# The calculation of nucleon electric dipole moments on Lattice

**Fangcheng He**
**(Stony Brook University)**

**In collaboration with Michael Abramczyk, Tom Blum, Taku Izubuchi, Hiroshi Ohki and Sergey Syritsyn**

# Outline

- Introduction to nucleon electric dipole moments
- The calculation of EDM on lattice
  - Form factor method
  - Background field method
- Summary

# Nucleon electric dipole moments

- **Matrix element in the CP violation vacuum**

Theta term

$$\langle N\,[\bar{q}\gamma^{\mu}q]\,\bar{N}\rangle_{\not{CP}} = \frac{1}{Z}\int \mathscr{D}U\,\mathscr{D}\bar{\psi}\mathscr{D}\psi N\,[\bar{q}\gamma^{\mu}q]\,\bar{N}e^{-S-iS_{\theta}}$$

$$S_{\theta} = \frac{\theta}{32\pi^2}\int d^4x Tr[F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)]$$

$$\langle p',\sigma'|J^{\mu}|p,\sigma\rangle_{\not{CP}} = \bar{u}_{p',\sigma'}\left[F_1(Q^2)\gamma^{\mu} + \left(F_2(Q^2)+iF_3(Q^2)\gamma_5\right)\frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N}\right]u_{p,\sigma},$$

Electric dipole moment $d_n = \frac{F_3(0)}{2m_n}$

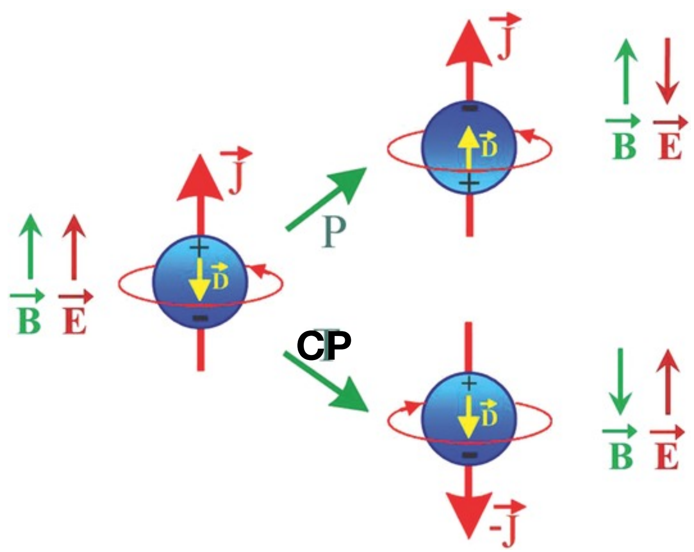
$$H = \mu\,\vec{\sigma}\cdot\vec{B} + d_n\,\vec{\sigma}\cdot\vec{E}$$

CP even    CP odd

CP violation

**Standard Model prediction**

$|d_n| \sim 10^{-31}\ e\cdot\text{cm}.$




- **Beyond Standard Model**
- **Baryongenesis**
- **Strong CP problem**

# Experimental measurement for EDM

## Recent EDM limits

$$d_n < 2.9 \times 10^{-26} e\,.\,cm$$

C. A. Baker, Phys. Rev. Lett. 97(2006)

$$d_n < 1.6 \times 10^{-26} e\,.\,cm$$

B. Graner, Phys. Rev. Lett. 116(2016)

$$d_n = (0.0 \pm 1.1_{stat} \pm 0.2_{sys}) \times 10^{-26} e\,.\,cm$$

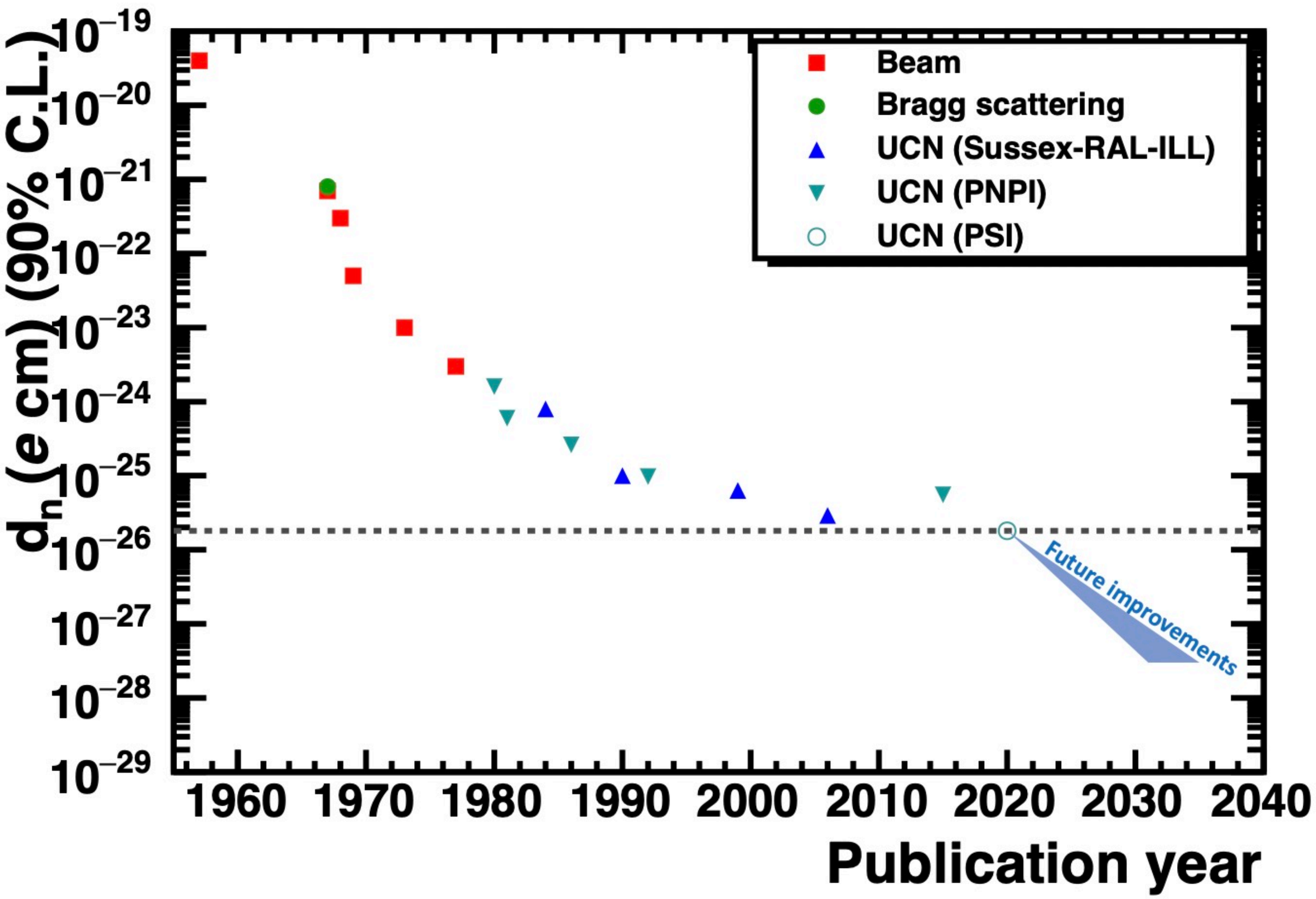
C. Abel et al, Phys. Rev. Lett. 124(2020)

**Standard Model prediction**

$|d_n| \sim 10^{-31}$ $e$·cm.

Evolution of EDM measurement

Snowmass 2021, 2203.08103



# Effective CPv operators

- **CP violated interactions**

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

**$\theta$ term** = $\frac{i}{32\pi^2}\Theta\, G_{\mu\nu}\tilde{G}_{\mu\nu}$

**Chromo interaction** $-\frac{i}{2}\sum_q d_q\, \bar{q}\sigma^{\mu\nu}\tilde{F}_{\mu\nu}q - \frac{i}{2}\sum_q d_q\, \bar{q}\sigma^{\mu\nu}\tilde{G}_{\mu\nu}q$ ,

**Weinberg term** $+\,\omega f^{abc} G_{\mu\nu,a} G^{\mu\beta,b} G^{\nu,c}{}_{\beta}$

$$d_{n,p} = d^{\theta}_{n,p}\theta_{\rm QCD} + d^{cEDM}_{n,p} c_{cEDM} + \dots$$

Determined by experiment

Non-perturbative QCD matrix elements

Determine QCD matrix elements

Determine the coupling constants $\theta_{QCD}$, $C_{cEDM}$....

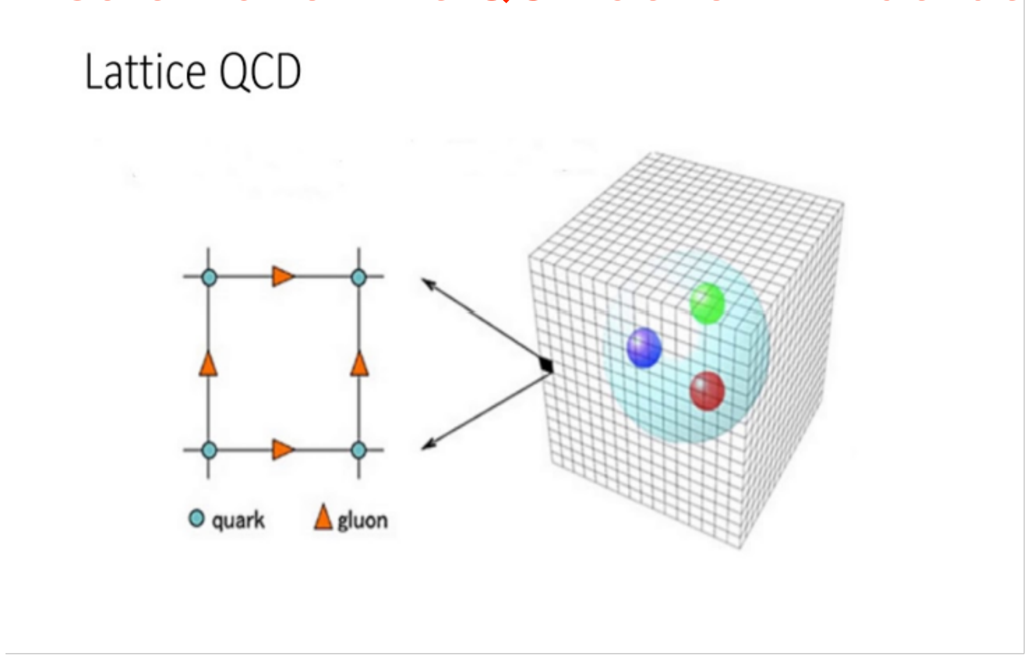
Lattice QCD

Constrain the new physics

# Lattice QCD

- **In lattice QCD method, the correlation functions are non-perturbatively calculated using path integral.**

