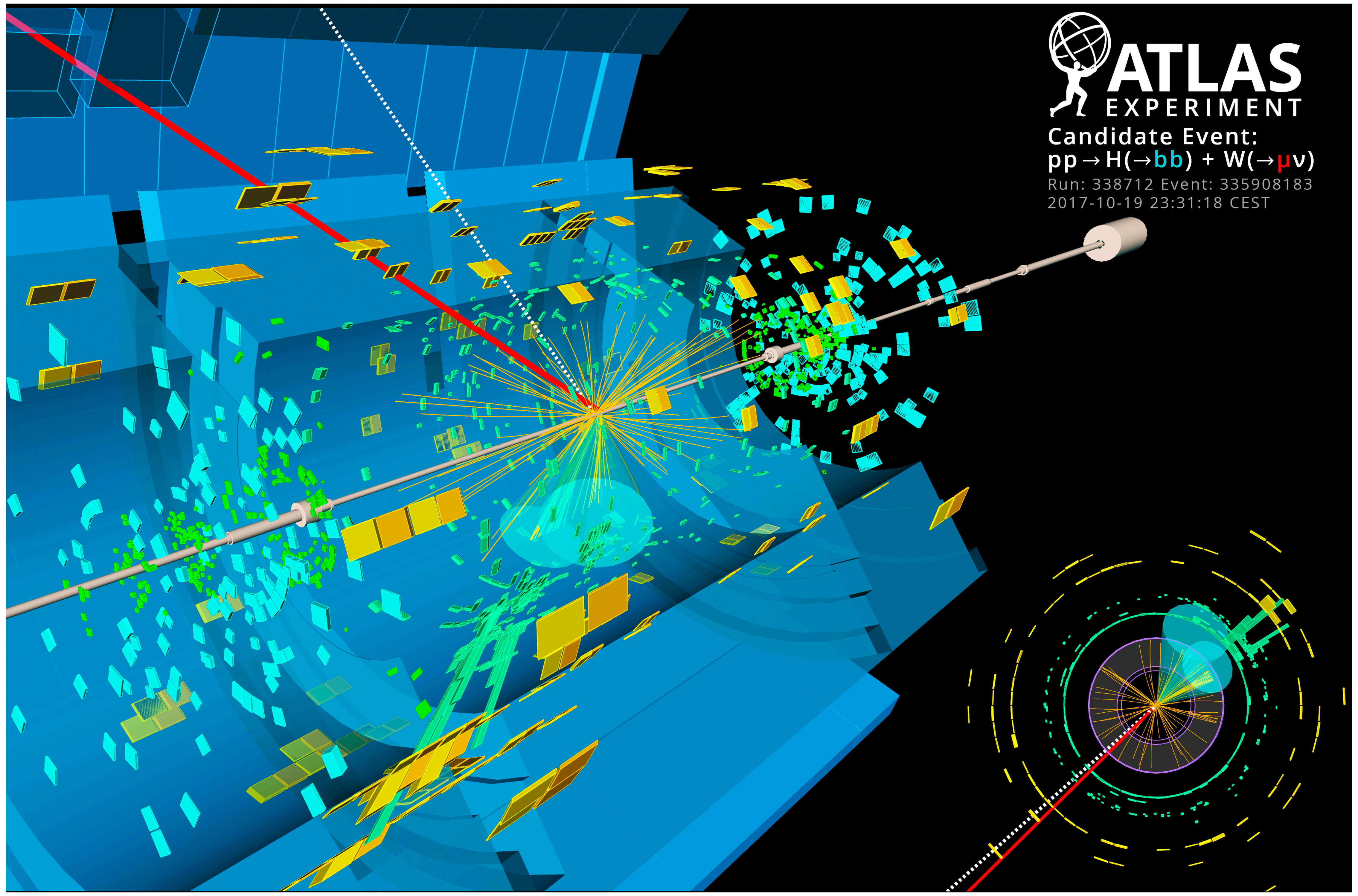
Nikhef

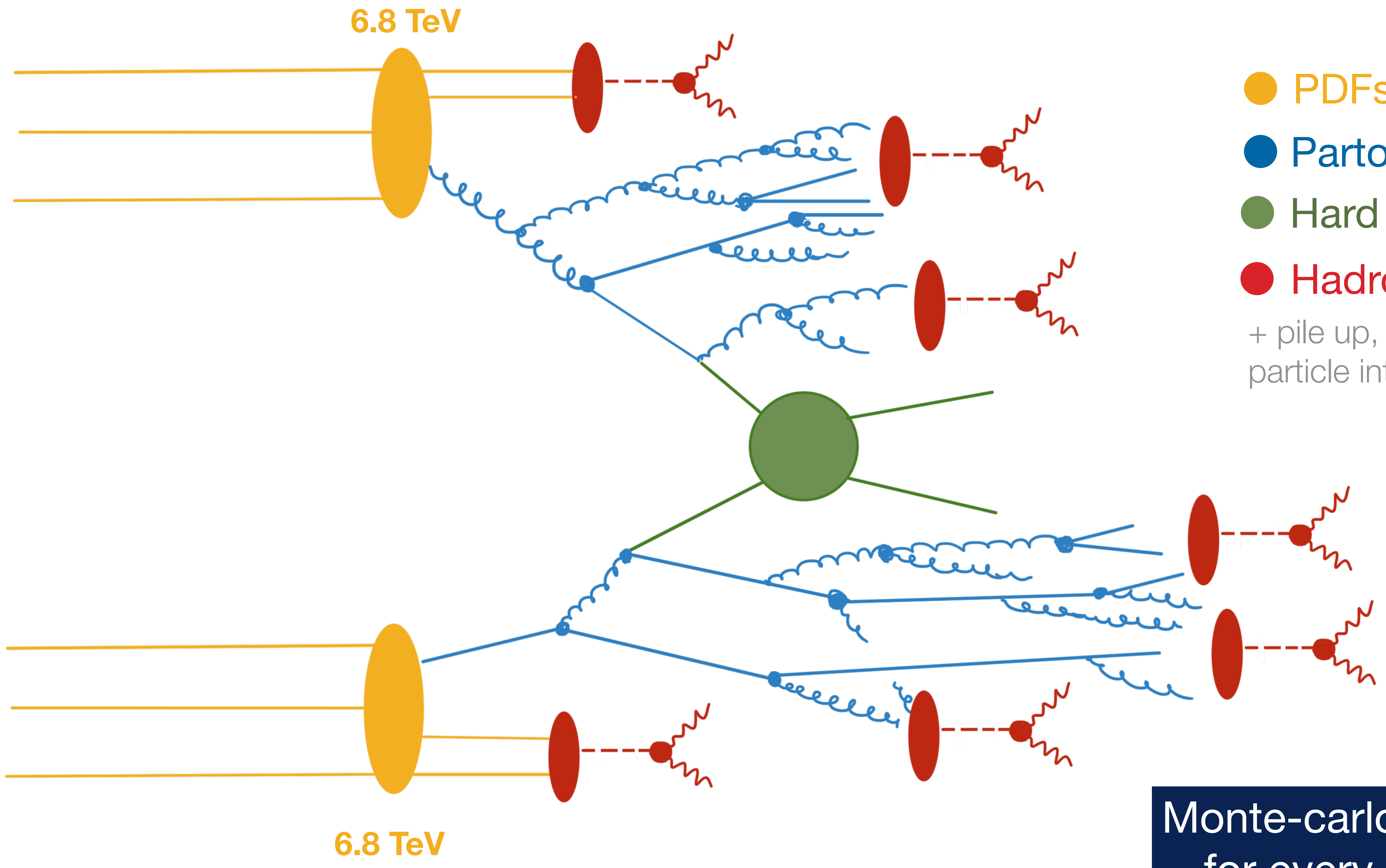


How to relate theory to what we see in actual experiments?

We use **Monte Carlo** generators!



Components of an LHC event



PDFs / beam remnants

Parton shower $\mathcal{O}(1-100)$ GeV

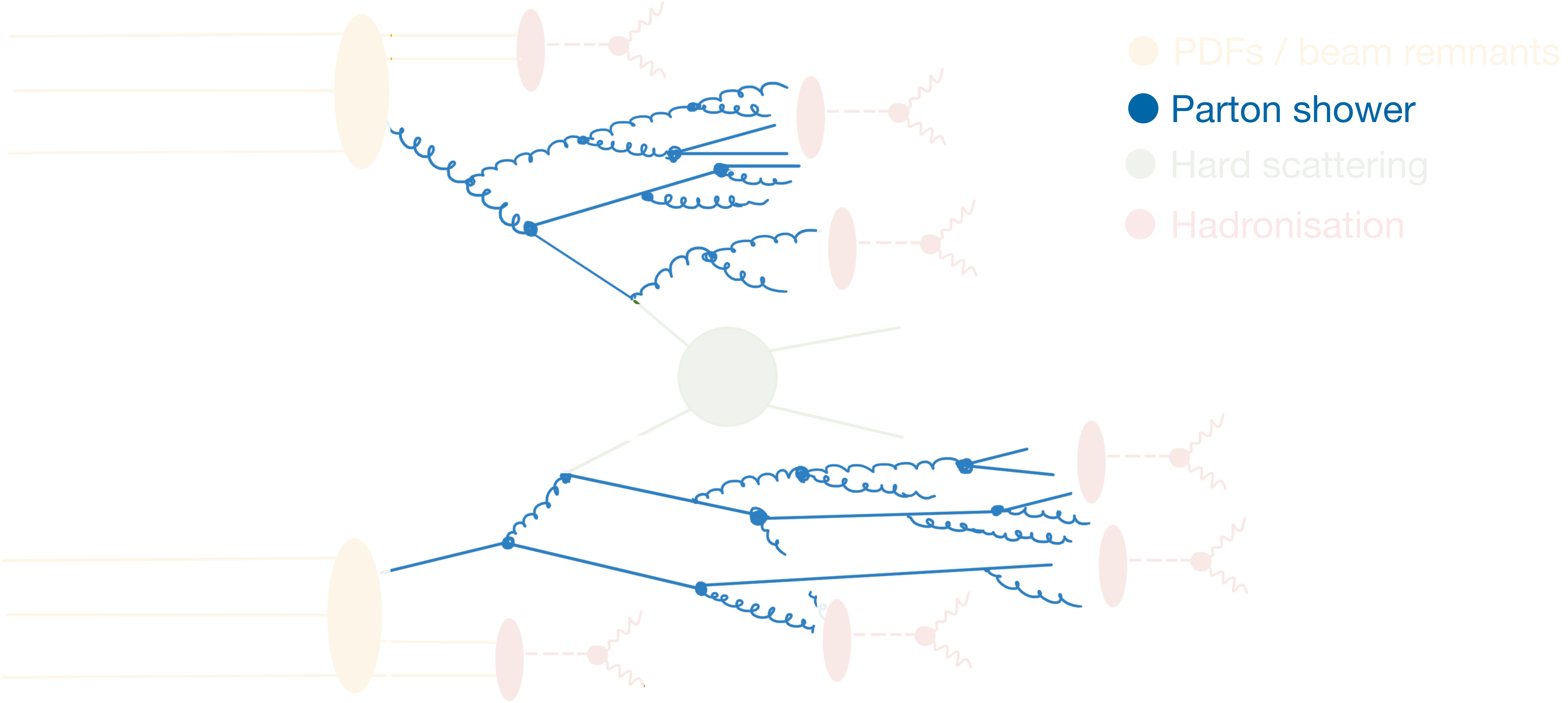
Hard scattering $\mathcal{O}(0.1-1)$ TeV

Hadronisation $\mathcal{O}(1)$ GeV

+ pile up, underlying event, multiple-particle interactions (MPI)…

Monte-carlo generators available for every step of the process

Components of an LHC event



Today's focus

Basics of a parton shower (PS)

QCD

- Described by the $SU(N_c = 3)$ group
- Quarks are in the fundamental representation (N_C generators)
- Anti-quarks in the anti-fundamental representation
- Gluons in the adjoint ($N_c^2 - 1$ generators)

Special type of shower: the dipole shower

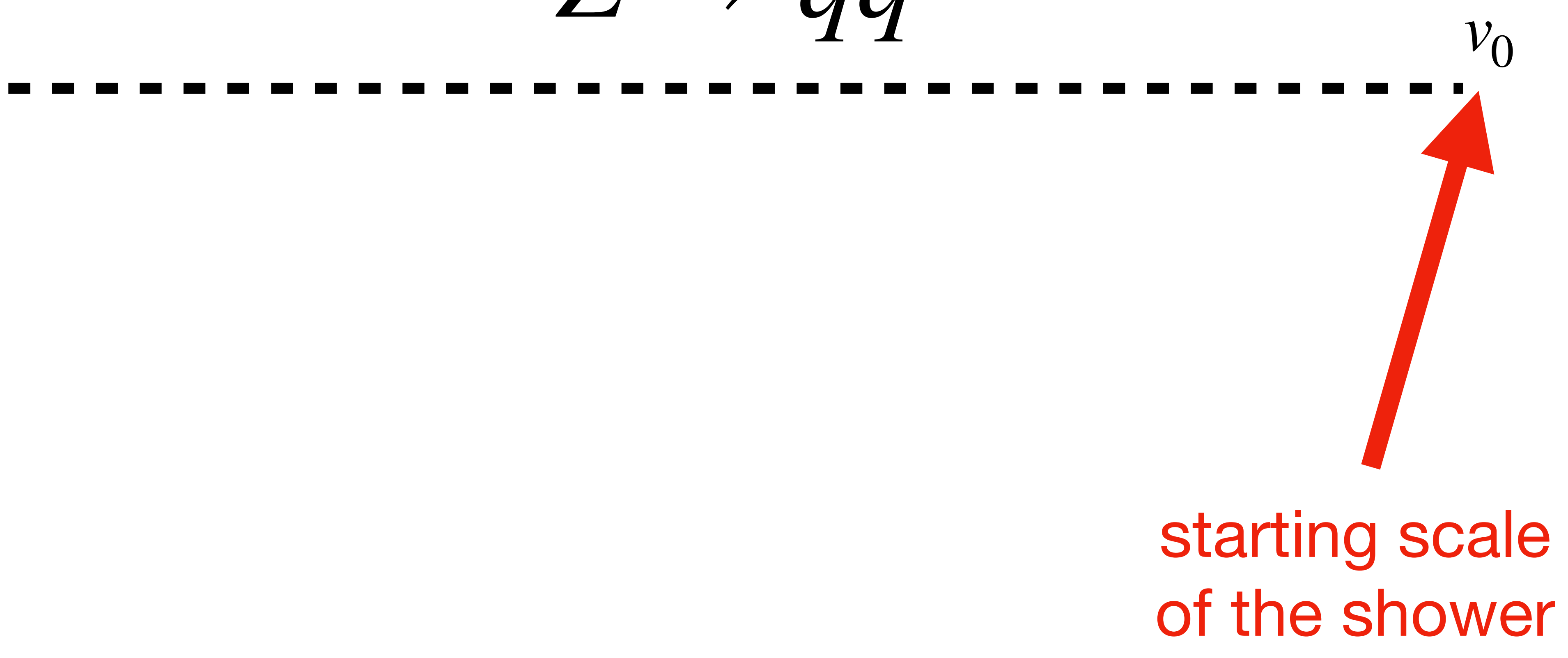
- Take the $N_c \to \infty$ limit
- (Anti-)quarks carry (anti-)colour
- Gluons carry one colour and one anti-colour charge
- Assign a colour connection between all colour charges

What is a parton shower?

Illustrated with a dipole shower for final-state emissions

$$Z \to q\bar{q}$$

Start with some partonic state
This spans an initial 'colour dipole'



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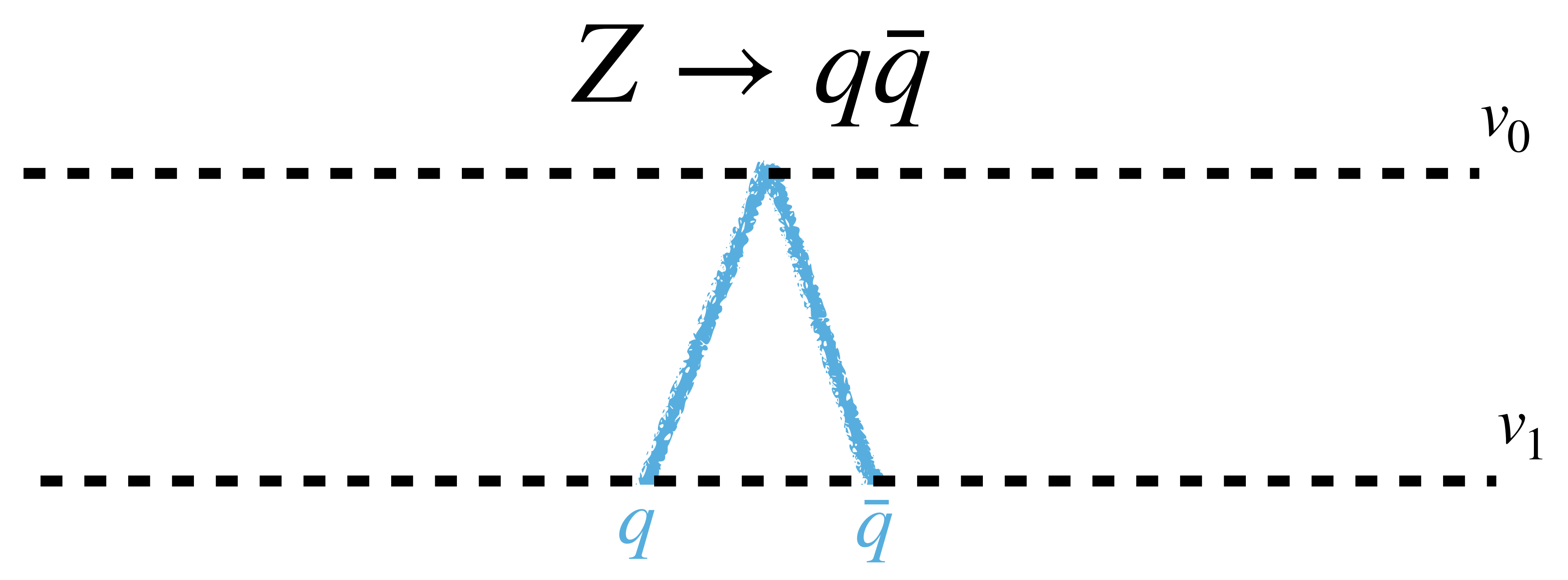

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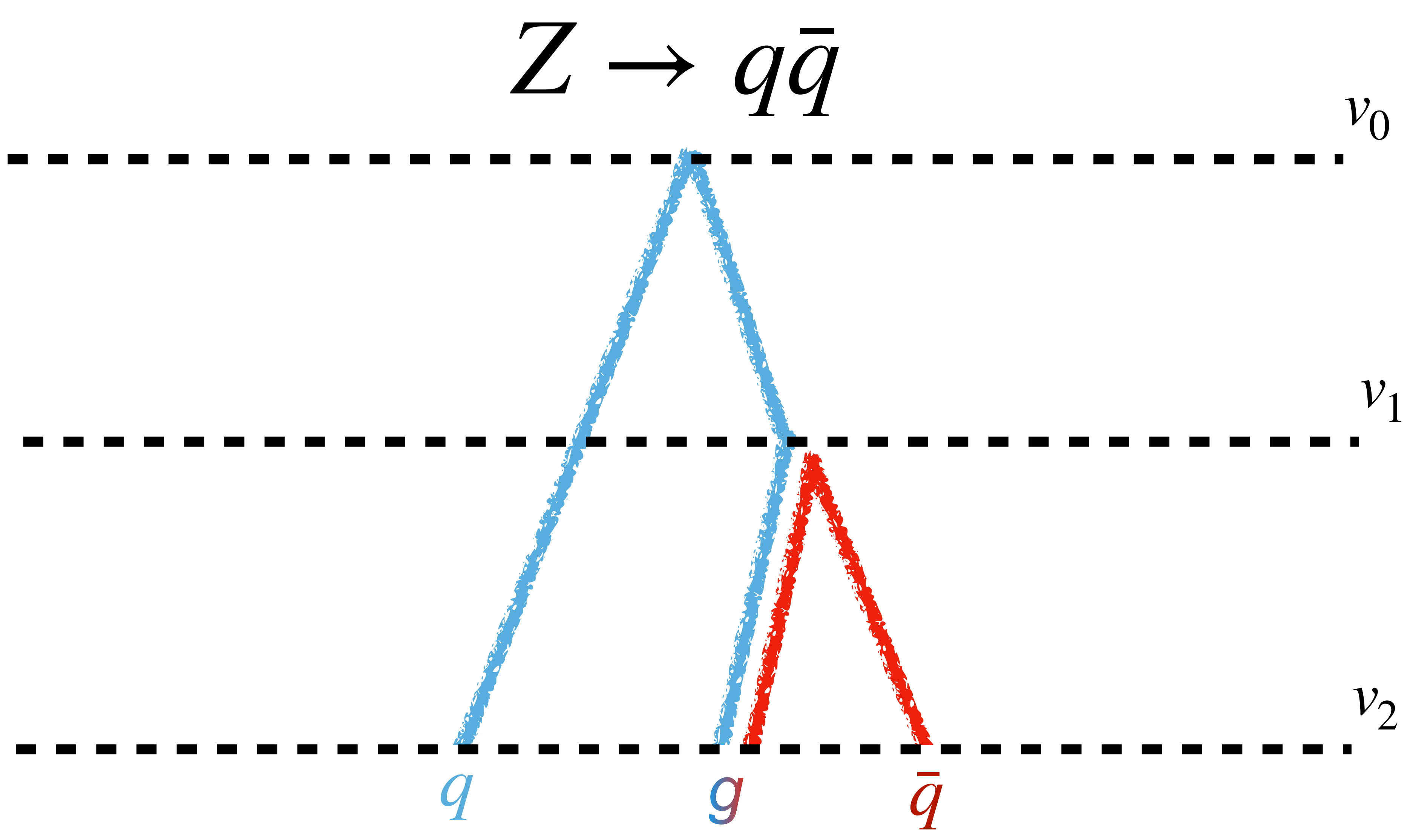
The state splits…
The new gluon is part of two (independent) dipoles



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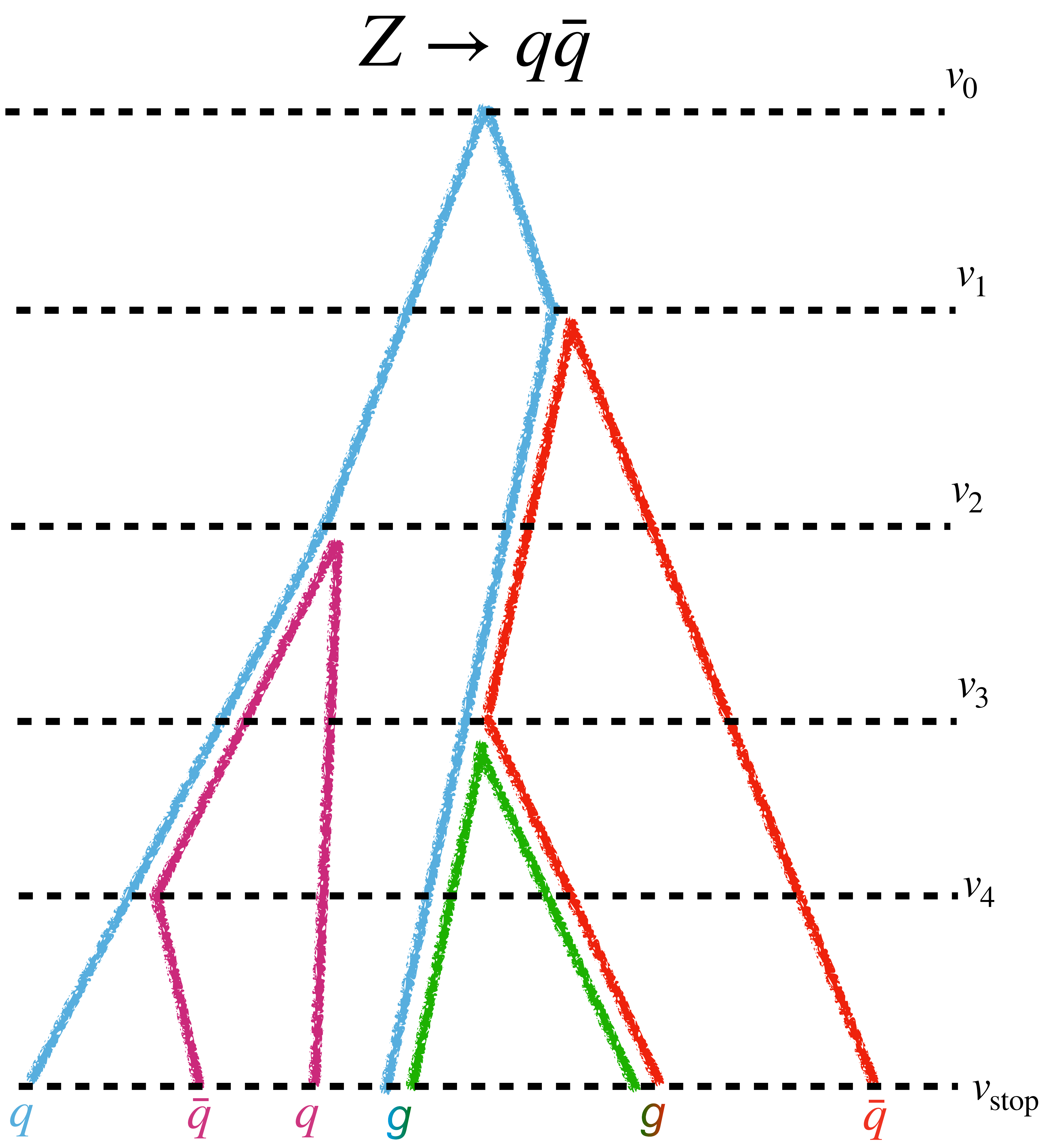
Process continues until it reaches a non-perturbative cut-off scale

End result: set of particles and their four momenta, from which any (well-defined) observable may be reconstructed



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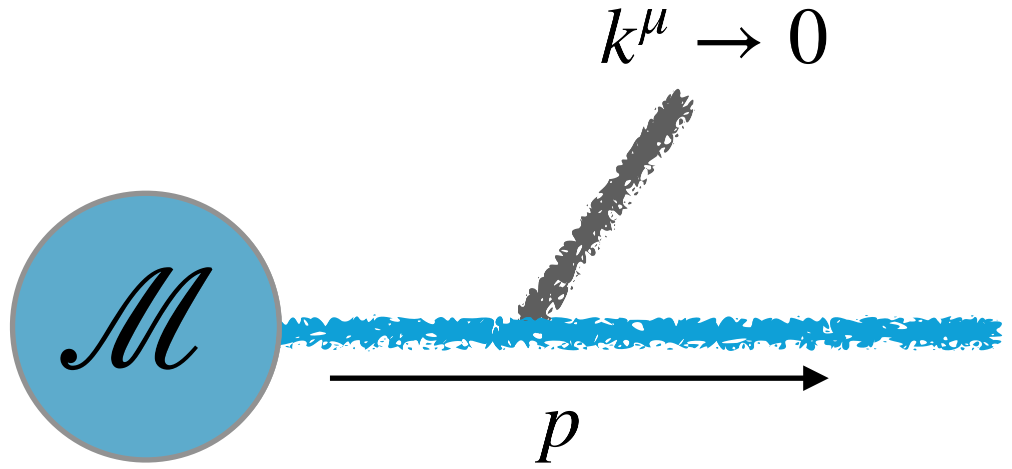
The splitting probability

Emission of a soft gluon: the eikonal Feynman rule

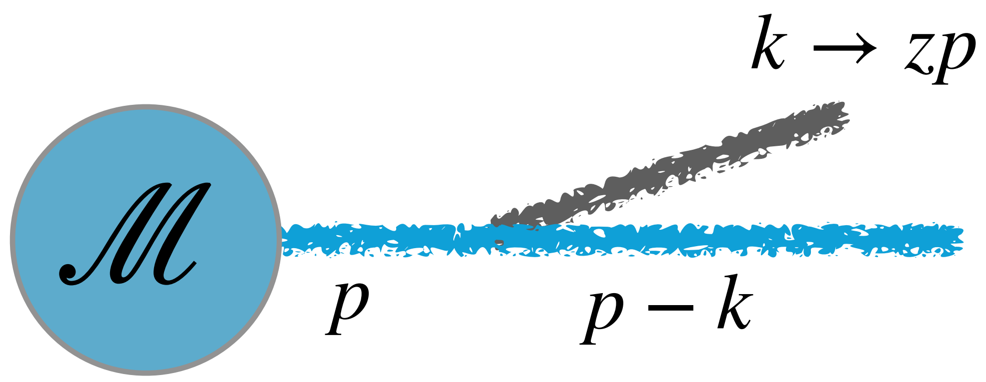


$$\propto g_s \frac{p^\mu}{p \cdot k} \mathbf{T} \otimes \mathscr{M}$$

$\mathbf{T}$ is a colour-generator

- Spin dependence is factorised
- Colour dependence is not

Emission of a collinear particle: Splitting functions $P_{(ij)a}$



$$\propto g_s \frac{1}{p \cdot k} P_{(ij),a}(z) \otimes \mathscr{M}_a$$

a is a spin index

- Colour dependence is factorised
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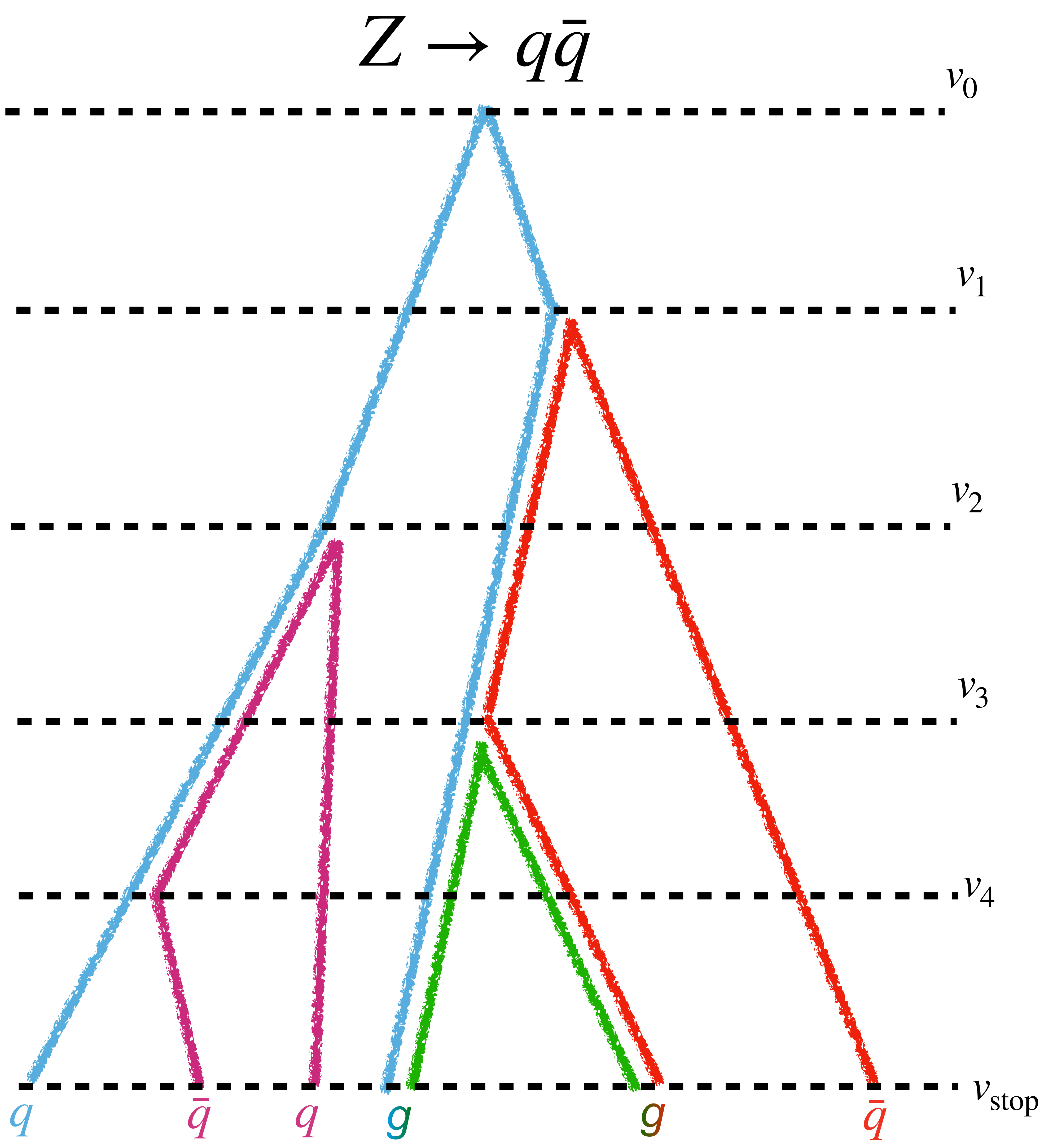

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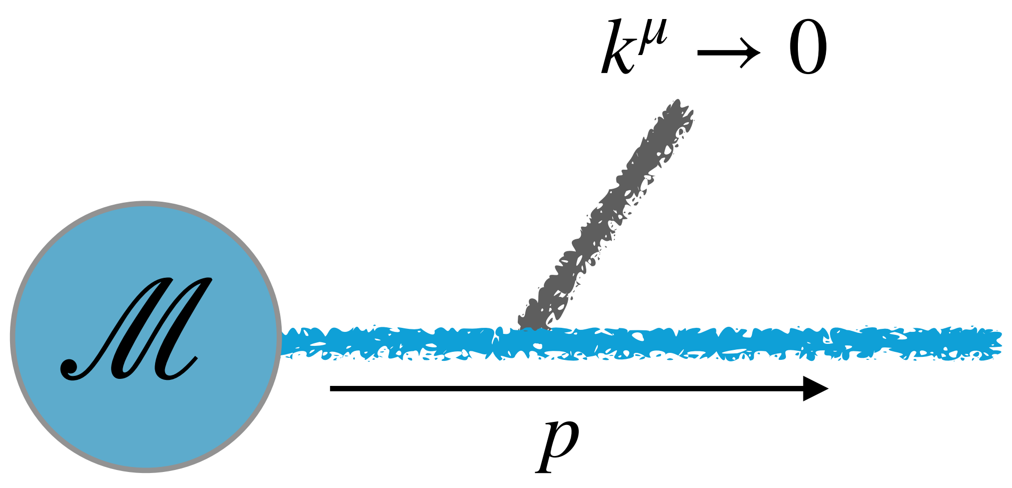

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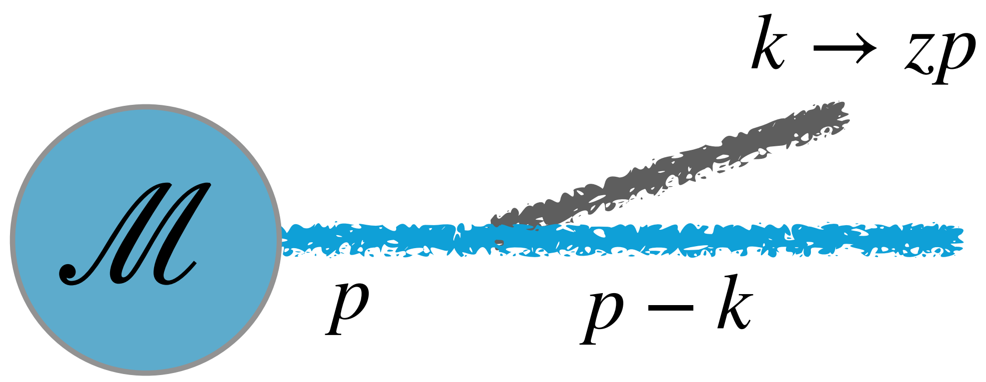

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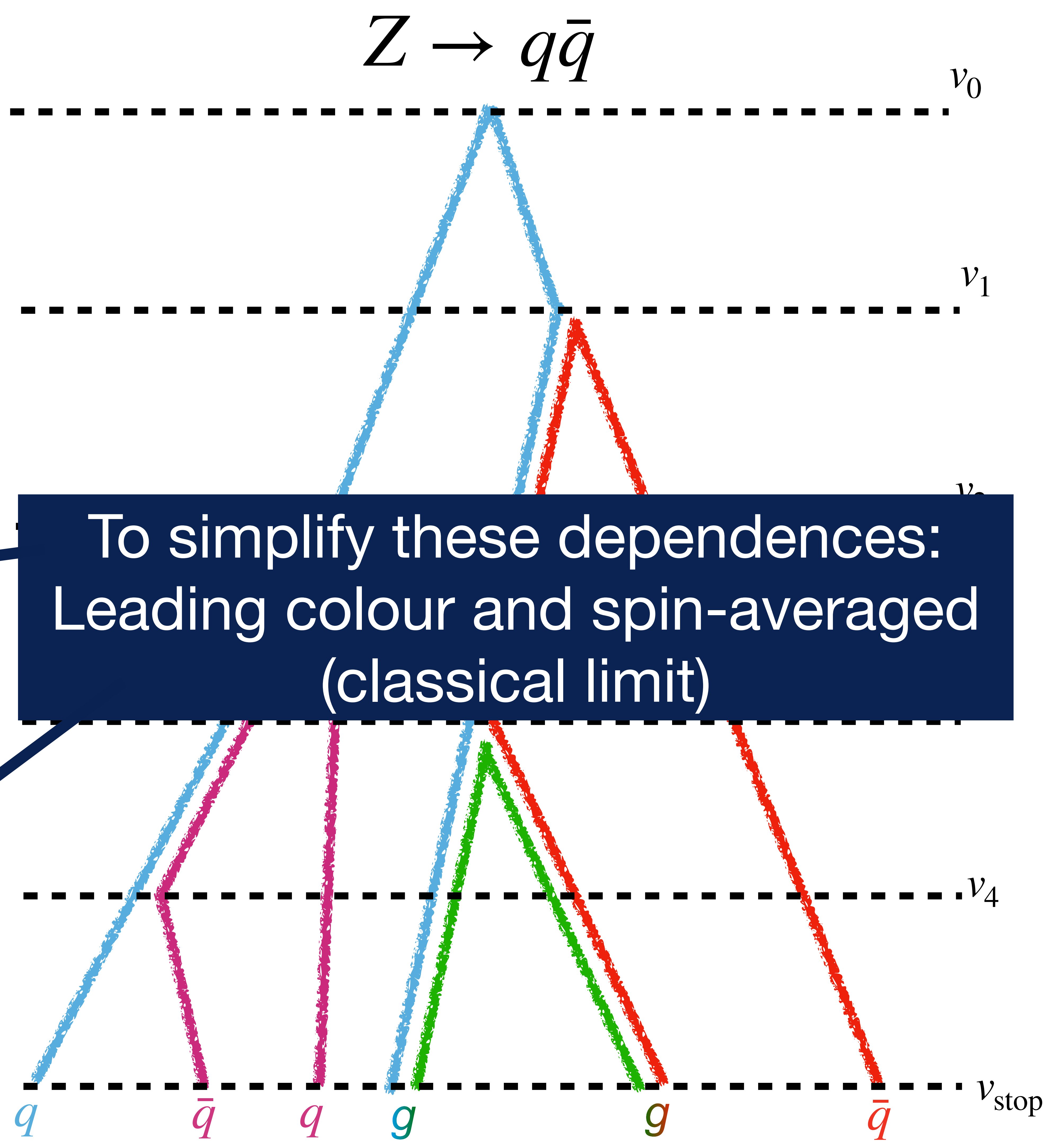

To simplify these dependences: Leading colour and spin-averaged (classical limit)

The splitting probability

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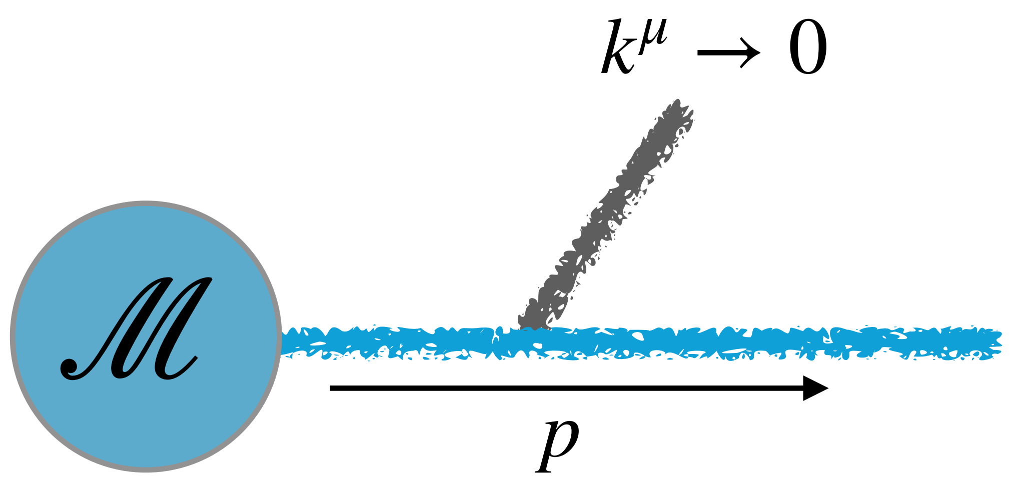

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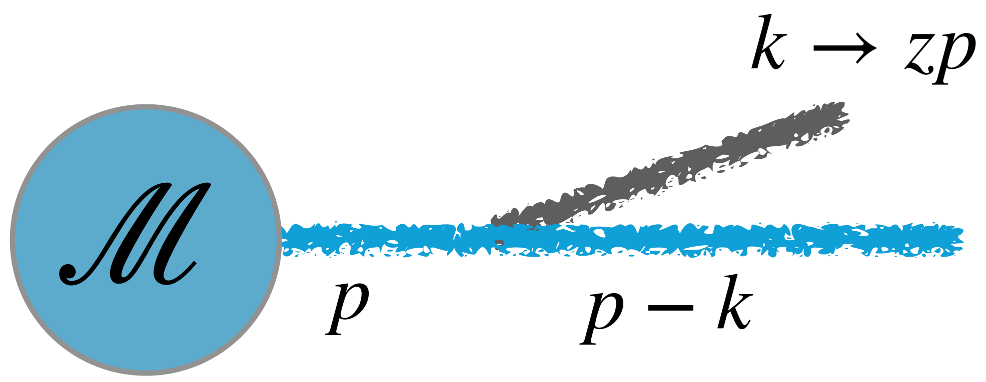

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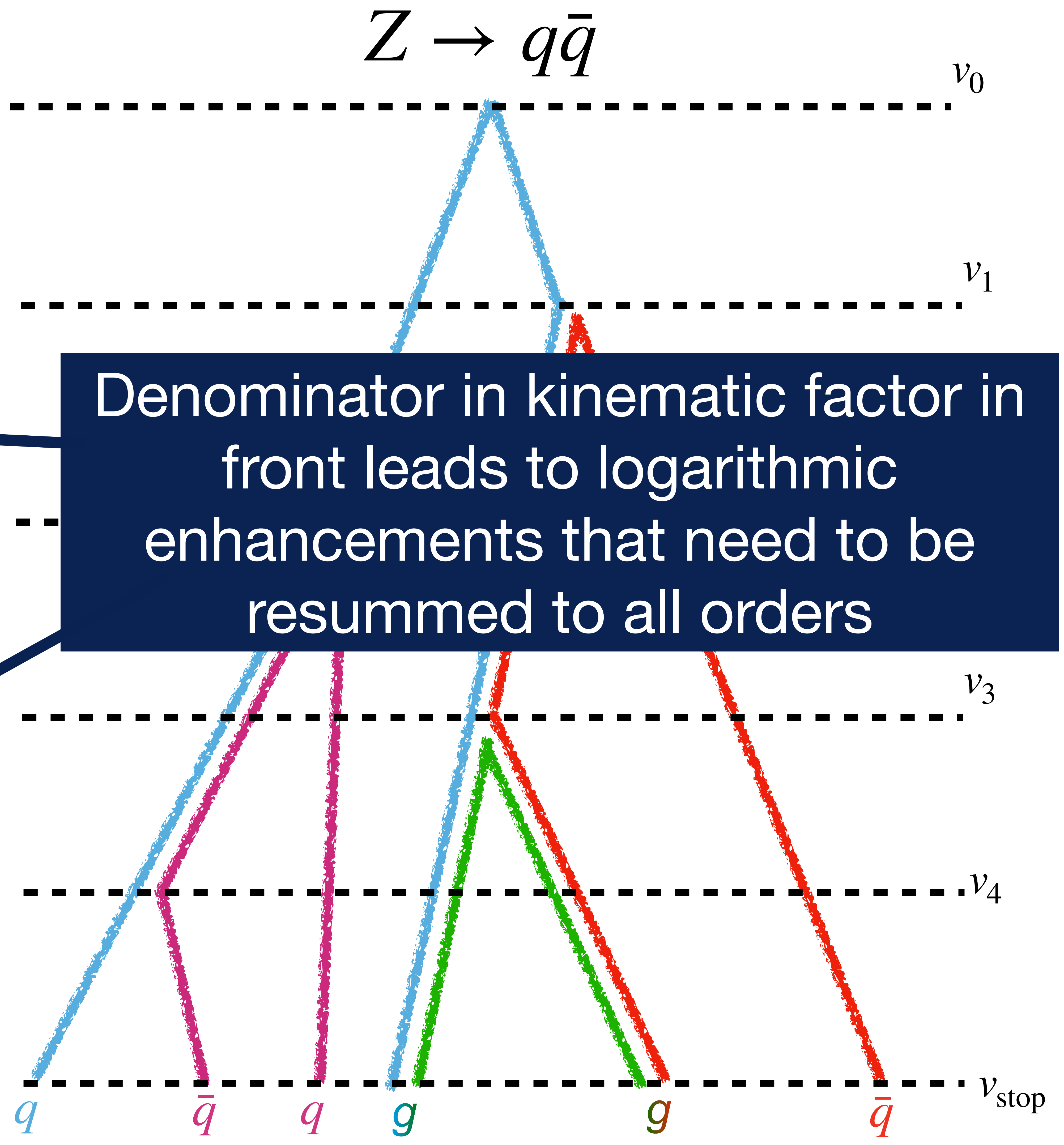


Denominator in kinematic factor in front leads to logarithmic enhancements that need to be resummed to all orders

PS algorithms - matter of making choices

DGLAP	**v.s.**	**Dipole/Antenna**
Pythia default		Pythia dipole
Herwig default		Herwig dipole
		Sherpa
		Dire
		Vincia

Kinematic map
How to go from n to $n+1$ partonic state?
global / local momentum conservation

Attribution of recoil
How to select an ‘emitter’?
dipole CM frame, event CM frame

Evolution variable v
Which emissions come first?
k_t *ordered, angular ordered, virtuality ordered…*

Parton showers: a crucial ingredient

Pythia 8

Herwig 7

Sherpa

An introduction to PYTHIA 8.2 #1
Torbjörn Sjöstrand (Lund U., Dept. Theor. Phys.), Stefan Ask (Cambridge U.), Jesper R. Christiansen (Lund U., Dept. Theor. Phys.), Richard Corke (Lund U., Dept. Theor. Phys.), Nishita Desai (U. Heidelberg, ITP) et al. (Oct 11, 2014)
Published in: *Comput.Phys.Commun.* 191 (2015) 159-177 • e-Print: 1410.3012 [hep-ph]
pdf links DOI cite 5,350 citations

PYTHIA 6.4 Physics and Manua 12,740 citations

A comprehensive guide to the physics and usage of PYTHIA 8.3 205 citations

Herwig++ Physics and Manual #1
M. Bahr (Karlsruhe U., ITP), S. Gieseke (Karlsruhe U., ITP), M.A. Gigg (Durham U., IPPP), D. Grellscheid (Durham U., IPPP), K. Hamilton (Louvain U.) et al. (Mar, 2008)
Published in: *Eur.Phys.J.C* 58 (2008) 639-707 • e-Print: 0803.0883 [hep-ph]
pdf links DOI cite 2,885 citations

Event generation with SHERPA 1.1 #1
T. Gleisberg (SLAC), Stefan. Hoeche (Zurich U.), F. Krauss (Durham U., IPPP), M. Schonherr (Dresden, Tech. U.), S. Schumann (Edinburgh U.) et al. (Nov, 2008)
Published in: *JHEP* 02 (2009) 007 • e-Print: 0811.4622 [hep-ph]
pdf links DOI cite 3,658 citations

Event Generation with Sherpa 2.2 721 citations

Do an amazing job at describing the phenomenology at colliders
(and sometimes even beyond colliders)

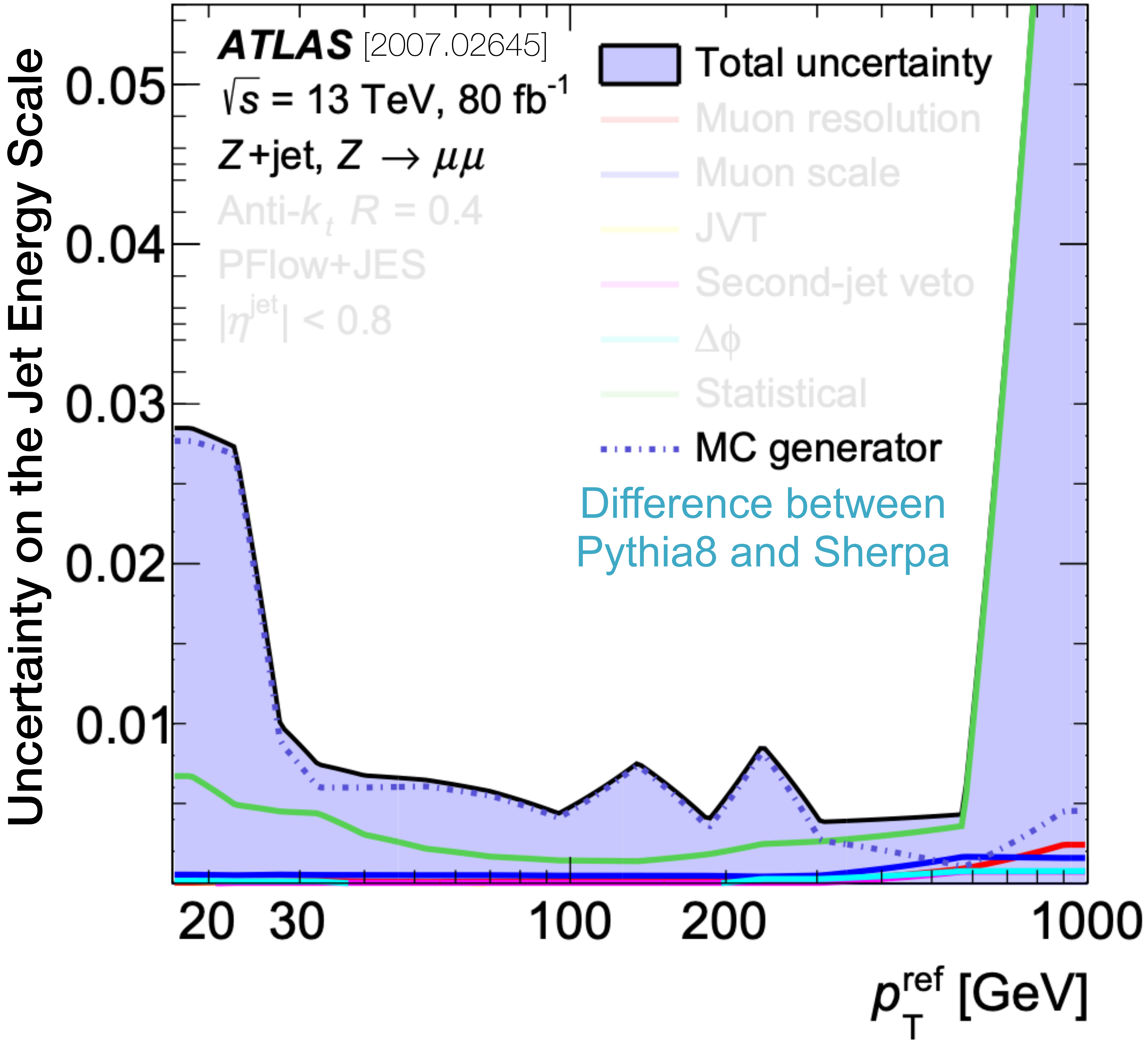
But differences matter…

A precise jet-calibration is important for many SM and BSM searches

Corrects directions and energies of measured jets to the objects produced by the MC

