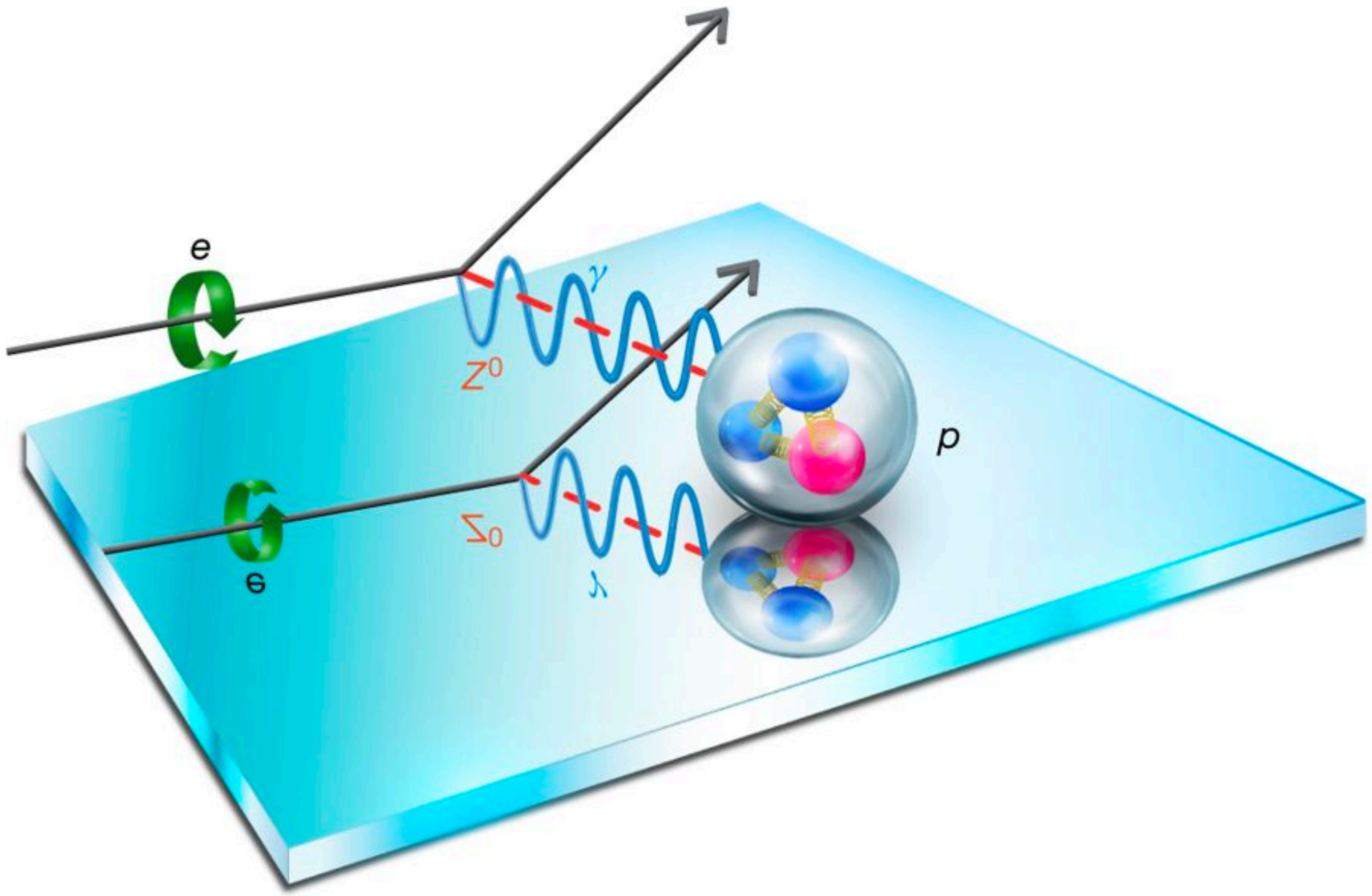
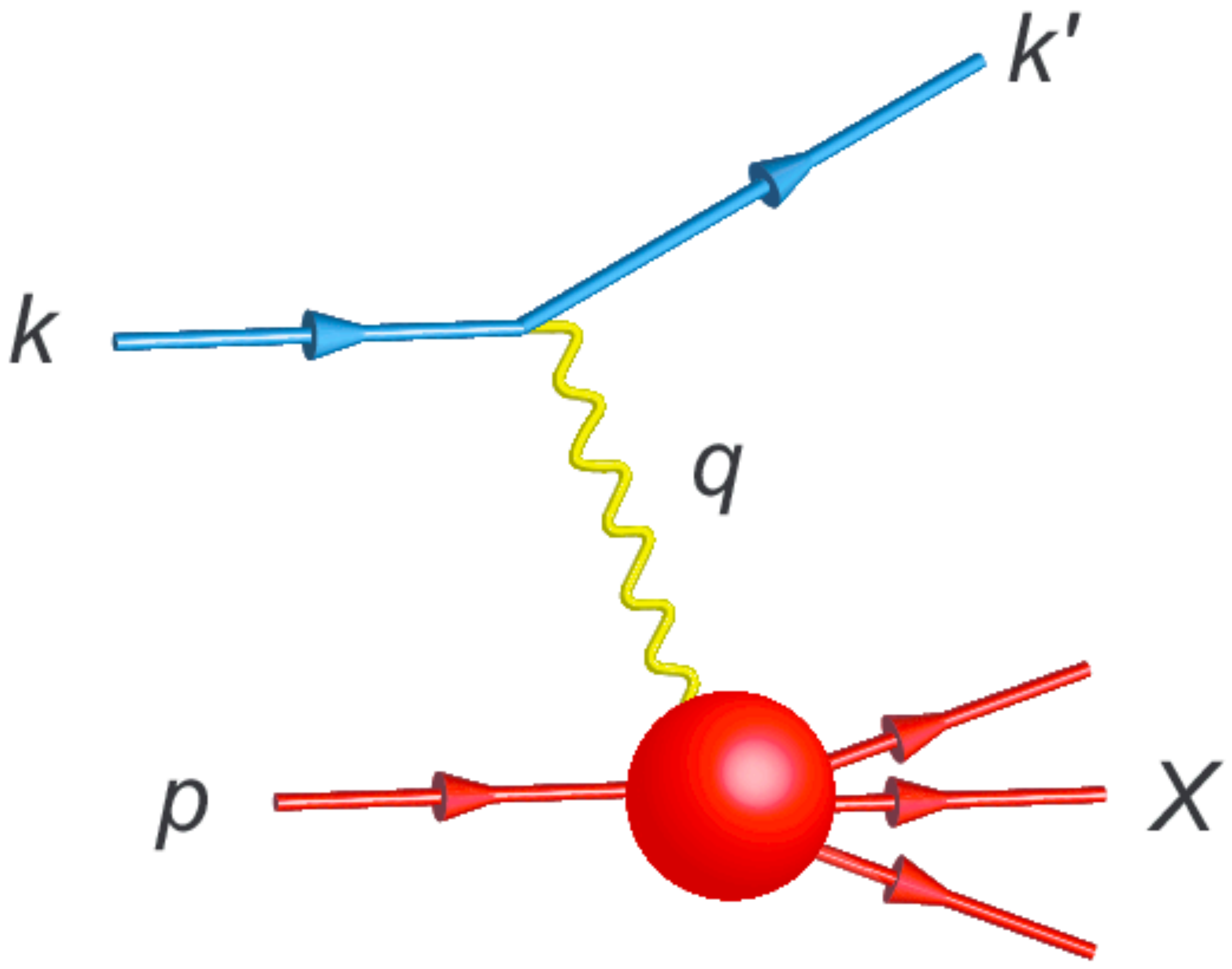
HIGHER ORDER ELECTROWEAK RADIATIVE CORRECTIONS USING COVARIANT APPROACH