**Discretization the QCD action in Euclidean space**

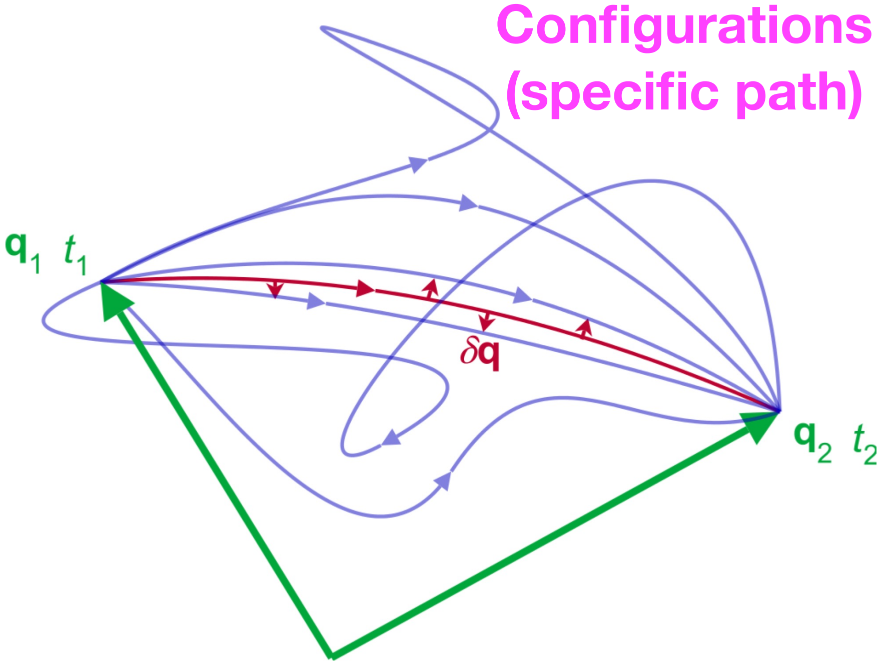



**Partition function**

$$Z = \int \mathcal{D}[U]\, e^{-S_G[U]} \prod_f \det(D[U] + m_f)$$

**The expectational value of operator**

$$\langle O_n \rangle = \frac{1}{Z} \int D[U] O_n[U, m_f] e^{-S_G[U]} \prod_f \det(D[U] + m_f)$$




**The configurations are distributed according to**

$$\frac{1}{Z} \exp(-S_G[U]) \prod_f \det(D[U] + m_f)$$

$$\langle O_n \rangle = \frac{1}{N} \sum_{n=1}^{N} O_n(U_n, m_f)$$

# Outline

- Introduction to nucleon electric dipole moments
- The calculation of EDM on lattice
  - Form factor method
  - Background field method
- Summary

# CP violation matrix element on Lattice

- **Expansion of coupling constants**

Aoki et al (2005); Berruto et al (2005); Shindler et al (2015) ; Alexandrou et al (2015) ; Shintani et al (2016); Dragos et al(2019); Alexandrou et al(2020); Bhattacharya et al (2021) ;Liang et al (2023)

$$e^{-S_{QCD}-i\theta Q} = e^{-S_{QCD}}\left[1 - i\theta Q + \mathcal{O}(\theta^2)\right]$$

$$\langle \mathcal{O}\ldots\rangle_{\not{CP}} = \langle \mathcal{O}\ldots\rangle_{CP-even} - i\theta\langle Q\cdot\mathcal{O}\ldots\rangle_{CP-even} + O(\theta^2)$$

$\not{CP}$ coupling

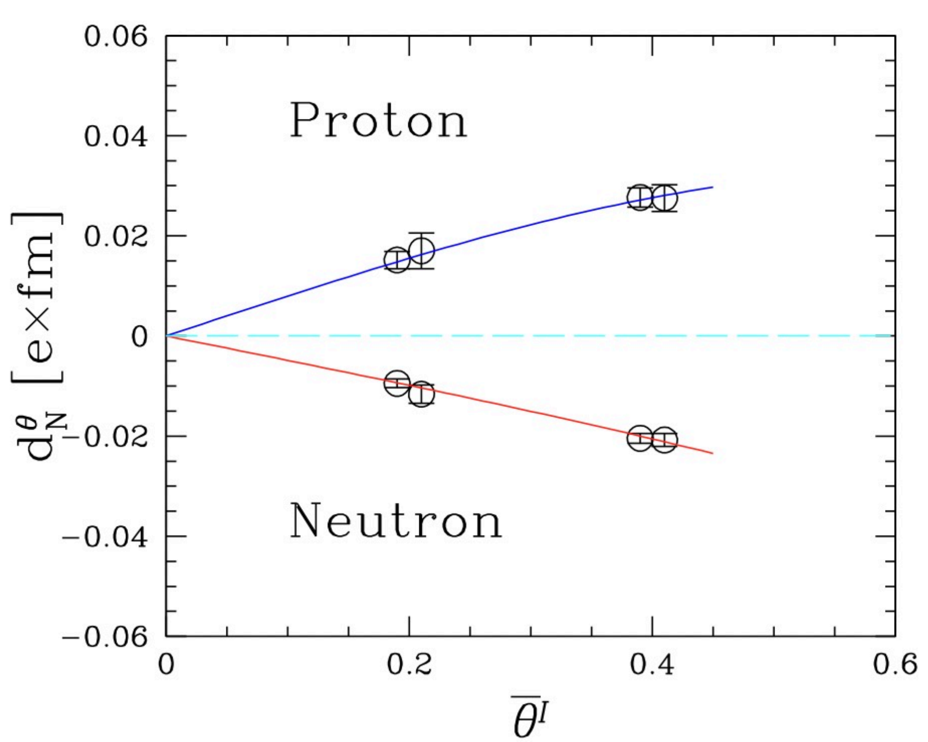
$\not{CP}$ operator: $G\tilde{G}$, cEDM, $GG\tilde{G}$(Weinberg), 4-quark

- **Dynamical simulation including imaginary phase term**

R. Horsley, et al (2008)
F.-K. Guo, et al (2015);

$$\langle \mathcal{O}\rangle \sim \int DU(\mathcal{O})e^{-S_{QCD}-\theta_I Q}$$

$d_n(\theta)$ **is linear dependent on** $\theta$ **when** $\theta$ **is small.**



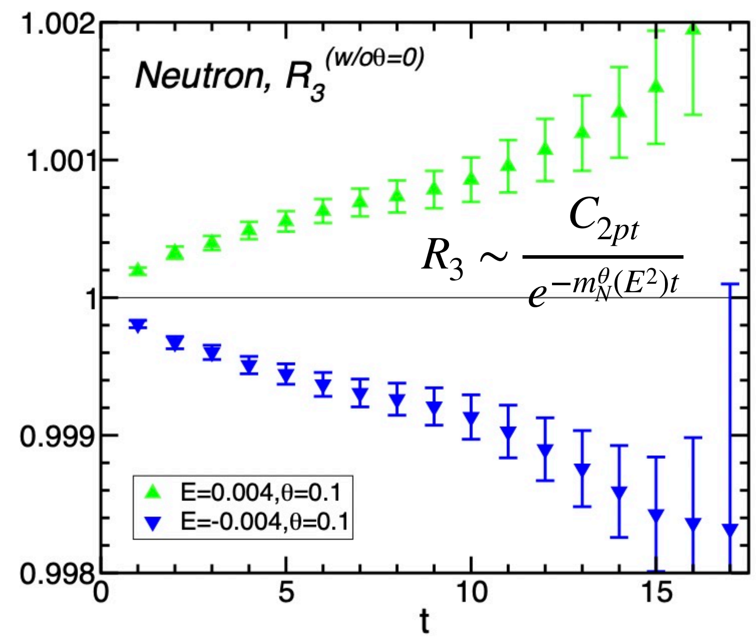
# The calculation of theta EDM on lattice

- **Background electric field method**

E. Shintani et al, 2005

$$\Delta E = \frac{d_n}{2}\vec{\sigma}\cdot\vec{E}$$

Energy shift in the background field



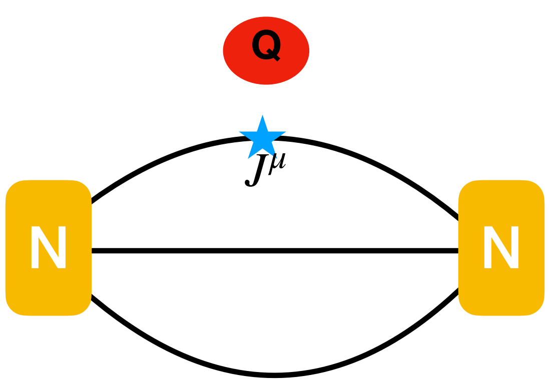

$$C_{2pt} \sim \langle N_\alpha(t)\bar{N}_\alpha(0)e^{i\theta Q}\rangle_{\vec{E}} \sim \exp\Big(-m_N^\theta(E^2)t - \frac{d_N(\theta,E^2)}{2}\vec{\sigma}\cdot\vec{E}t\Big)\Big]$$

**CP violated nucleon 2pt in the background field**

- **Form factor is widely used to extract EDM on lattice QCD, one needs to calculate the "3pt correlation function" with topological charge.**

**Vector current** $J^\mu = \bar{q}\gamma^\mu q$

$$\langle N J^\mu \bar{N}\rangle_{\not{CP}} = \langle N J^\mu \bar{N}\rangle + i\theta\langle N J^\mu \bar{N} Q\rangle$$




$$\langle p',\sigma'|J^\mu|p,\sigma\rangle_{\not{CP}} = \bar{u}_{p',\sigma'}\Big[F_1(Q^2)\gamma^\mu + \big(F_2(Q^2) + iF_3(Q^2)\gamma_5\big)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]u_{p,\sigma},$$

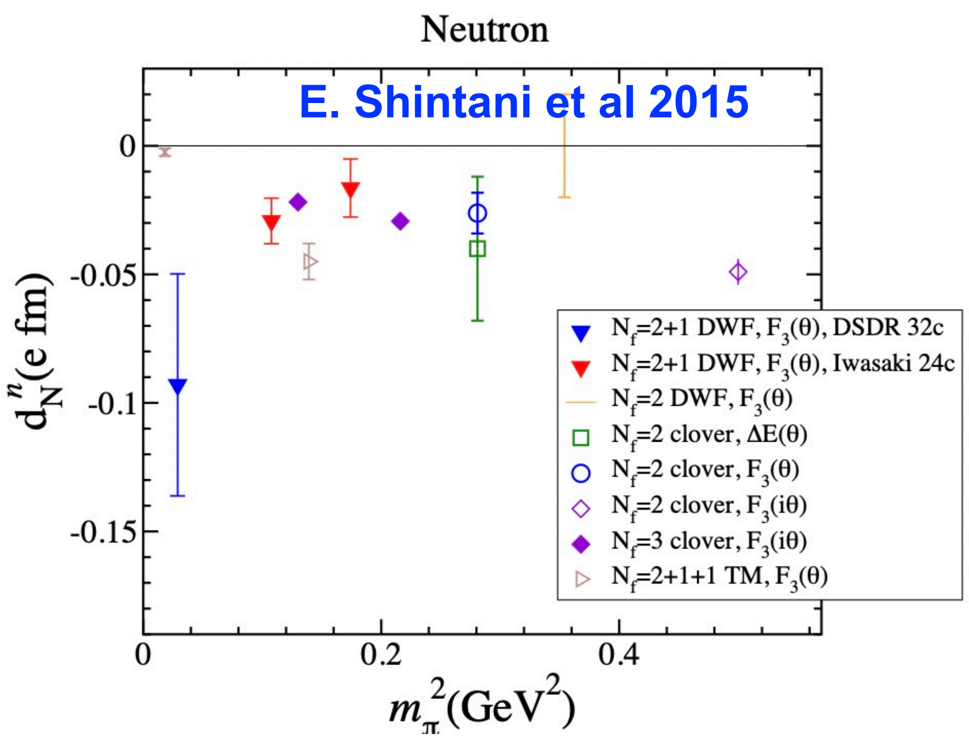
**Dirac form factor** **Electric dipole form factor**