Method is robust to effects from pile-up and underlying event…

Leading uncertainty originates from different **parton-shower** modeling



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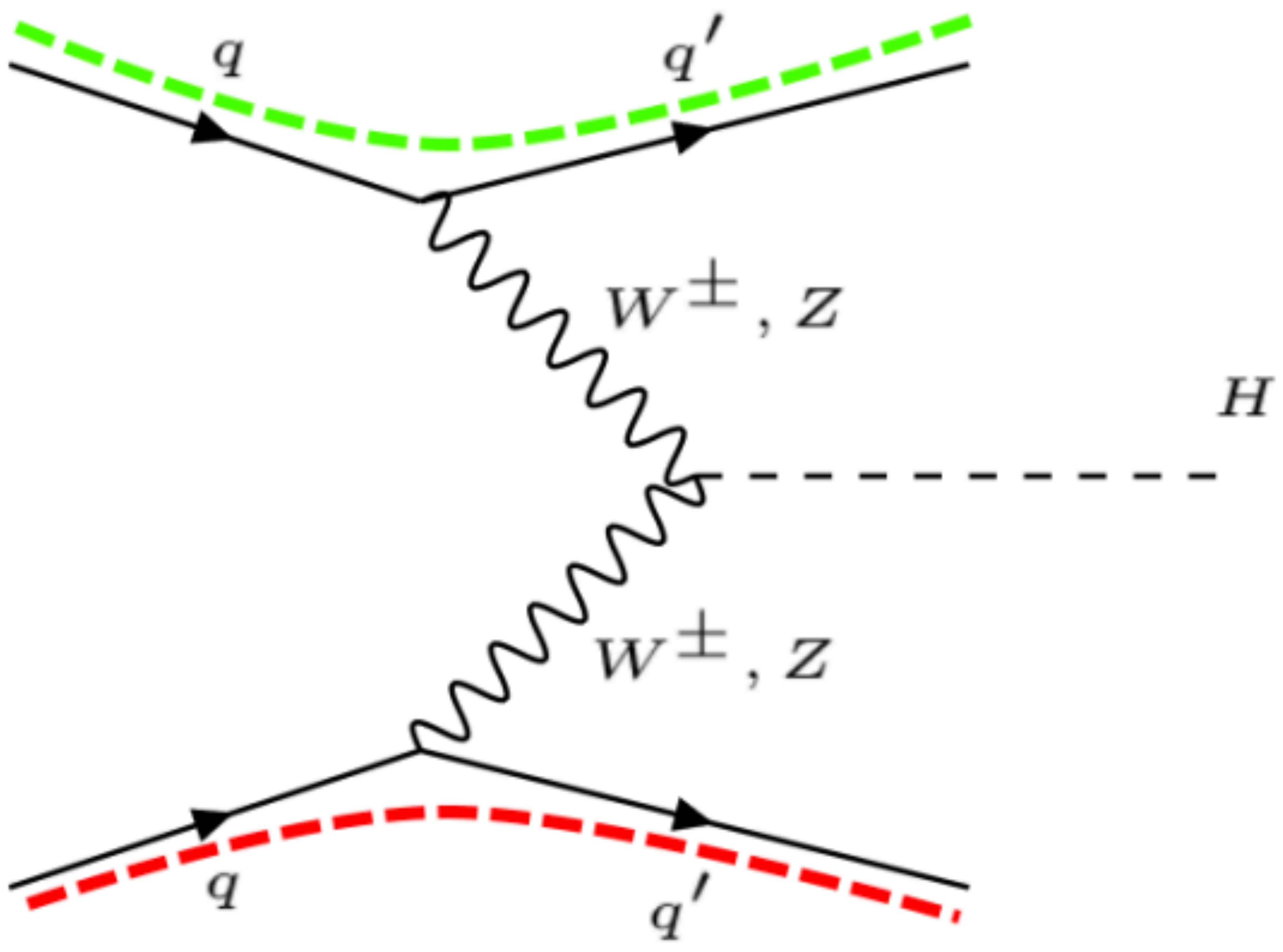
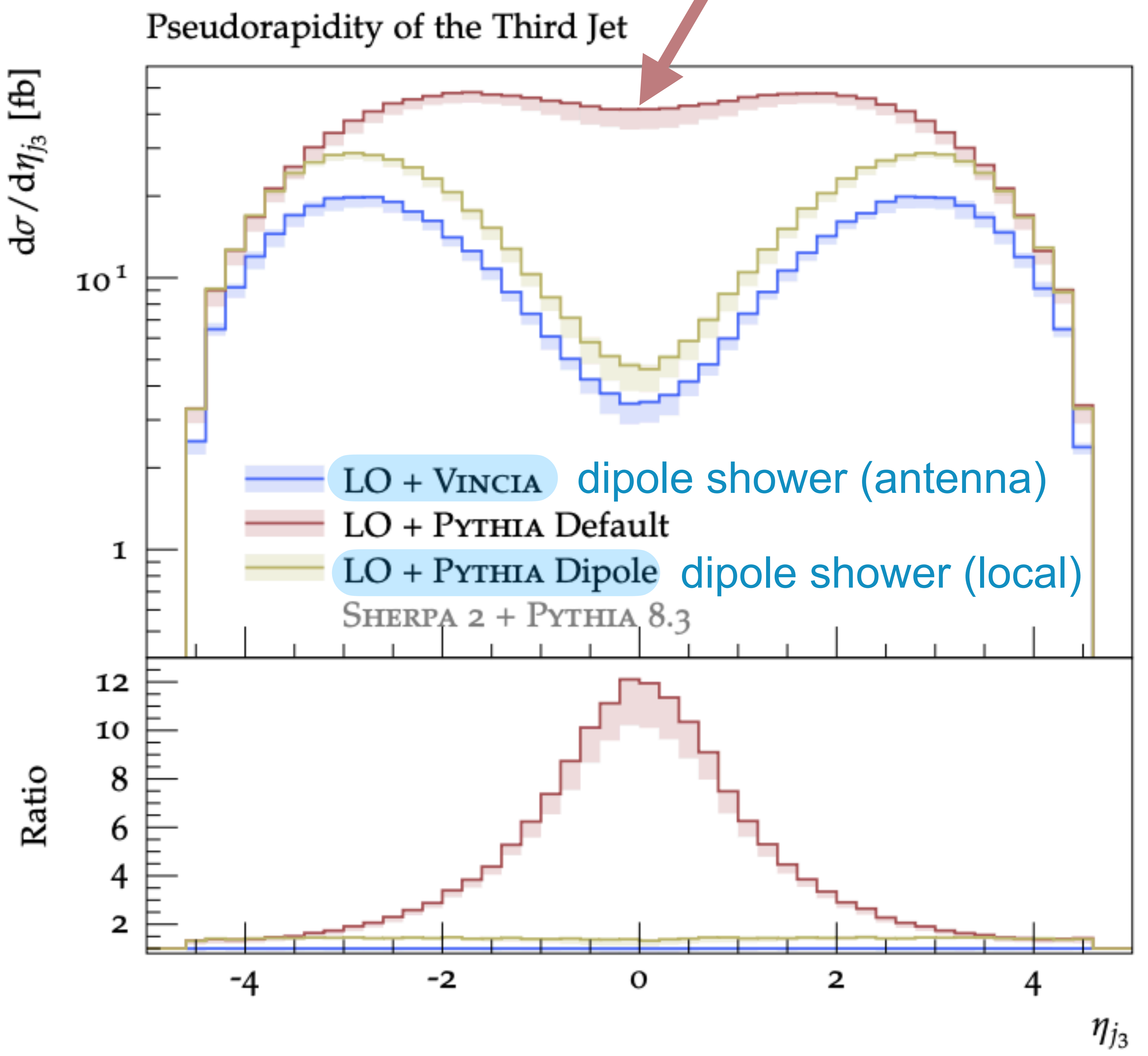
But differences matter…

VBF production of $h + 2j$

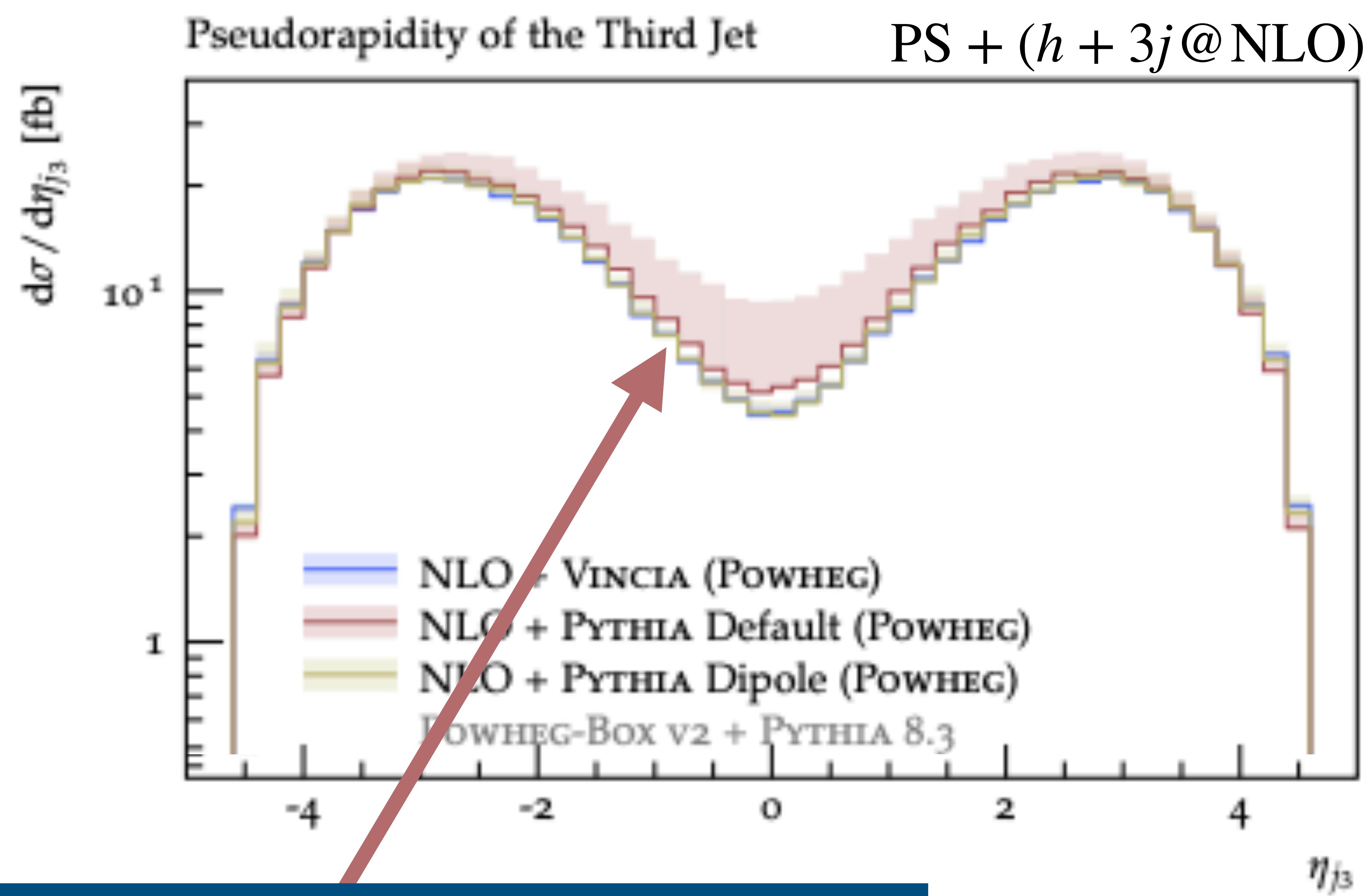


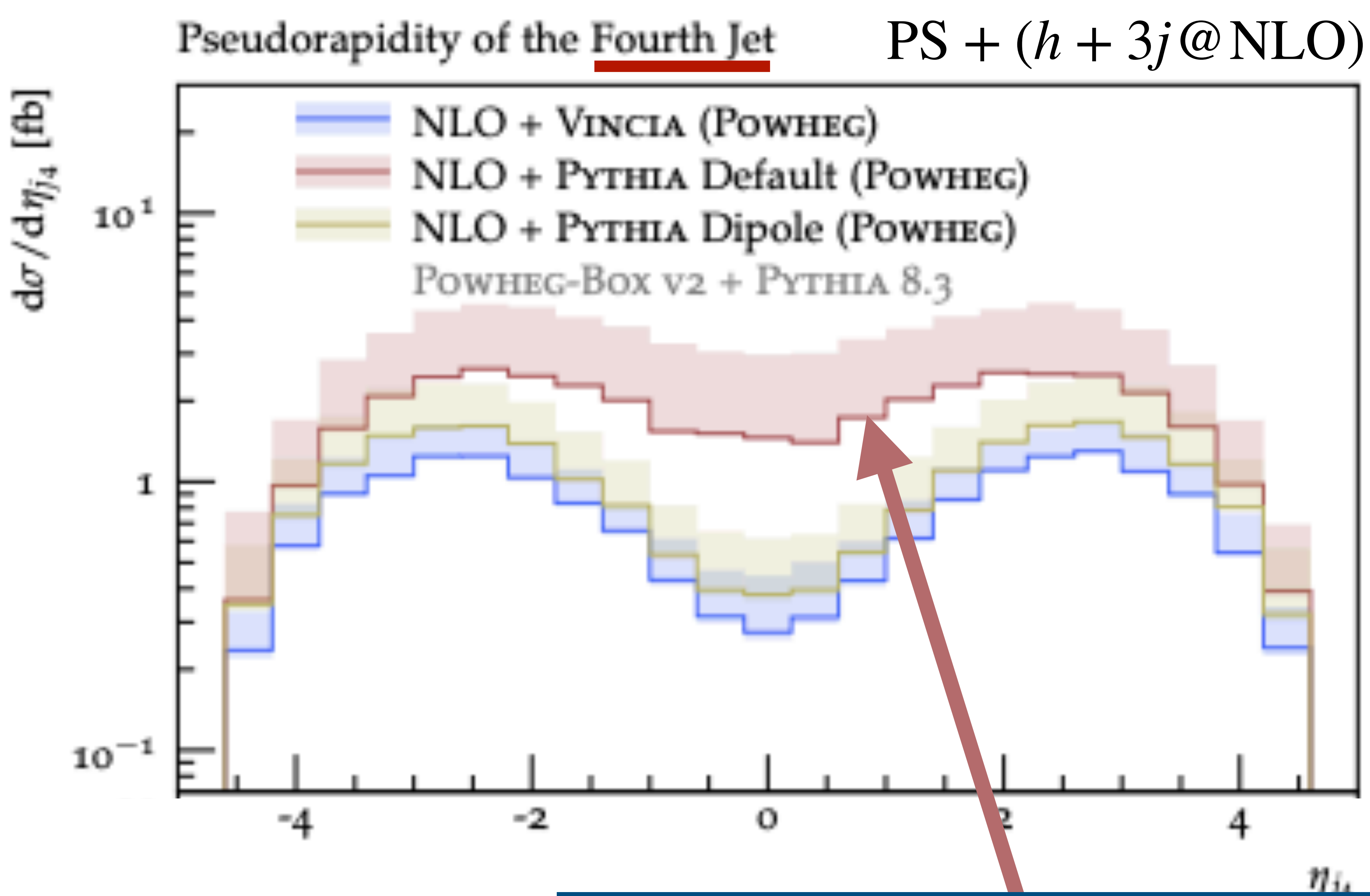
Colour coherence strongly **suppresses** radiation in central rapidity region

Pythia's default (global) shower unphysically fills this central region!



Matching for VBF (Powheg-box + PS)





Matching fixes the rapidity distribution of the 3rd jet…

But we again see a huge discrepancy for four-jet observables!

Important message:
Matching does not magically fix your shower

MC@NLO + Pythia/Herwig in [2003.12435]

Progress in improving the PS accuracy

- Assessing the logarithmic accuracy of a shower **Focus of this talk**
  Herwig [1904.11866, 2107.04051], Deductor [2011.04777], Forshaw, Holguin, Plätzer [2003.06400]
  PanScales [1805.09327, 2002.11114, 2207.09467, 2305.08645], Alaric [2208.06057, 2307.00728], …

- Triple collinear / double soft splittings
  Dulat, Höche, Krauss, Gellersen, Prestel [1705.00982, 1705.00742, 1805.03757, 2110.05964]
  Li & Skands [1611.00013], Löschner, Plätzer, Simpson Dore [2112.14454], PanScales [2307.11142]

- Matching to fixed-order
  NLO; i.e. Frixione & Webber [0204244], Nason [0409146], …
  NNLO; i.e. UNNLOPS [1407.3773], MiNNLOps [1908.06987], Vincia [2108.07133], …
  NNNLO; Prestel [2106.03206], Bertone, Prestel [2202.01082]

- Colour (and spin) correlations
  Forshaw, Holguin, Plätzer, Sjödahl [1201.0260, 1808.00332, 1905.08686, 2007.09648, 2011.15087]
  Deductor [0706.0017, 1401.6364, 1501.00778, 1902.02105], Herwig [1807.01955], Plätzer & Ruffa [2012.15215]
  PanScales [2011.10054, 2103.16526, 2111.01161], …

- Electroweak corrections
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], …

Disclaimer: list is not exhaustive

Addressing the accuracy of a parton shower

For a ***given*** observable, one may address the question of accuracy systematically

At fixed order

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + \ldots$$

At all orders using analytic resummation

$$\Sigma^{\mathrm{NLL}}(\lambda \equiv \alpha_s L) = \exp(\underbrace{\frac{1}{\alpha_s} g_1(\lambda)}_{\mathcal{O}(1/\alpha_s)} \quad + \quad \underbrace{g_2(\lambda)}_{\mathcal{O}(1)} \quad + \quad \ldots) \qquad \Sigma^{\mathrm{NDL}}(\xi \equiv \alpha_s L^2) = h_1(\xi) + \sqrt{\alpha_s} h_2(\xi) + \ldots$$

in resummation regime where $\alpha_s L = \mathcal{O}(1)$

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in resummation regime where $\alpha_s L = \mathcal{O}(1)$

Conversely, showers produce a set of particles with specified four momenta, from which any well-defined observable can be constructed

How to design showers that are NLL/NDL accurate for *all* observables?

The PanScales family

Gavin Salam
Oxford

Gregory Soyez
Saclay

Keith Hamilton
UCL

Mrinal Dasgupta
Manchester

Pier Monni
CERN

Silvia Ferrario Ravasio
CERN

Alba Soto Ontoso
CERN

Alexander Karlberg
CERN

Basem El-Menoufi
Monash

Jack Helliwell
Oxford

Ludo Scyboz
Monash

Melissa van Beekveld
Nikhef

+ past members
Frederic Dreyer
Emma Slade
Rok Medves
Rob Verheyen
Scarlett Woolnough

Dasgupta, Dreyer, Hamilton, Monni, Salam [1805.09327]

PanScales NLL/NDL correctness requirements

Resummation

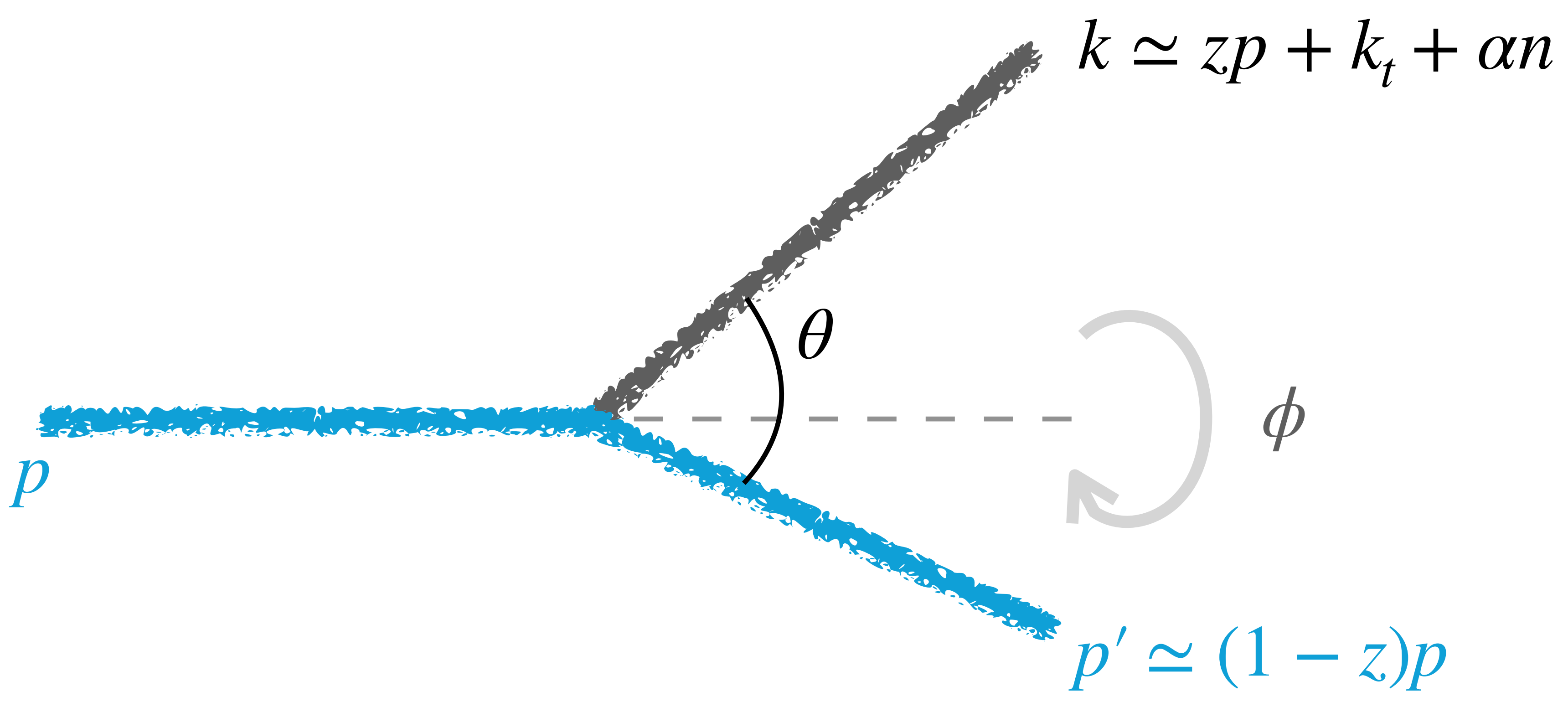
Require single-logarithmic accuracy for suitably defined observables

- global event shapes ($\alpha_s^n L^n$) Probe the structure of double-log Sudakov resummation in the shower
- parton distribution / fragmentation functions ($\alpha_s^n L^n$) Probe the hard-collinear region
- non-global observables ($\alpha_s^n L^n$) Probe the soft wide-angle region
- particle/jet multiplicity ($\alpha_s^n L^{2n-1}$) Probe nested emissions in the soft and collinear regions

Test the basic underlying concept

Require correctness of effective matrix elements generated by the shower for well-separated emissions (only thing one can do if a resummation cannot be formulated)

Phase space for final-state emissions



Described in terms of **shower variables**:

- The transverse momentum $k_t = E\theta$, which can be linked to the **evolution variable** $v \simeq k_t e^{-\beta|\eta|}$
- $\eta = -\ln\tan\theta/2$ the **pseudorapidity**
- ϕ the **azimuthal angle** (trivial for spin-averaged splitting functions)

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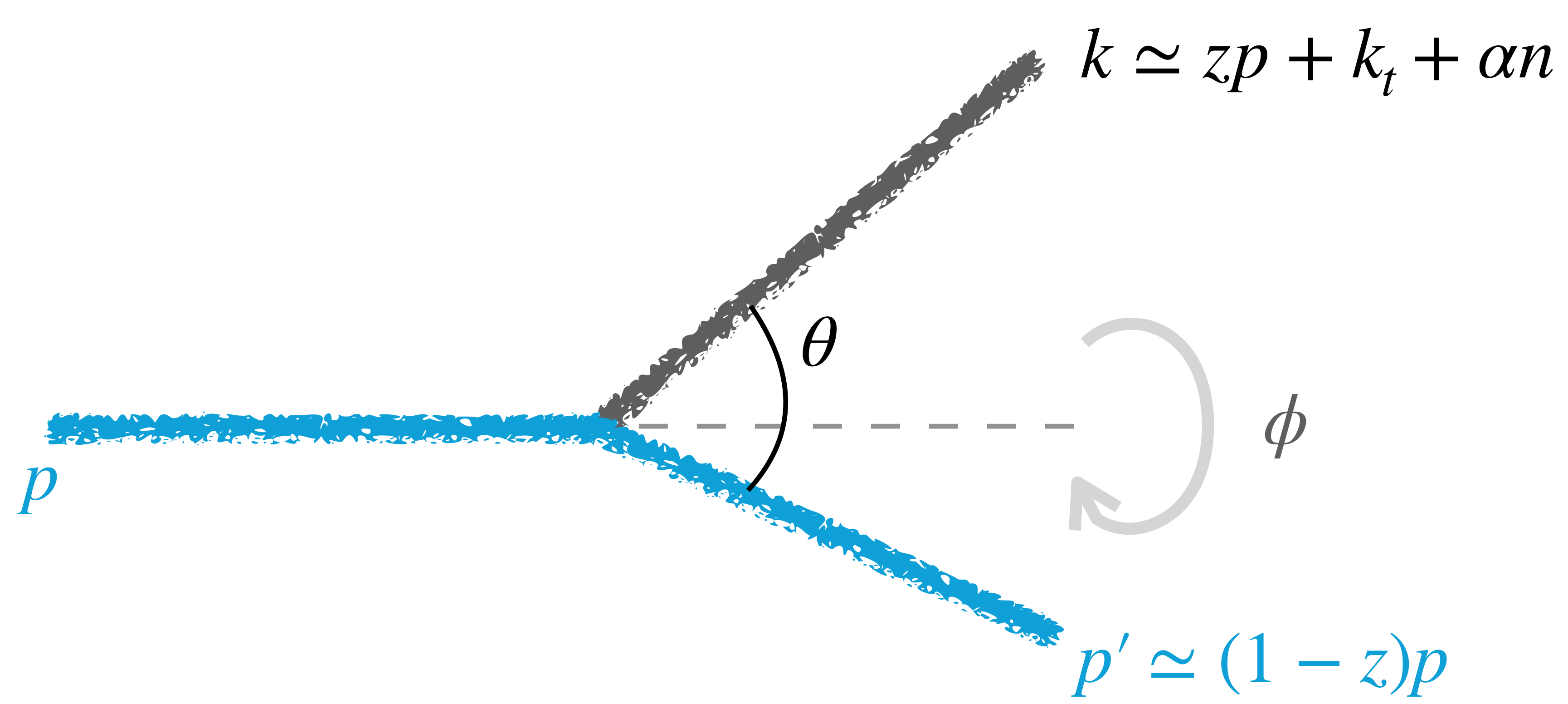

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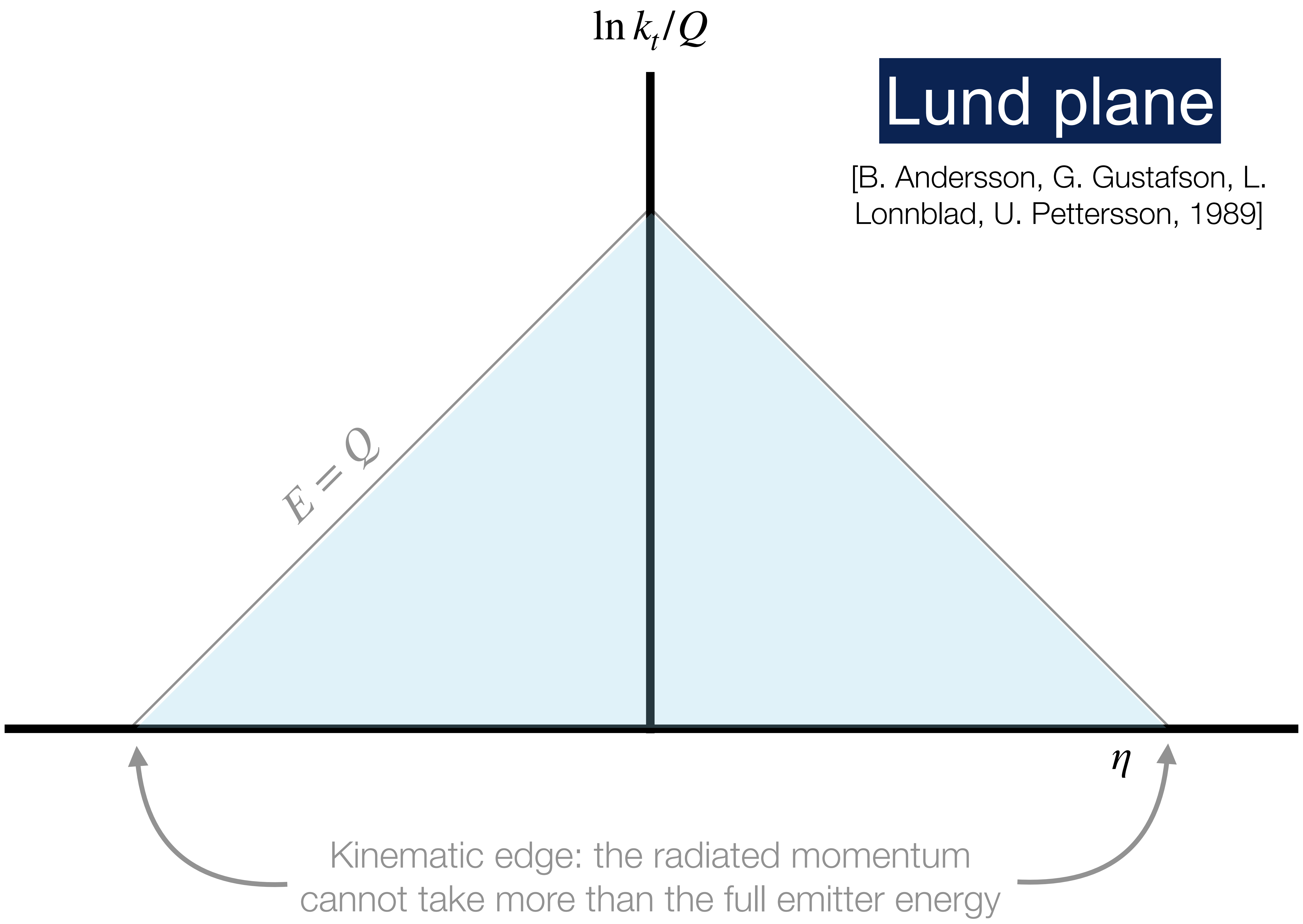
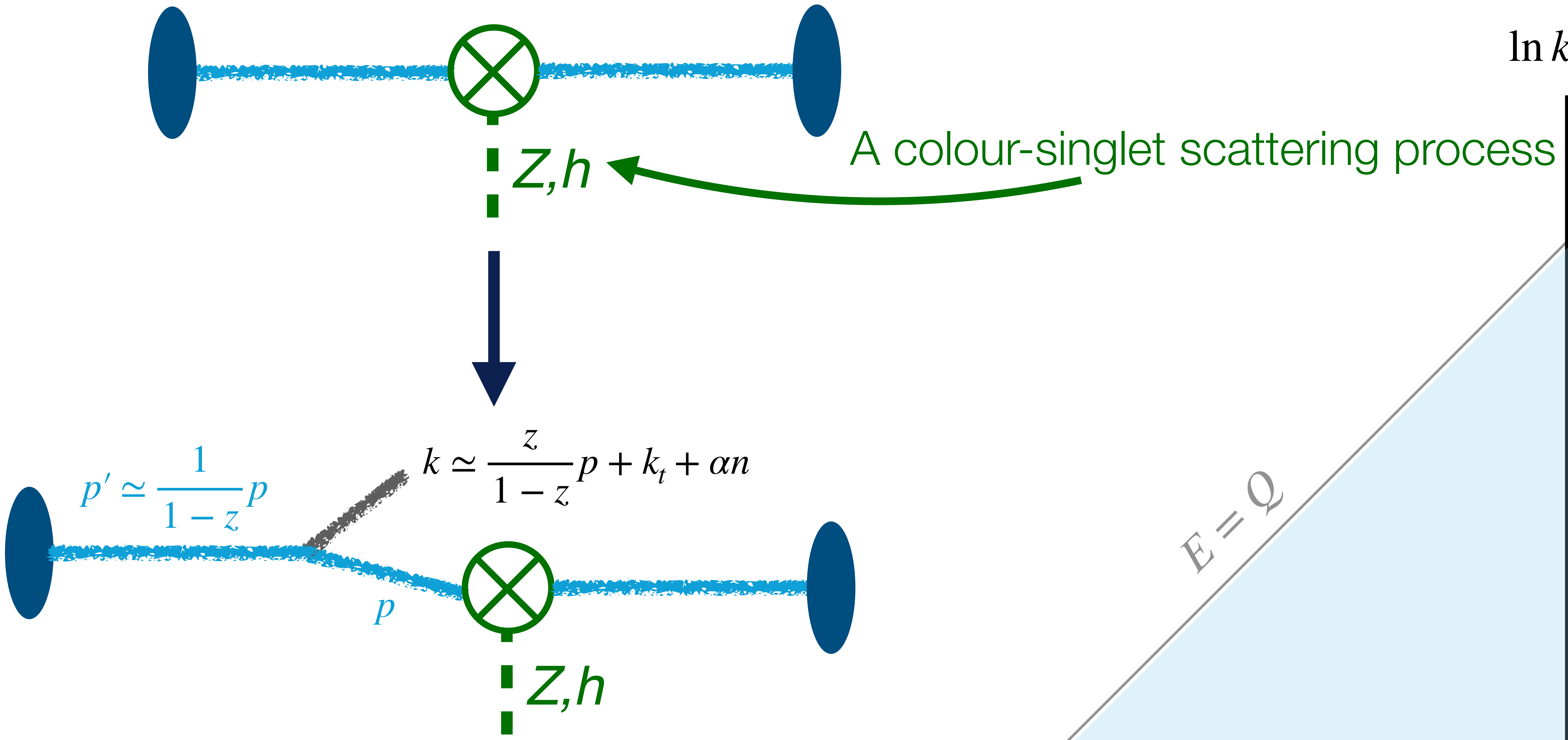
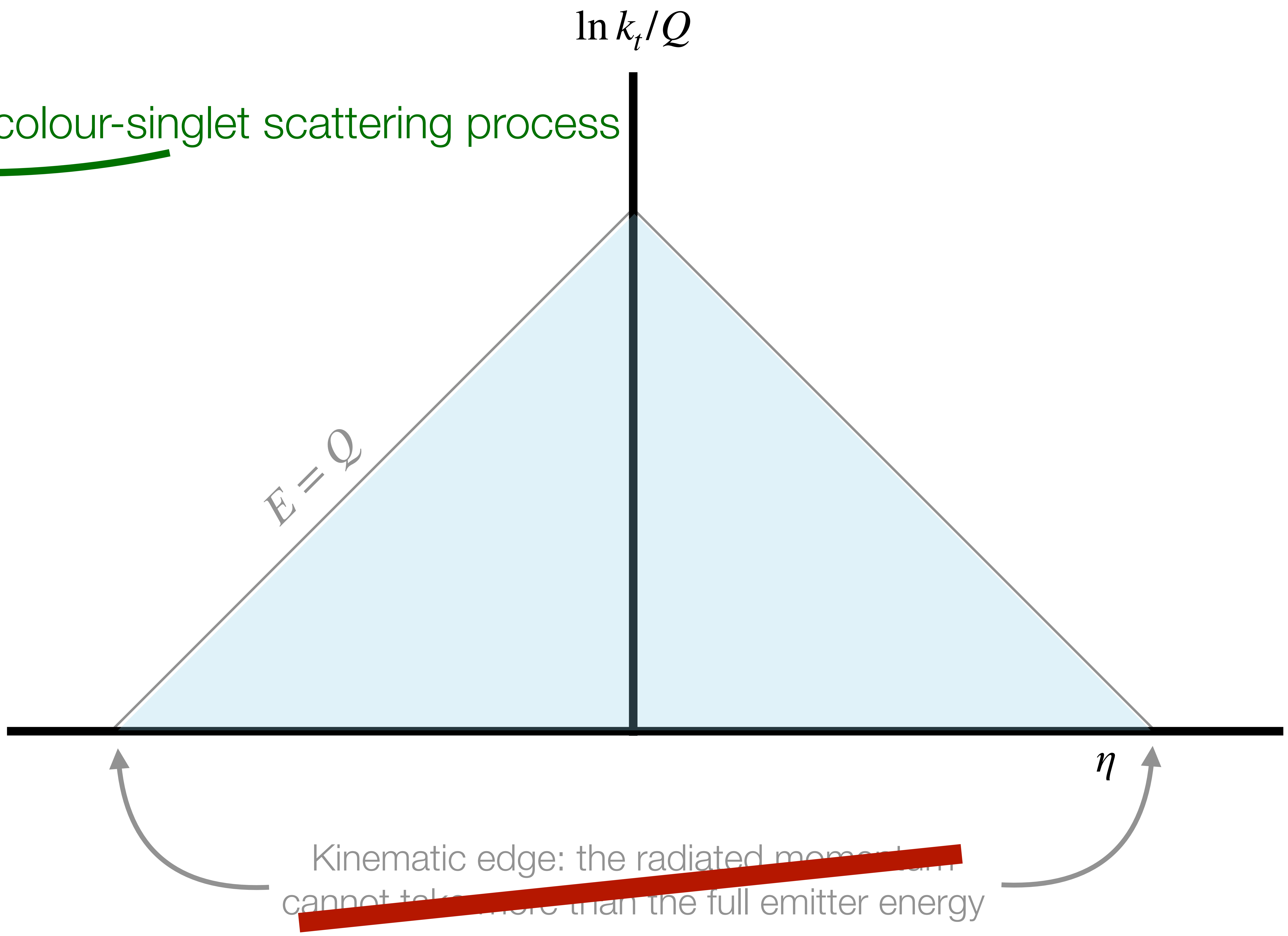


Phase space for initial-state emissions



This is called backward evolution

[T. Sjostrand, 1985 / T.D. Gottschalk, 1986]



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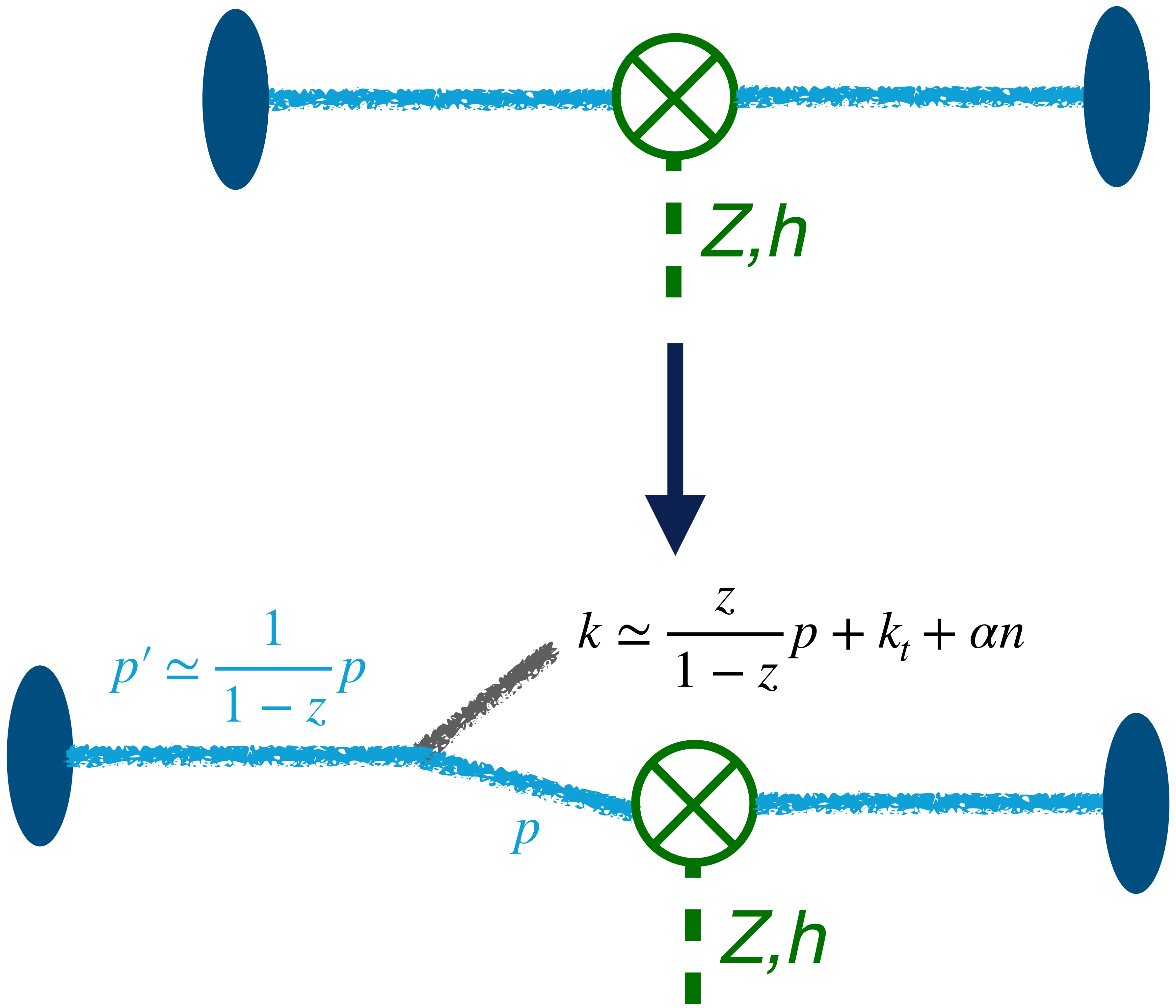

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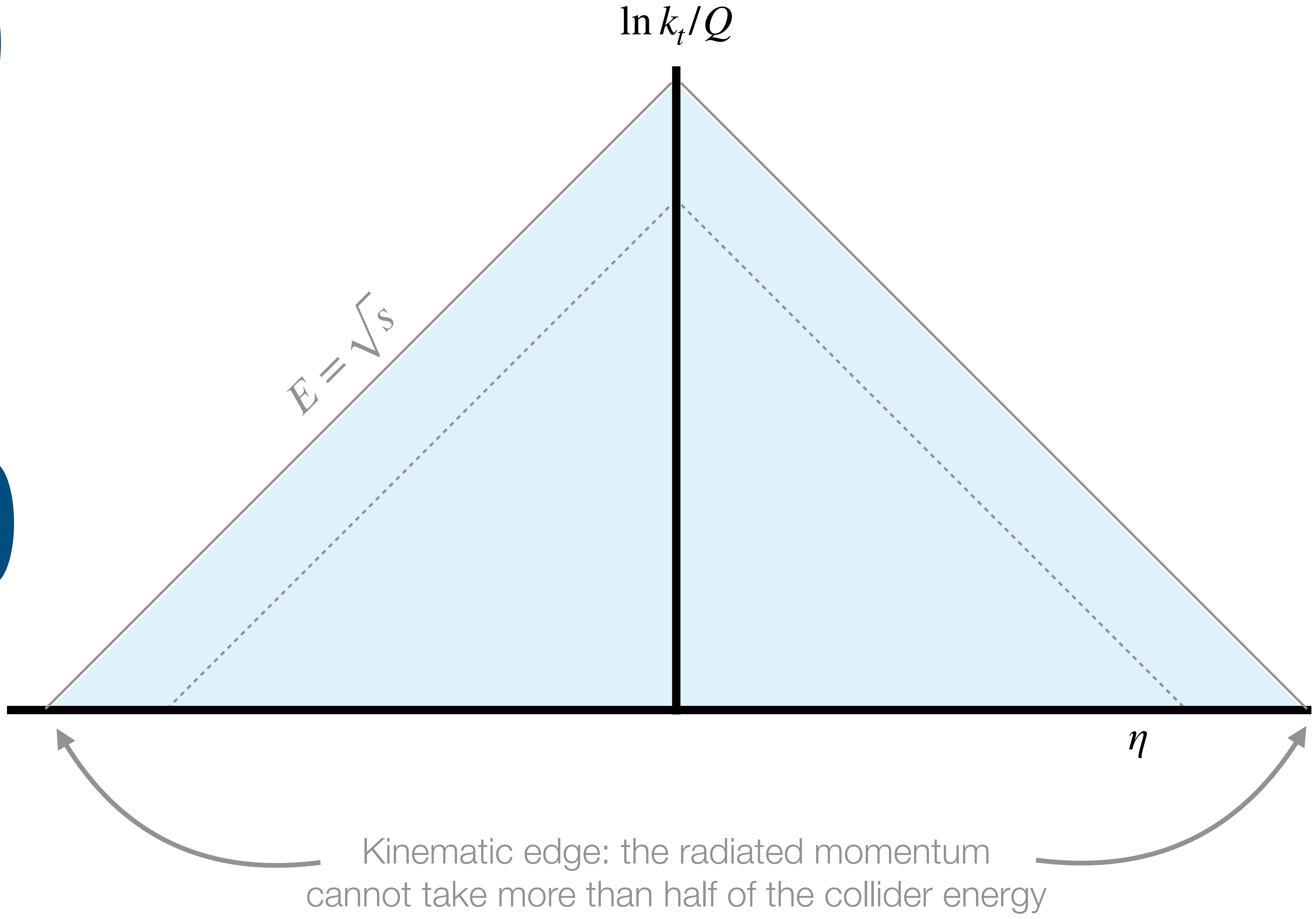
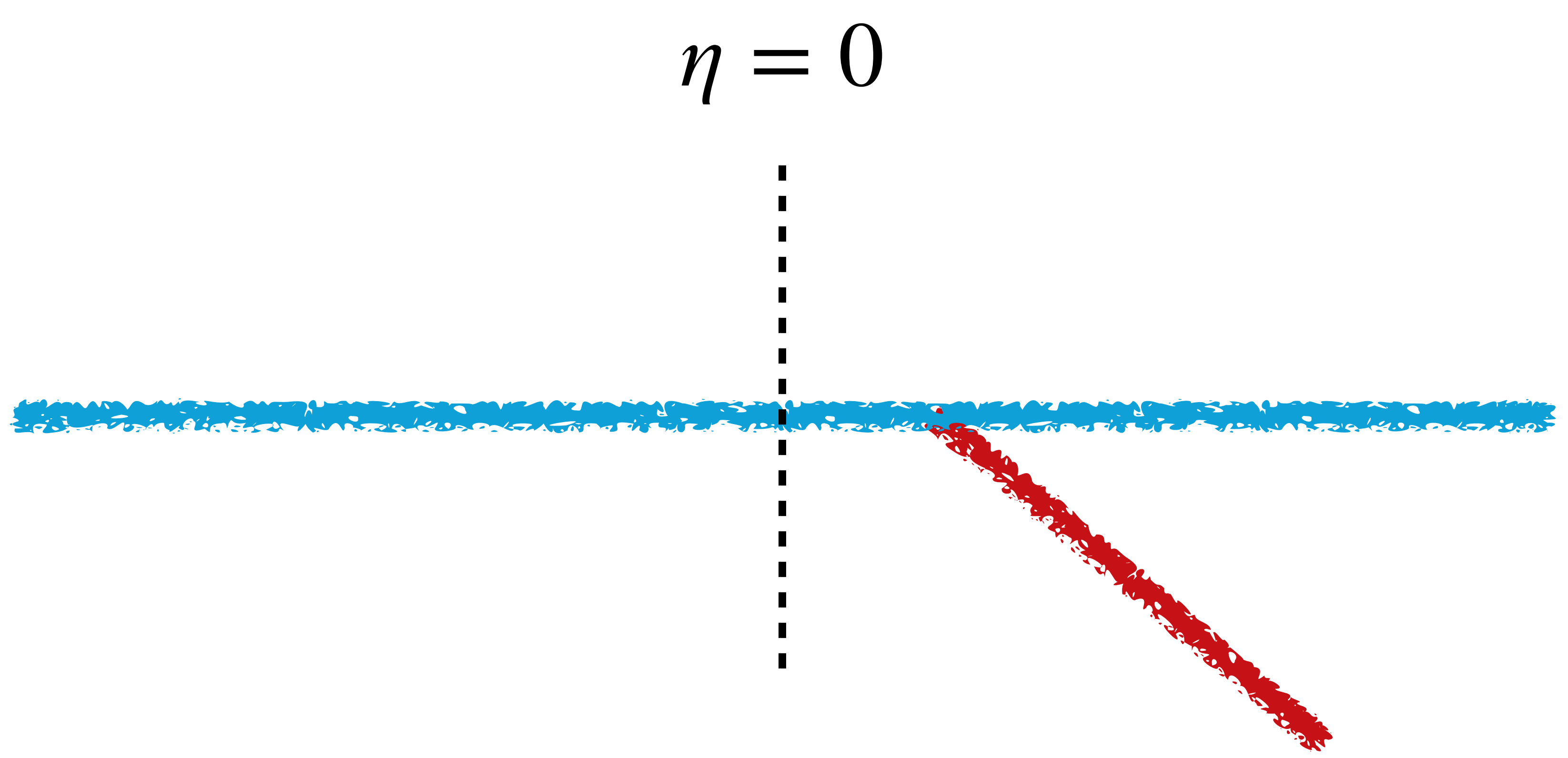
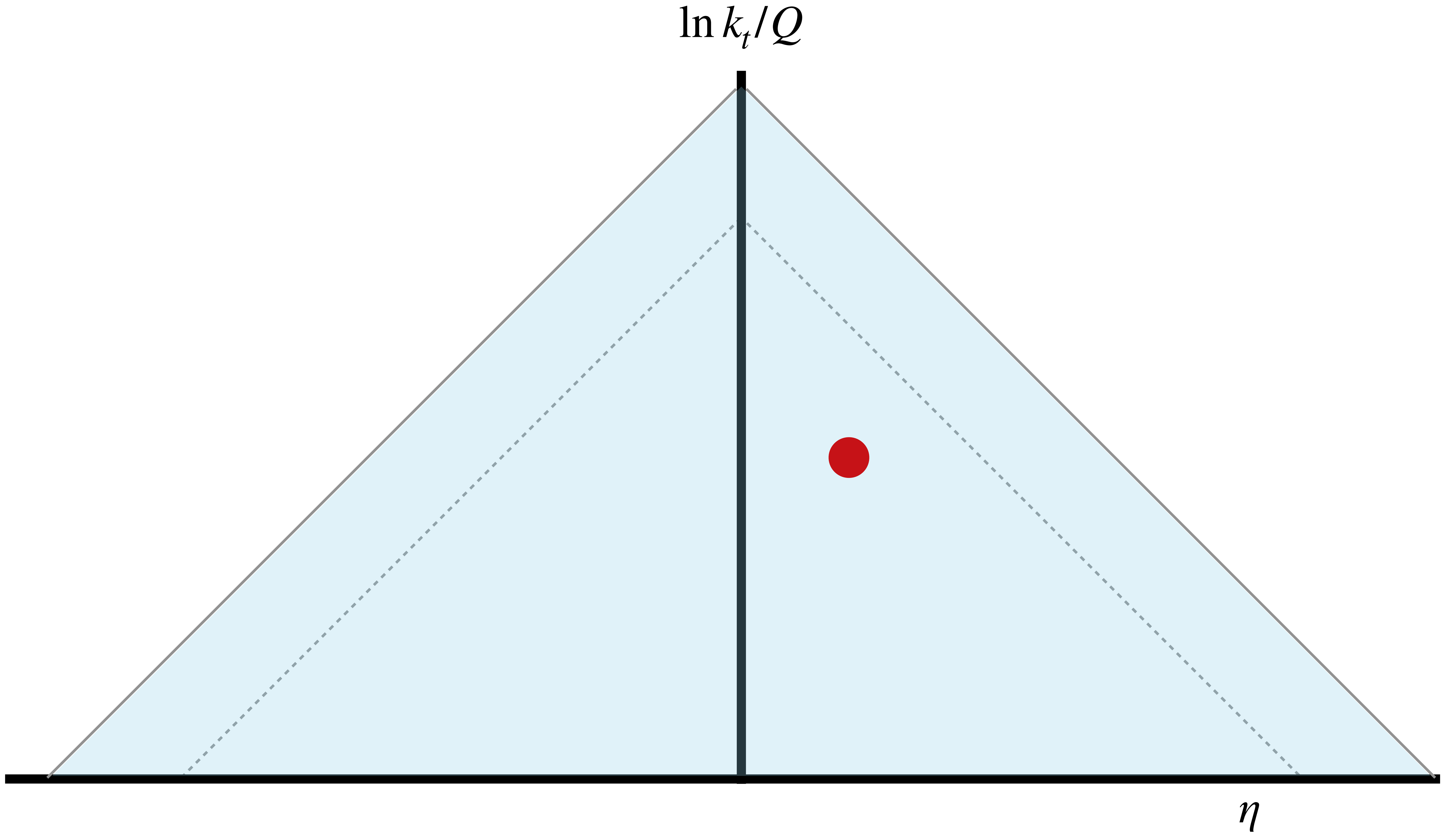