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MOTIVATION

- The theory of Standard Model (SM) → unifies Electromagnetic, Weak and Strong interactions → can make predictions that match experiments to one part in ten billion.

- SM limitations → don't include gravity, dark matter/dark energy existence, hierarchies of scale related to Higgs boson etc.

- Theoretical door open for Beyond the SM (BSM) physics to be observed at TeV scale → **but** till date no concrete evidence of BSM at 13 TeV centre-of-mass energy at LHC.

- **Low energy precision physics** becomes important → provides a way to reach mass scales not directly accessible at existing high-energy colliders.

- We are doing precision physics with full electroweak Parity Violating Asymmetry (A_{PV}) → achieve by calculating the higher order corrections up to NNLO (α^4) using Covariant/ leptonic tensor approach.

PARITY VIOLATING ASYMMETRY

- Formula: $A_{PV} = \dfrac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$, where: $\sigma_R \propto |M_R|^2$ and $\sigma_L \propto |M_L|^2$

- For QED, $|M_{\gamma R}| = |M_{\gamma L}|$, numerator contains just weak+electroweak cross terms.

- Denominator contains just QED terms as m_Z (90 GeV)> m_{e^-}(0.5 MeV)

- $A_{PV} = \dfrac{|M_{ZZ}|_R^2 - |M_{ZZ}|_L^2 + |M_{\gamma Z}|_R^2 - |M_{\gamma Z}|_L^2}{|M_{\gamma\gamma}|_R^2 + |M_{\gamma\gamma}|_L^2}$

FULL **ELECTROWEAK** e^-p SCATTERING

- Elastic e^-p scattering is studied up to the NNLO level considering all SM particles in the loop.

- A longitudinally polarized e^- scatters off an unpolarized proton target

Covariant Approach

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(m, k_1)

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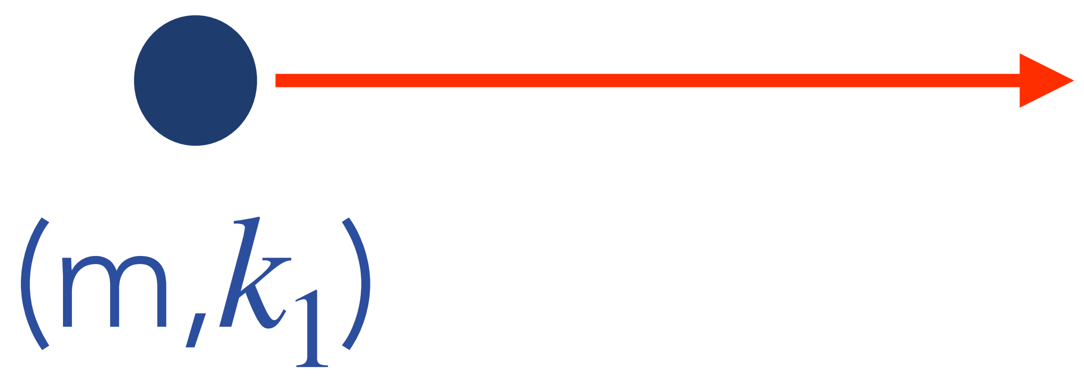

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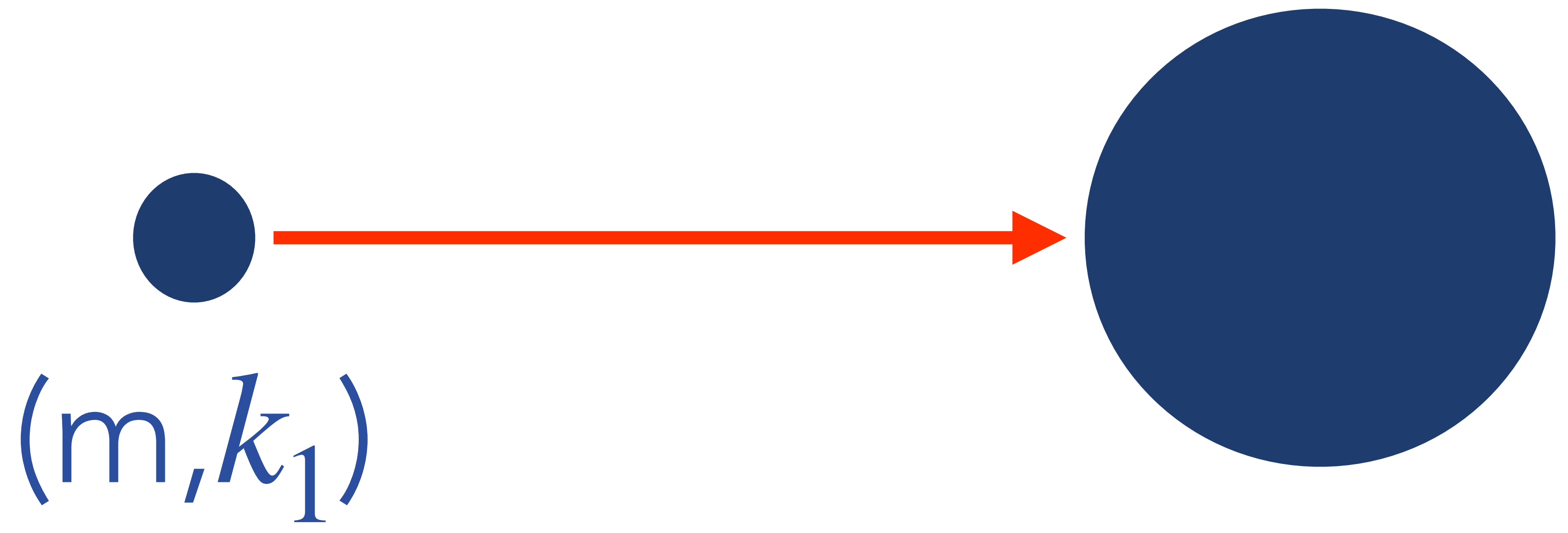