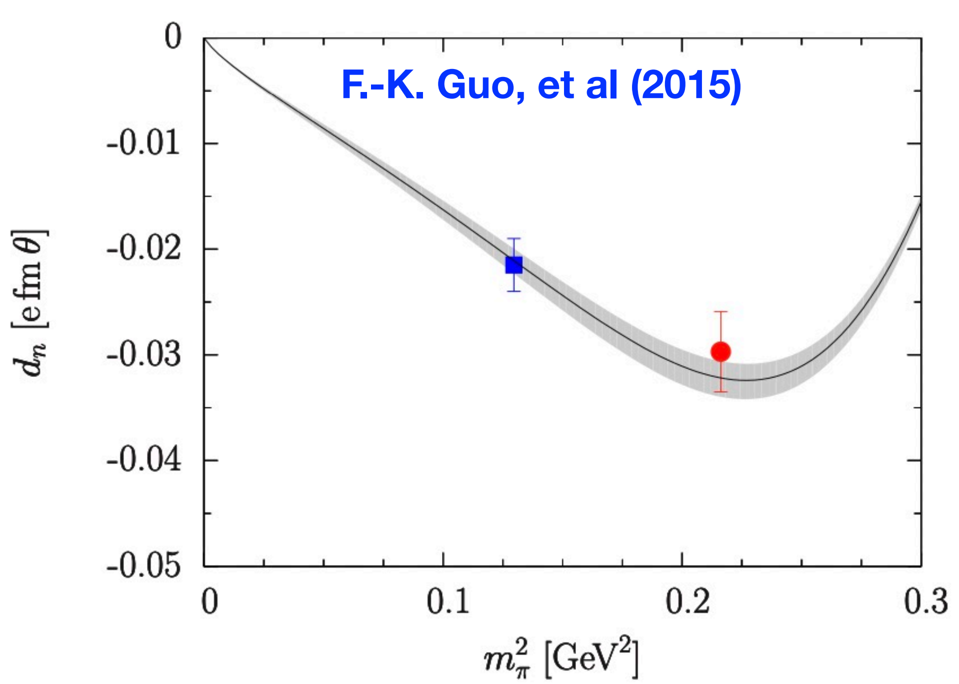
**Pauli form factor**

# Lattice vs phenomenology

- **The comparison of lattice results (pre-2017) with phenomenological results**

**Lattice results**




**Phenomenological results**

| method | value |
|---|---|
| ChPT/NDA | $\sim 0.002$ e fm |
| QCD sum rules [1,2] | $0.0025 \pm 0.0013$ e fm |
| QCD sum rules [3] | $0.0004^{+0.0003}_{-0.0002}$ e fm |

[1]. M. Pospelov, A. Ritz (2000)
[2]. M. Pospelov, A. Ritz (1999)
[3]. J. Hisano, J.Y. Lee, N. Nagata, Y. Shimizu(2012)

**Lattice results** $d_n/\theta \sim 10^{-2} e\,.fm$

**Phenomenological results** $d_n/\theta \sim 10^{-3} e\,.fm$

**The lattice results are an order magnitude larger than the phenomenological results.**

# Mixing between dipole and Pauli form factors

M. Abramczyk, et al 2017

- **Dirac spinor in CPv vacuum and CP even vacuum**

$$\langle 0|N|p,\sigma\rangle_{\not{CP}} = \tilde{u}_{p,\sigma} \qquad \tilde{u}_{p,\sigma} = e^{i\alpha\gamma_5} u_{p,\sigma}$$

- **The matrix element in CPv vacuum**

$$\langle p',\sigma'|J^\mu|p,\sigma\rangle_{\not{CP}} = \bar{\tilde{u}}_{p',\sigma'}\left[\tilde{F}_1(Q^2)\gamma^\mu + \left(\tilde{F}_2(Q^2) + i\tilde{F}_3(Q^2)\gamma_5\right)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]\tilde{u}_{p,\sigma},$$

$$= \bar{u}_{p',\sigma'}\left[F_1(Q^2)\gamma^\mu + \left(F_2(Q^2) + iF_3(Q^2)\gamma_5\right)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]u_{p,\sigma},$$

$\tilde{F}_3$ is used to define EDM before 2017

$$d_n = \frac{\tilde{F}_3(0)}{2m_N}$$

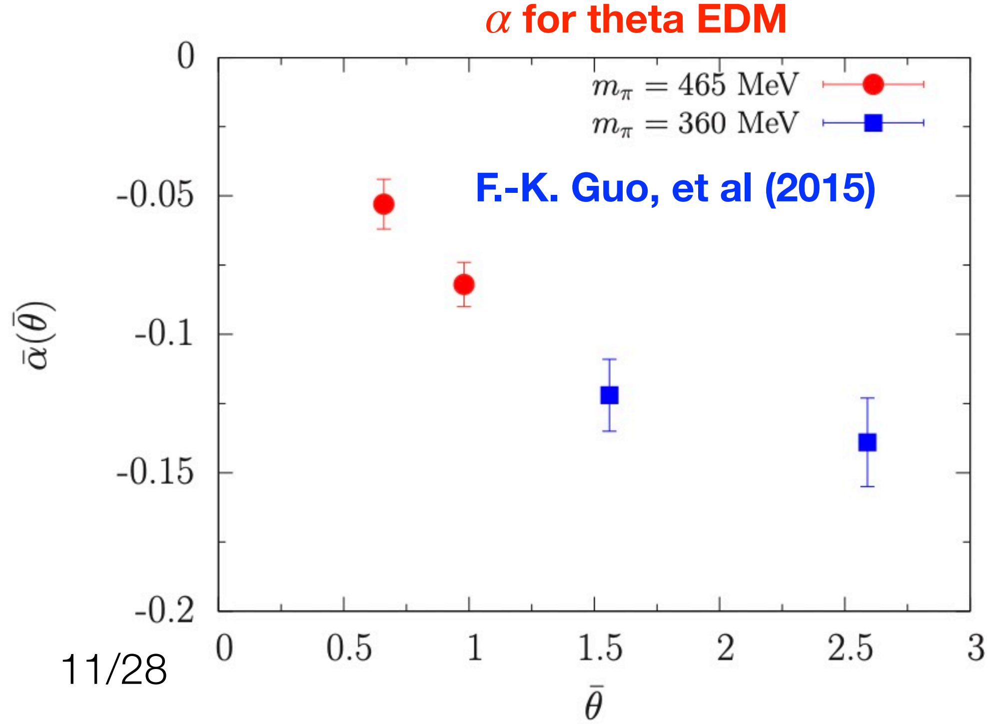
- **The "old definition" of EDFF $\tilde{F}_3$, which is used in lattice calculation prior to 2017 includes a spurious contribution from Pauli form factor.**

Relation between $\tilde{F}_3$ and $F_3$

$$\tilde{F}_3 = F_3 - 2\alpha F_2$$

Correct EDM $\quad d_n = \dfrac{F_3(Q^2 \to 0)}{2m_N}$

$\alpha$ for theta EDM



# Lattice results after subtracting the mixing term

- **Correction to the electric dipole form factor**

$$F_3 = \tilde{F}_3 + 2\alpha F_2.$$

**Correct EDFF** **"Old definition" of EDFF**

$\Theta$-nEDM before and after correction **M. Abramczyk, et al 2017**

| | | | $m_\pi$ [MeV] | $m_N$ [GeV] | $F_2$ | $\alpha$ | $\tilde{F}_3$ | $F_3$ |
|---|---|---|---|---|---|---|---|---|
| [ETMC 2016] | [10] | $n$ | 373 | 1.216(4) | −1.50(16)[b] | −0.217(18) | −0.555(74) | 0.094(74) |
| [Shintani et al 2005] | [5] | $n$ | 530 | 1.334(8) | −0.560(40) | −0.247(17)[a] | −0.325(68) | −0.048(68) |
| | | $p$ | 530 | 1.334(8) | 0.399(37) | −0.247(17)[a] | 0.284(81) | 0.087(81) |
| [Berruto et al 2006] | [6] | $n$ | 690 | 1.575(9) | −1.715(46) | −0.070(20) | −1.39(1.52) | −1.15(1.52) |
| | | $n$ | 605 | 1.470(9) | −1.698(68) | −0.160(20) | 0.60(2.98) | 1.14(2.98) |
| [Guo et al 2015] | [8] | $n$ | 465 | 1.246(7) | −1.491(22)[c] | −0.079(27)[d] | −0.375(48) | −0.130(76)[d] |
| | | $n$ | 360 | 1.138(13) | −1.473(37)[c] | −0.092(14)[d] | −0.248(29) | 0.020(58)[d] |

**The results after subtracting the mixing term are consistent with zero but very noisy**

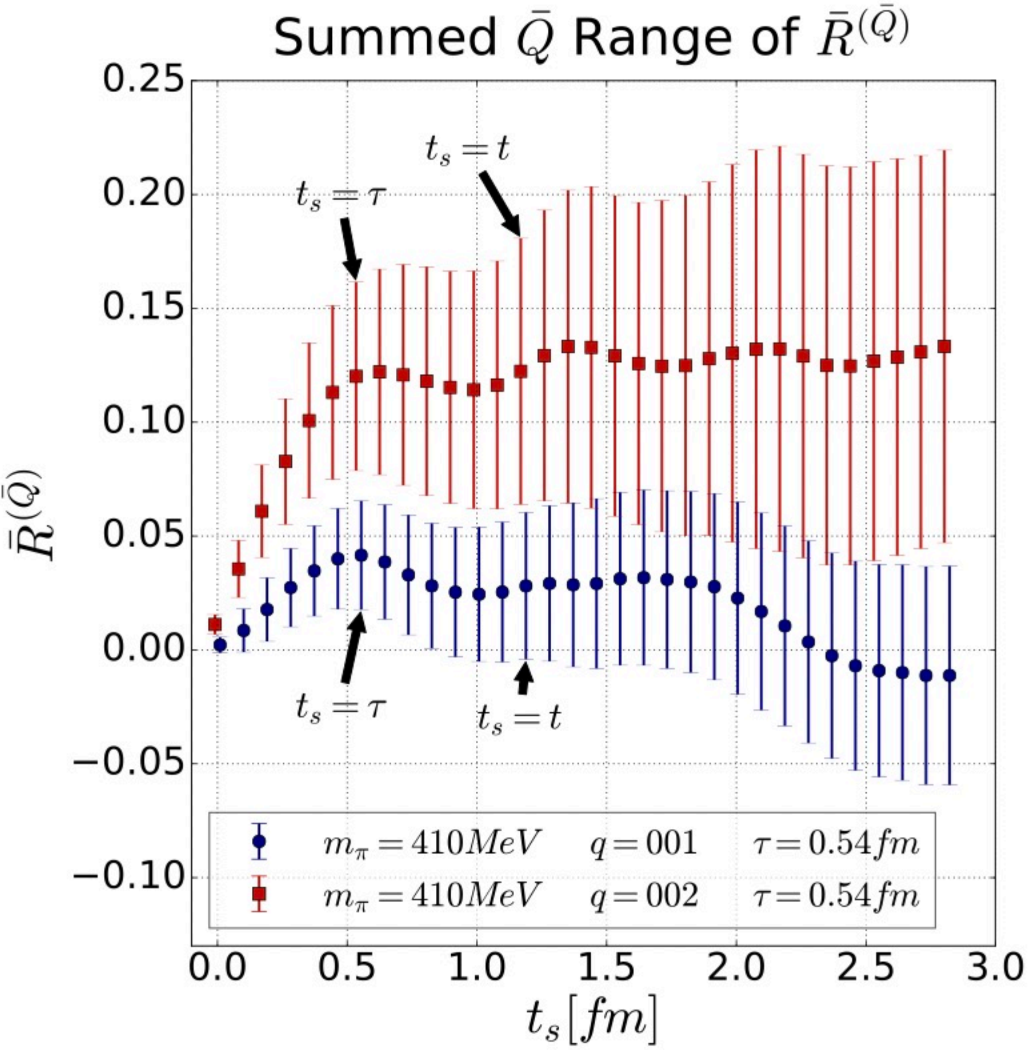
# Variance reduction

**Correlation function for EDM** $d_N \sim \left\langle \sum_{\vec{y}} \sum_{\tau_Q=0}^{T} Q(\vec{y},\tau_Q)(\sum_{\vec{x}} N(\vec{x},t) \sum_{\vec{z}} J^\mu(\vec{z};\tau) N(0)) \right\rangle$