Emissions illustrated in Lund plane



Colour dipole (e.g. a $q\bar{q}$ pair) emitting a parton with $\eta > 0$



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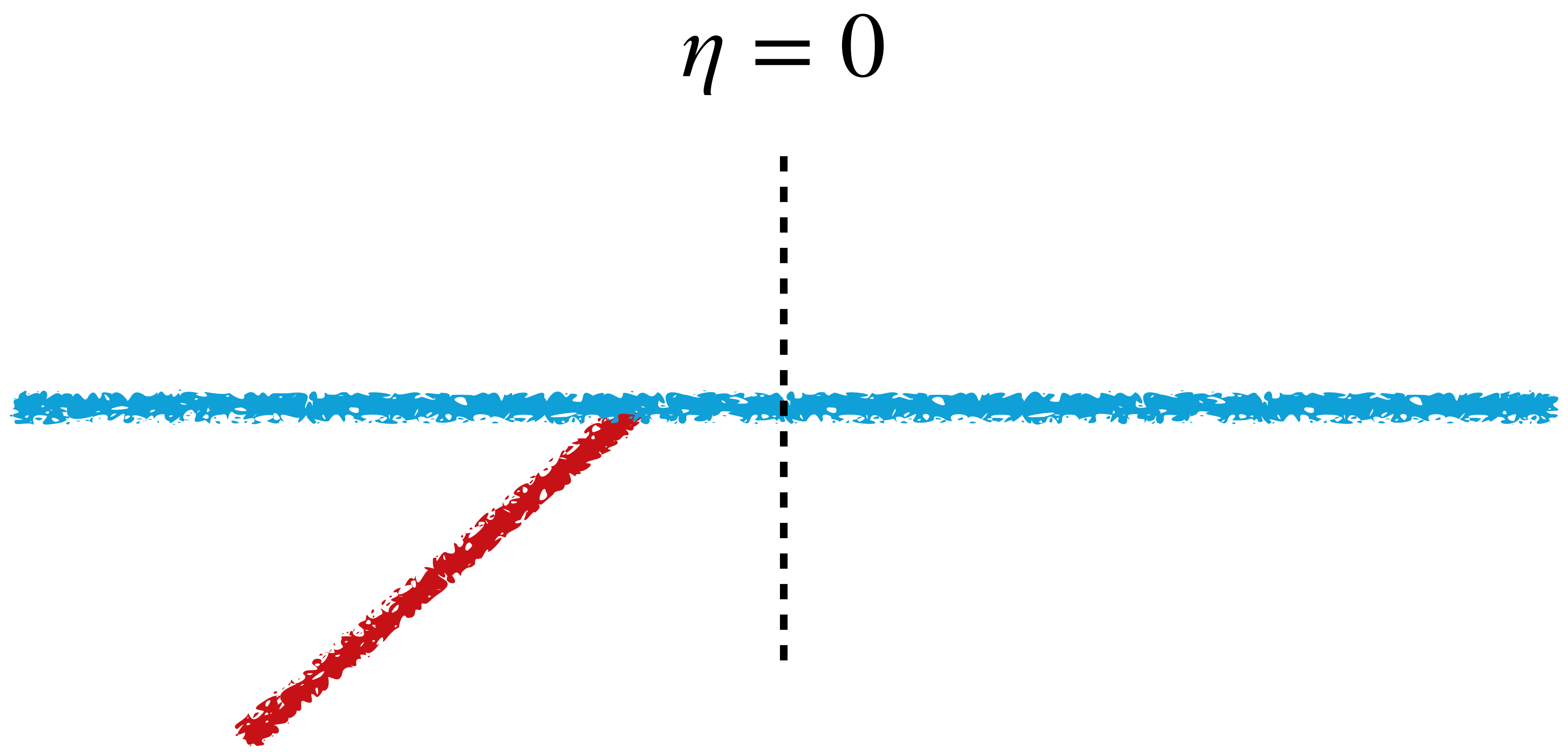
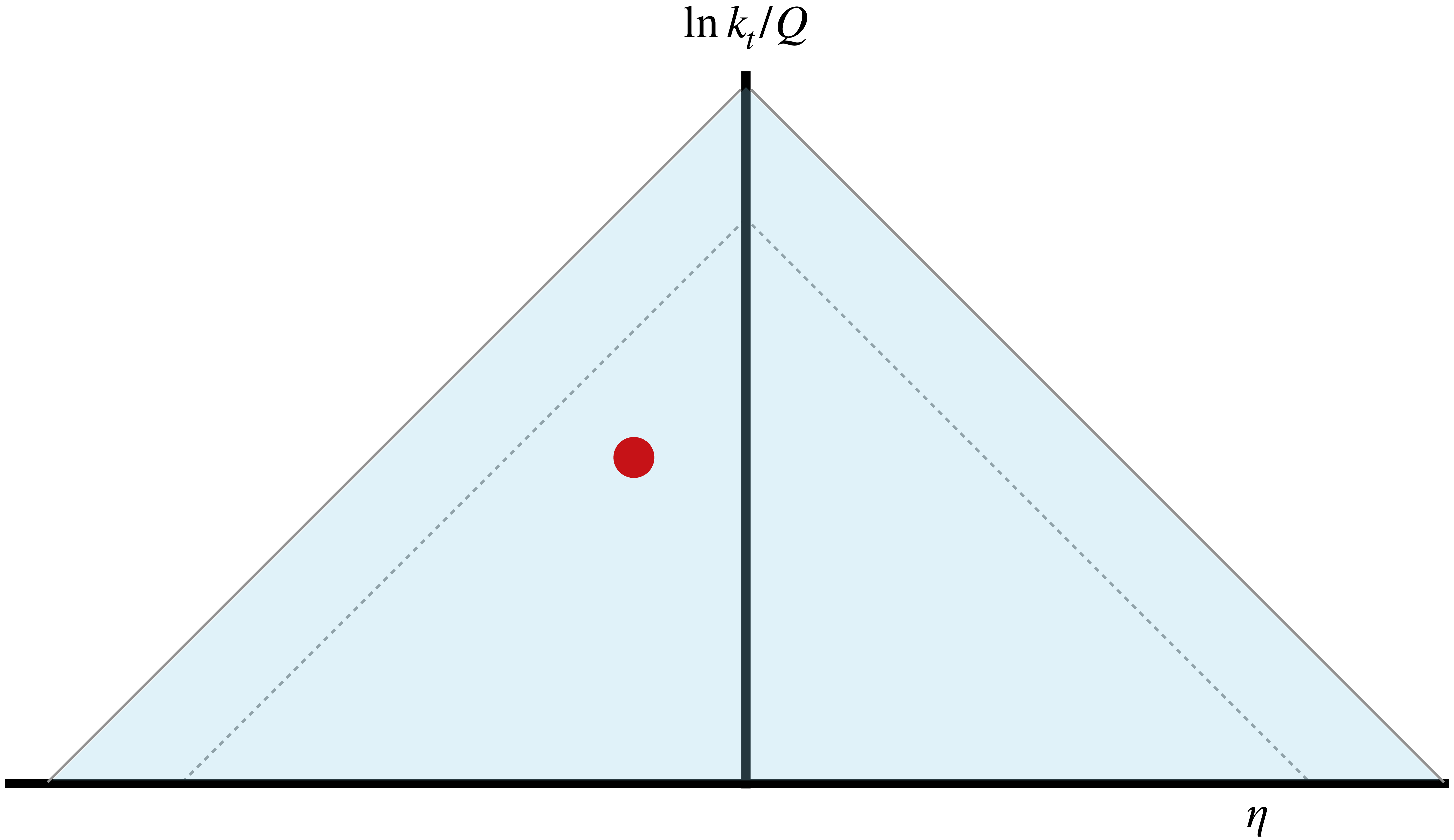


Colour dipole (e.g. a $q\bar{q}$ pair) emitting a parton with $\eta < 0$



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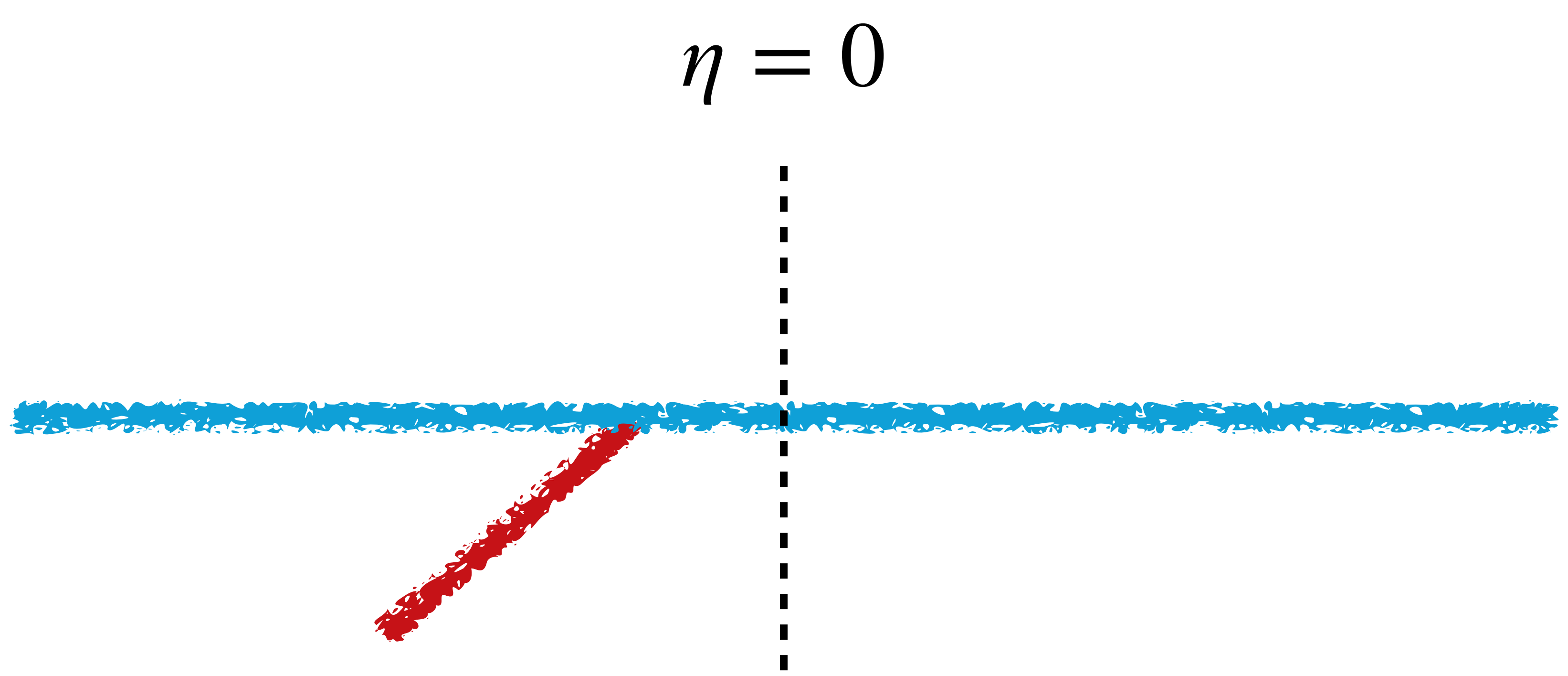
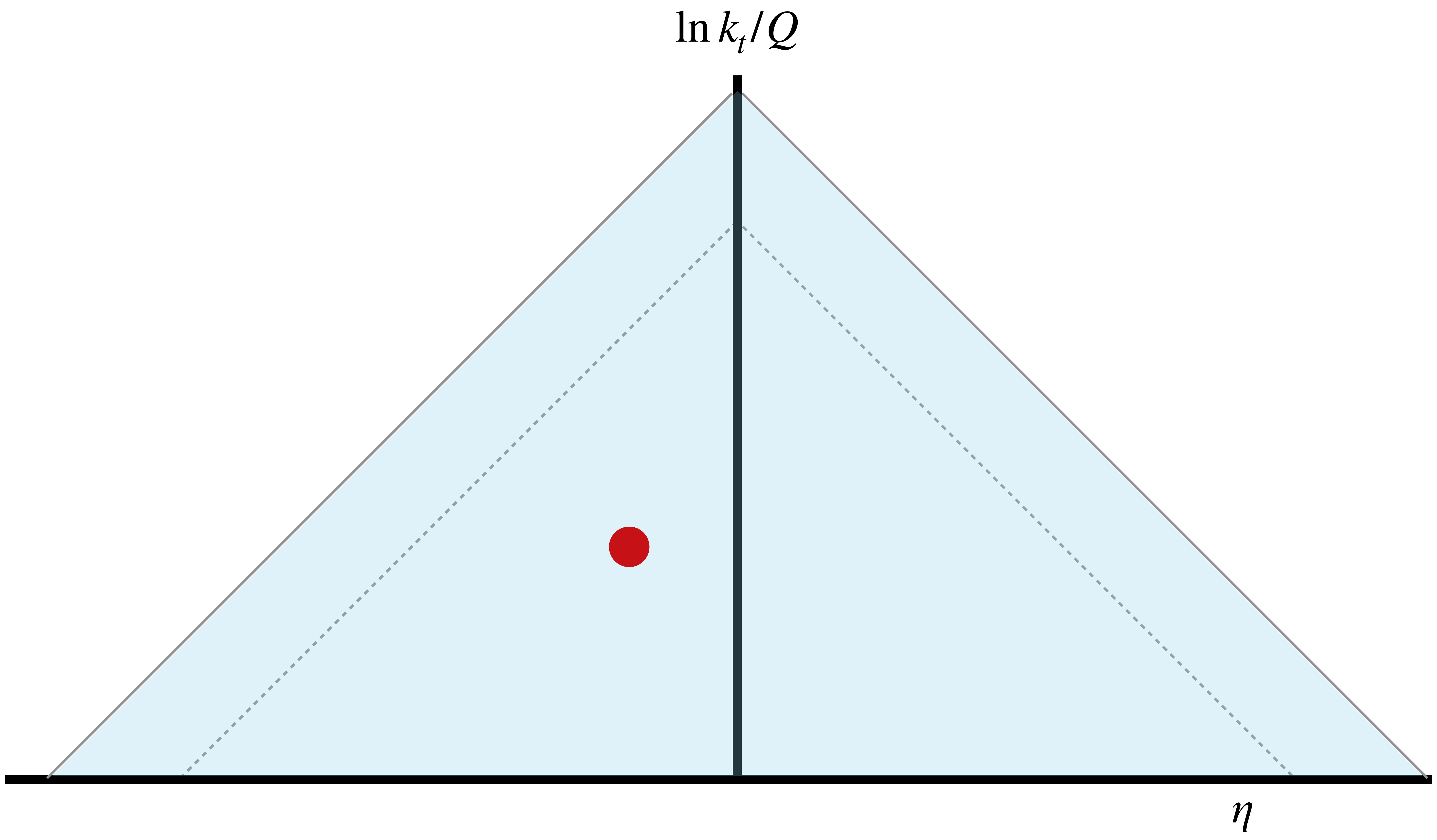


Softer emissions move down in the Lund plane (their $|k_t|$ drops)



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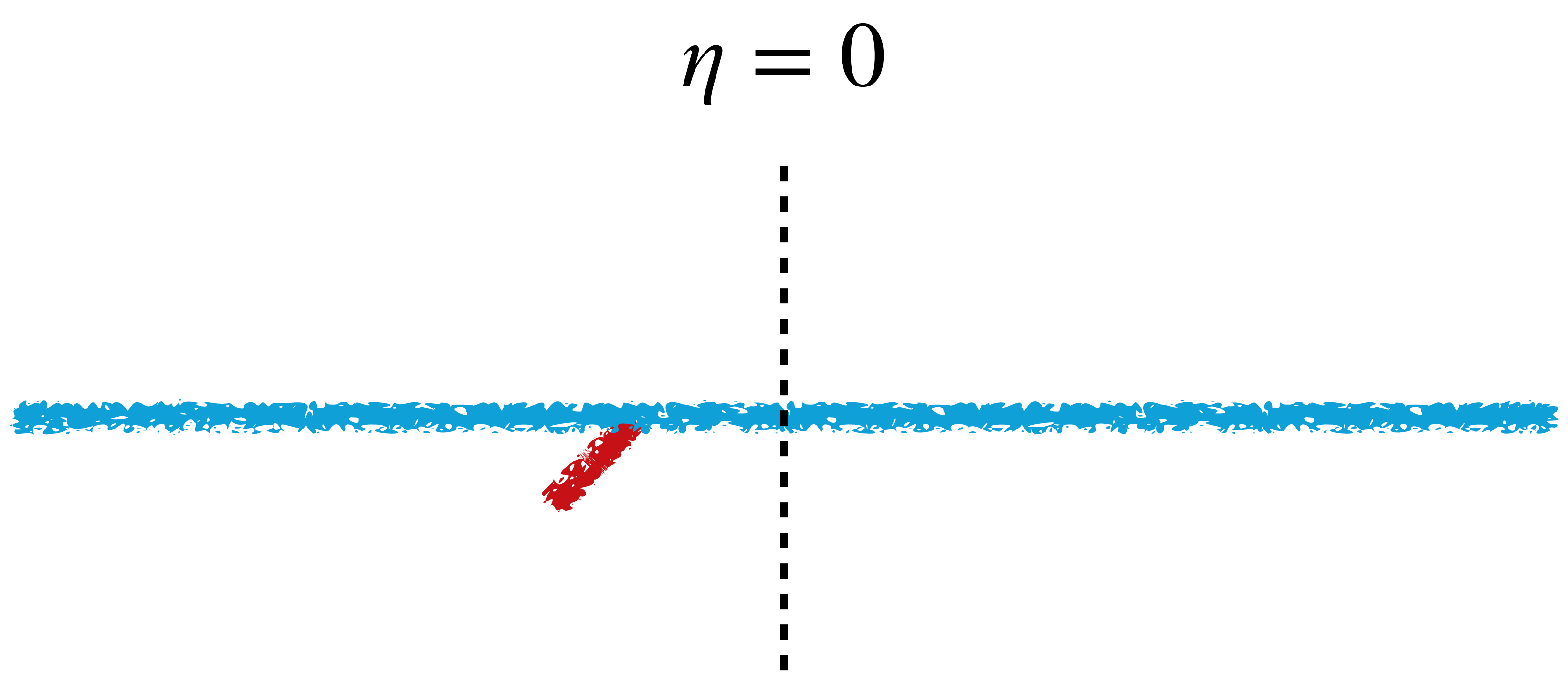

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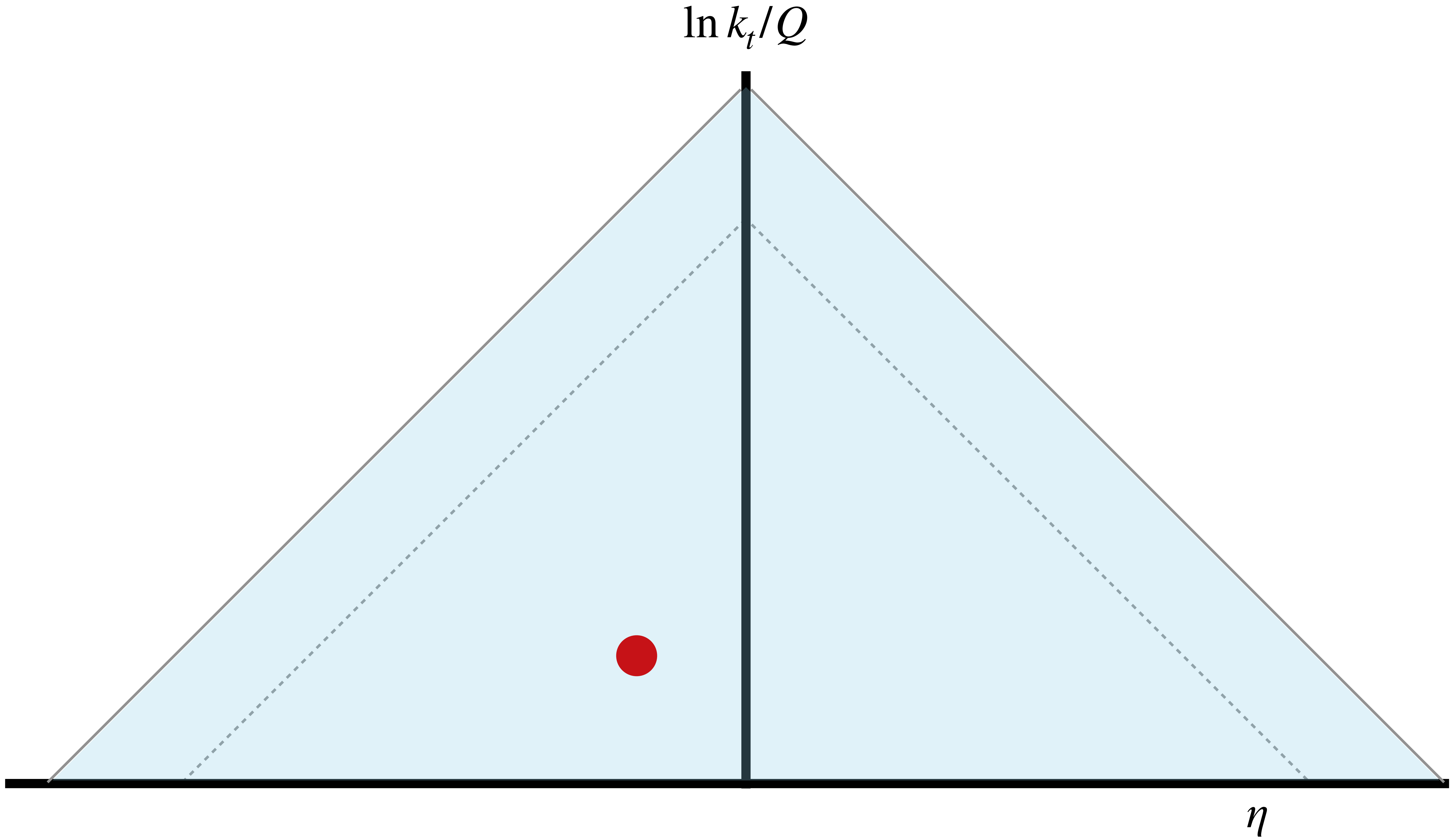

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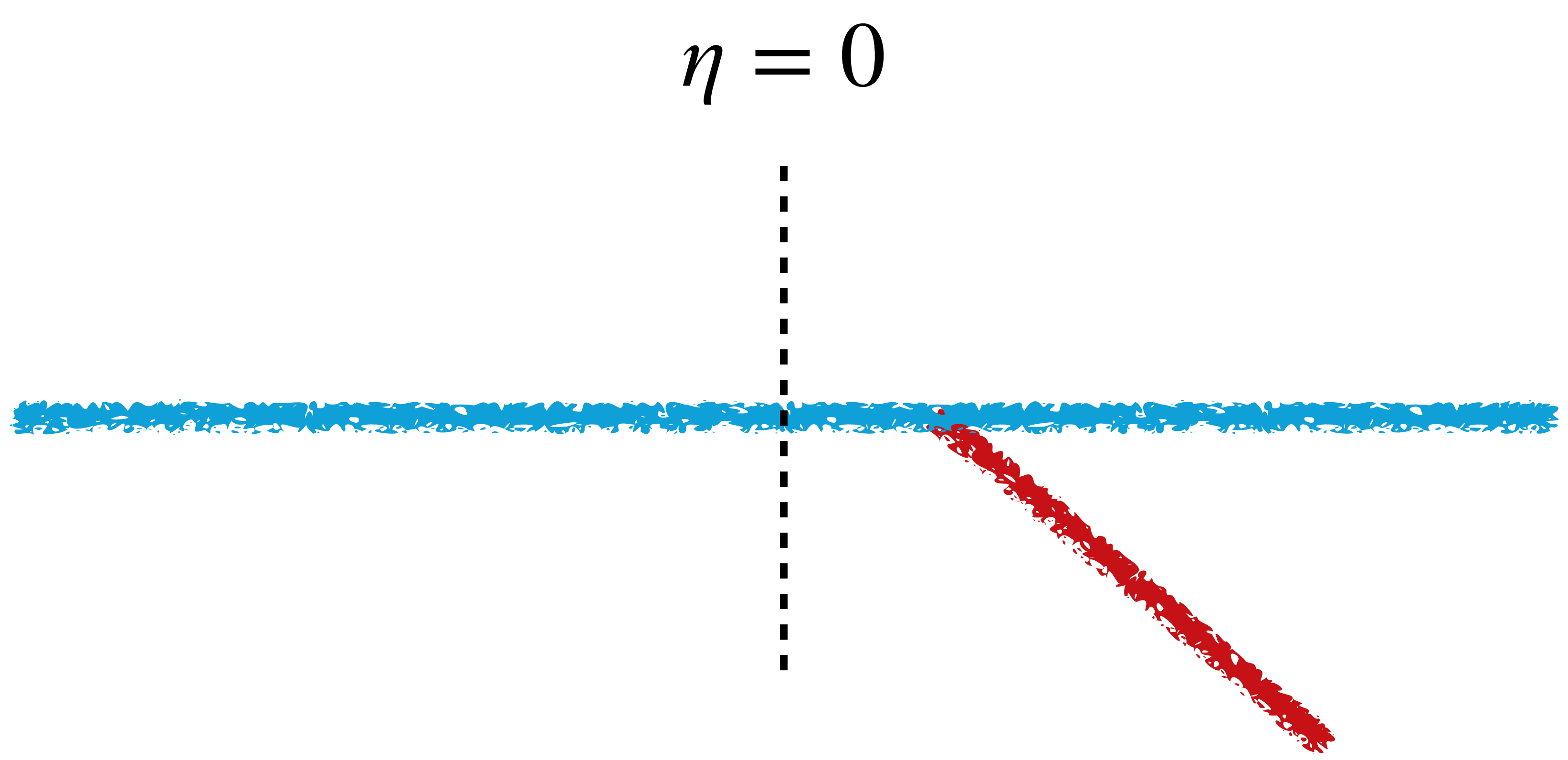

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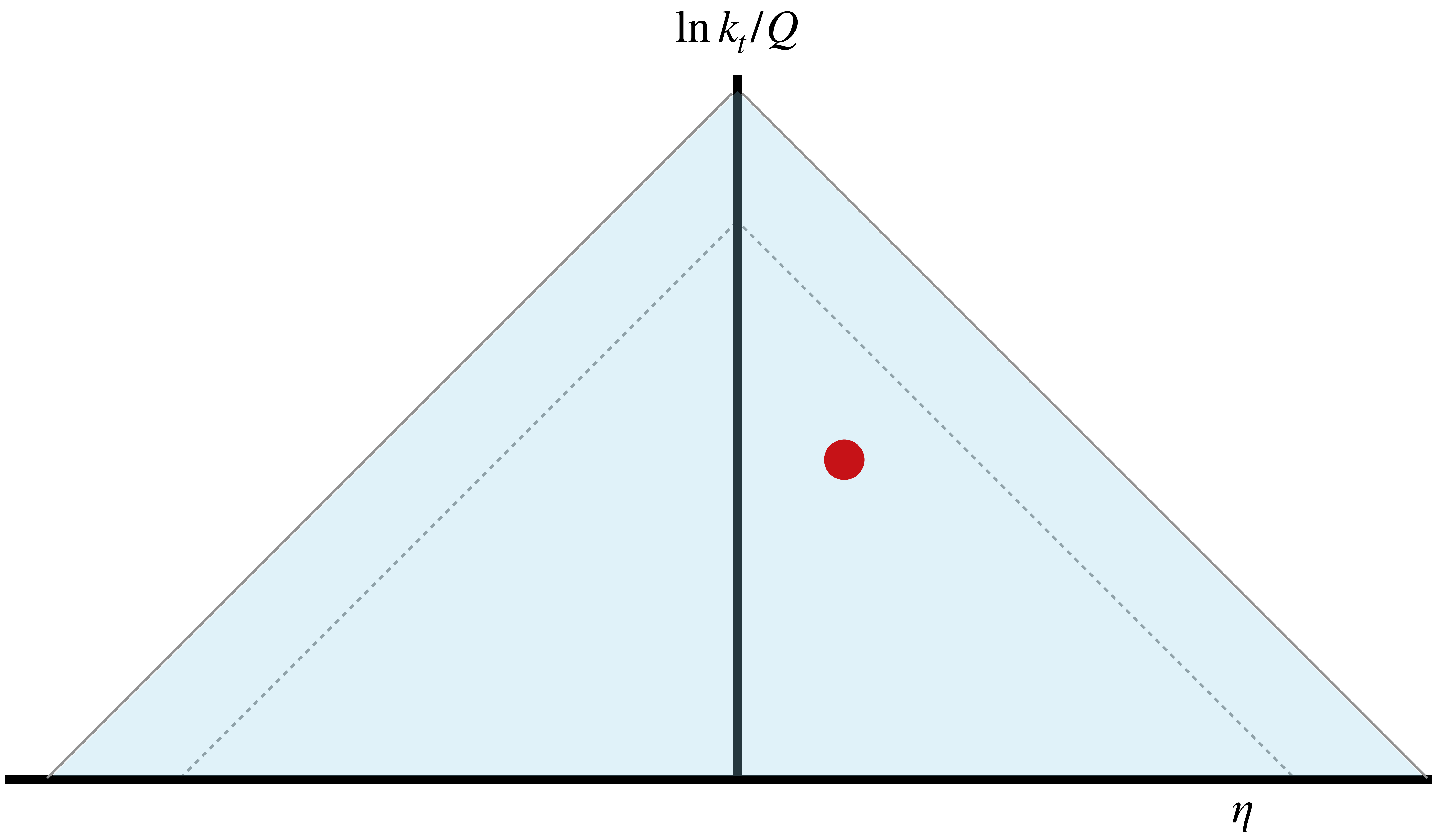

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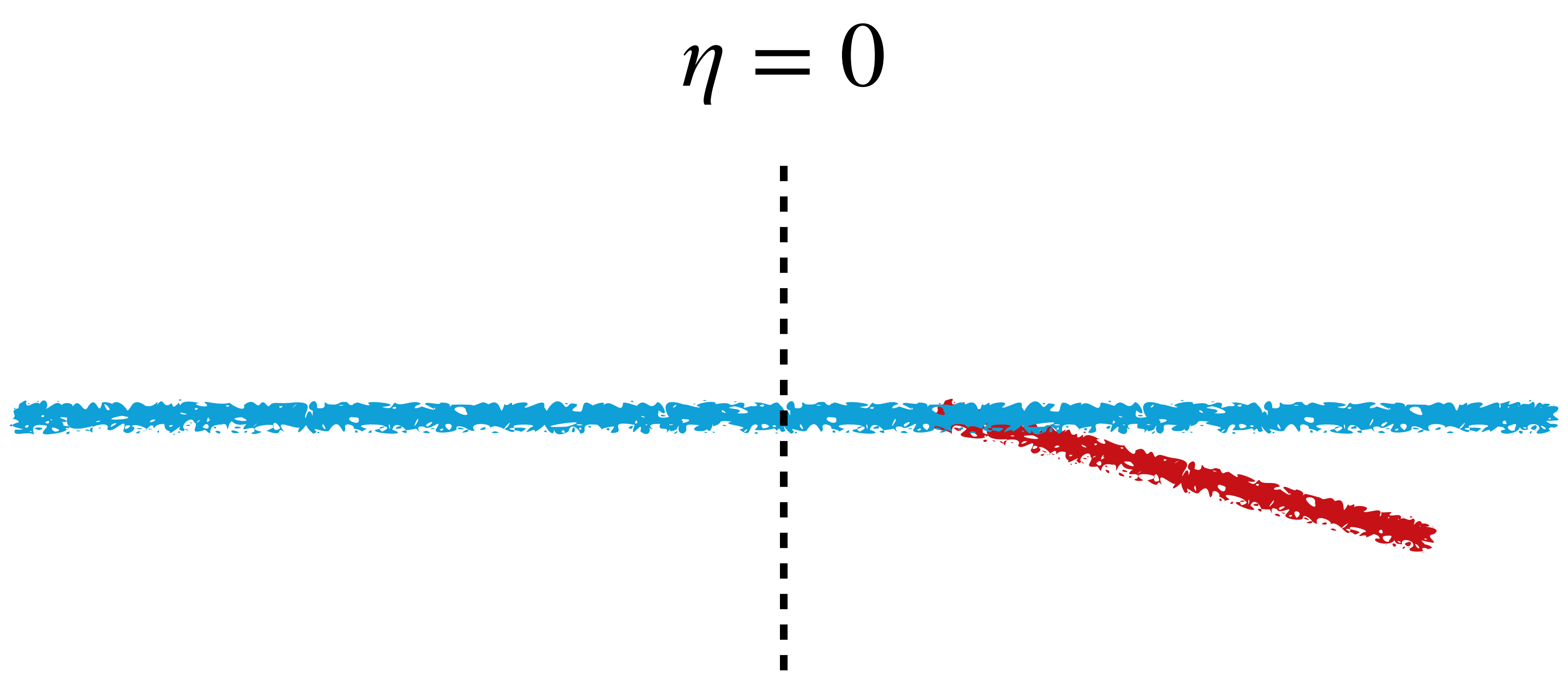

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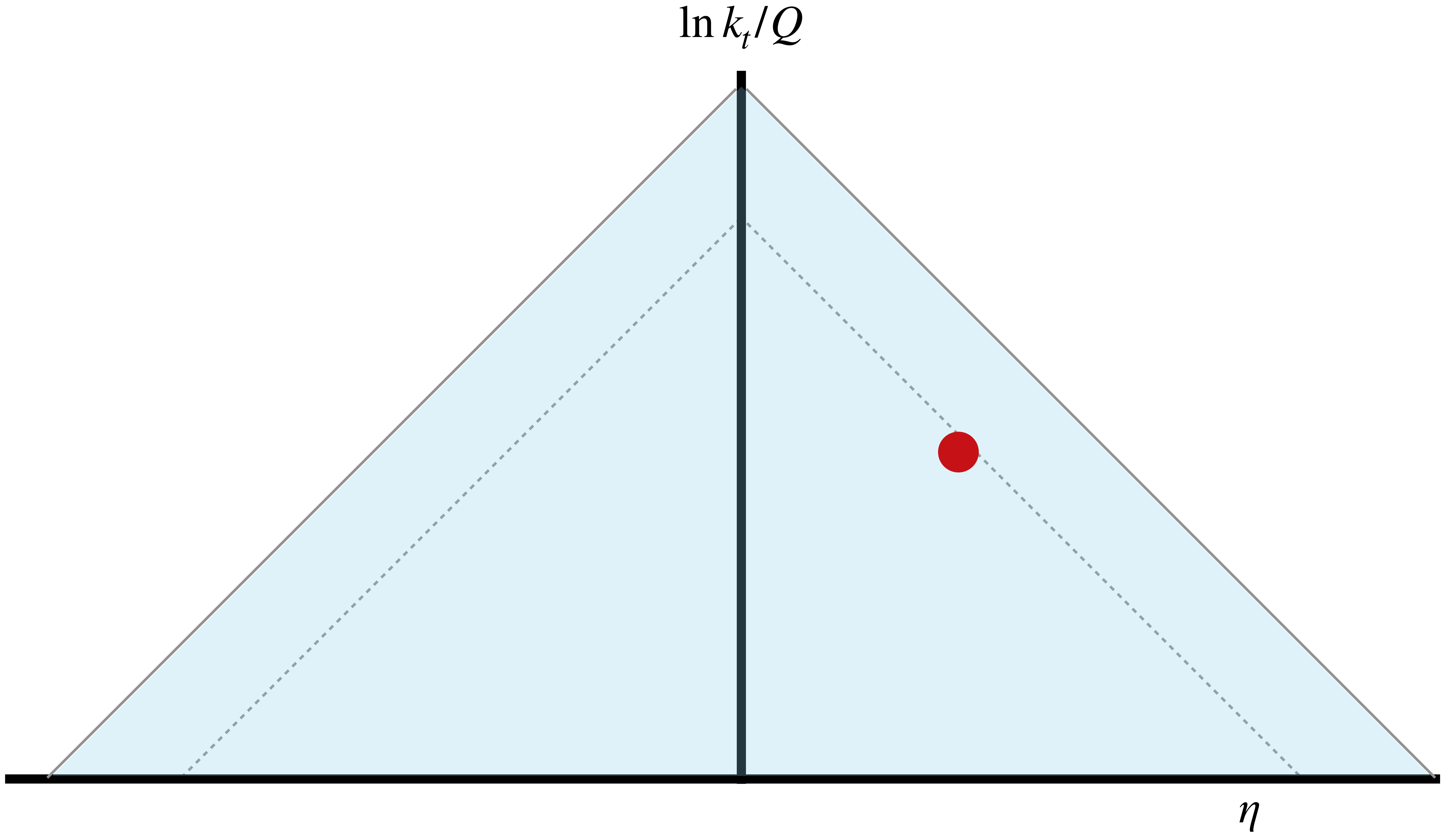
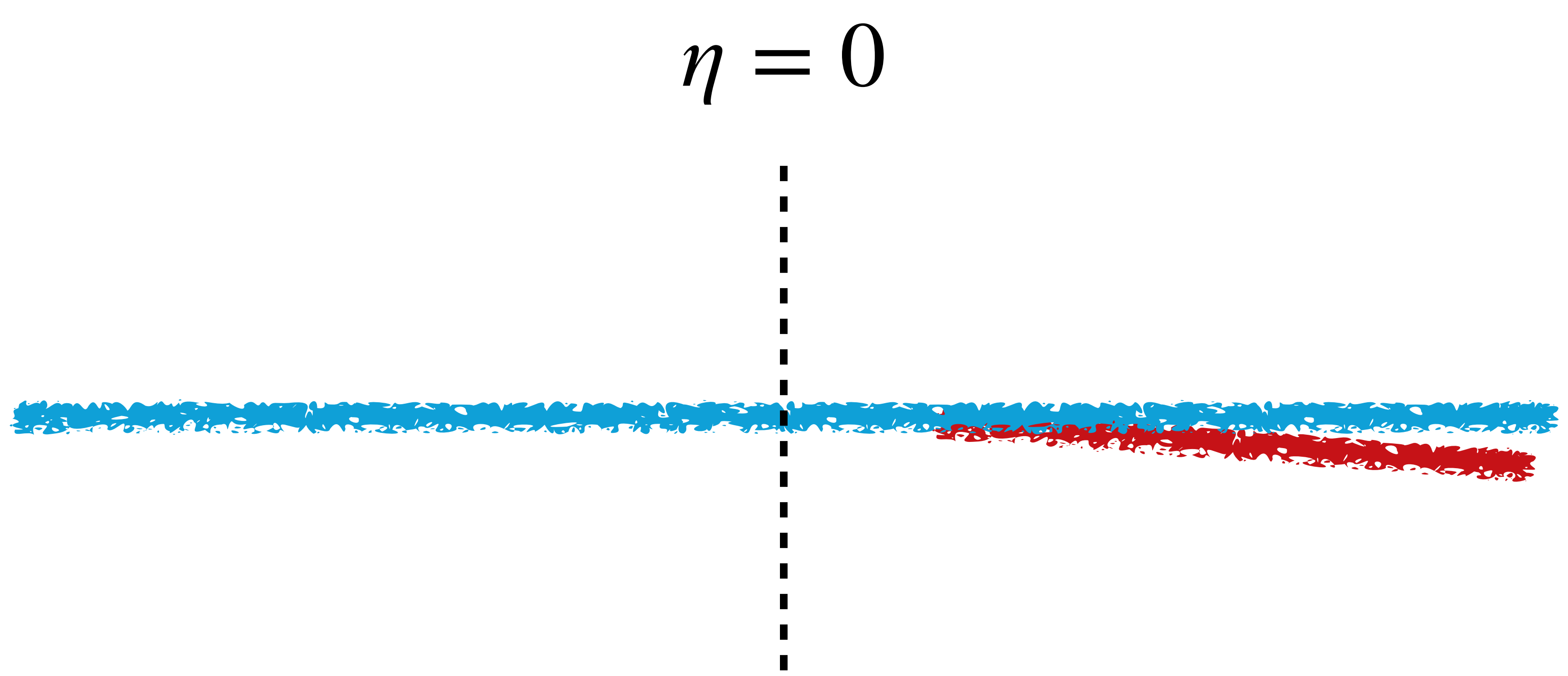
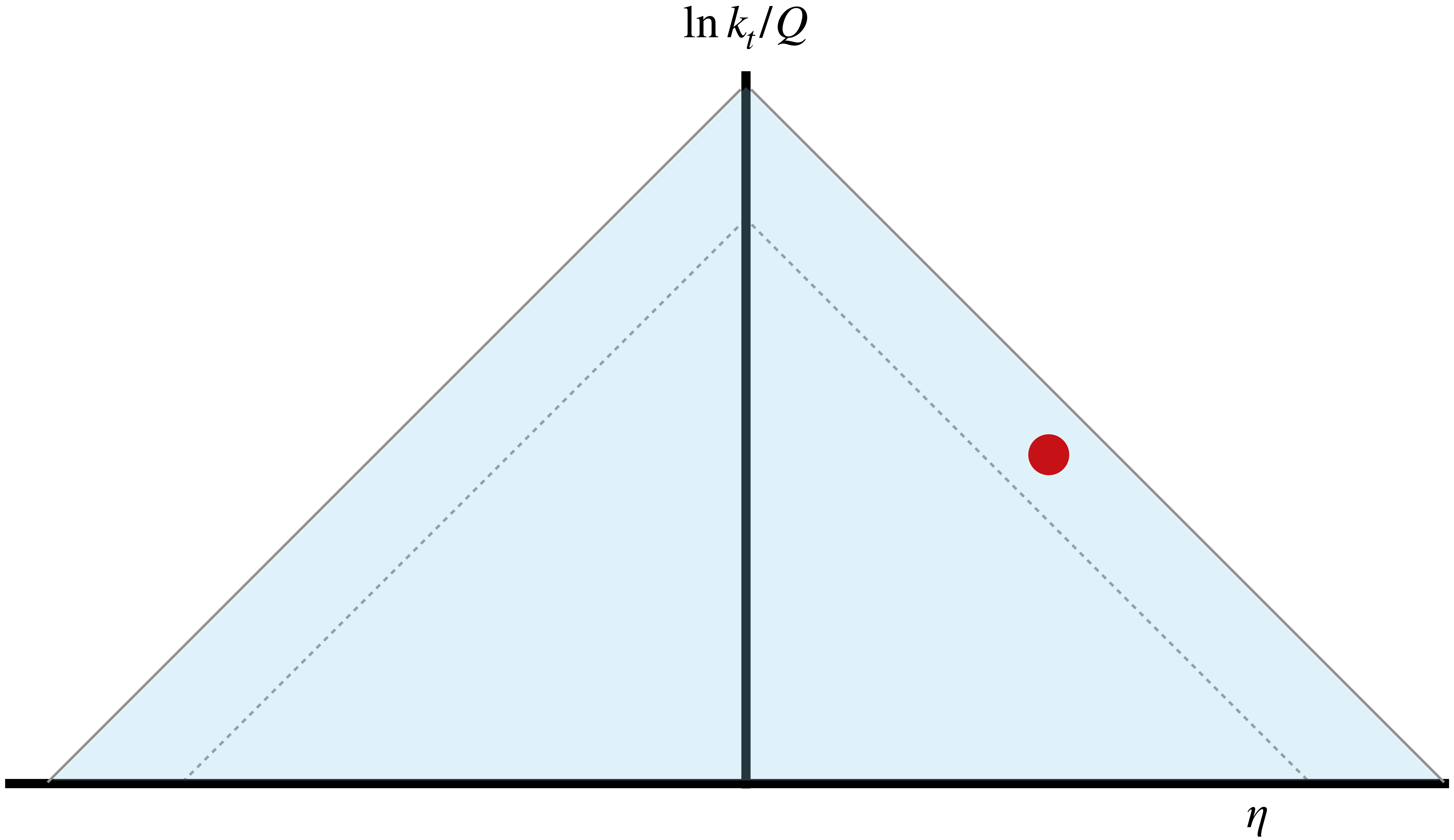


Emissions illustrated in Lund plane



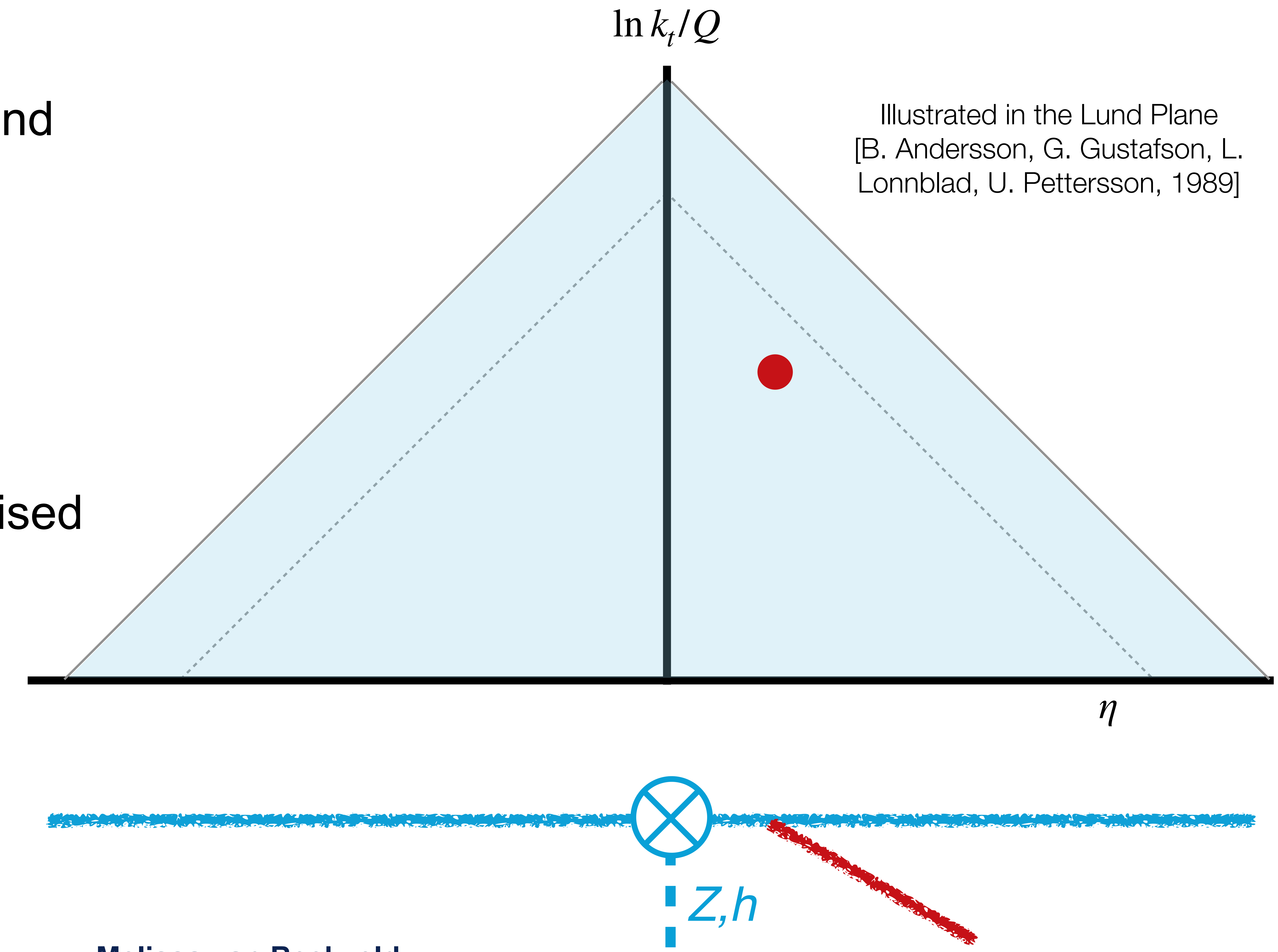
Collinear emissions move out of the Lund plane (their $|\eta|$ increases)



Testing the underlying principle

- QCD amplitudes factorise in soft and collinear limits
- Shower has the factorised $1 \to 2$ eikonal/splitting functions implemented
- Shower must reproduce the factorised amplitude when emissions are 'sufficiently' independent

Any particle emitted after particle 1 may not influence the kinematics of particle 1!



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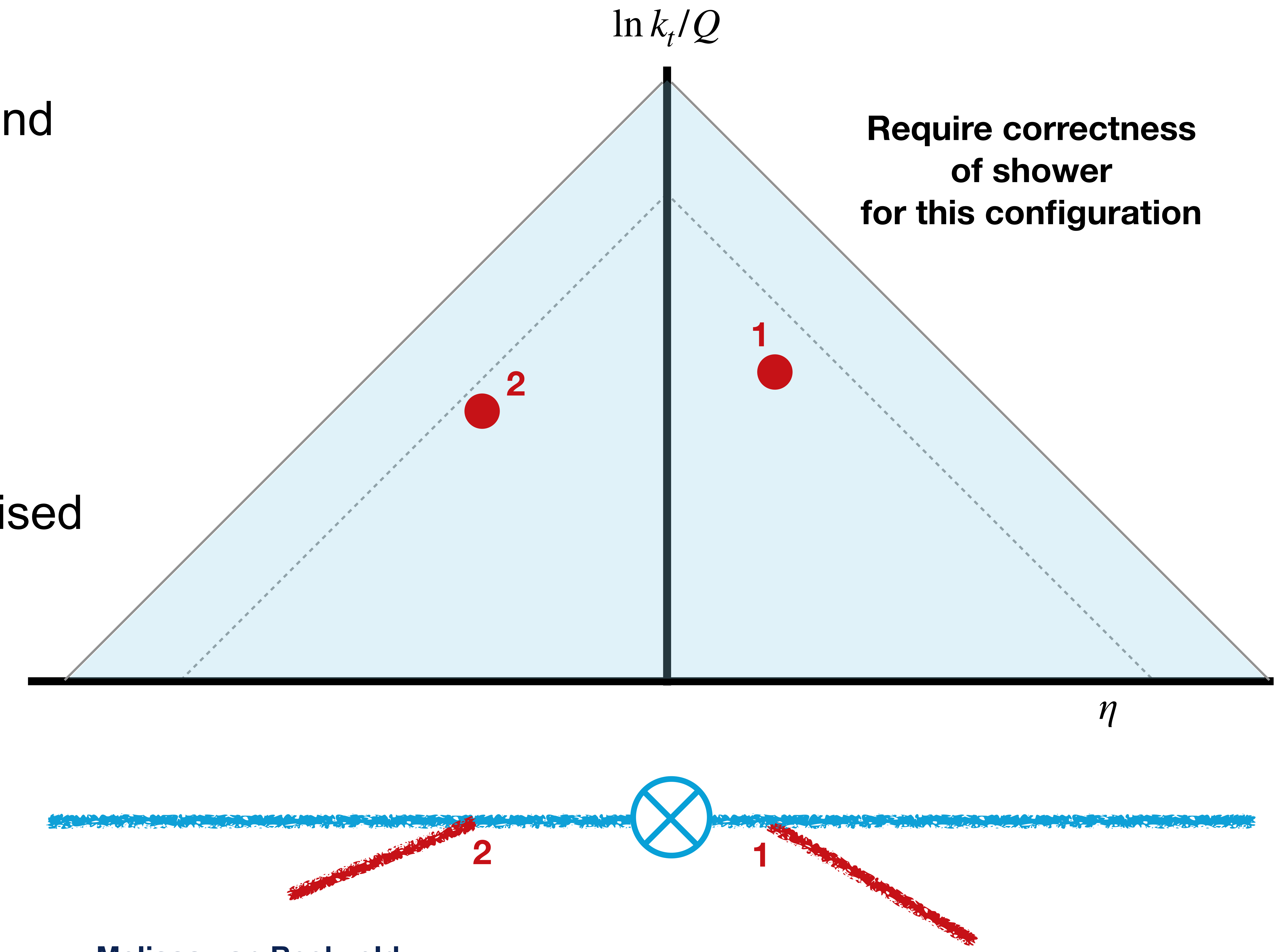

Require correctness of shower for this configuration

Any particle emitted after particle 1 may not influence the kinematics of particle 1!

