Generalized Parton Distributions from Lattice QCD

1st CFNS Postdoc Meet

Center for Frontiers in Nuclear Science

Stony Brook

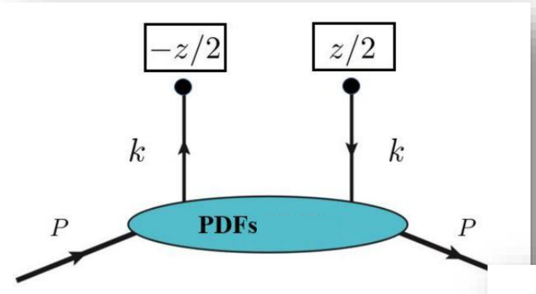
Shohini Bhattacharya
RIKEN BNL

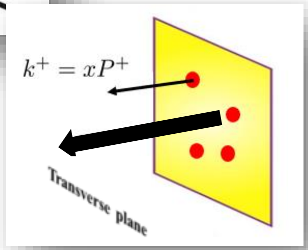
19th October 2023

Non-perturbative functions in QCD

Parton motion
Nucleon motion

Snapshots of the nucleons





Parton Distribution Functions

PDFs (x)

Non-perturbative functions in QCD

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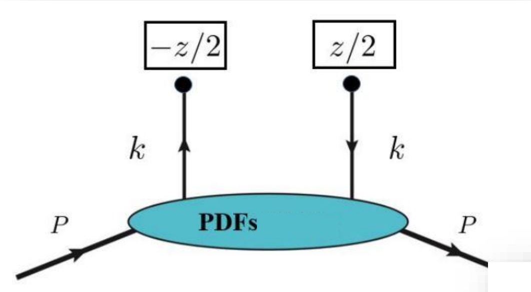
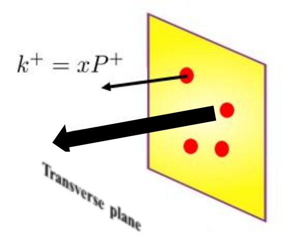
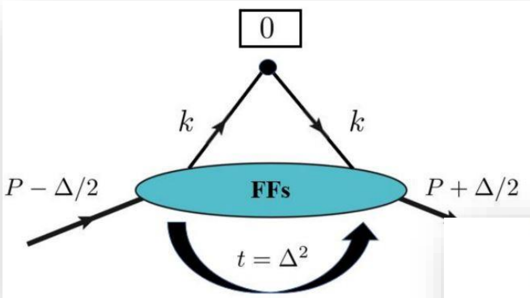


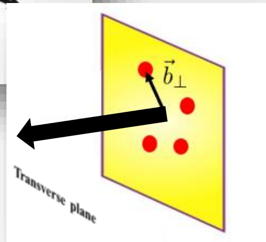


Form **F**actors

PDFs (x)

FFs (Δ)





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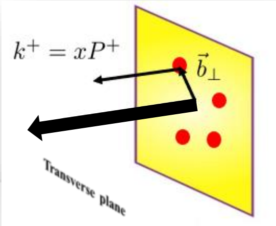
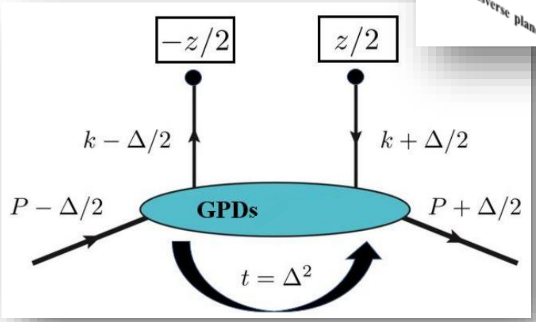
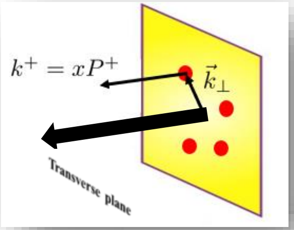
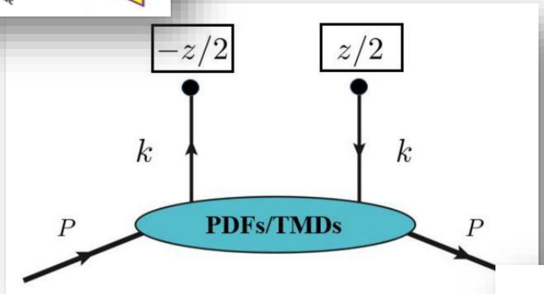
Generalized Parton Distributions

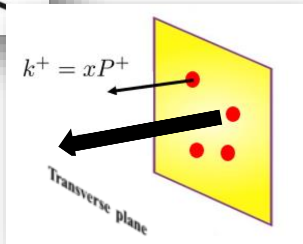




Transverse Momentum-dependent Distributions



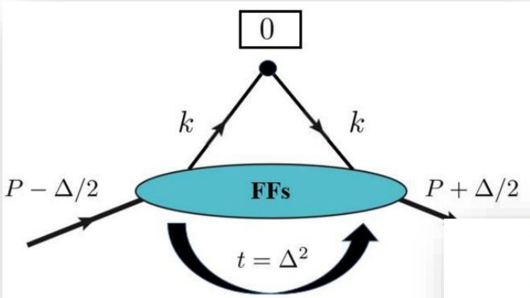


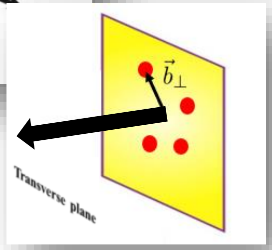


TMDs $(x, \vec{k}_\perp)$ — $\int d^2\vec{k}_\perp$ → **PDFs** (x)

GPDs (x, Δ) — $\Delta = 0$ → **PDFs** (x)

GPDs (x, Δ) — $\int dx$ → **FFs** (Δ)





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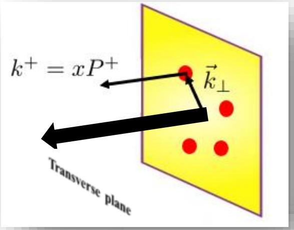
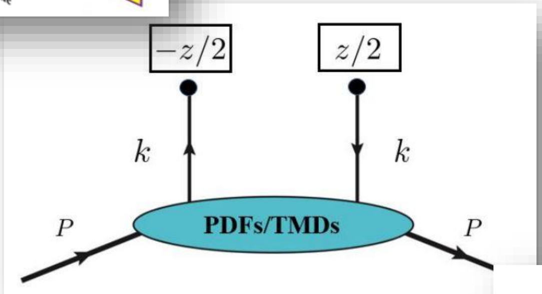
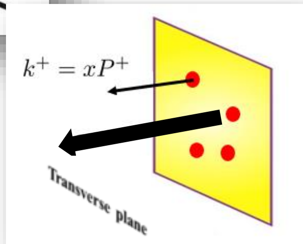
Generalized Transverse Momentum-dependent Distributions







GTMDs $(x, \vec{k}_\perp, \Delta)$

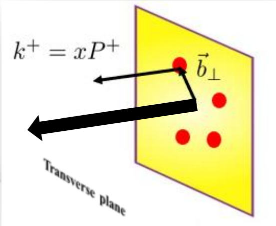
$\Delta = 0$ → **TMDs** $(x, \vec{k}_\perp)$

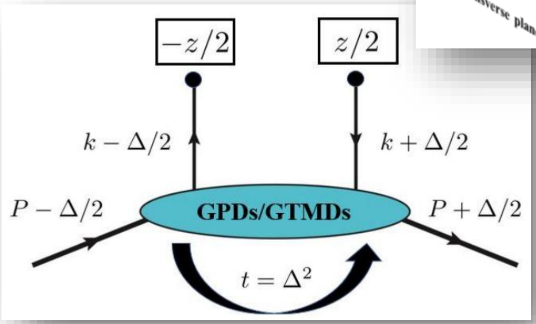
$\int d^2\vec{k}_\perp$ → **GPDs** (x, Δ)

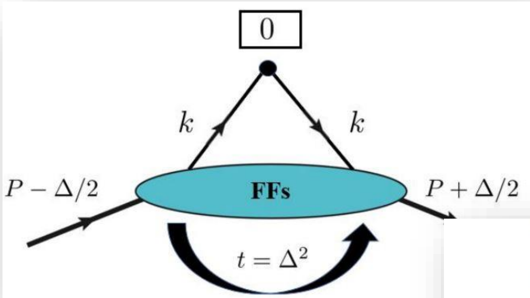
TMDs $(x, \vec{k}_\perp)$ — $\int d^2\vec{k}_\perp$ → **PDFs** (x)

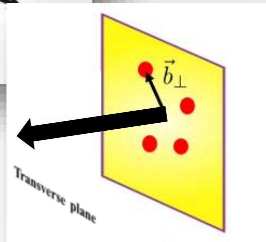
GPDs (x, Δ) — $\Delta = 0$ → **PDFs** (x)

GPDs (x, Δ) — $\int dx$ → **FFs** (Δ)



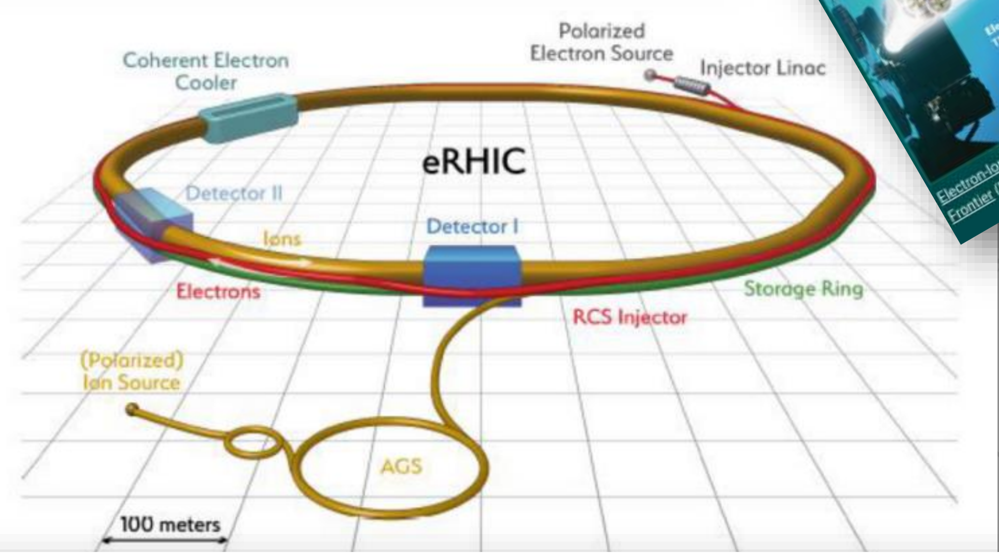






Efforts detailed in a decade worth of reports:

Electron-Ion Collider (EIC)

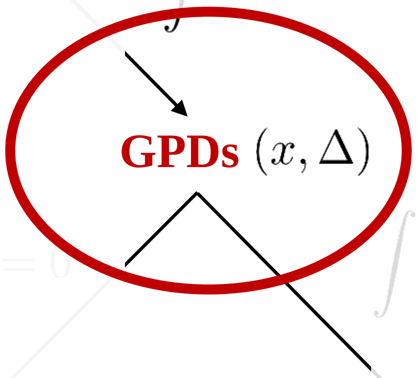


Nucleon tomography (mapping partonic distributions) is one of the major goals of the EIC

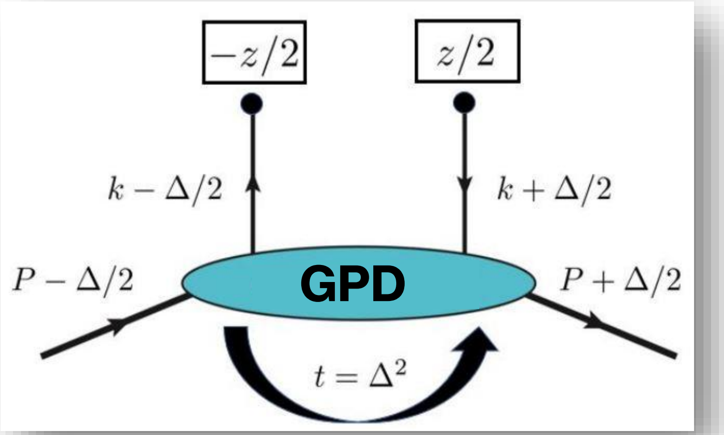
Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC

Outline

- What are GPDs?
- Lattice results of GPDs: state-of-art
- Summary



What are Generalized Parton Distributions?

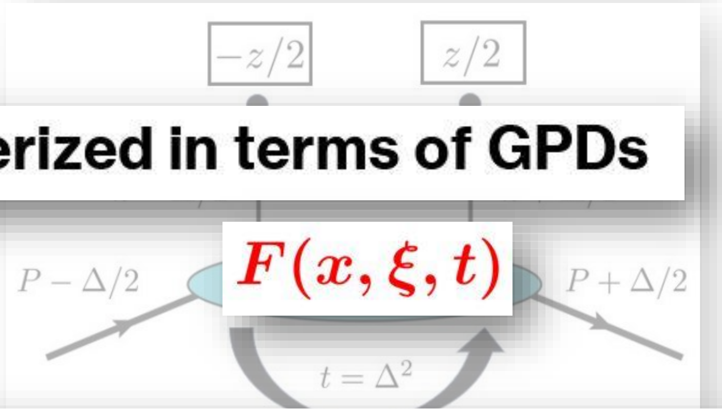


GPD correlator for quarks: Graphical representation

Definition of GPD correlator for quarks:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

What are Generalized Parton Distributions?



Correlator parameterized in terms of GPDs

$F(x,\xi,t)$

- x : "average" longitudinal momentum fraction carried by parton
- ξ : skewness parameter; longitudinal momentum transfer to nucleon
- t : momentum transfer squared

Definition of (

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

What are Generalized Parton Distributions?

Example:

At twist 2 there are 8 GPDs

$z/2$

$F(x,\xi,t)$

GPD corr… …entation

Twist-2 GPDs

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$i\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

Definition of GPD correlator for quarks:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

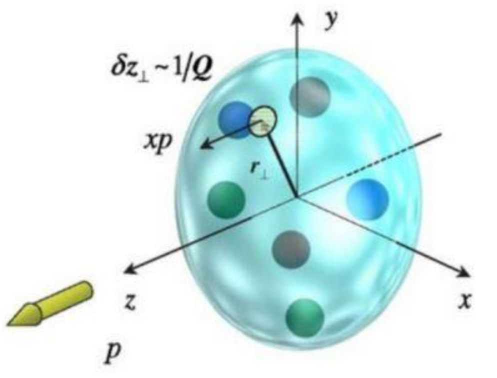
Motivation for studying GPDs

1) **3D imaging** (Burkardt, 0005108 ...)

IMPACT PARAMETER SPACE INTERPRETATION FOR GENERALIZED PARTON DISTRIBUTIONS

MATTHIAS BURKARDT*

Department of Physics, New Mexico State University
Las Cruces, New Mexico 88011, U.S.A. †



3D quark/gluon dist.

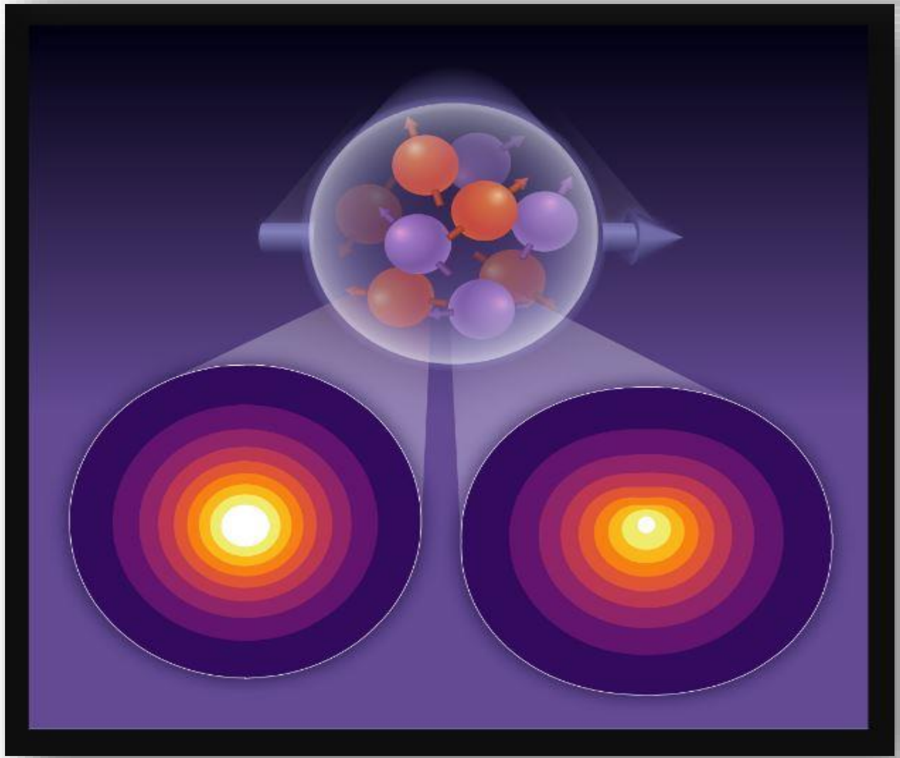
$$\boxed{F(x,\xi=0,\Delta_\perp) \xrightarrow{\mathcal{FT}} f(x,r_\perp)}$$

Motivation for studying GPDs

1) **3D imaging** (Burkardt, 0005108 ...)

Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[3] Xiang Gao,[4,*] Andreas Metz,[3] Joshua Miller,[3] Swagato Mukherjee,[5] Peter Petreczky,[5] Fernanda Steffens,[6] and Yong Zhao[4]

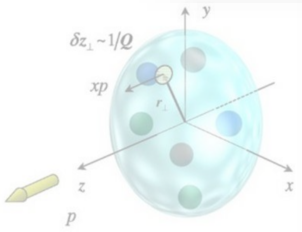


Lattice QCD results of impact-parameter distributions

Motivation for studying GPDs

3D imagin

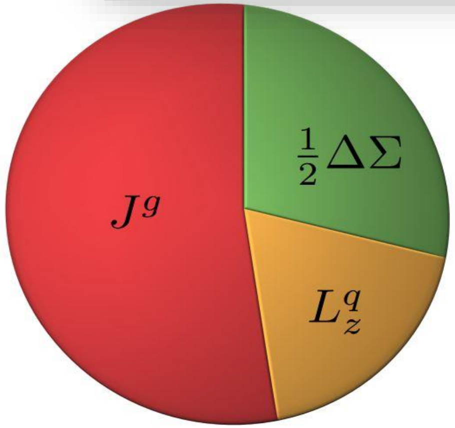
2) Spin sum rule & orbital angular momentum (Ji, 9603249)



3D quark/gluon dist.

GAUGE-INVARIANT DECOMPOSITION OF NUCLEON SPIN AND ITS SPIN-OFF *

Xiangdong Ji



$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma(\mu) + L_z^q(\mu)}_{J^q} + J^g(\mu)$$

Example:

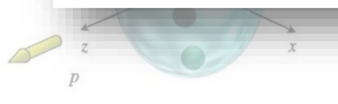
$$\boxed{J^q = \int_{-1}^{1} dx\, x\big(H^q + E^q\big)\Big|_{t=0}}$$

Motivation for studying GPDs

3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)

On "dual" parametrizations of generalized parton distributions

M.V. Polyakov[a,b], A.G. Shuvaev[a]



3D quark/gluon dist.

Exploits relations between GPDs & Gravitational Form Factors:

Gravitational Form Factors:

$$\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)$$

$$A + \xi^2 D = \int_{-1}^{1} dx\, xH \qquad B - \xi^2 D = \int_{-1}^{1} dx\, xE$$

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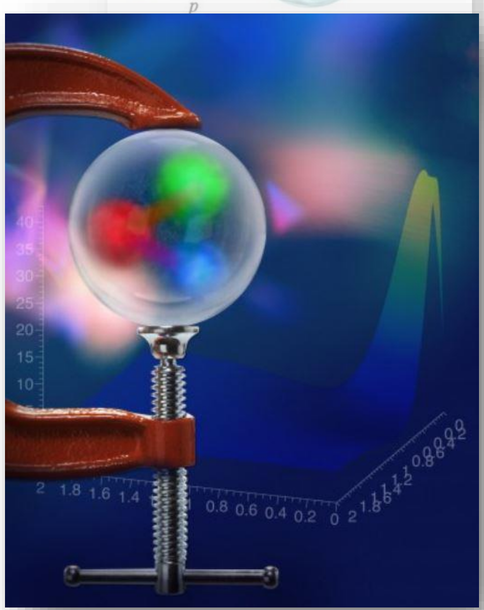
On "dual" parametrizations of generalized parton distributions

M.V. Polyakov[a,b], A.G. Shuvaev[a]



Courtesy: JLab media

LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

QUARKS FEEL THE PRESSURE IN THE PROTON

Motivation for studying GPDs

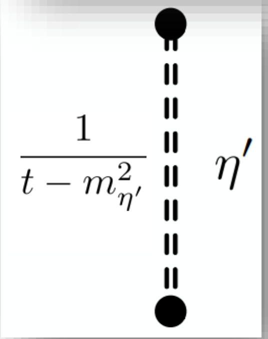
4) Mass generations & chiral symmetry breaking

(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)

Chiral and trace anomalies in Deeply Virtual Compton Scattering: QCD factorization and beyond

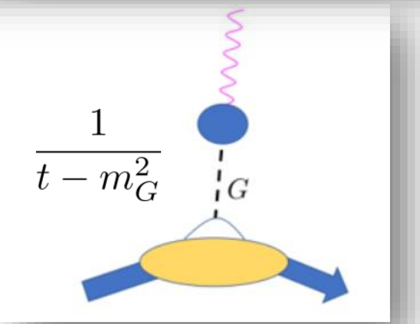
Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

- **Clarified QCD factorization for the first time within $\Lambda^2_{\mathrm{QCD}} \ll t \ll Q^2$ regime: Crucial topic for ongoing & future experiments including at EIC**
- **Unraveled profound & previously undiscovered connections between chiral/trace anomalies & GPDs**



Eta meson mass generation:

$$\boxed{\tilde{E}(x) \sim \frac{1}{t - m^2_{\eta'}}}$$



Glueball mass generation:

$$\boxed{H(x), E(x) \sim \frac{1}{t - m^2_G}}$$

Motivation for studying GPDs

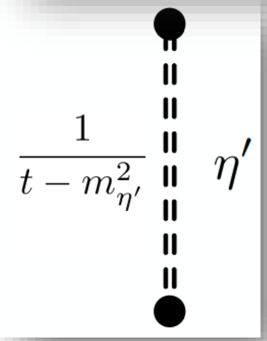
4) Mass generations & chiral symmetry breaking

(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)

Chiral and trace anomalies in Deeply Virtual Compton Scattering: QCD factorization and beyond

Novel avenue of GPD research

Profound physical implication of anomaly poles:
Touches questions on mass generations, Chiral symmetry breaking, ...



Eta meson mass generation:

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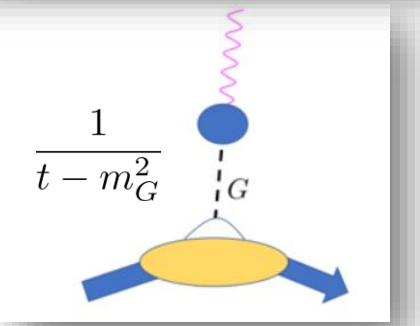

Glueball mass generation:

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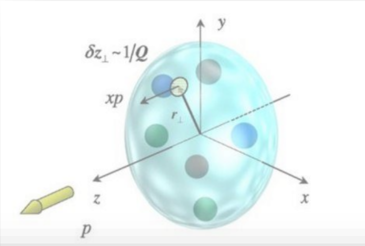


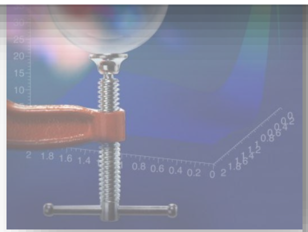
2) **Spin sum rule & orbital angular momentum** (Ji, 9603249)

Example:

$$\boxed{J_q = \int_{-1}^{1} dx\, x (H_q + E_q)\big|_{t=0}}$$

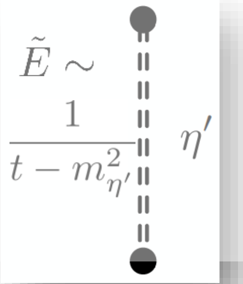
We have numerous compelling reasons to engage in GPD studies!

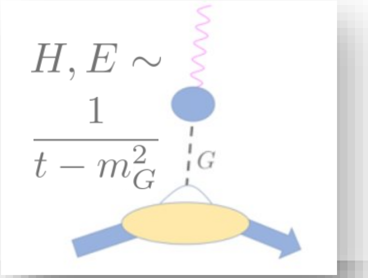
3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)



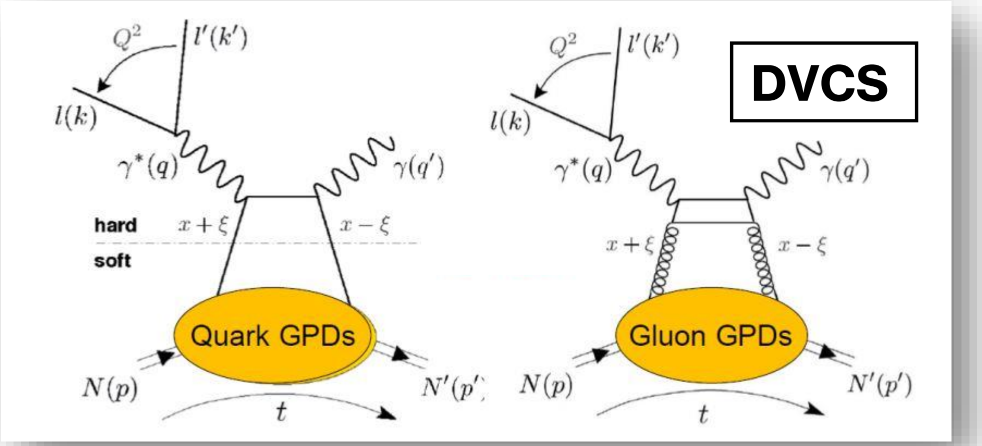
4) **Mass generations & chiral symmetry breaking**

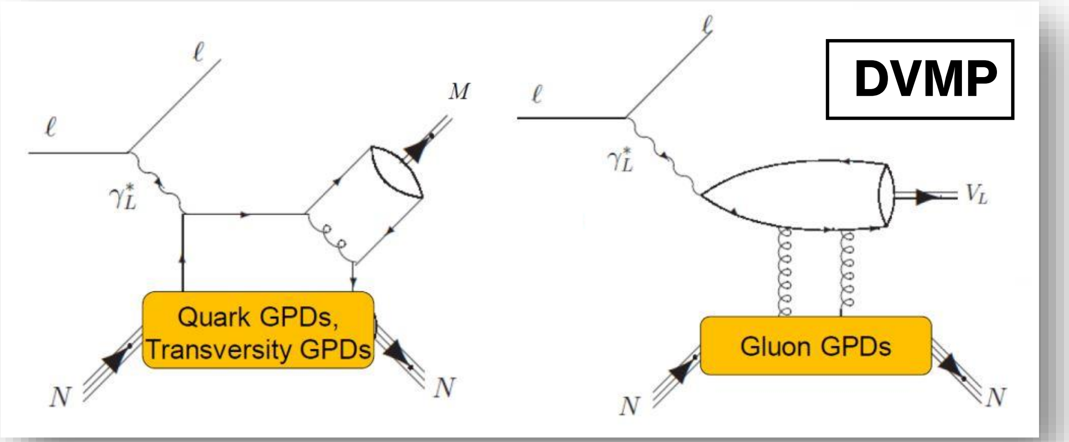
(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)





Physical processes sensitive to GPDs

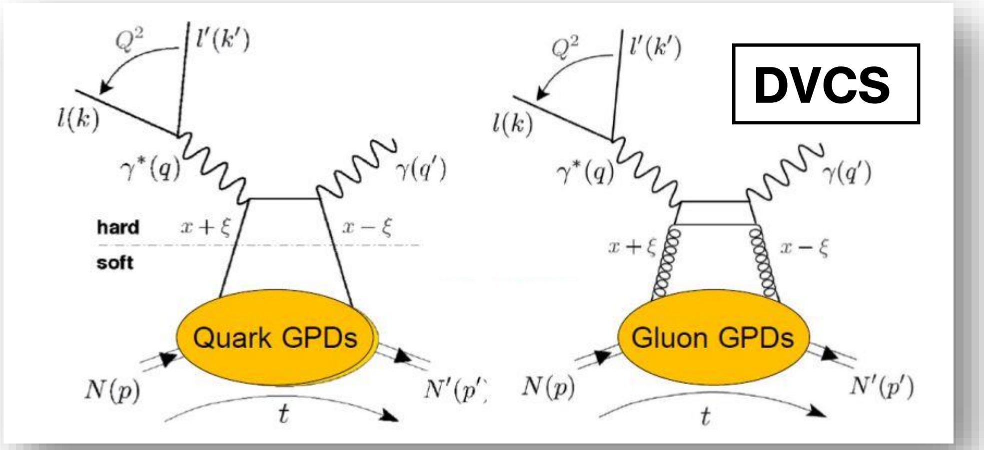


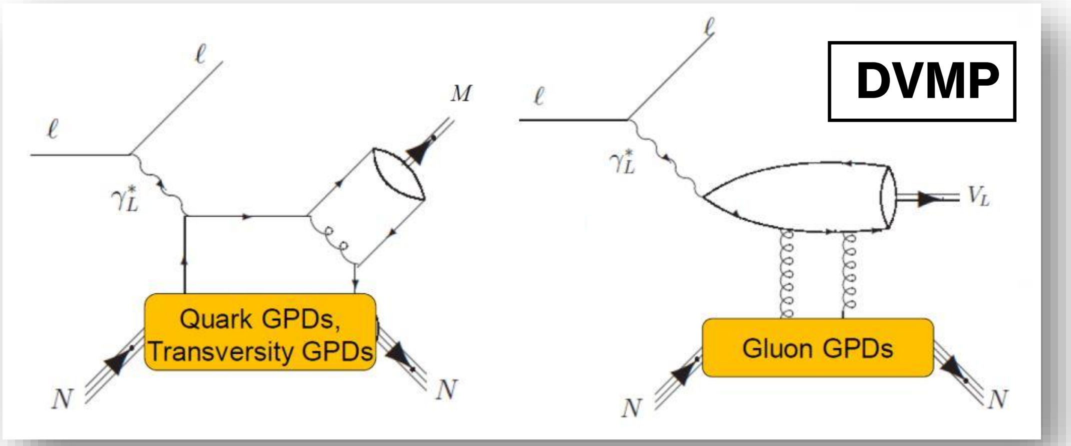


Courtesy: Hyon-Suk Jo, KPS Meeting

No access to x-dependence of GPDs

Physical processes sensitive to GPDs





Courtesy: Hyon-Suk Jo, KPS Meeting

No access to x-dependence of GPDs

Complementarity: Lattice results can be integrated into global analysis of experimental data

Physical processes sensitive to GPDs

Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

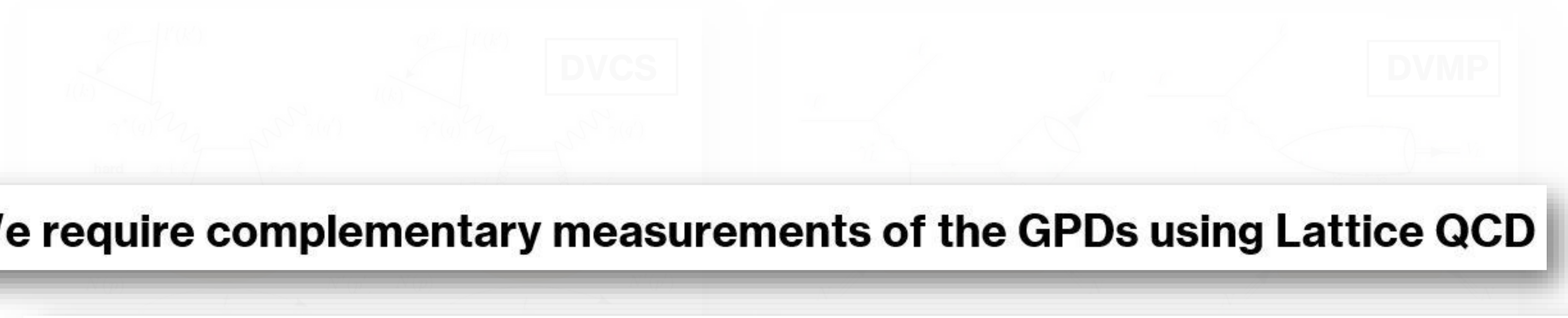
Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,[1] B. Pire,[2] L. Szymanowski,[1] and J. Wagner[1]

(References not exhaustive)

Access to x-dependence of GPDs

Physical processes sensitive to GPDs



We require complementary measurements of the GPDs using Lattice QCD

In recent years, significant breakthroughs have been made in our ability to access the x-dependence of GPDs

Ex...
m... extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,[1] B. Pire,[2] L. Szymanowski,[1] and J. Wagner[1]

(References not exhaustive)

Access to x-dependence of GPDs

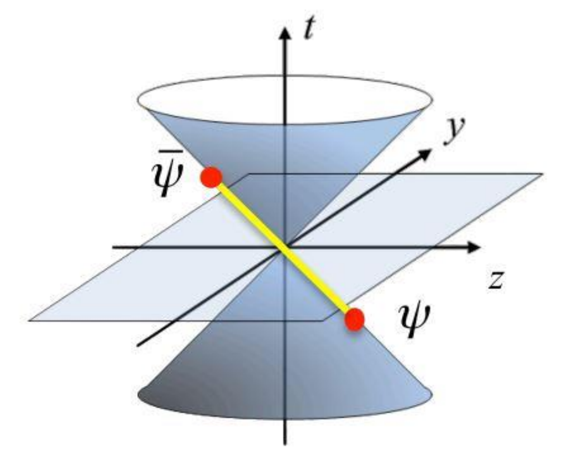
Calculating Parton Distributions in Lattice QCD

“Physical” distributions

Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x, \Delta; \lambda, \lambda') = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p'; \lambda' | \bar{\psi}(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \psi(\tfrac{z}{2}) | p; \lambda \rangle \Big|_{z^+ = \vec{z}_\perp = 0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **Cannot be computed on Euclidean lattice**



Calculating Parton Distributions in Lattice QCD

“Physical” distributions

Parton Physics on Euclidean Lattice

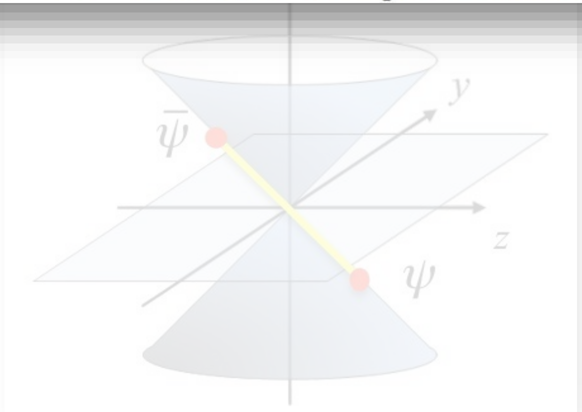
Xiangdong Ji[1,2]

[1]*INPAC, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, 200240, P. R. China*
[2]*Maryland Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

(Dated: May 8, 2013)

Abstract

I show that the parton physics related to correlations of quarks and gluons on the light-cone can be studied through the matrix elements of frame-dependent, equal-time correlators in the large momentum limit. This observation allows practical calculations of parton properties on an