Topological charge density $Q = G\tilde{G}$

**Cut-off in the time direction** Dragos et al(2019)

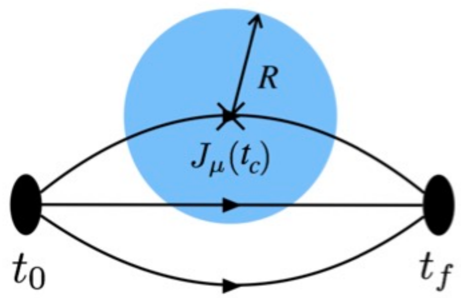
$$d_N \sim \langle \sum_{\vec{y}} \sum_{\tau_Q=0}^{\tau_Q<t_s} Q(\vec{y},\tau_Q)(\sum_{\vec{x}} N(\vec{x},t) \sum_{\vec{z}} J^\mu(\vec{z};\tau) N(0)) \rangle$$

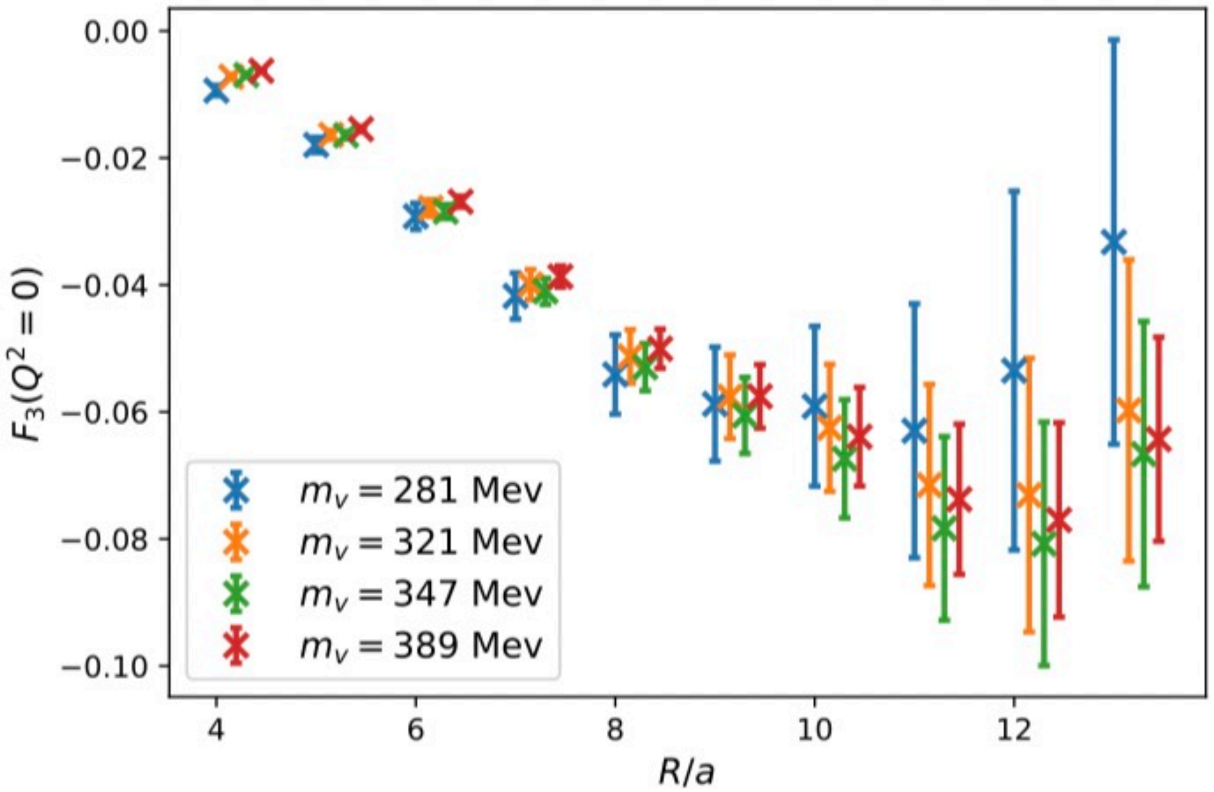



**Signal saturates at** $t_s = \tau$

**Four dimension cut-off** Liang et al (2023)

$$(\tau_Q - \tau)^2 + (\vec{y} - \vec{z})^2 < R^2$$




**Signal saturates at** $R \sim 10a$

# Recent lattice results

- **Recent results about theta EDM (after 2017)**

| | Neutron EDM(e.fm) | Proton EDM(e.fm) |
|---|---|---|
| Dragos et al(2019); | $d_n/\theta = -0.00152(71)$ | $d_p/\theta = 0.0011(10)$ |
| Alexandrou et al(2020); | $\lvert d_n/\theta \rvert = 0.0009(24)$ | — |
| Bhattacharya et al (2021) ; | $d_n/\theta = -0.003(7)(20)$ | $d_p/\theta = 0.024(10)(30)$ |
| Liang et al (2023) | $d_n/\theta = -0.00148(14)(31)$ | $d_p/\theta = 0.0038(11)(8)$ |

| method | value |
|---|---|
| ChPT/NDA | $\sim 0.002$ e fm |
| QCD sum rules [1,2] | $0.0025 \pm 0.0013$ e fm |
| QCD sum rules [3] | $0.0004^{+0.0003}_{-0.0002}$ e fm |

**Using the correct definition of $F_3$, the lattice results are more consistent with the phenomenological results**

# Outline

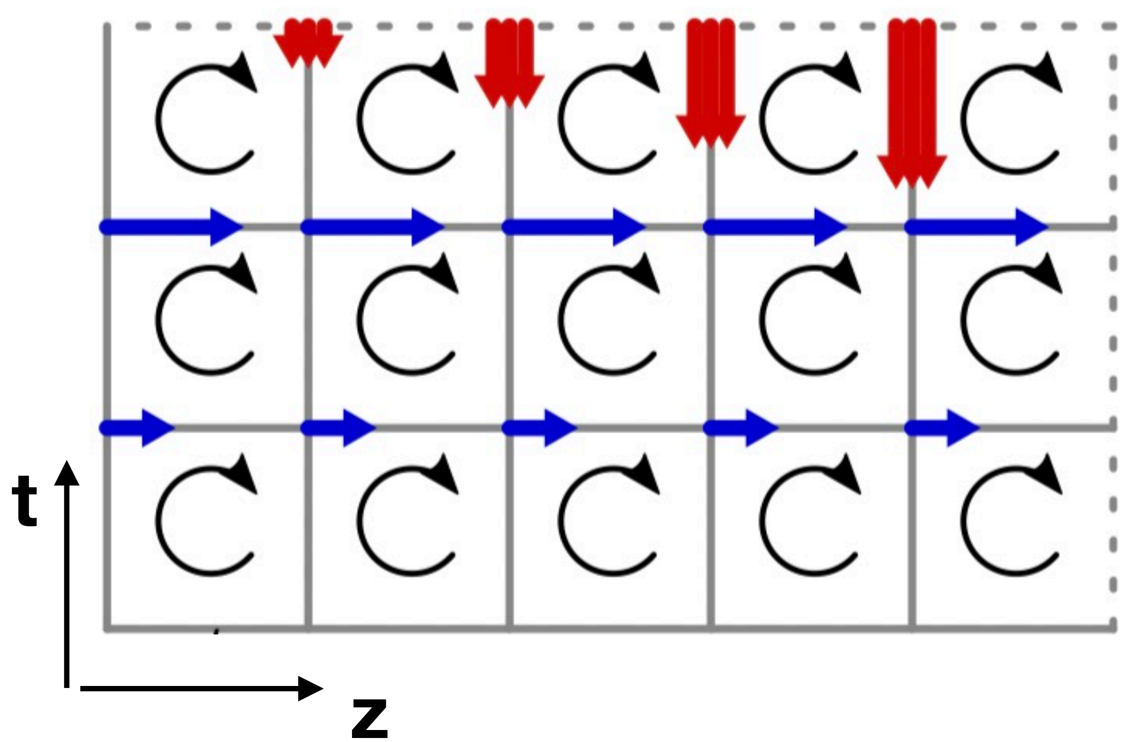
- Introduction to nucleon electric dipole moments
- The calculation of EDM on lattice
  - Form factor method
  - Background field method
- Summary

# Background electric field method

**Neutron energy shift in background electric field** $\Delta E = d_n \overrightarrow{S} \cdot \overrightarrow{\epsilon}$

E. Shintani et al, 2005

**The constant background electric field on Lattice**




**The setup of U(1) gauge link**

$$U_\mu \rightarrow e^{iqA_\mu} U_\mu$$

$$A_z(z, t) = -\epsilon_z t$$

$\epsilon_z$ **: Strength of background field**

$$A_t(z, L_t - 1) = \epsilon_z z \times L_t$$

**Quantization condition** $\epsilon_z = \dfrac{6\pi}{L_t L_x} n$ $\quad n = \pm 1, \pm 2, \ldots$

# Topological charge under gradient flow

[M.Luscher, JHEP08:071; 1006.4518]

**Gradient flow** $$\frac{d}{dt_{\rm GF}} B_\mu(t_{\rm GF}) = D_\mu G_{\mu\nu}(t_{\rm GF}) \,, \quad B_\mu(0) = A_\mu$$

**Tree level results of gradient flow** $$B_\mu(x, t_{\rm GF}) \propto \int d^4y \, \exp\left[-\frac{(x-y)^2}{4t_{\rm GF}}\right] A_\mu(y)$$

**Topological charger with gradient flow time $t_{GF}$** $$\tilde{Q}(t_{\rm GF}) = \int d^4x \, \frac{g^2}{32\pi^2} \left[G_{\mu\nu}\widetilde{G}_{\mu\nu}\right]\Big|_{t_{\rm GF}}$$

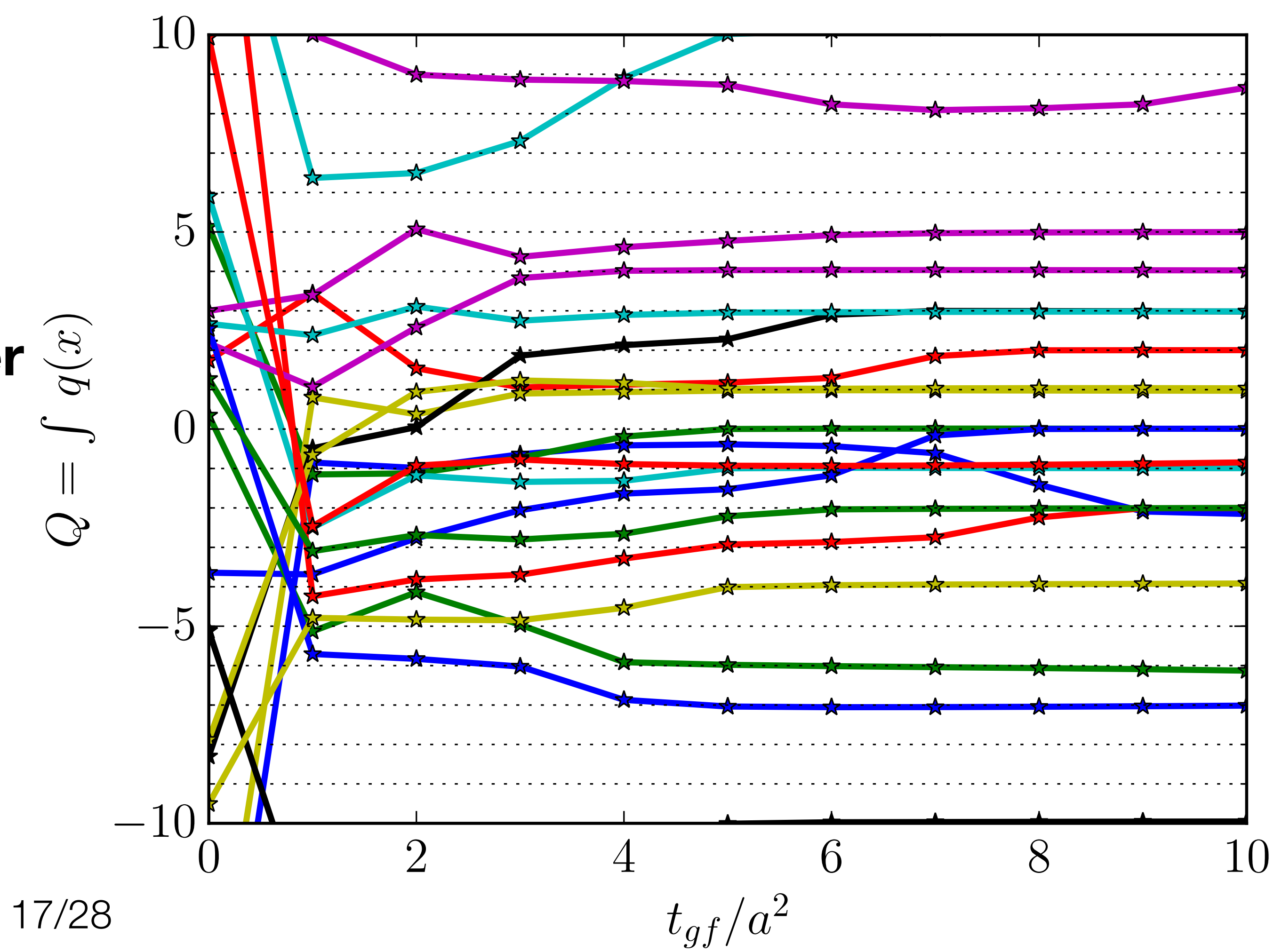
- **effective scale $\Lambda_{UV} \to (t_{GF})^{-1/2}$**
- **remove the UV fluctuation in Q**