Testing the underlying principle

- QCD amplitudes factorise in soft and collinear limits
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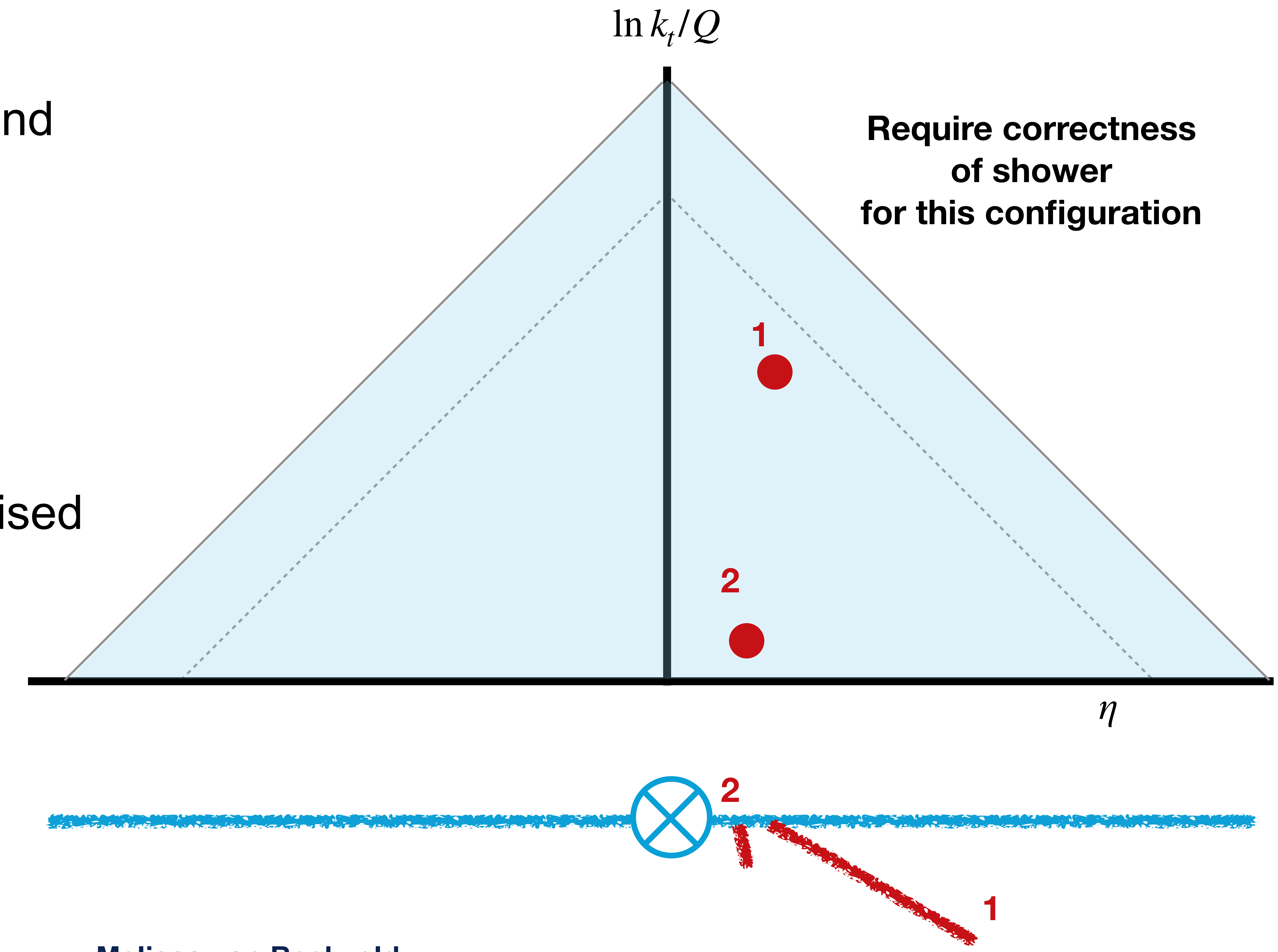
- Shower must reproduce the factorised amplitude when emissions are 'sufficiently' independent



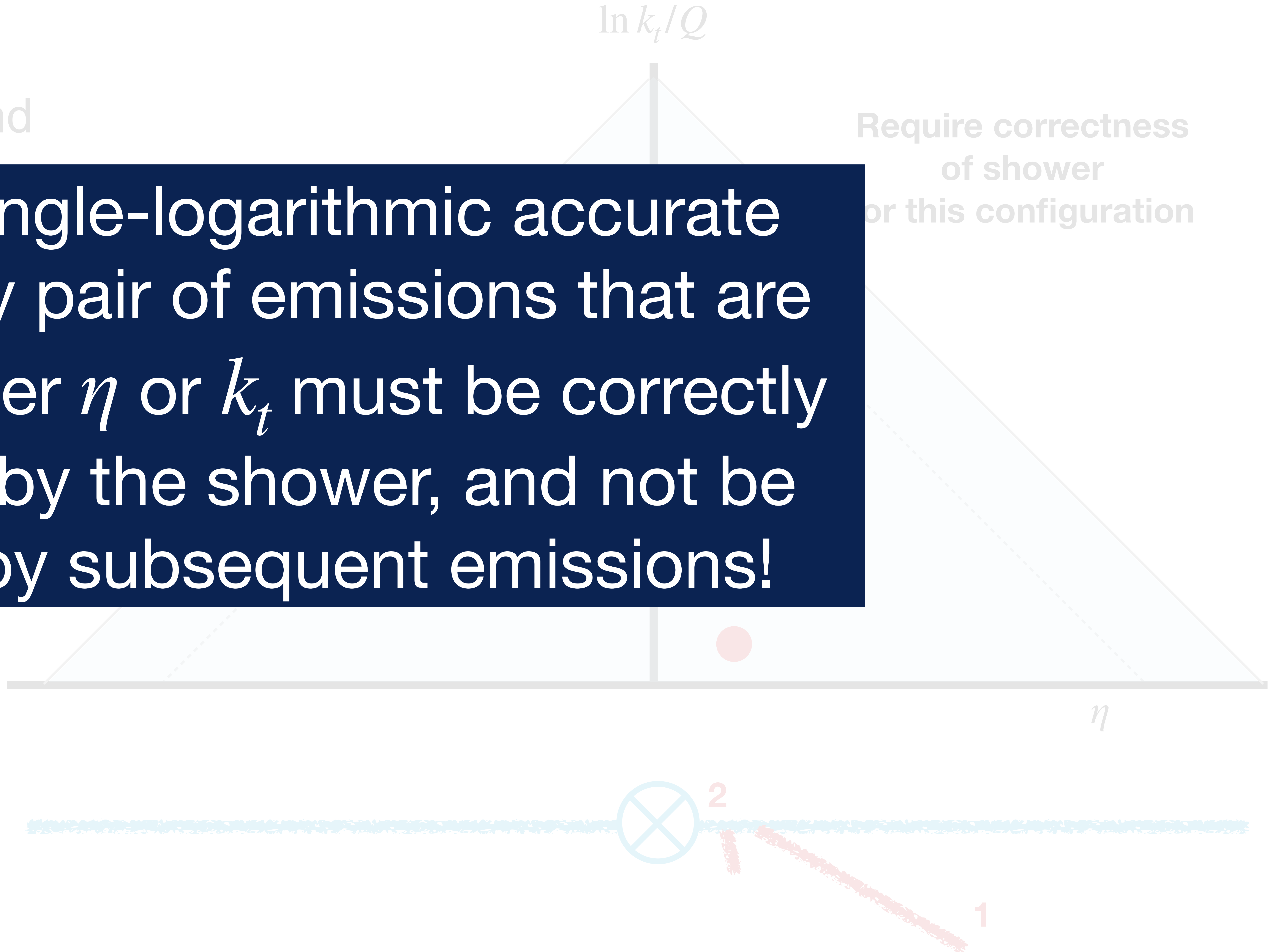
Any particle emitted after particle 1 may not influence the kinematics of particle 1!

Testing the underlying principle

- QCD amplitudes factorise in soft and collinear limits
- Shower has the … eikonal/splitting … implemented
- Shower must rep… amplitude when … 'sufficiently' independent

To get a single-logarithmic accurate shower, any pair of emissions that are close in either η or k_t must be correctly generated by the shower, and not be modified by subsequent emissions!

Any particle emitted after particle 1 may not influence the kinematics of particle 1!



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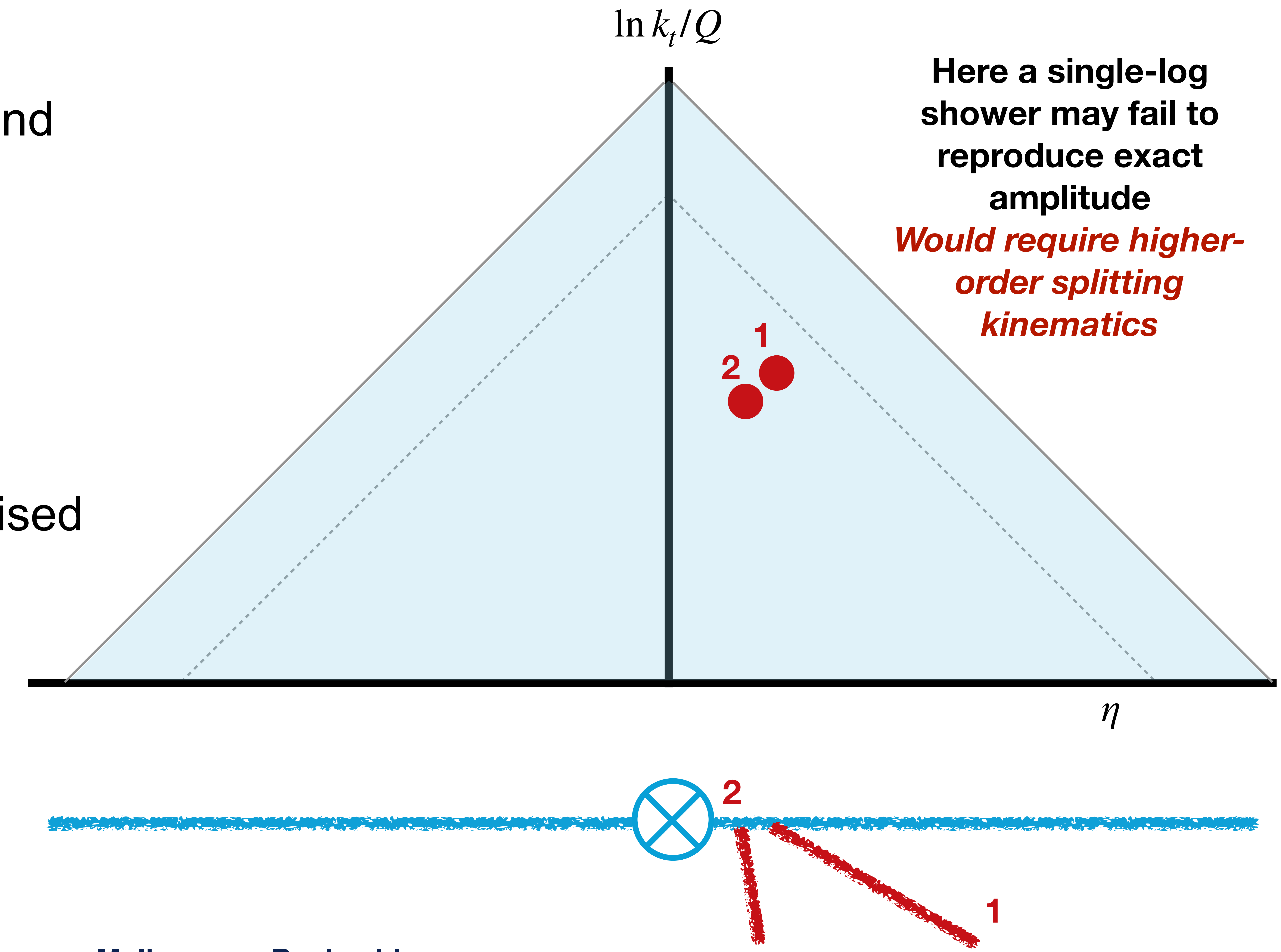
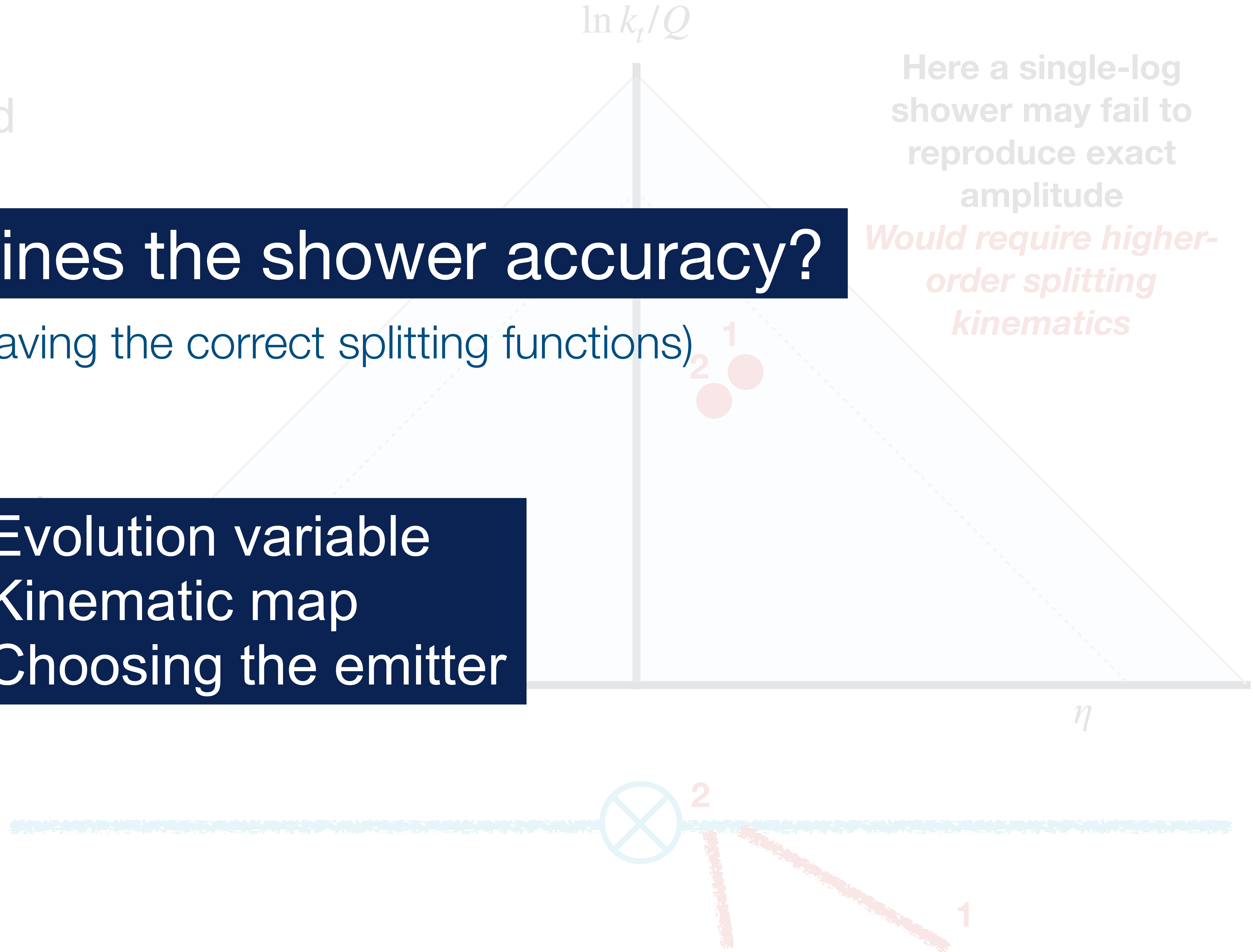


Here a single-log shower may fail to reproduce exact amplitude

Would require higher-order splitting kinematics

Testing the underlying principle

- QCD amplitudes factorise in soft and collinear limits
- Shower has the … eikonal/splitting function… implemented
- Shower must reproduce the fac… amplitude when emissions are 'sufficiently' independent



What determines the shower accuracy?

(apart from having the correct splitting functions)

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

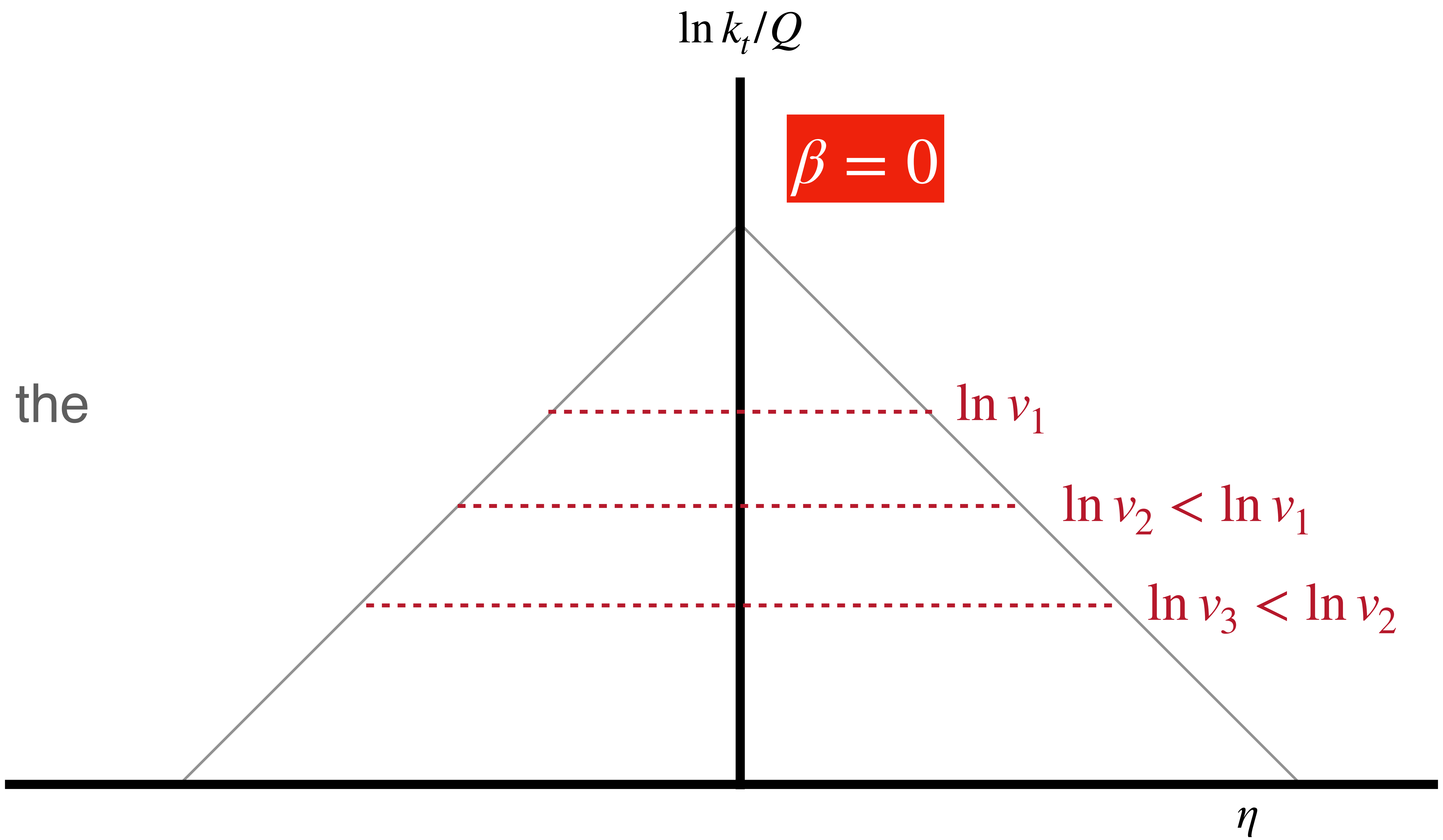
What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

A parton shower orders emissions

The evolution variable v tells us which emissions come first, and which later in the showering process

We use the definition $v \simeq k_t e^{-\beta|\eta|}$



Transverse-momentum ordered with $\beta_{PS} = 0$

Choice for most dipole parton showers

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

A parton shower orders emissions

The evolution variable v tells us which emissions come first, and which later in the showering process

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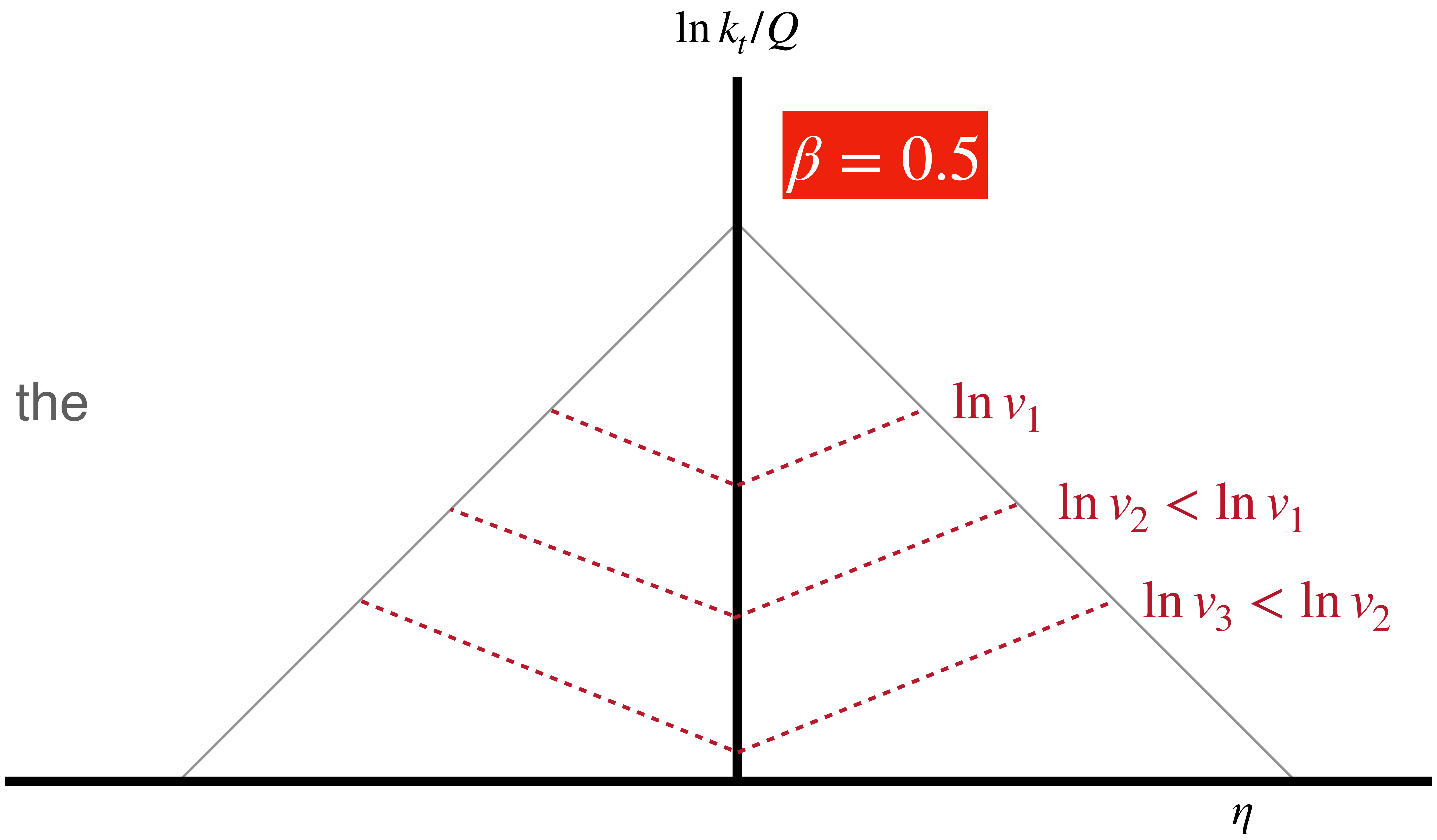
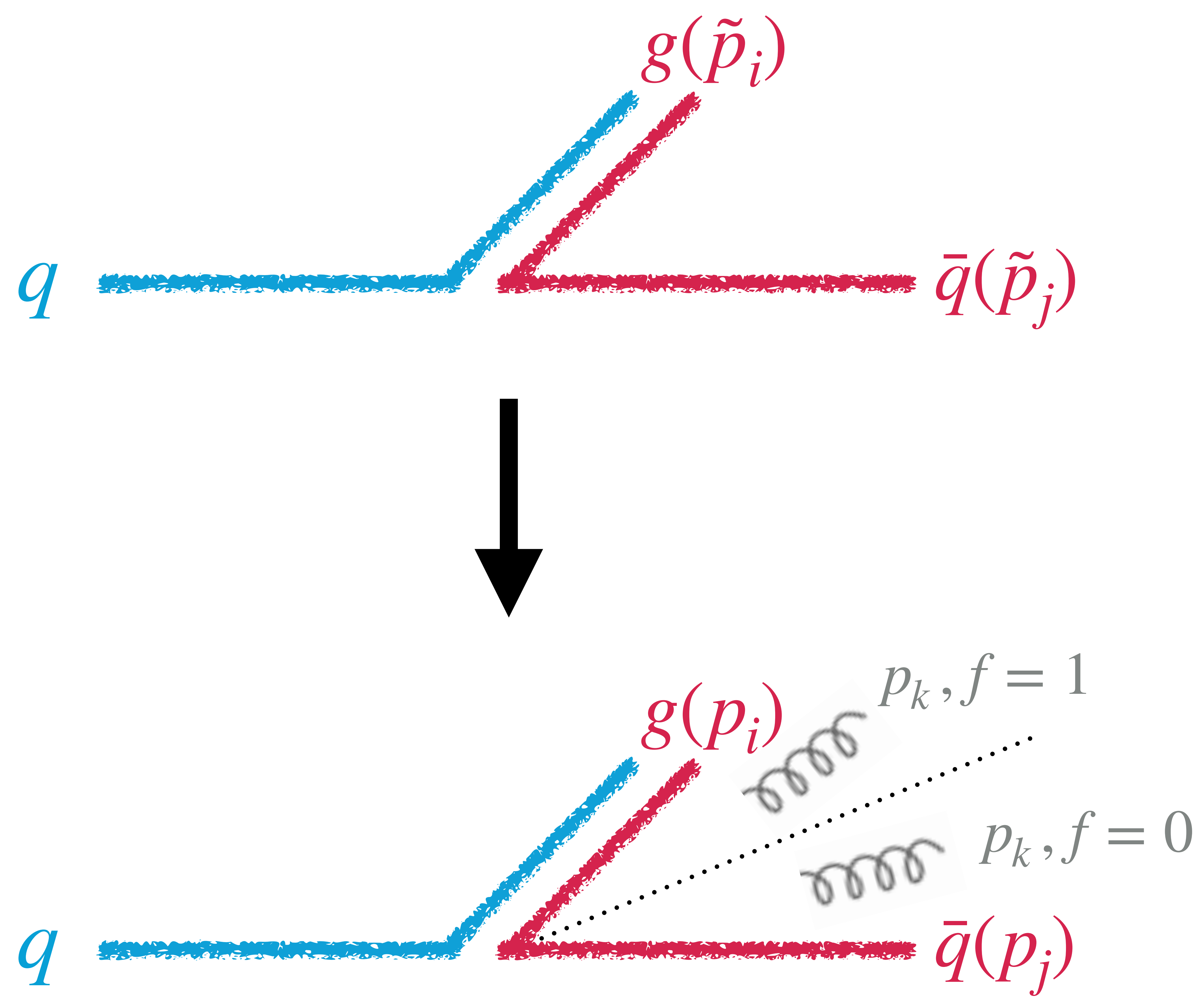


Introduce some angular dependence with $\beta > 0$

Angular-ordering (e.g. as implemented in Herwig) will not be considered here

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter



Local kinematic map

$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j + f k_\perp$$

$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j + (1 - f) k_\perp$$

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp$$

Mapping coefficients depend on

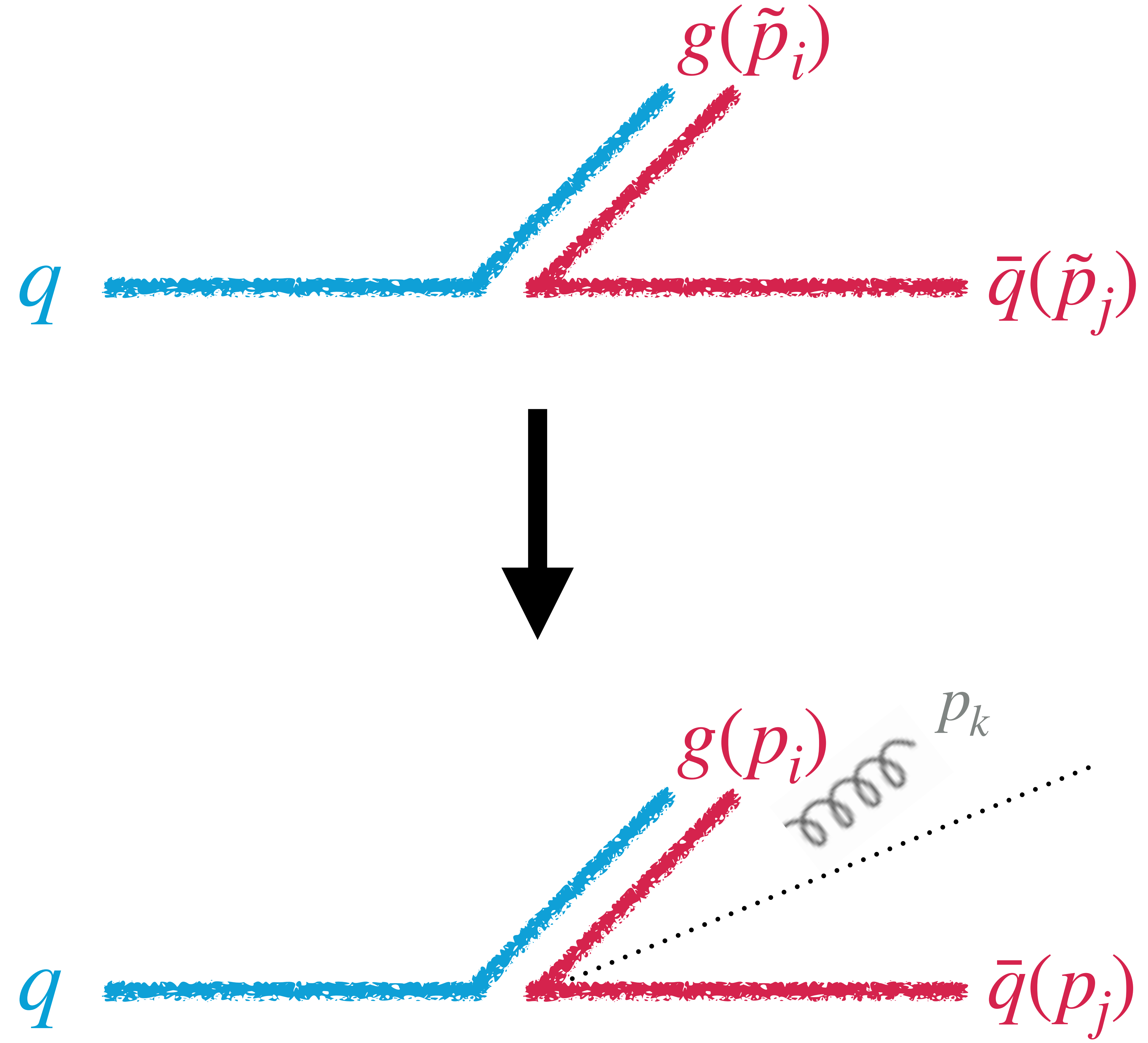
- Evolution variable $\ln v$
- Rapidity η

Dipole: step function for f

Antenna: smooth transition for f

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter



Global kinematic map

$$p_i = a_i \tilde{p}_i$$

$$p_j = b_j \tilde{p}_j$$

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp$$

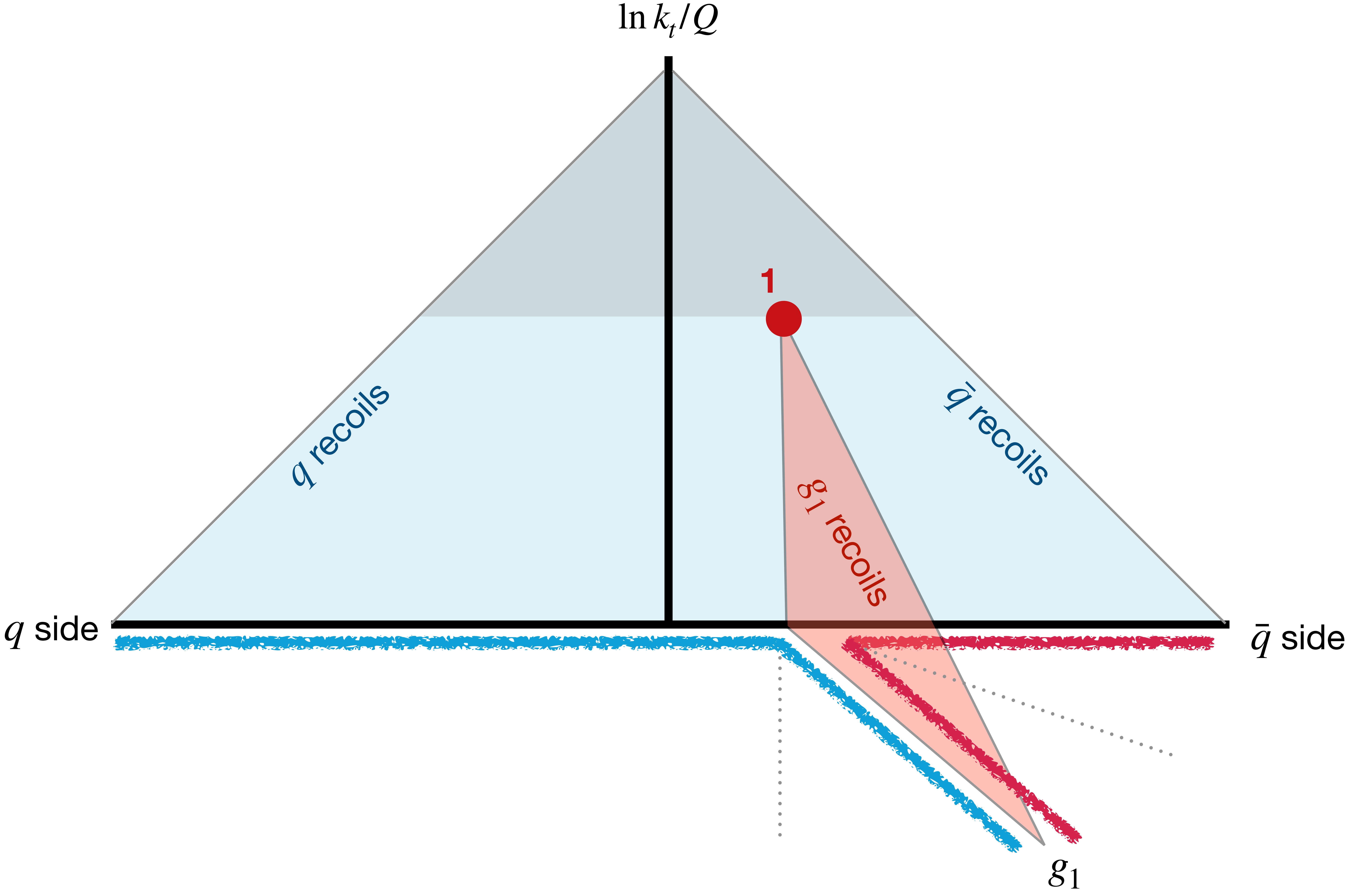
Boost (part of) event after each emission to restore momentum conservation

Choice: global in some/all $+/-$ and $\perp$ components

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

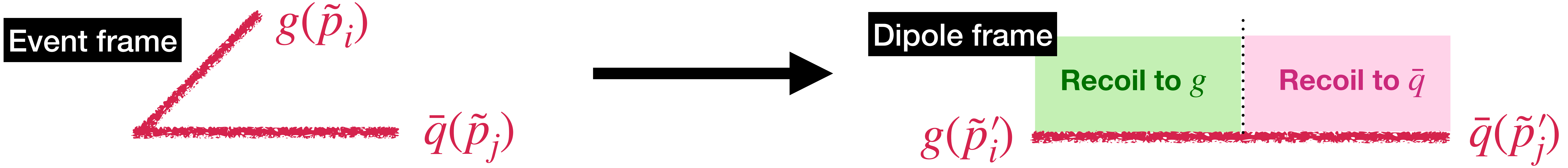
Expected attribution of recoil



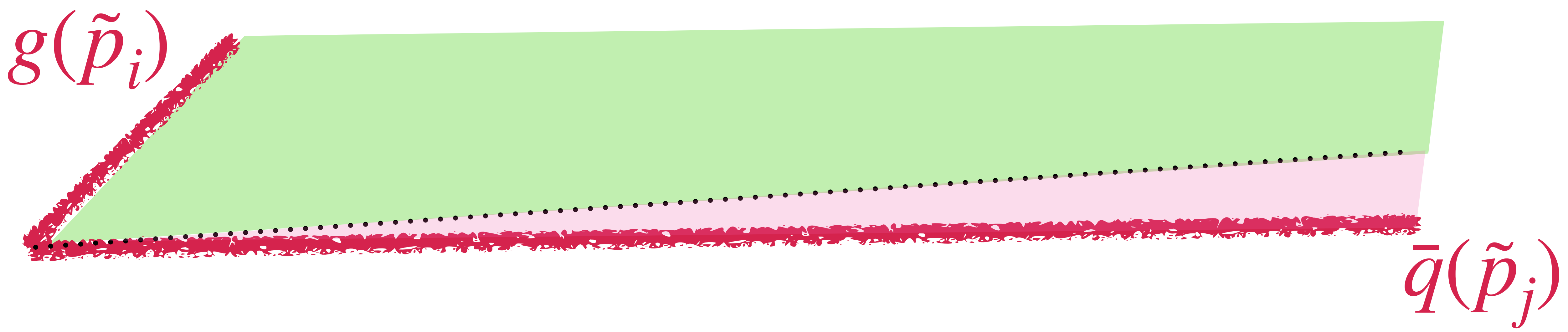
What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

Standard dipole showers distinguish the emitter from the spectator at $\eta = 0$ in the CM dipole frame



Boosting back to the event frame…



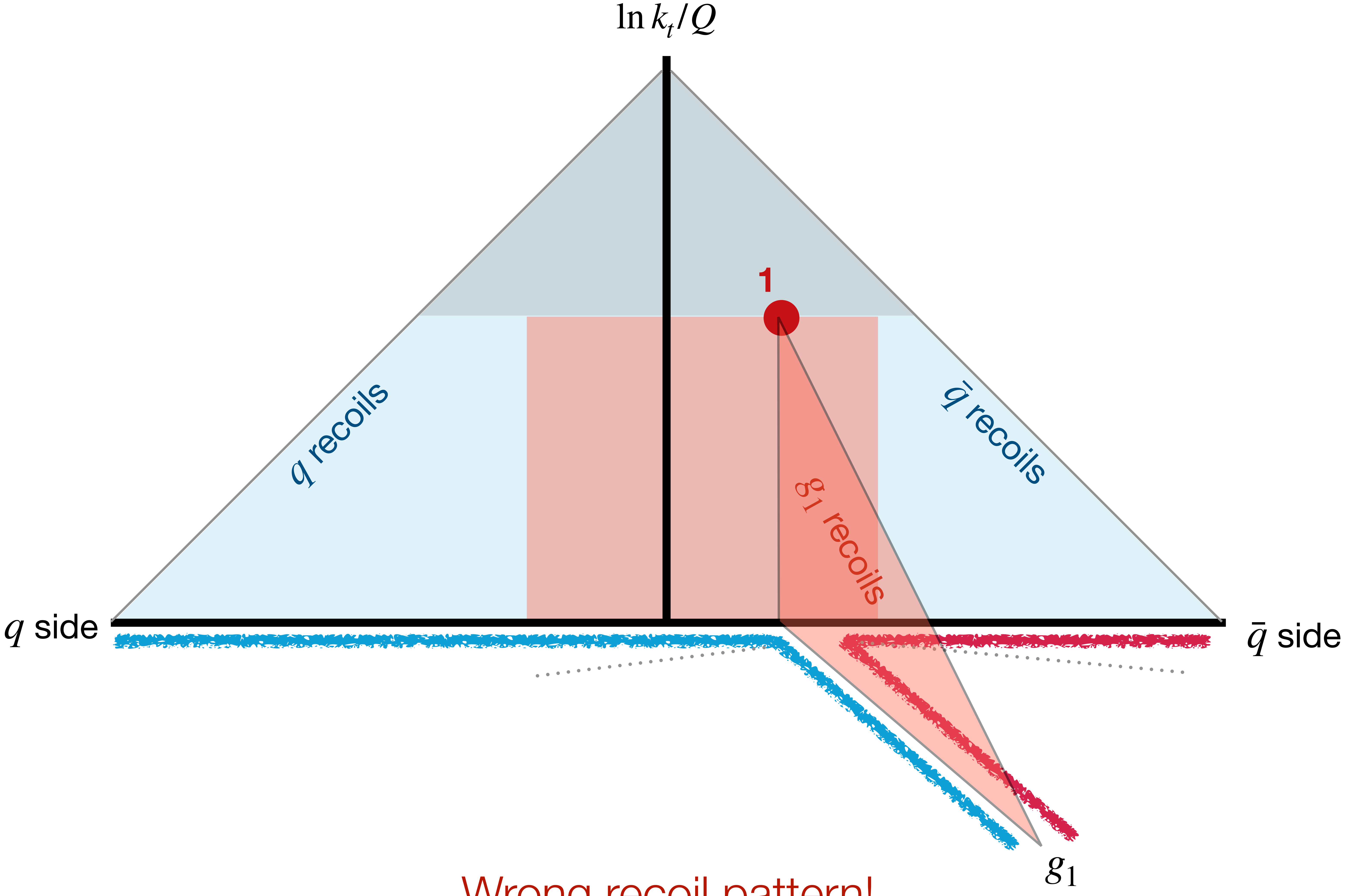
Leads to an incorrect (and quite unphysical) recoil picture!

Physical attribution of recoil

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

Recoil attribution for transverse-momentum ordered local shower (choosing emitter in dipole frame)

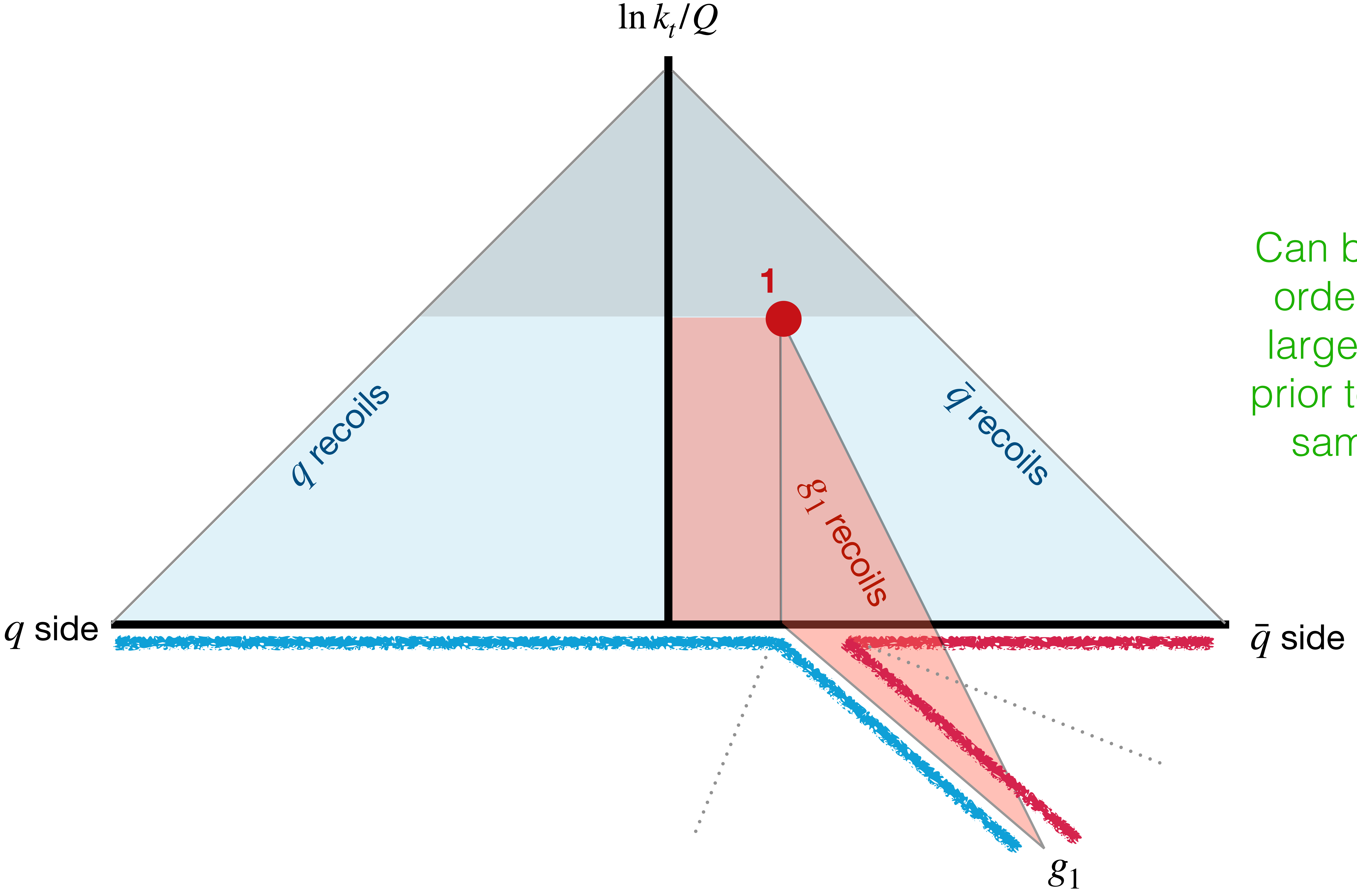


Wrong recoil pattern!

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

Recoil attribution for transverse-momentum ordered local shower (choosing emitter in event frame)



Can be fixed using a different ordering variable, such that large-angle emissions come prior to small-angle ones (with same k_t), or a global map

Less wrong, but still not correct recoil pattern!

PanScales NLL correctness requirements

1. Test of the basic underlying physics principle
Require correctness of effective matrix elements generated by the shower for well-separated emissions

2. Resummation
Require single-logarithmic (NLL/NDL) accuracy for suitably defined observables

Knobs to turn that affect the logarithmic accuracy

1. Evolution variable
2. Kinematic map
3. Attribution of recoil

How does a standard dipole shower (i.e. Sherpa or Pythia) behave?

A standard dipole shower: **dipole-k_t**

1. Evolution variable: transverse momentum (k_t)

2. Kinematic map:

   a) Local Dates back to Gustafson, Petterson [Nucl. Phys. B 306 (1988)], Catani, Seymour [hep-ph/9605323], many variations available

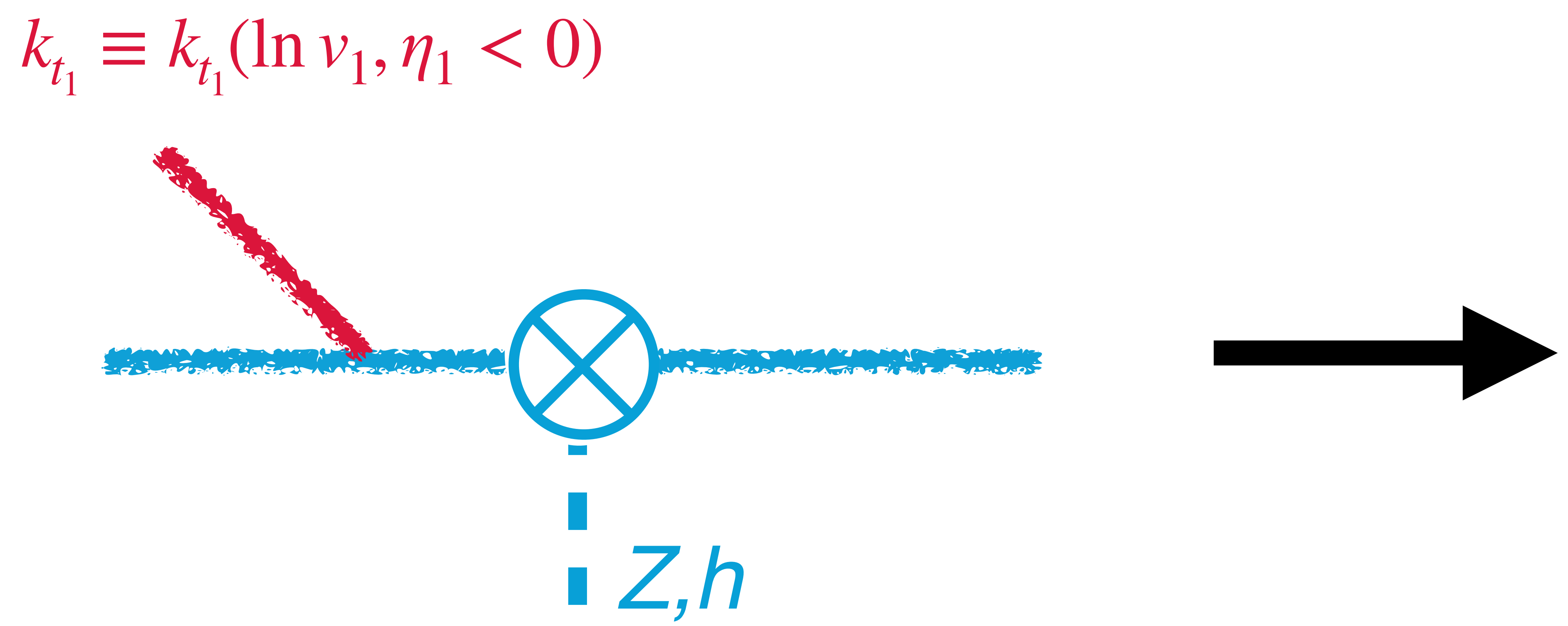
   For every emission the momentum is locally conserved
   This means that the e.g. the Z-boson p_t almost never gets a kick!
   → not in line with the NLL prediction Plätzer, Gieseke [0909.5593], Nagy, Soper [0912.4534]

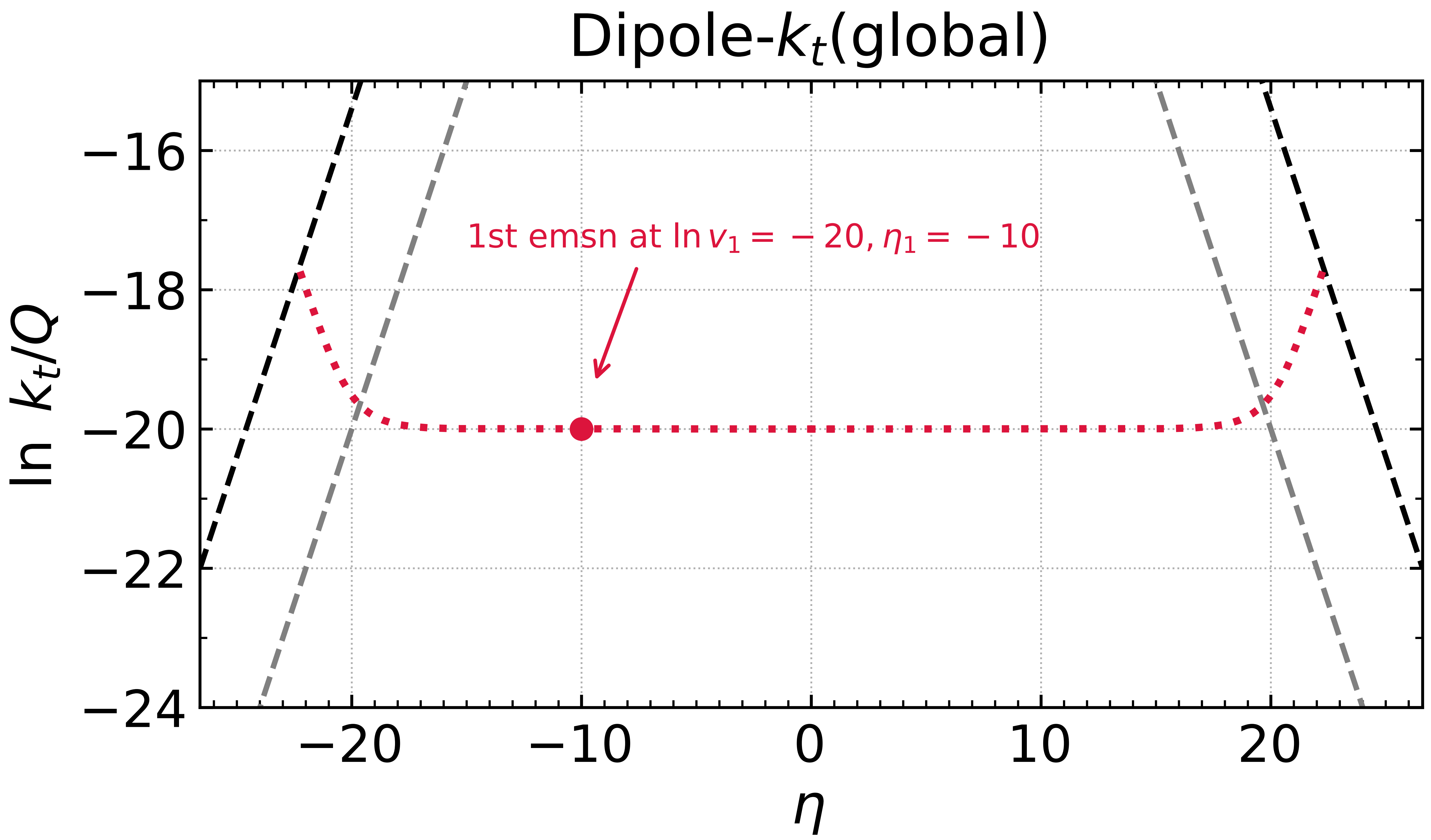
   b) Global Plätzer, Gieseke [0909.5593], Höche, Prestel [1506.05057] [Pythia8 (global ISR) & Deductor have different solutions]

   The Z-boson absorbs the k_t imbalance induced by the global map through a boost
   Claimed to fix the Z-p_t distribution

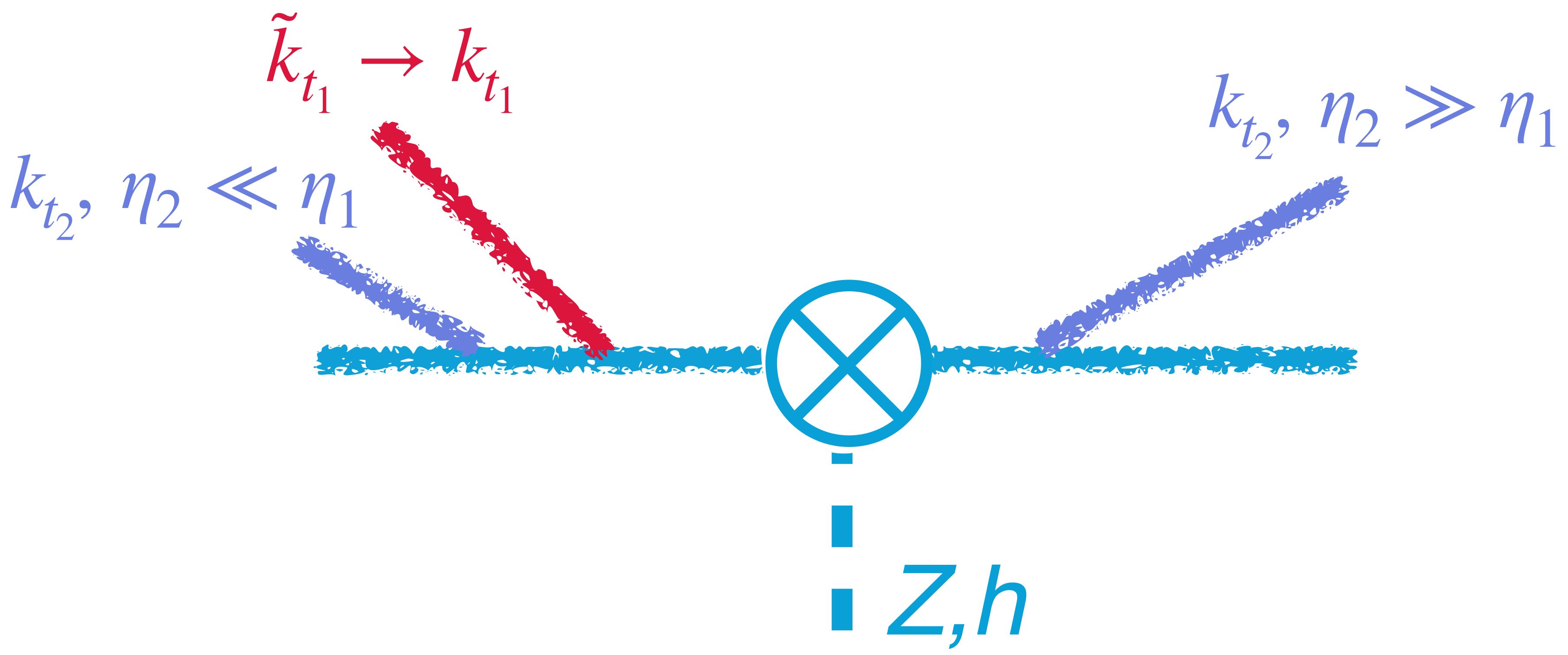
3. Attribution of recoil: dipole CM frame

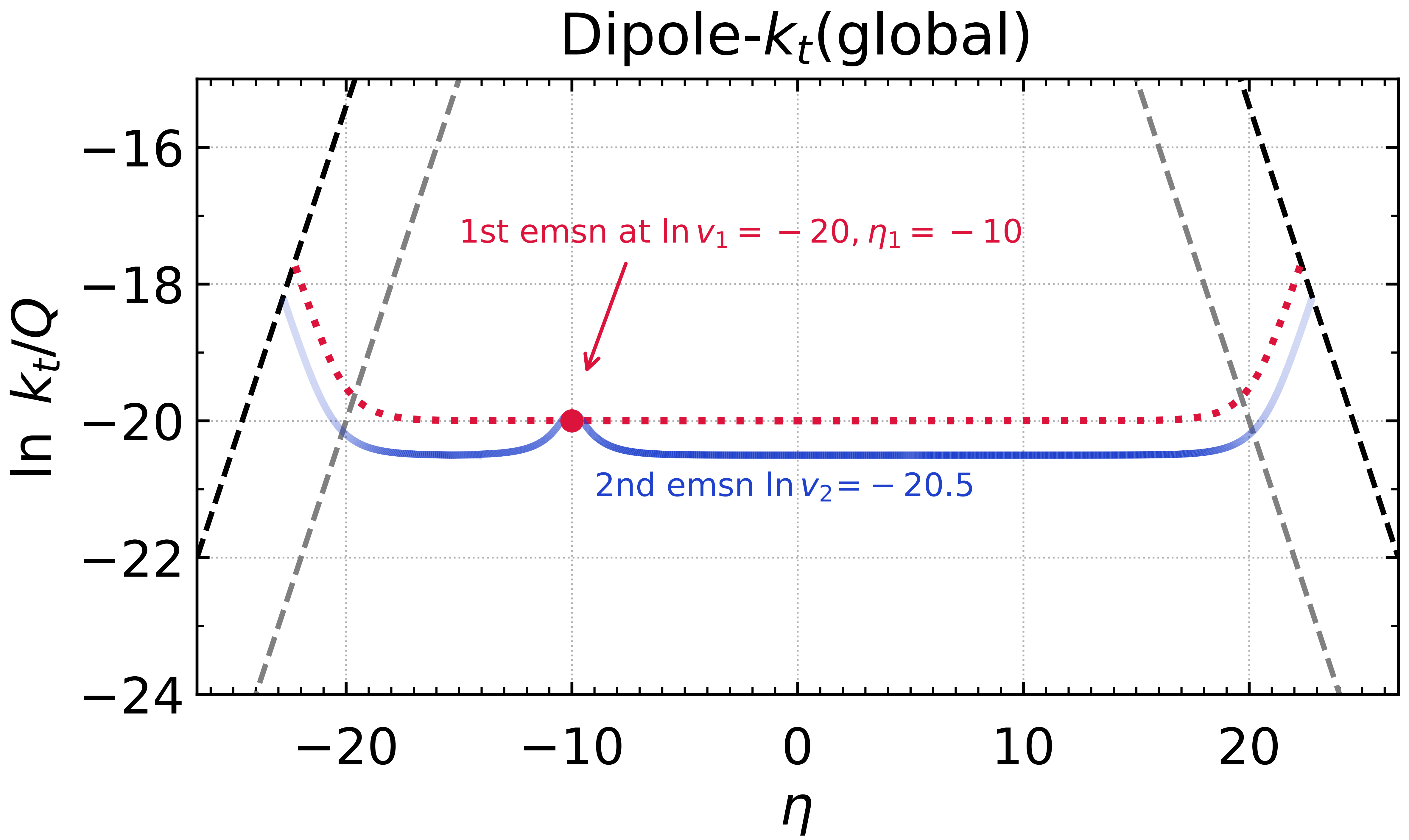
Choice of evolution variable (1) + kinematic map (2) determine *phase-space contours* in the **Lund plane**





How does a **second** emission affect the **first** emission's momentum?