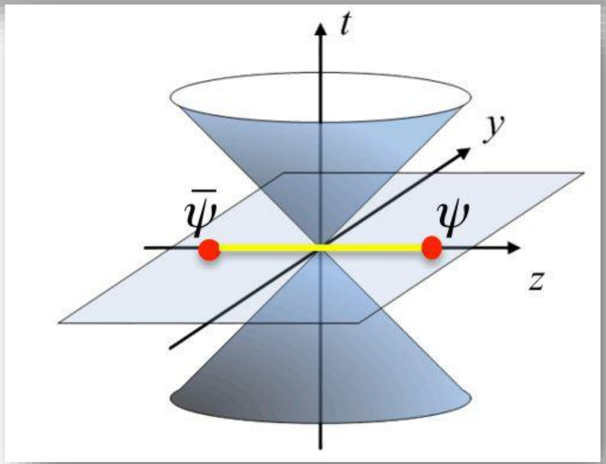


“Auxiliary” distributions

Correlator for quasi-GPDs (Ji, 2013) $\quad -\infty \leq x \leq \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int\frac{dz^3}{2\pi}e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=z_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **Can be computed on Euclidean lattice**



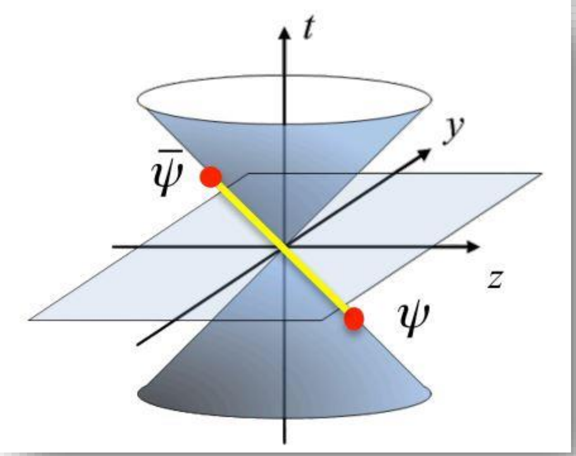
Calculating Parton Distributions in Lattice QCD

“Physical” distributions

Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**

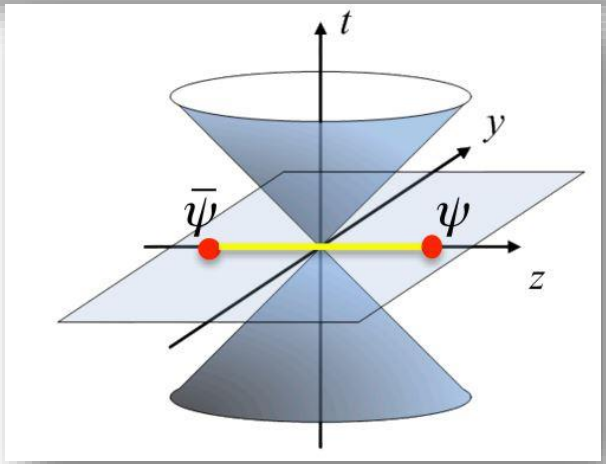


“Auxiliary” distributions

Correlator for quasi-GPDs (Ji, 2013) $-\infty \leq x \leq \infty$

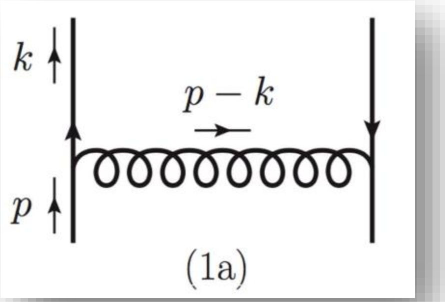
$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**



?

Essence of the quasi-distribution approach (Example: PDF)



Light-cone PDF:

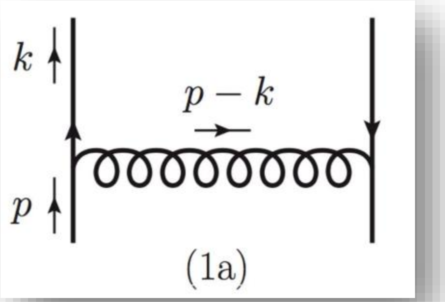
$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2}{x\boldsymbol{m_g^2}} - 2\right)}$$

$$\int_{\mathbf{0}}^{\boldsymbol{\infty}} dk_\perp$$

$$0 < x < 1$$

$$\mathcal{P}_{\rm UV} = \frac{1}{\epsilon_{\rm UV}} + \ln 4\pi - \gamma_E$$

Essence of the quasi-distribution approach (Example: PDF)



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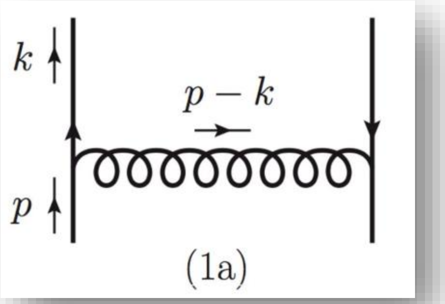
$\int_0^\infty dk_\perp$

Quasi PDF:

$$\boxed{f_1(x,p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases}(1-x)\ \ln\dfrac{x}{x-1}+1 & x>1\\[2ex] (1-x)\ \ln\dfrac{4(1-x)p_3^2}{m_g^2}+x & 0<x<1\\[2ex] (1-x)\ \ln\dfrac{x-1}{x}-1 & x<0\end{cases}}$$

Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



Light-cone PDF:

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$$\int_0^\infty dk_\perp$$

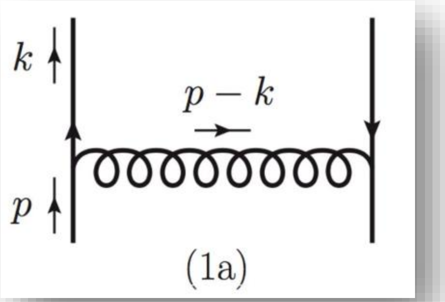
Quasi PDF:

Support outside physical region $0 < x < 1$

$$f_1(x,p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\ln\dfrac{x}{x-1} + 1 & x > 1 \\ (1-x)\ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}$$

Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



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Absence of UV divergence: They manifest only after $\int dx$

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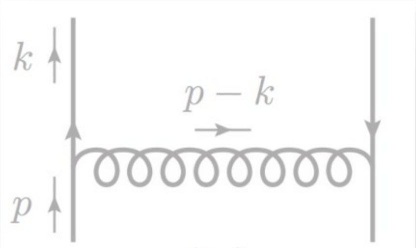

(1a)

Light-cone PDF:

$$f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2}{\ldots} - 2\right) \qquad 0 < x < 1$$

By construction, if one boosts the quasi-observable to infinite-momentum frame, then it reduces to the light-cone observable

$$\int_0^\infty dk_\perp$$

Absence of UV divergence: They manifest only after $\int dx$

Quasi PDF:

Support outside physical region $0 < x < 1$

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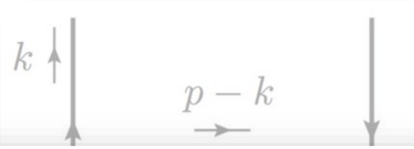
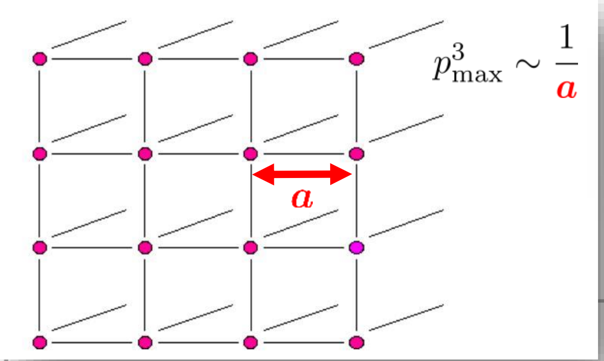
Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)





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By construction, if one boosts the quasi-observable to infinite-momentum frame, then it reduces to the light-cone observable

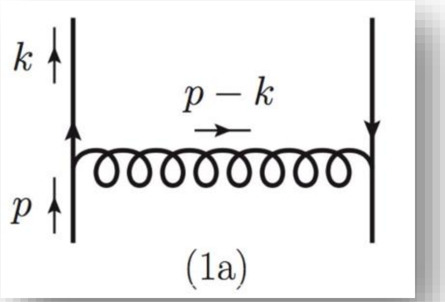
$\int_0^\infty dk_\perp$

Absence of UV divergence: They manifest only after $\int dx$

In lattice computations, UV cut-offs (Λ) are given by the finite lattice spacing a ($\Lambda \sim a^{-1}$), and one (naturally) deals with UV renormalization before taking the limit $P^3 \to \infty$. The limits $\Lambda \to \infty$ and $P^3 \to \infty$ do not commute, which leads to non-trivial differences in the UV behavior of the quasi-PDFs and light-cone PDFs.

$$f_1(x, p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\ln\frac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ln\frac{x-1}{x} - 1 & x < 0 \end{cases}$$

Essence of the quasi-distribution approach (Example: PDF)



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IR pole structure of light-cone & quasi-PDFs are same

Calculating Parton Distributions in Lattice QCD

Matching formula: **(PDF)**

Matching coefficient

$$\tilde{q}(x,\mu,P^3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P^3}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^3)^2}, \frac{M_N^2}{(P^3)^2}\right)$$

Xiong, Ji, Zhang, Zhao/ Stewart, Zhao/ Izubuchi, Ji, Jin, Stewart, Zhao ...

Essence of the quasi-PDF approach

IR pole structure of light-cone & quasi-PDFs are same

Calculating Parton Distributions in Lattice QCD

Matching formula: **(GPD)**

Matching coefficient

$$\tilde{q}(x,\xi,t,\mu,P^3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{P^3}\right) q(y,\xi,t,\mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(P^3)^2},\frac{M_N^2}{(P^3)^2},\frac{t}{(P^3)^2}\right)$$

GPD matching known up to one-loop order (non-singlet & singlet)

References: (not exhaustive)

Connecting Euclidean to light-cone correlations: From flavor nonsinglet in forward kinematics to flavor singlet in non-forward kinematics

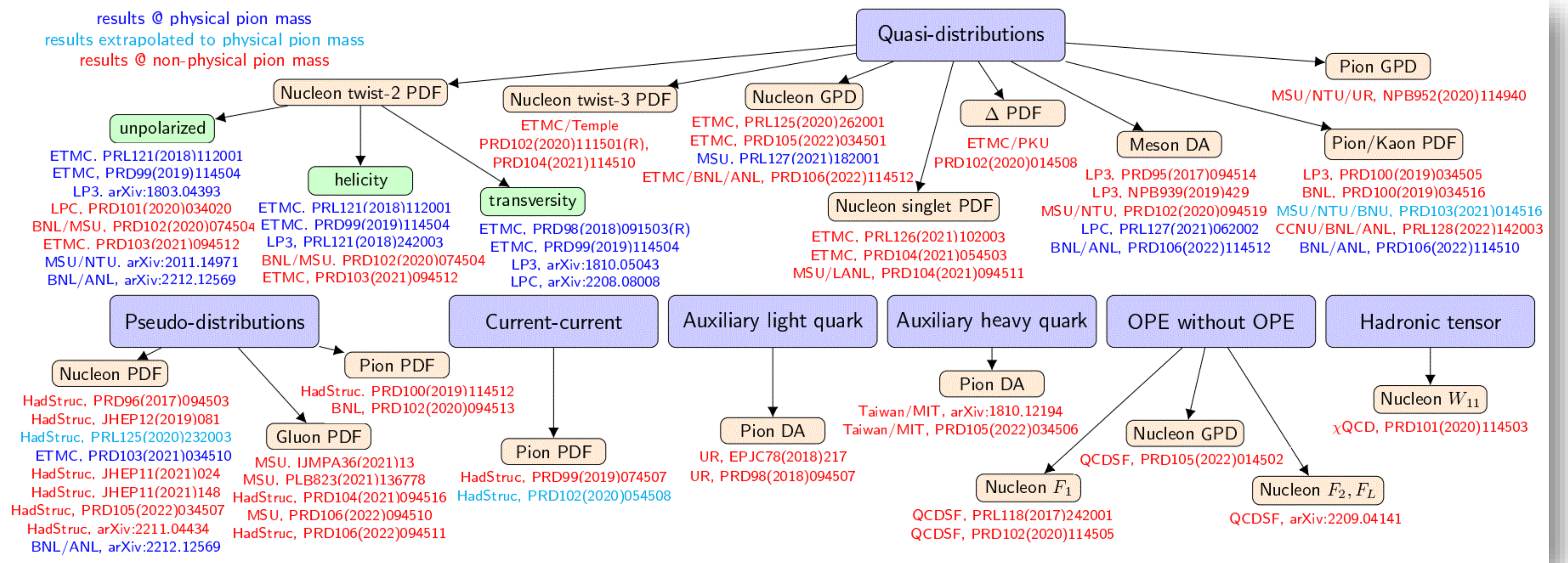
Yao Ji,[a] Fei Yao[b] and Jian-Hui Zhang[c,b]

One-Loop Matching for Generalized Parton Distributions

Xiangdong Ji,[1,2,3] Andreas Schäfer,[4] Xiaonu Xiong,[5,6] and Jian-Hui Zhang[1,4]

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

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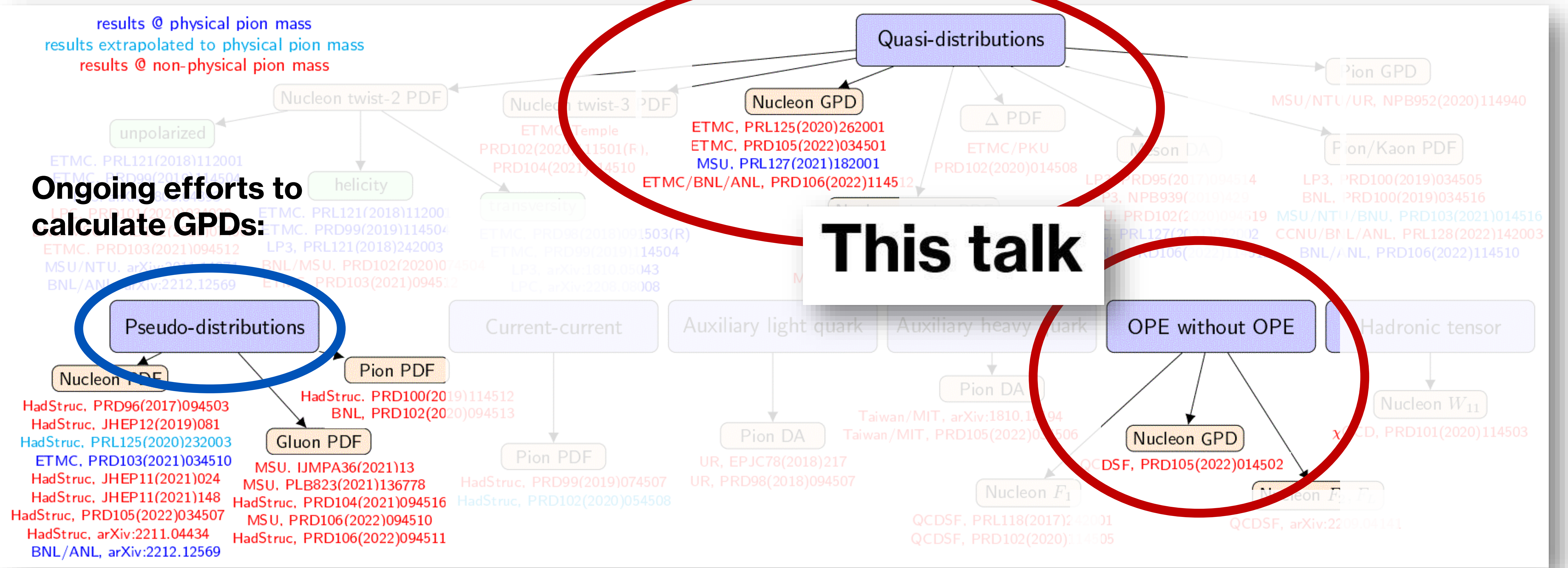


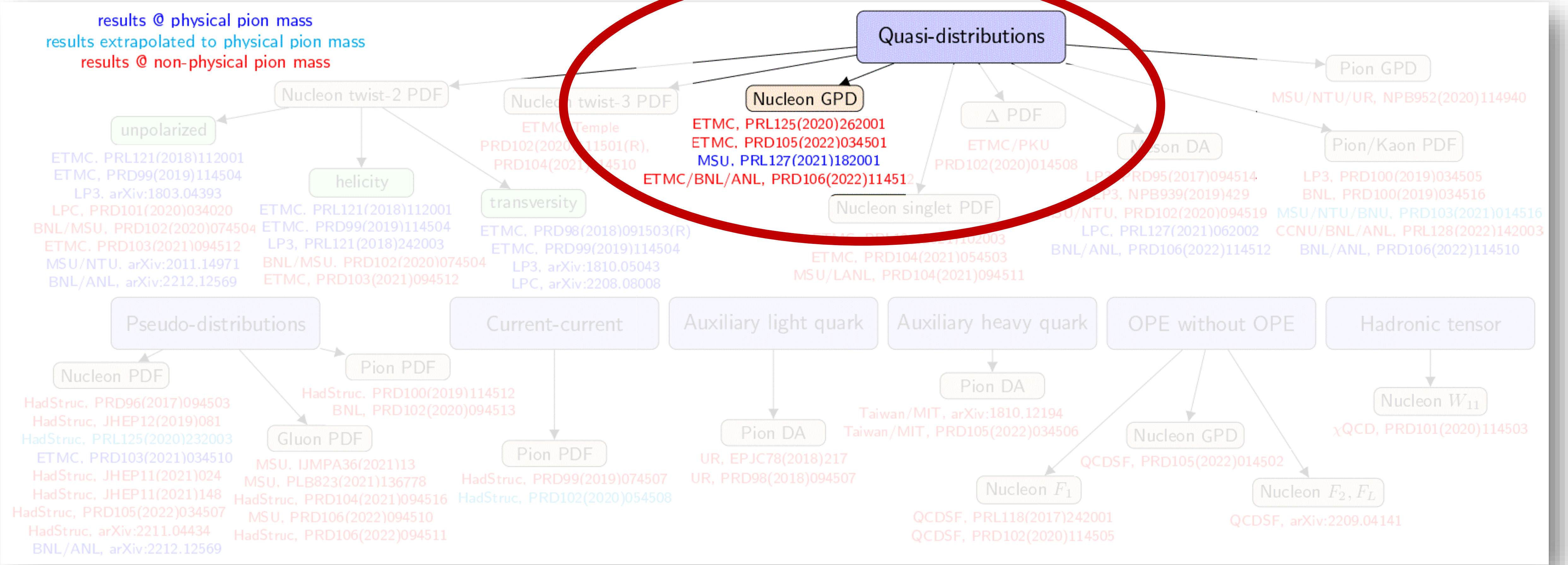
Ongoing efforts to calculate GPDs:

This talk

Compilation by Cichy, 2110.07440

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

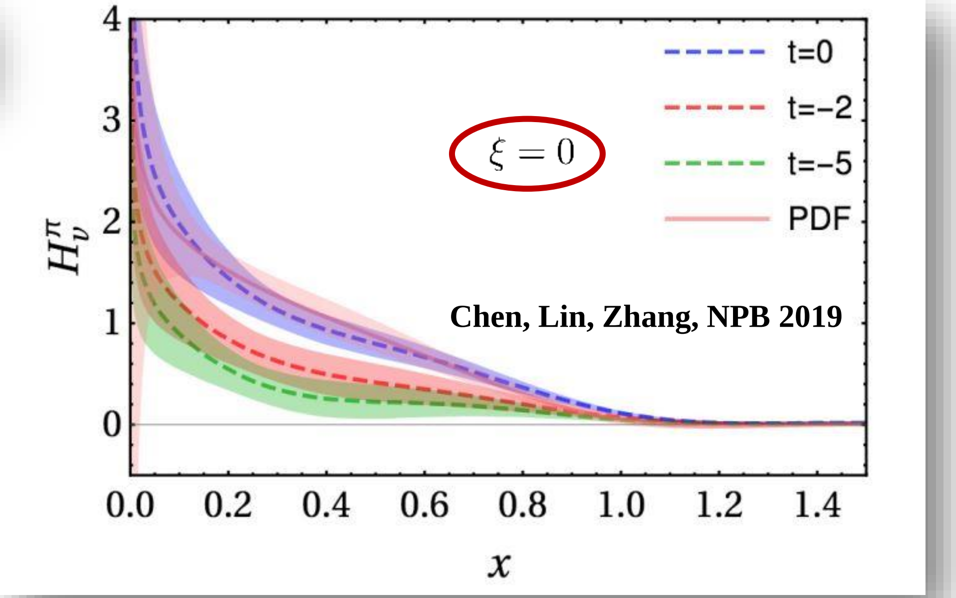
Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

First Lattice QCD results of the x-dependent GPDs

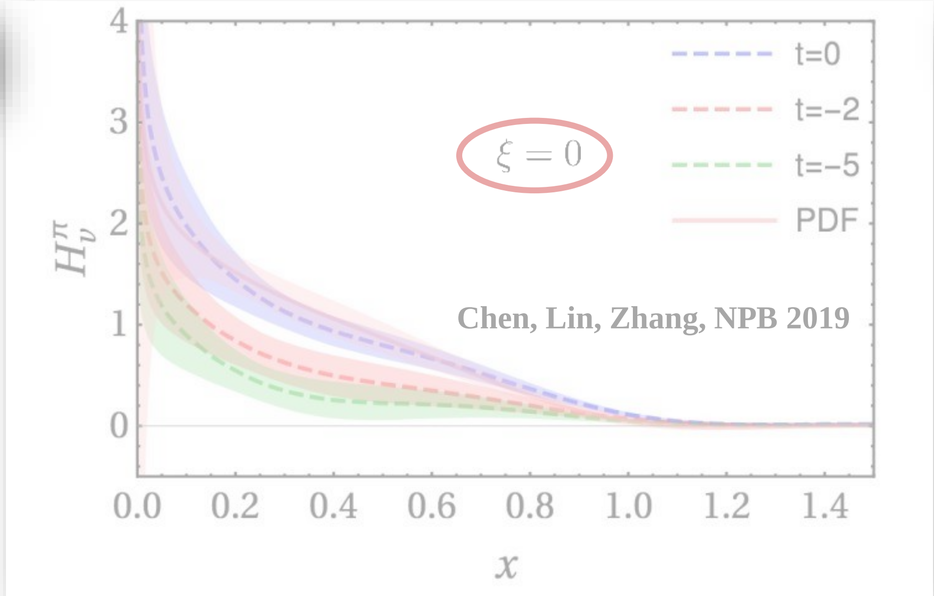
pion



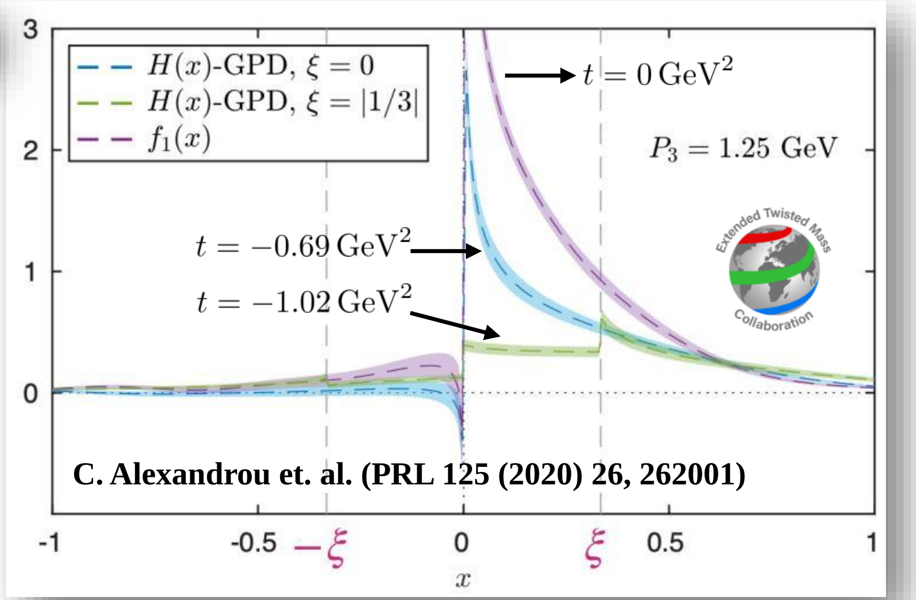
As t increases, the distribution flattens

First Lattice QCD results of the x-dependent GPDs

pion



proton

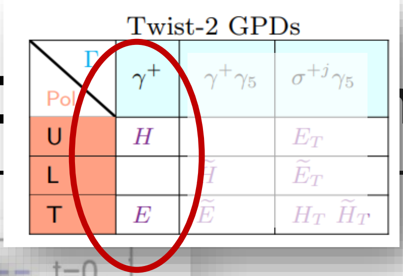


ERBL/DGLAP: Qualitative differences

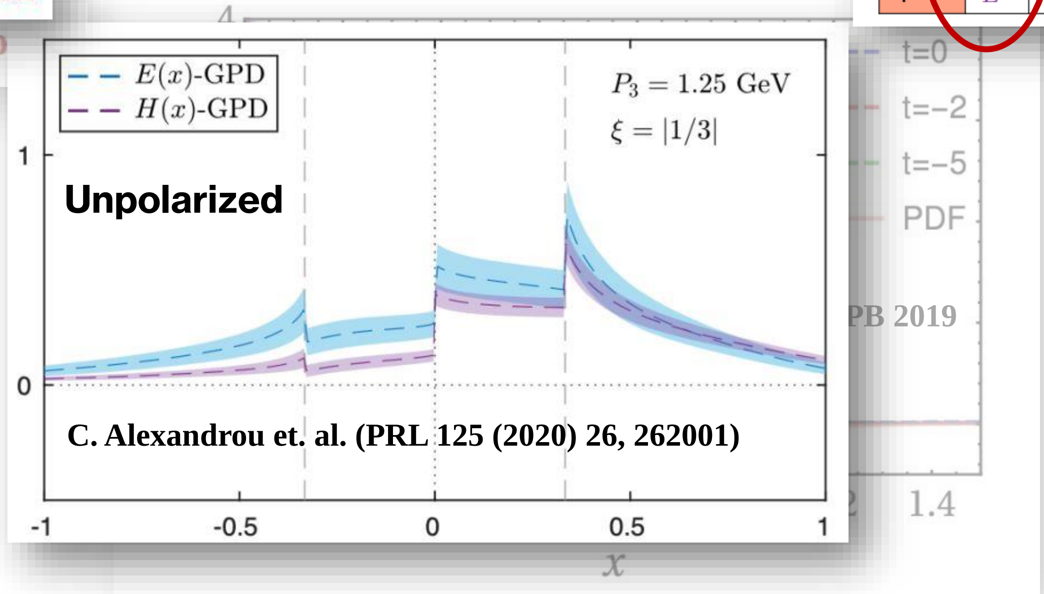
As $x \to 1$, qualitative behavior in agreement with power counting analysis

(F. Yuan, 0311288)

First Lattice QCD ... he x-dependent GPDs



proton

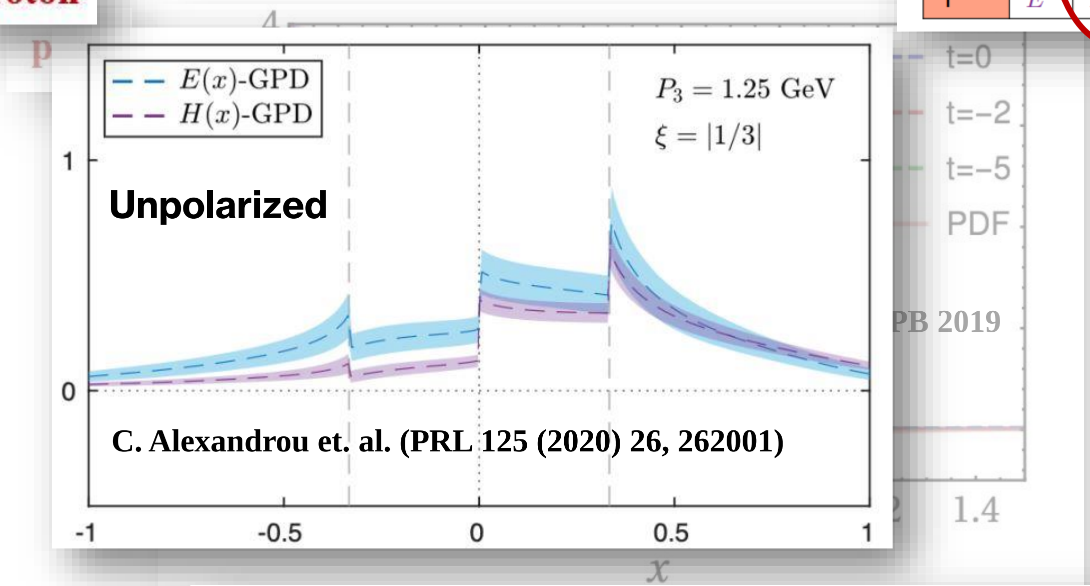


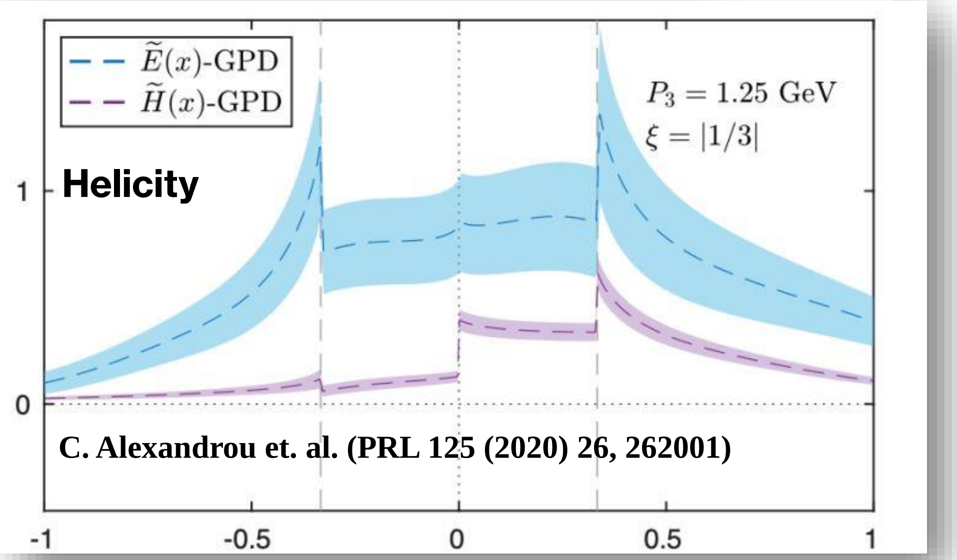
First Lattice QCD he x-dependent GPDs

Twist-2 GPDs

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

proton





First Lattice QCD ... he x-dependent GPDs

Twist-2 GPDs

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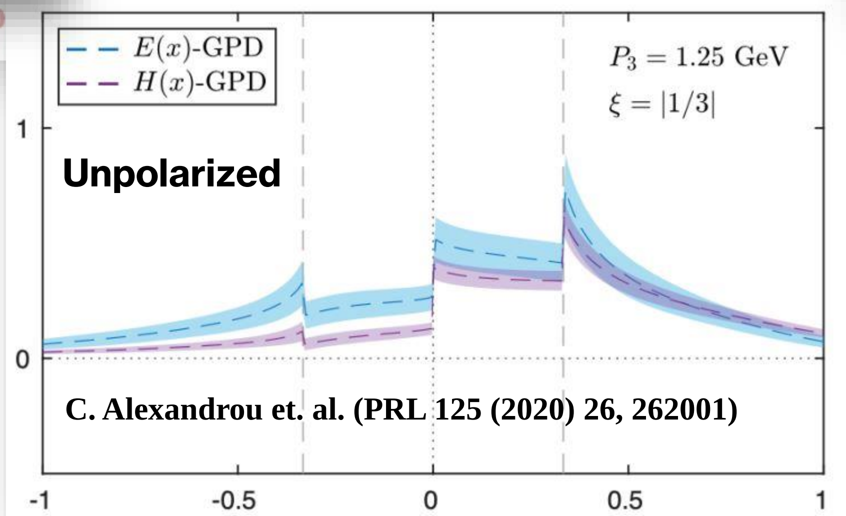
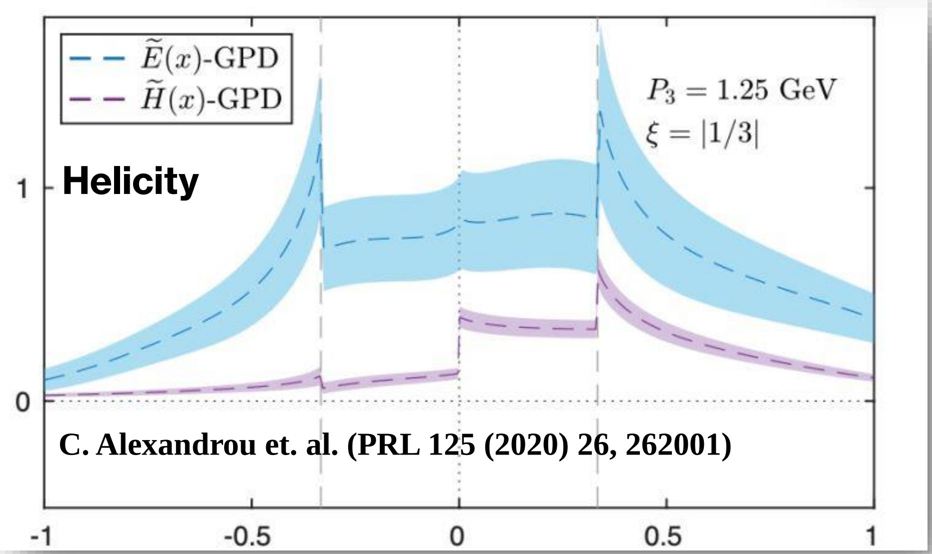
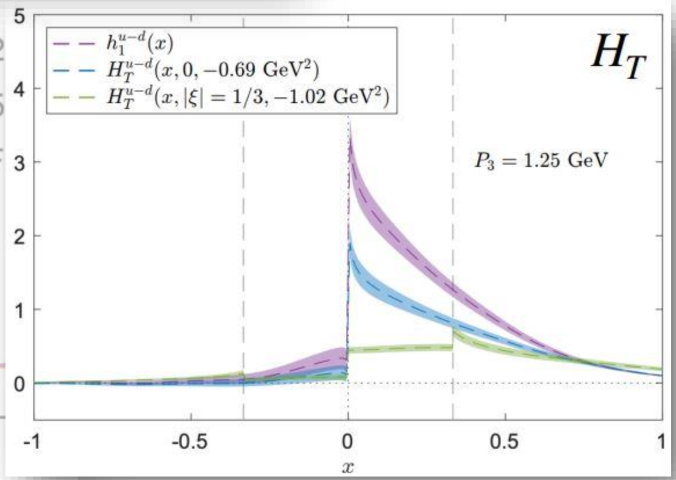
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

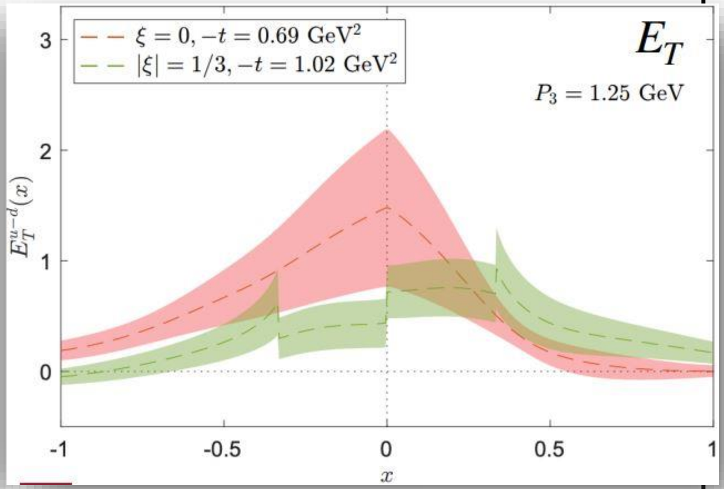
proton

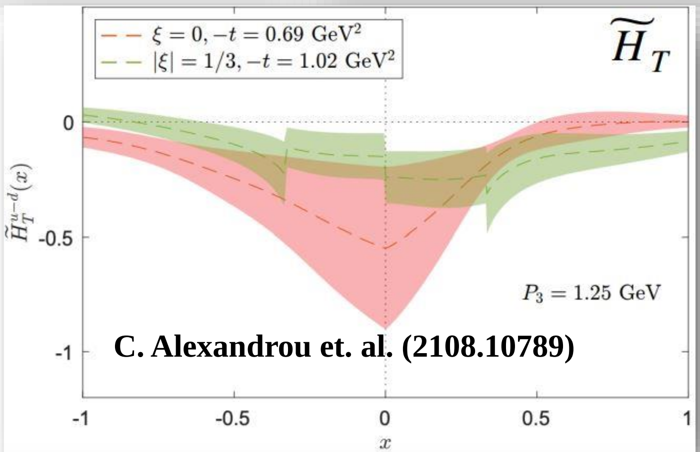




Transversity







GPD $\widetilde{E}_T$ is small/zero within uncertainties (not shown)

First exploration of twist-3 GPDs

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

Why twist 3?