→ **Q tends to be integer number**

- **diffusion of top.charge density**

$$q(x)q(0) \sim exp[-(x-y)^2/8t_{GF}]$$

Topological charge Q versus gradient flow



# Numerical results of EDM

- **The EDM can be extracted from the energy shift of 2pt in the background electric field** (T. Izubuchi et al 2020)

$$C^{2\text{pt},\vec{E}}_{\not{CP}}(\vec{0},t) = \langle N(t)\bar{N}(0)e^{i\theta Q}\rangle_E = C_{2\text{pt},\vec{E}}(\vec{0},t) + C^{Q}_{2\text{pt},\vec{E}}(\vec{0},t)$$

$$= |Z_N|^2\left(\frac{1+\gamma_4}{2} - i\frac{\kappa}{2m^2}\gamma_3\gamma_4\varepsilon_z\right)e^{-m_N t} + |Z_N|^2\left(i\alpha\gamma_5 - \frac{1+\gamma_4}{2}\Sigma_Z\delta E t + \frac{\kappa}{m^2}\Sigma_Z\gamma_5\varepsilon_z\right)e^{-m_N t}$$

- **2pt with Tp topological charge**

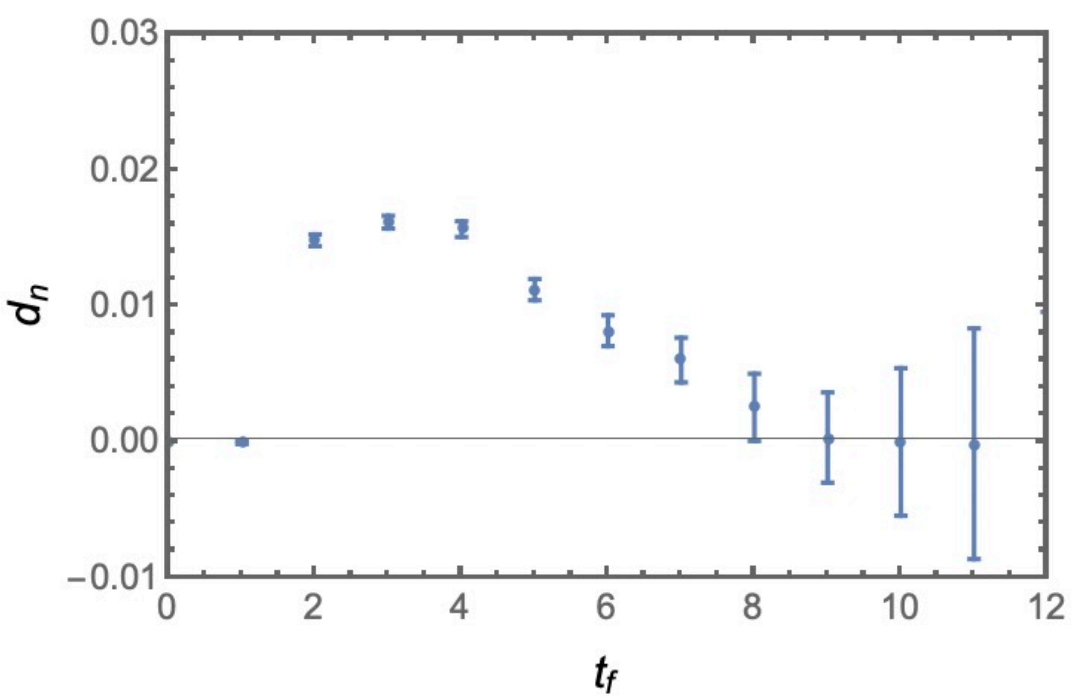
$$C^{Q}_{2\text{pt},\vec{E}}(0,t) = \sum_{\vec{y}}\langle N(\vec{y},t)\left(\sum_{\tau_q=0}^{T}\sum_{\vec{x}}[Q(\vec{x},\tau_q)]\right)\bar{N}(\vec{0},0)\rangle_{\vec{E}}$$

$\delta E = d_n \epsilon_z$

$\Sigma_Z : -i\gamma_x\gamma_y$

- **The extraction of EDM**

$$d_n \propto \frac{\text{Tr}[\Sigma_Z C^{Q}_{2pt,\vec{E}}(0,t_f)]}{\text{Tr}[C_{2pt,\vec{E}}(0,t_f)]} - \frac{\text{Tr}[\Sigma_Z C^{Q}_{2pt,\vec{E}}(0,t_f-1)]}{\text{Tr}[C_{2pt,\vec{E}}(0,t_f-1)]}$$

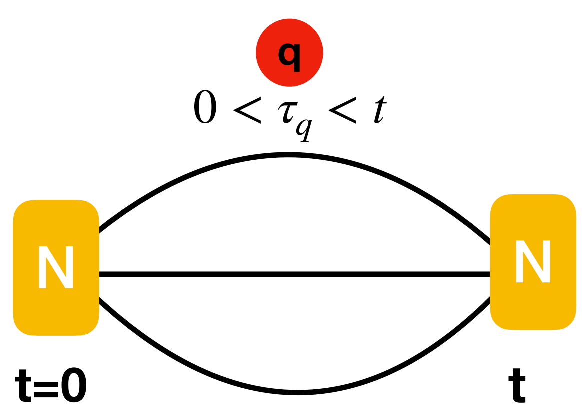


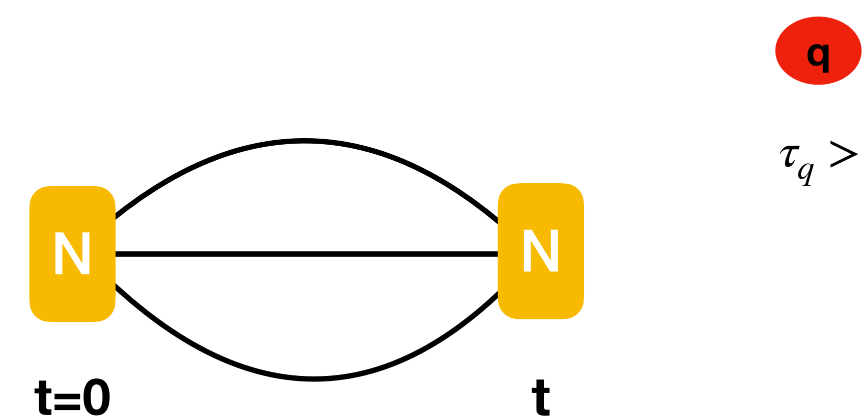
# Spectrum decomposition

- **The spectrum decomposition of 2pt with topological charge**

$$C^{Q}_{2\mathrm{pt},\vec{E}}(0,t) = \sum_{\vec{y}} \langle N(\vec{y},t) \left( \sum_{\tau_q=0}^{T} \sum_{\vec{x}} [Q(\vec{x},\tau_q)] \right) \bar{N}(\vec{0},0) \rangle_{\vec{E}} = \cdot\, |Z_N|^2 \left( i\alpha\gamma_5 - \frac{1+\gamma_4}{2}\Sigma_Z \delta E t + \frac{\kappa}{m^2}\Sigma_Z \gamma_5 \varepsilon_z \right) e^{-m_N t}$$

$$C^{Q}_{2\mathrm{pt},\vec{E}}(0,t) = \sum_{\vec{y}} \langle N(\vec{y},t) \left( \sum_{\tau_q=0}^{t} \sum_{\vec{x}} [Q(\vec{x},\tau_q)] \right) \bar{N}(\vec{0},0) \rangle_{\vec{E}} \quad + \quad O(e^{-E_s t})$$




$$\sim \; t \sum_{\vec{y}} \langle N(\vec{y},t) \sum_{\vec{x}} Q(\vec{x}) \bar{N}(\vec{0},0) \rangle_{\vec{E}}$$

**EDM is related to the local operator in the ground state**

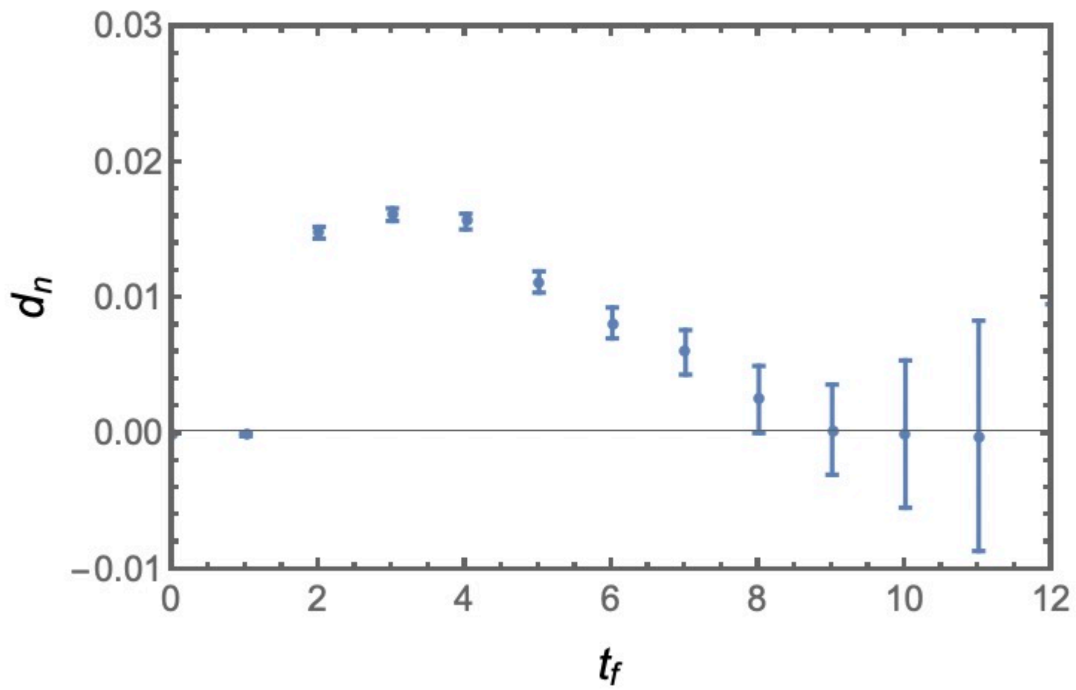
$$\langle N\uparrow | \sum_{\vec{x}} Q(\vec{x}) | N\uparrow \rangle_E = d_n \epsilon_z$$