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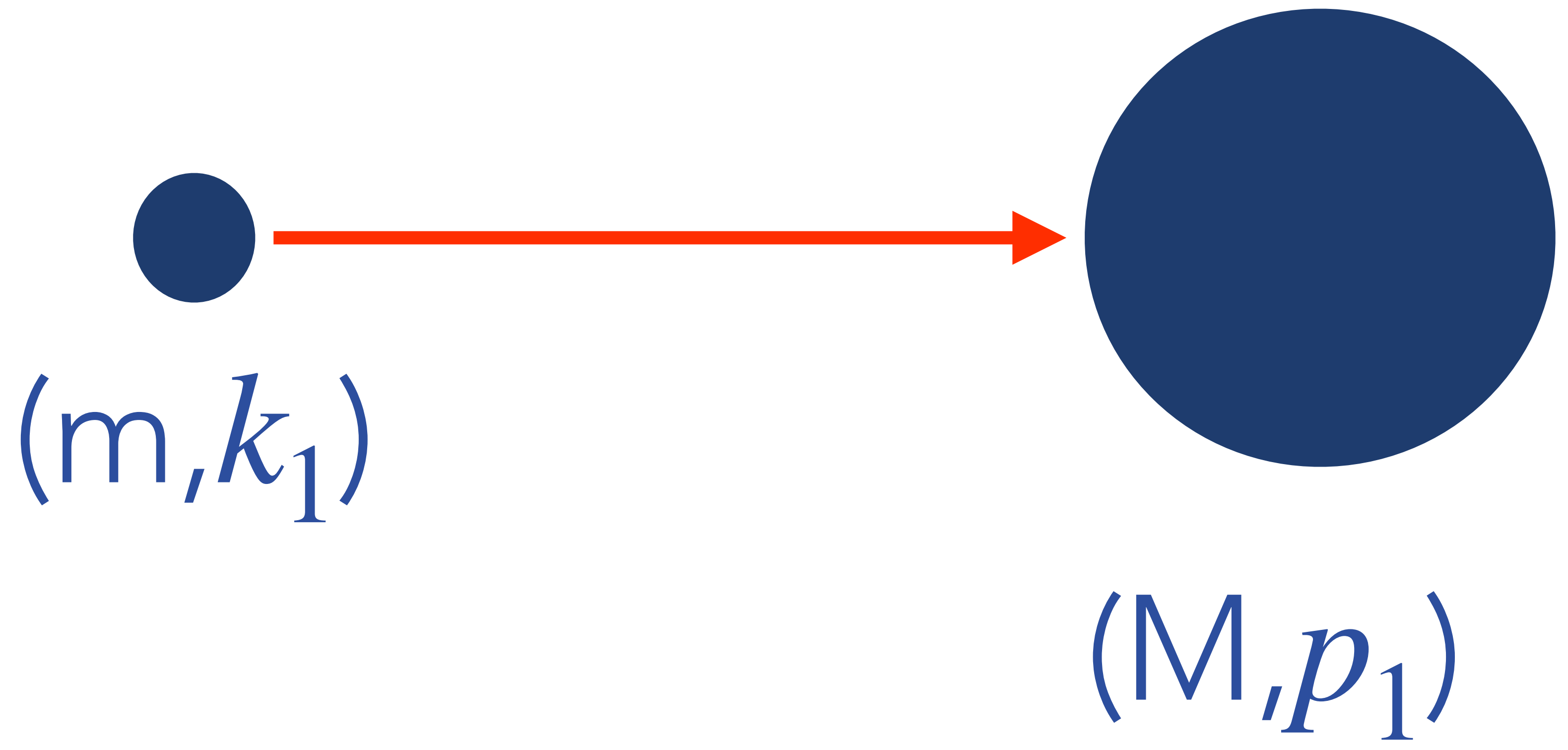