- **Can be as sizeable as twist 2**
- **Contain information about quark-gluon-quark correlations inside hadrons …**

First exploration of twist-3 GPDs

Definition:

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

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$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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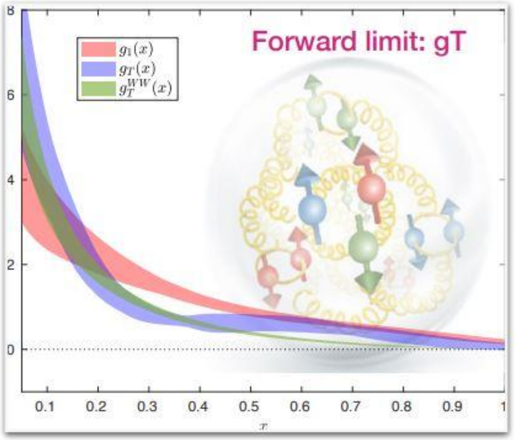
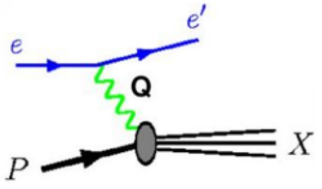


[S. Bhattacharya et al., PRD 102 (2020) 11]

PRD 102 (2020) 11, 111501 [Editor's suggestion]

New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$

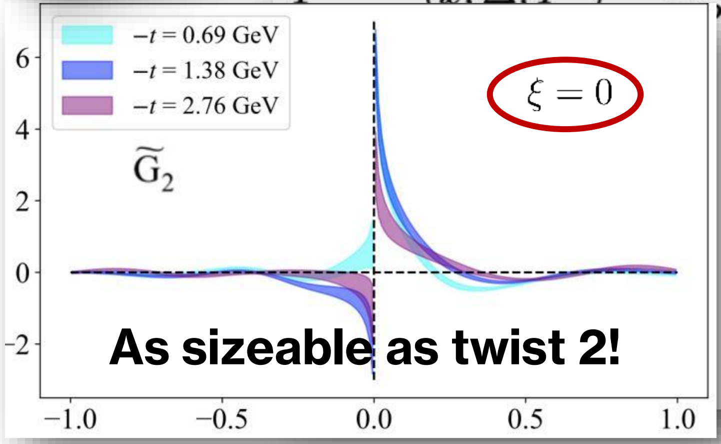
Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

Twist-3 PDF	Processes	Data
$g_T(x)$		For instance: Hall A, 2016/ Hall C, 2018

First exploration of twist-3 GPDs

proton

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$



[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

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$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{\ldots^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$

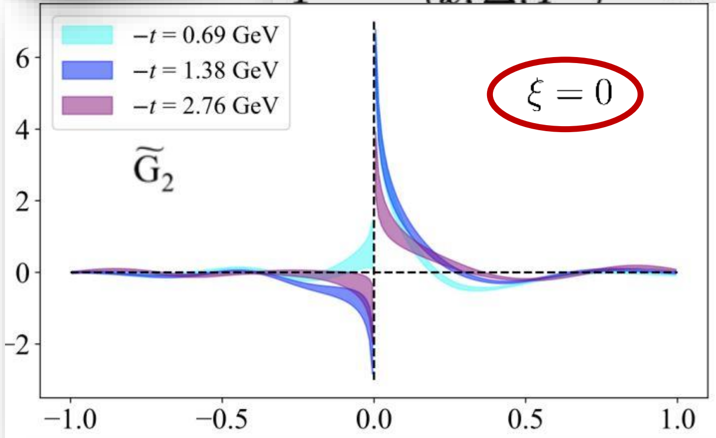
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t \ldots$$

$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

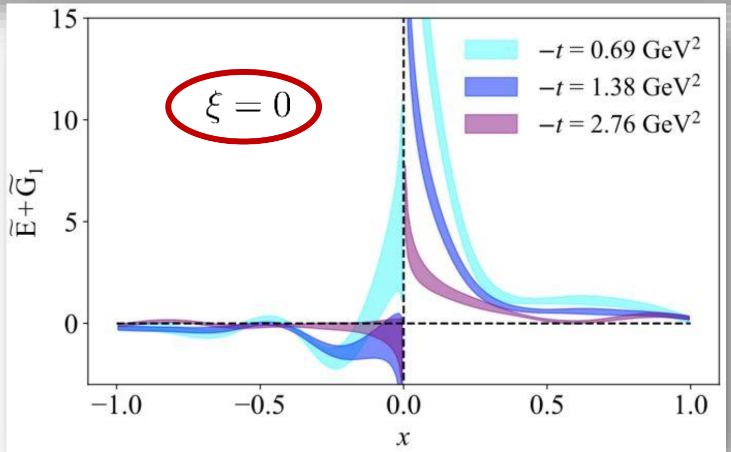
GPD $\widetilde{E}$ can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]



Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness:



First exploration of twist-3 GPDs

proton

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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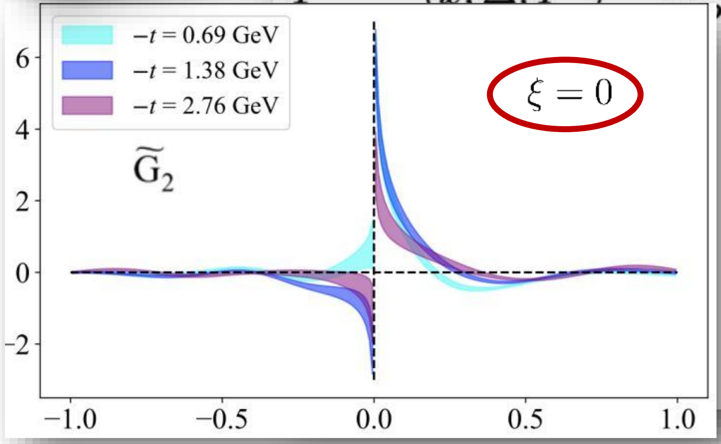

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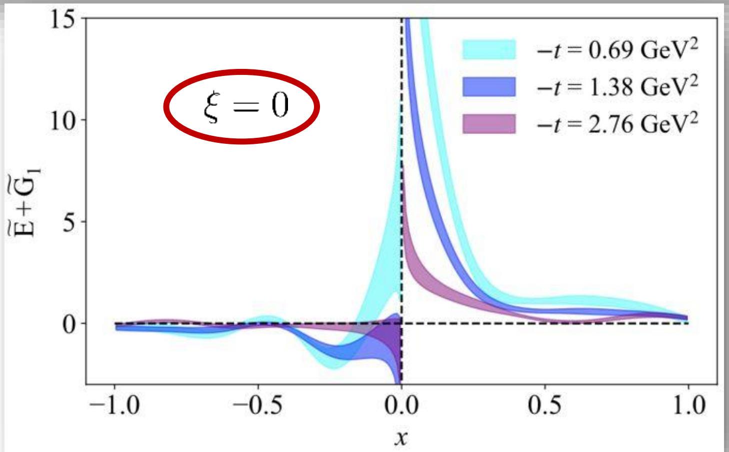

First indication of pion pole from Lattice QCD!

$$\tilde{E}_u - \tilde{E}_d \sim \frac{1}{l^2 - m_\pi^2}$$

(Penttinen, Polyakov, Goeke)

First exploration of twist-3 GPDs

proton:

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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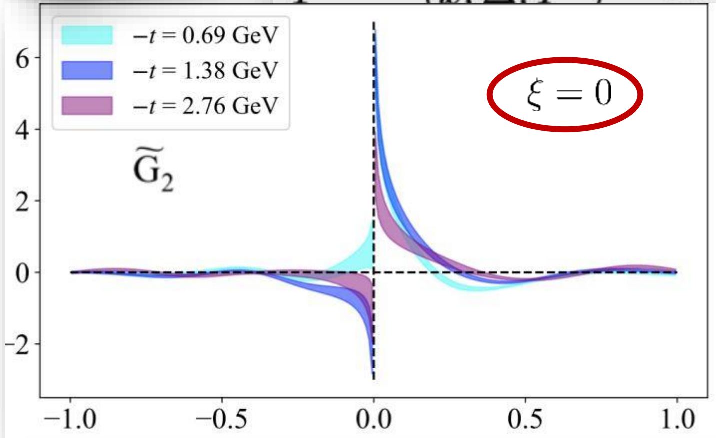
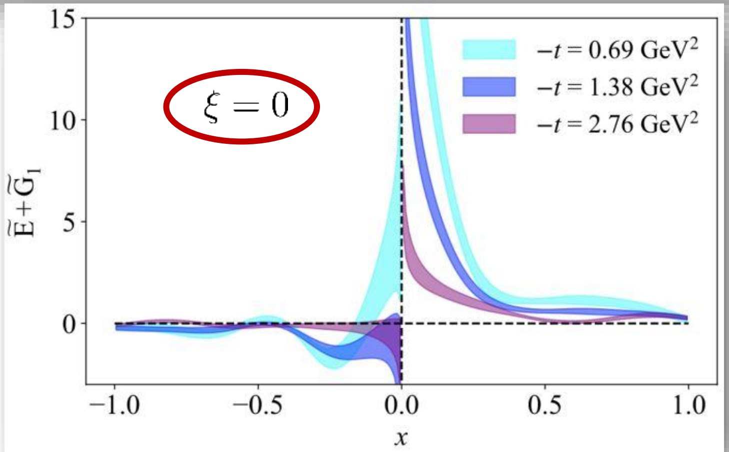
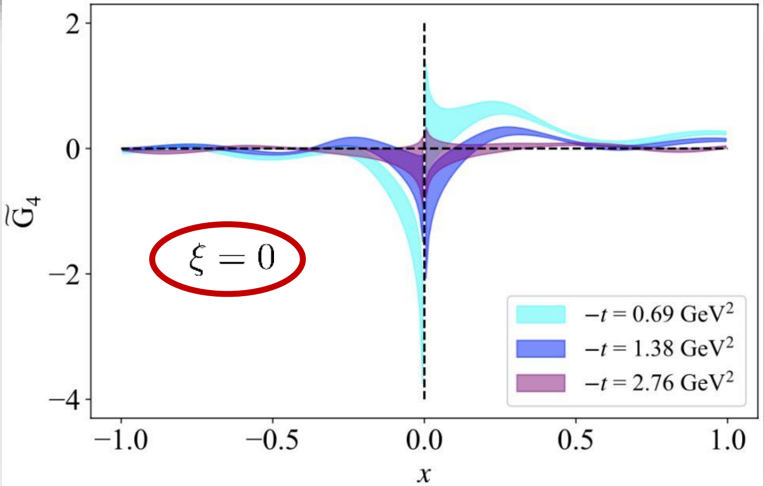


Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness:





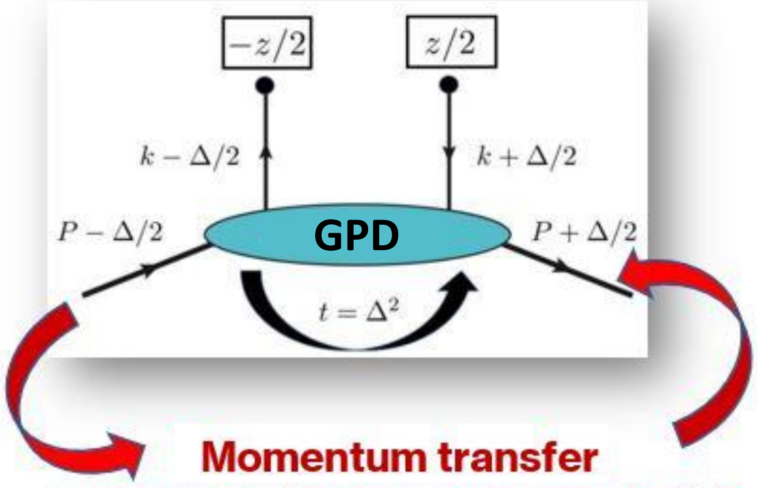
GPD $\widetilde{G}_3$ is zero within uncertainties (not shown)

But little hiccup ...

GPDs from asymmetric frames

Traditionally, GPDs have been calculated from "symmetric frames"

Practical drawback



Momentum transfer
symmetric between source & sink

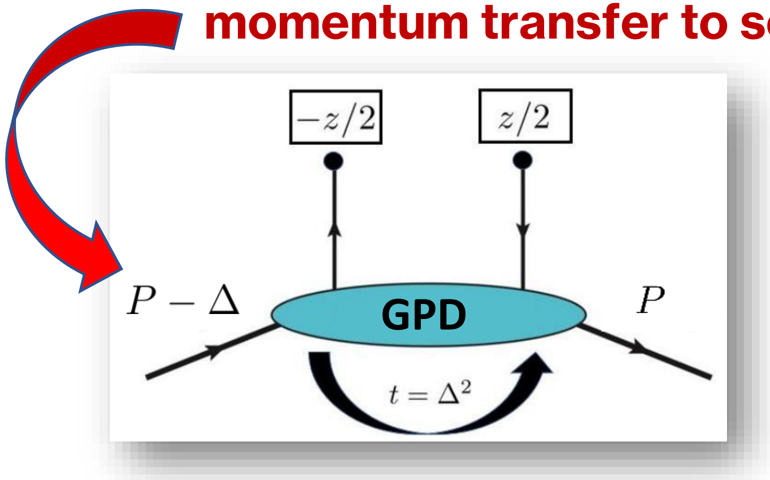
Lattice QCD calculations of GPDs in symmetric frames are expensive

In symmetric frame, full new calculation required for each momentum transfer (Δ)

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source



Perform Lattice QCD calculations of GPDs in asymmetric frames:

- **Reduction in computational cost**
- **Access to broad range of t (enabling creation of high-resolution partonic maps)**

GPDs from asymmetric frames

This talk

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

source

In Preparation

Generalized Parton Distributions from Lattice QCD
with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Joshua Miller,[3,‡] Swagato Mukherjee,[5] Peter Petreczky,[5] Aurora Scapellato,[3] Fernanda Steffens,[6] and Yong Zhao[4]

Major theoretical advances (2209.05373):

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source

Major theoretical advances:

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- Elimination of power corrections potentially allowing faster convergence to light-cone GPDs

GPDs from asymmetric frames

Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}\boldsymbol{A_1}+mz^{\mu}\boldsymbol{A_2}+\frac{\Delta^{\mu}}{m}\boldsymbol{A_3}+im\sigma^{\mu z}\boldsymbol{A_4}+\frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5}+\frac{P^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_6}+mz^{\mu}i\sigma^{z\Delta}\boldsymbol{A_7}+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'}=\langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$

Features:

- **8** linearly-independent Dirac structures
- **8** Lorentz-invariant (frame-independent) amplitudes $\boldsymbol{A_i\equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)}$

GPDs from asymmetric frames

Example

Lorentz covariant formalism

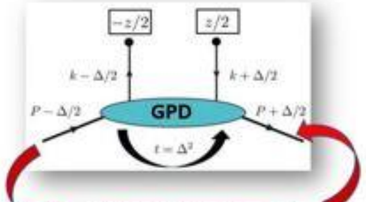
Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}\boldsymbol{A_1} + mz^\mu \boldsymbol{A_2} + \frac{\Delta^\mu}{m}\boldsymbol{A_3} + im\sigma^{\mu z}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_6} + mz^\mu i\sigma^{z\Delta}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Vect

Featu

Traditional definition (symmetric frame):



Momentum transfer symmetric between source & sink

$$F^0_{\lambda,\lambda'}\big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\big|_s\right]u_s(p_s,\lambda)$$

Quasi-GPDs are intrinsically frame-dependent

GPDs from asymmetric frames

Lorentz covariant formalism

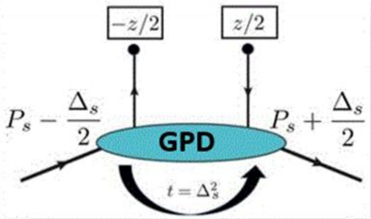
Novel para... n-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^\mu(z, P, \Delta) = \bar{u}(p_f, \lambda')\left[\frac{P^\mu}{m} A_1 + m z^\mu \ldots \frac{P^\mu i\sigma^{z\Delta}}{m} A_6 + m z^\mu i\sigma^{z\Delta} A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m} A_8\right] u(p_i, \lambda)$$