# Results using local topological(tp) charge

- The comparison of results obtained using local and global topological charge

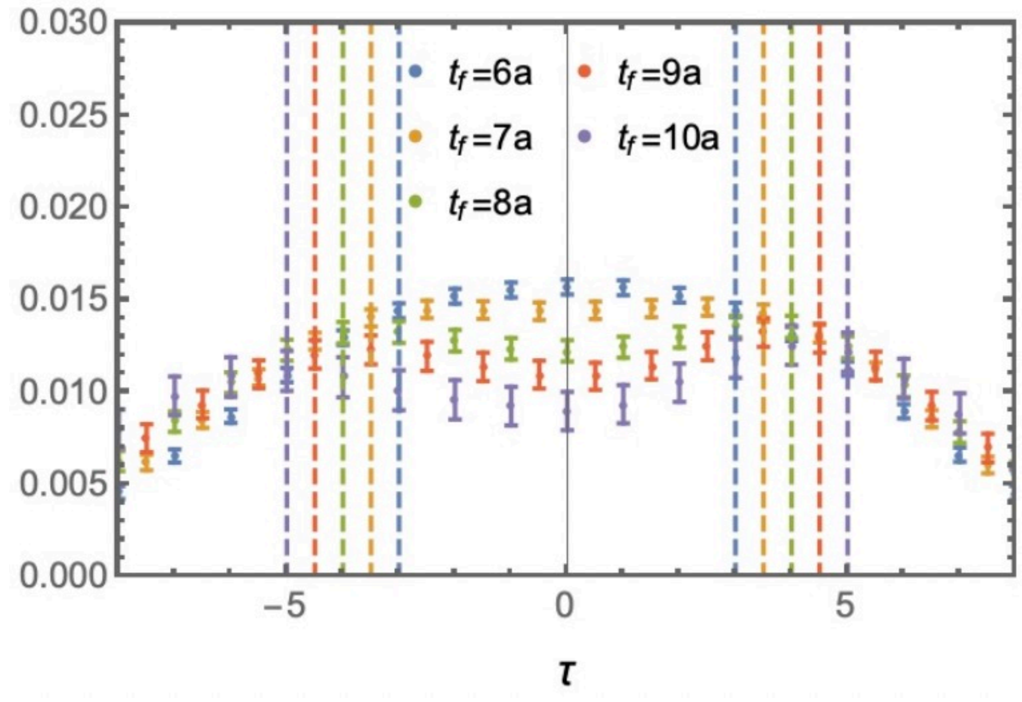
**EDM using Global tp charge**

$$d_n(t_f)\epsilon_z t_f \sim \langle N(t_f) \sum_{\tau} \sum_{\vec{x}} Q(\vec{x}, \tau) N(0) \rangle_E$$




**EDM using local tp charge**

$$d_n(t_f, \tau)\epsilon_z \sim \langle N(t_f) \sum_{\vec{x}} Q(\vec{x}, \tau) N(0) \rangle_E$$

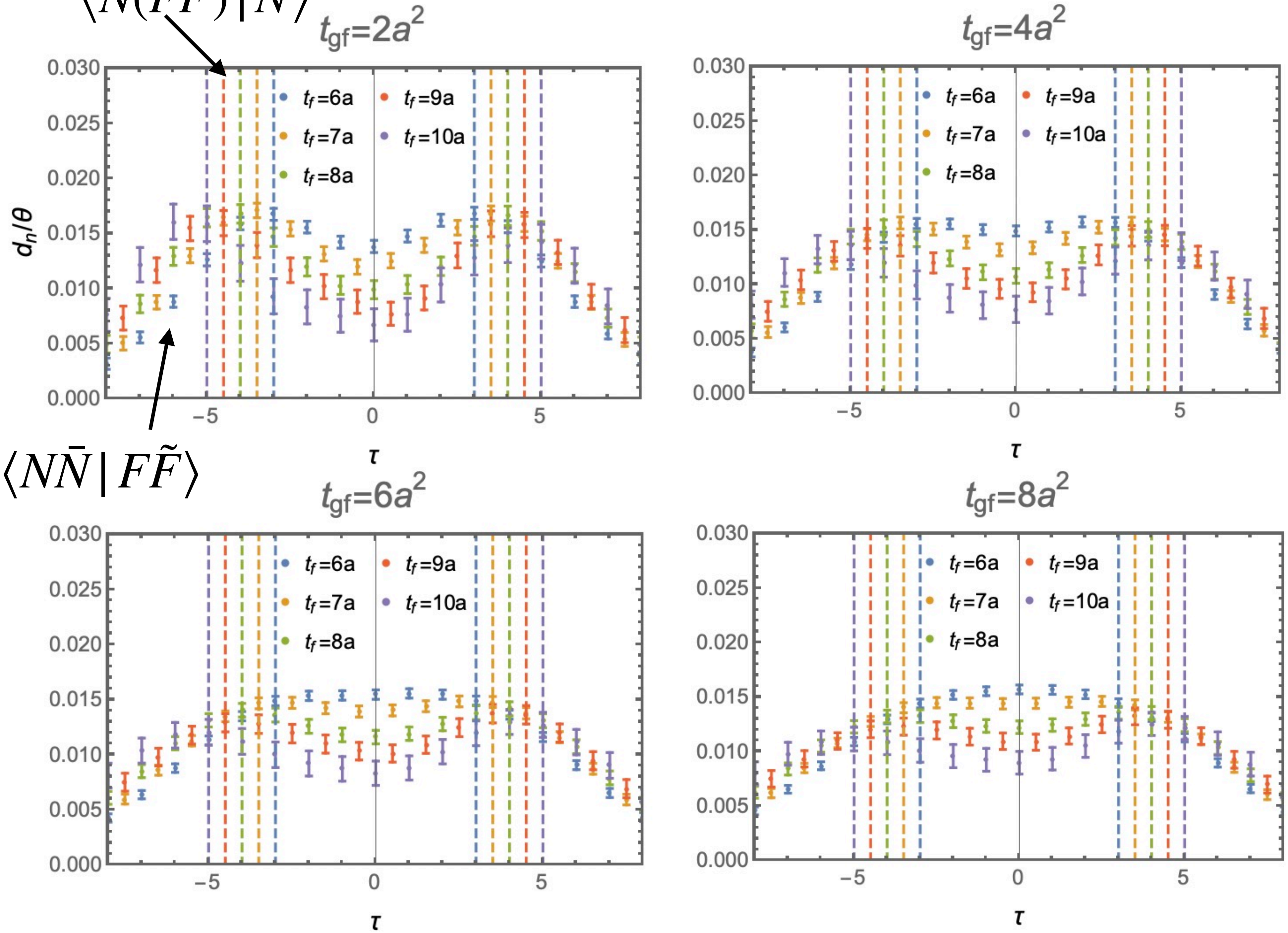



**The results obtained using the local operator have much better signal**

# Gradient flow dependence

**Contact term**

$\langle N(F\tilde{F})\,|\,N\rangle$




$\langle N\bar{N}\,|\,F\tilde{F}\rangle$

- **The noise is suppressed at larger gradient flow time.**
- **The plateau will be shifted due to the diffusion.**

**Gradient flow diffusion**

$$\langle \tilde{q}(\tau, t_{gf})\tilde{q}(0,t_{gf})\rangle \propto e^{-C\frac{\tau^2}{t_{gf}}}$$

$$C_3(t_2^{gf};\tau,t_{sep}) = K(t_2^{gf}-t_1^{gf};|\tau-\tau'|) \overset{\tau'}{\otimes} C_3(t_1^{gf};\tau',t_{sep})$$

**Diffusion kernel**

# The extraction of gradient flow diffusion effect

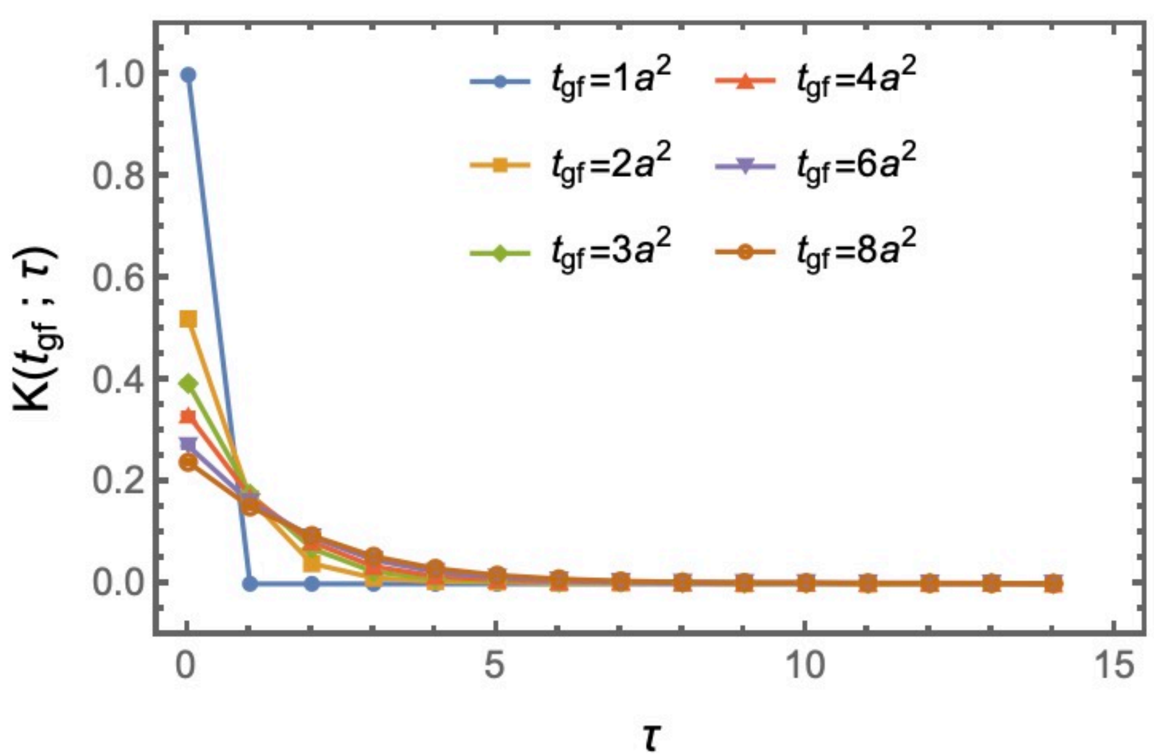
- **The diffusion effect in the gradient flow**

$$\tilde{q}(t_2^{gf};\tau) = \int dt' K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|)\tilde{q}(t_1^{gf};\tau') \xrightarrow{\text{Fourier transformation}} \tilde{q}(t_2^{gf};\omega) = K(t_2^{gf} - t_1^{gf};\omega)\tilde{q}(t_1^{gf};\omega)$$

**The diffusion kernel can be extracted through**

$$K(t_2^{gf} - t_1^{gf};\tau) = \widetilde{\mathrm{FT}}_{\omega\to t}\left[\sqrt{\frac{\mathrm{FT}_{\tau_2\to\omega}[\langle\tilde{q}(t_2^{gf};0)\tilde{q}(t_2^{gf};\tau_2)\rangle]}{\mathrm{FT}_{\tau_1\to\omega}[\langle\tilde{q}(t_1^{gf};0)\tilde{q}(t_1^{gf};\tau_1)\rangle]}}\right]$$