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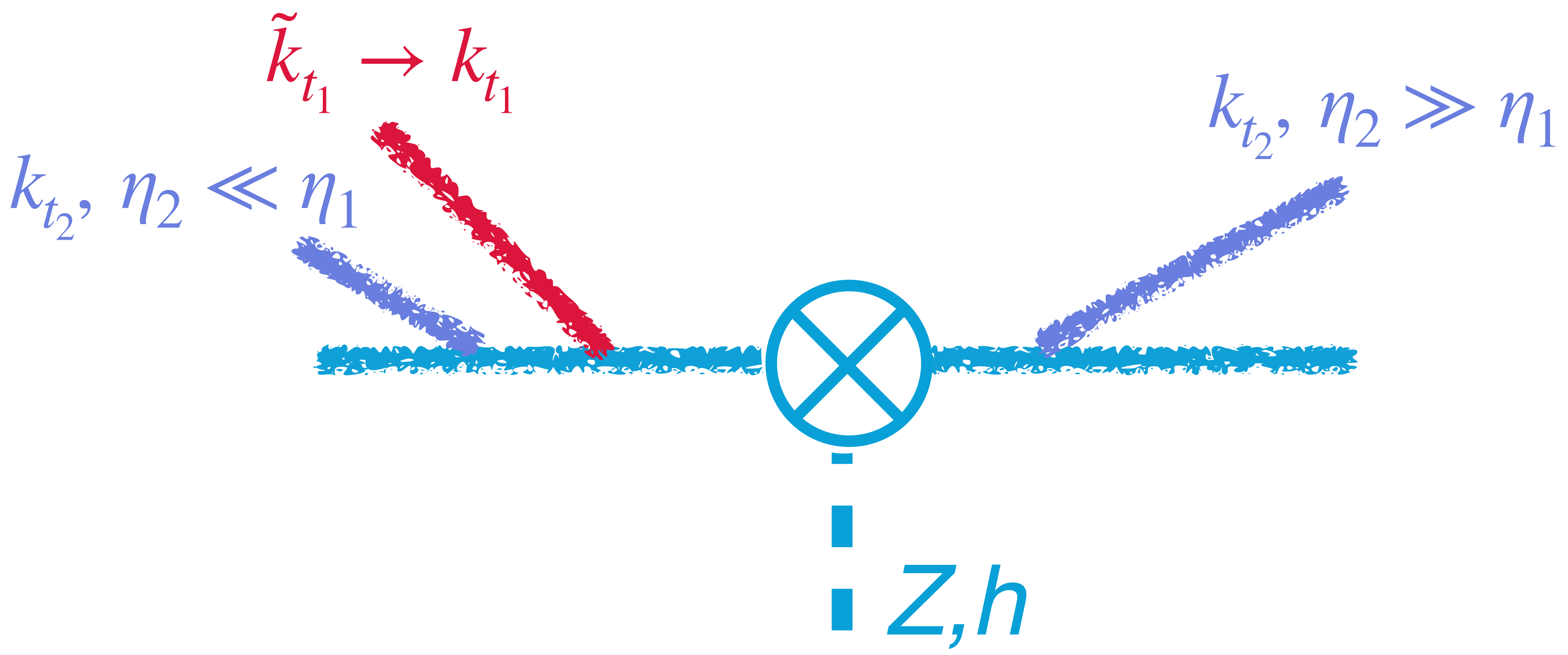

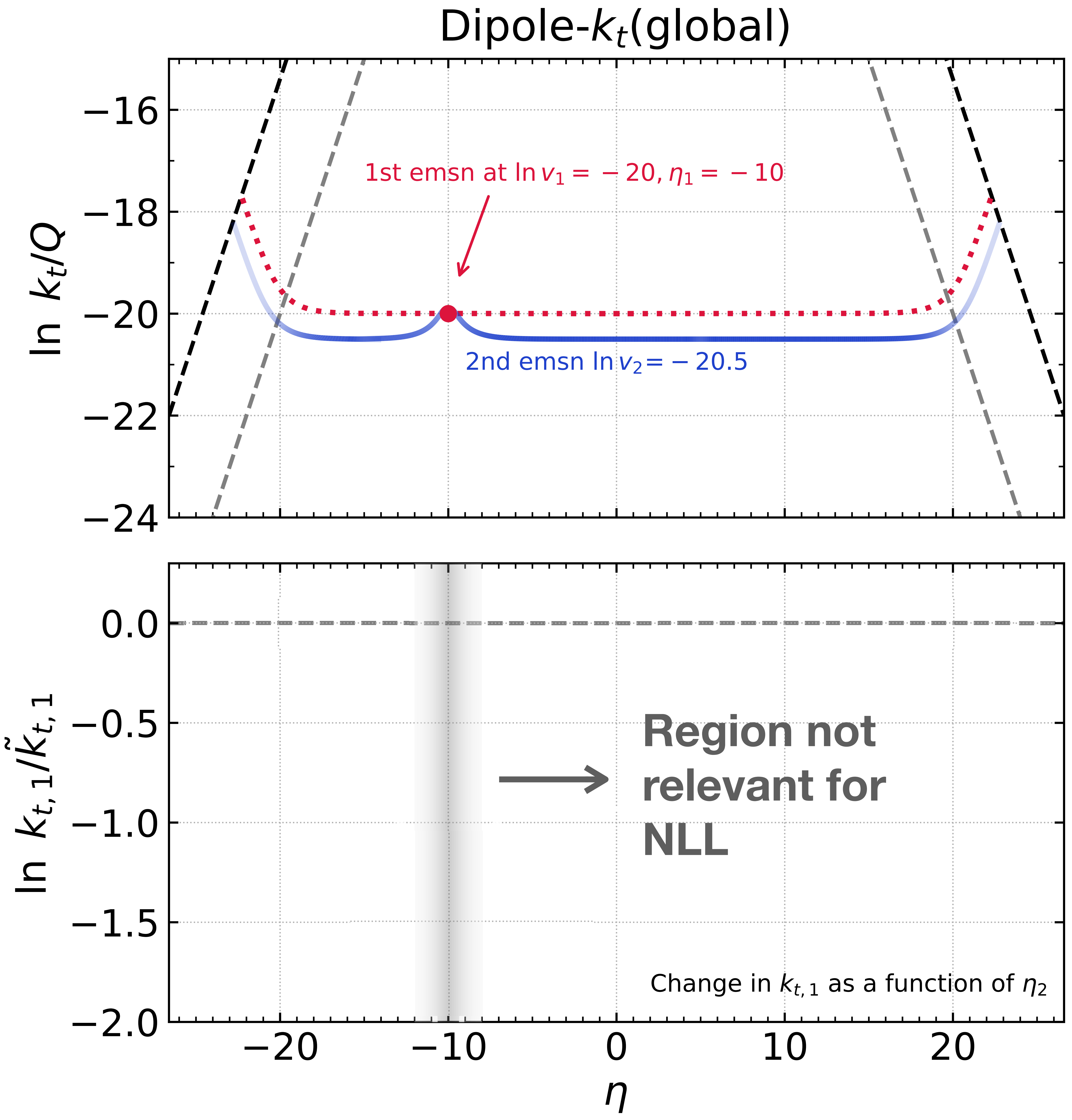

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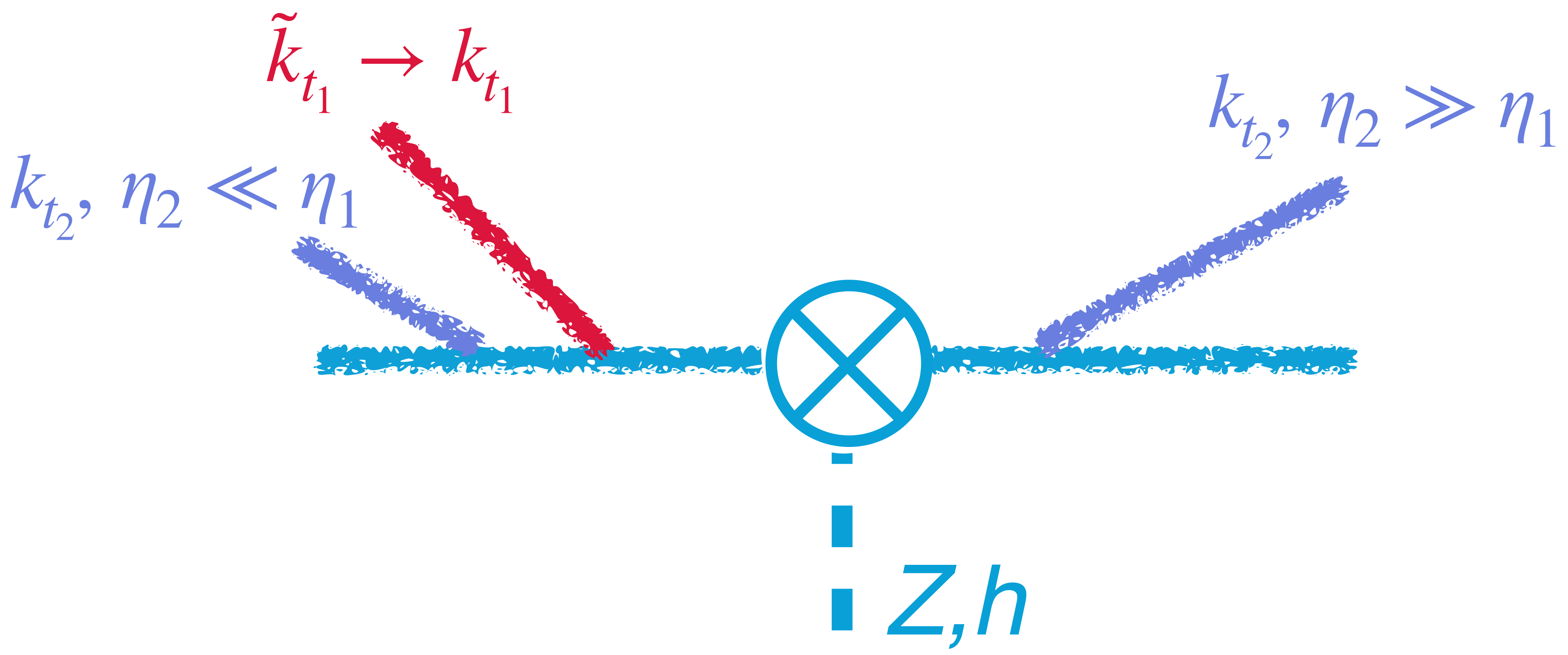

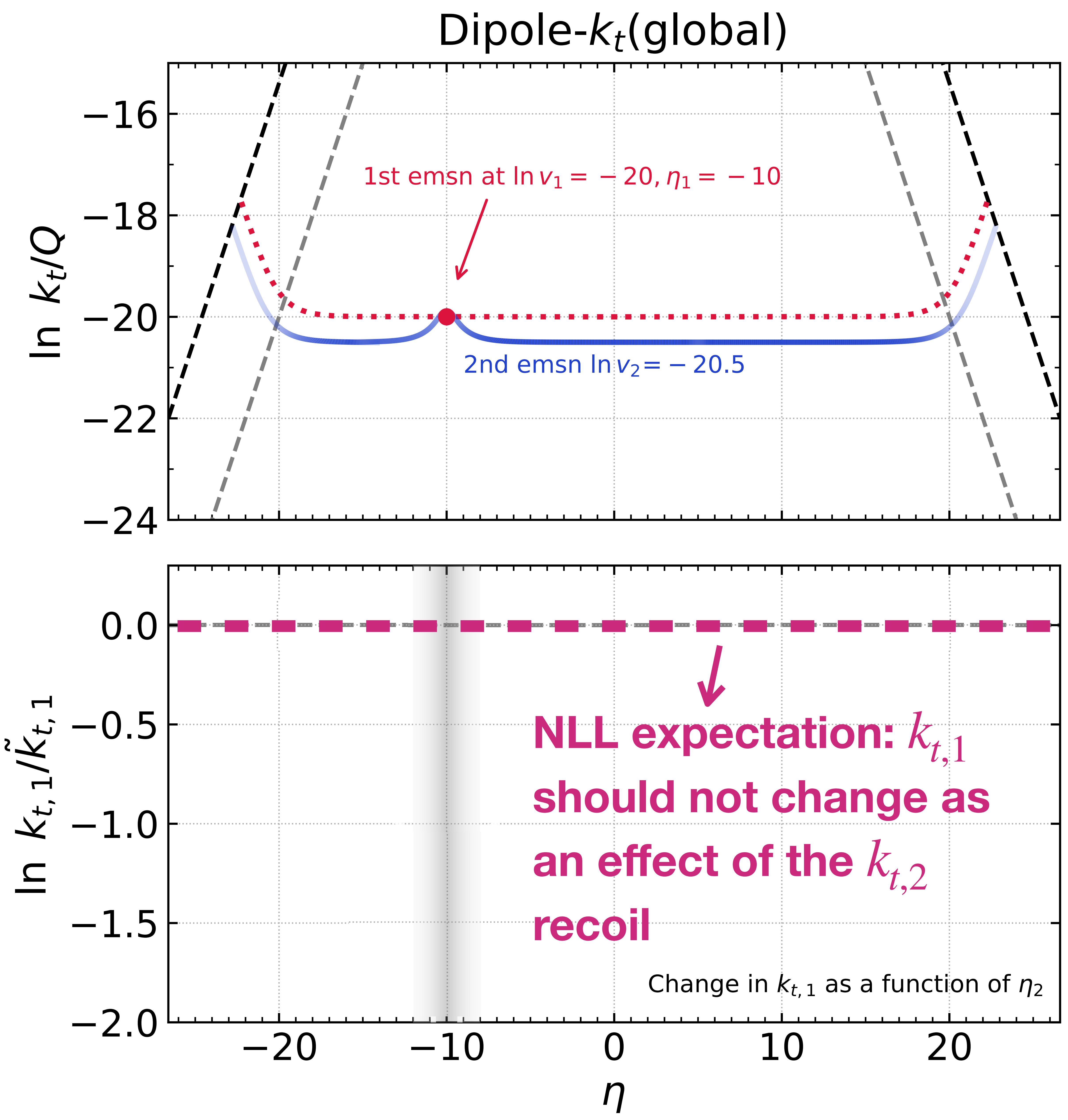

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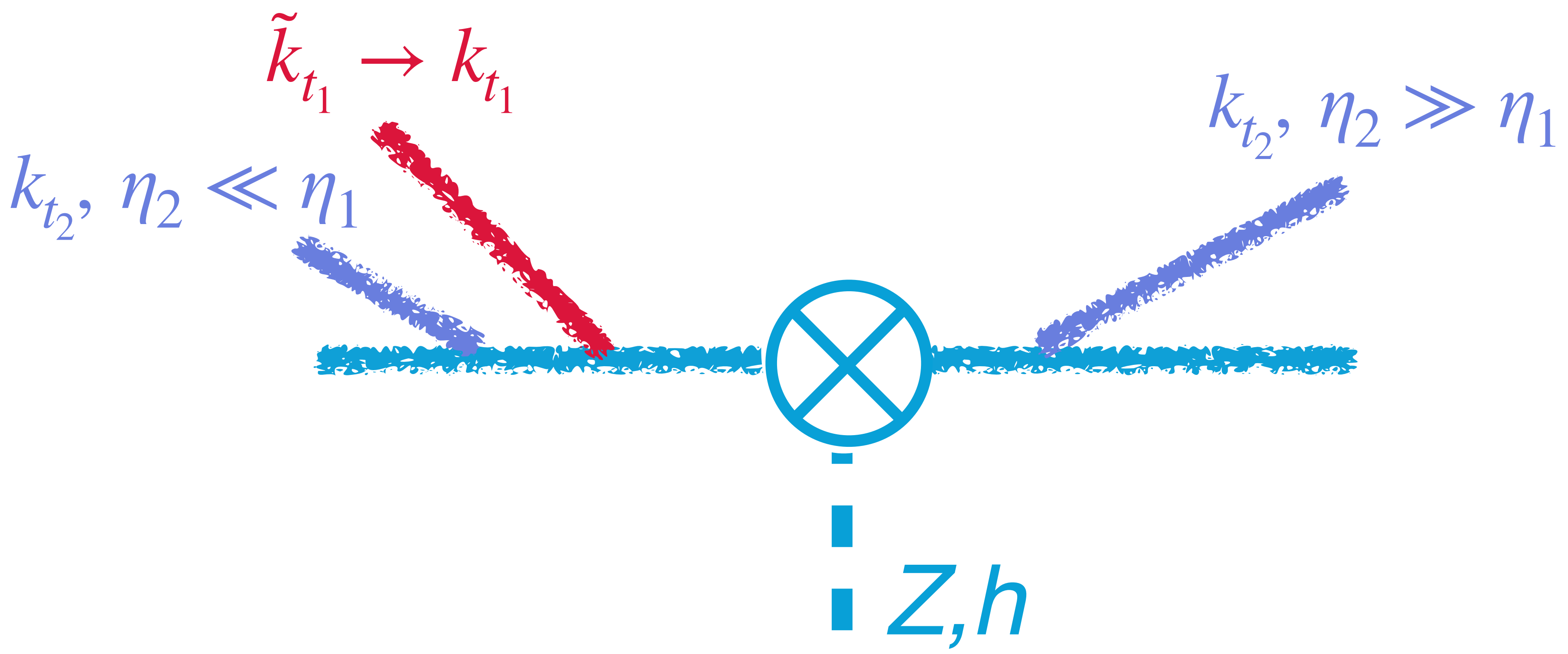
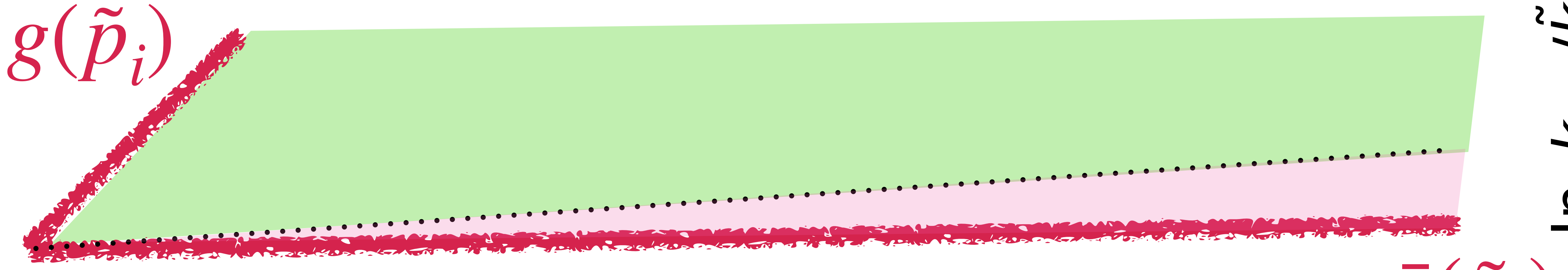
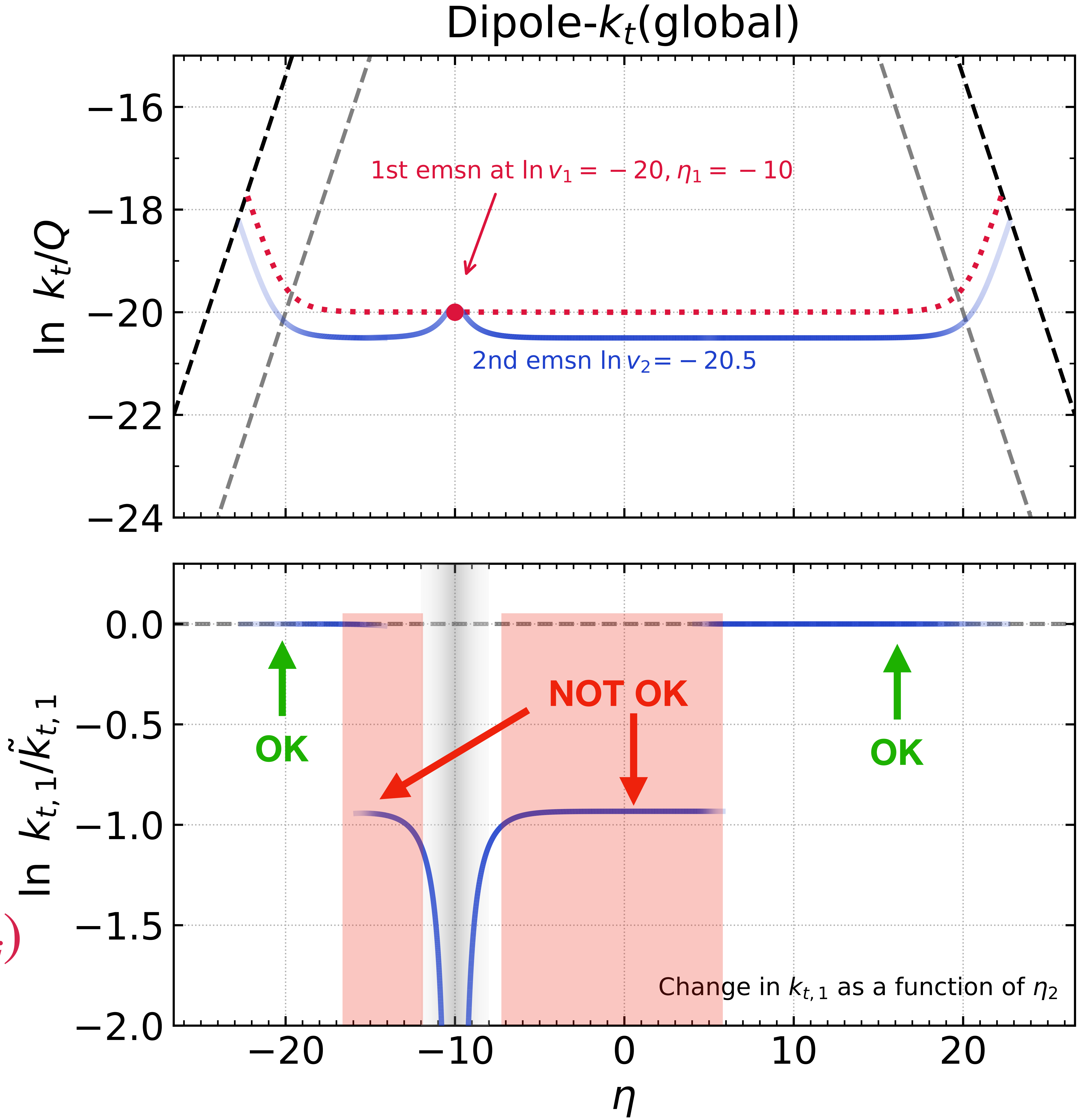


Direct consequence of CM dipole separation



Wrong in rapidity region

$$\frac{1}{2}\left(\eta_1 + \ln\frac{k_{t_1}}{Q}\right) < \eta_2 < \frac{1}{2}\left(\eta_1 - \ln\frac{k_{t_1}}{Q}\right)$$



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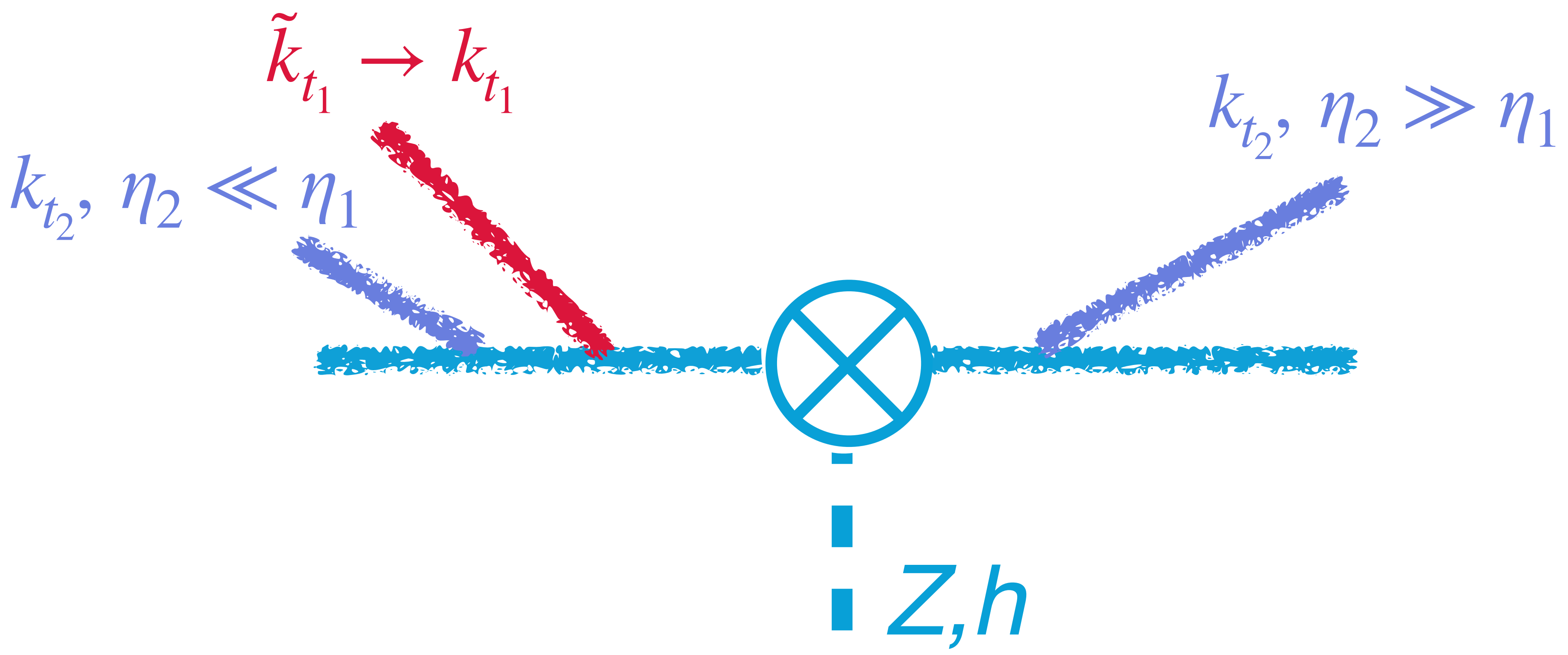

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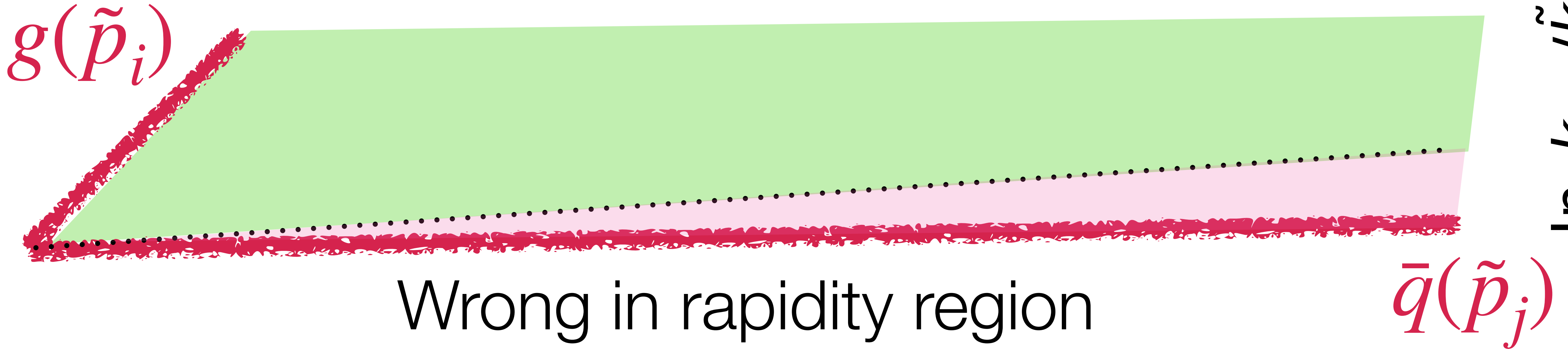

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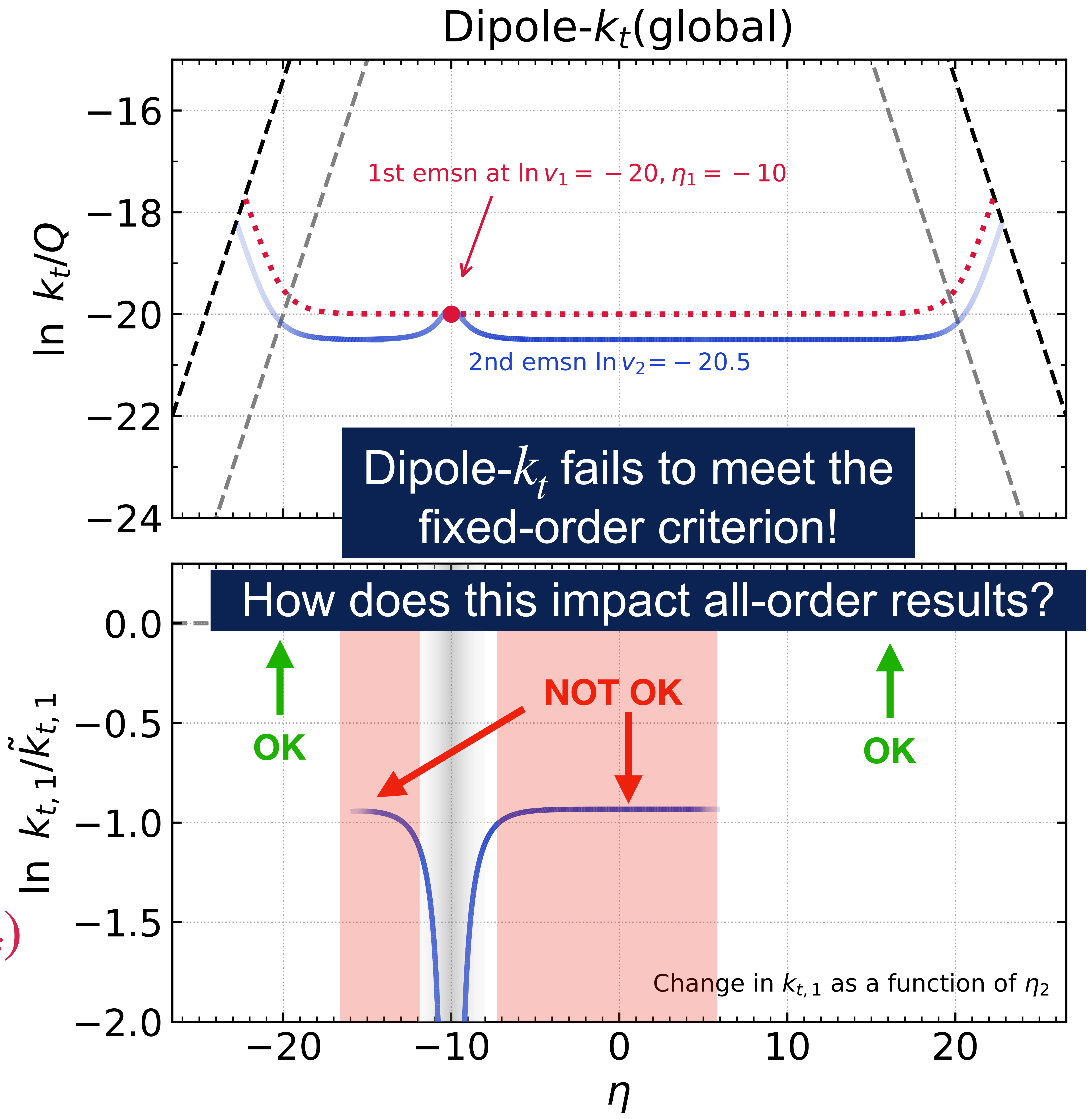


What is the all-order consequence?

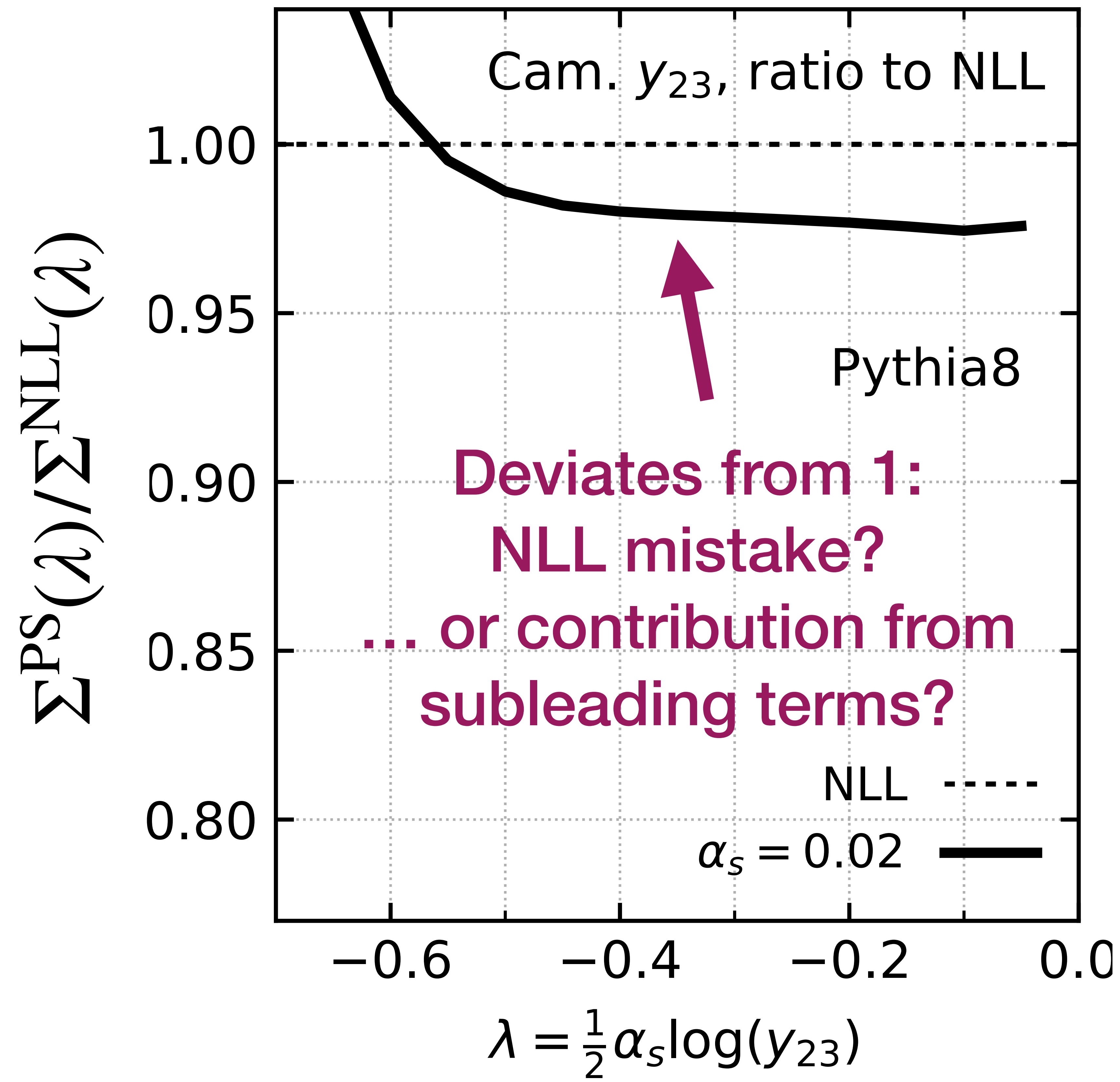
Consider e.g. Cambridge y_{23}

Observable with standard resummation at NLL of the form

$$\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s) = \exp\left[-Lg_1(\lambda) + g_2(\lambda)\right]$$

with $\lambda = \alpha_s \ln \sqrt{y_{23}}$

Tested by taking $\dfrac{\Sigma^{\mathrm{PS}}(\alpha_s L)}{\Sigma^{\mathrm{NLL/NDL}}(\alpha_s L)}$?



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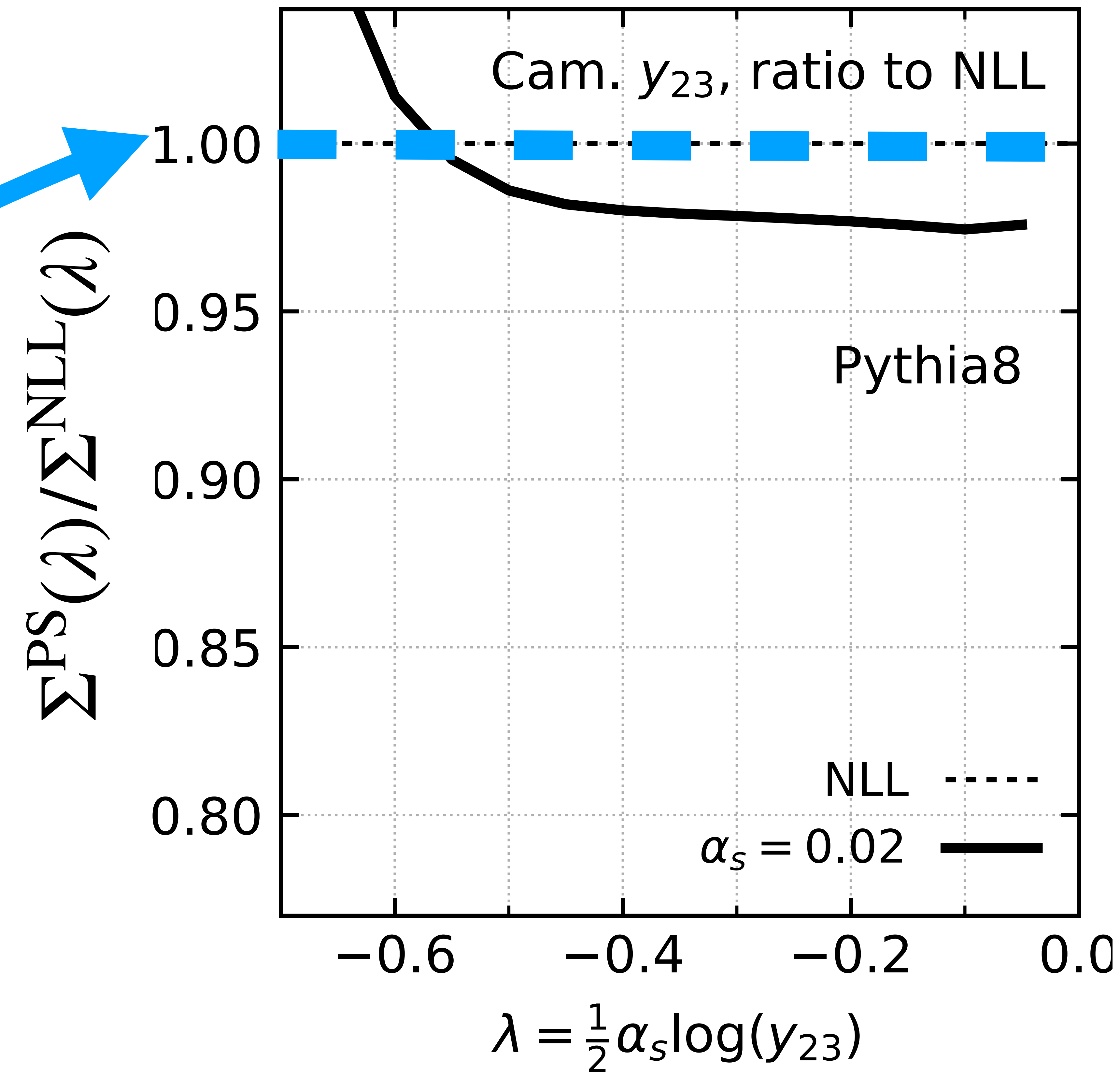
~~**Tested by taking** $\frac{\Sigma^{\mathrm{PS}}(\alpha_s L)}{\Sigma^{\mathrm{NLL/NDL}}(\alpha_s L)}$ **?**~~

Tested by taking $\lim_{\alpha_s \to 0} \frac{\Sigma^{\mathrm{PS}}(\alpha_s L)}{\Sigma^{\mathrm{NLL/NDL}}(\alpha_s L)}$

Should tend to 1 if the shower is NLL



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Tested by [redacted] $\frac{[redacted]}{\Sigma^{\rm NLL/NDL}(\alpha_s L)}$ **?**

Tested by taking $\lim_{\alpha_s \to 0} \frac{\Sigma^{\rm PS}(\alpha_s L)}{\Sigma^{\rm NLL/NDL}(\alpha_s L)}$

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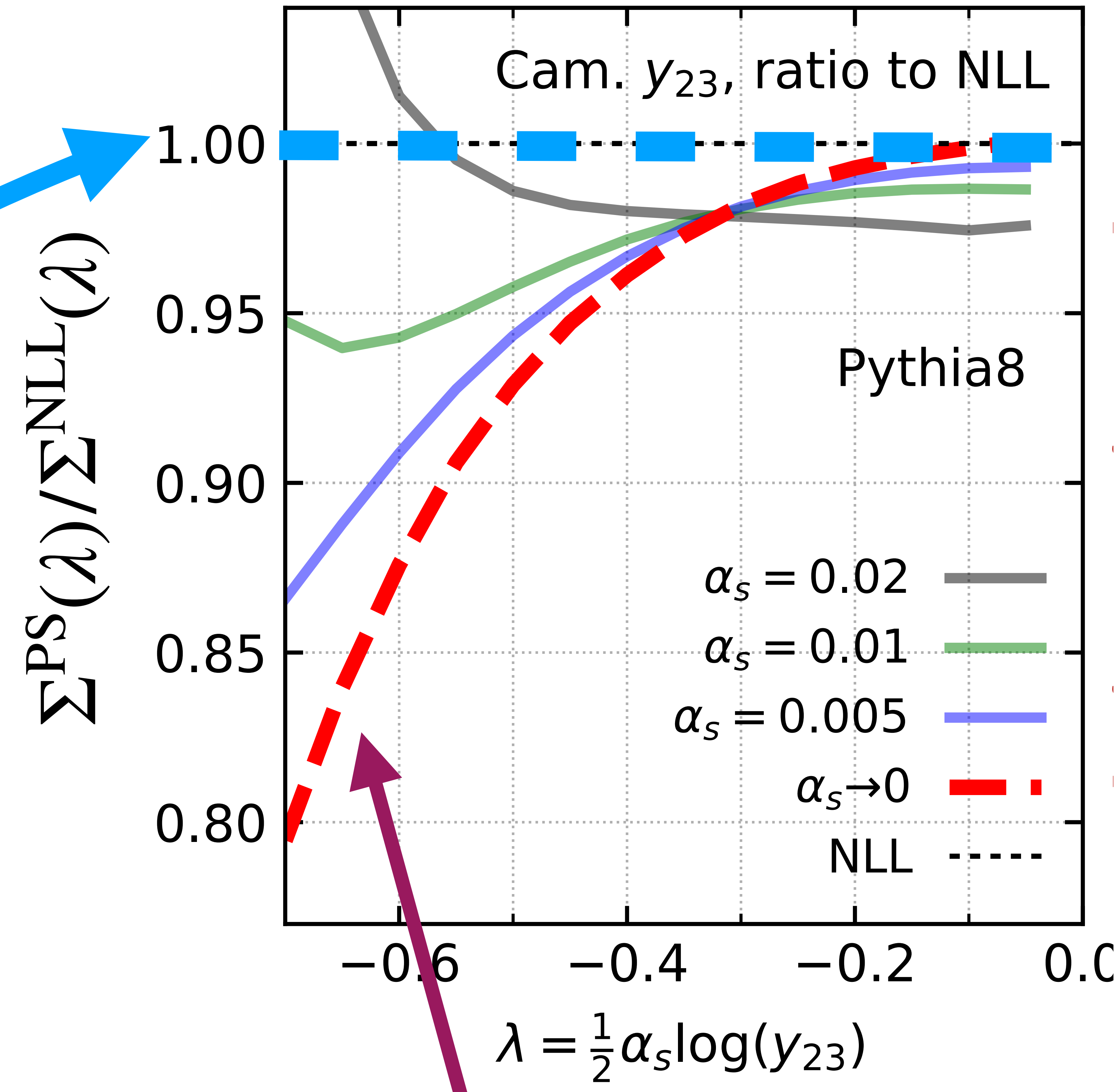
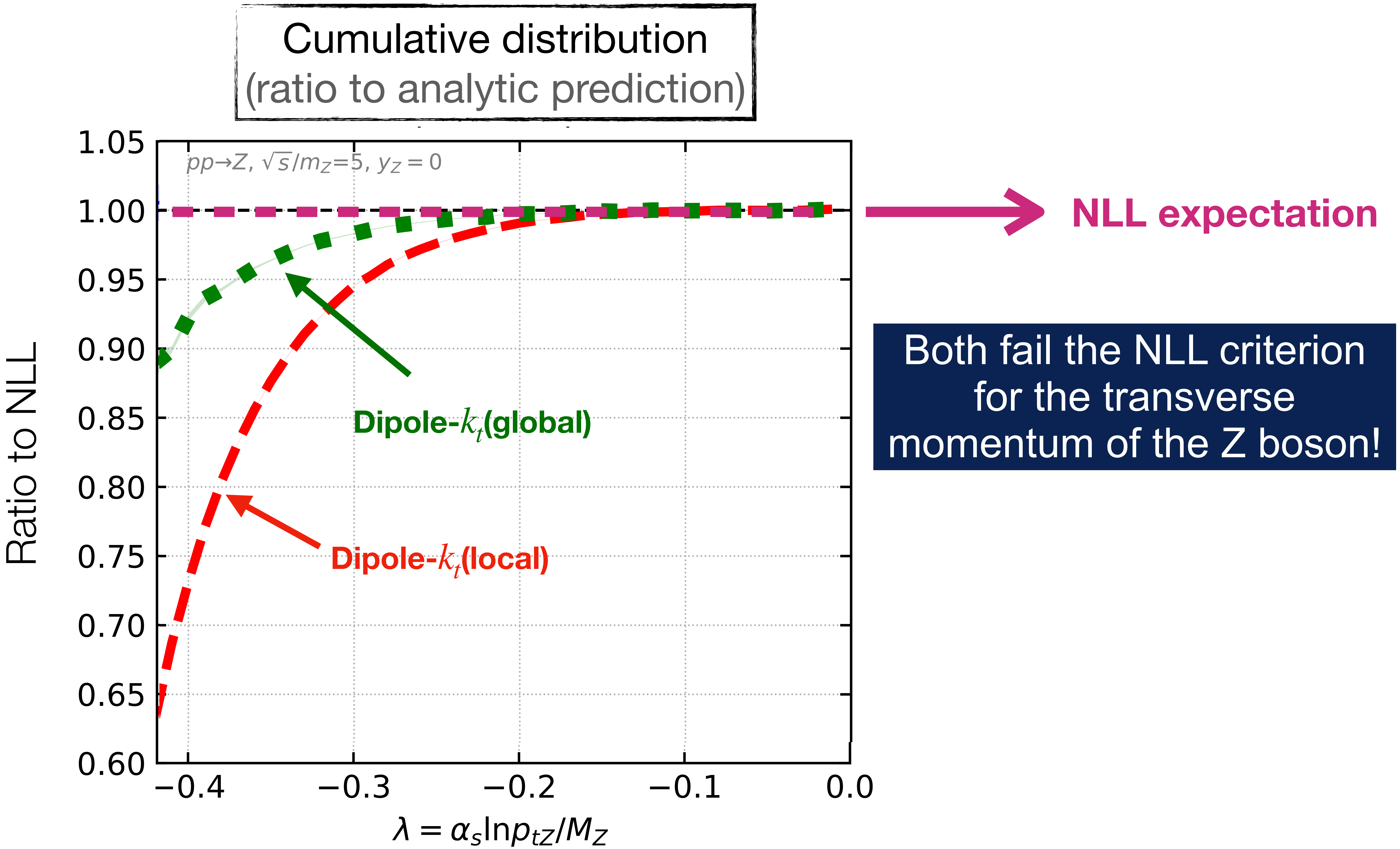


Clear deviation from 1 in the $\alpha_s \to 0$ limit!

Transverse momentum of the Z boson



Both fail the NLL criterion for the transverse momentum of the Z boson!

