NC STATE UNIVERSITY

Center for Frontiers in Nuclear Science

QCD factorization: from small to large x

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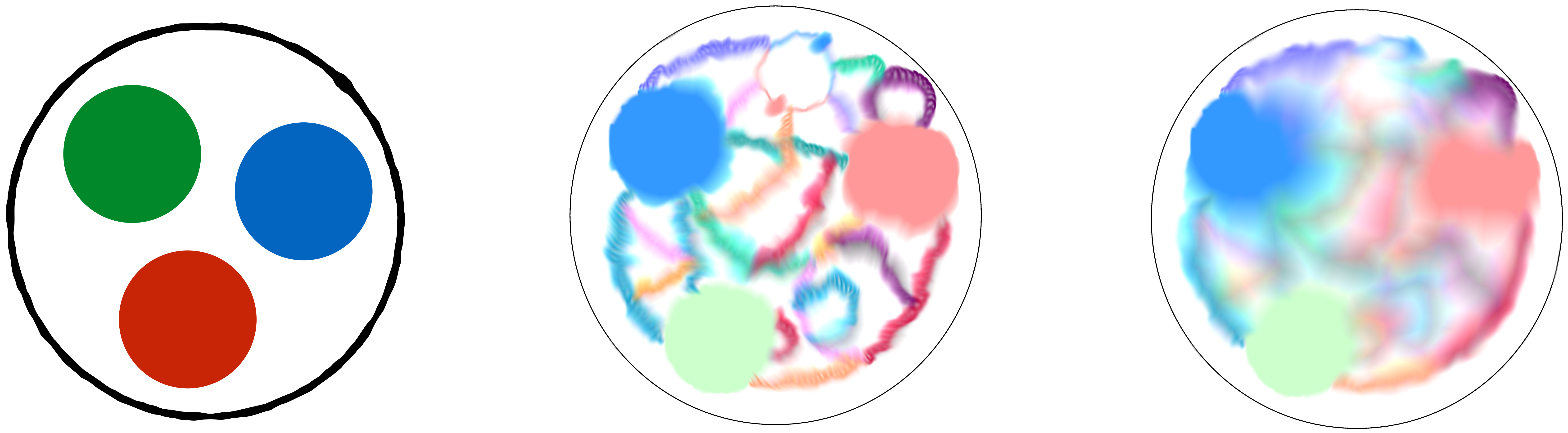
CFNS PostDoc Meeting, October 17th, 2023

Hadron as a many-body parton system

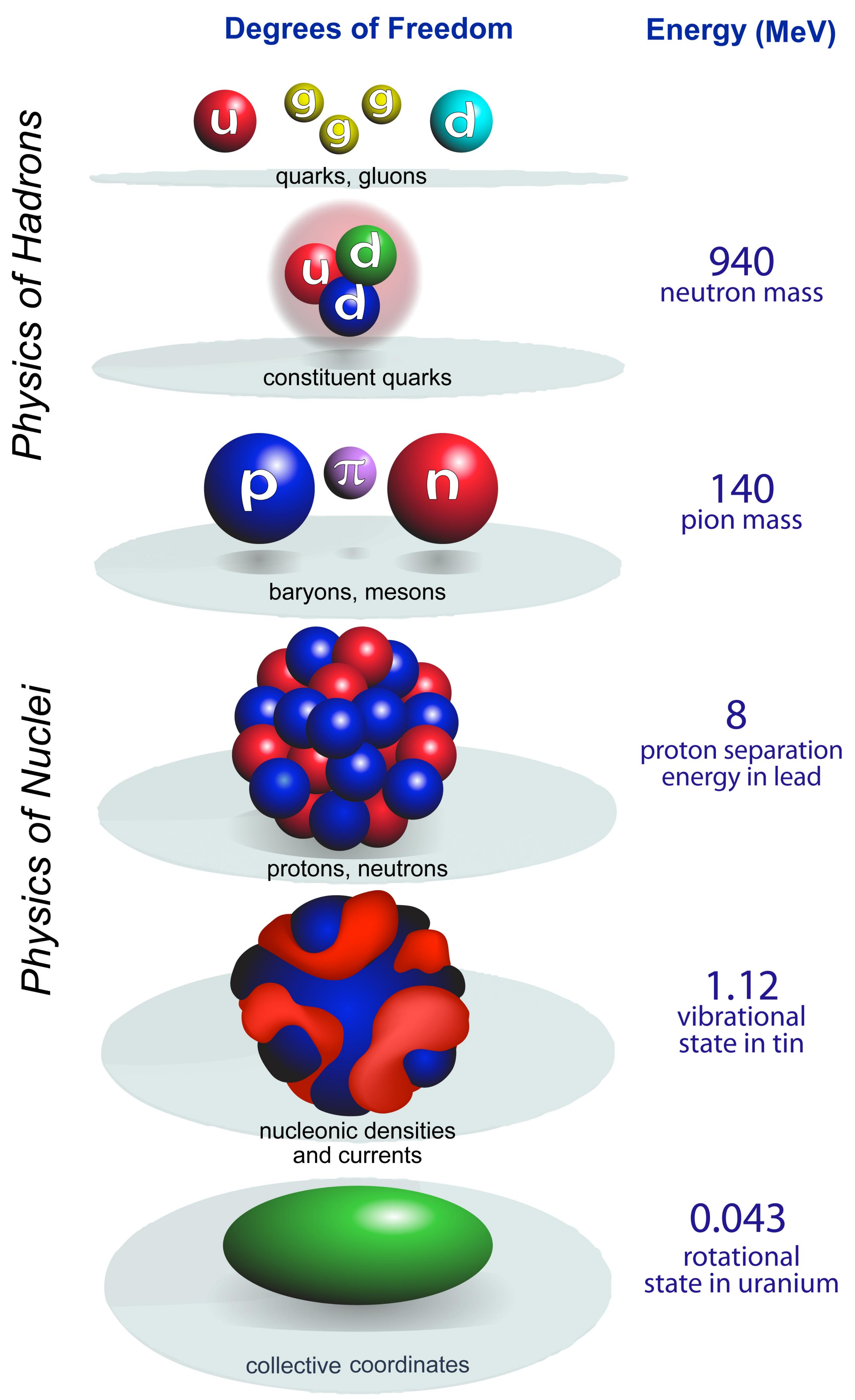


$1\ fm \sim 5\ GeV^{-1}$

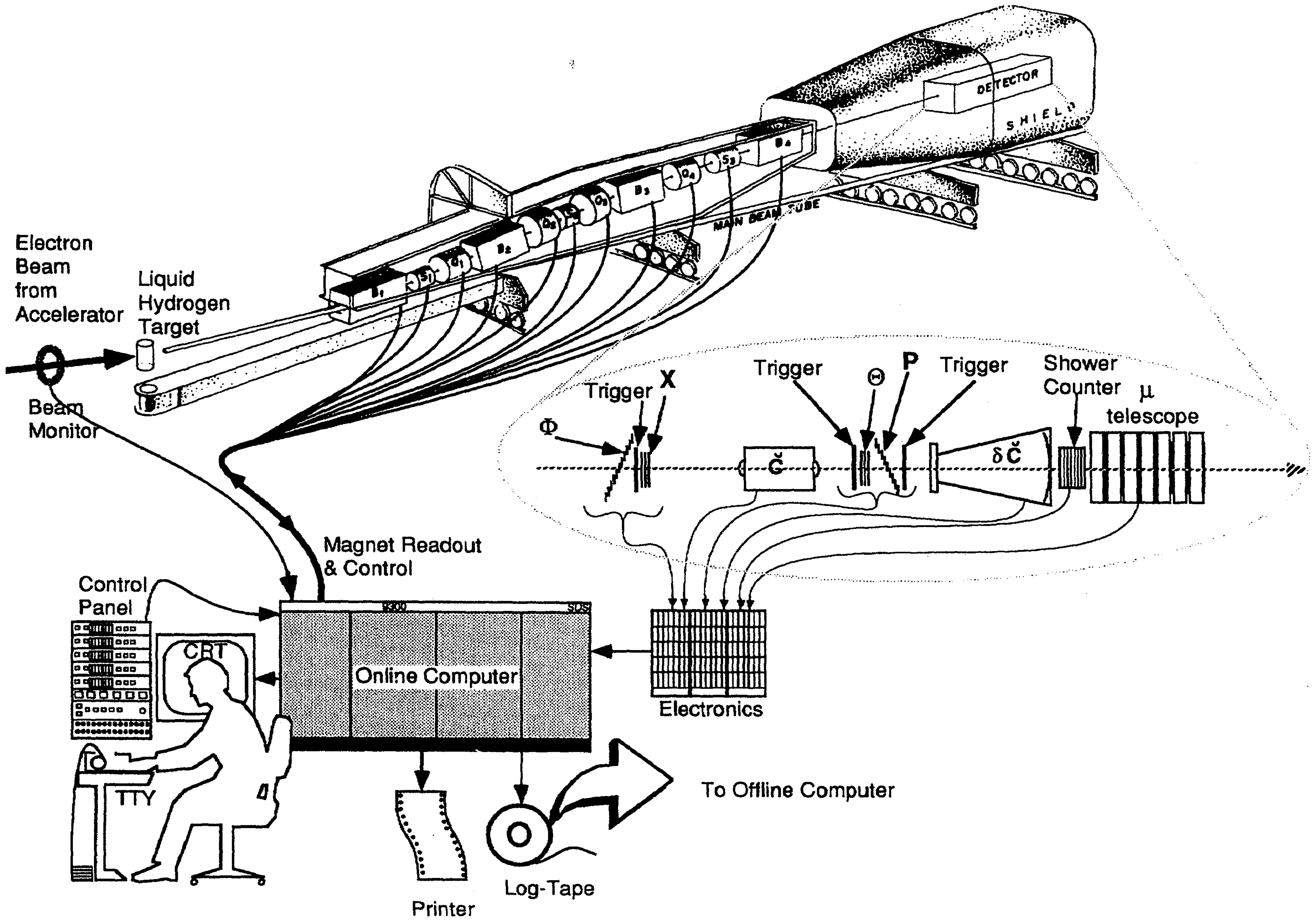
- The typical hadron scale is small
- Due to running of the coupling constant one has to understand the hadron as a strongly bonded many-body parton system
- Hadron is characterized by complex dynamics of parton interactions
- Cannot distinguish individual quarks and gluons
- How do we study this system?

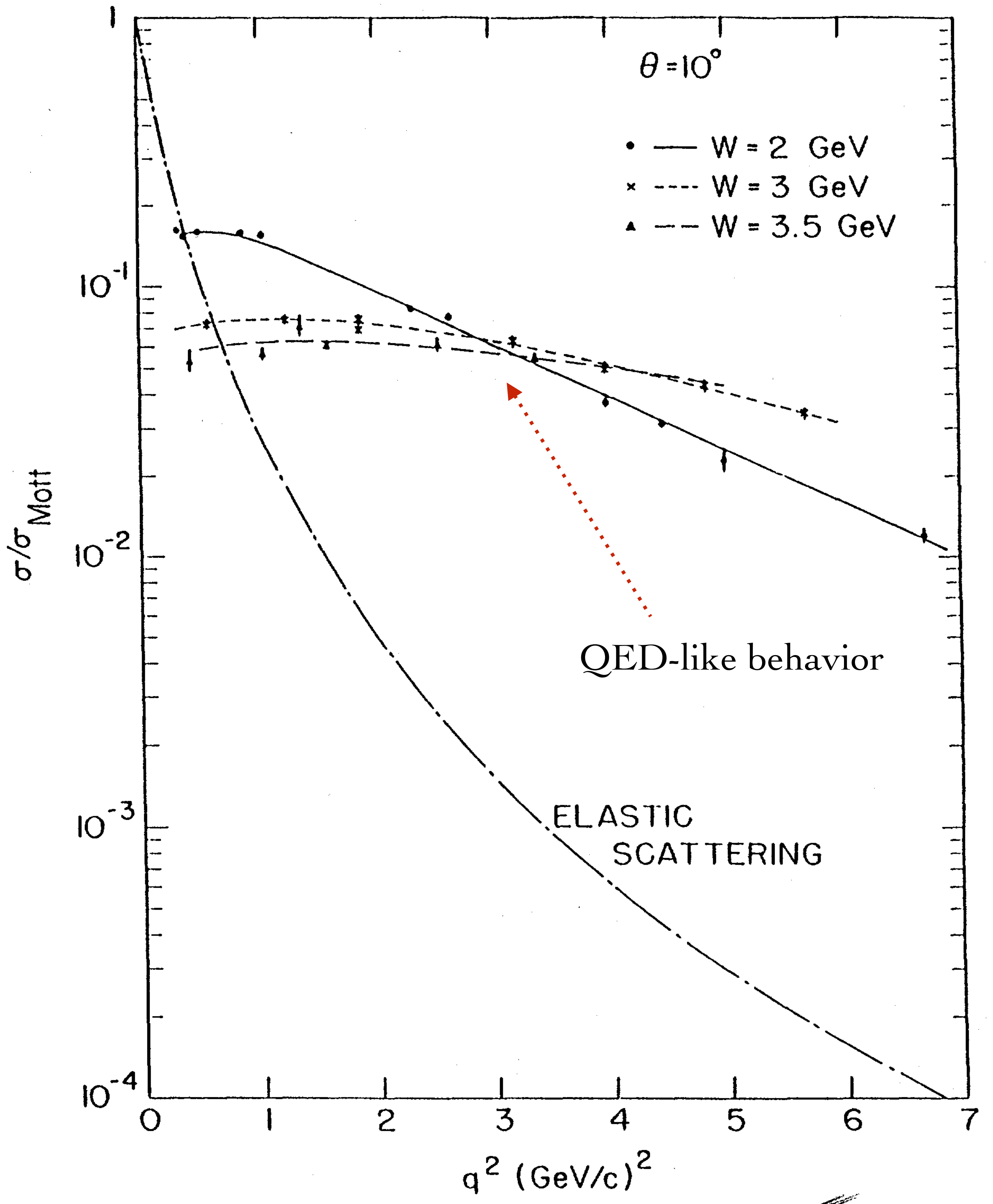
The system at different scales

- The system exhibits different properties at different scales with relevant degrees of freedom
- An appropriate effective model should be used for each energy scale (ex: Lund model)
- Lattice calculations proved to be very efficient with, however, certain limitations
- Potential application of quantum computers
- The typical scale is small. But what if we introduce a large external scale?

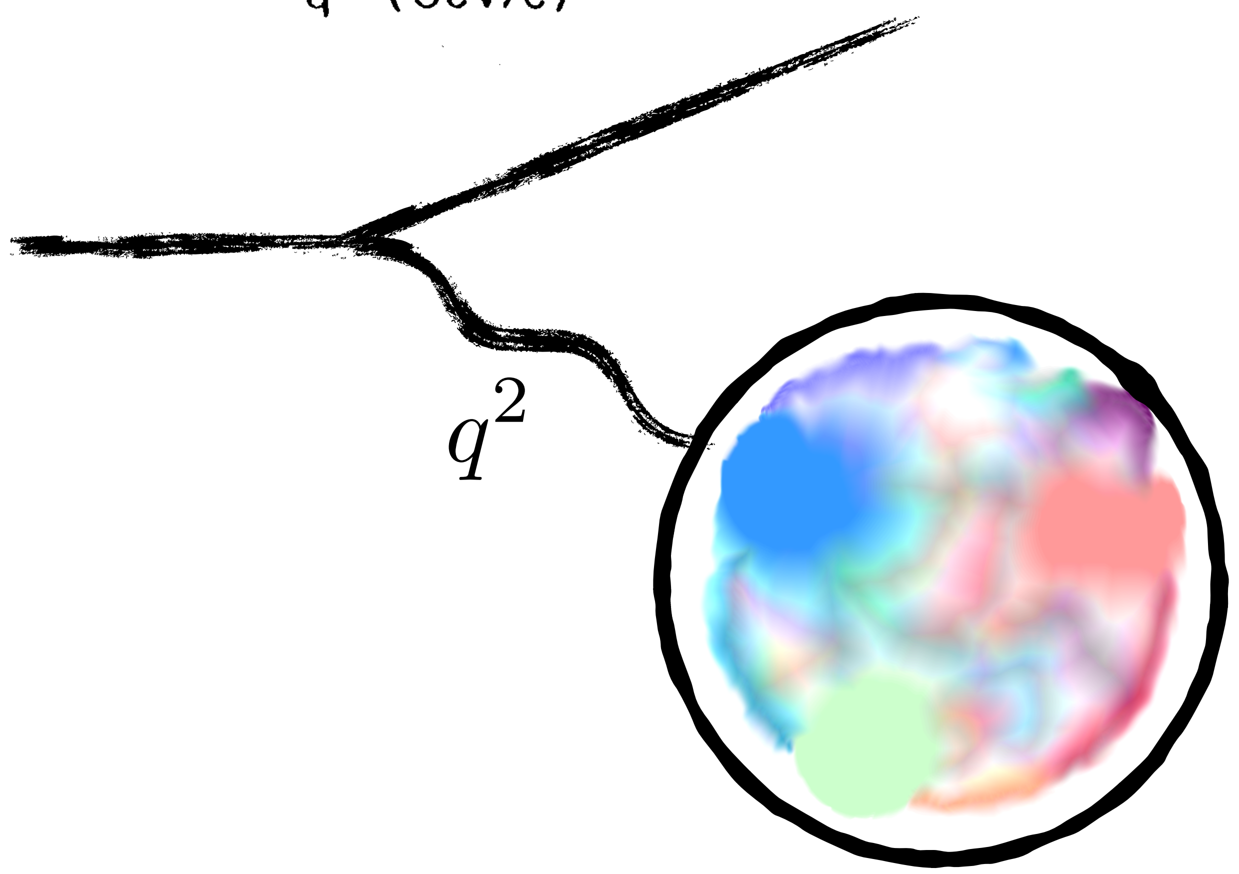


High-energy probe, external scale



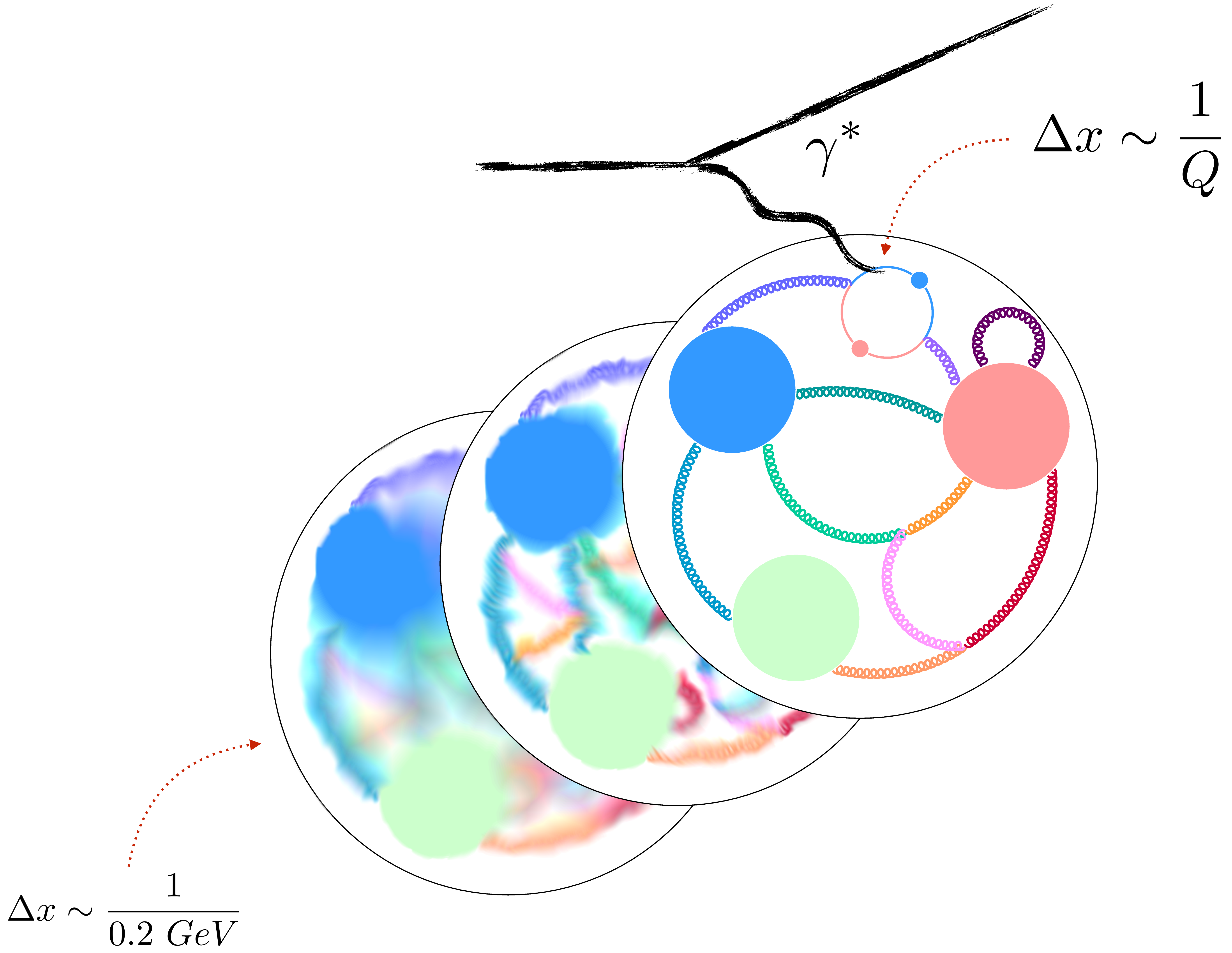


- How does the strongly bonded system respond to a high-energy probe?
- The existence of perturbative phase in the system was discovered
- It became the foundation of the parton model and perturbative QCD