**Diffusion kernel under gradient flow**




**Normalization** $\sum_{\tau} K(t_{gf};\tau) = 1$

**The correlation length become larger with increasing $t_{gf}$**

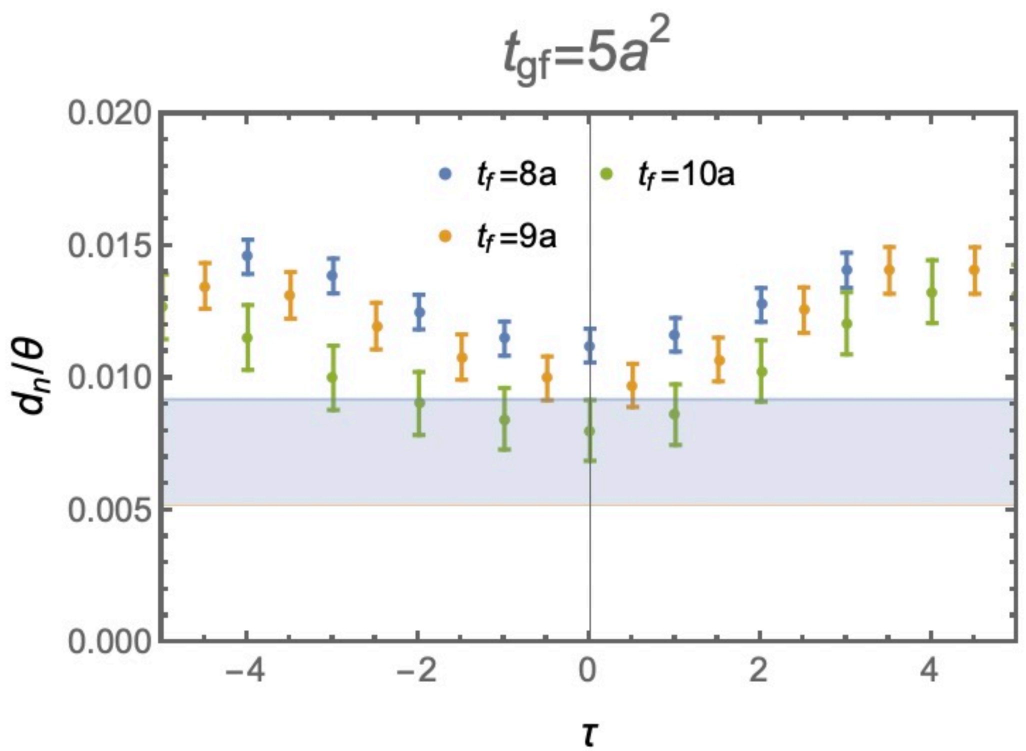
**The correlation will be zero when $\tau > 6$**

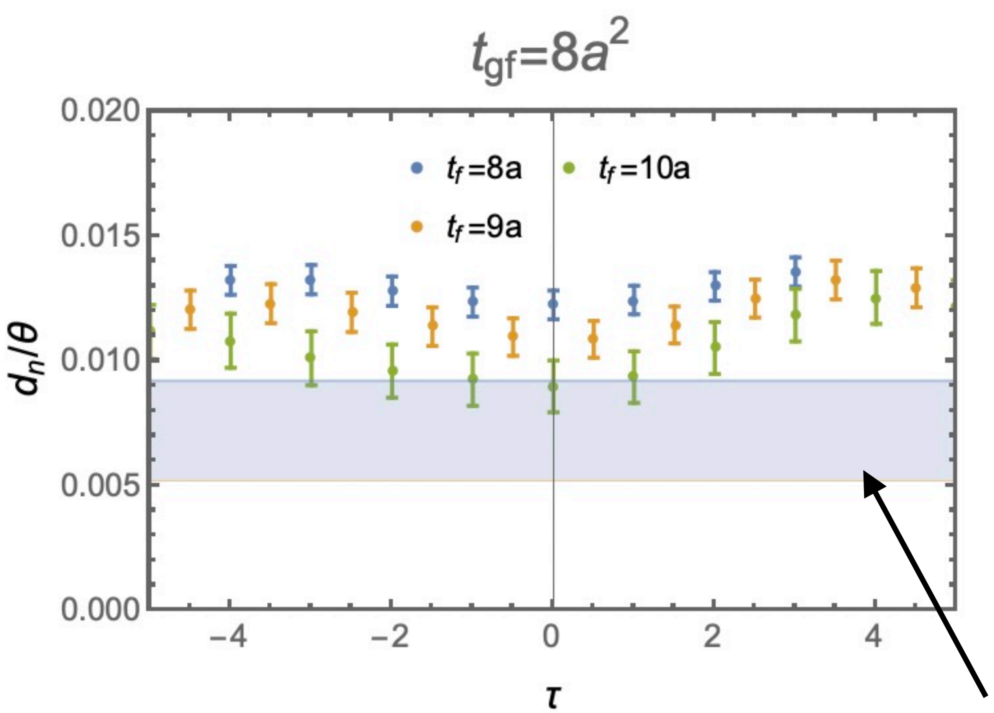
# Fit ansatz and results of ground state

- **Fit ansatz including smearing effect**

**Gradient flow diffusion effect** $\tilde{q}(t_2^{gf};\tau) = \int dt' K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|)\tilde{q}(t_1^{gf};\tau')$

**3pt including diffusion effect** $\tilde{C}_3(t_2^{gf}; t, t_f) = \sum_{\tau'} K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|) C_3(t_1^{gf}; \tau', t_f)$




**Result of Ground state**

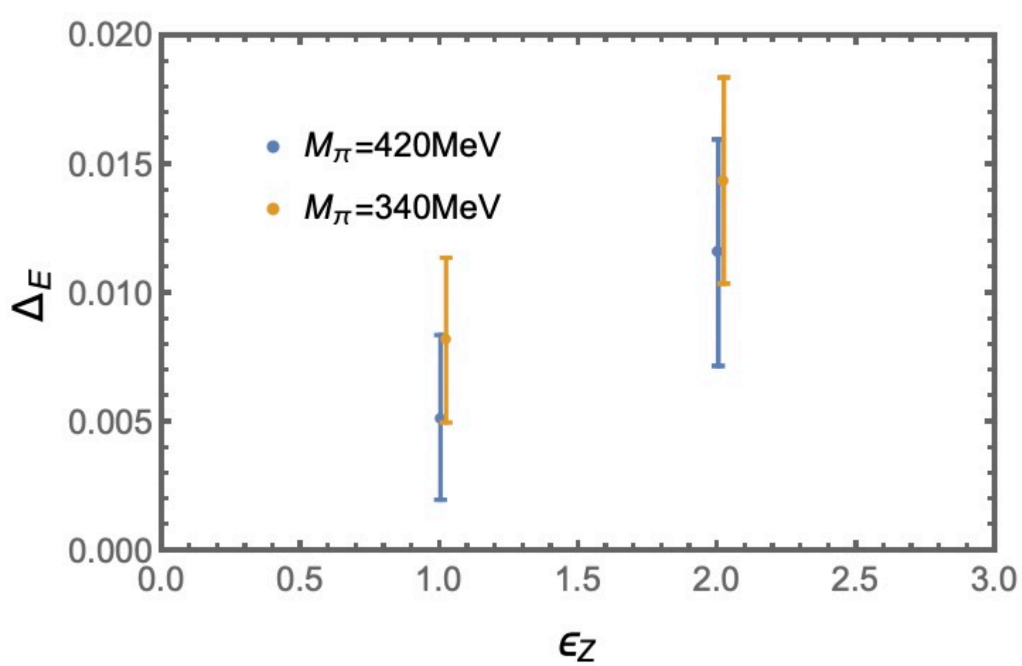
$d_n = 0.007(2)$

# Numerical results

- **The information of configurations we used**

| Lattice size | Lattice spacing | Pion mass | Statistics |
|---|---|---|---|
| $24^3 \times 64$ | 0.1105fm | 340MeV | 1400cfgs |
| $24^3 \times 64$ | 0.1105fm | 420MeV | 1100cfgs |

**Energy shift versus electric field strength**




$$\Delta_E = \langle N\uparrow | \sum_{\vec{x}} q(\vec{x}) | N\uparrow \rangle_E = \frac{d_n}{\theta}\epsilon_z$$

| Pion mass | $d_n/\theta$ |
|---|---|
| 340MeV | 0.007(2) |
| 420MeV | 0.006(2) |