Introducing NLL-accurate showers

PanGlobal

1. Evolution variable
   $v \simeq k_t e^{-\beta_{PS}|\eta|}$ with $0 \leq \beta_{PS} < 1$
   ($\beta_{PS} = 0$ is standard k_t-ordering)

2. Kinematic map
   Global $\perp$
   Local $+/-$
   Transverse-momentum imbalance is absorbed by the hard system (Z/h)

3. Attribution of recoil
   hard-system CM frame

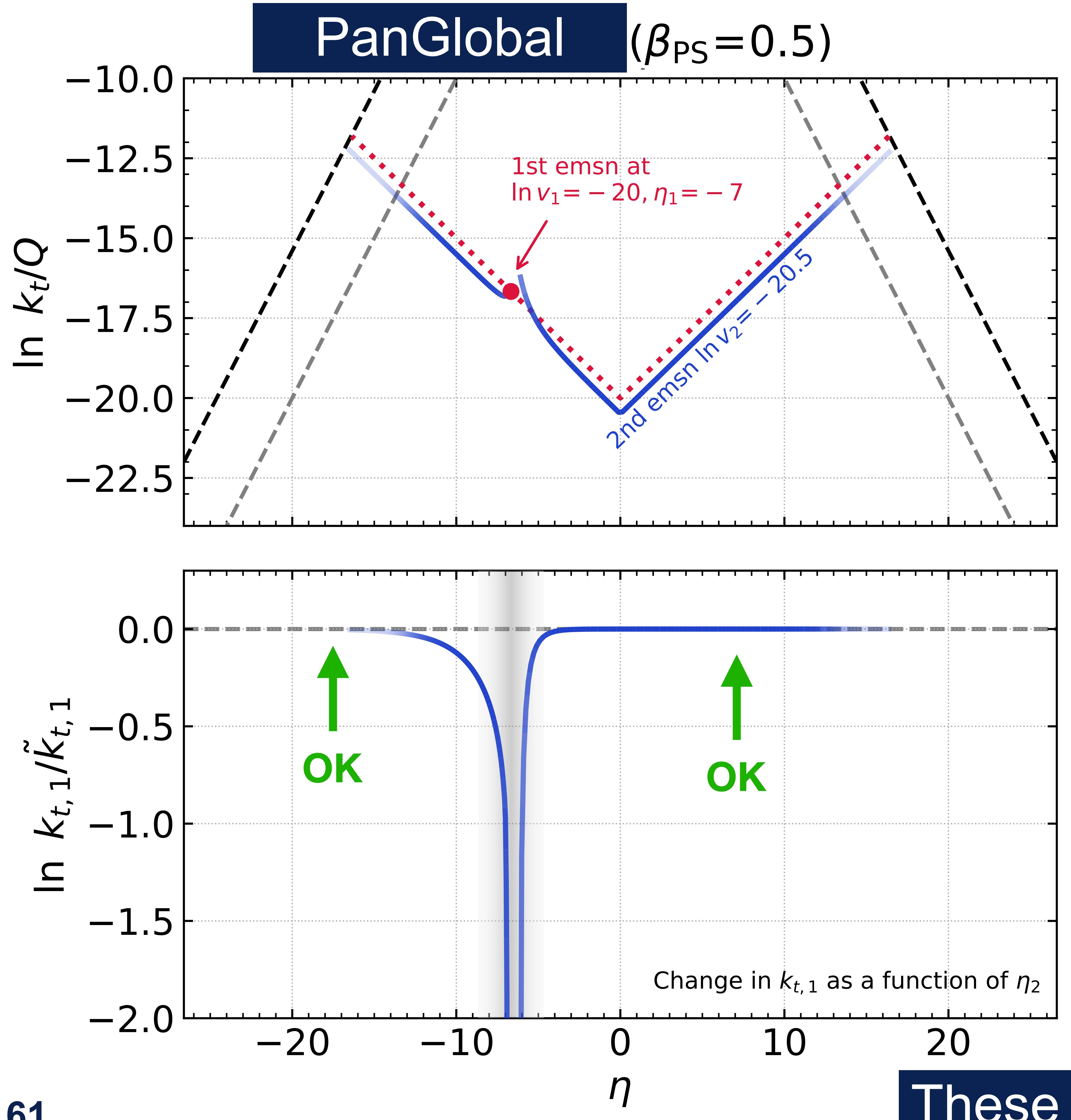
PanLocal

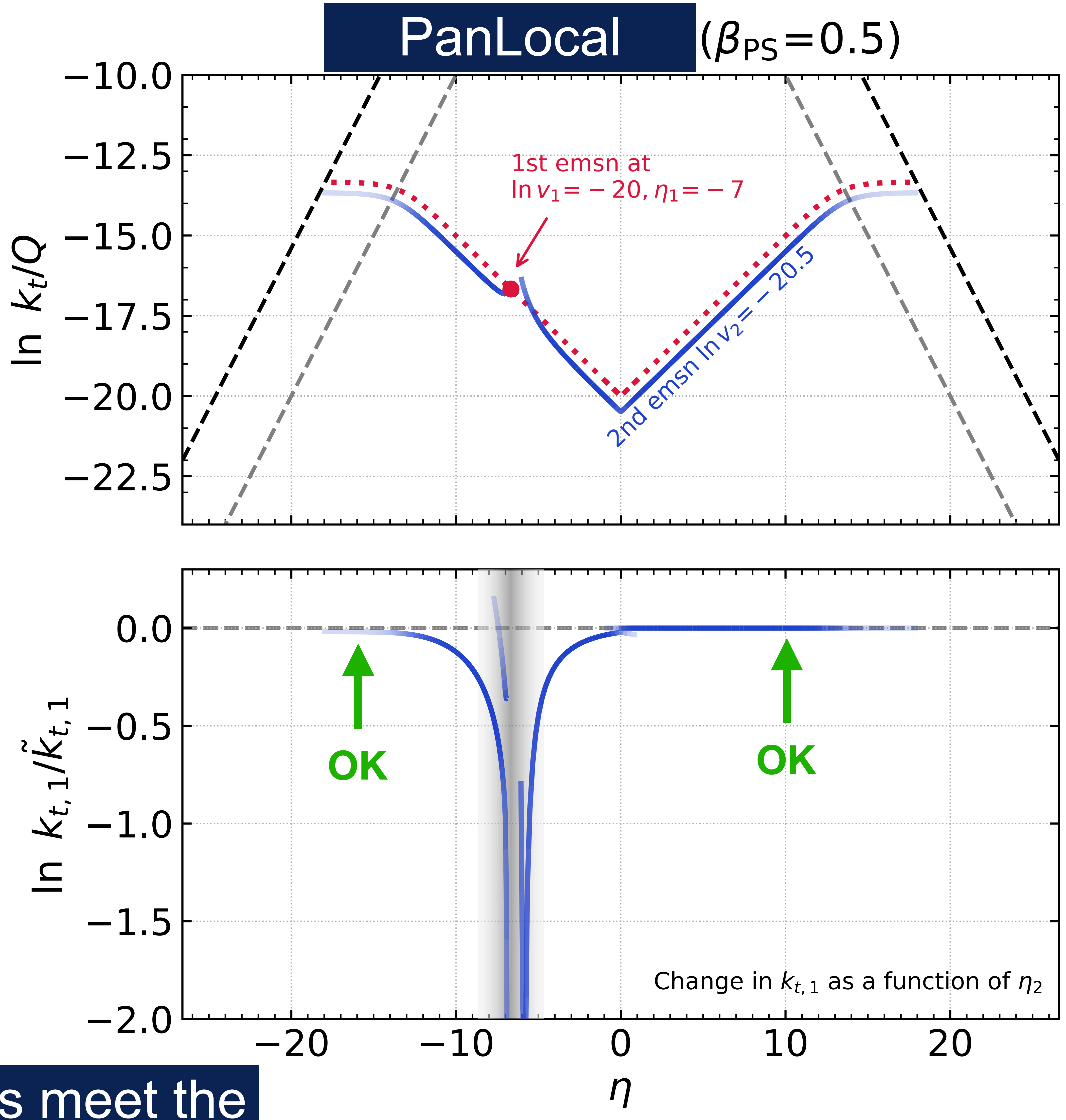
1. Evolution variable
   $v \simeq k_t e^{-\beta_{PS}|\eta|}$ with $0 < \beta_{PS} < 1$

2. Kinematic map
   Local $\perp$
   Local $+/-$
   Initial-state particles that gain a k_t component are realigned with the beam axis with a boost

3. Attribution of recoil
   hard-system CM frame

Introducing NLL showers: PanGlobal and PanLocal

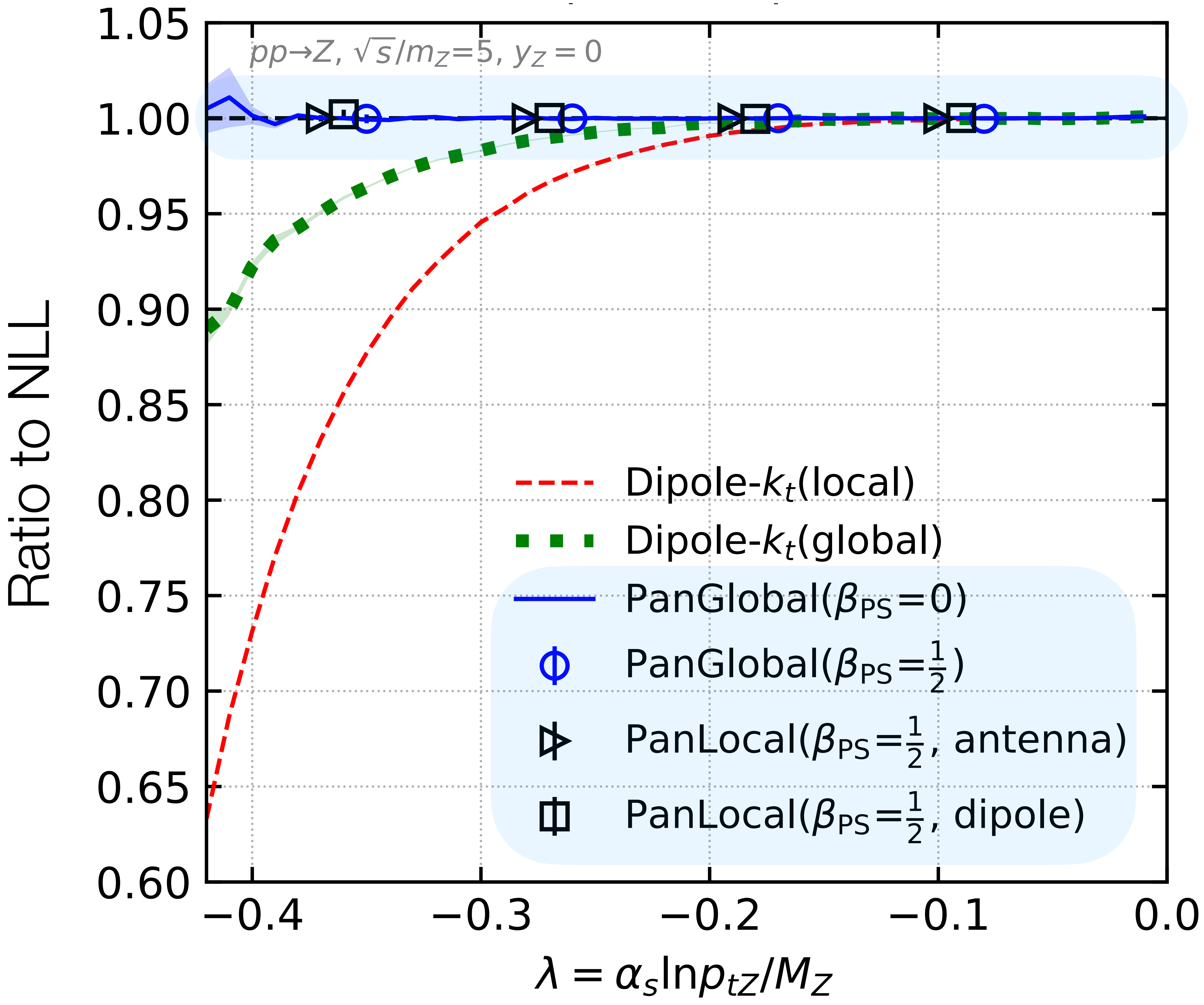




These showers meet the fixed-order criterion

Transverse momentum of the Z boson

Cumulative distribution
(ratio to analytic prediction)



In line with NLL prediction

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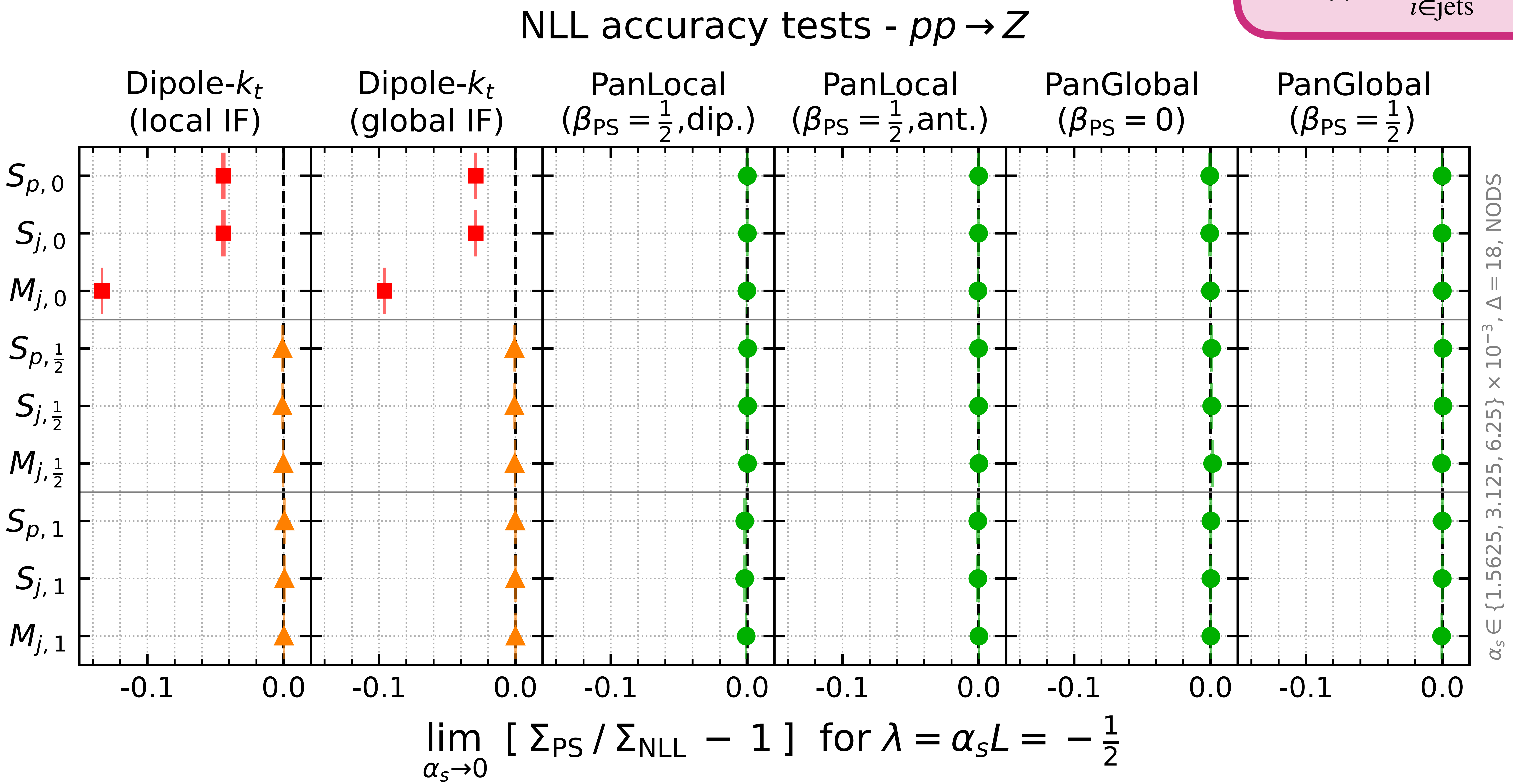
General global observables

$$S_{p/j,\beta} = \sum_{i \in f/\text{jets}} p_{\perp,i}\, e^{-\beta|\eta_i|}$$

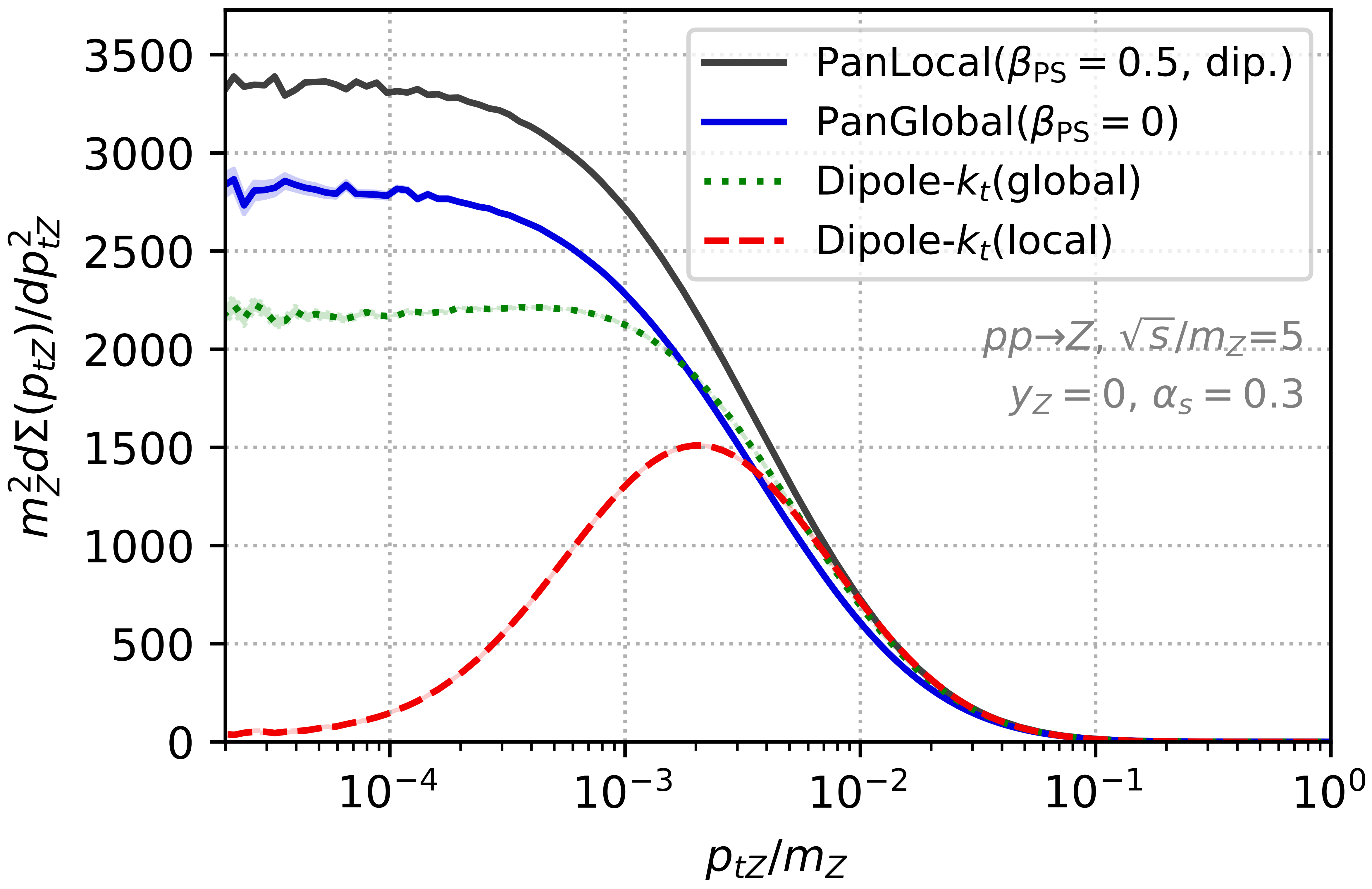
$$M_{j,\beta} = \max_{i \in \text{jets}} \left[p_{\perp,i}\, e^{-\beta|\eta_i|}\right]$$



Transverse momentum of the Z boson

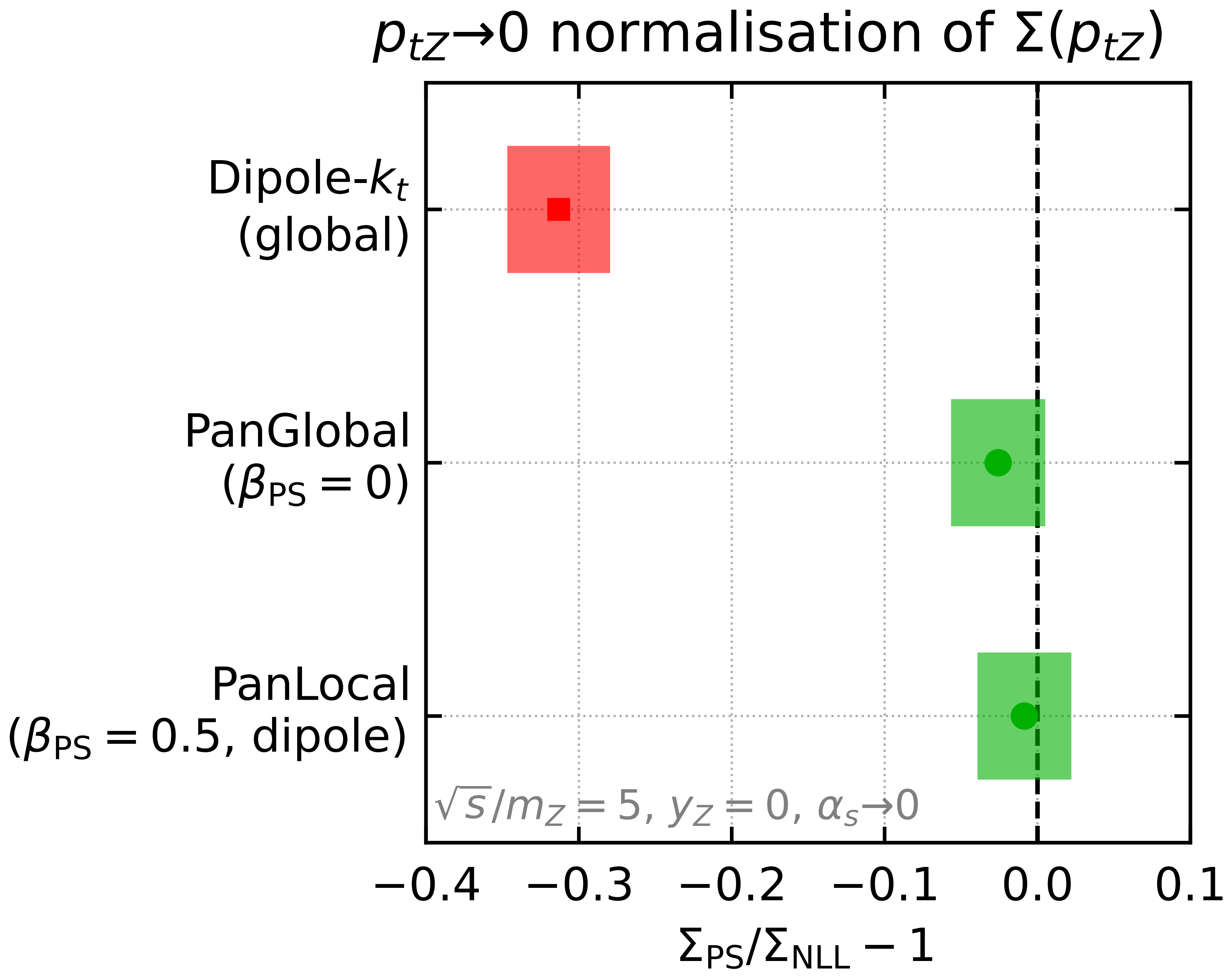
Scaling at small p_t

The Sudakov suppression is compensated by azimuthal cancellations at small p_t
Leads to a power-law fall-off



Parisi, Petronzio [NPB 154 (1979) 427-440]

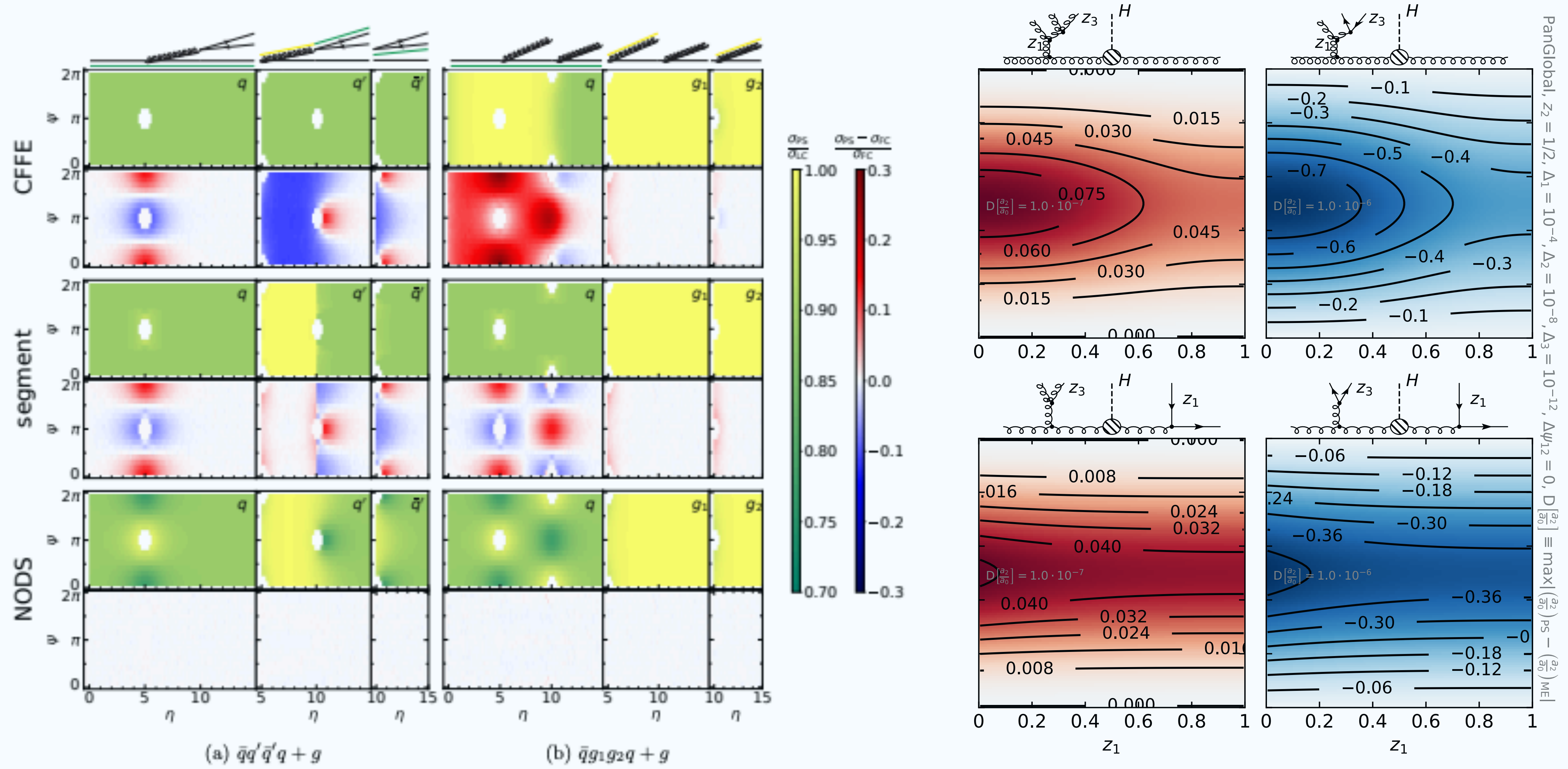
$$\frac{d\Sigma}{dp_{tZ}^2} = \int_0^\infty \frac{db}{2} b\, J_0(b p_{tZ})\, \Sigma_V(b_0/b)$$



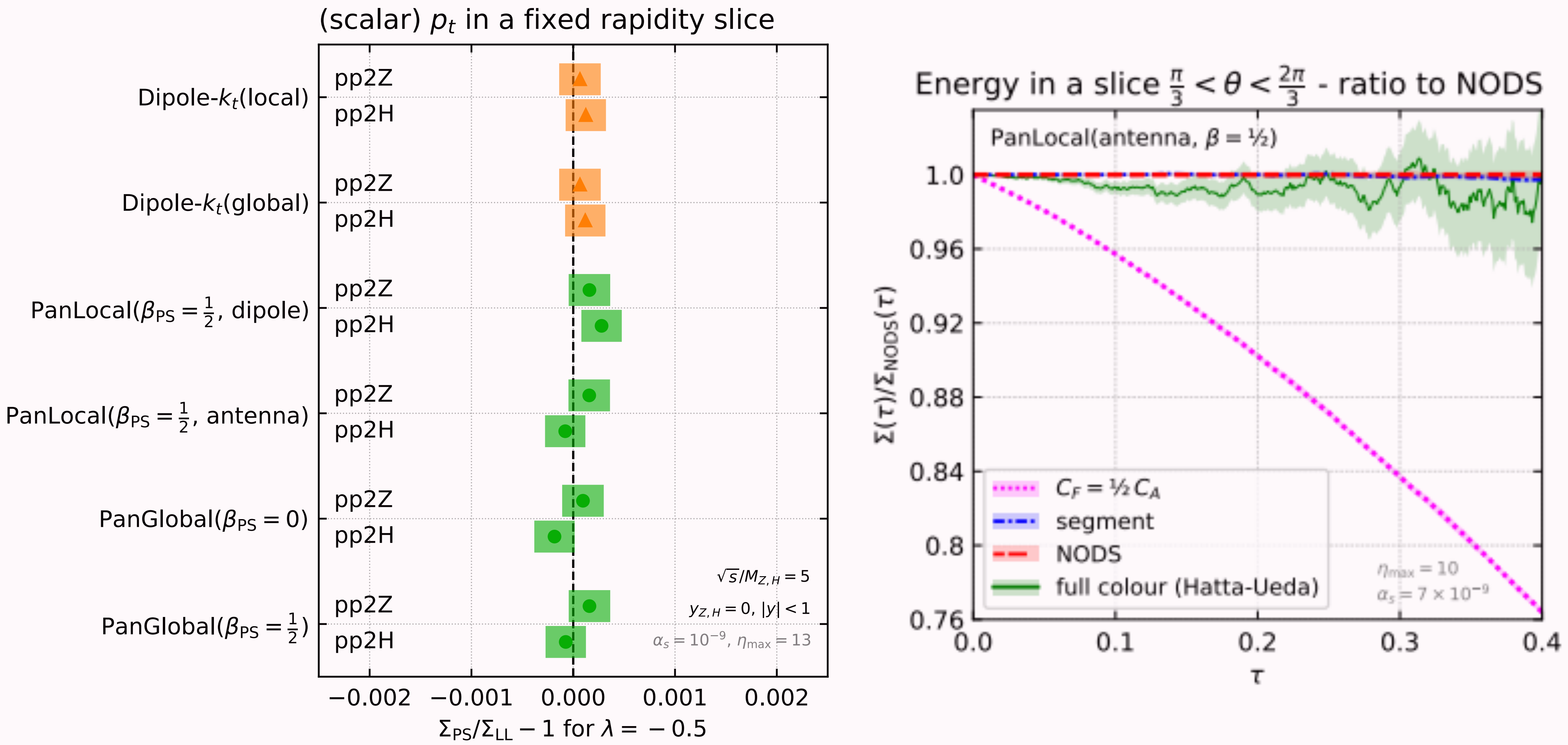
But there is more to test!

[2002.11114, 2103.16526, 2011.10054, 2111.01161, 2205.02237, 2207.09467]

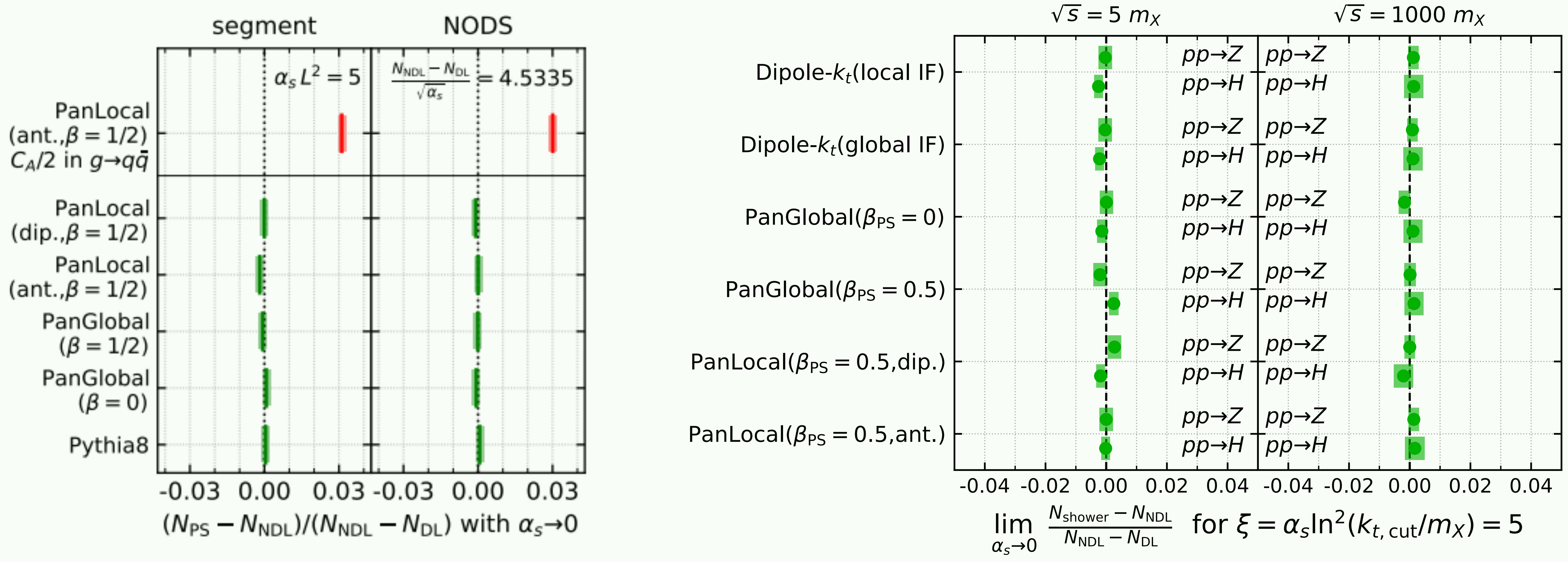
Fixed-order checks



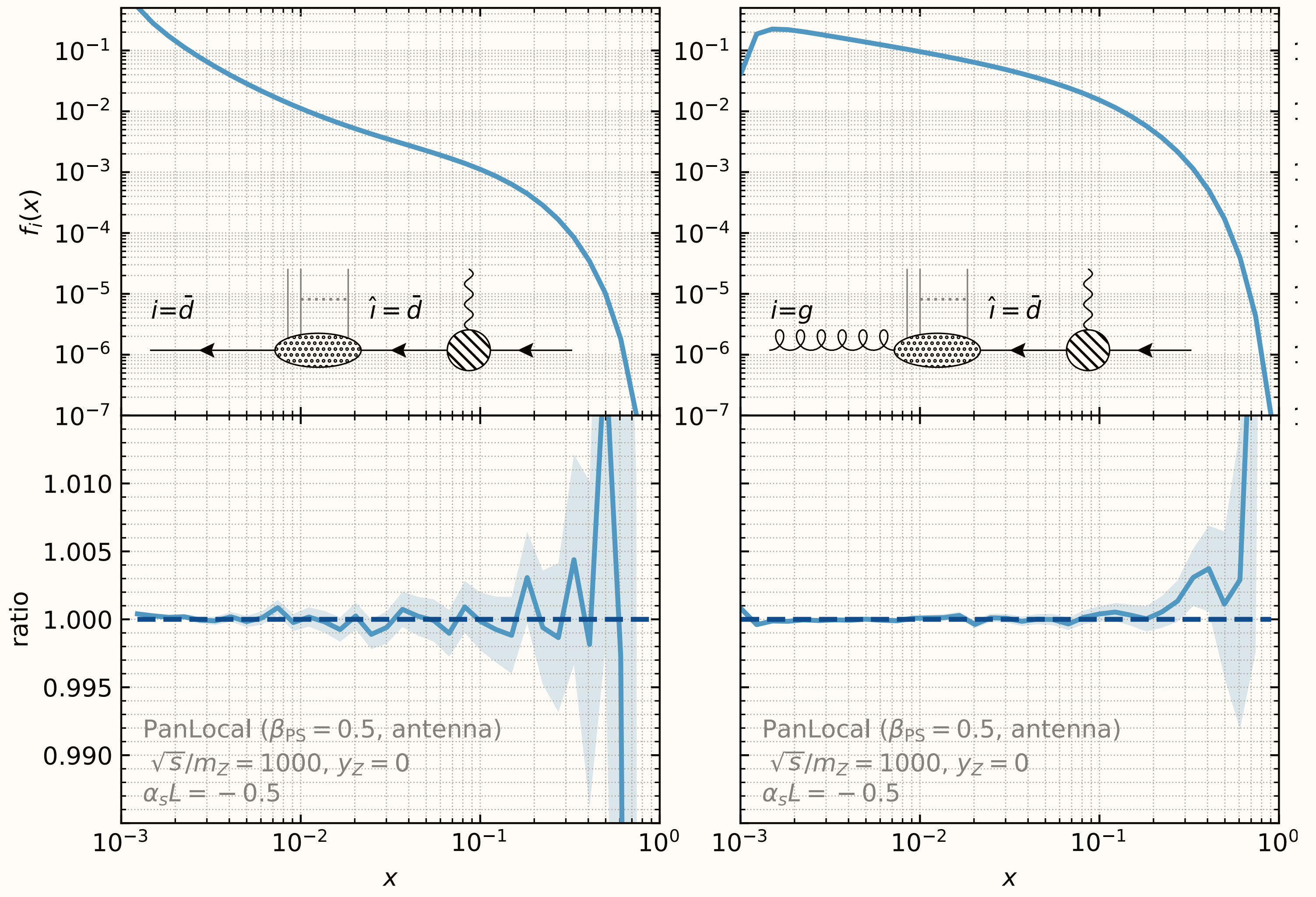
Non-global observables



Multiplicity



DGLAP evolution



Overview of other NLL dipole-shower solutions

		Ordering	Kinematic map: Dipole-local	Kinematic map: Global	Tests
PanScales showers [2002.11114, 2207.09467, 2305.08645]	**PanLocal** (Dipole and antenna)	$0 < \beta < 1$	$+, -, \perp$		Fixed- and all-order numerical tests for different observables for e^+e^-, DIS and pp (colour singlet)
	PanGlobal	$0 \leq \beta < 1$	$+, -$	$\perp$	
Alaric [2208.06057, 2307.00728]		$\beta = 0$	$+$	$-, \perp$	Analytical & numerical tests for global event shapes, massive maps implemented
Deductor [2011.04777]	**Deductor k_t**	$\beta = 0$	$+$ (Also formulation with $+, -, \perp$)	$-, \perp$	Analytical and to some extent numerical for thrust
	Deductor Λ	$\beta = 1$	$+$	$-, \perp$	
Manchester-Vienna [2003.06400]		$\beta = 0$	$+$	$-, \perp$	Analytical for thrust and multiplicity

Showers also differ on the implementation of the splitting functions and how the global imbalance is redistributed

Towards phenomenology

Towards phenomenology

Results up to now shown in asymptotic limit - *what happens at physical scales?*

Renormalisation scale uncertainty implemented through

$$\alpha_s(x_r\mu_{r,0})\left(1+\frac{K\alpha_s(x_r\mu_{r,0})}{2\pi}+2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

with $\mu_{r,0} = k_{t,\text{approx}},\ x_r \in \left[\frac{1}{2},\ 1,\ 2\right]$

Towards phenomenology

Results up to now shown in asymptotic limit - *what happens at physical scales?*

Renormalisation scale uncertainty implemented through

$$\alpha_s(x_r\mu_{r,0})\left(1+\frac{K\alpha_s(x_r\mu_{r,0})}{2\pi}+2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

Usual shower emission strength

Towards phenomenology

Results up to now shown in asymptotic limit - *what happens at physical scales?*

Renormalisation scale uncertainty implemented through

$$\alpha_s(x_r\mu_{r,0})\left(1+\frac{K\alpha_s(x_r\mu_{r,0})}{2\pi}+2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

Include if NLL shower

Factor $(1-z)$ *ensures this is only active for soft emissions*

Towards phenomenology

Results up to now shown in asymptotic limit - *what happens at physical scales?*

Renormalisation scale uncertainty implemented through

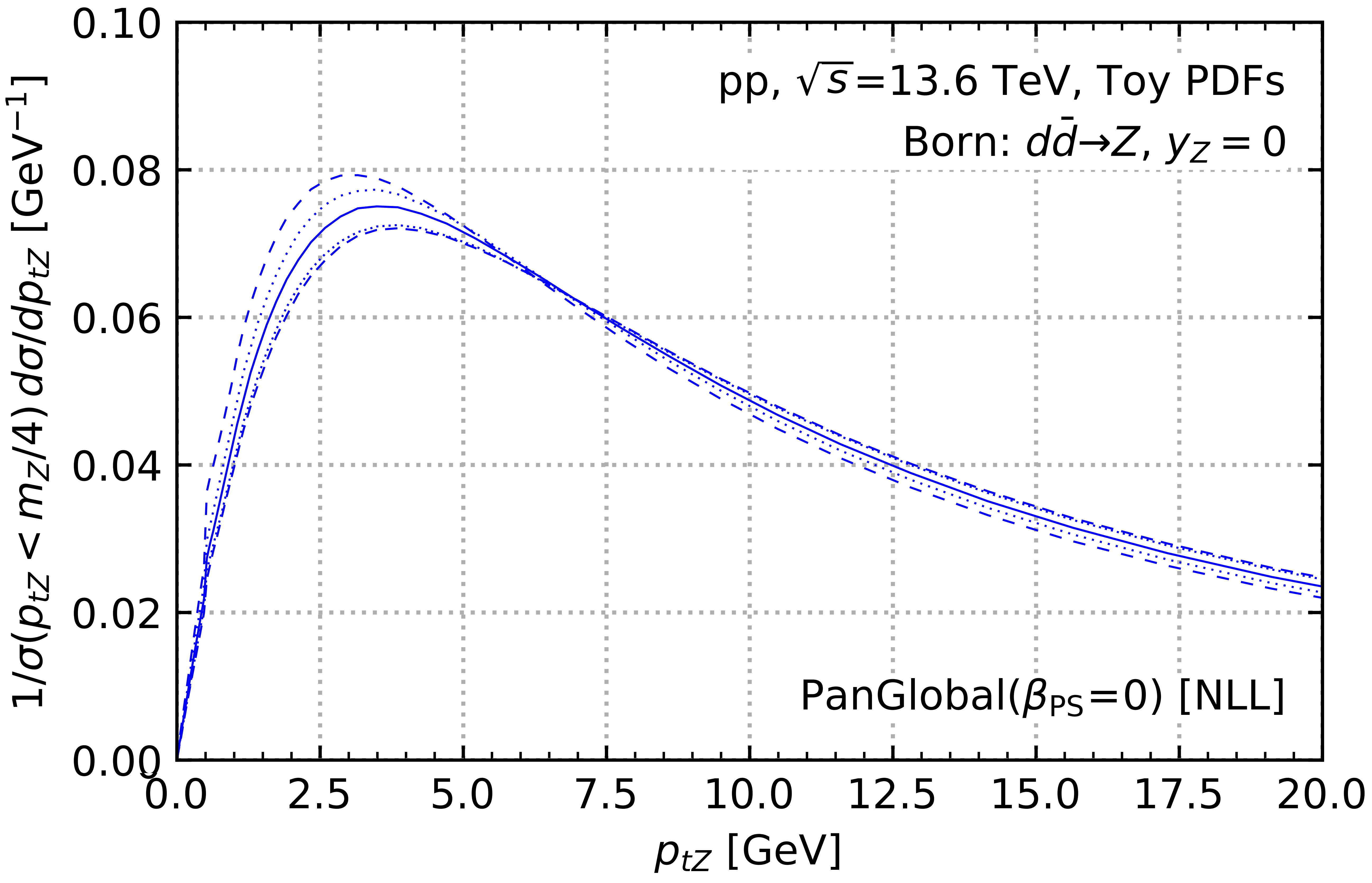
$$\alpha_s(x_r\mu_{r,0})\left(1+\frac{K\alpha_s(x_r\mu_{r,0})}{2\pi}+2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

Factorisation scale uncertainty implemented through

$$\mu_F = x_f\mu_{F,0} = x_f\,Q\left(\frac{v}{Q}\right)^{1/(1+\beta)}$$

Take $x_f \in \left[\frac{1}{2},\,1,\,2\right]$

Towards phenomenology - Z pt distribution



Caveats:

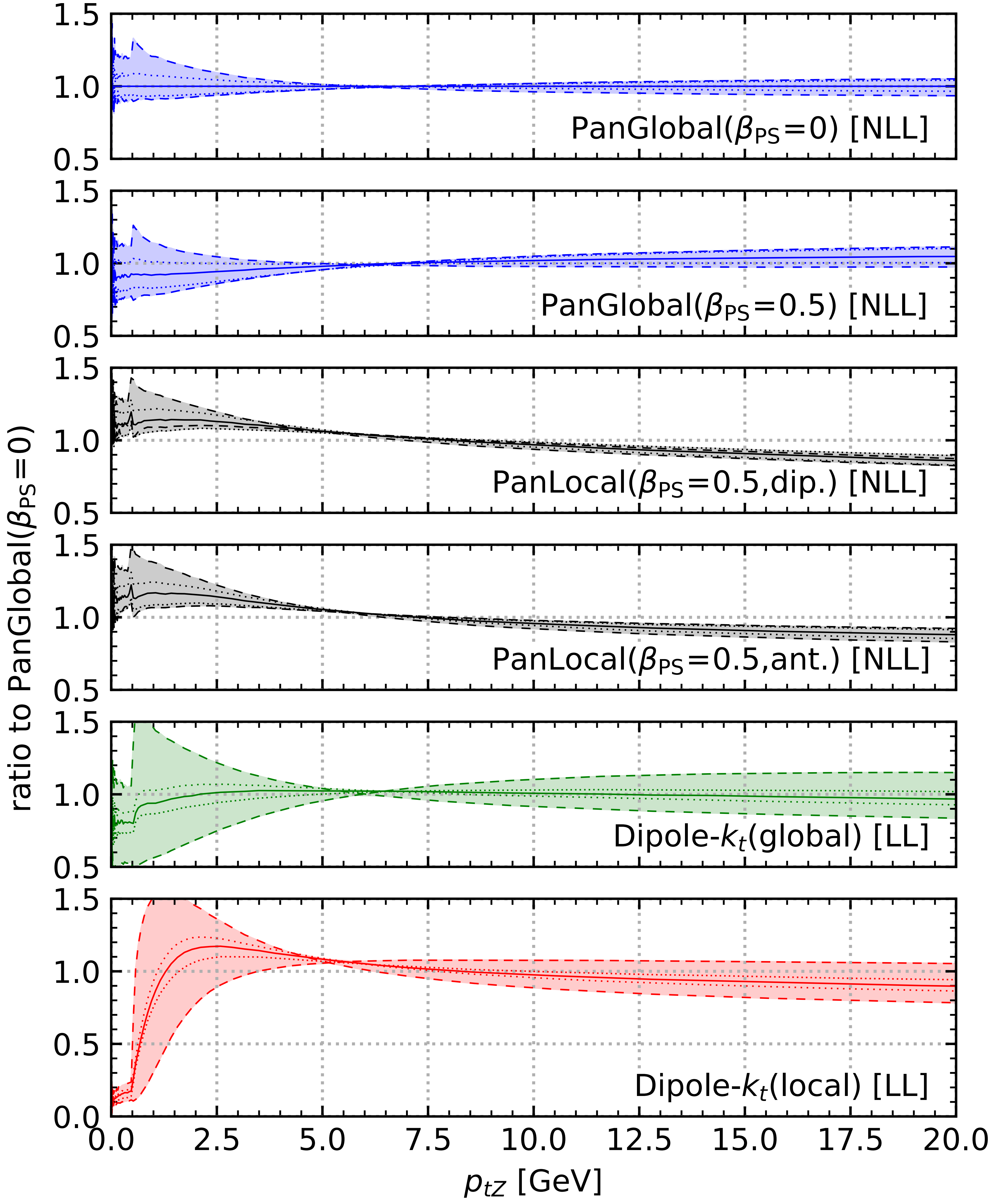
We use a fixed underlying Born event

We use 5-flavour toy PDF set

No quark-mass / flavour thresholds

No hadronisation / underlying event

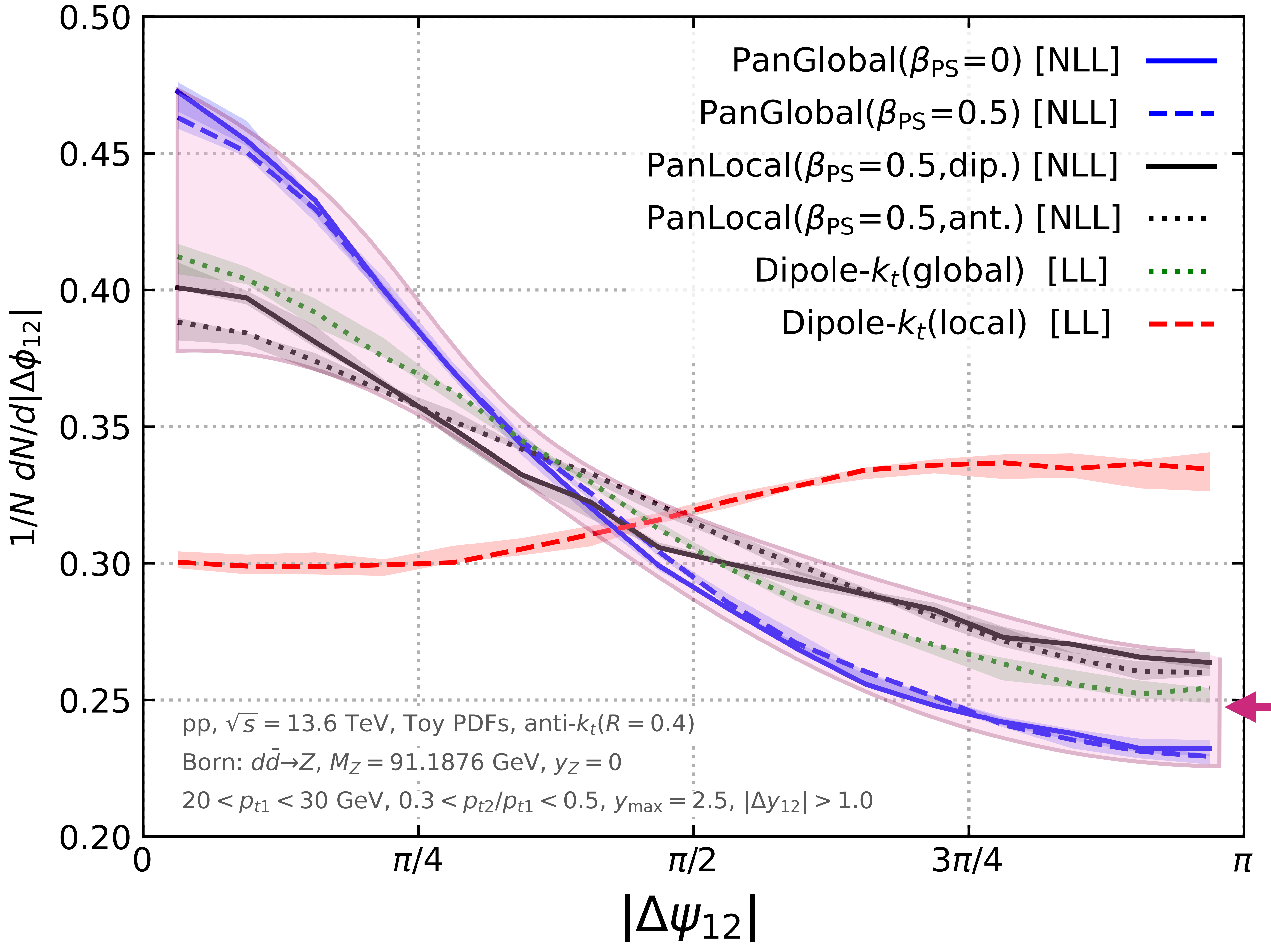
Towards phenomenology - Z pt distribution

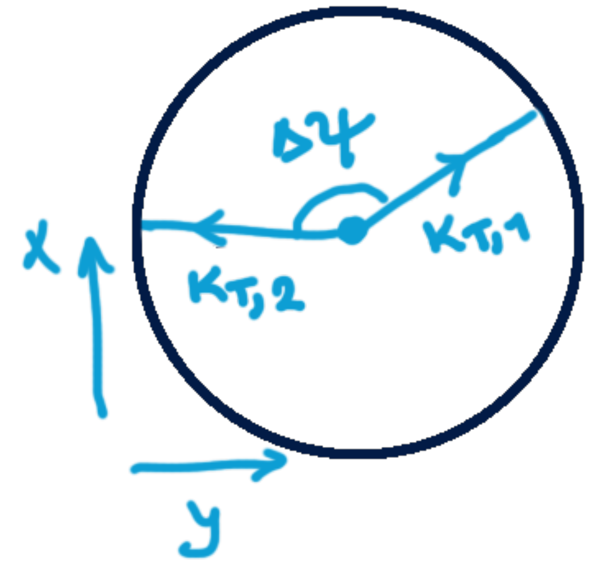


- Scale uncertainty LL showers > NLL showers
- Differences between the NLL showers - consequence of different treatment of beyond-NLL terms
- Dipole-kt(local) shows different scaling behaviour in low-pt region
- Dipole-kt(global) similar to PanGlobal

Differences are relatively small except at very small p_{tZ} (related to the absence of azimuthal cancelations)