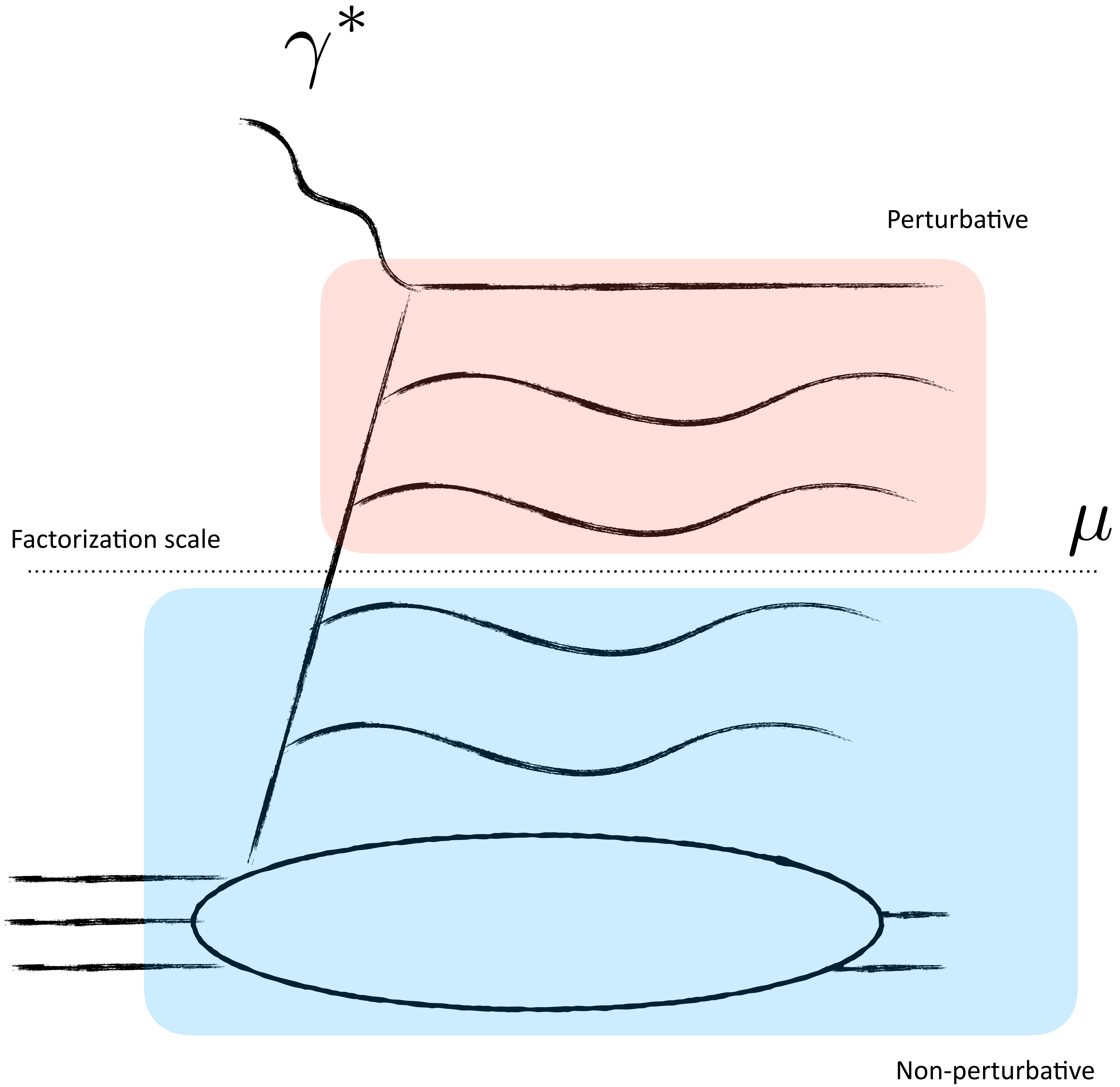
Hadron as a many-body parton system

- Interacting with external probe the hadron reveals different types of parton dynamics, we have separation of different scales
- There is a strong correlation between different phases, i.e. perturbative and non-perturbative components.
- The dense QCD medium strongly interacts with the probe
- If we study perturbative phase at a large energy scale, we can extract information about a non-perturbative many-body parton structure of the hadron



Factorization

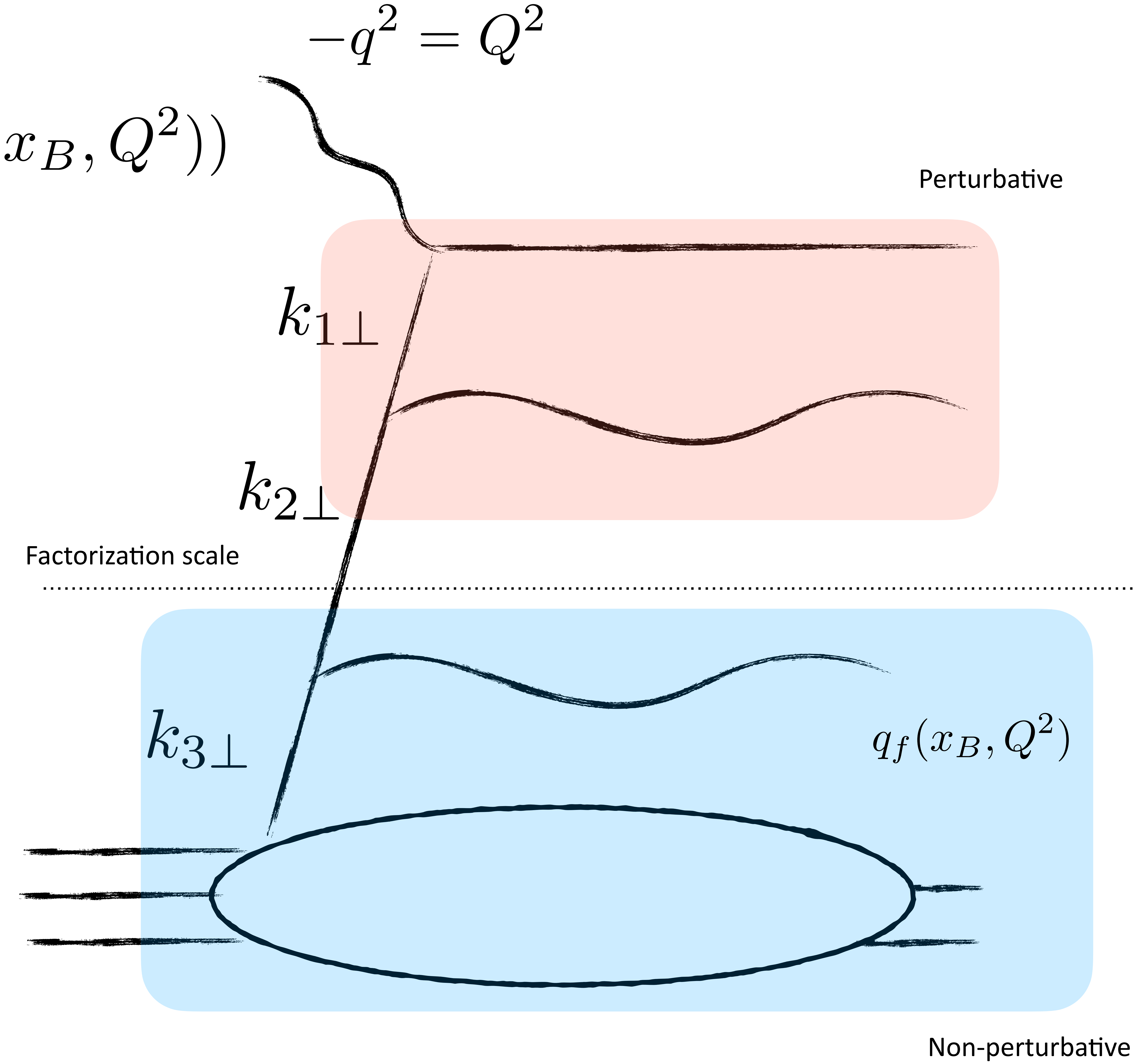
- The perturbative phase is under control - different types of reactions, different kinematic limits
- The phases are not independent. The information about the perturbative phase can be accessed indirectly
- The non-perturbative phase interacts with the perturbative one in a dynamical way - evolution equations
- This is formalized in terms of factorization theorems. Details of the factorization scheme are essential!



Deep inelastic scattering (DIS)

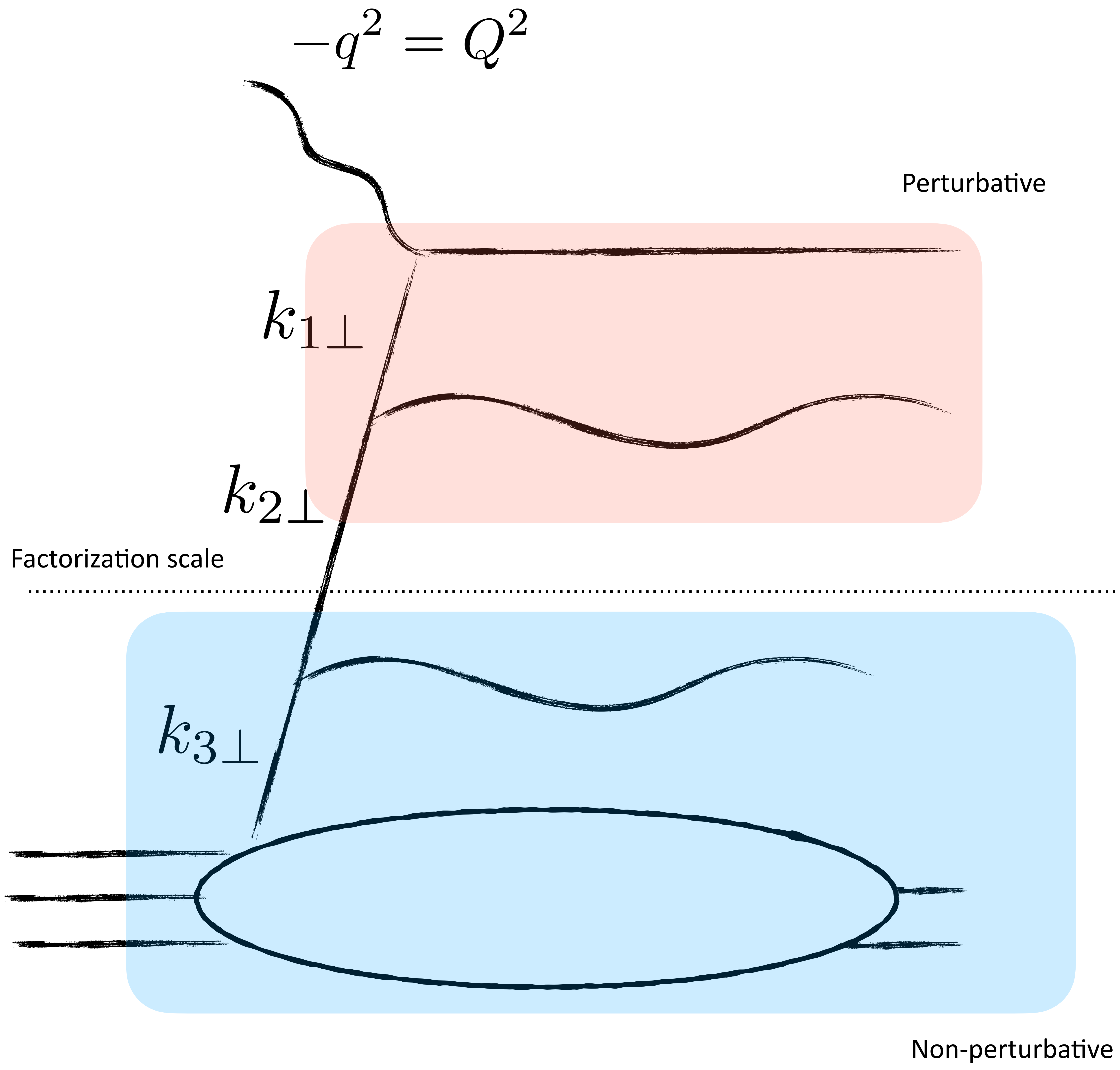
$$F_2(x_B, Q^2) = x_B \sum_f e_f^2 (q_f(x_B, Q^2) + \bar{q}_f(x_B, Q^2))$$

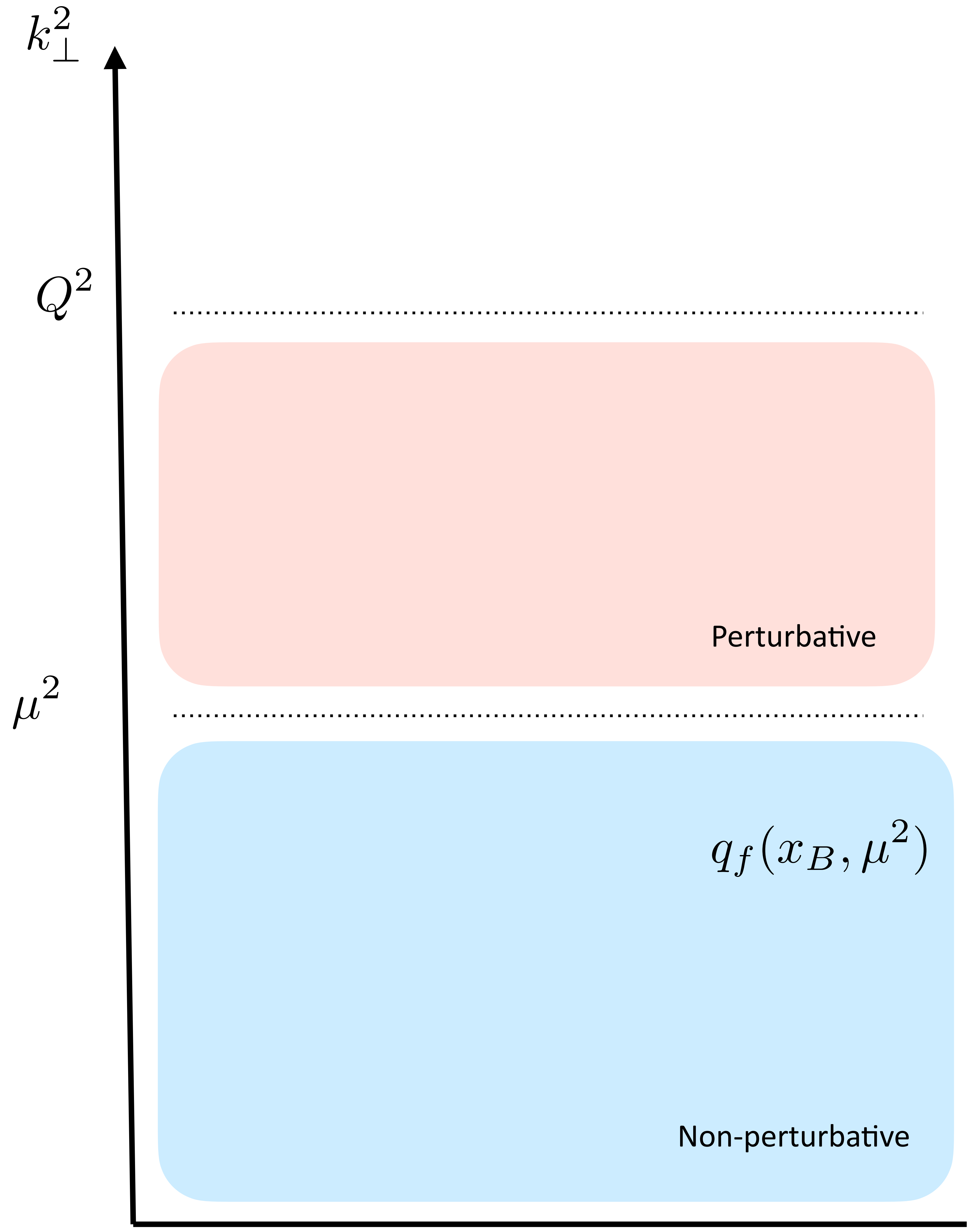
- Ordering of the transverse momenta $k_{1\perp} \gg k_{2\perp} \gg k_{3\perp}$ etc.
- Factorization in the transverse momentum
- The background gluons can be approximated with collinear particles
- It's convenient to choose the factorization scale as Q^2
- Resummation of transverse logarithms $\ln Q^2$



Collins, Foundations of perturbative QCD

Deep inelastic scattering (DIS)



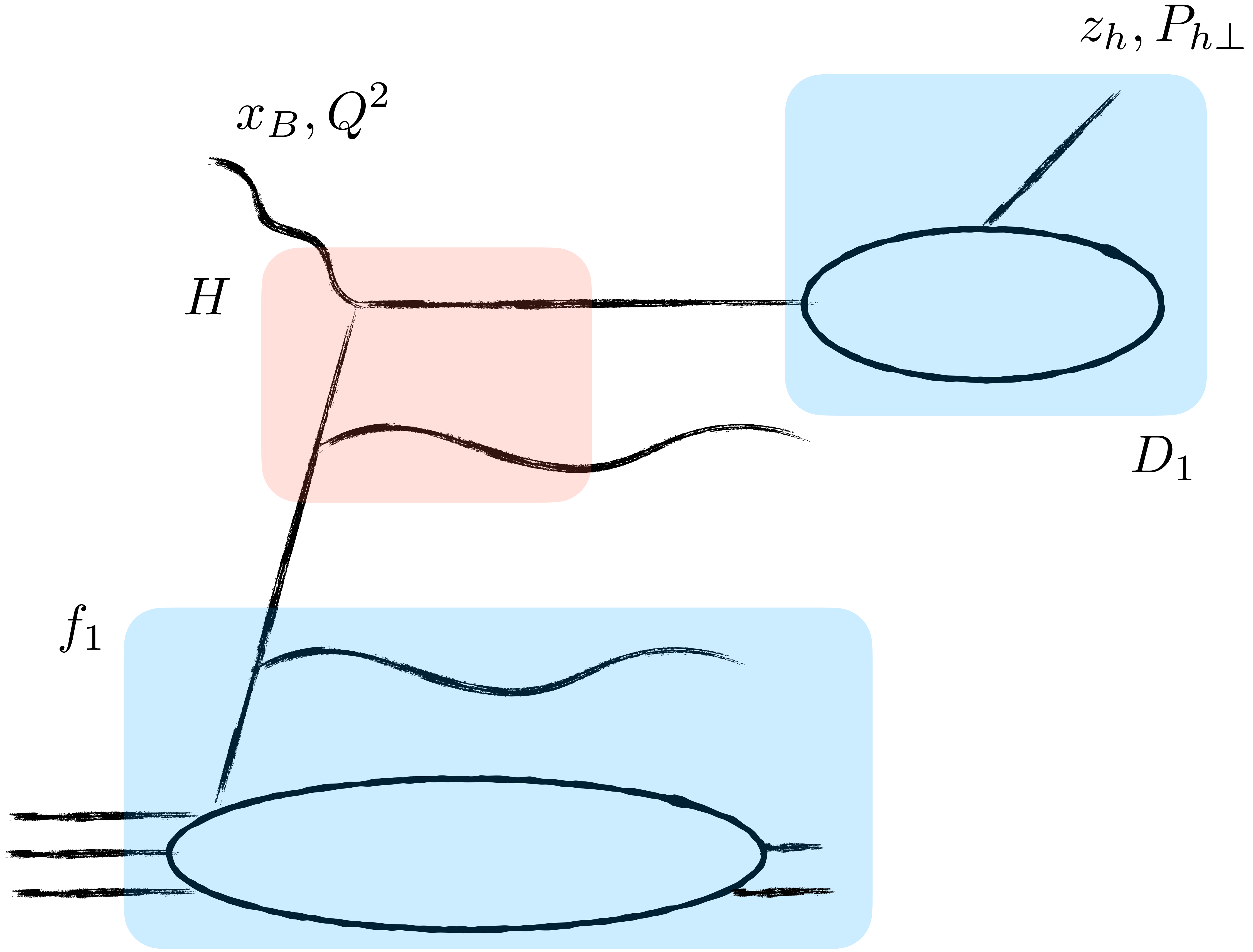


Semi-inclusive deep inelastic scattering (SIDIS)

$$F_{UU,T}(x_B, z_h, P_{h\perp}^2, Q^2) \propto H(Q^2,\mu^2) \int d^2p_\perp d^2k_\perp \delta^2(p_\perp - k_\perp - P_{h\perp}/z_h)$$
$$\times\, x_B \sum_a e_a^2 f_1^a(x_B, p_\perp^2, \mu^2, \zeta) D_1^a(z, k_\perp^2, \mu^2, \zeta_h)$$

- The TMD factorization structure is more involved comparing to DIS
- Resummation of transverse L_b and rapidity logarithms $\ln \nu$
- TMD distribution functions depend on two scales

TMD Handbook (2023)



Semi-inclusive deep inelastic scattering (SIDIS). Factorization

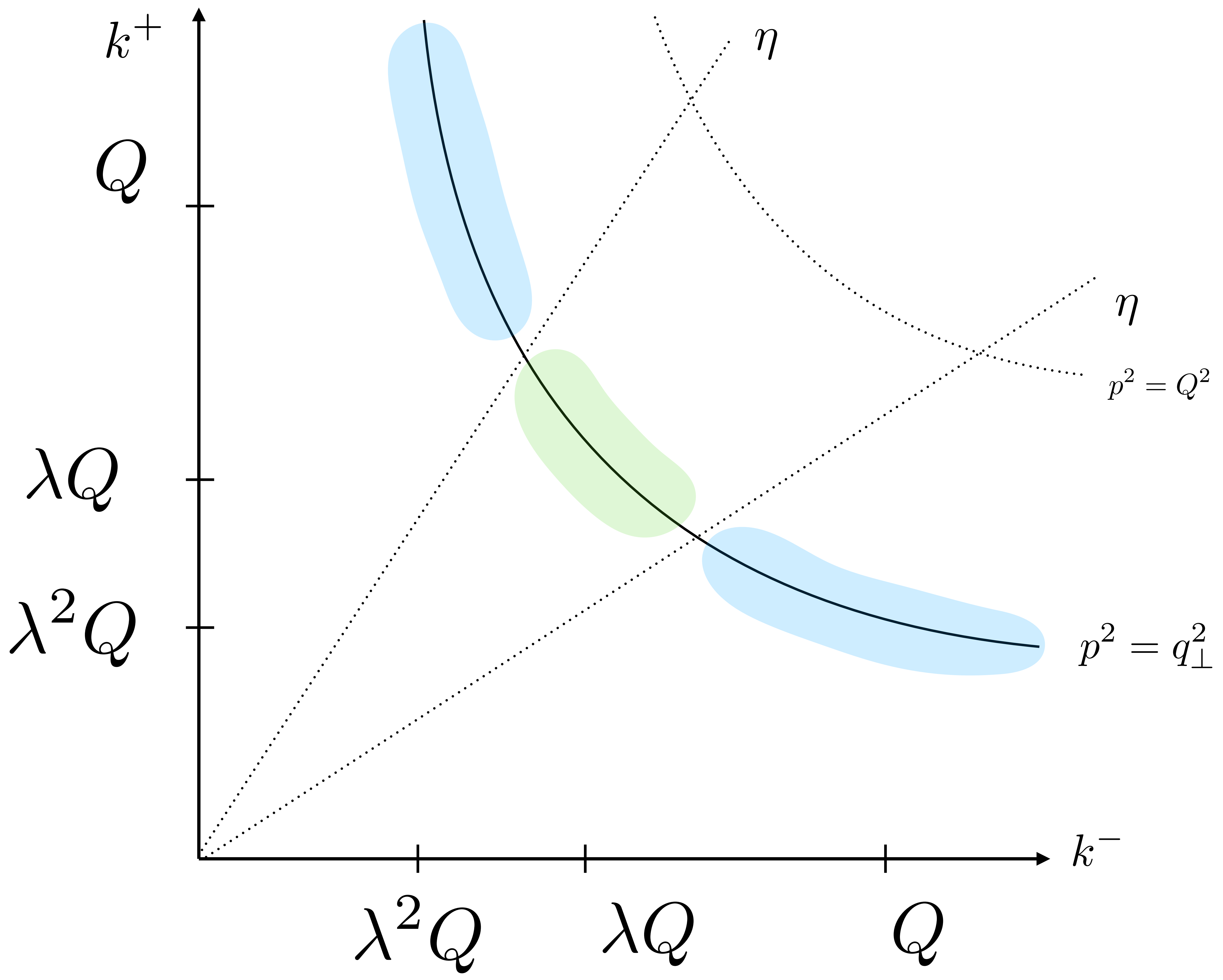
Collinear: $p^\mu \sim Q(1, \lambda^2, \lambda)$