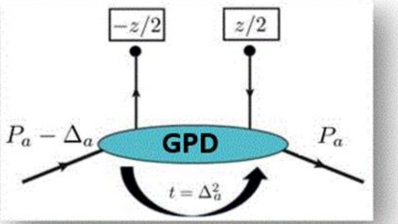
Main point:

$$H^s_{\mathrm{Q}(0)} = \sum_i A_i$$

Calculate quasi-GPD in symmetric frame through matrix elements of asymmetric frame



Symmetric frame



Asymmetric frame

- **8** Lorentz ... $A_i(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2)$

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source

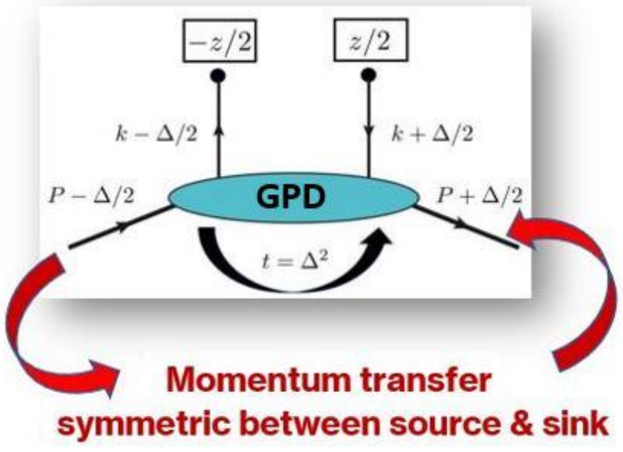
Major theoretical advances:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
- Elimination of power corrections potentially allowing faster convergence to light-cone GPDs

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Example: Symmetric frame



Quasi-GPD:

$$H^s_{\mathrm{Q}(0)}(z,P^s,\Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z} A_3$$

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}} A_3 - \frac{m^2 \Delta^{0,s} z^3}{2P^{0,s}P^{3,s}} A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s} z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3 (\Delta^s_\perp)^2}{2P^{3,s}}\right] A_6$$
$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2 \Delta^{3,s} z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s} z^3 (\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right] A_8 \,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Contamination from additional amplitudes or explicit power corrections

Quasi-GPD: (Symmetric frame)

$$H^s_{Q(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GI

You can think of eliminating additional amplitudes by the addition of other operators

(γ^1, γ^2)

Contamination from additional amplitudes or explicit power corrections

Quasi-GPD: (Symmetric frame)

$$H^s_{Q(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}} A_3 - \frac{m^2 \Delta^{0,s} z^3}{2P^{0,s} P^{3,s}} A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s} z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3 (\Delta^s_\perp)^2}{2P^{3,s}}\right] A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s} P^{3,s}} - \frac{(\Delta^{0,s})^2 \Delta^{3,s} z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s} z^3 (\Delta^s_\perp)^2}{2P^{0,s} P^{3,s}}\right] A_8 \,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Main finding

Schematic structure of (operator-level) Lorentz-invariant definition of quasi-GPD:

$$\boxed{H_{\mathrm{Q}} \to c_0 \langle \bar{\psi} \gamma^0 \psi \rangle + c_1 \langle \bar{\psi} \gamma^1 \psi \rangle + c_2 \langle \bar{\psi} \gamma^2 \psi \rangle}$$

Here, c's are frame-dependent kinematic factors that cancel additional amplitudes such that quasi-GPD has the same functional form as light-cone GPD (Lorentz invariant)

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$\boxed{H(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3}$$

$$A_i \equiv A_i(z^2 = 0)$$

Lorentz-invariant definition of quasi-GPD:

$$\boxed{\mathcal{H}(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3}$$

$$A_i \equiv A_i(z^2 \neq 0)$$

Same functional forms

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$H(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3$$

$$A_i \equiv A_i(z^2 = 0)$$

Lorentz-invariant definition of quasi-GPD:

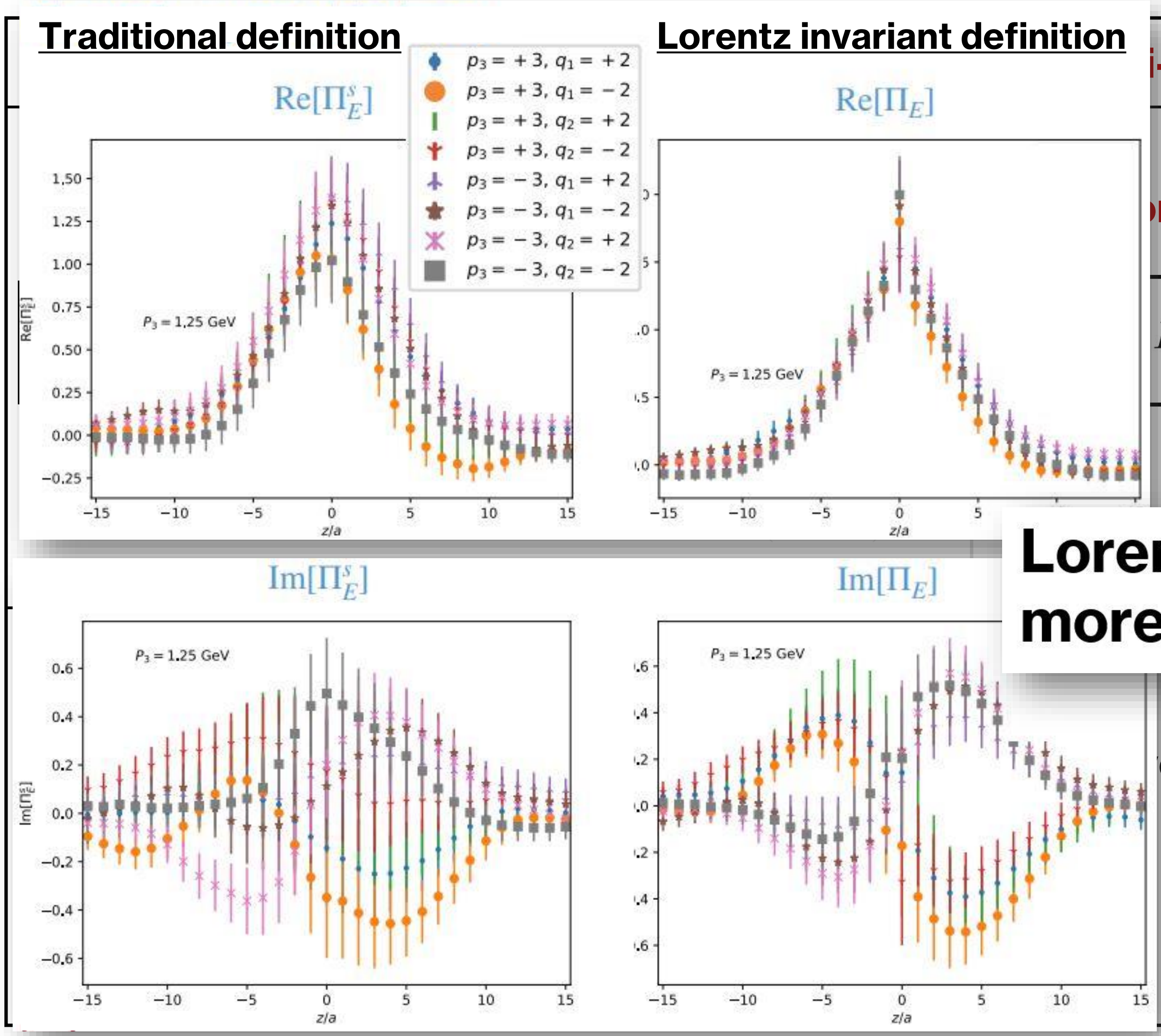
$$\mathcal{H}(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3$$

$$A_i \equiv A_i(z^2 \neq 0)$$

Feature:

- **Lorentz-invariant definition of quasi-GPDs may converge faster**

GPDs from asymmetric frames



i-GPDs

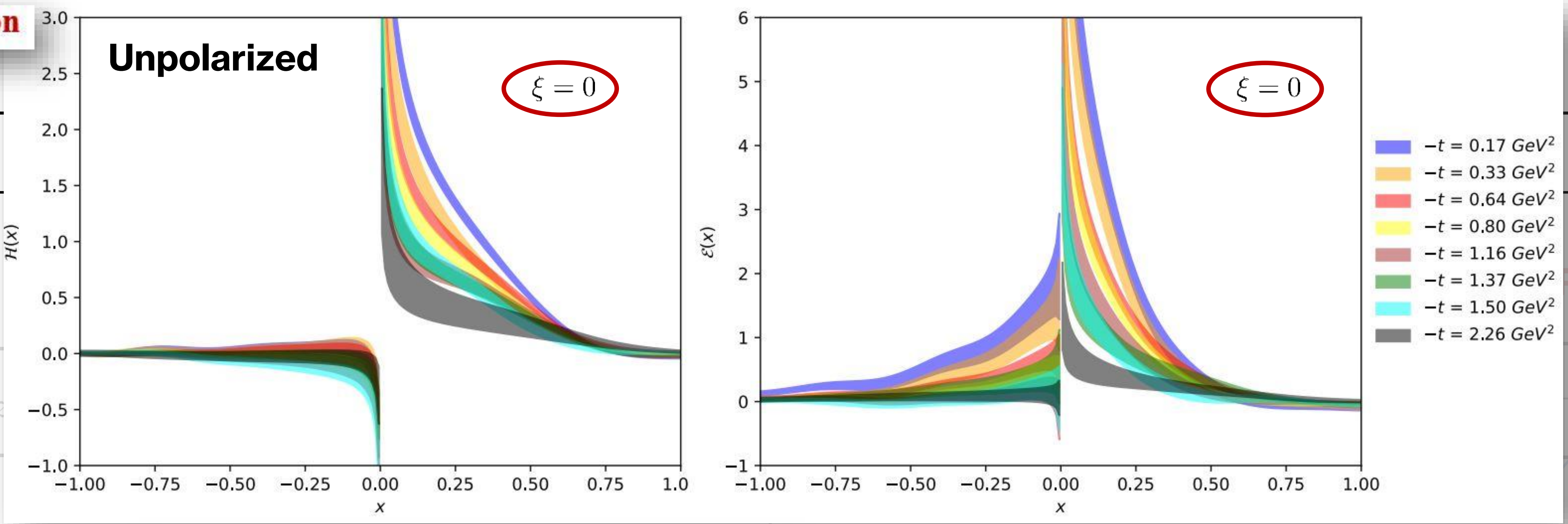
rentz-invariant definition of quasi-GPD:

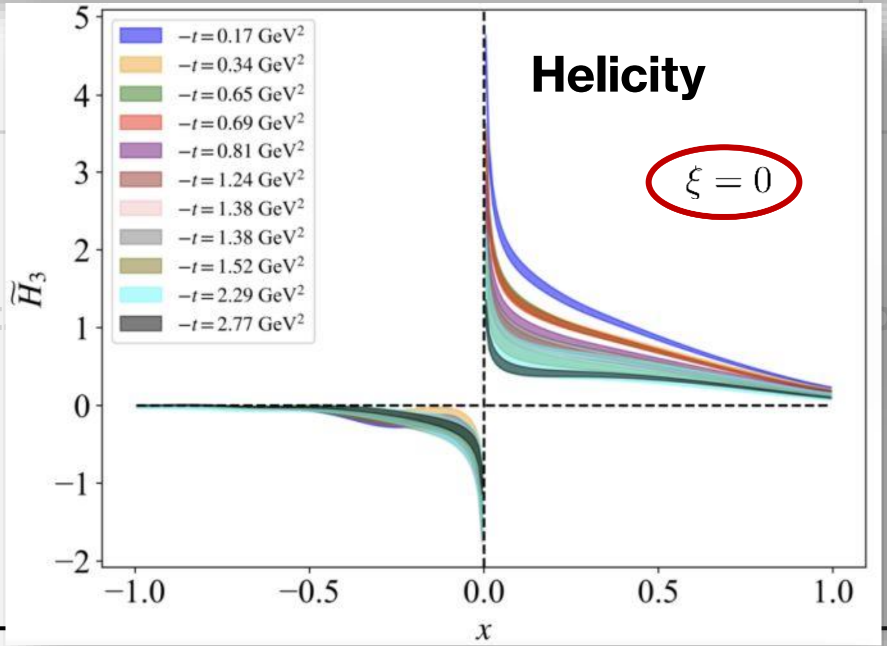
$$P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3$$

Lorentz invariant definition leads to more precise results for GPD E

erge faster

proton





Feature:

GPDs derived from asymmetric frames within the amplitude formalism

GPDs from asymmetric frames

New definition of quasi-GPDs

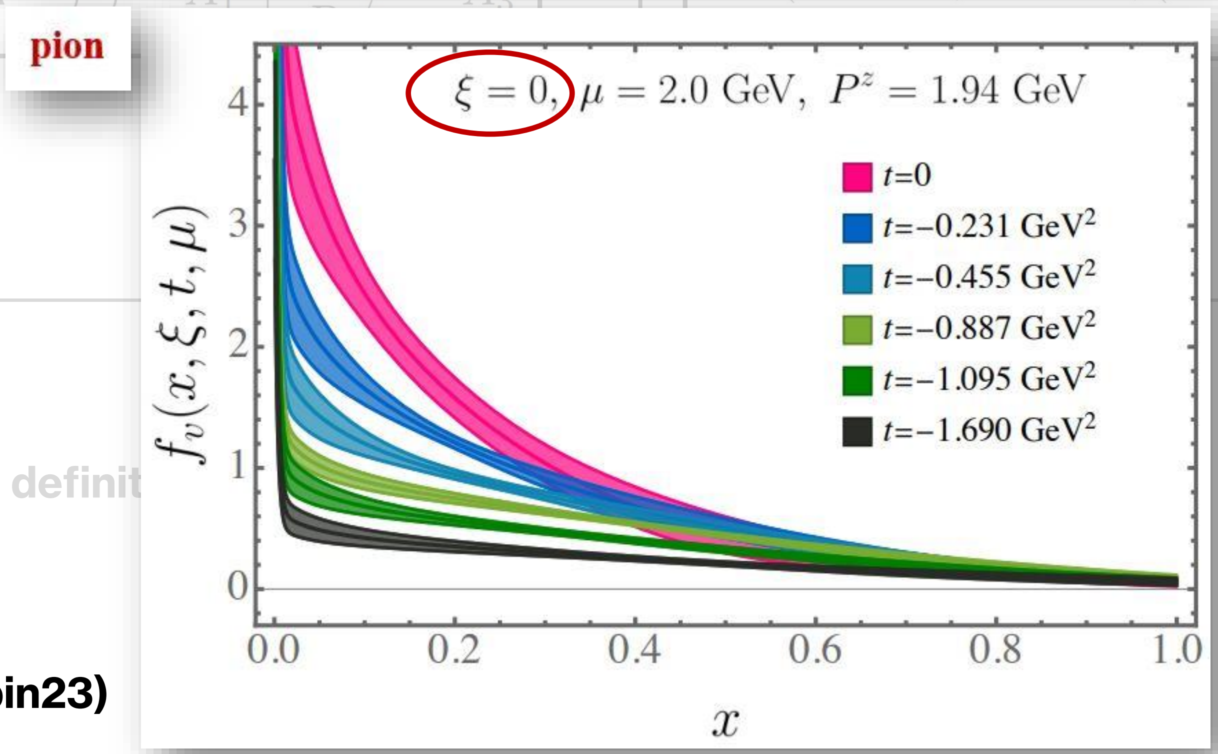
GPDs derived from asymmetric frames within the amplitude formalism

Ligh… …tion of quasi-GPD:

$A_i \equiv A_i(z^2 \neq 0)$

Feature:

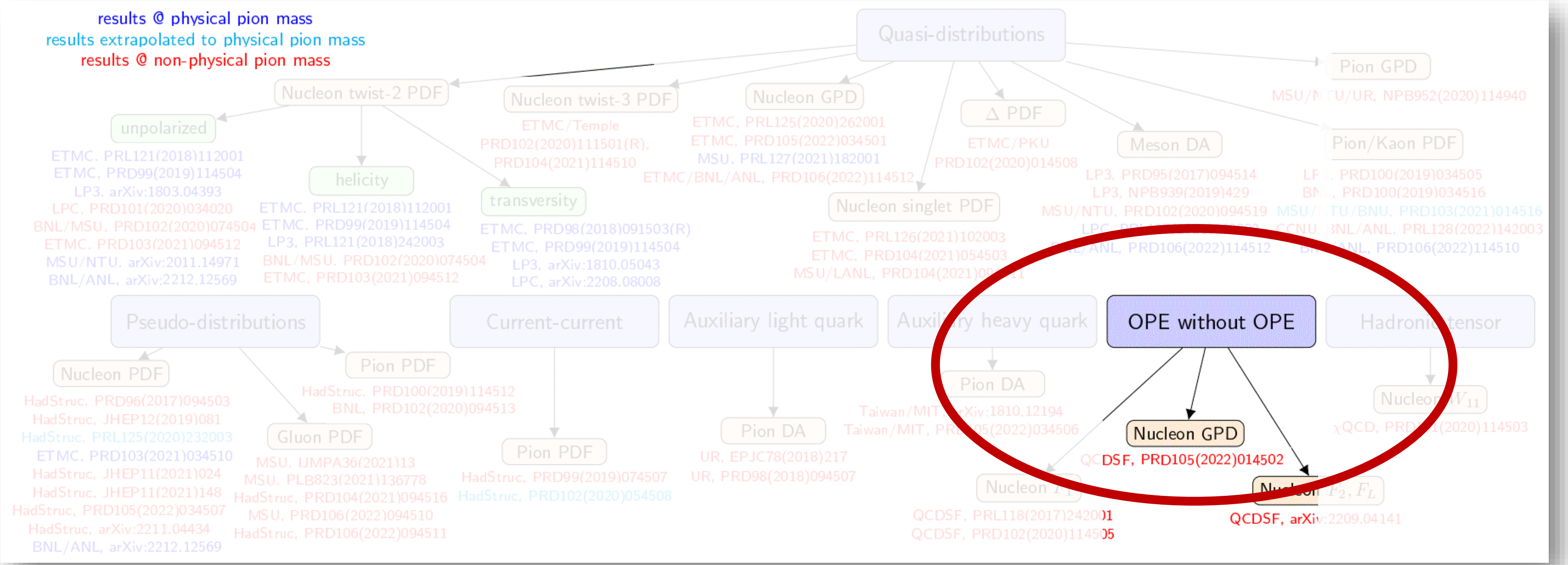
- Lorentz-invariant definit…



Qi Shi (Talk at Spin23)