Towards phenomenology - $\Delta\Psi_{12}$

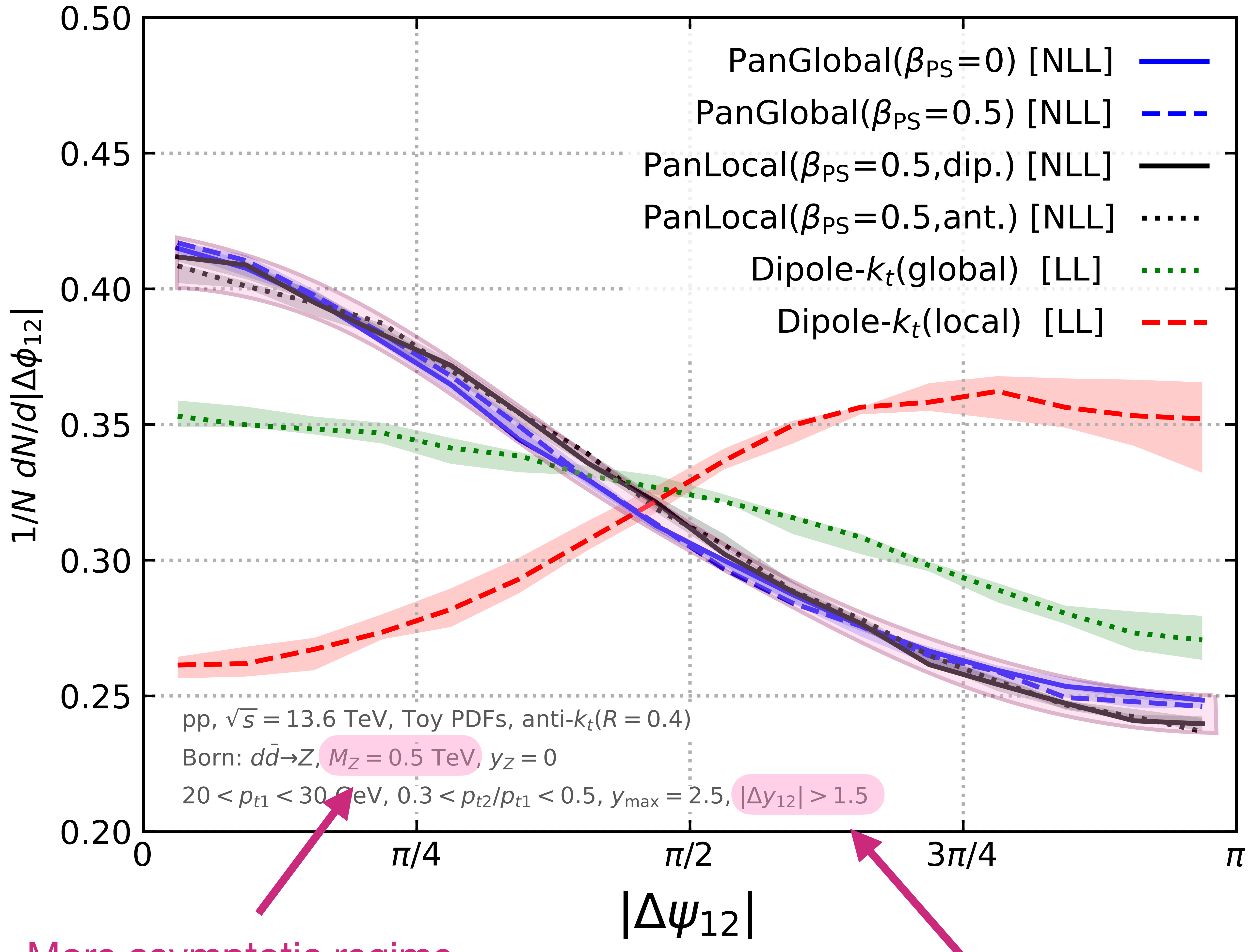


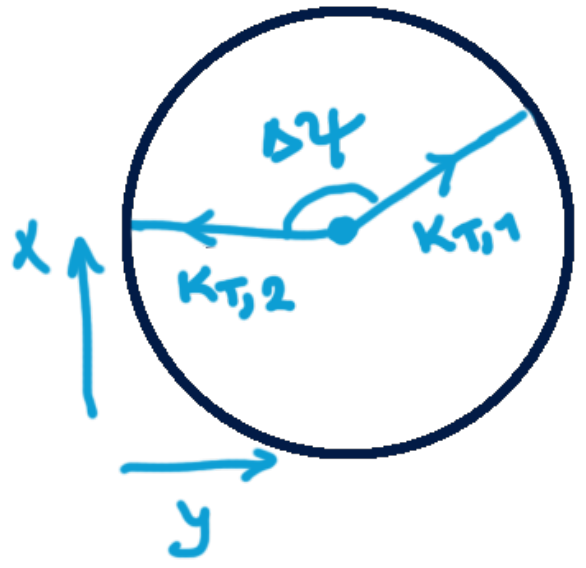


Spread of NLL showers
(Dipole-kt global is contained)

[2207.09467]

Towards phenomenology - $\Delta\Psi_{12}$





Dipole-kt global now falls outside the spread

More asymptotic regime

Less double-soft contamination

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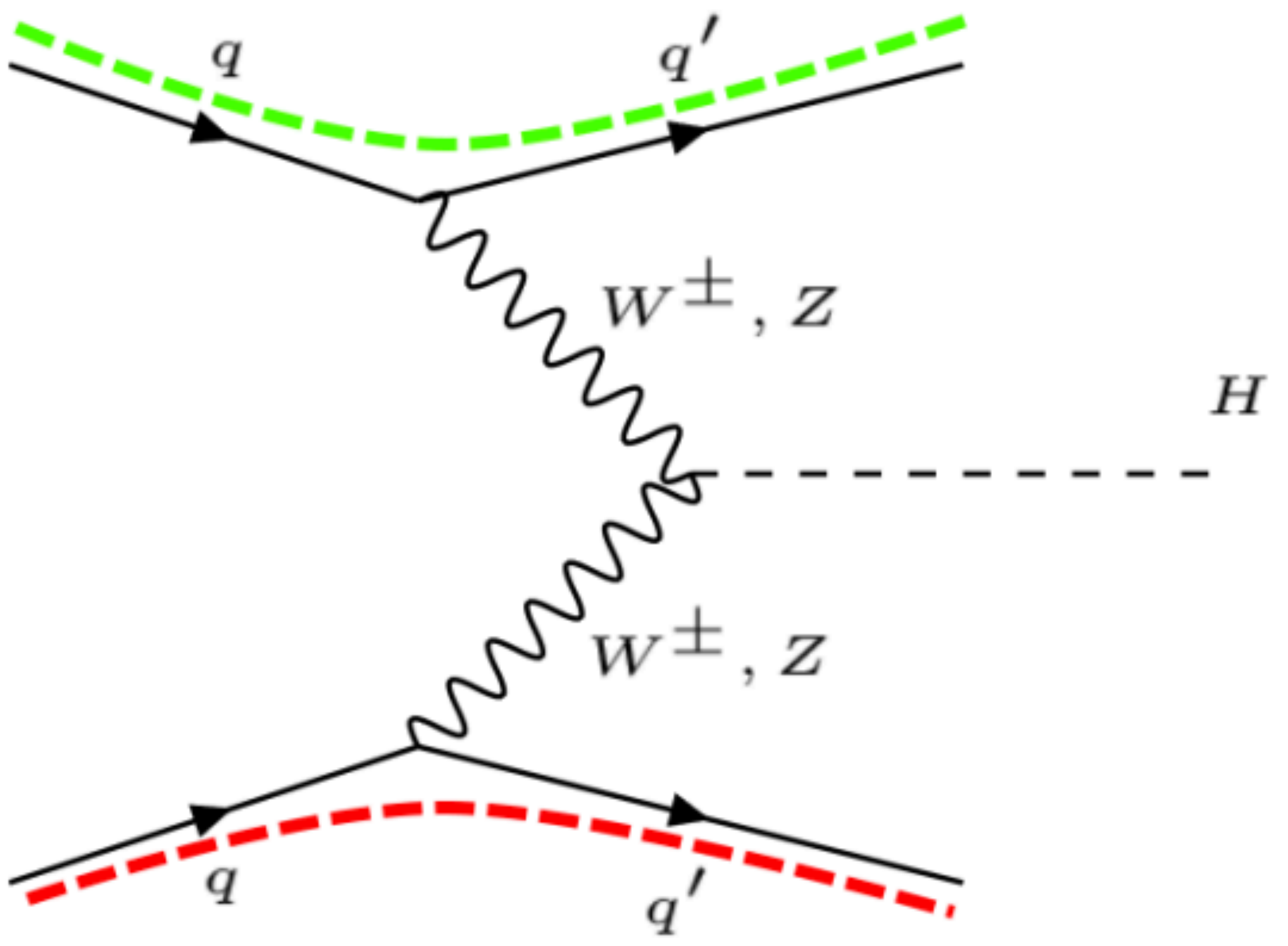
[2207.09467]

Towards LHC phenomenology - VBF

- Hard process generated with Pythia at LO accuracy (no beam remnants, hadronisation or multi-parton interaction)
- NNPDF 4.0 LO PDF set
- Shower starting scale is set separately for the two DIS chains
- VBF cuts: at least two jets with $p_{T,j} > 25$ GeV, $|\eta_j| < 4.5$, $\Delta\eta_{j_1 j_2} > 4.5$, $\eta_{j_1}\eta_{j_2} < 0$, $m_{j_1 j_2} > 600$ GeV



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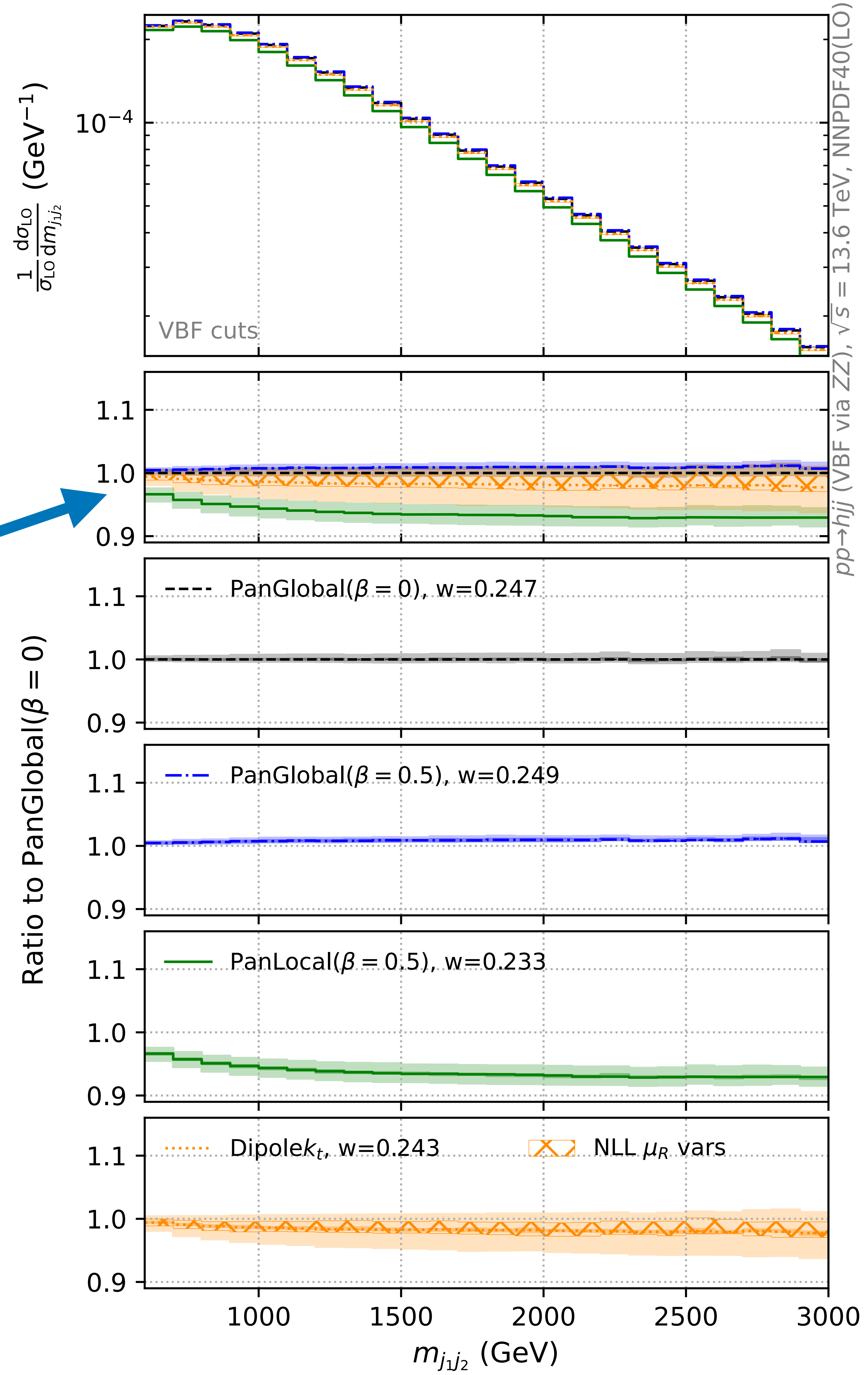
For observables that are non-vanishing at LO, the LL shower lies in-between the spread of NLL showers



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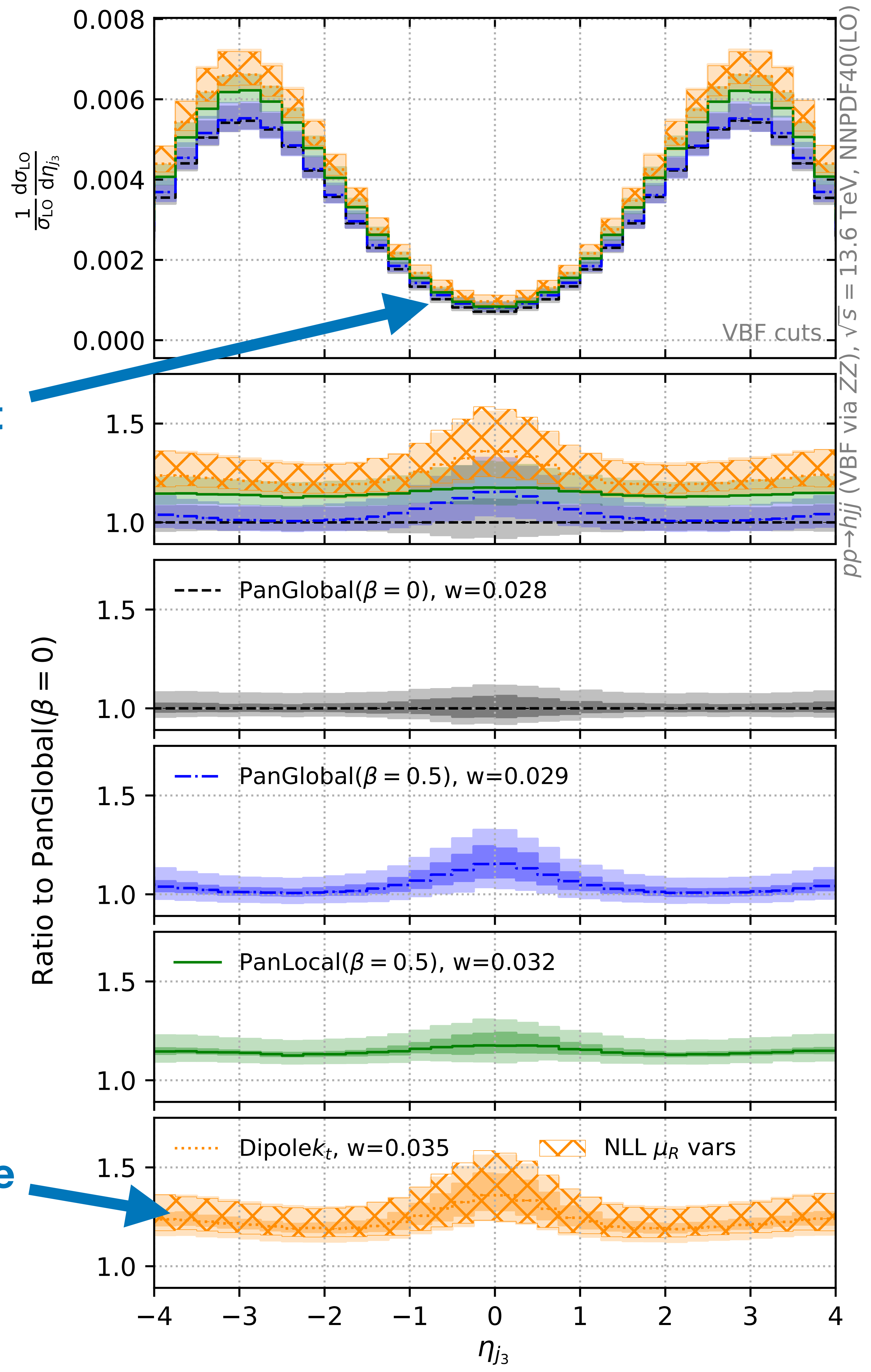
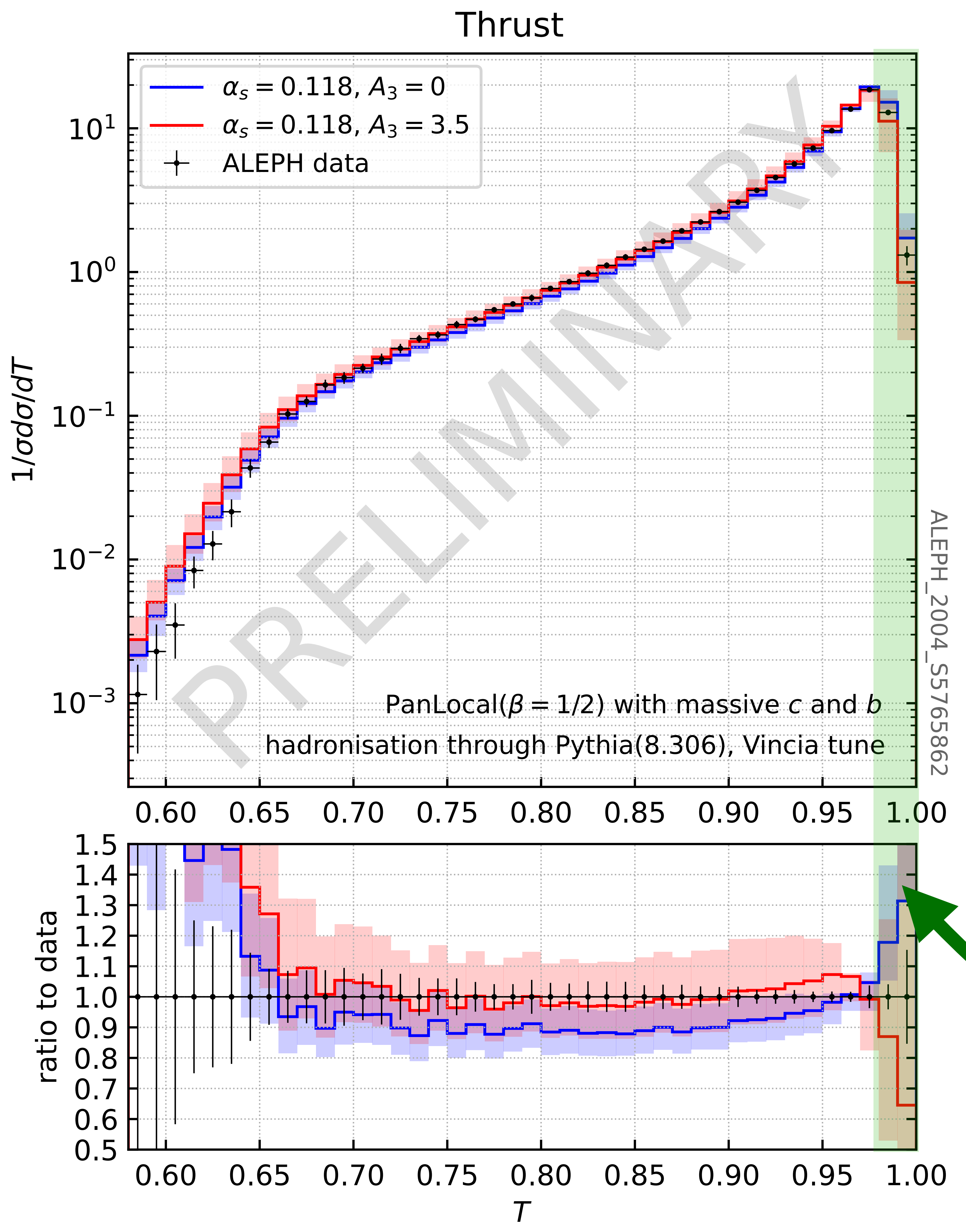
All feature right suppression in central rapidity region



Shows largest difference with the NLL showers

Towards LEP phenomenology



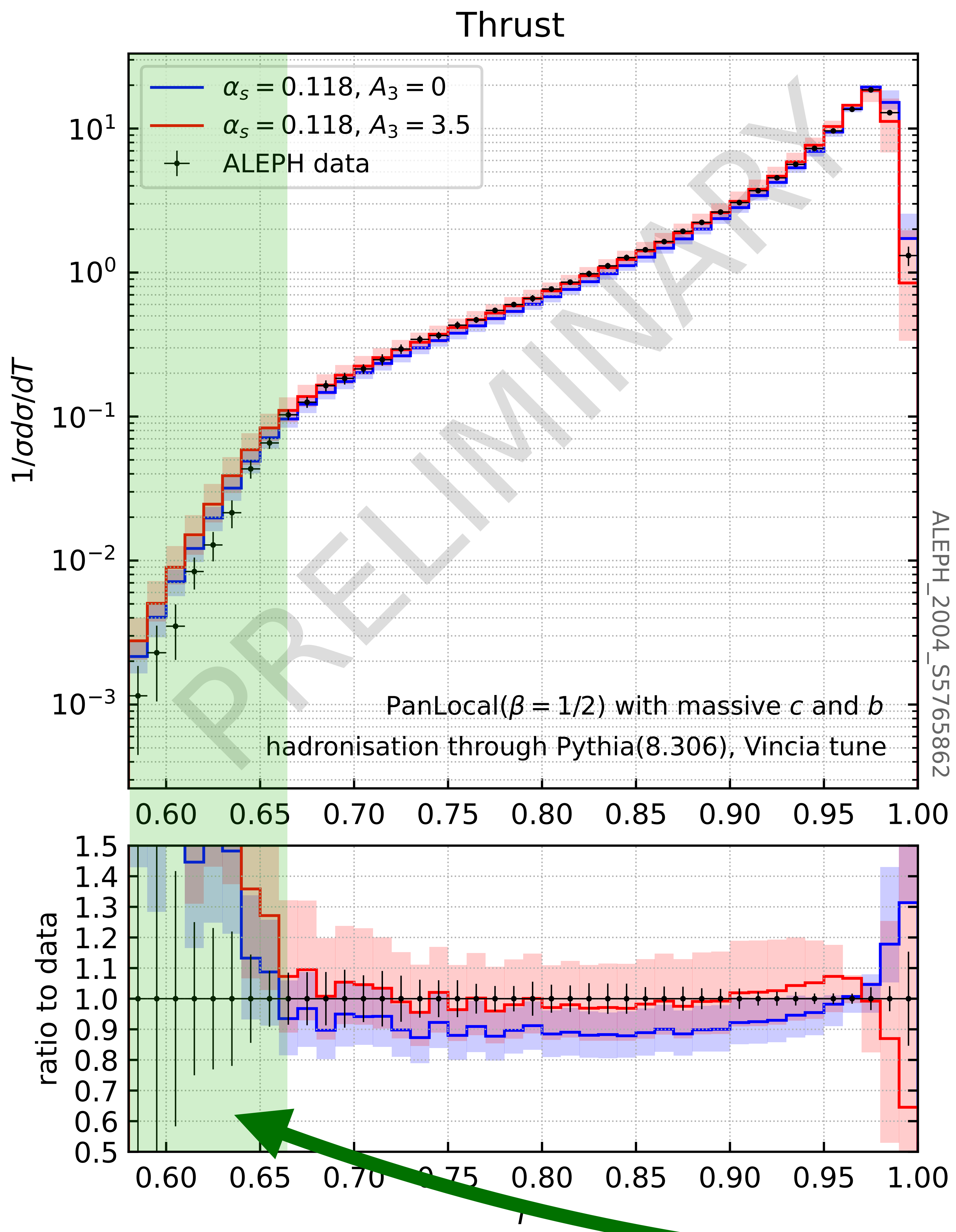
- PanLocal$(\beta = 0.5)$ dipole shower
- Heavy quarks ($m_c = 1.5$ GeV, $m_b = 4.8$ GeV)
- Matching to NLO
- Renormalisation-scale uncertainties included

$$\alpha_s^{(\mathrm{CMW})} = \alpha_s(x_r\mu_{r,0})\left(1 + \frac{K_{\mathrm{CMW}}\alpha_s(x_r\mu_{r,0})}{2\pi} + 2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

- Enhanced coupling - $\alpha_s = \alpha_s^{(CMW)} + A_3\alpha_s^3$
- Hadronisation from Pythia8 with the Vincia tune

Hadronisation region
(tuning of the shower is needed)

Towards LEP phenomenology



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- Renormalisation-scale uncertainties included

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- Enhanced coupling - $\alpha_s = \alpha_s^{(CMW)} + A_3\alpha_s^3$
- Hadronisation from Pythia8 with the Vincia tune

Poor description in the 4-jet region - need for 2-jet at NNLO?

Conclusions

- Parton showers will continue to play an indispensable role in any (future) particle physics experiment

- PanScales NLL showers for massless partons in e^+e^- (matched to NLO), pp and DIS collisions are now available
  - Next steps: NLO matching, including massive partons, processes with a complicated colour structure
  - Public code is coming soon! (timescale: ~2 months)

- Actively working towards NNLL showers
  - Double-soft emissions are under control [2307.11142]
  - Working towards a triple-collinear implementation [2307.15734]
  - We need to have reference calculations to check our shower e.g.
    - Next-to-leading non-global logarithms [2104.06416]
    - NNDL multiplicity [2205.0286]
    - NNLL groomed jet observables [2007.10355, 2211.03820]

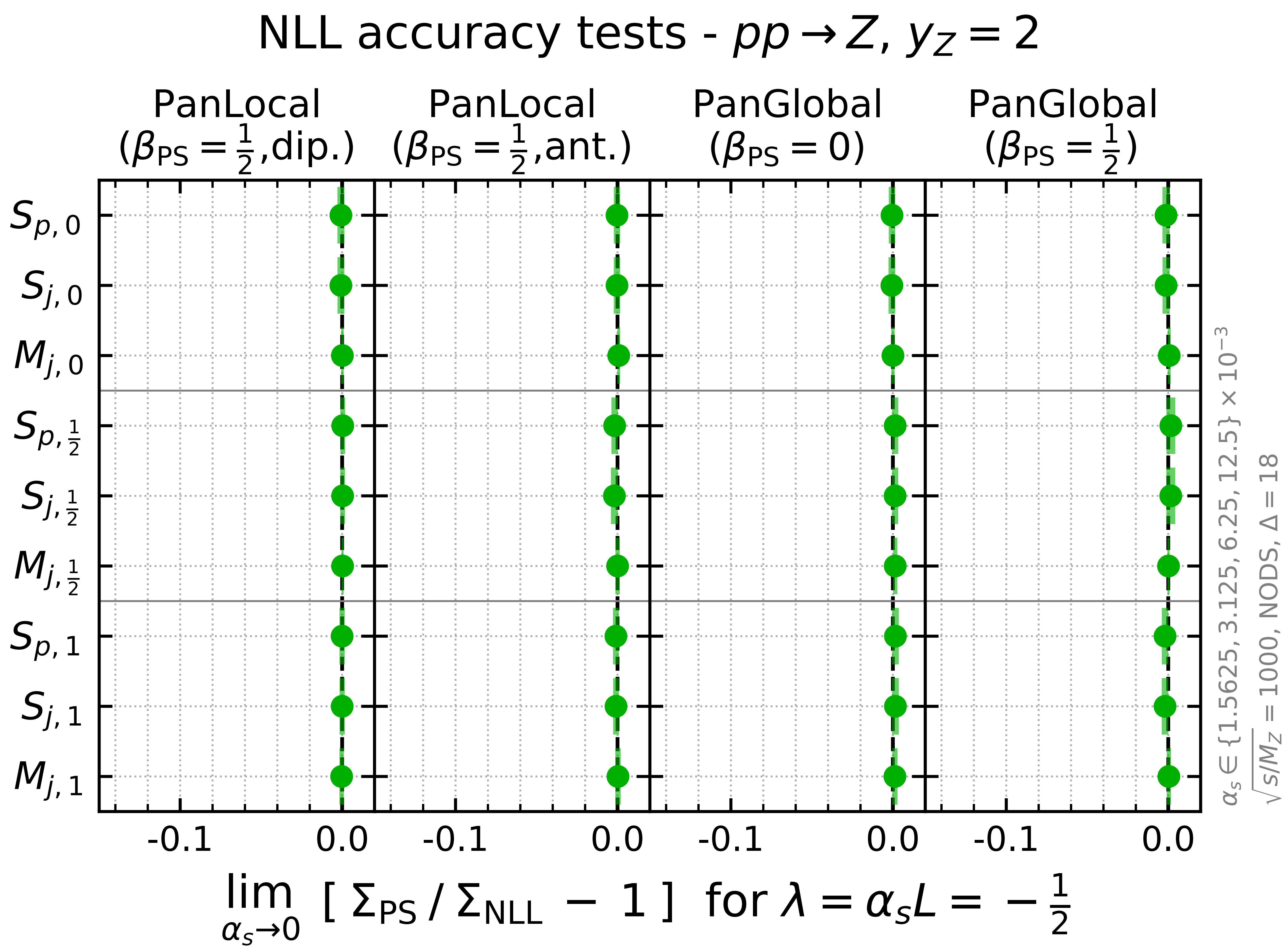
- Interested in exploring the question of NLP corrections…

Back up

Mapping between λ and physical quantities

Q [GeV]	$\alpha_s(Q)$	$p_{t,\min}$ [GeV]	$\xi = \alpha_s L^2$	$\lambda = \alpha_s L$	τ
91.2	0.1181	1.0	2.4	−0.53	0.27
91.2	0.1181	3.0	1.4	−0.40	0.18
91.2	0.1181	5.0	1.0	−0.34	0.14
1000	0.0886	1.0	4.2	−0.61	0.36
1000	0.0886	3.0	3.0	−0.51	0.26
1000	0.0886	5.0	2.5	−0.47	0.22
4000	0.0777	1.0	5.3	−0.64	0.40
4000	0.0777	3.0	4.0	−0.56	0.30
4000	0.0777	5.0	3.5	−0.52	0.26
20000	0.0680	1.0	6.7	−0.67	0.45
20000	0.0680	3.0	5.3	−0.60	0.34
20000	0.0680	5.0	4.7	−0.56	0.30

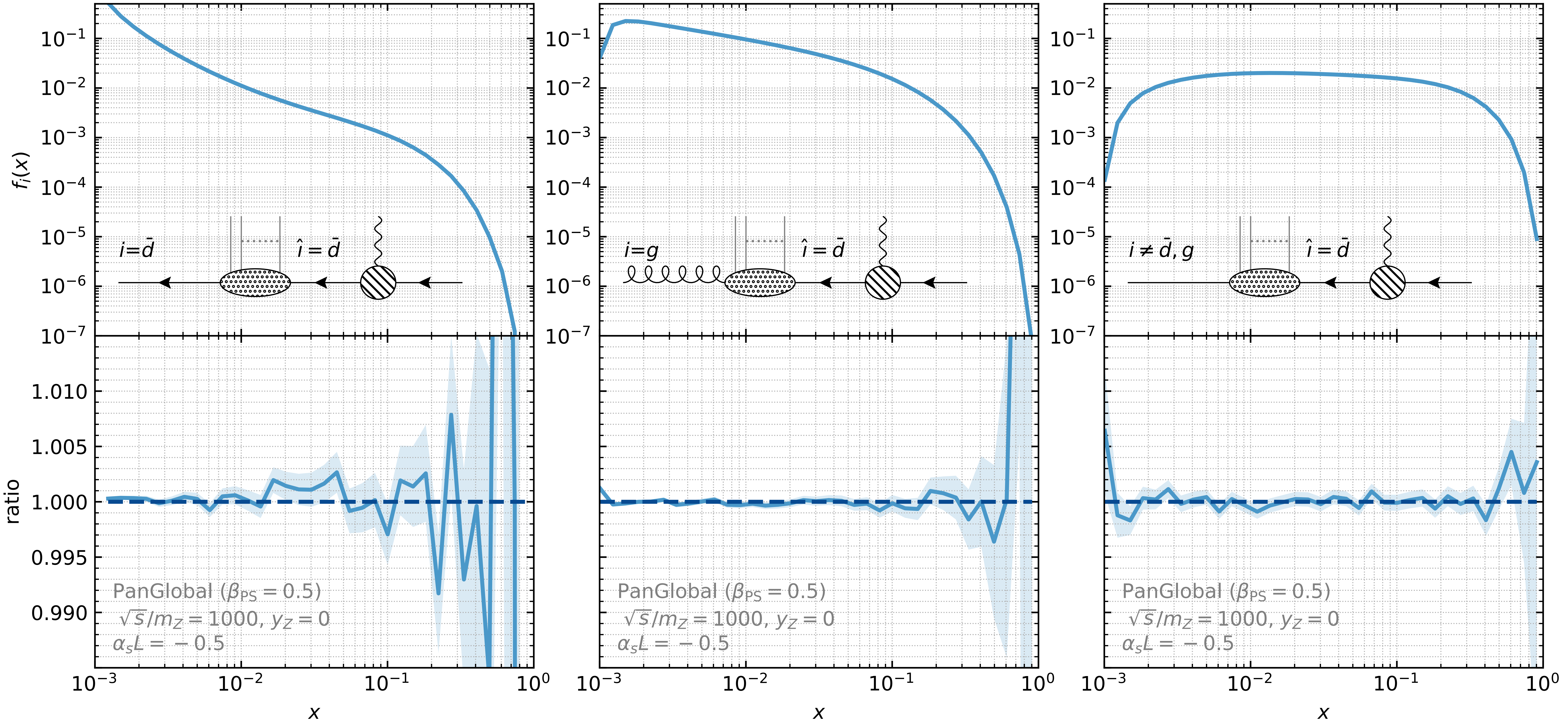
Global event shapes for $y_Z \neq 0$



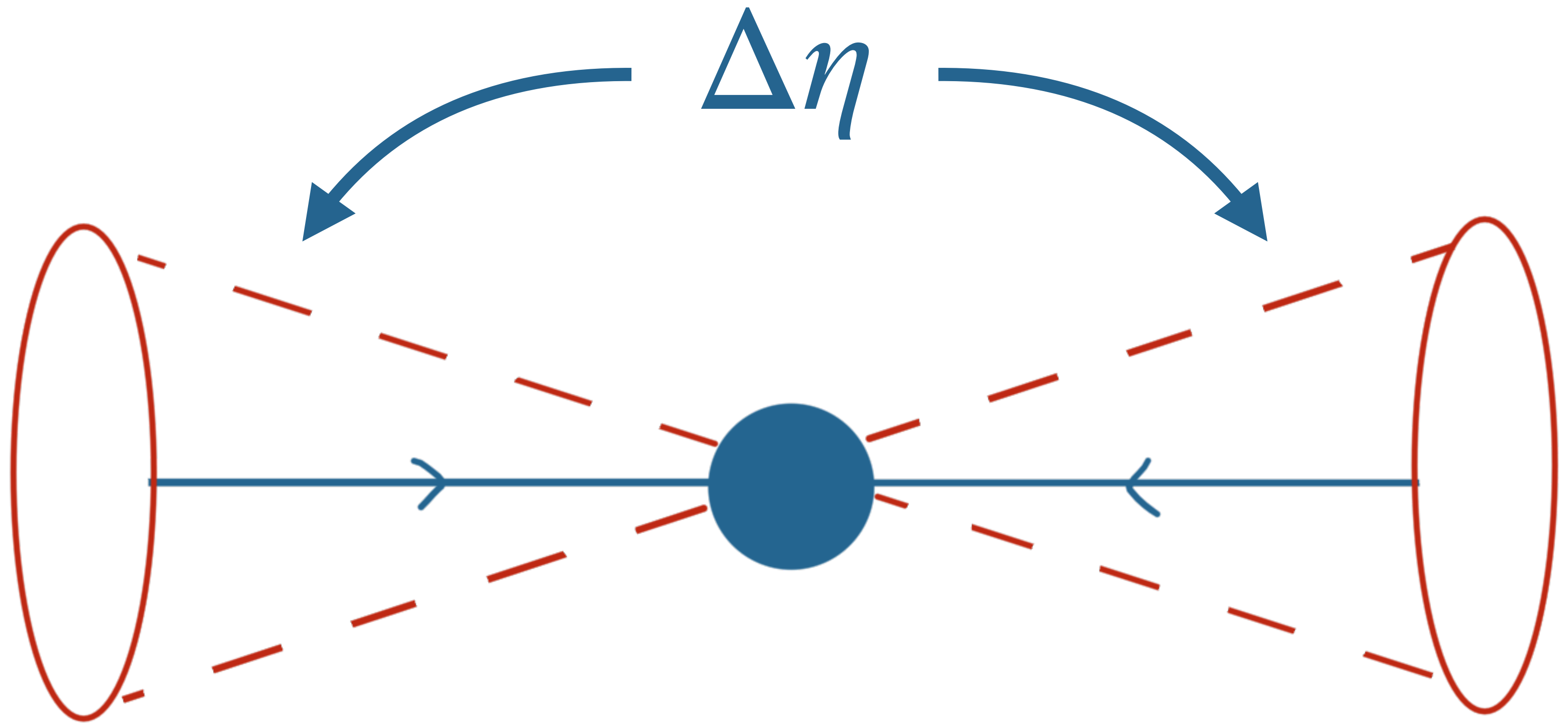
Parton distribution functions

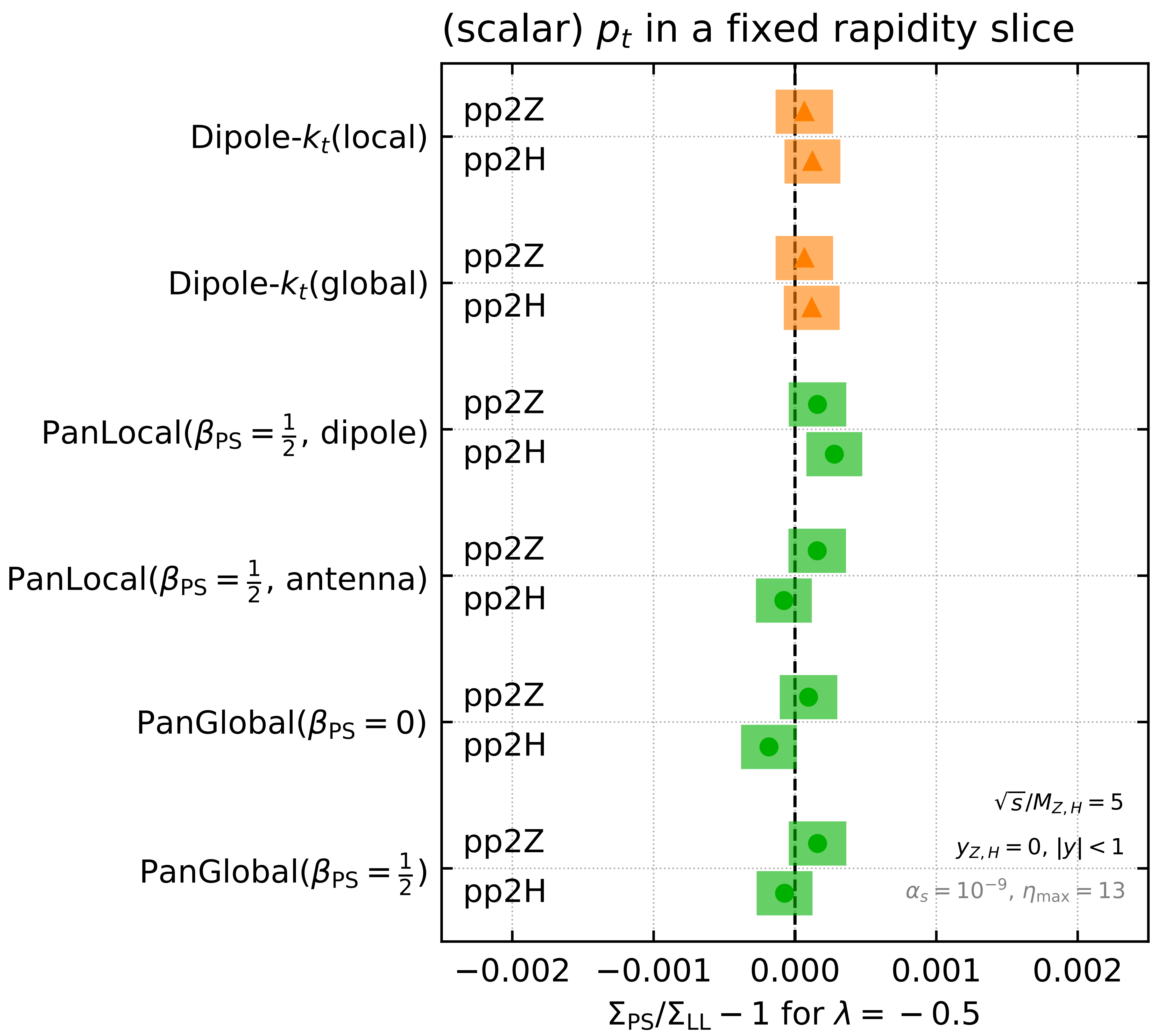
DGLAP expectation

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma_i}{\mathrm{d}x} = \frac{1}{f_{\hat{i}}(\hat{x}, m_Z^2)}\int_{\hat{x}}^{1}\frac{\mathrm{d}z}{z}D_{\hat{i}i}(z,\alpha_s L)f_i\left(\frac{\hat{x}}{z}, p_{t,\mathrm{cut}}^2\right)\delta\left(\frac{\hat{x}}{z}-x\right)$$

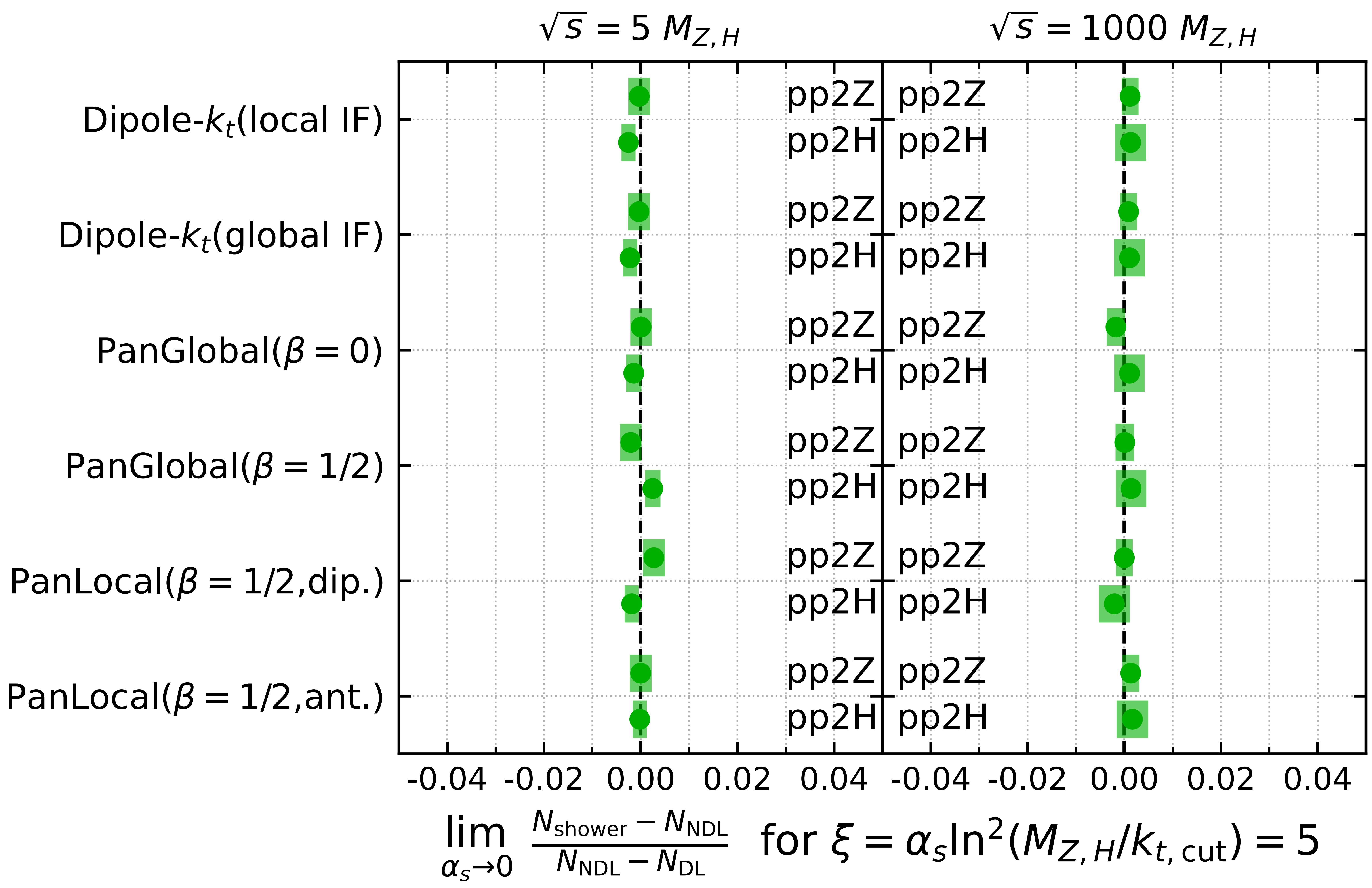


Non-global observable: rapidity gap

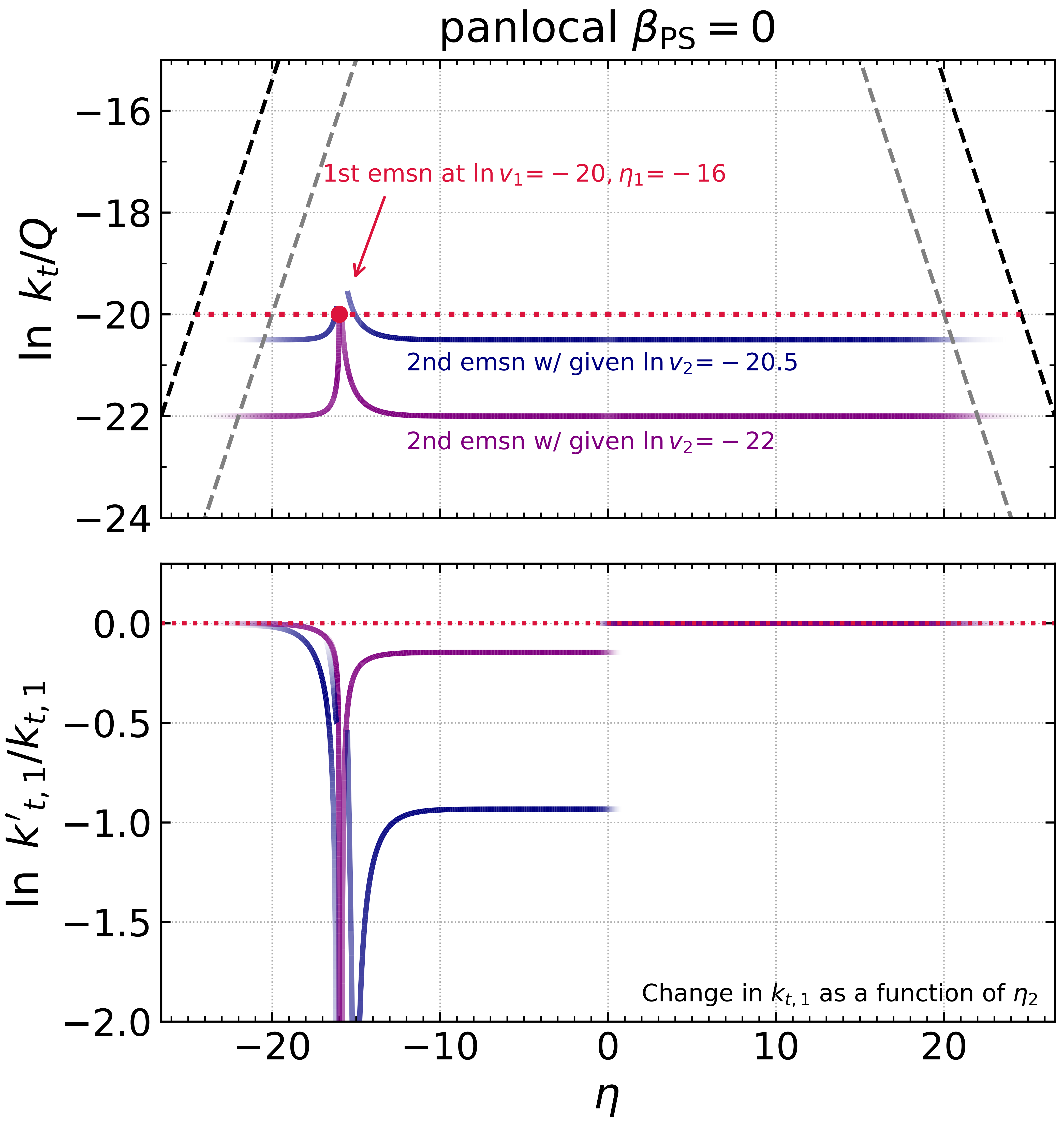




Particle multiplicity

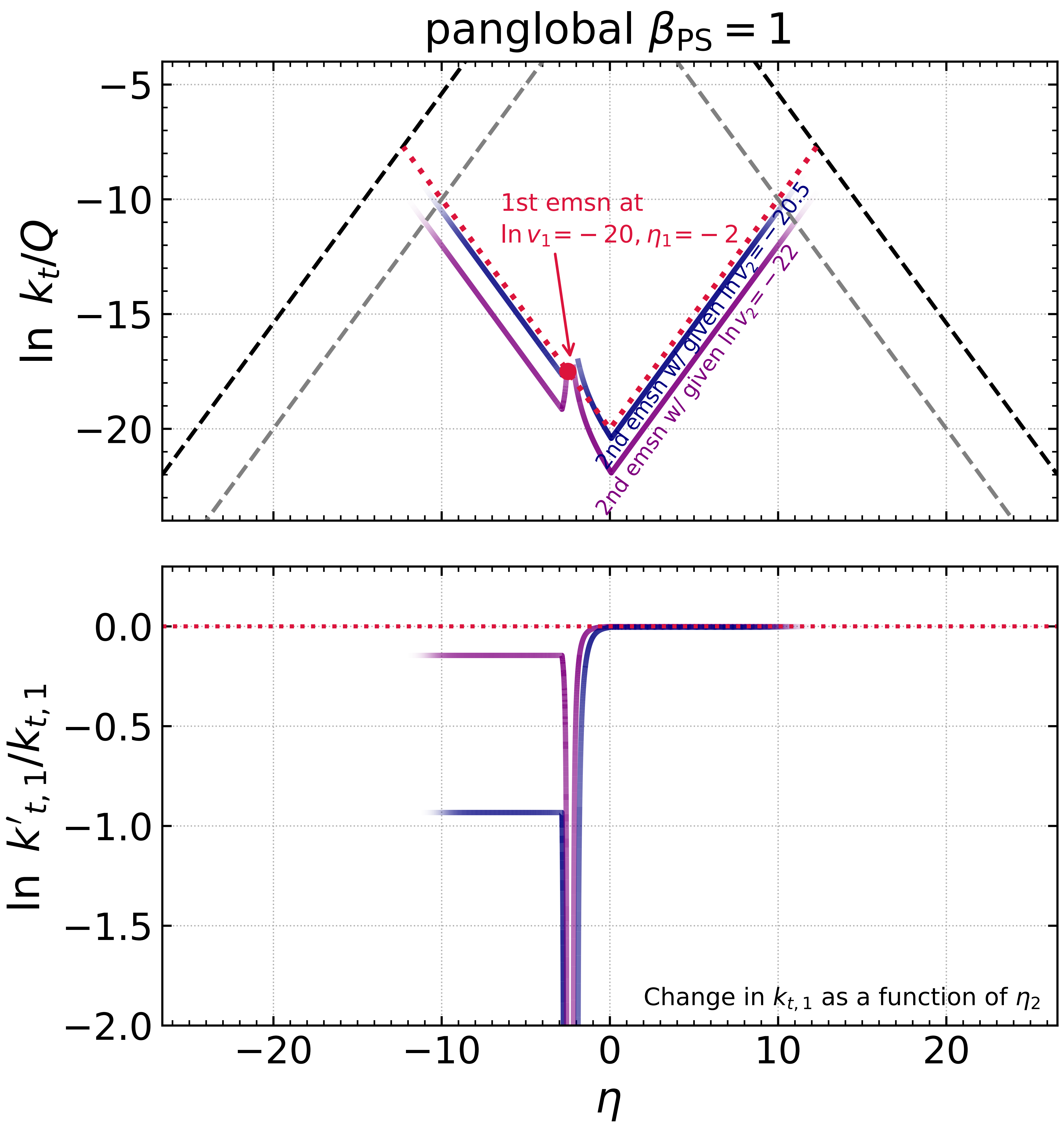


PanLocal issue for $\beta_{\mathrm{PS}} = 0$



- Recoil is taken from the first gluon even when emissions are separated in rapidity
- Separation of dipole in event CM frame is not enough to cure dipole-showers with local maps from locality issue, the transverse momentum ordering is problematic here
- Only when emissions are ordered in angle ($\beta_{\mathrm{PS}} > 0$) we solve this
- Then commensurate k_t emissions are ordered in angle, so they take their recoil from the hard system (after boost)