# Chiral extrapolation

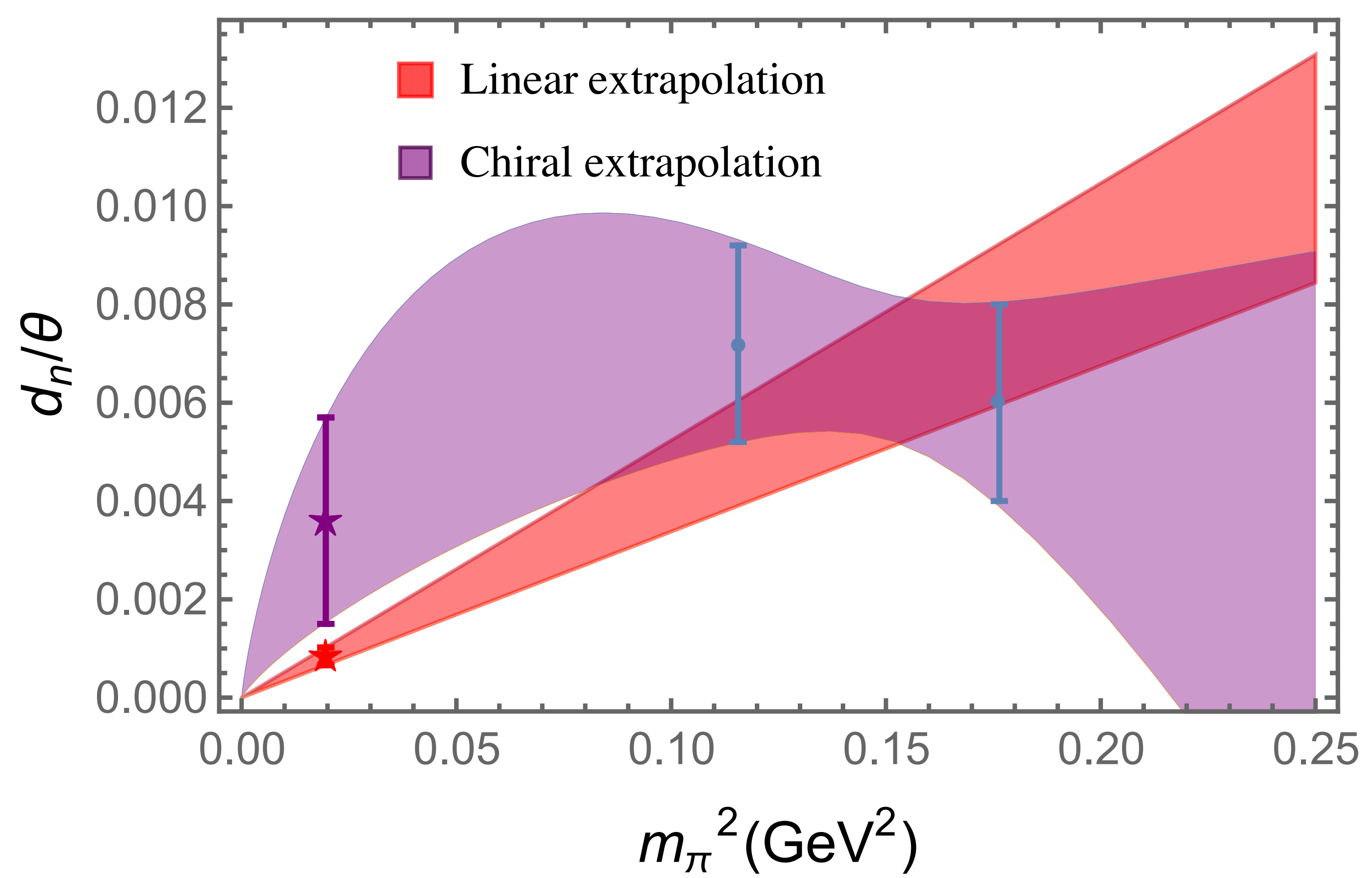
Talk by F. He at Lattice 2023

- **The chiral extrapolation of EDM to the physical point**

**Linear extrapolation:** $d_n = c_0 m_\pi^2$

**ChPT extrapolation:** $d_n = c_1 m_\pi^2 + c_2 m_\pi^2 log(m_\pi^2)$

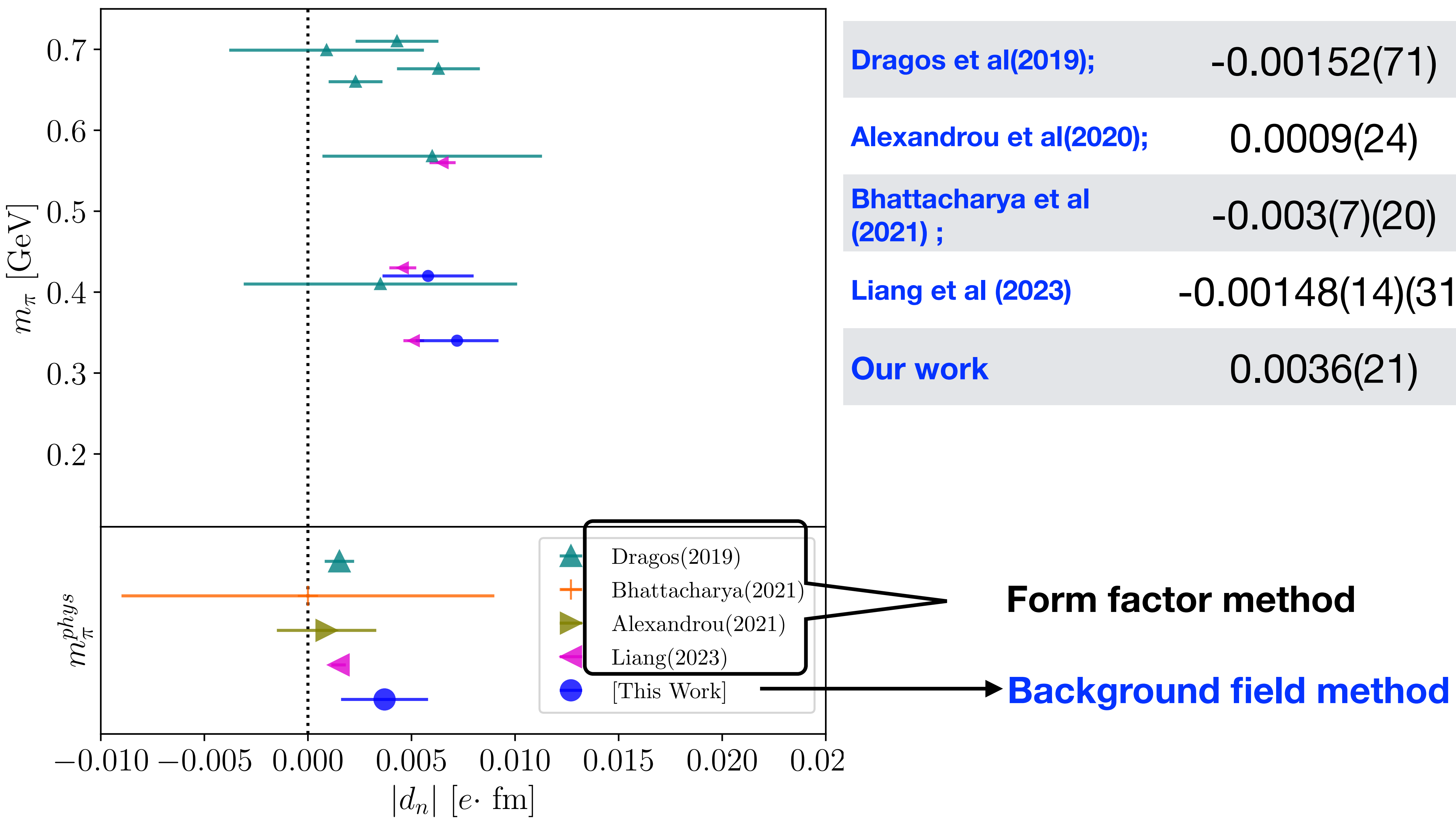
**The EDM should vanish in the chiral limit**




| Extrapolation form | Result at the physics point |
|---|---|
| Linear extrapolation | $d_n/\theta = 0.00084(18)$ |
| ChPT extrapolation | $d_n/\theta = 0.0036(21)$ |

# Summary of neutron θ-EDM from Lattice QCD

- **The summary of $\theta$-EDM calculation from Lattice QCD**




| | Neutron EDM | Proton EDM |
|---|---|---|
| Dragos et al(2019); | -0.00152(71) | 0.0011(10) |
| Alexandrou et al(2020); | 0.0009(24) | — |
| Bhattacharya et al (2021) ; | -0.003(7)(20) | 0.024(10)(30) |
| Liang et al (2023) | -0.00148(14)(31) | 0.0038(11)(8) |
| Our work | 0.0036(21) | — |

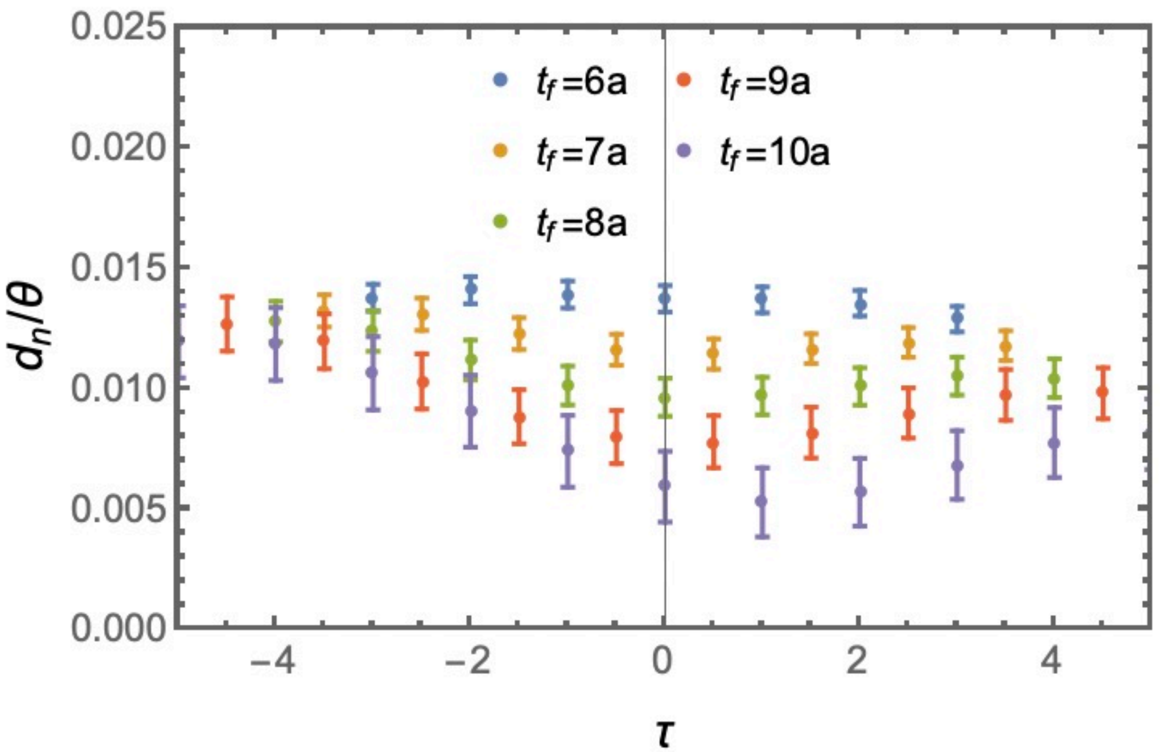
# Topological charge using Fermionic definition

- **partially conserved axial current (PCAC) relation**

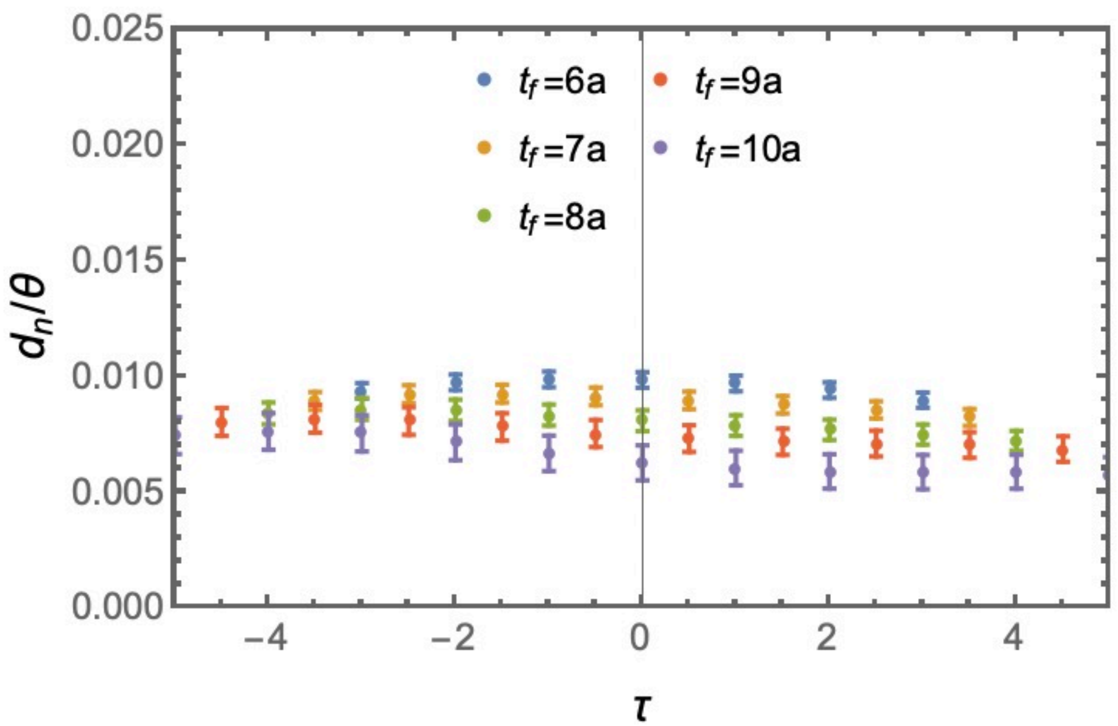
$$\partial_\mu A^\mu = -2i\frac{1}{32\pi^2}tr_c(F\tilde{F})(x) + 2im_q\bar{q}\gamma_5 q(x),$$

**Topological charge** $$Q = \sum_x \frac{1}{32\pi^2}tr_c(F\tilde{F})(x) = \sum_x m_q\bar{q}\gamma_5 q(x),$$

**EDM using gluonic definition**




**EDM using Fermionic definition**




**The results using the topological charge defined by fermionic operator have better signal.**

# Summary

- **Summary:**
  - **Lattice calculation of EDM is challenging. The lattice results prior to 2017 suffered spurious mixing problem.**
  - **We introduce and compare the form factor method and background field method, the results obtained using different methods are consistent.**
  - **The EDM calculated using the topological charge defined by fermionic operator has better signal.**

**Thank you for you attention!**