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



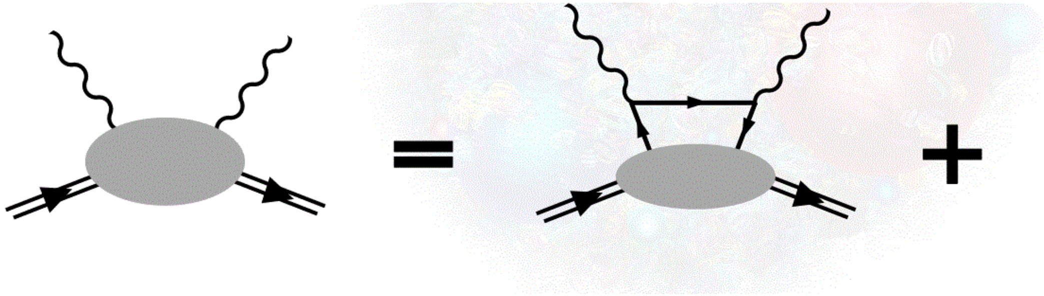
Compilation by Cichy, 2110.07440

Compton amplitude in Lattices

Generalised parton distributions from the off-forward Compton amplitude in lattice QCD

A. Hannaford-Gunn,[1] K. U. Can,[1] R. Horsley,[2] Y. Nakamura,[3] H. Perlt,[4]
P. E. L. Rakow,[5] G. Schierholz,[6] H. Stüben,[7] R. D. Young,[1] and J. M. Zanotti[1]
(CSSM/QCDSF/UKQCD Collaborations)

Example: **Forward Compton amplitude**

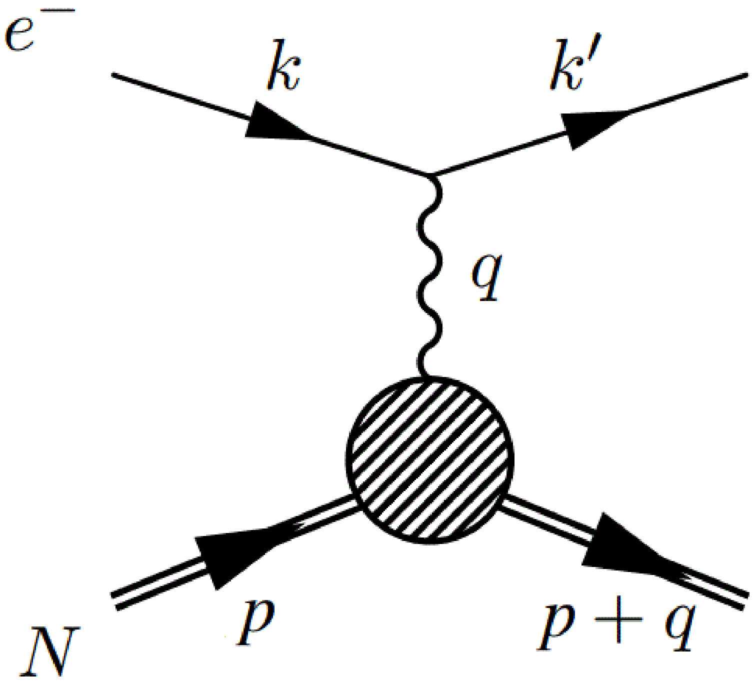


$$+\ \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{1}{Q^2}\right)$$

Courtesy: Utku Can

Compton amplitude in Lattices

Deep Inelastic Scattering (DIS)



DIS & Hadronic Tensor:

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) \frac{F_2(x, Q^2)}{p\cdot q}$$

Structure Functions

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Same Lorentz decomposition as the Hadronic tensor

Compton amplitude in Lattices

Forward Compton amplitude:

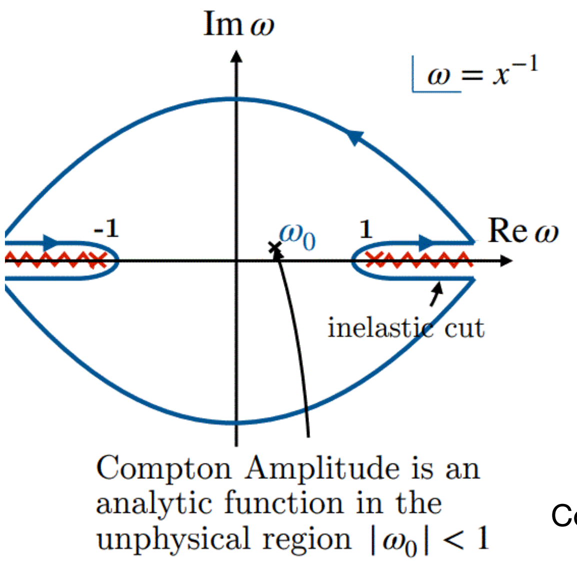
$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$
$$= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Dispersion relations connecting Compton SFs to DIS SFs:

$$\underbrace{\mathcal{F}_1(\omega,Q^2) - \mathcal{F}_1(0,Q^2)}_{\equiv \overline{\mathcal{F}}_1(\omega,Q^2)} = 2\omega^2\int_0^1 dx \frac{2x\,F_1(x,Q^2)}{1-x^2\omega^2 - i\epsilon}$$

$$\mathcal{F}_2(\omega,Q^2) = 4\omega\int_0^1 dx \frac{F_2(x,Q^2)}{1-x^2\omega^2 - i\epsilon}$$



Courtesy: Utku Can

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Compton amplitude approach gives access to moments of DIS SFs:

Example:

$$\mathcal{F}_2(\omega,Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1}M_{2n}^{(2)}(Q^2), \text{ where } M_{2n}^{(2,L)}(Q^2) = \int_0^1 dx\, x^{2n-2}F_{2,L}(x,Q^2)$$

Compton amplitude in Lattices

Off-forward is very similar

$$T_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\left[-\left(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\right)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\right] + \ldots$$

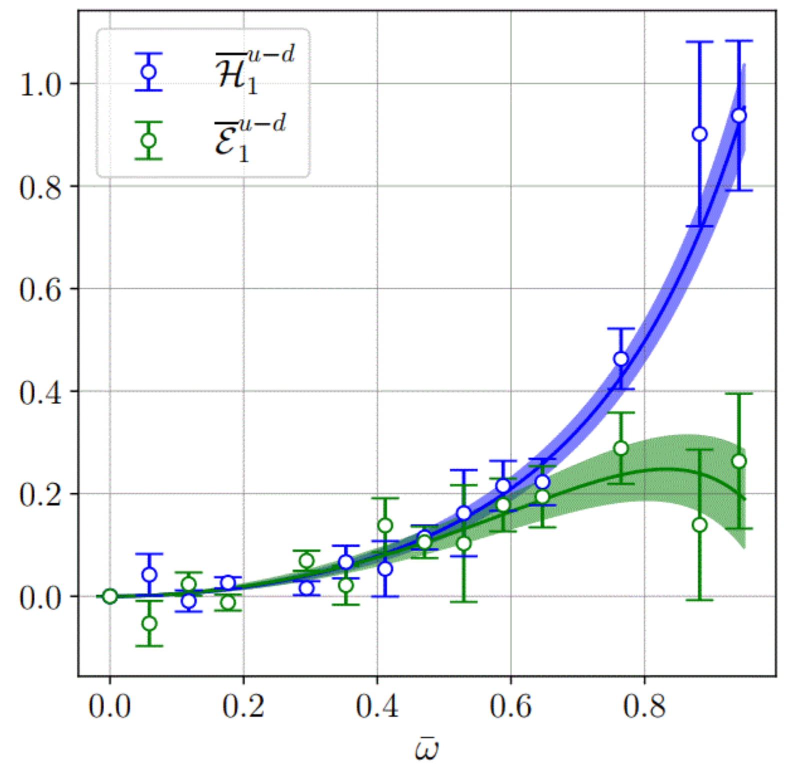
Compton amplitude:

This approach gives access to moments GPDs:

Compton amplitude approac ... Fs:

Example:

$$\mathcal{F}_2(\omega, Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1} \ldots (x, Q^2)$$



Isovector results for $t = -0.57$ GeV^2.

Summary

- **Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators**

- **Impact of approach(es) largest where experiments are difficult → GPDs**

Summary

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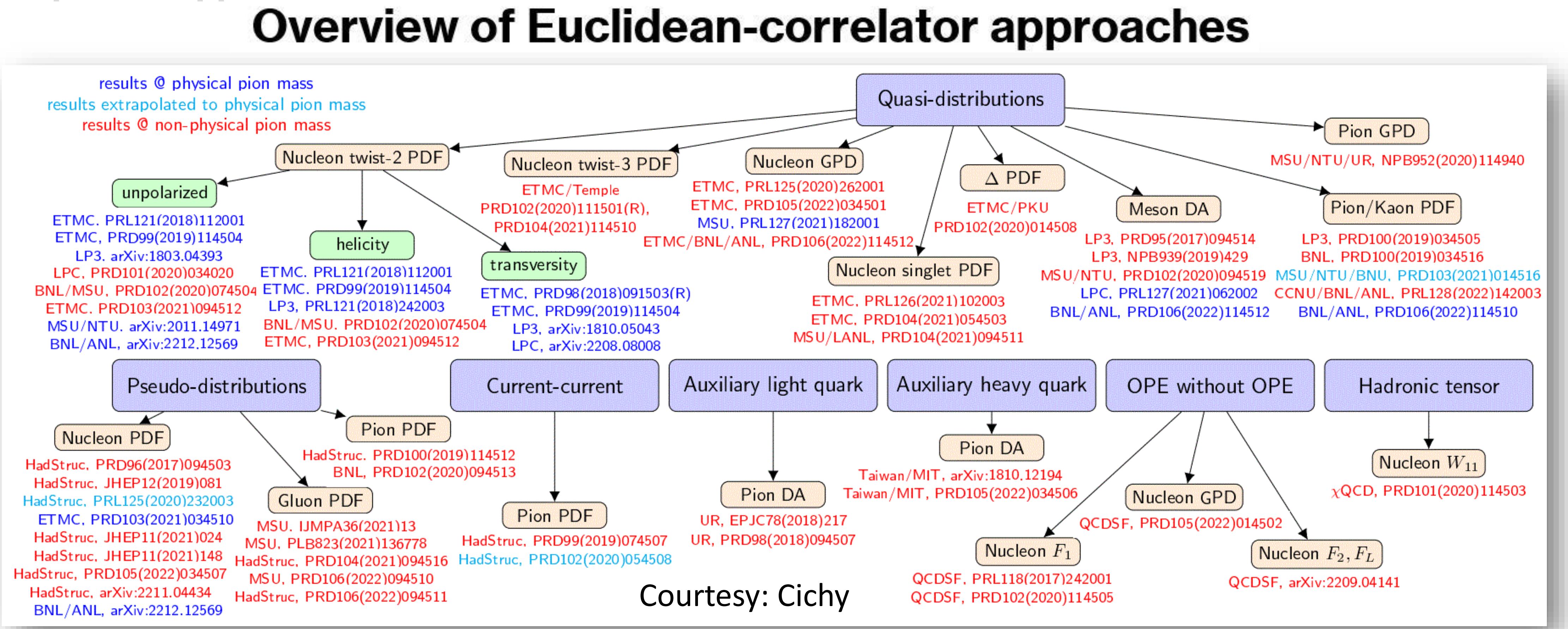
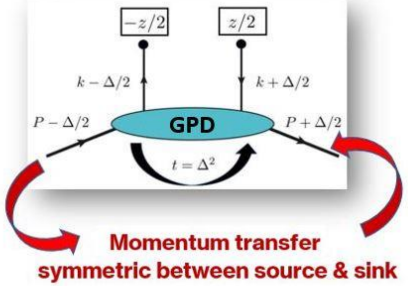
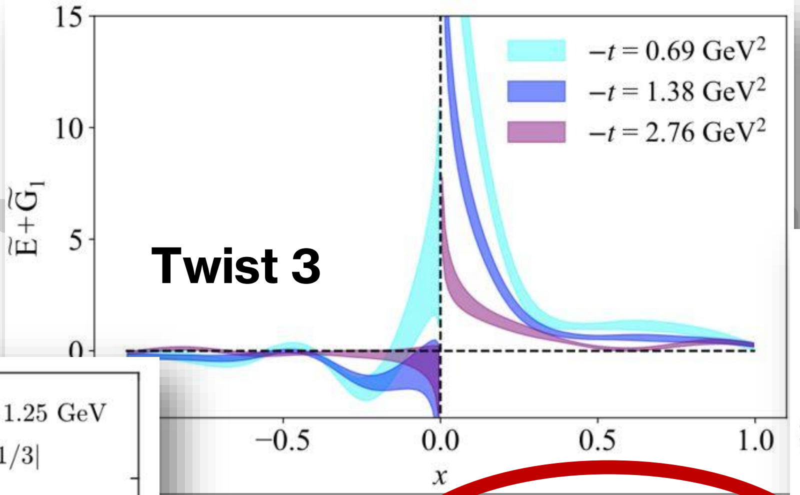
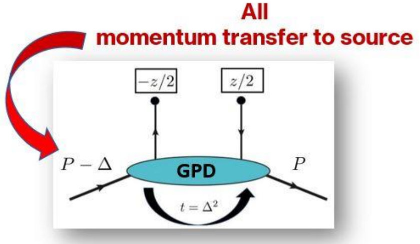
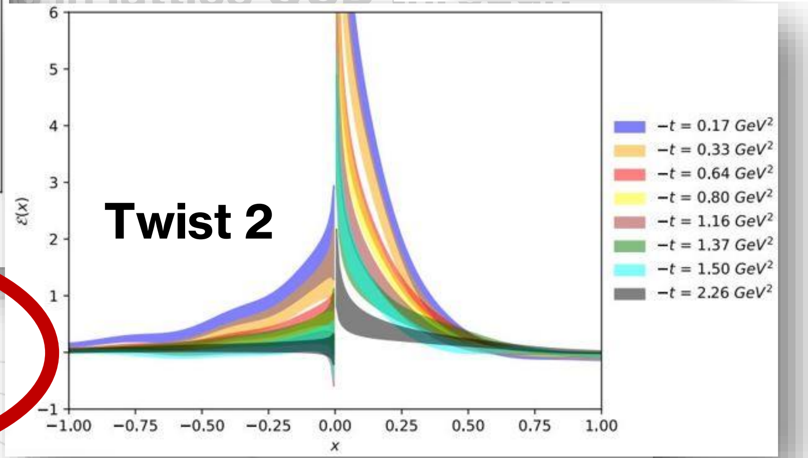


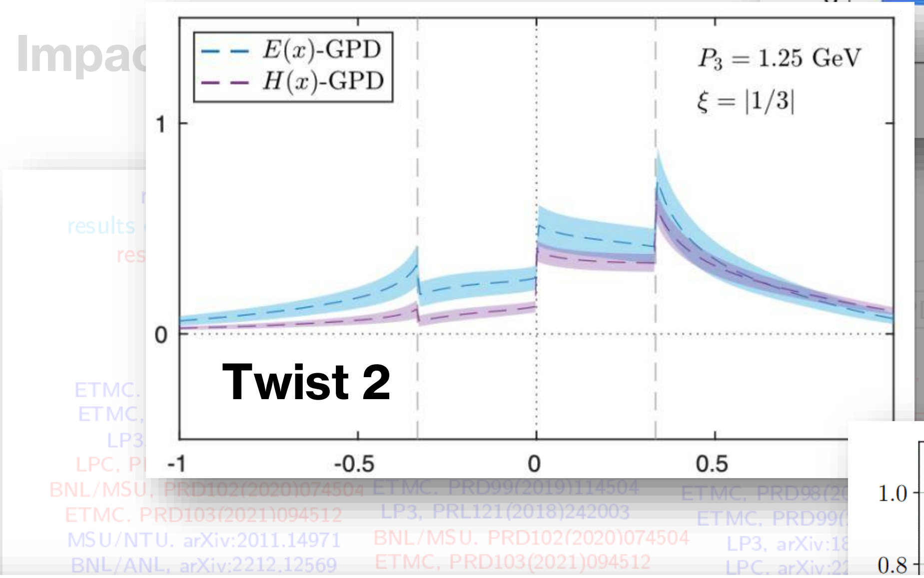
Significant progress!

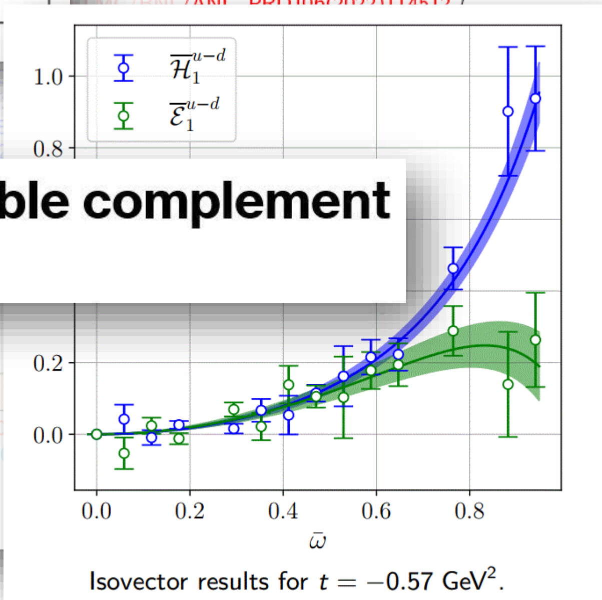












Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC

Backup slides

What are Generalized Parton Distributions?