Anti-collinear: $p^\mu \sim Q(\lambda^2, 1, \lambda)$

Soft: $p^\mu \sim Q(\lambda, \lambda, \lambda)$

- To compensate the overlap between collinear modes, a soft factor S should be introduced

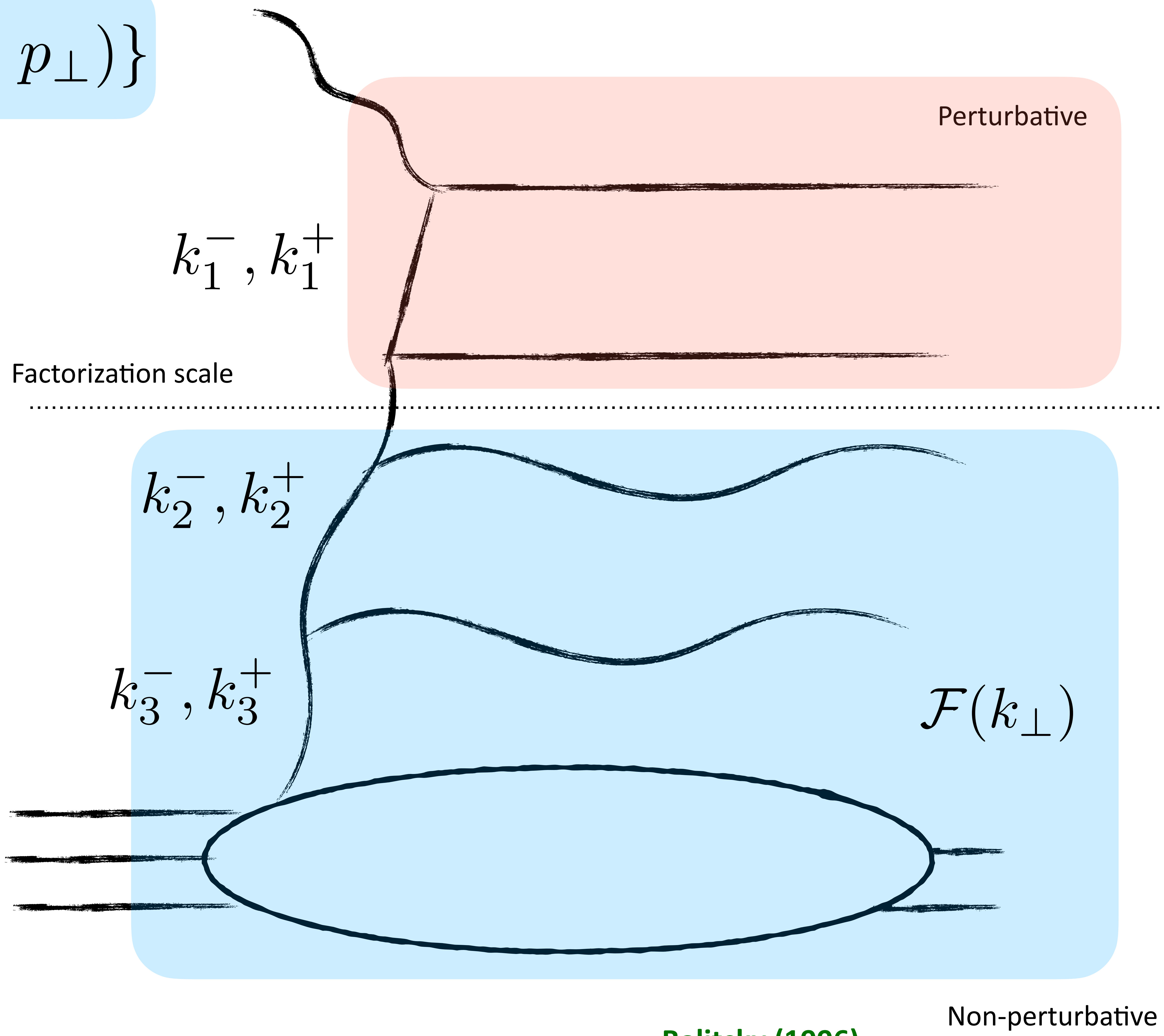
$$f^{TMD} = \sqrt{S} B^{TMD}$$



DIS at small-x

$$\sigma \propto \int \frac{d^2 p_\perp}{4\pi^2} I(p_\perp, q_\perp) Tr\{V(p_\perp)V^\dagger(q_\perp - p_\perp)\}$$

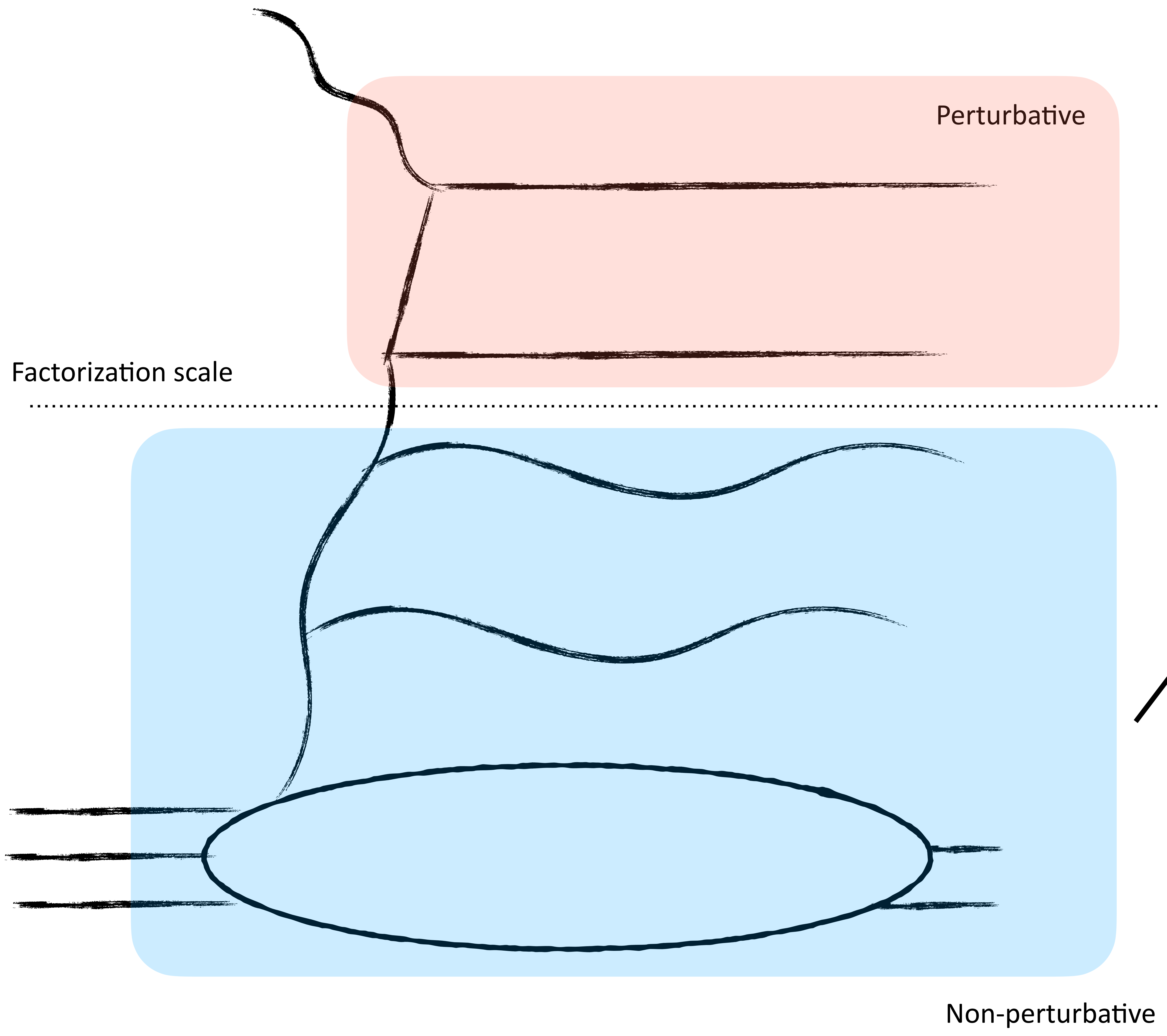
- Ordering of the longitudinal momenta $k_1^- \gg k_2^- \gg k_3^-; k_1^+ \ll k_2^+ \ll k_3^+$
- Factorization in the longitudinal momentum fraction
- The background gluons are described with an unintegrated distribution which depends on a transverse momentum
- It's convenient to choose the factorization scale as x_B
- Resummation of rapidity logarithms $\ln 1/x_B$

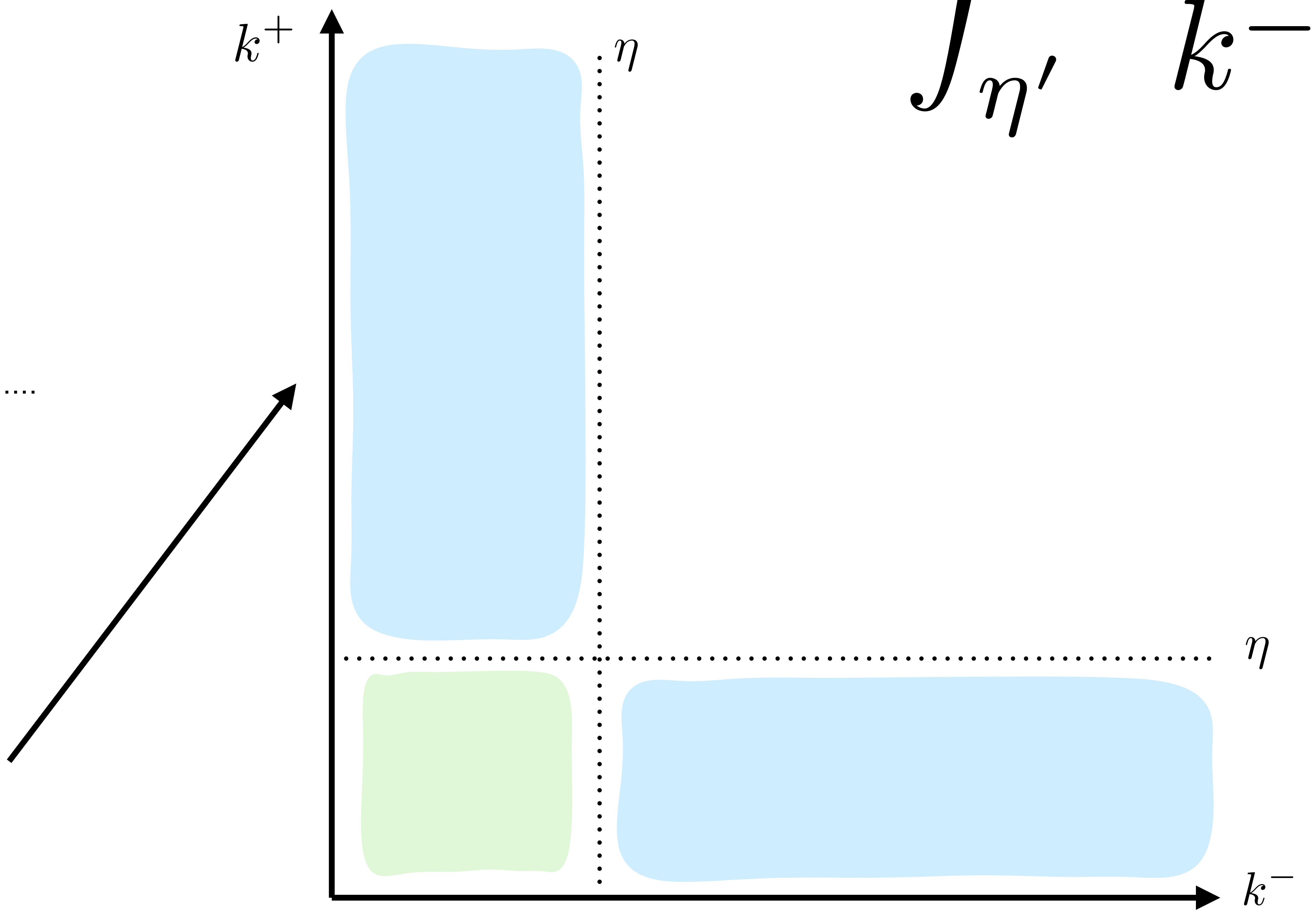


Balitsky (1996)

Rapidity factorization with rigid cut-offs

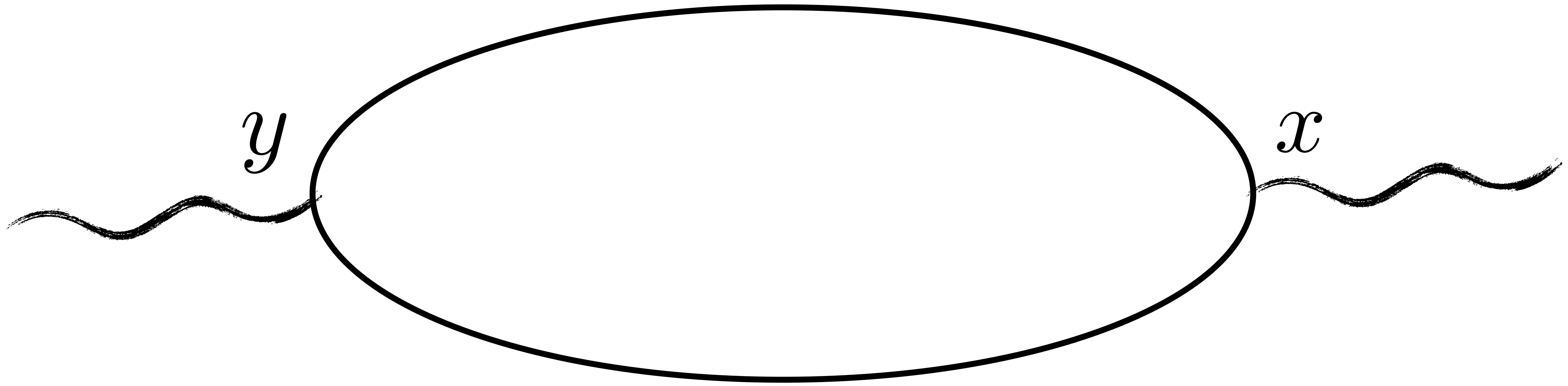
$$\int_{\eta'}^{\eta} \frac{dk^-}{k^-}$$





- Factorization in the longitudinal momentum fraction which is strictly ordered

DIS at small-x

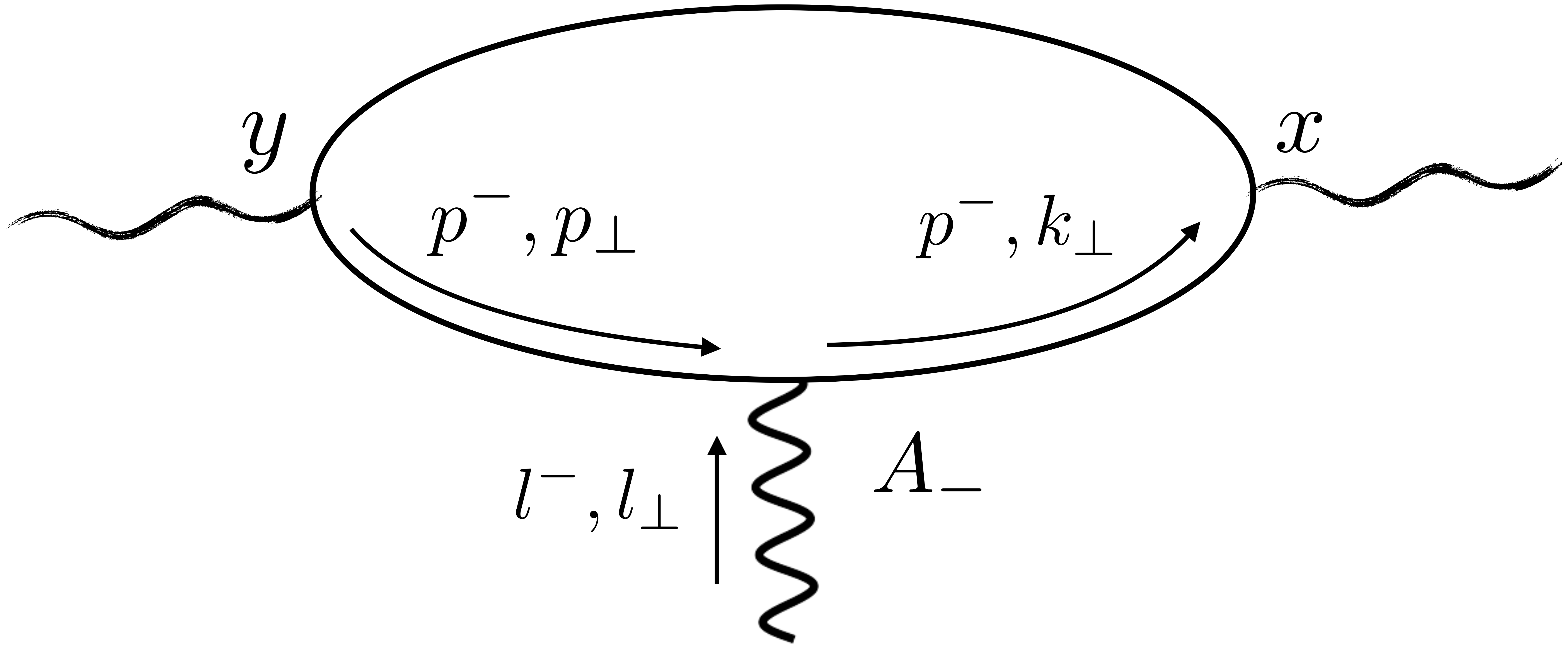


$$(x|\frac{1}{p^2+i\epsilon}|y) = \int_{-\infty}^{\infty}\frac{dp^-}{2\pi}e^{-ip^-(x-y)^+}\int_{-\infty}^{\infty}\frac{dp^+}{2\pi}e^{-ip^+(x-y)^-}(x_\perp|\frac{1}{2p^+p^- - p_\perp^2 + i\epsilon}|y_\perp)$$

$$= \frac{1}{2\pi}\int_0^{\infty}\frac{dp^-}{2p^-}e^{-ip^-(x-y)^+}\int_{-\infty}^{\infty}\frac{dp^+}{2\pi}e^{-ip^+(x-y)^-}(x_\perp|\frac{1}{p^+ - \frac{p_\perp^2}{2p^-} + i\epsilon}|y_\perp)$$

$$= -\frac{i}{2\pi}\int_0^{\infty}\frac{dp^-}{2p^-}e^{-ip^-(x-y)^+}(x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-}e^{i\frac{p_\perp^2}{2p^-}y^-}|y_\perp)$$

DIS at small-x



- Rapidity factorization

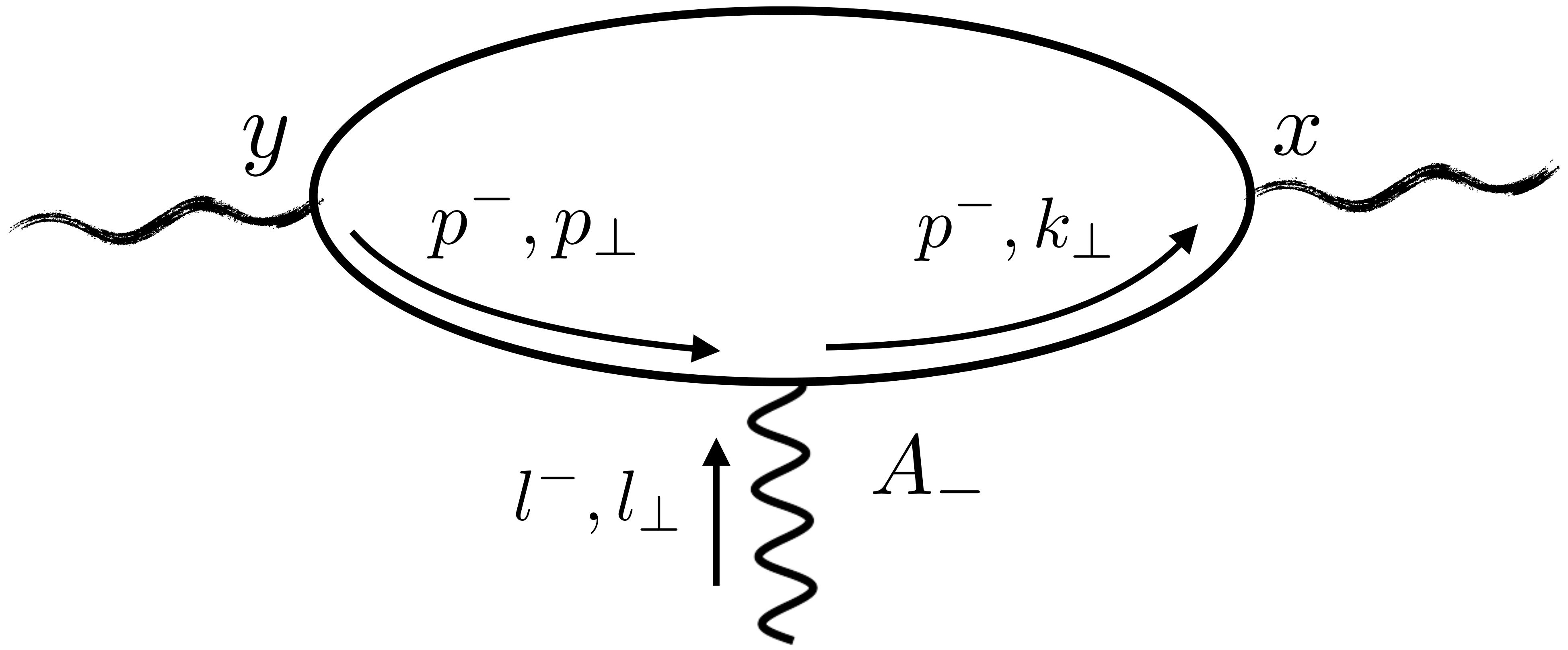
$$p^- \gg l^-$$

$$\frac{p_\perp^2}{2p^-} z^- \sim \frac{p_\perp^2}{2p^-} \frac{2l^-}{l_\perp^2} \sim \frac{l^-}{p^-} \ll 1$$

$$(x|\frac{1}{P^2 + i\epsilon}|y) = -\frac{i}{2\pi} \int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+} (x_\perp| e^{-i\frac{p_\perp^2}{2p^-}x^-} e^{i\frac{p_\perp^2}{2p^-}y^-} |y_\perp)$$

$$-\frac{i}{2\pi} \int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+} (x_\perp| e^{-i\frac{p_\perp^2}{2p^-}x^-} \left(ig \int_{y^-}^{x^-} dz^- e^{i\frac{p_\perp^2}{2p^-}z^-} A_-(z^-) e^{-i\frac{k_\perp^2}{2p^-}z^-} \right) e^{i\frac{k_\perp^2}{2p^-}y^-} |y_\perp) + O(g^2)$$