Issue for $\beta_{\rm PS} = 1$



- For IF dipoles, momentum of first emission is rescaled by $b_j = 1 - \beta_k$ in map
- For $\beta = 1$ this equates to $1 - \frac{\tilde{s}_i}{\tilde{s}_{ij}}\frac{v}{Q}$ and becomes independent of $\bar{\eta}$
- Consider change in first emitted parton:

$$p_{k,1} = \tilde{p}_j \to b_j p_{k,1} = \left(1 - \frac{\tilde{s}_i}{\tilde{s}_{ij}}\frac{v_2}{Q}\right) p_{k,1}$$

- With $\frac{\tilde{s}_i}{\tilde{s}_{ij}} = \frac{2\tilde{p}_i \cdot Q}{2\tilde{p}_i \cdot \tilde{p}_j} = \frac{1}{b_{k,1}}$ and $b_{k,1} = \beta_{k,1} = \frac{v_1}{Q}$

$$\frac{k_{\perp,1}}{k_{\perp,1 \text{ after } 2}} = \left(1 - \frac{v_2}{v_1}\right)$$

Colour tests

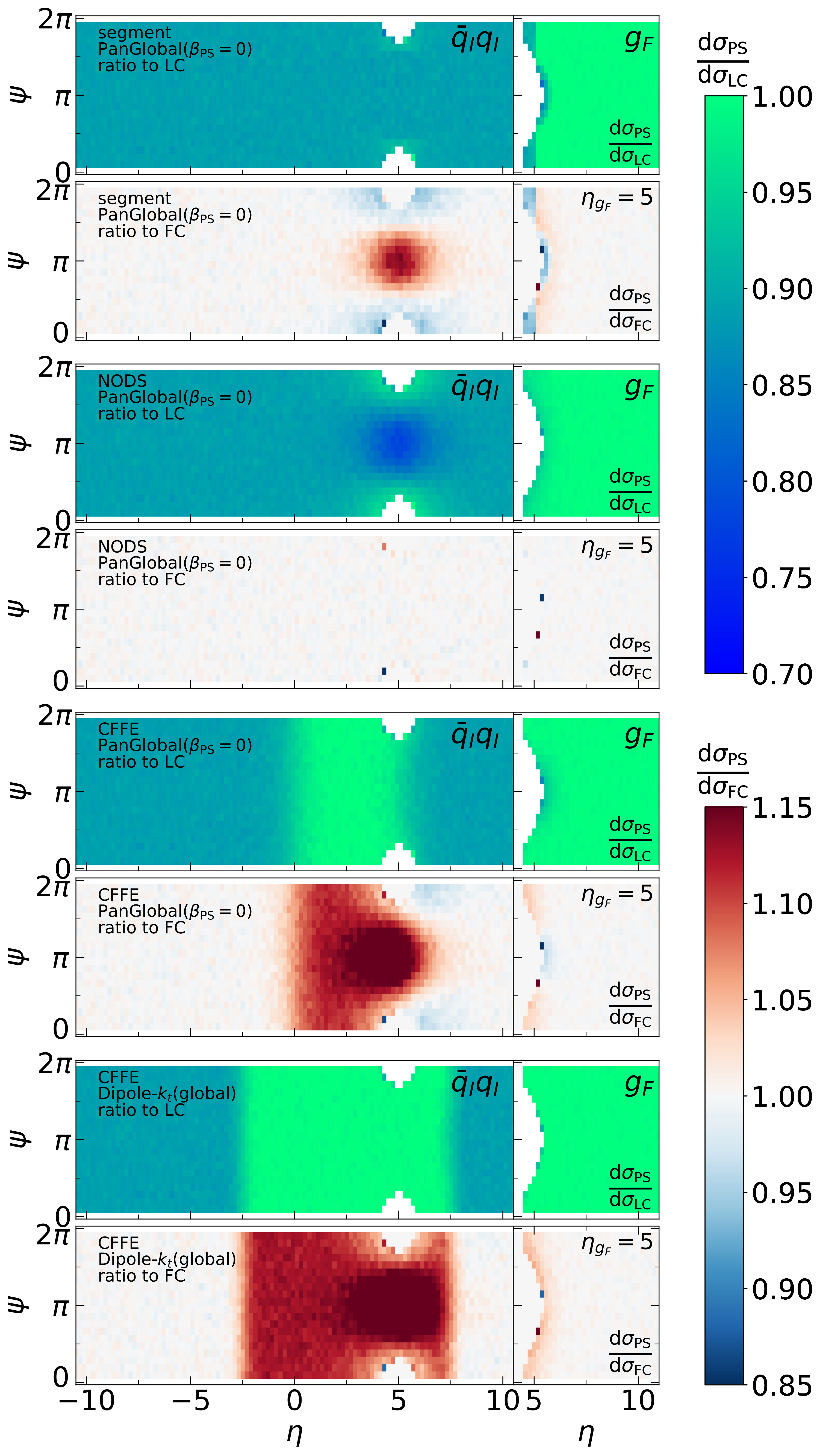
Test of the differential matrix element

Here primary $\bar{q}q$ Lund plane and the new g Lund leaf

LC = leading colour (standard)
FC = full colour

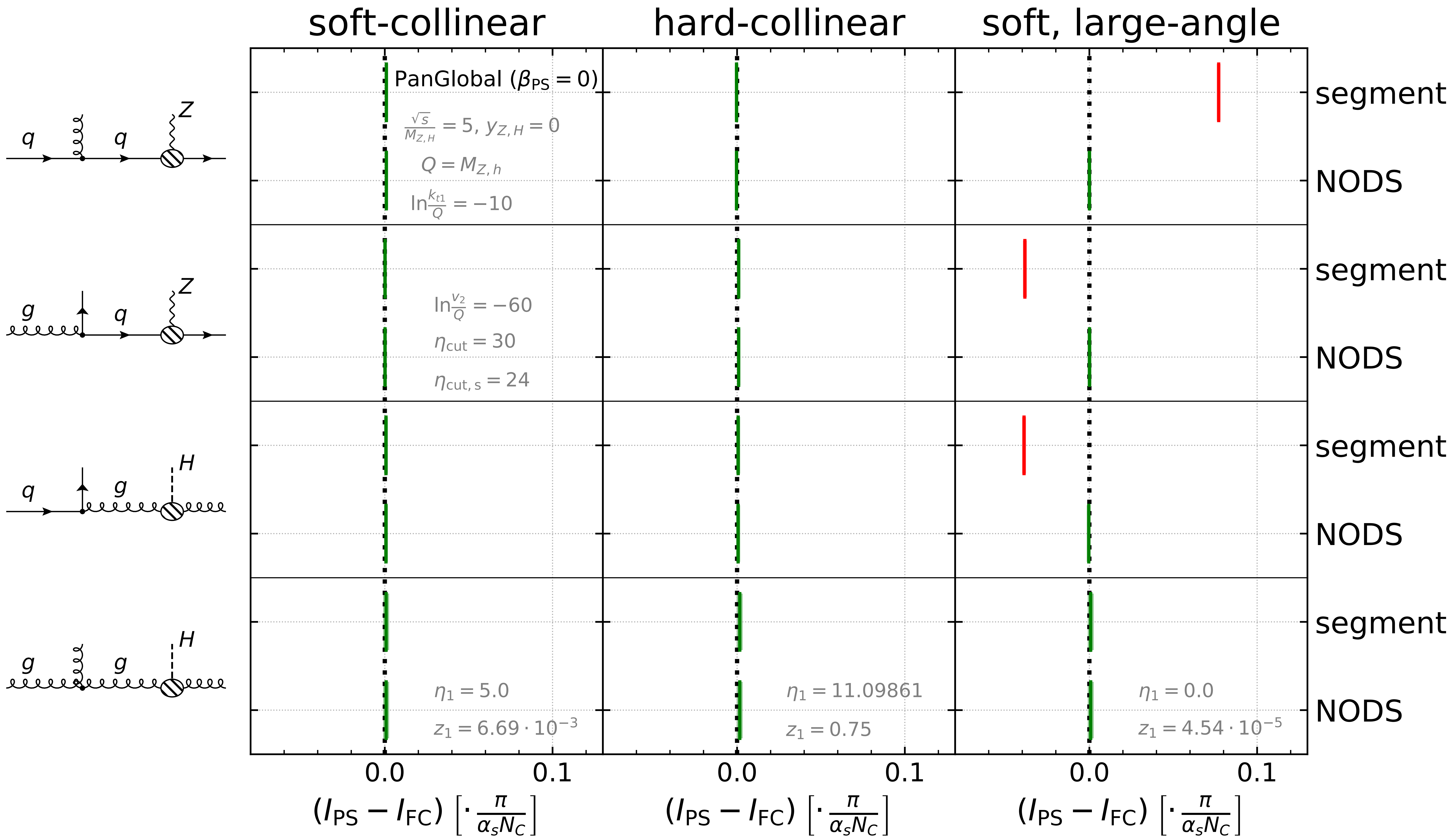
CFFE = standard colour treatment

Segment and NODS two ways to improve the colour handling in the PanScales showers



Colour tests

$$I_{\mathrm{FC}}^{Zg_1} \equiv \int \frac{d\Omega}{2\pi} \frac{|\mathcal{M}_{q\bar{q}g_1g_2}|^2}{|\mathcal{M}_{q\bar{q}g_1}|^2}$$

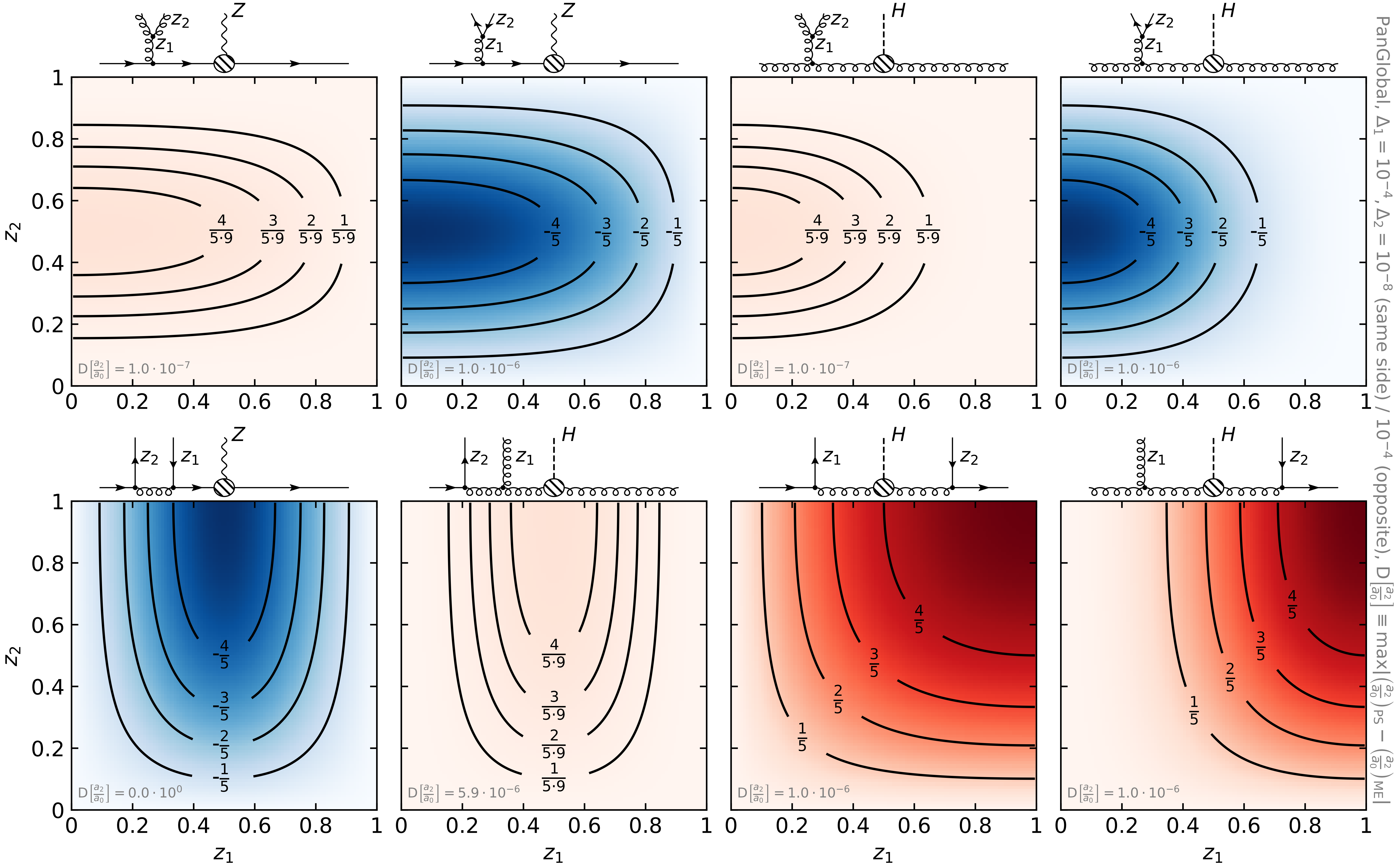


Test of the integrated rate of emissions

Spin tests

$$\frac{d\sigma}{d\Delta\psi_{ij}} \propto a_0 \left(1 + \frac{a_2}{a_0}\cos(2\Delta\psi_{ij})\right)$$

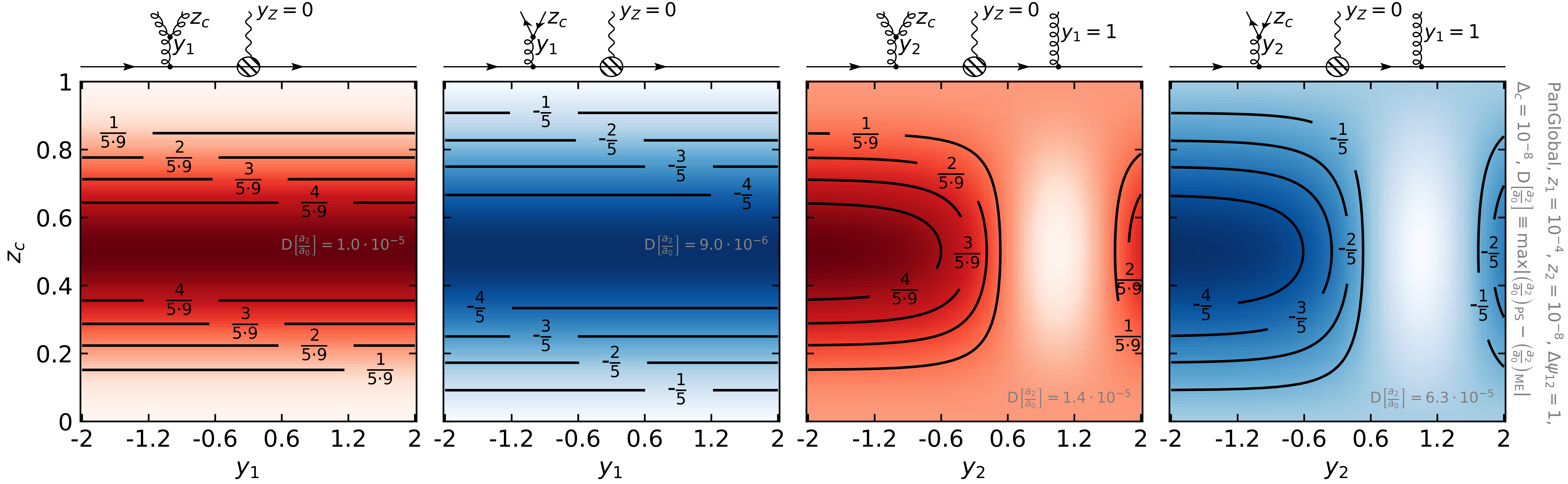
Two collinear emissions



Spin tests

$$\frac{d\sigma}{d\Delta\psi_{ij}} \propto a_0 \left(1 + \frac{a_2}{a_0}\cos(2\Delta\psi_{ij})\right)$$

One collinear, one soft emission



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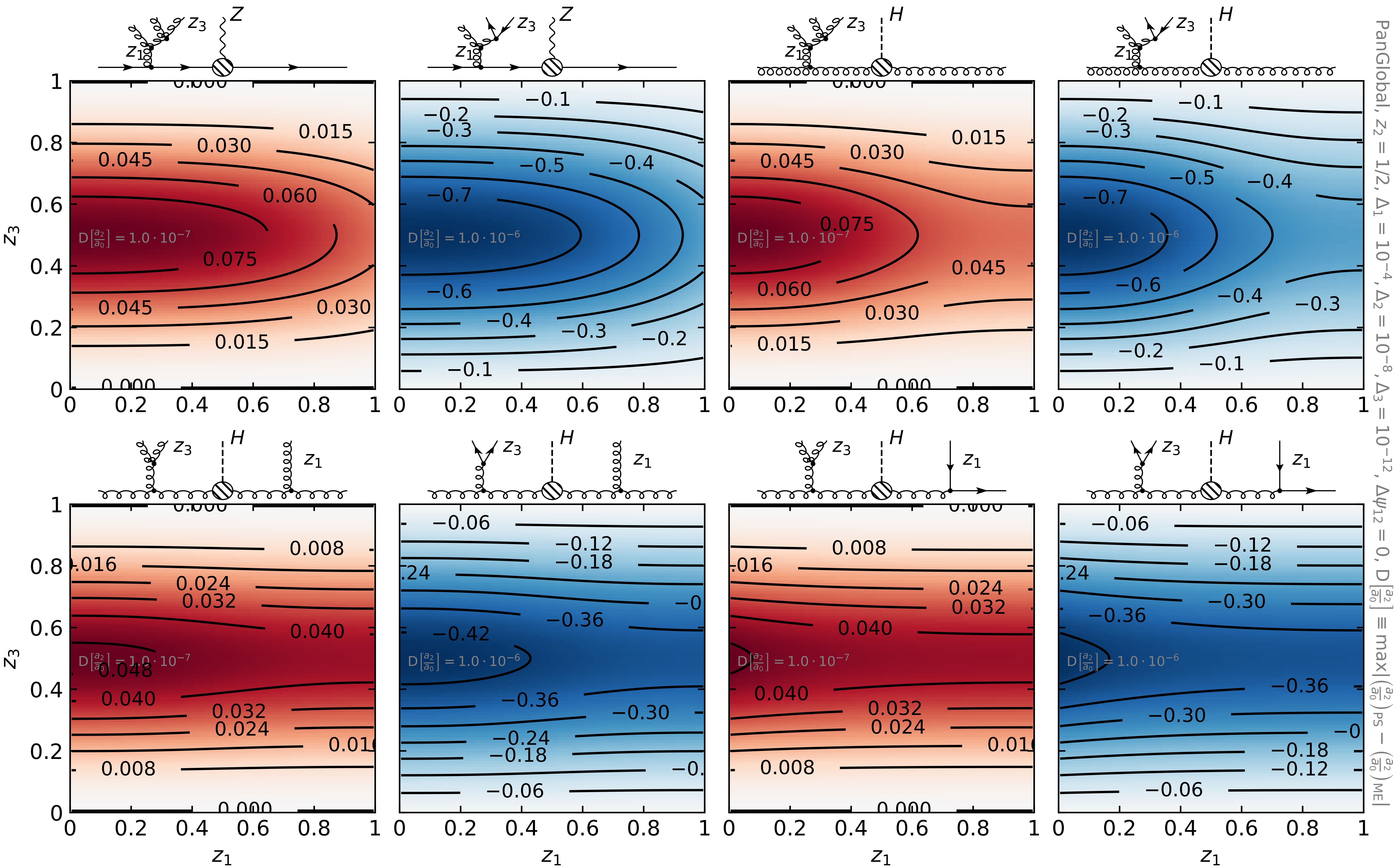
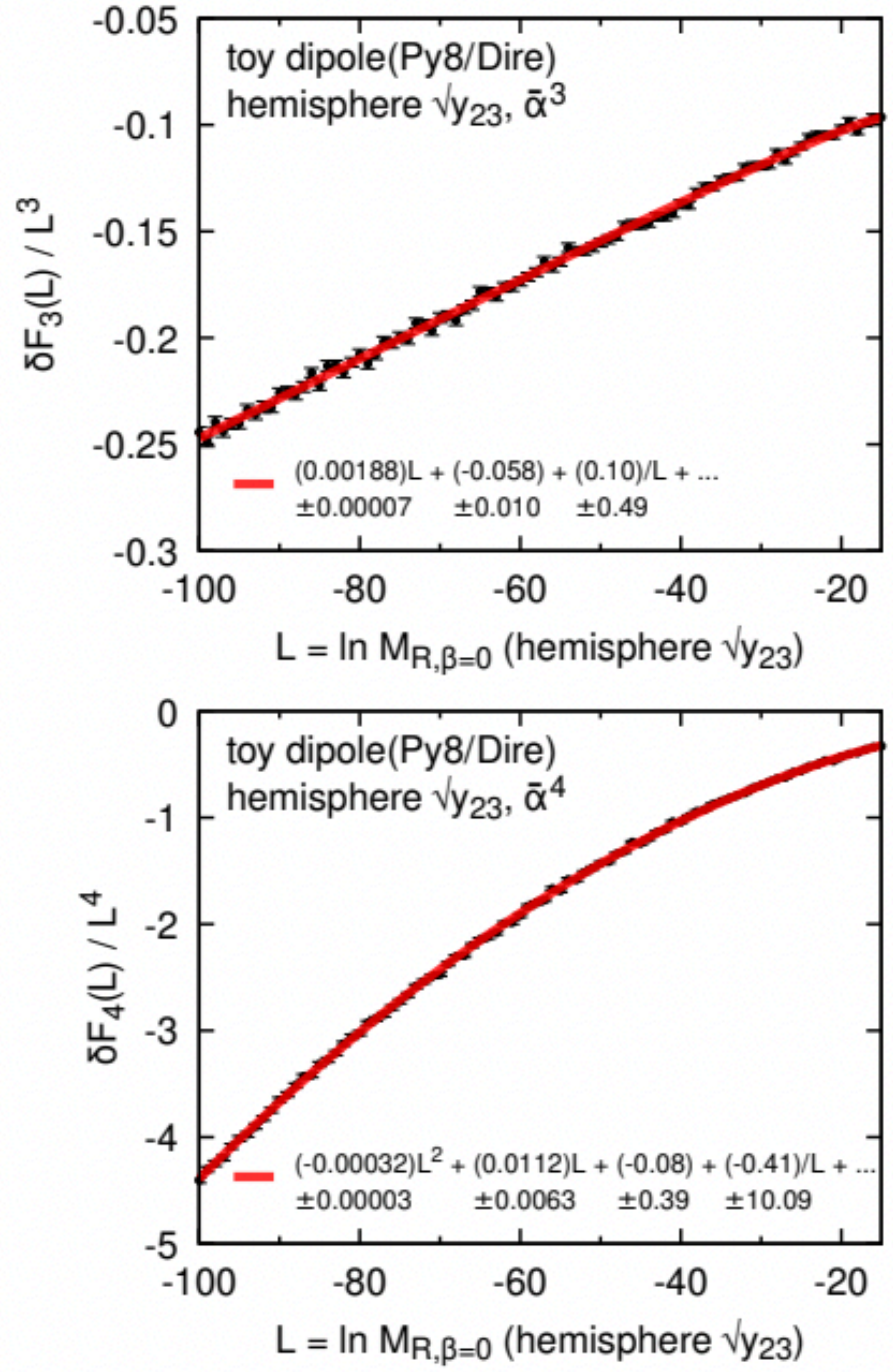
Spin tests

$$\frac{d\sigma}{d\Delta\psi_{13}} \propto a_0 \left(1 + \frac{a_2}{a_0}\cos(2\Delta\psi_{13}) + \frac{b_2}{a_0}\sin(2\Delta\psi_{13})\right)$$

Three collinear emissions

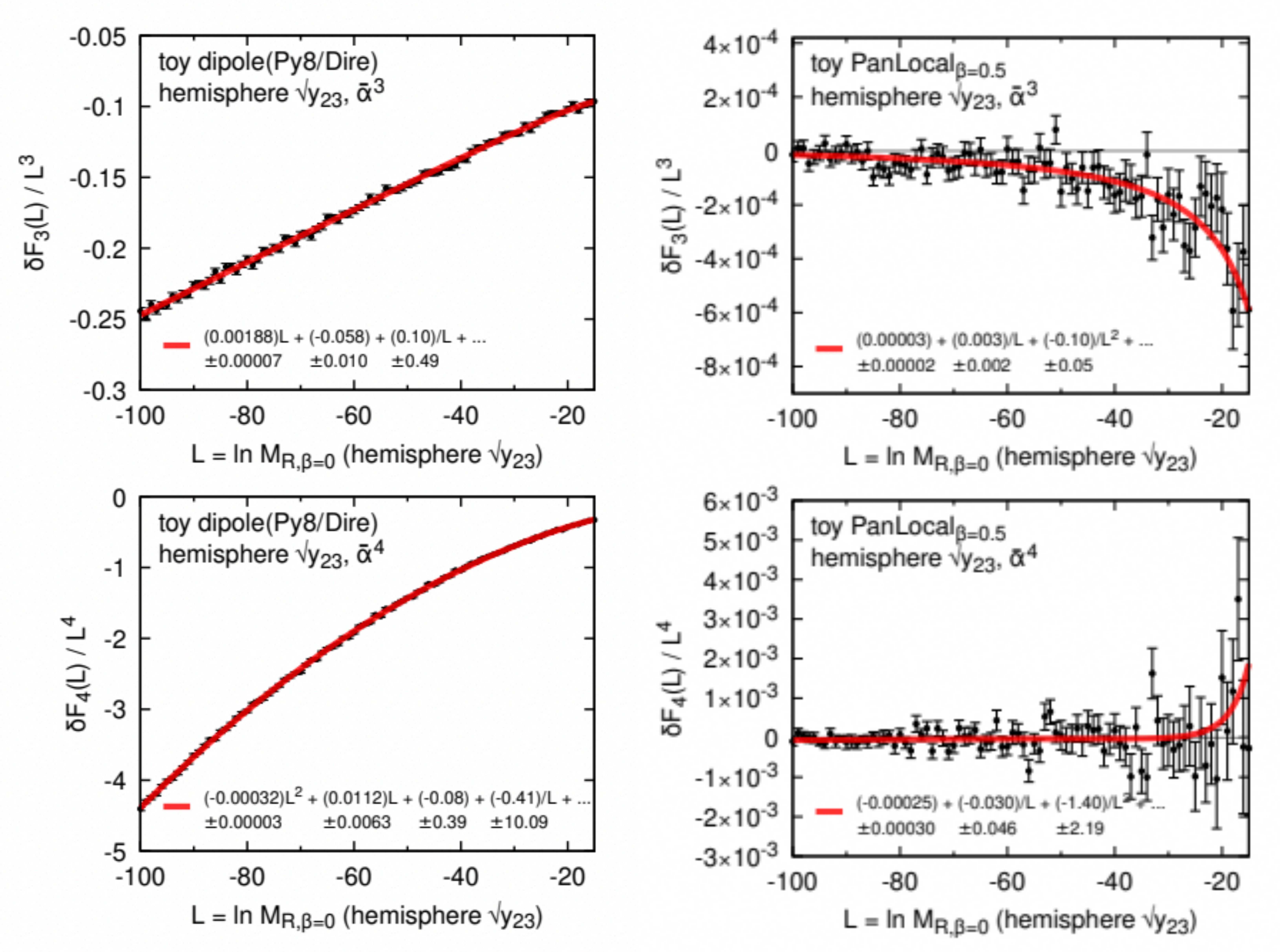


Super-leading logarithms



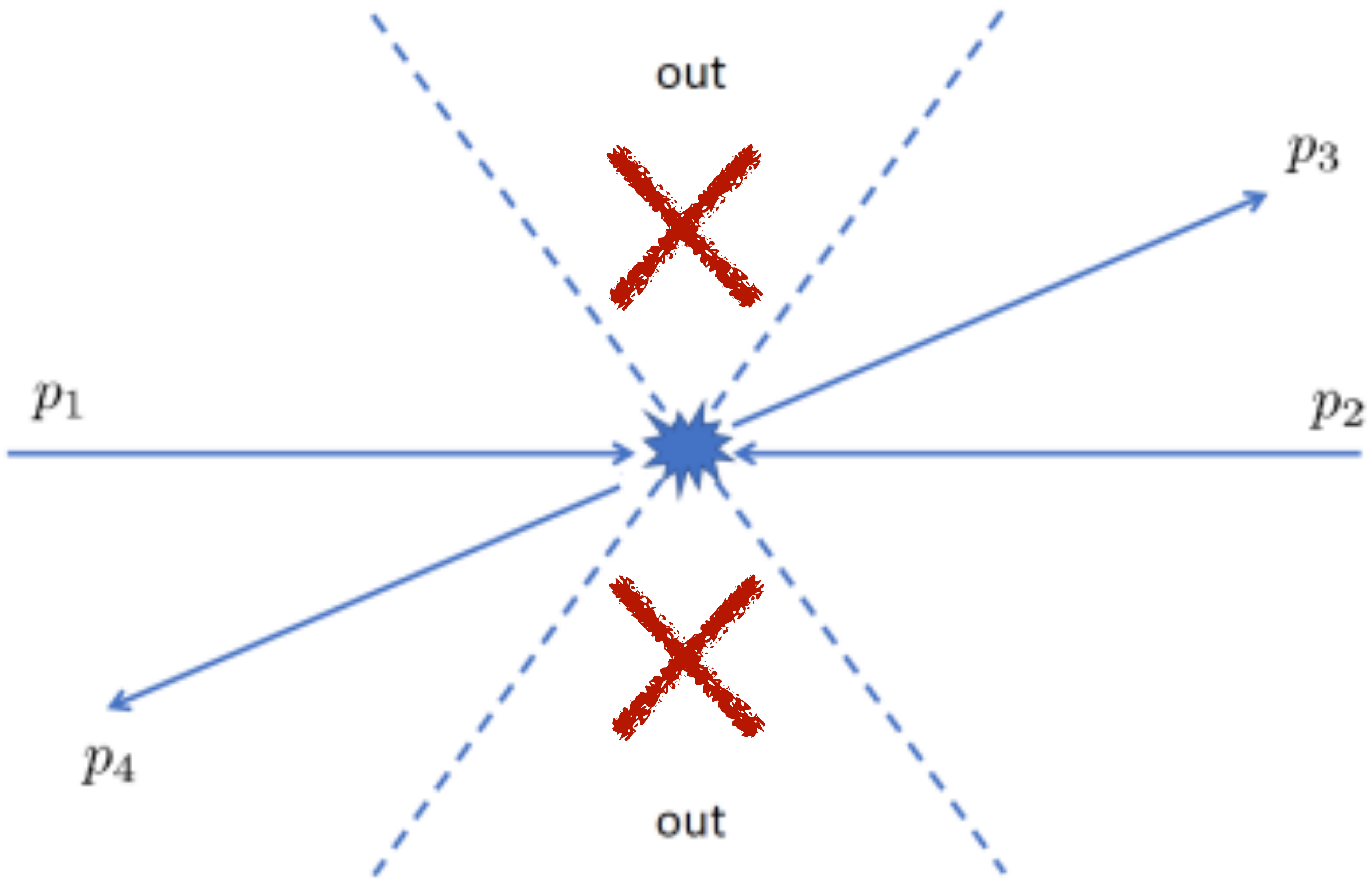
- Consider $M_{R,0}$, max $p_\perp$ of emissions in the right hemisphere (sensitive to super-leading logs at $\mathcal{O}(\alpha_s^3)$)
- Take toy-model approach with only soft primary emissions and fixed coupling
- Take difference between CEASAR result and toy shower $\delta F_n(L)$, n = order in α_s, where $F = \sum \alpha_s^n F_n$ has terms of $\alpha_s^n L^m$ with $m \leq n$
- Clearly a discrepancy at fixed-order for standard dipole showers
- Vanishes at all orders because it is numerically comparable to the NNLL terms -> orange points

Super-leading logarithms



- Discrepancy not there for PanScales family of showers

Subleading colour corrections - jet veto in $h + 2j$



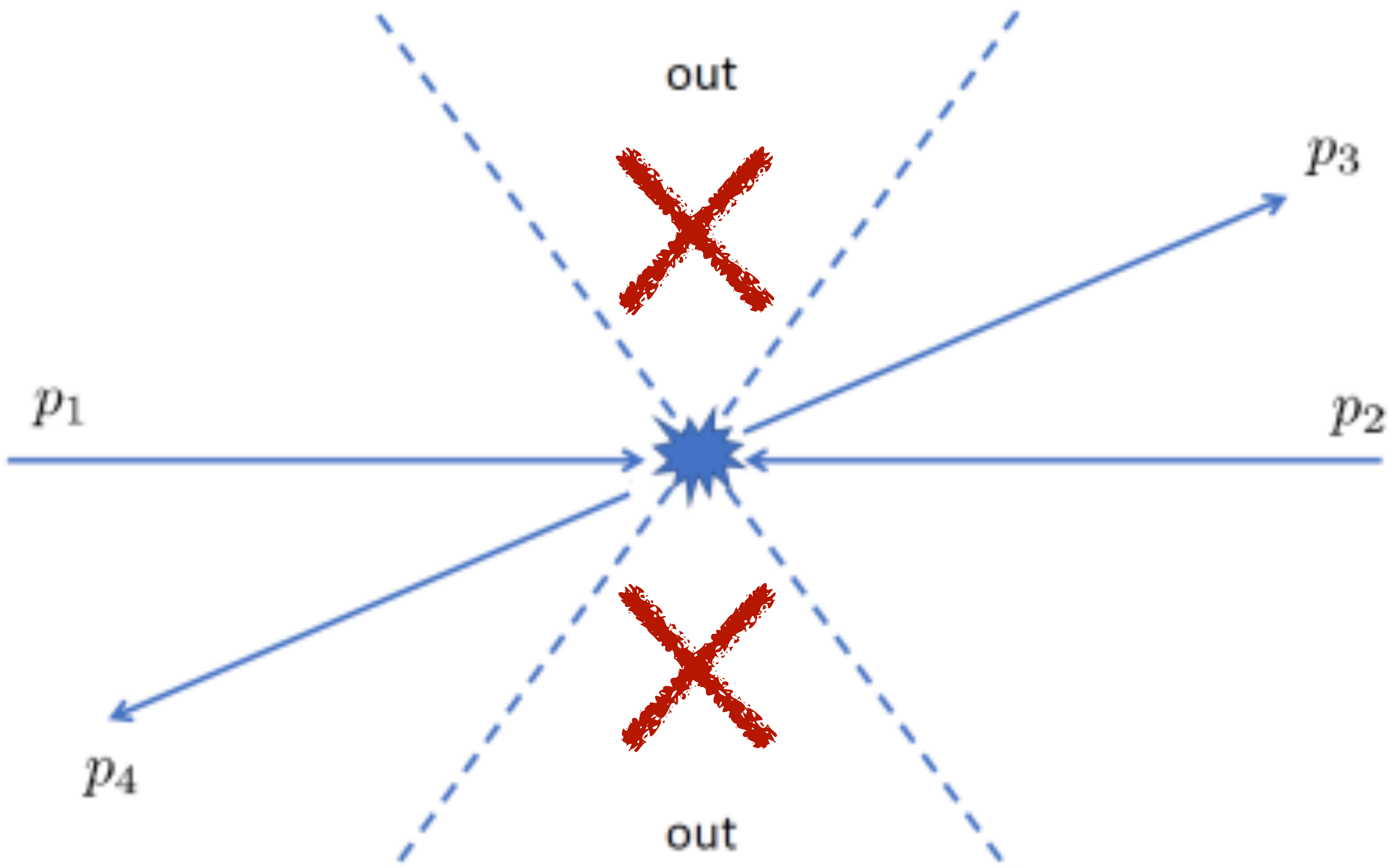
Non-global observable: sensitive to wide-angle soft gluon emissions in restricted regions of phase space

Soft gluons are sensitive to **colour flow** of underlying process i.e.

$$qq \to qqH$$

has an **octet** and a **singlet** channel

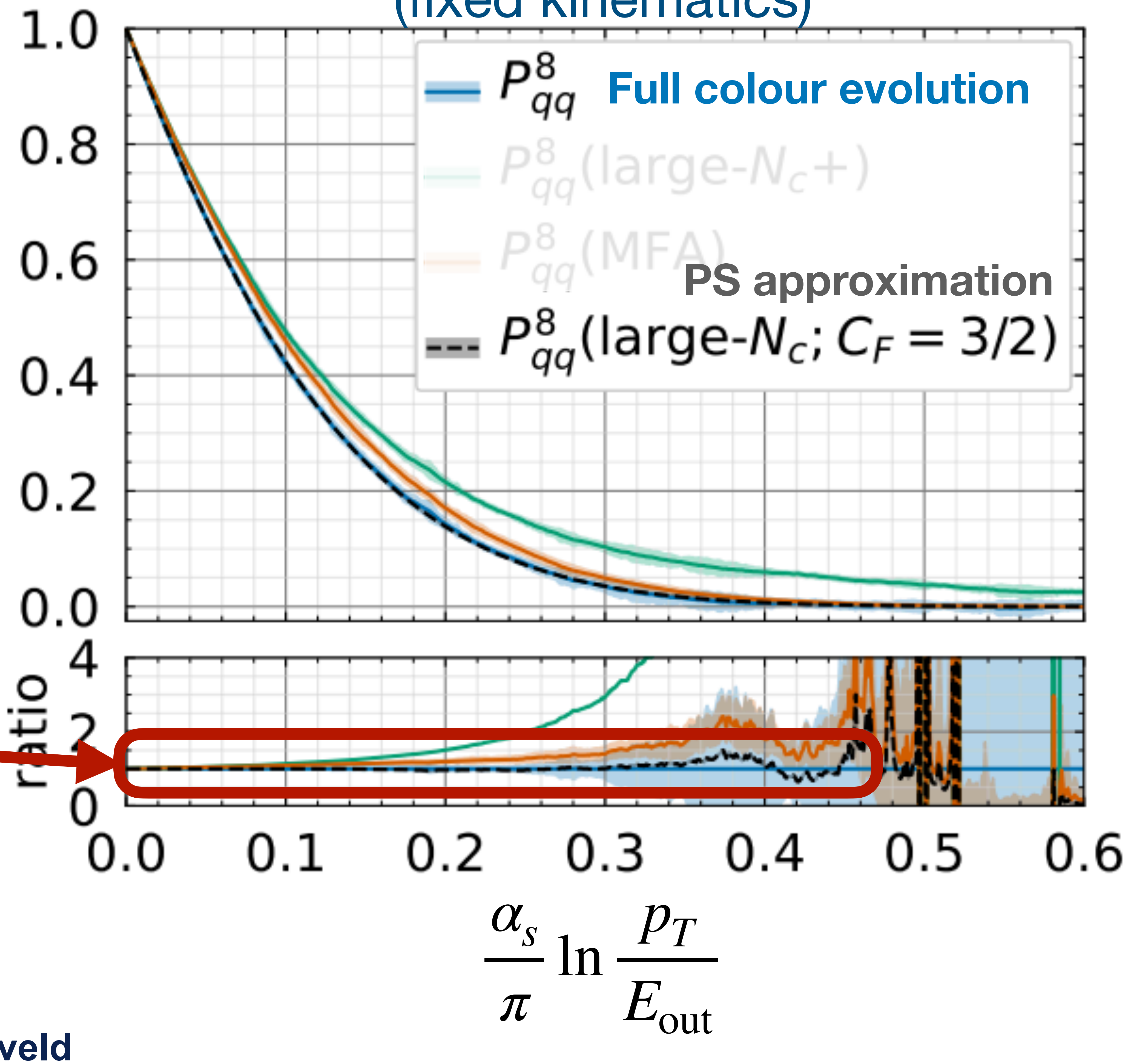
Subleading colour corrections - jet veto in $h + 2j$



Puzzling agreement between large-N_c and full colour, observed in **all** channels!

Maybe good news for the large-N_c parton showers, but need to understand what is happening here…

Gap survival probability for octet channel
(fixed kinematics)



Including higher-logarithmic effects

Including higher-logarithmic effects

Discussion so far is based on the factorisation in a single unresolved limit

What about double-unresolved configurations?

Triple-collinear splitting functions

Catani, Grazzini [9810389, 9908523]

$$|M_{1,2,3,\ldots,k,\ldots}(p_1,p_2,p_3,\ldots)|^2 \stackrel{123-\text{coll}}{\longrightarrow}$$

$$\left(\frac{8\pi\mu^{2\varepsilon}\alpha_s}{s_{123}}\right)^2 \mathcal{T}^{ss'}_{123,\ldots}(p_{123},\ldots)\, P^{ss'}_{123}(p_1,p_2,p_3)$$

Double-soft emissions

Campbell, Glover [9710255]
Catani, Grazzini [9908523]

$$|M_{1,2,3,\ldots,n}(p_1,p_2,p_3,\ldots,p_n)|^2 \stackrel{12-\text{soft}}{\longrightarrow}$$

$$\left(4\pi\mu^{2\varepsilon}\alpha_s\right)^2 \sum_{i,j=3}^{n} \mathcal{I}_{ij}(p_1,p_2)\, |M^{(i,j)}_{3,\ldots,n}(p_3,\ldots,p_n)|^2$$

These corrections need to be included to get to NNLL/NNDL accuracy

Analytic ingredients - new hard collinear terms

One important and new ingredient for a fully differential shower is $B_2(z)$

Consider the Sudakov for transverse-momentum resummation

Parisi, Petronzio [NPB 154 (1979) 427-440]

$$S(Q,b) = \exp\left(-\int_{\bar{b}^2/b^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2}\left[A(\alpha_s(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_s(q^2))\right]\right)$$

Both obey a perturbative expansion in α_s

$$A(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_n \qquad B(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_n$$

$A_1,\ B_1,\ A_2$ are observable independent
(they only depend on the emitting particle)

Dasgupta, El-Menoufi [2109.07496]

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B_2 is observable-dependent, i.e. for a quark emitter

$$B_2^q = -\gamma_q^{(2)} + C_F b_0 X_v$$

Catani, de Florian, Grazzini [0008184, 0407241]

$B_2^{q/g}$ needs to be included in a differential manner $\rightarrow$ $B_2^{q/g}(z)$

Dasgupta, El-Menoufi [2109.07496]

$B_2(z)$ for quark channels

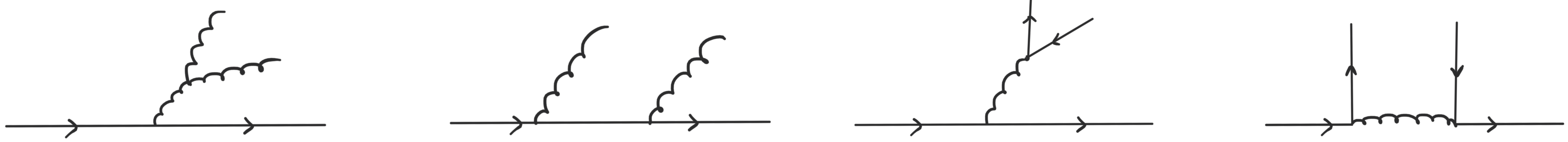
1. Integrate the triple-collinear contributions over 2 energies and 1 angular variable $(\theta, \rho, k_T, \ldots)$

Non-abelian $C_F C_A$

Abelian C_F^2

$g \to q^{(\prime)}\bar{q}^{(\prime)}$ splitting $C_F T_R n_F$

Identical quarks interference $C_F\left(C_F - \frac{C_A}{2}\right)$



2. Isolate the pure NNLL terms (subtract iterated LO splittings and K_{CMW} contributions)

+ virtual corrections

Result: $B_2^q(z)$ differential in z, θ for all channels

$$\int_0^1 \mathrm{d}z \left[B_2^{q,C_F C_A}(z) + B_2^{q,C_F^2}(z) + B_2^{q,C_F T_R n_F}(z) + B_2^{q,\mathrm{id}}(z)\right] = -\gamma_q^{(2)} + C_F b_0 X_v = B_2$$

Observable-dependence depends on the scale of the coupling through the angular variable that is fixed

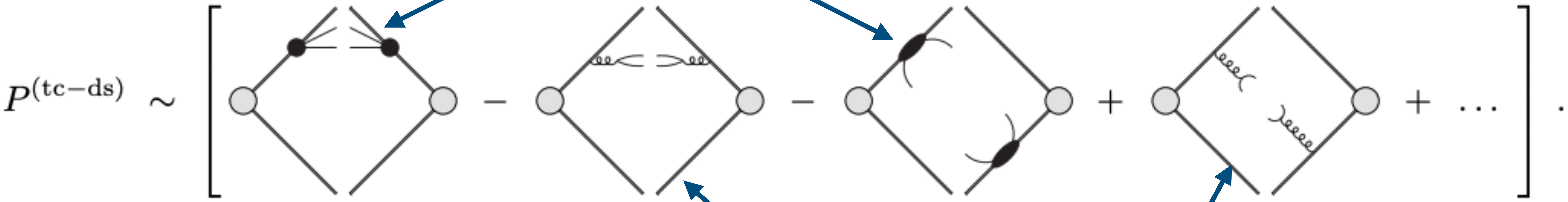
To be done: get $B_2^g(z)$, implement this in a shower, understand cross-talk with double-soft…

Implementing higher-order splitting kernels

Consider quark-pair emissions in the triple-collinear (tc) and double-soft (ds) limits

Need to remove overlapping singularities and contributions obtained by LO iteration

Complete MEs in the tc and ds limits
(latter with a minus sign to remove the double counting)



Diagrams obtained iterating LO splittings

Result is fully finite through introduction of integrated subtraction terms and factorization counter terms

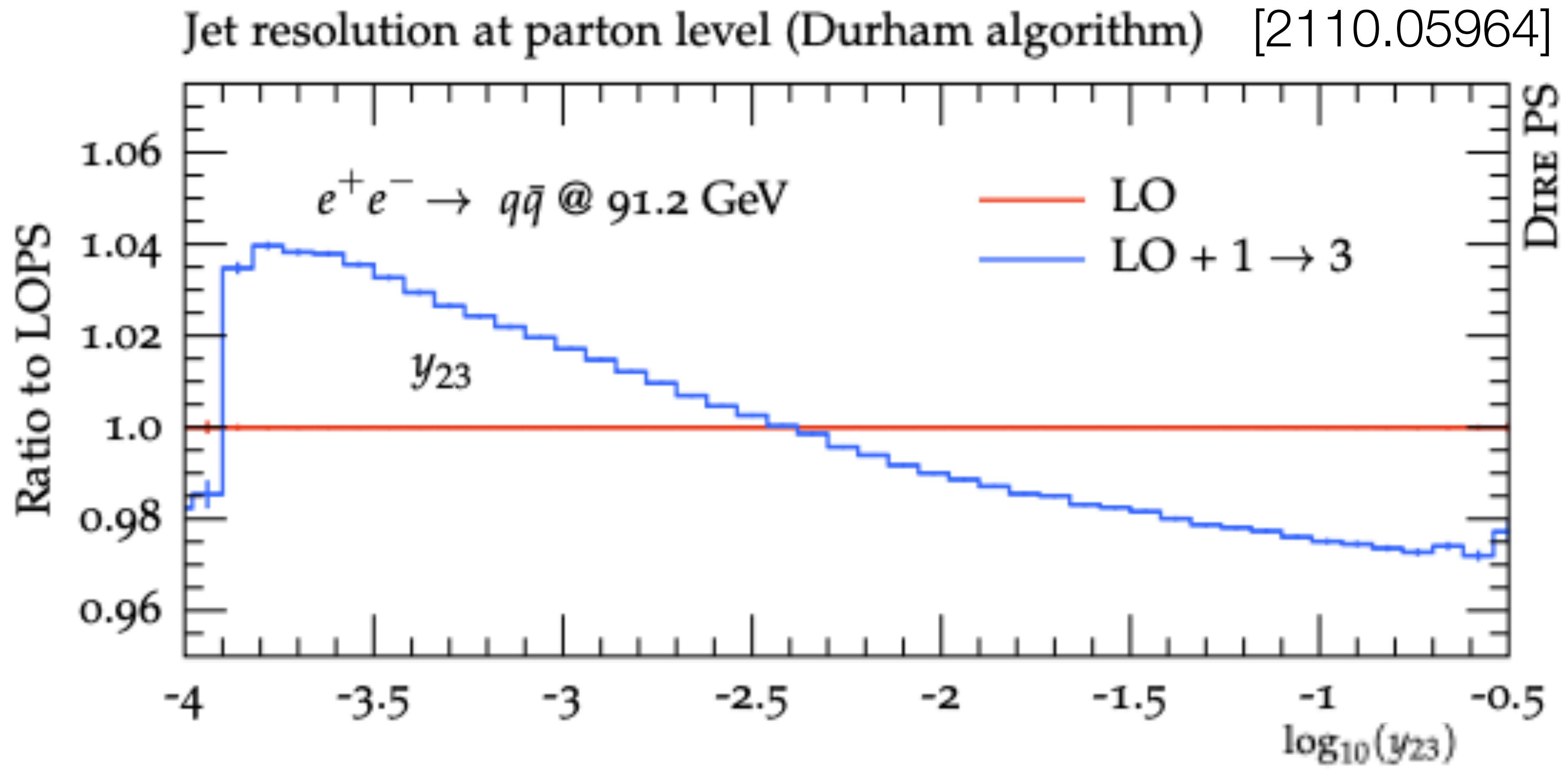
Generate emissions using the $1 \to 3$ branching kernels in a $2 \to 4$ ‘tripole’

Note that this is not an NNLL shower, i.e. the kinematic map has the issues pointed out before

Implementing higher-order splitting kernels

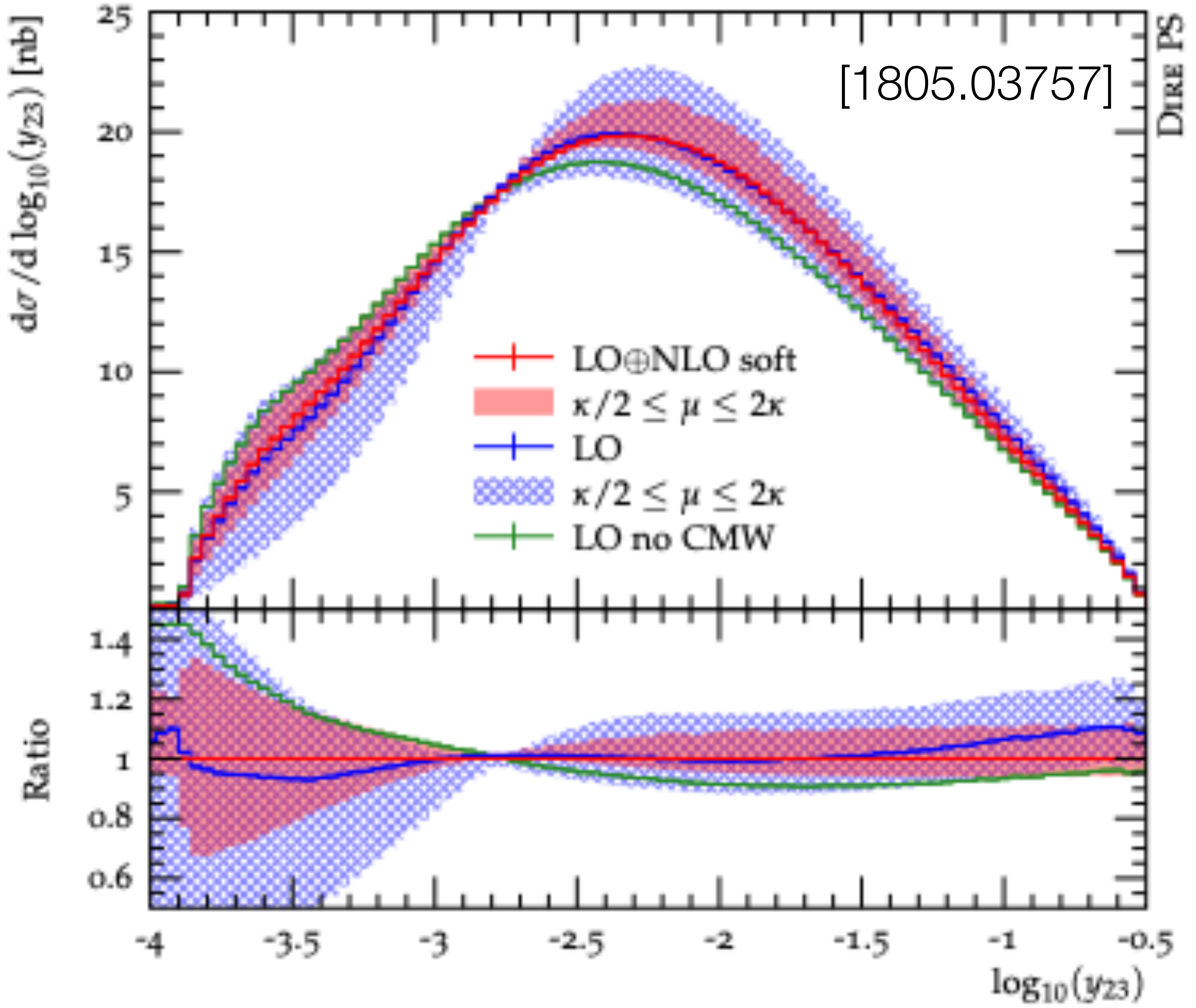
- Dire with soft-subtracted triple-collinear $q \to qq\bar{q}$ splittings

- K_{CMW} included in the coupling (not in differential form)



tc corrections shift the y_{23} distribution wrt the LO shower

- Dire with only double-soft corrections (all channels)



ds corrections have a similar effect as the soft-subtracted tc terms on the y_{23} distribution