Example:

At twist 2 there are 8 GPDs

Twist-2 GPDs

Γ / Pol.	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
T	E	$\widetilde{E}$	$H_T\ \widetilde{H}_T$

$F(x,\xi,t)$

GPD c

There are a total of 32 GPDs

Definition of GPD correlator:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

Physical processes sensitive to GPDs

(list not exhaustive)

Shadow GPDs

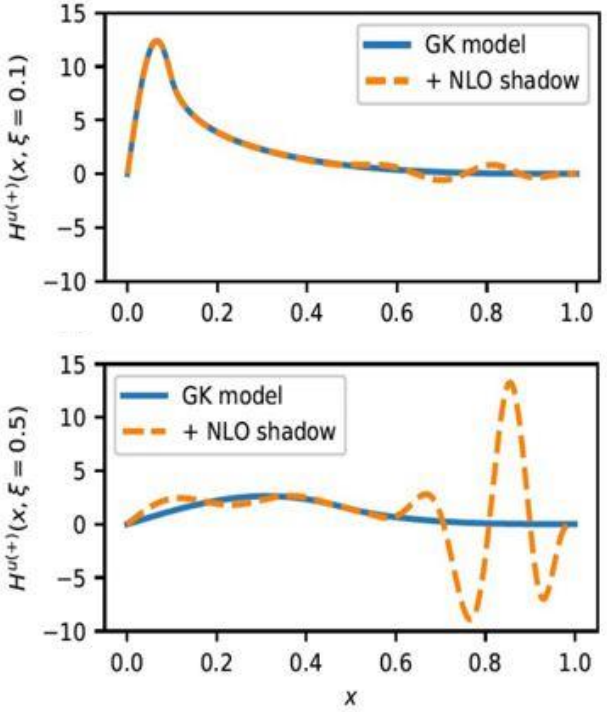
Deconvolution problem of deeply virtual Compton scattering

V. Bertone,[1,*] H. Dutrieux,[1,†] C. Mezrag,[1,‡] H. Moutarde,[1,§] and P. Sznajder[2,‖]

$$F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$$

with $\displaystyle \int_{-1}^{1} \mathrm{d}x \, \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0$

Blue and dashed
Fit the same CFFs!



Dyna[...] Progress of Lattice QCD calculations of PDFs/GPDs

Check out!

...rs:

result[...]
results extra[...]
results ([...]

un[...]

ETMC, PRL1[...]
ETMC, PRD[...]
LP3, arXiv[...]
LPC, PRD101[...]
BNL/MSU, PRD[...]
ETMC, PRD10[...]
MSU/NTU, ar[...]
BNL/ANL, ar[...]

P[...]

Nucleon P[...]

HadStruc, PRD96[...]
HadStruc, JHEP[...]
HadStruc, PRL125[...]
ETMC, PRD103[...]
HadStruc, JHEP[...]
HadStruc, JHEP[...]
HadStruc, PRD105([...]
HadStruc, arXiv:2[...]
BNL/ANL, arXiv:2212.12569

Hindawi
Advances in High Energy Physics
Volume 2019, Article ID 3036904, 68 pages
https://doi.org/10.1155/2019/3036904

Hindawi

Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy[1] **and Martha Constantinou**[2]

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[2]*Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

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GPD
[...], NPB952(2020)114940

Kaon PDF
[...]00(2019)034505
[...]100(2019)034516
[...]U, PRD103(2021)014516
[...]NL, PRL128(2022)142003
[...]PRD106(2022)114510

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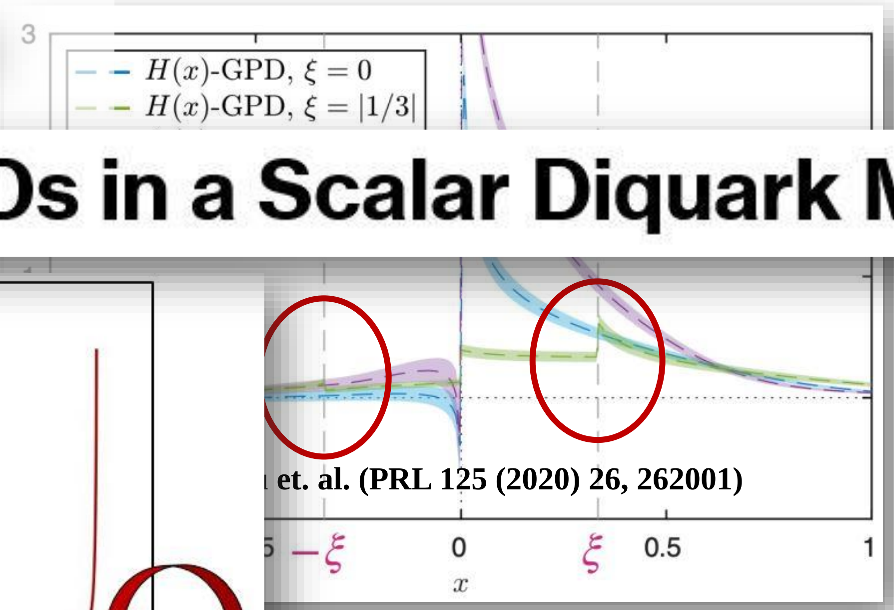
Nucleon W_{11}
[...], PRD101(2020)114503

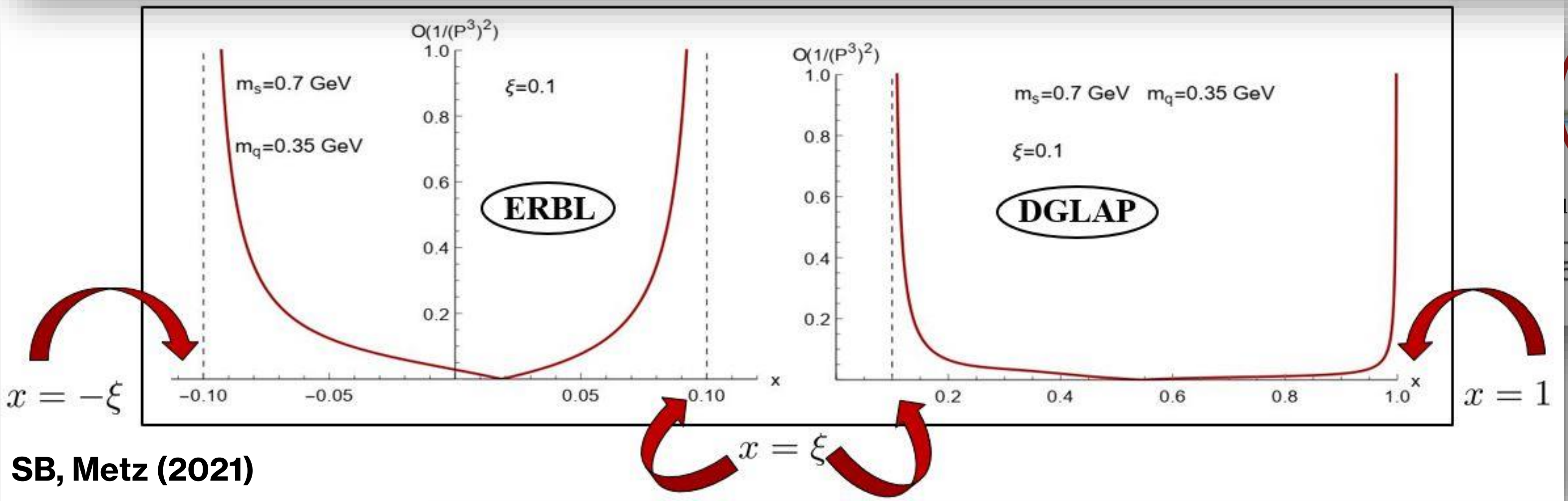
[...]141

Compilation by Cichy, 2110.07440

First Lattice QCD results of the x-dependent GPDs



Power corrections for quasi-GPDs in a Scalar Diquark Model



SB, Metz (2021)

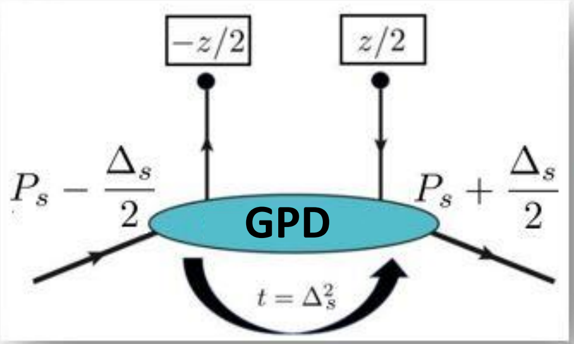
et. al. (PRL 125 (2020) 26, 262001)

Our prediction regarding the structure of divergence: $q_{\mathrm{Q}}(x) \approx \mathcal{O}\left(\frac{1}{(x+\xi)(x-\xi)(1-x)P_3^2}\right)$

GPDs from asymmetric frames

Historic definitions of quasi-GPDs H & E are not manisfestly Lorentz invariant

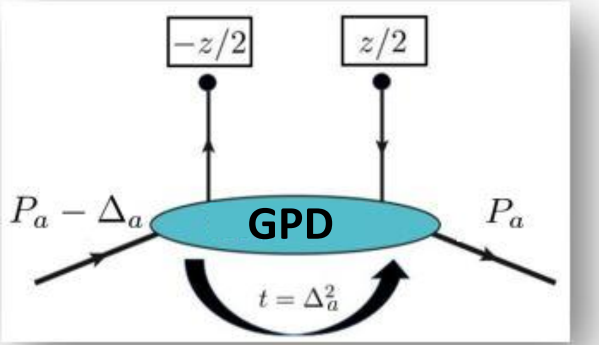
Think about how γ^0 transforms under Lorentz transformation



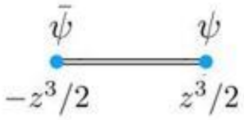
Symmetric frame

“Transverse” Lorentz

transformation



Asymmetric frame



“Transverse” with respect to Wilson Line

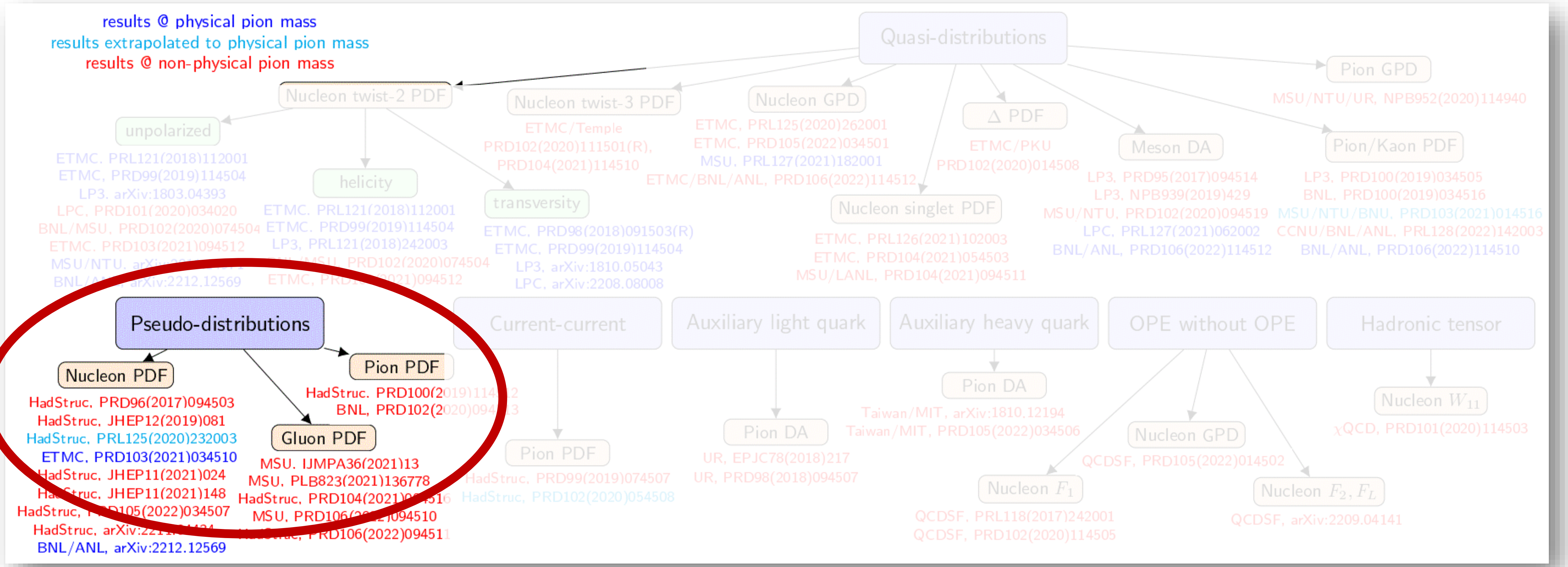
$$\boxed{F_s^0 = \gamma F_0^a - \gamma\beta F_\perp^a}$$

$$\beta = -\sqrt{\frac{E_i^a - E_f^a}{E_i^a + E_f^a}} < 0$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



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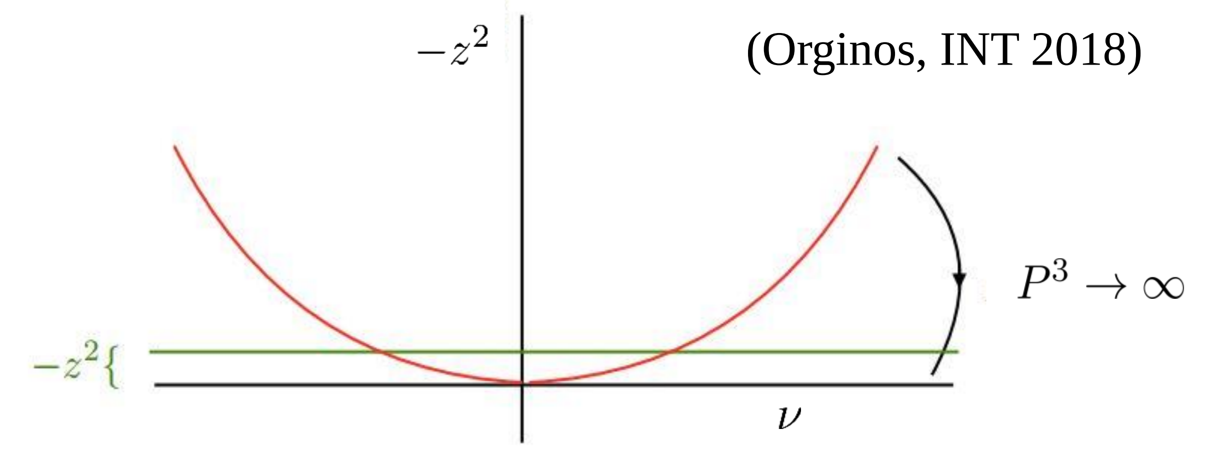
Pseudo-GPD approach

Generalized Parton Distributions and Pseudo-Distributions

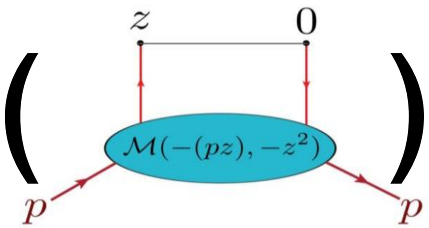
A. V. Radyushkin[1,2]

Sketch of the approach:



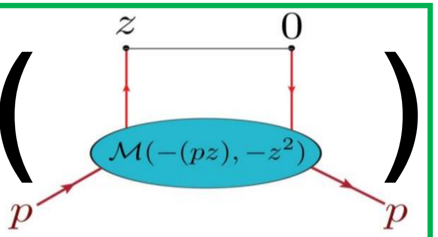
Quasi-PDF : Fixed P^3

$$Q(x, P^3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu} \left(\quad \right)$$



Pseudo-PDF : Fixed z^2

$$P(x, -z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu} \left(\quad \right)$$



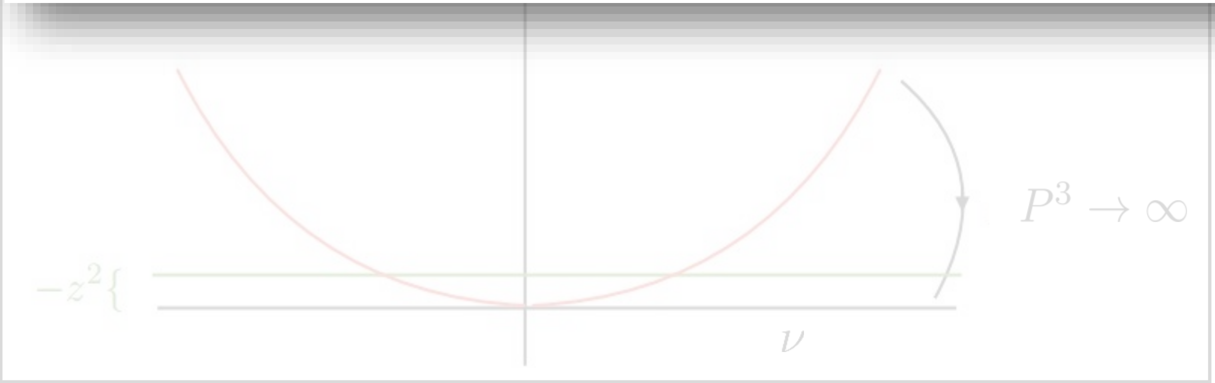
Pseudo-GPD approach

Generalized Parton Distributions and Pseudo-Distributions

A. V. Radyushkin[1,2]

Quasi-PDF : (fixed P^3)

Progress is steadily advancing
& we anticipate forthcoming results regarding GPDs from the pseudo-GPD approach



Pseudo-PDF : (fixed z^2)

$$P(x,-z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu}$$