DIS at small-x



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Expansion parameter

$$(x|\frac{1}{P^2 + i\epsilon}|y) = -\frac{i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+}(x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-} e^{i\frac{p_\perp^2}{2p^-}y^-}|y_\perp)$$

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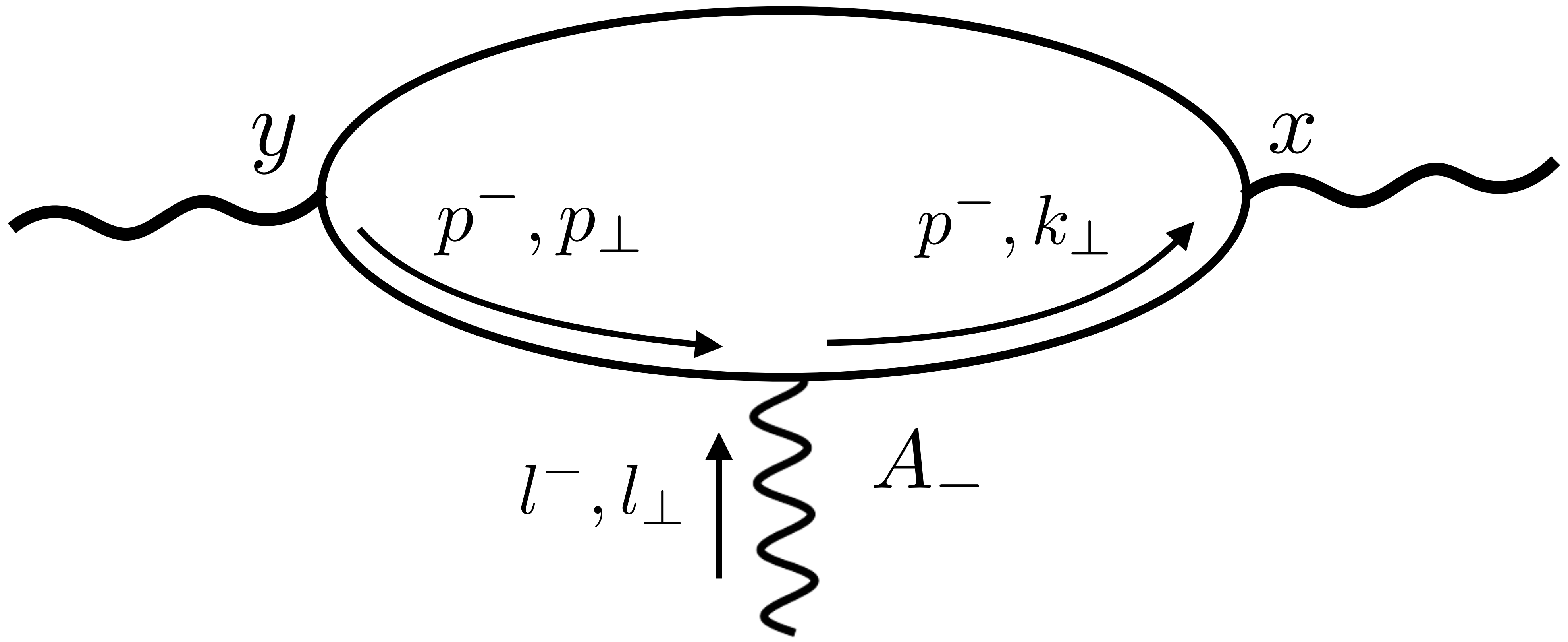
DIS at small-x



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$$-\frac{i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+} (x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-} \left(ig\int_{y^-}^{x^-} dz^- e^{i\frac{p_\perp^2}{2p^-}z^-} A_-(z^-) e^{-i\frac{k_\perp^2}{2p^-}z^-}\right) e^{i\frac{k_\perp^2}{2p^-}y^-}|y_\perp) + O(g^2)$$

$$= -\frac{i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+} (x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-} \exp\left\{ig\int_{y^-}^{x^-} dz^- A_-(z^-)\right\} e^{i\frac{p_\perp^2}{2p^-}y^-}|y_\perp)$$

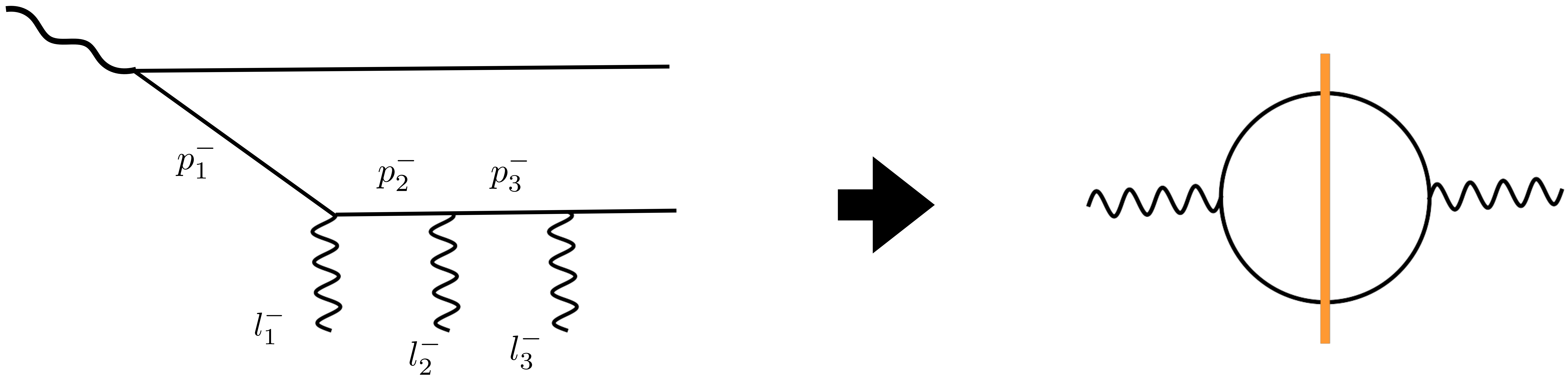
- Straight Wilson line $V(l_\perp)$ along the light-cone. A gauge phase acquired by a quark propagating in an external gluon field. No “kicks" in the transverse direction!

Transverse momentum at small-x

- Small-x: propagation along the light-cone direction -> shock-wave picture
- Due to wide separation in longitudinal momentum fraction one can obtain a large logarithm
- BFKL/BK evolution: resummation of longitudinal logs

$$\frac{l^-}{p^-} \ll 1$$

$$\ln x_B \sim \int^{1/x_B} \frac{dp^-}{p^-}$$

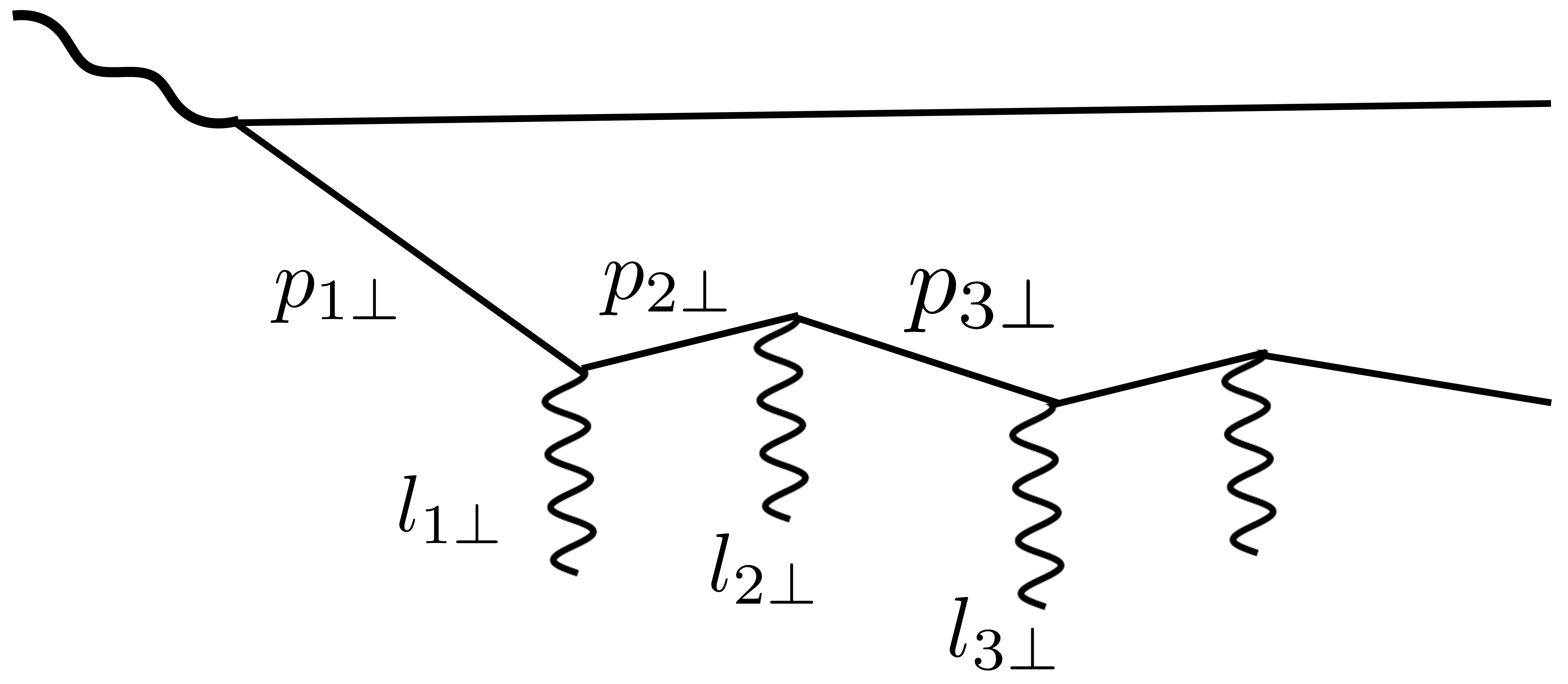


Transverse momentum at large-x

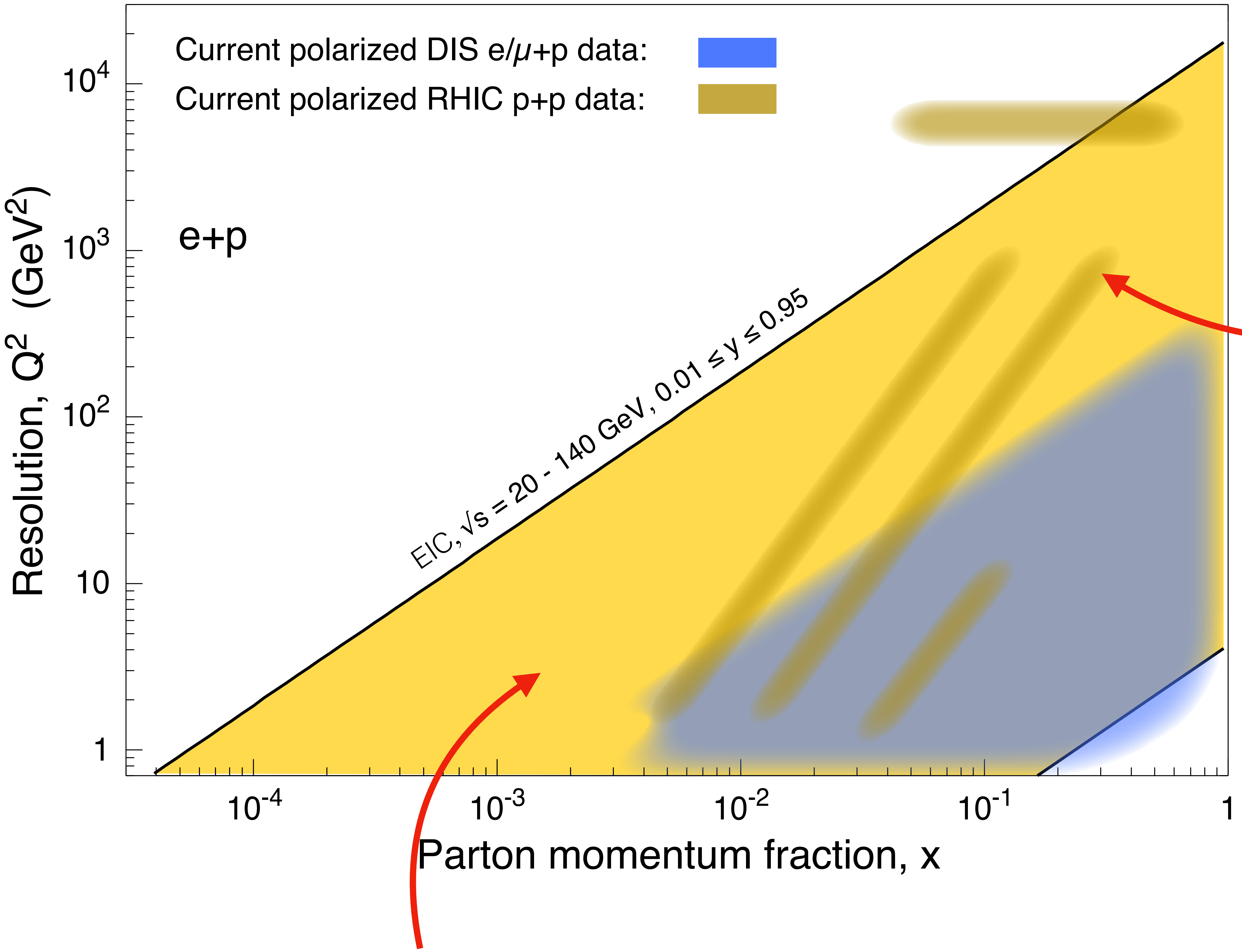
- Large-x : transverse “kicks” from the background field
- Systematically can be taken into account -> twist expansion
- Due to wide separation separation in transverse momentum, large logarithms of transverse momentum appear
- DGLAP evolution equation. Resummation of transverse logs
- Different factorization scheme -> different calculation

$$\frac{l_\perp}{p_\perp} \ll 1$$

$$\ln Q^2 \sim \int^{Q^2} \frac{dp_\perp^2}{p_\perp^2}$$



Transverse momentum at large-x



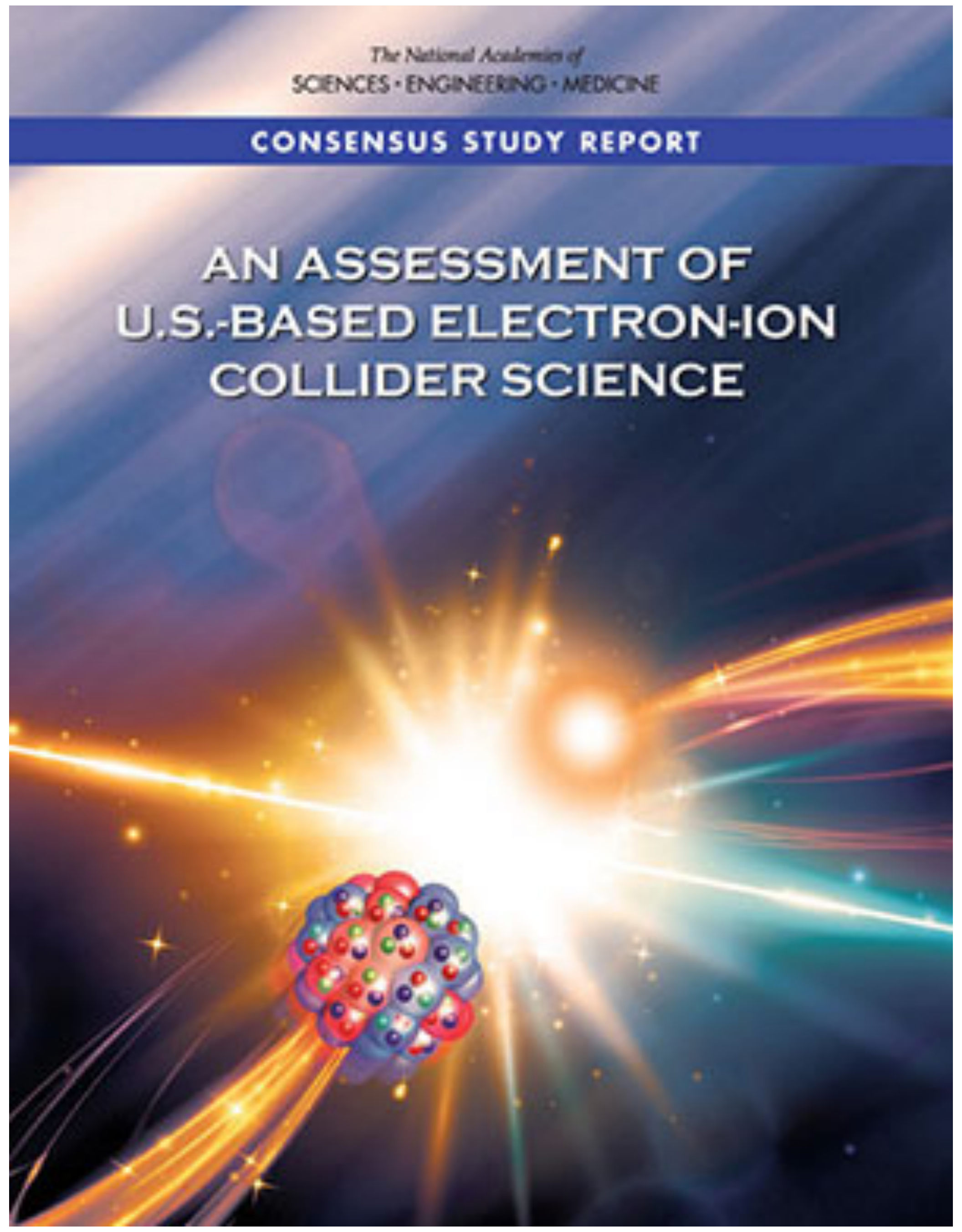
- Large Q -> large transverse integrals
- DIS at large x: rapidity logs are suppressed
- Rapidity logs are essential in SIDIS

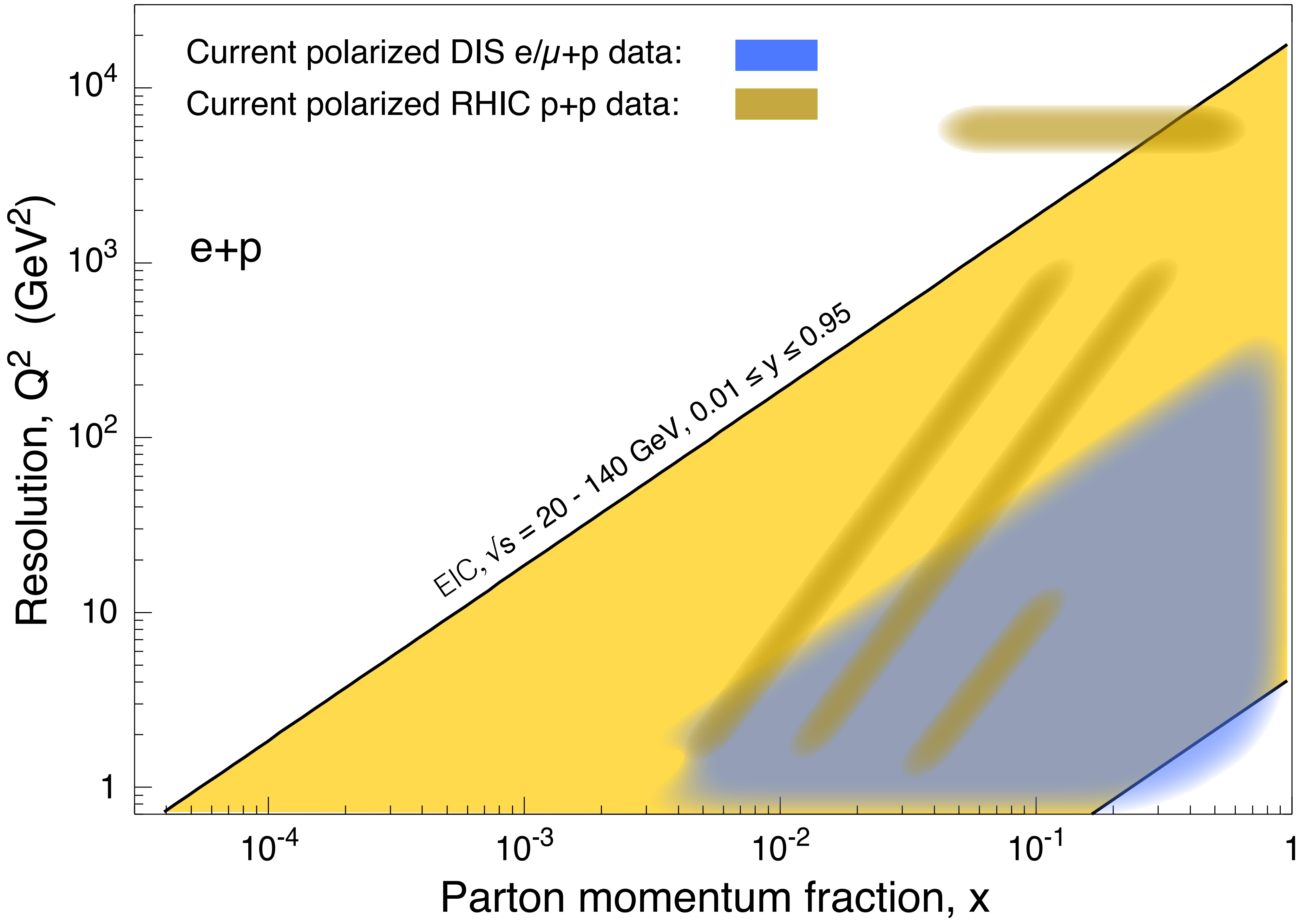
- Low Q, not enough phase space for a large transverse integral
- Wide separation in rapidity -> large rapidity logs (logs of xB)

Search of saturation at EIC

Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?





- In EIC kinematics transverse logs will always be present -> No “pure” small-x regime.
- This is going to jeopardize saturation search.
- We need a factorization scheme which systematically takes into account transverse and longitudinal logs

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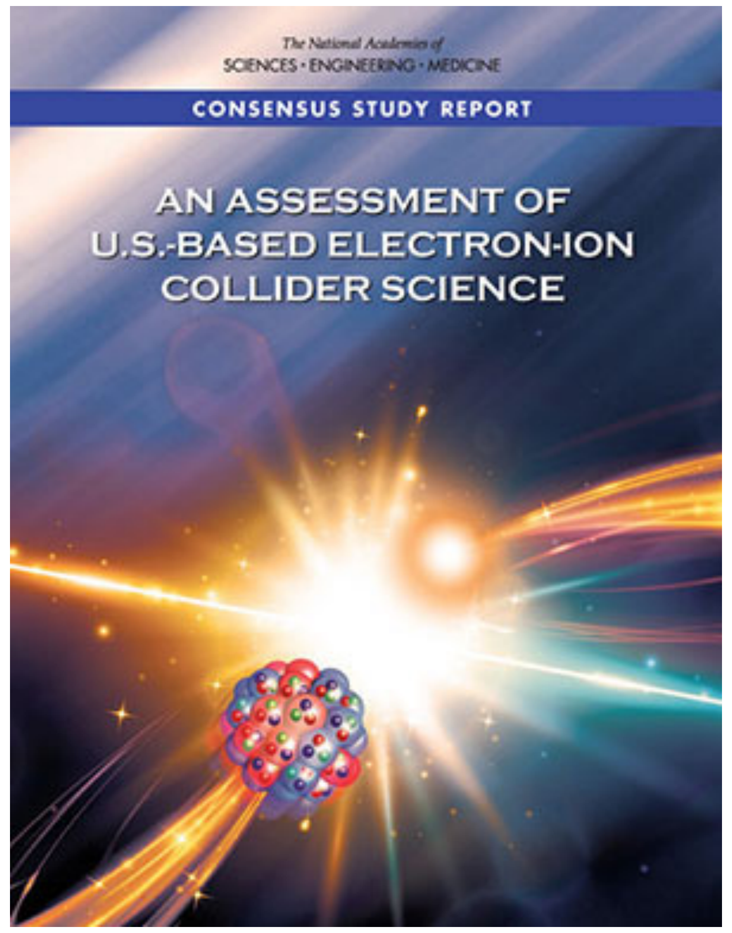
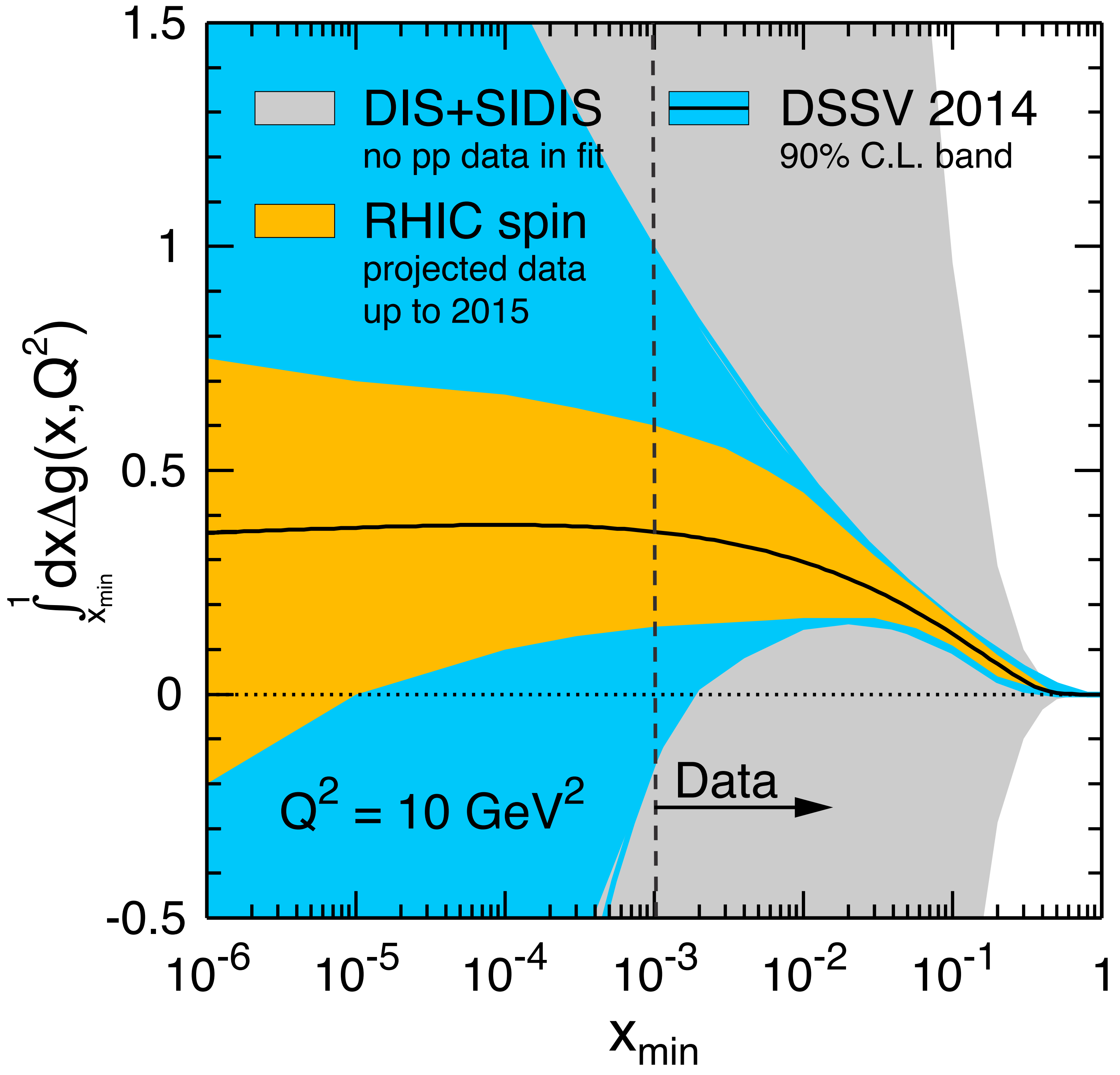




D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)
Aschenauer et al., arXiv:1708.01527

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton's spin: $\Delta\Sigma \approx 0.32$, which is much smaller than predicted by the quark model $\Delta\Sigma \approx 0.6$ - spin puzzle

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B \, \left(\Delta q_f(x_B, Q^2) + \Delta \bar{q}_f(x_B, Q^2)\right)$$

Integration from zero!

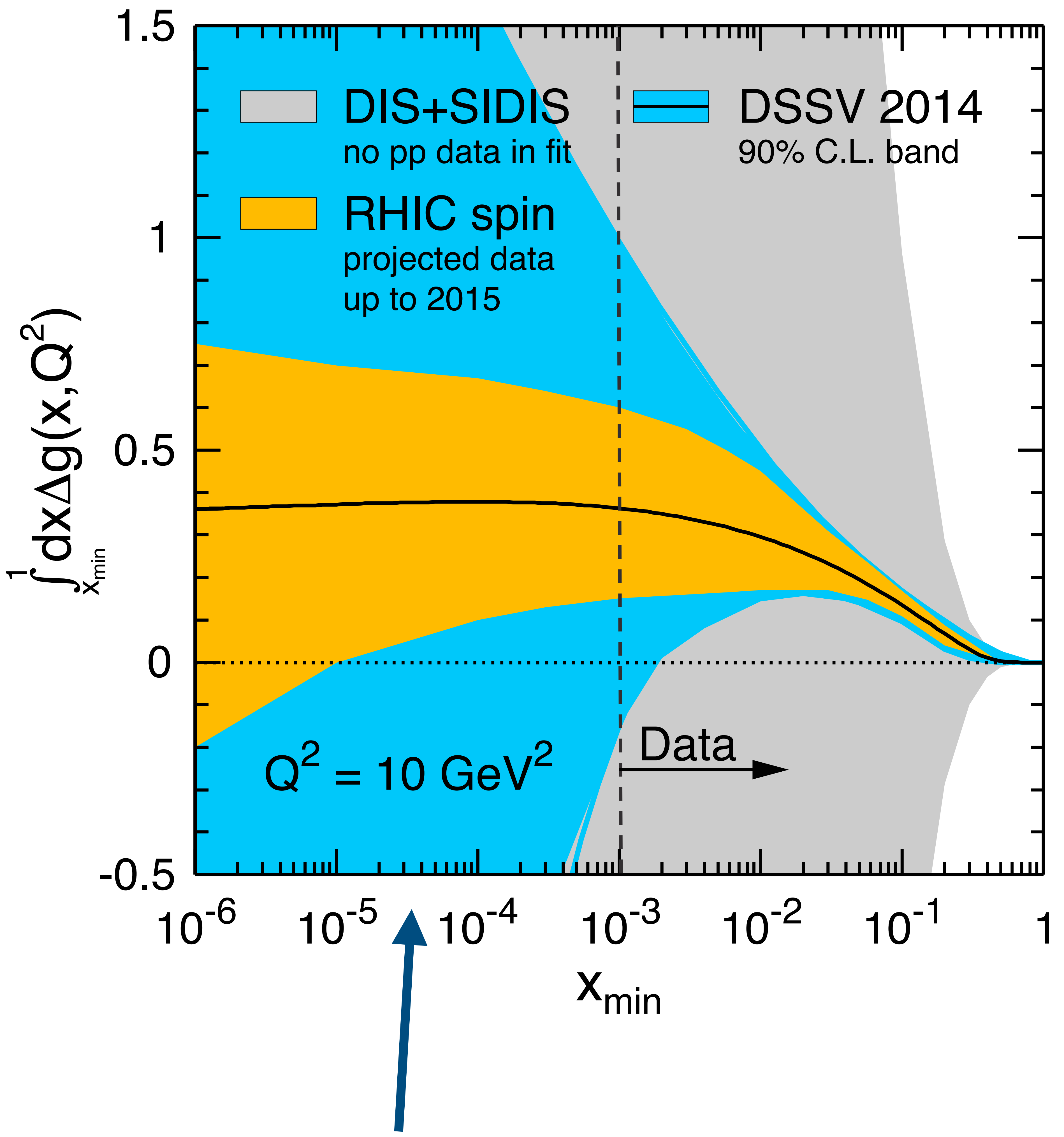
- Spin of the proton -> contribution of small-x?
- Contribution of the region which cannot be accessed experimentally!
- Theory input: small-x evolution evolution equations

Extractions based on DGLAP

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0 \\ -\frac{1}{2N_f}\Delta P_{\Sigma\Sigma}(a_s) & 0\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix}$$

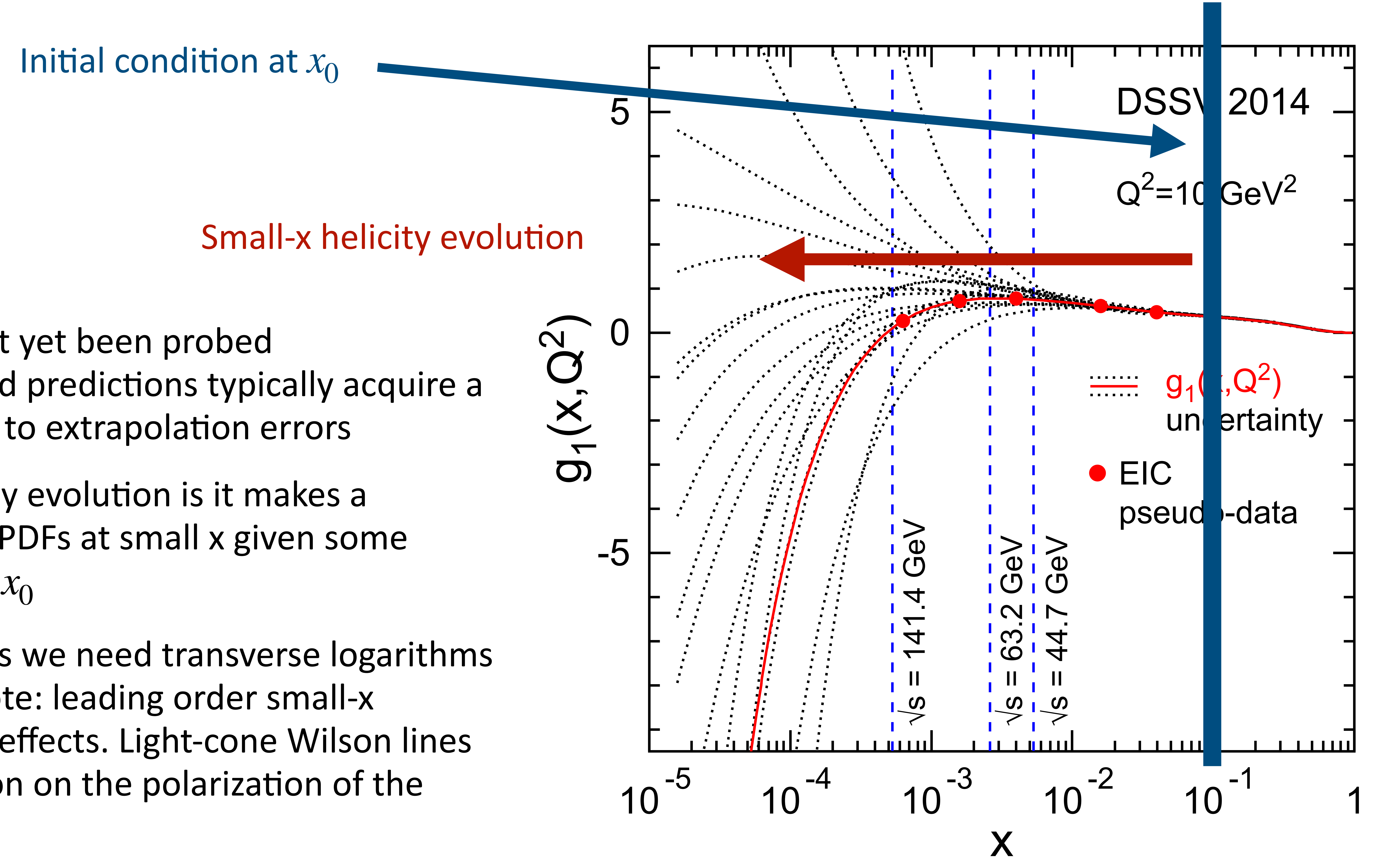
$$\Delta\Gamma(Q^2) \equiv a_s(Q^2)\Delta G(Q^2)$$

- The standard way to address the proton spin puzzle is by extracting the hPDFs from experimental data using collinear factorization along with the spin-dependent DGLAP evolution equations
- Since the DGLAP equations evolve PDFs in Q^2, they cannot truly predict the x dependence of PDFs
- The x dependence is greatly affected by the functional form of the PDF parametrization at the initial momentum scale Q_0^2, which gives the initial conditions for the DGLAP evolution



No data ⇒ no initial condition for DGLAP

DGLAP vs. small-x helicity evolution

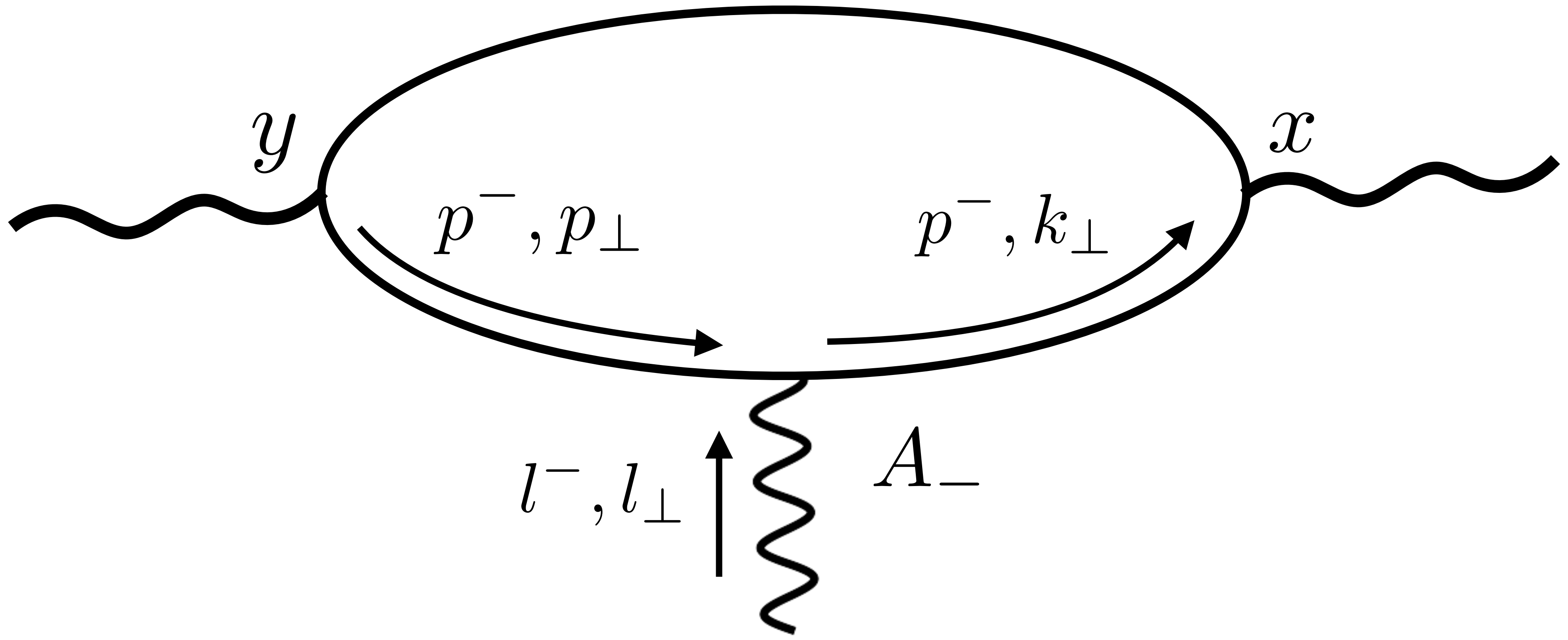


- In the x region which has not yet been probed experimentally, DGLAP-based predictions typically acquire a broad uncertainty band due to extrapolation errors
- The benefit of small-x helicity evolution is it makes a genuine prediction for the hPDFs at small x given some initial conditions at a higher x_0
- To obtain reliable predictions we need transverse logarithms in the small-x evolution! (Note: leading order small-x evolution doesn't have spin effects. Light-cone Wilson lines don't contain any information on the polarization of the target)

Approach 1: extend the classical rapidity factorization scheme. Can we do that?

$$\frac{l^-}{p^-} \ll 1$$

- Rapidity cut-off of a longitudinal integral $\int_{\eta'}^{\eta} \frac{dp^-}{p^-}$ can be used to resume both longitudinal and transverse logarithms



- Exact result for the propagator in the background field contains transverse phases
- In the leading order rapidity factorization we neglect the phases
- Interplay between transverse and longitudinal variables. Expand in powers of $1/p^-$ -> corrections to the leading order factorization

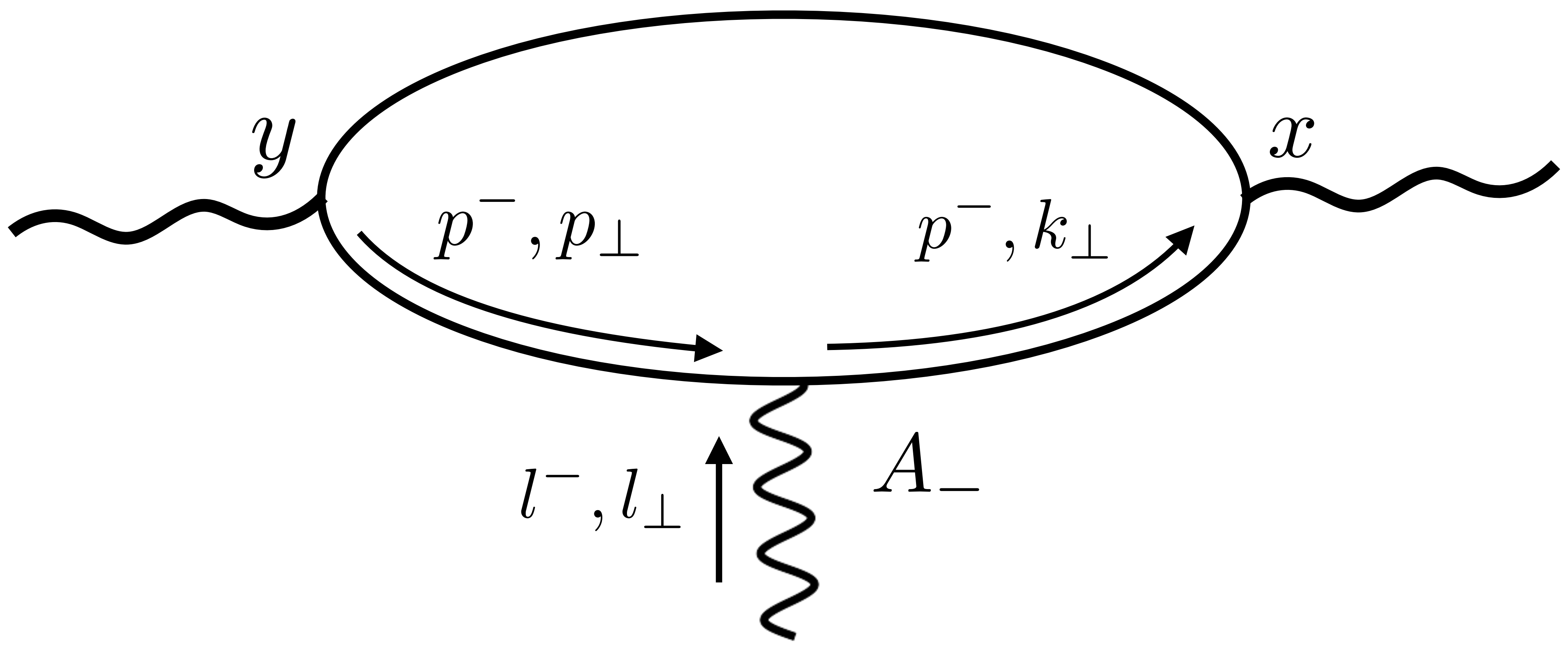
$$(x|\frac{1}{P^2+i\epsilon}|y) = -\frac{i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+}(x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-} e^{i\frac{p_\perp^2}{2p^-}y^-}|y_\perp)$$

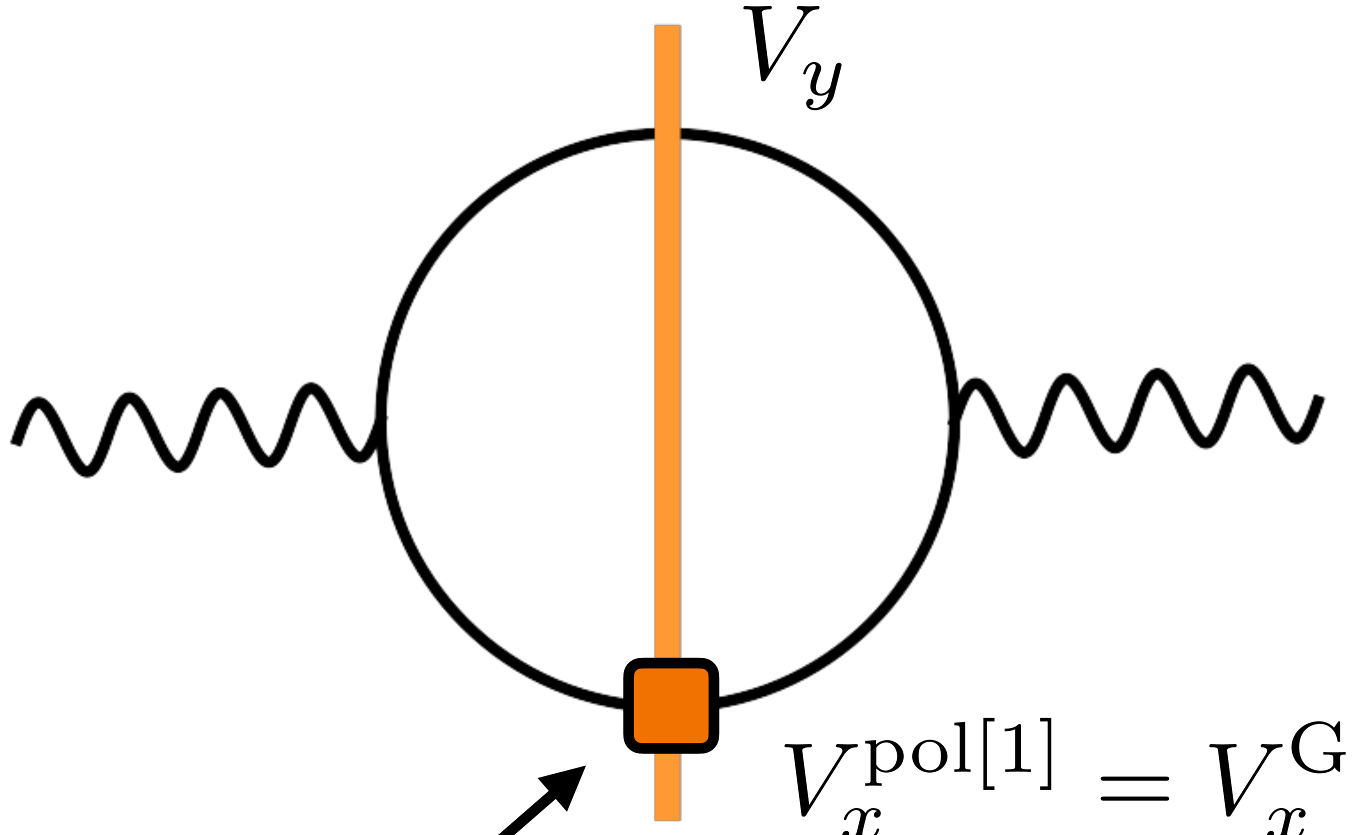
$$-\frac{i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+}(x_\perp|e^{-i\frac{p_\perp^2}{2p^-}x^-}\Big(ig\int_{y^-}^{x^-} dz^- e^{i\frac{p_\perp^2}{2p^-}z^-} A_-(z^-) e^{-i\frac{k_\perp^2}{2p^-}z^-}\Big) e^{i\frac{k_\perp^2}{2p^-}y^-}|y_\perp) + O(g^2)$$

Balitsky, Tarasov (2015)

Sub-eikonal corrections

$$\frac{l^-}{p^-} \ll 1$$





$$V_x^{\mathrm{pol}[1]} = V_x^{\mathrm{G}[1]} + V_x^{\mathrm{q}[1]}$$

- Correction to the leading order factorization. Dominates spin dependent effects

$$(x|\frac{1}{P^2+i\epsilon}|y) = -\frac{i}{2\pi}\int_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+}$$

$$\times(\underline{x}|e^{-i\frac{p_\perp^2}{2p^-}x^-}\left\{V - \frac{ig}{2p^-}\int_{-\infty}^{\infty} dz^- z^- V[\infty, z^-]\{P^k, F_{-k}\}V[z^-, -\infty] + O\left(\frac{1}{(p^-)^2}\right)\right\}e^{i\frac{p_\perp^2}{2p^-}y^-}|\underline{y})$$

Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)

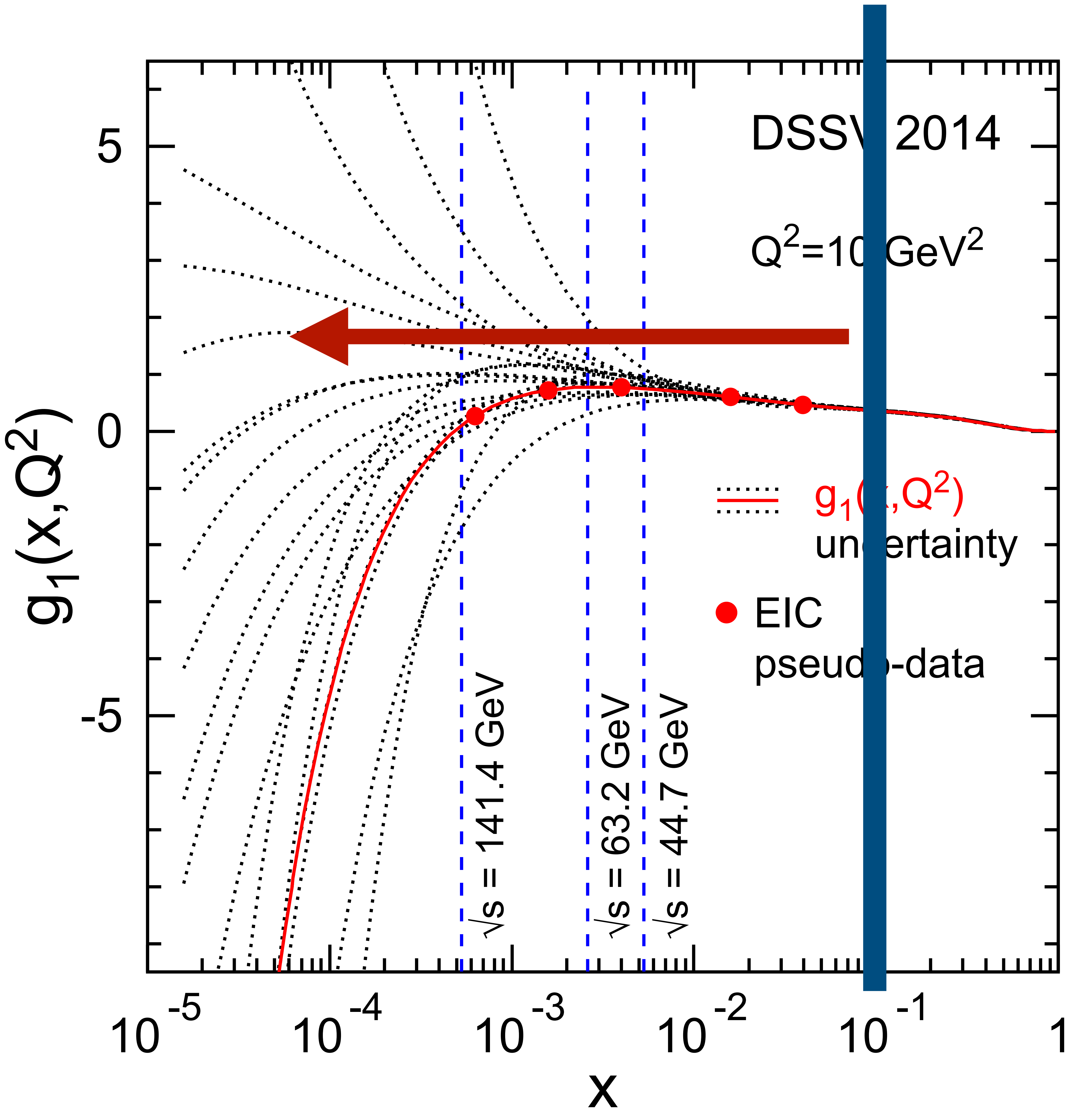
KPS-CTT evolution

Kovchegov, Pitonyak, Sievert (2016-2019)

Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)

- Helicity dependent KPS-CTT evolution equations
- Sums up powers of $\alpha_s \ln 1/x$ and $\alpha_s \ln 1/x \ln Q^2$
- Double log originates in an interplay of the transverse and longitudinal integrals
- Contains mixing between different types of operators (amplitudes)
- Consistent with small-x DGLAP evolution
- The equations are closed in the large-N_c and large-N_c & N_f limits.
- Large-N_c equations have been solved numerically (CKTT 2022) and analytically (J. Borden and Y. V. Kovchegov, 2023). The result is in agreement with the BER result:

$$\Delta\Sigma(x,Q^2) \sim \Delta G(x,Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$



Experimental data

Data set (A_1)	Target	N_{pts}	χ^2/N_{pts}
SLAC (E142) [137]	^{3}He	1	0.60
EMC [142]	p	5	0.20
SMC [143, 145]	p	6	1.29
	p	6	0.53
	d	6	0.67
	d	6	2.26
COMPASS [146]	p	5	1.02
COMPASS [147]	p	17	0.74
COMPASS [148]	d	5	0.88
HERMES [149]	n	2	0.73
Total		59	0.91

Data set ($A_\parallel$)	Target	N_{pts}	χ^2/N_{pts}
SLAC(E155) [140]	p	16	1.28
	d	16	1.62
SLAC (E143) [139]	p	9	0.56
	d	9	0.92
SLAC (E154) [138]	^{3}He	5	1.09
HERMES [150]	p	4	1.54
	d	4	0.98
Total		63	1.19

Dataset (A_1^h)	Target	Tagged Hadron	N_{pts}	χ^2/N_{pts}
SMC [144]	p	h^+	7	1.03
	p	h^-	7	1.45
	d	h^+	7	0.82
	d	h^-	7	1.49
HERMES [154]	p	π^+	2	2.39
	p	π^-	2	0.01
	p	h^+	2	0.79
	p	h^-	2	0.05
	d	π^+	2	0.47
	d	π^-	2	1.40
	d	h^+	2	2.84
	d	h^-	2	1.22
	d	K^+	2	1.81
	d	K^-	2	0.27
	d	$K^+ + K^-$	2	0.97
HERMES [155]	^{3}He	h^+	2	0.49
	^{3}He	h^-	2	0.29
COMPASS [152]	p	π^+	5	1.88
	p	π^-	5	1.10
	p	K^+	5	0.42
	p	K^-	5	0.31
COMPASS [153]	d	π^+	5	0.50
	d	π^-	5	0.78
	d	h^+	5	0.90
	d	h^-	5	0.86
	d	K^+	5	1.50
	d	K^-	5	0.78
Total			104	1.01

$$5 \times 10^{-3} < x < 0.1 \equiv x_0$$

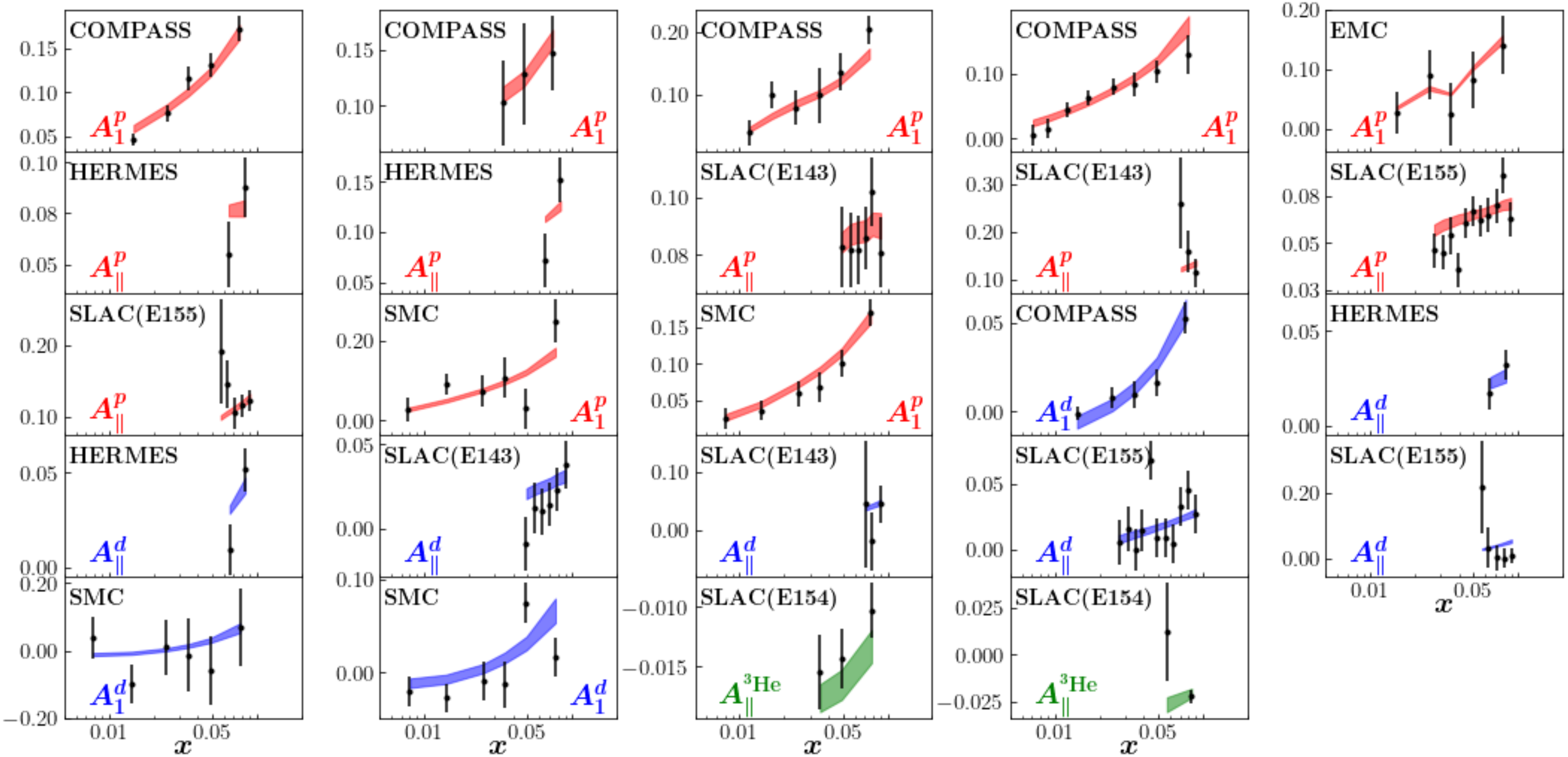
$$1.69 \text{ GeV}^2 < Q^2 < 10.4 \text{ GeV}^2$$

$$0.2 < z < 1.0$$

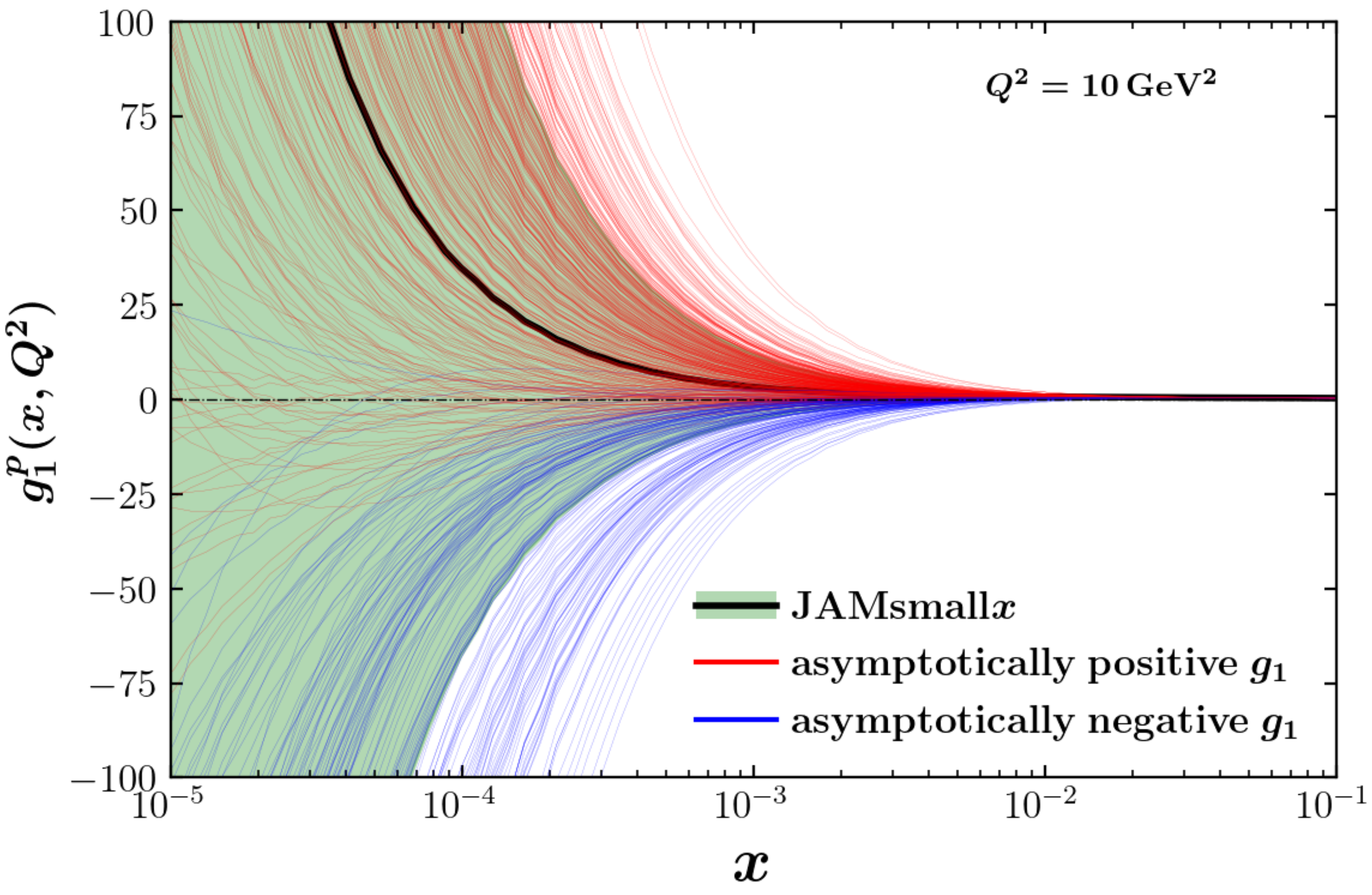
$$N_{\text{pts}} = 226$$

D. Adamiak, N. Baldonado, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato, M. D. Sievert, A. Tarasov, and Y. Tawabutr (2023)

Data versus theory

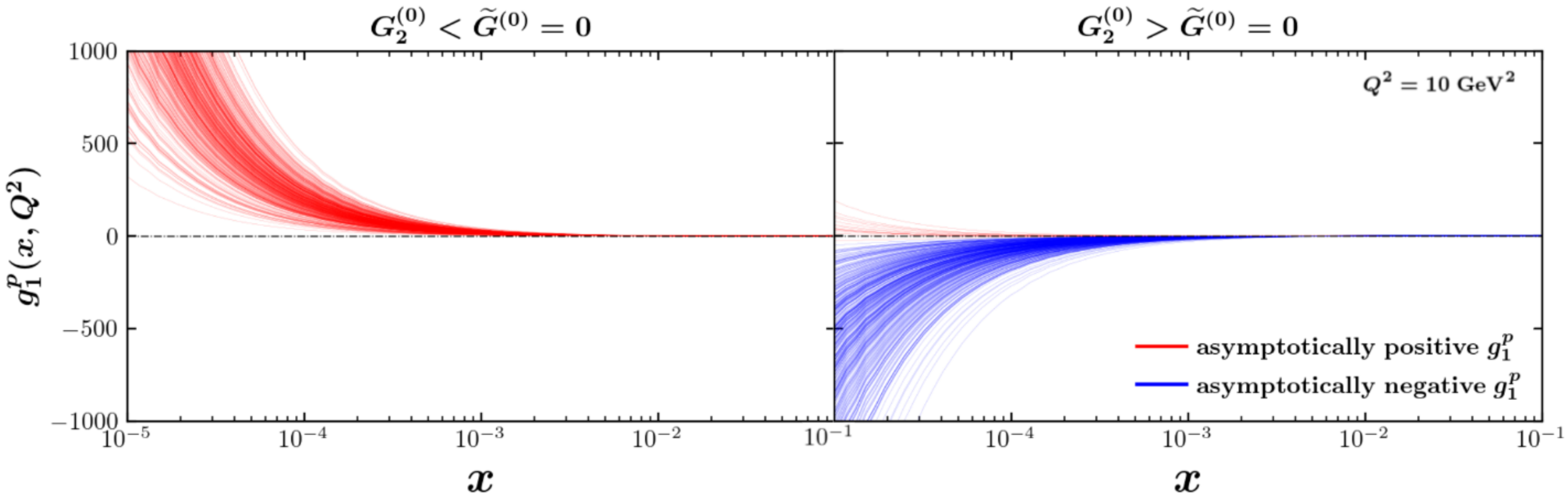


g_1 structure function (x dependence)



- 500 replicas
- Each replica represents an individual fit of the experimental data
- largely unconstrained at smaller x
- g_1 is well constrained in the region where there is experimental data
- Evolution equations guarantees that the small x behavior of g_1 must be exponential in $\ln(1/x)$

Sign of g_1 structure function



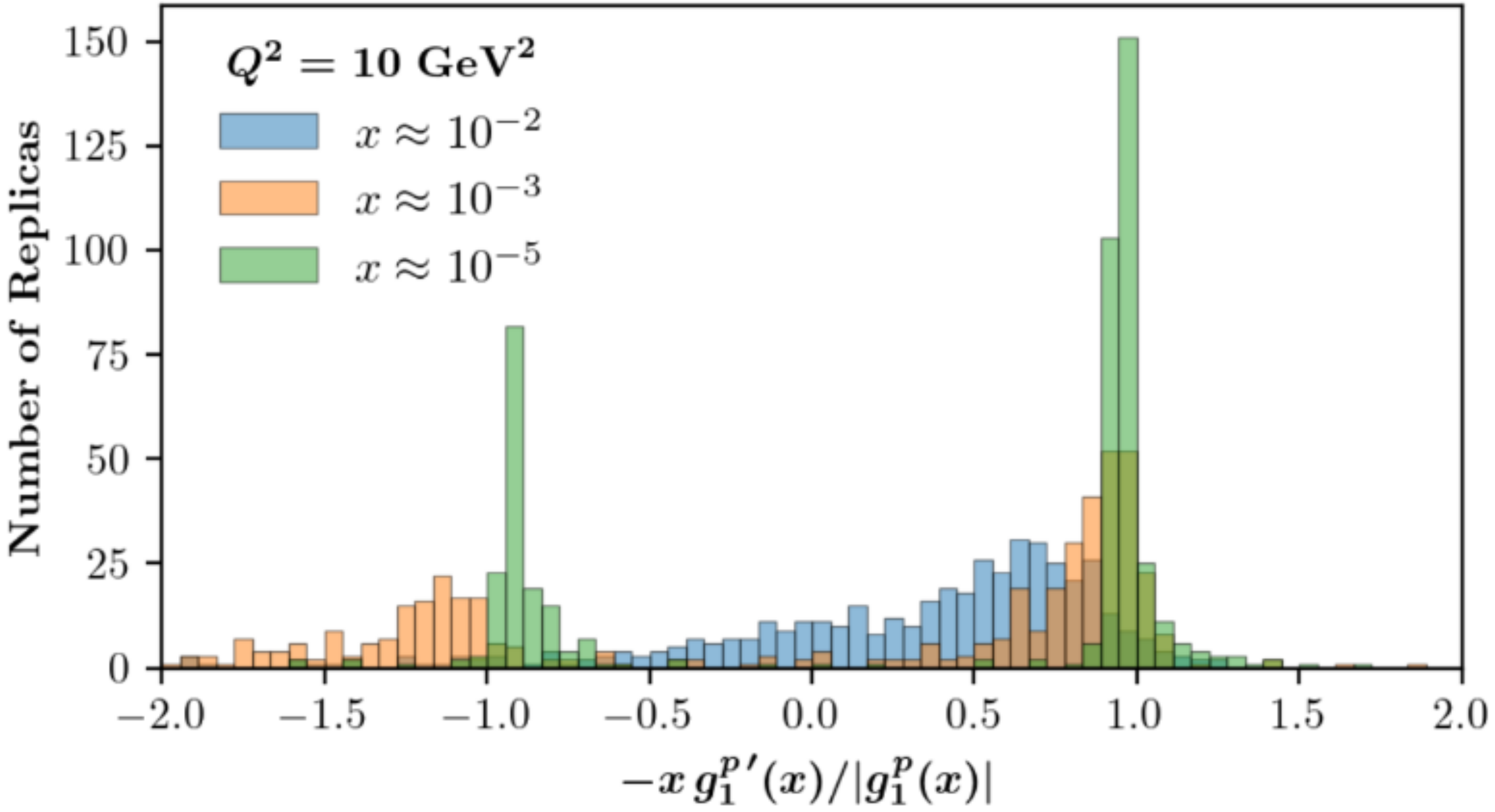
- The major difficulty in constraining g_1 is caused by the insensitivity of the data to the G_2 and $\tilde{G}$ amplitudes

Asymptotic behavior

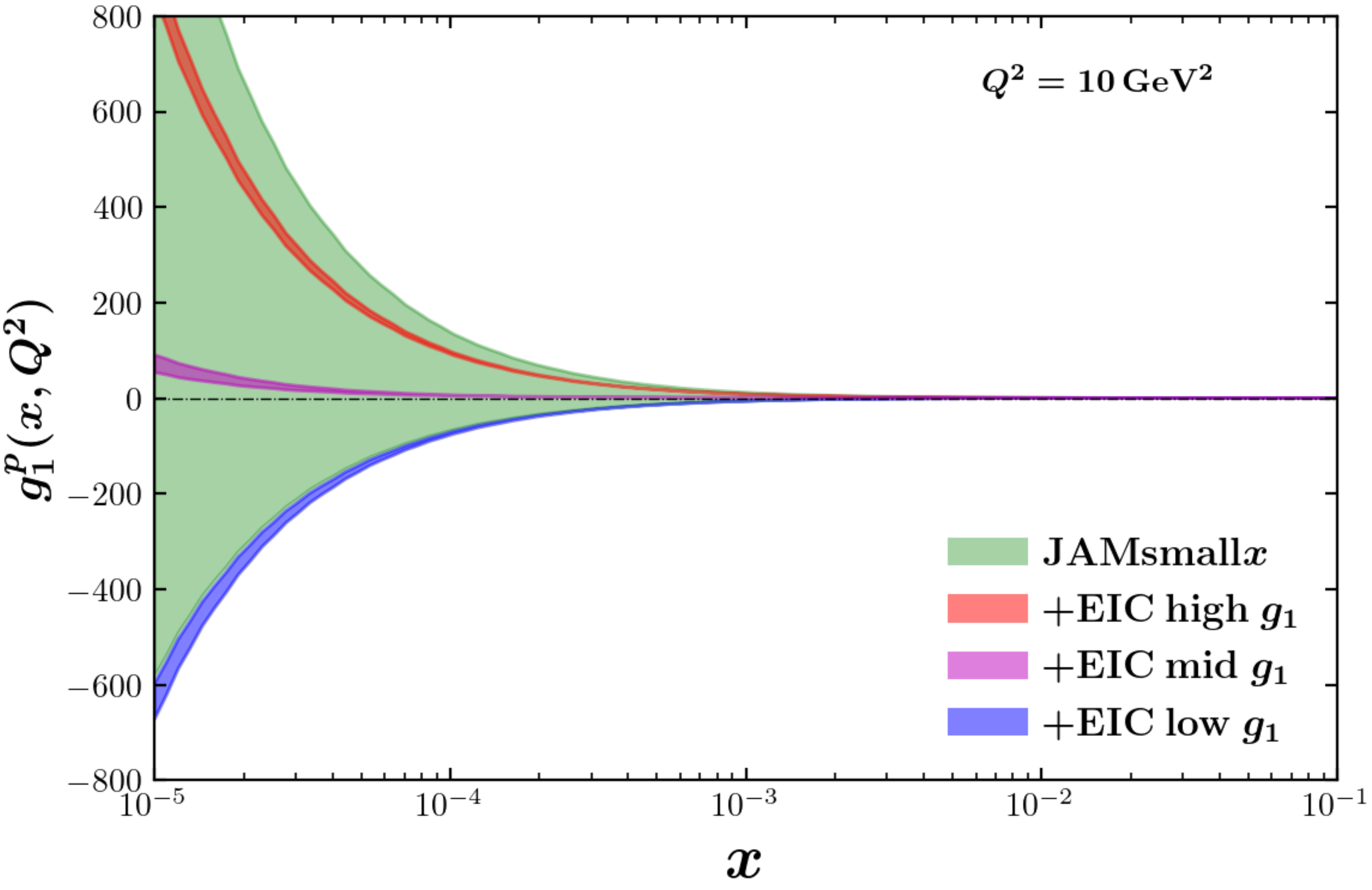
- Evolution equations that we use guaranty the asymptotic behavior

$$\lim_{x\to 0} g_1^p(x) \equiv g_1^{p\,(0)}\, x^{-\alpha_h(x)}$$

$$\alpha_h(x) \equiv \frac{1}{g_1^p(x)} \frac{\mathrm{d}\, g_1^p(x)}{\mathrm{d}\ln(1/x)}$$

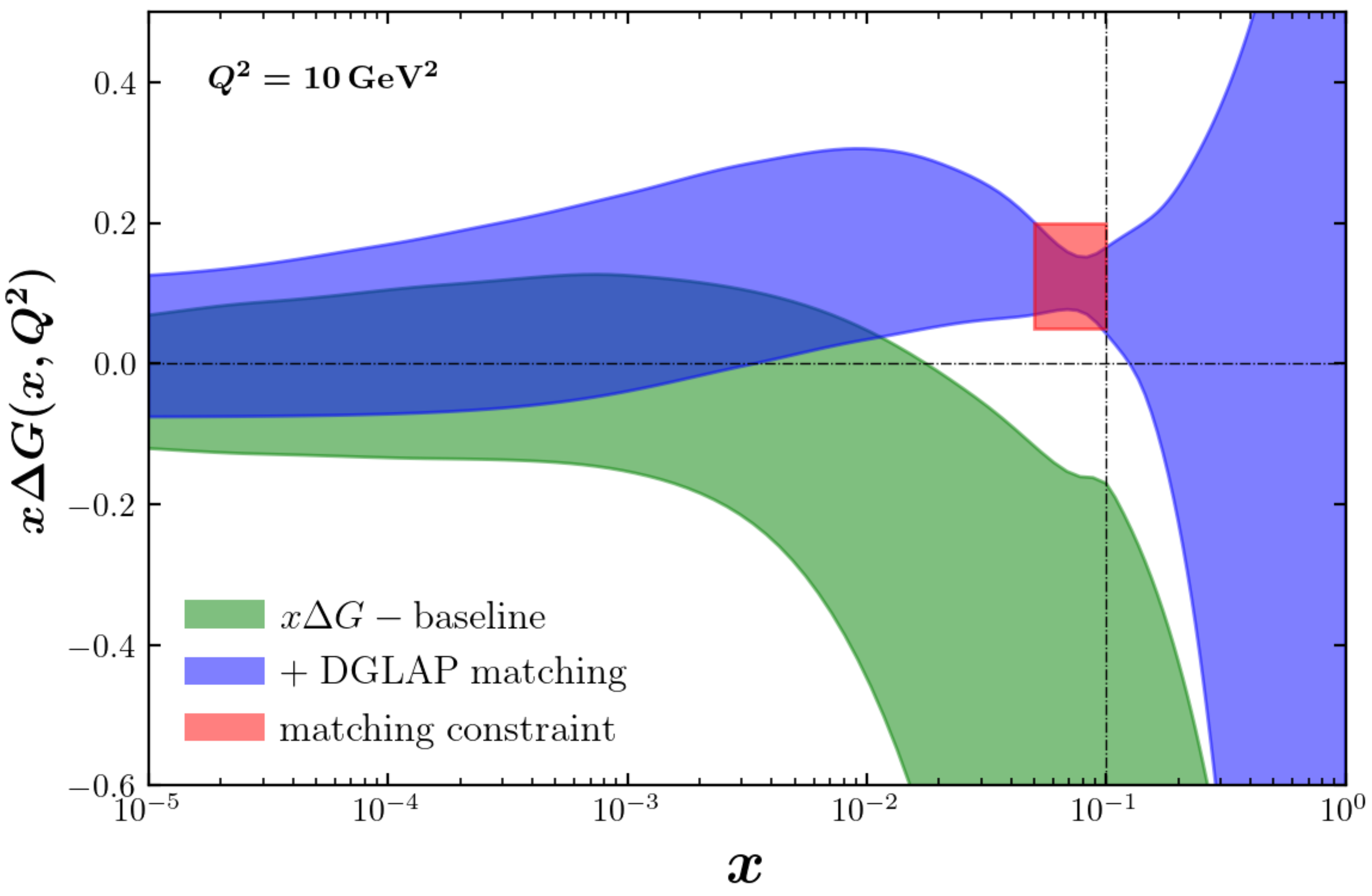


Impact of EIC data

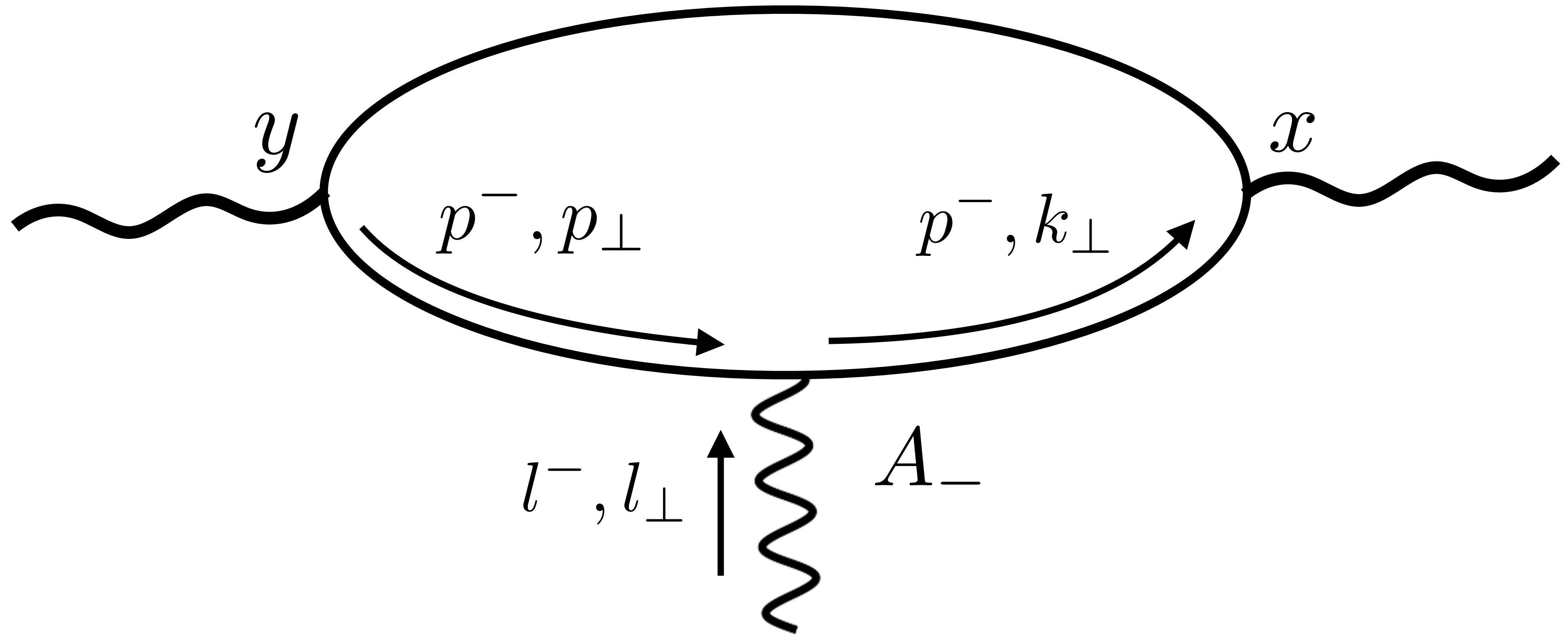


- In order to study the impact of lower x measurements on our ability to predict the behavior of g_1 and the hPDFs at even smaller x, we utilized EIC pseudodata for the kinematic region of $10^{-4} < x < 10^{-1}$ and $1.69\ GeV^2 < Q^2 < 50\ GeV^2$

Initial conditions at large-x



- Large uncertainties do to initial conditions
- Our equations contain small-x limit of DGLAP but we use it for large x data
- We need to include a single transverse logarithms $\alpha_S \ln Q^2$. Full DGLAP evolution.
- Hard, but can be done.

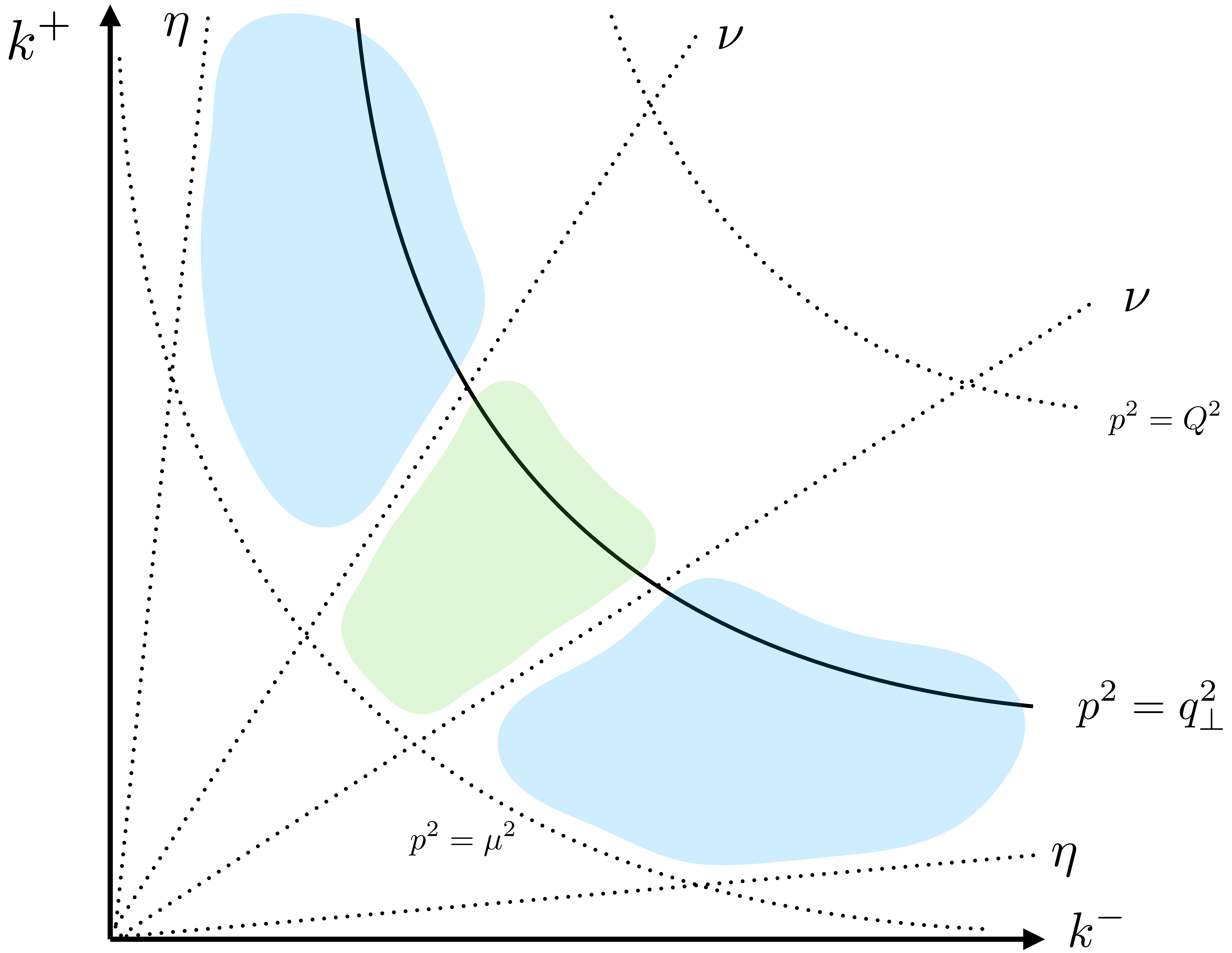


$$\int_{y^-}^{x^-} dz^- e^{i\frac{p_\perp^2}{2p^-}z^-} A_-(z^-) e^{-i\frac{k_\perp^2}{2p^-}z^-}$$

Don't expand transverse phases!

Approach 2: Extension of the TMD factorization to the small x regime

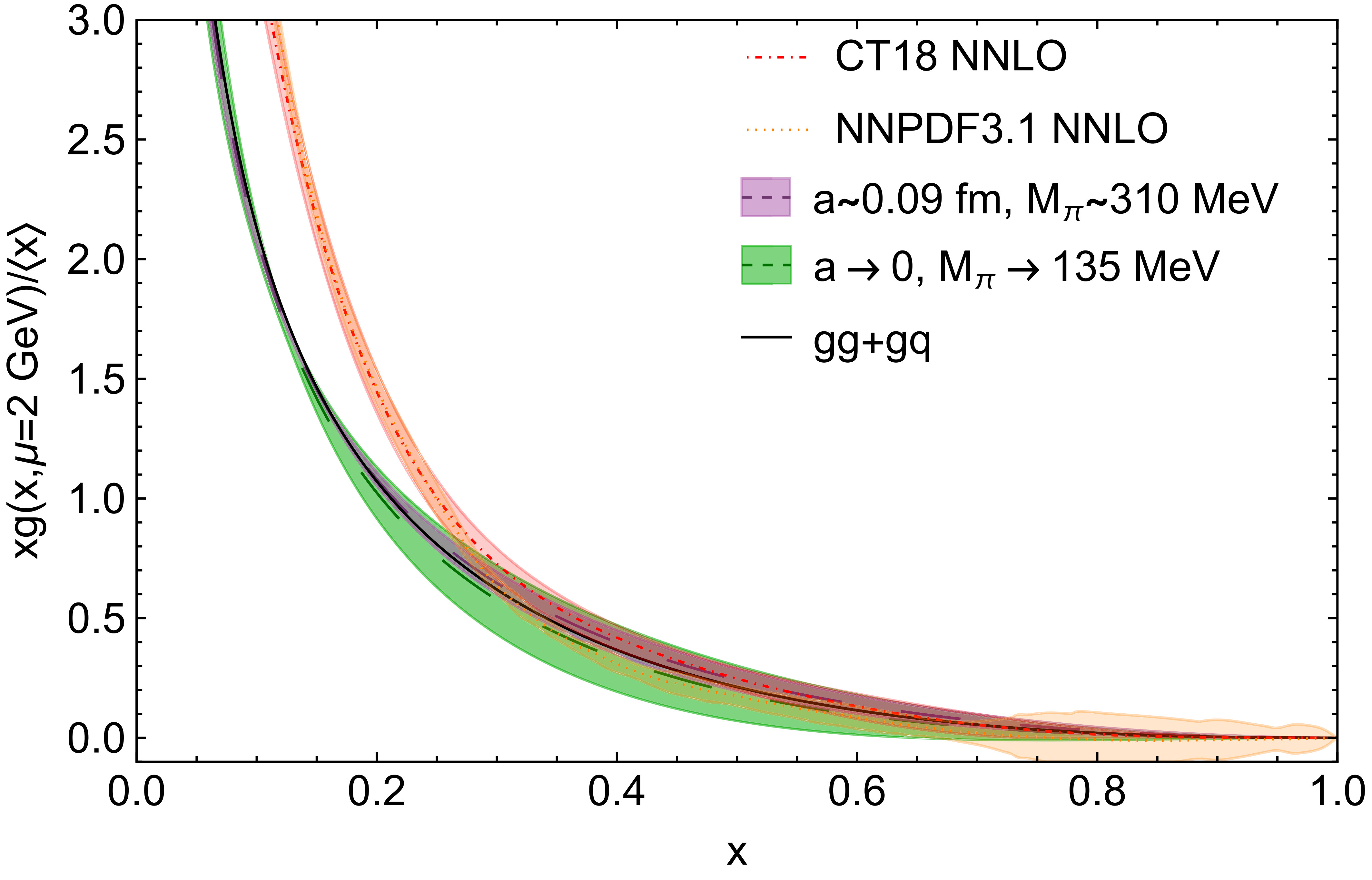
- TMD factorization resums both transverse and longitudinal (rapidity) logarithms
- At small-x we consider dipole operators which are essentially the TMD operators



Swagato Mukherjee, Vladi Skokov, AT, Shaswat Tiwari, in preparation

Initial condition for small-x evolution from lattice

- There is a lot of progress in lattice extraction of distribution functions (matrix element of operators)
- Works well in the region of large x
- This techniques can be used to define initial conditions for small-x evolution
- For this we need a proper matching procedure. PDF is defined in a particular factorization scheme!
- Need a proper factorization scheme for small-x



Fan, Good, Li (2023)

First moment of the structure function

- Does the eikonal/light-cone expansion always work? Can we write it down for all observables and study their asymptotic in a particular kinematic limit or the expansion breaks down?
- The expansion breaks down for topological quantities. This effects are not local!

The helicity can be extracted from the first moment of the g_1 structure function

$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\Sigma(Q^2)\right)\left(1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right) + O\left(\frac{\Lambda^2}{Q^2}\right)$$

Integration from zero!

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N}\sum_f \langle P, S|\bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f |P, S\rangle \equiv \frac{1}{M_N}\langle P, S| J_5^\mu(0) |P, S\rangle$$

Quark contribution to the proton spin is defined by the isosinglet axial vector current J_5^μ

Anomaly equation

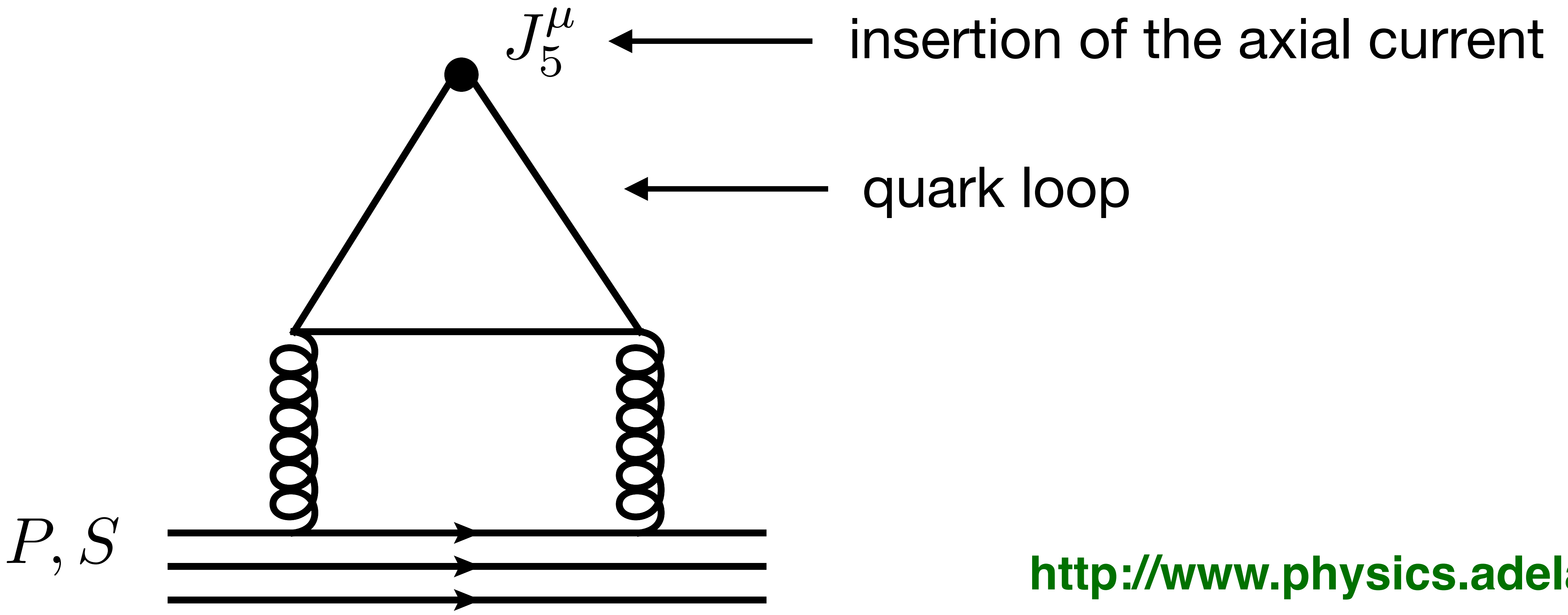
The fundamental property of the J_5^μ current is the anomaly equation:

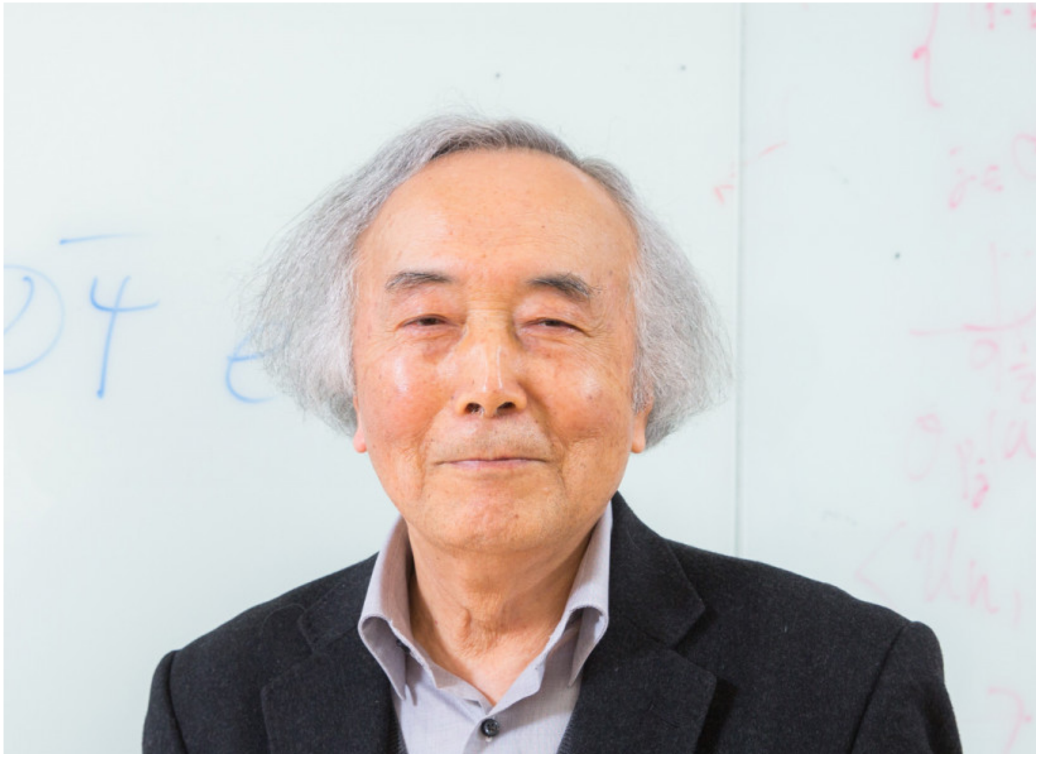
$$\partial^\mu J_\mu^5(x) = \frac{n_f \alpha_s}{2\pi} \, \mathrm{Tr}\left(F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)\right) = 2\, n_f \, \partial_\mu K^\mu$$

The isosinglet current couples to the topological charge density in the polarized proton!

The anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations. Topological properties of the QCD vacuum! K. Fujikawa, PRL. 42, 1195 (1979)

In the leading order the coupling is generated by the triangle diagram:

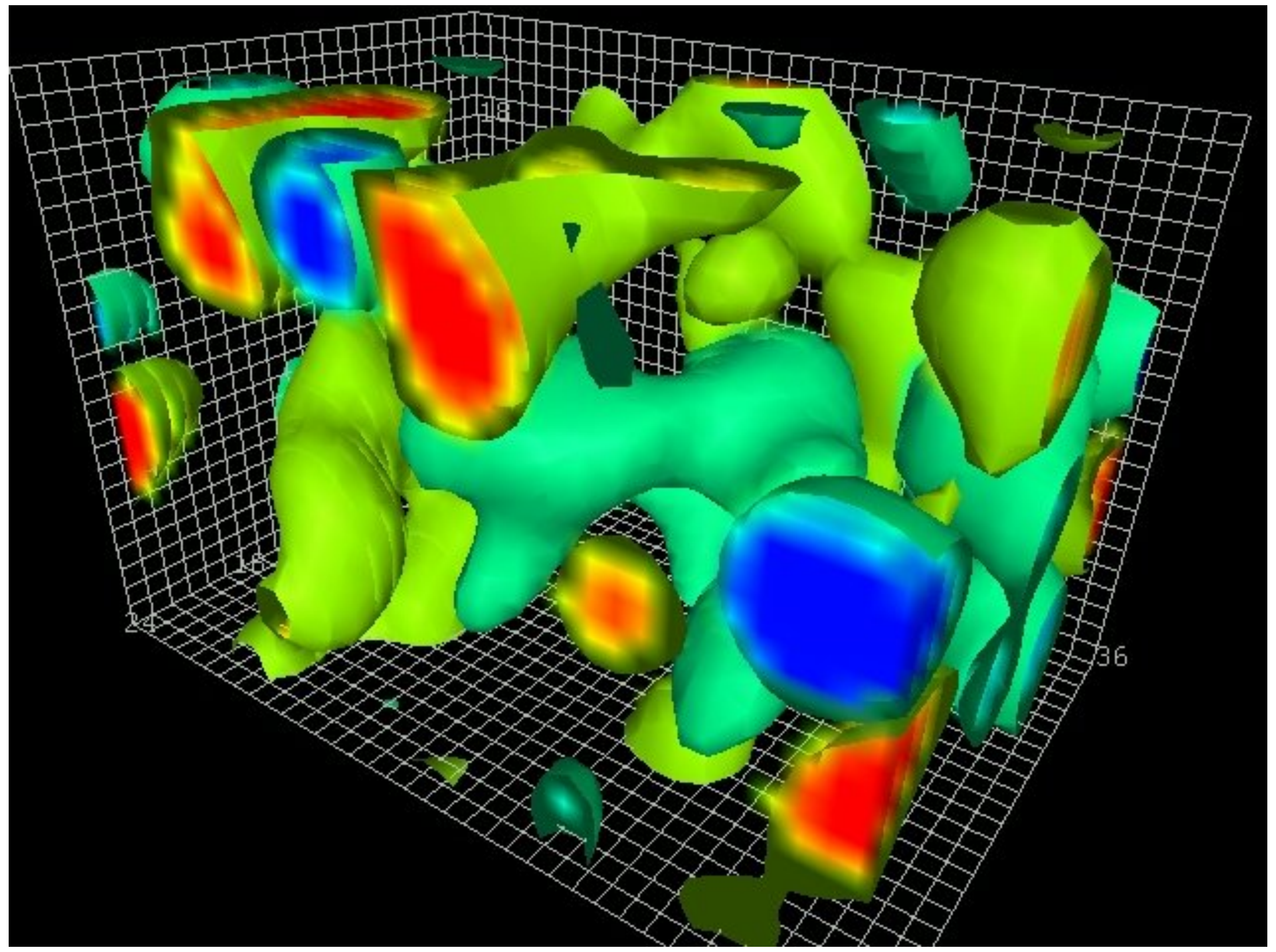




Kazuo Fujikawa

Chern-Simons current:

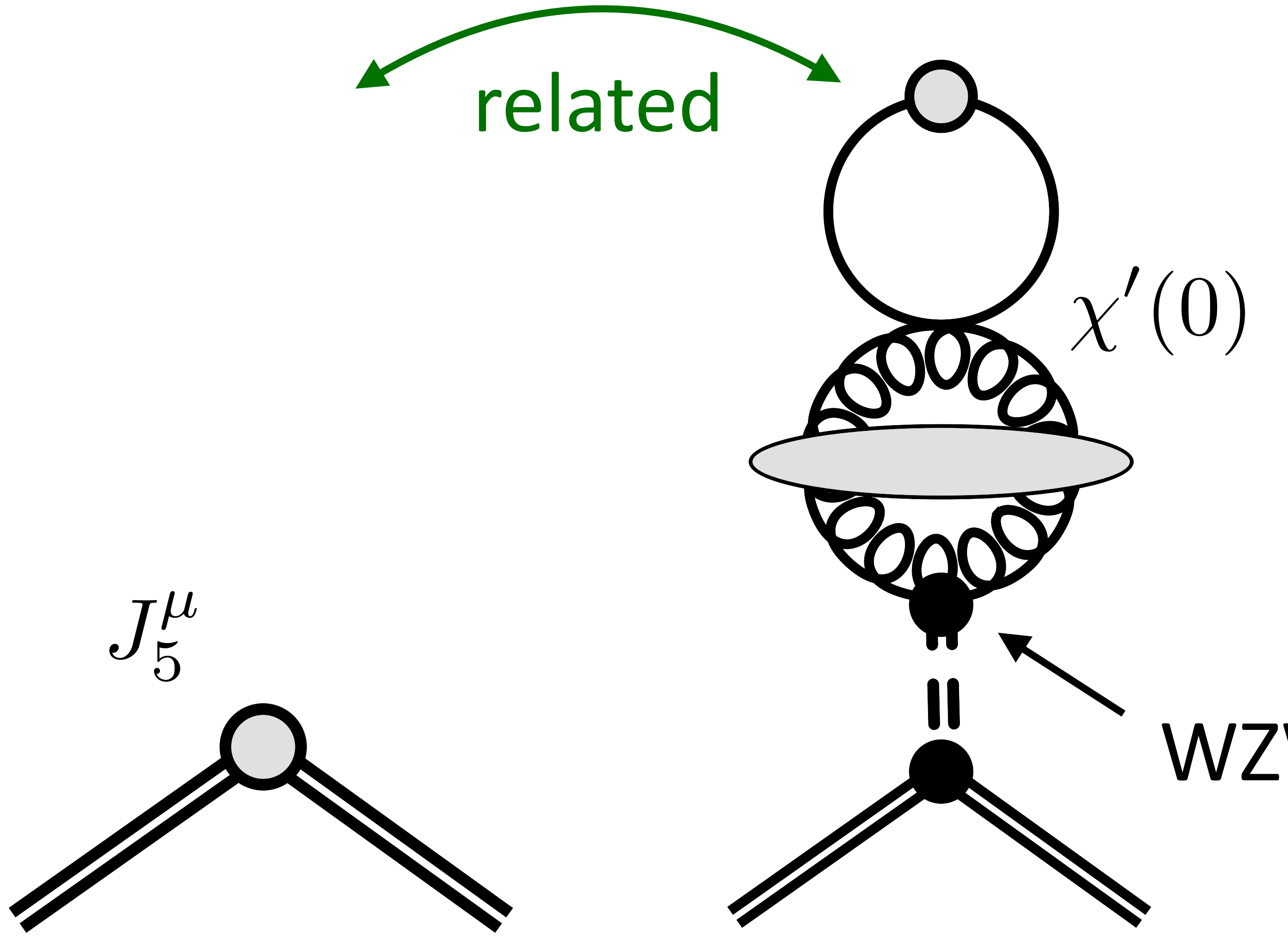
$$K_\mu = \frac{\alpha_S}{4\pi}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma\right)\right]$$



Topological charge density

http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

Infrared pole cancelation



From the cancellation of the anomaly pole, using Goldberger-Treiman relation one can relate helicity and the QCD topological susceptibility - topological screening

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\sqrt{\chi'(0)}$$

Shore, Veneziano (1992)

Tarasov, Venugopalan (2022)

- The first moment of g_1 is determined by the non-perturbative effects
- Can this relation be checked on the lattice?

$$\langle P,S|J_5^\mu|P,S\rangle = M_N S^\mu \Sigma(Q^2) = 2M_N S^\mu a_0$$

$$a^0|_{Q^2=10GeV^2} = 0.33 \pm 0.05$$

Gives a natural resolution of the spin crisis

In agreement with COMPASS ($a^0{}_{Q^2=3GeV^2} = 0.35 \pm 0.08$) and HERMES data ($a^0{}_{Q^2=5GeV^2} = 0.330 \pm 0.064$)

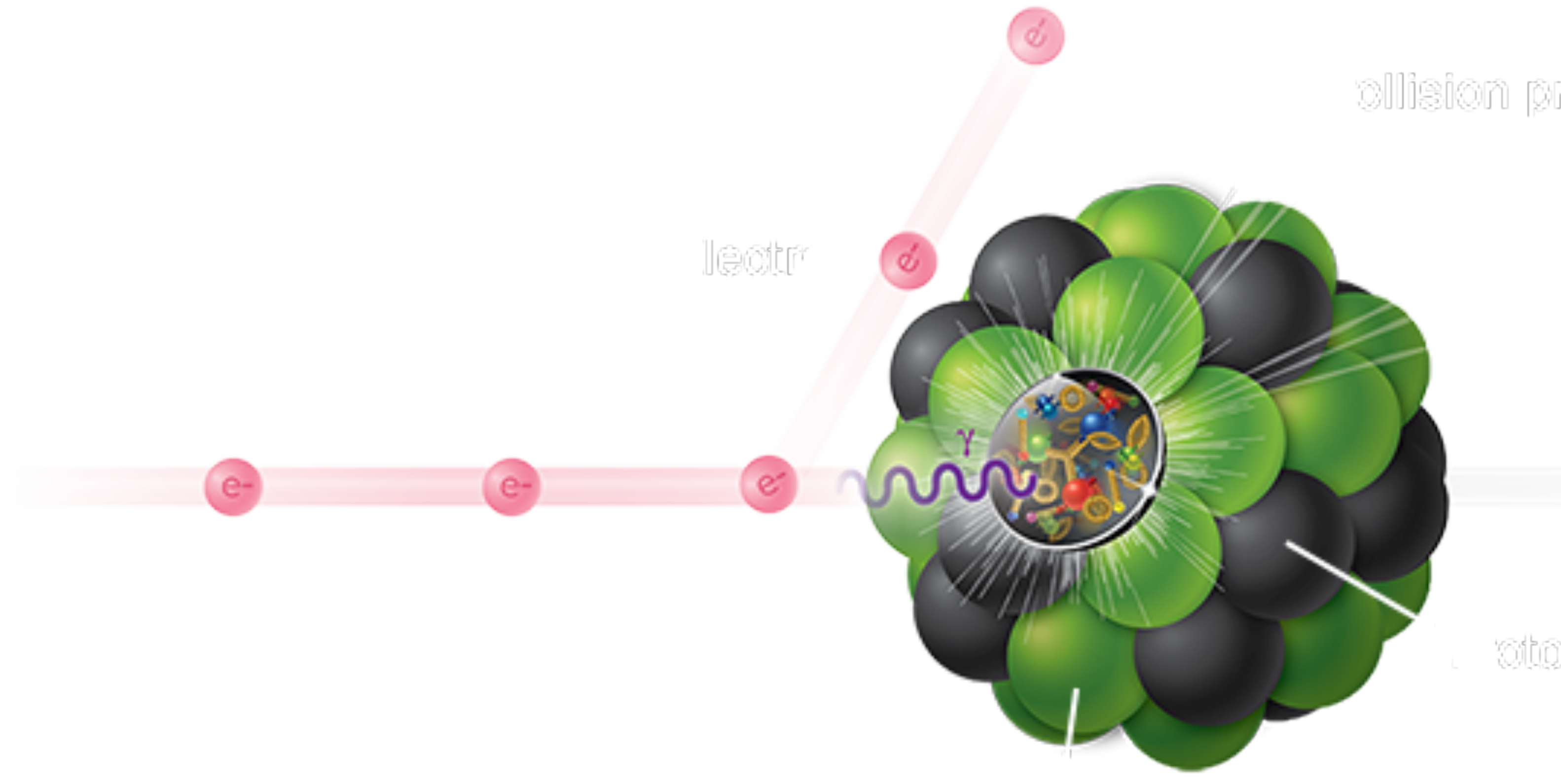
Shore (2007), Narison (2021)

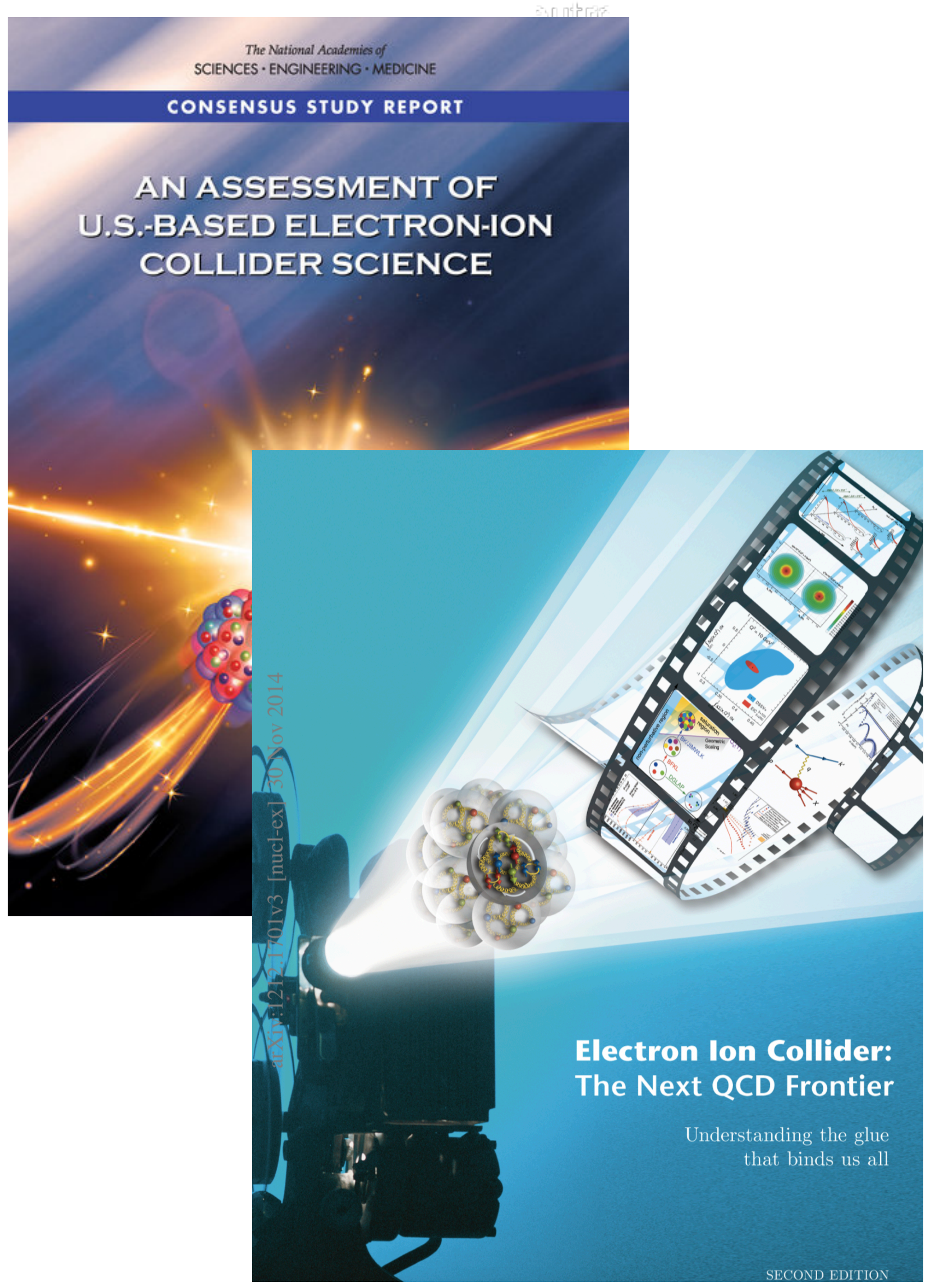

Shore


Veneziano

Summary

- In a high-energy reaction many observables can be factorized into a perturbative and non-perturbative parts
- The structure of the factorization depends on the type of the observable and kinematic of the scattering reaction
- In many problems we need to understand a transition between different types of factorization
- Problem: how to systematically extend the region of validity of the existing factorization schemes
- Example: spin at small-x. Experimentally is not accessible. We need a theoretical input. Interplay between large and small x
- Lattice extractions of initial conditions for small-x evolution equations
- Anomaly: interplay between perturbative and non-perturbative effects





Thank you!