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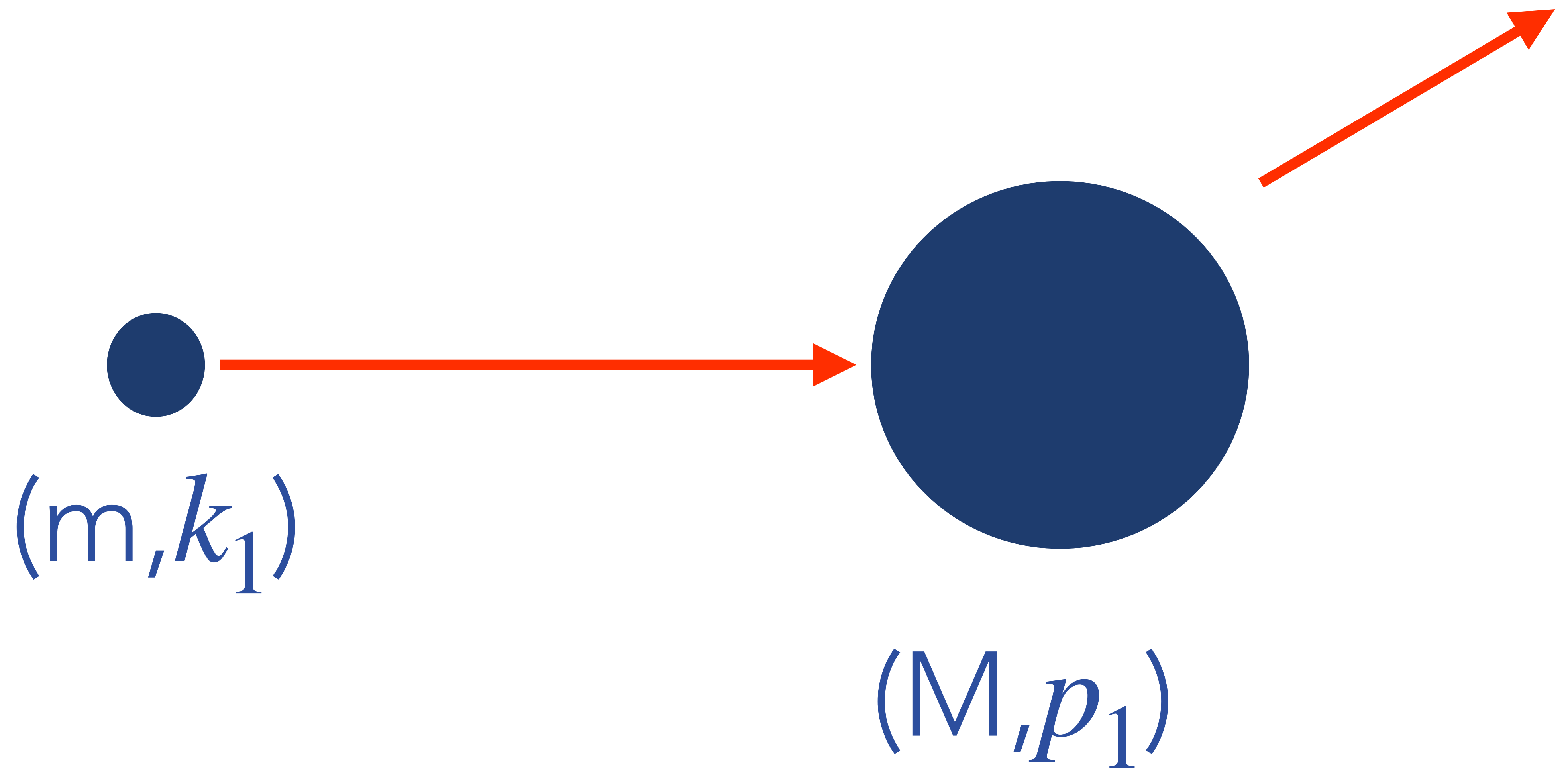

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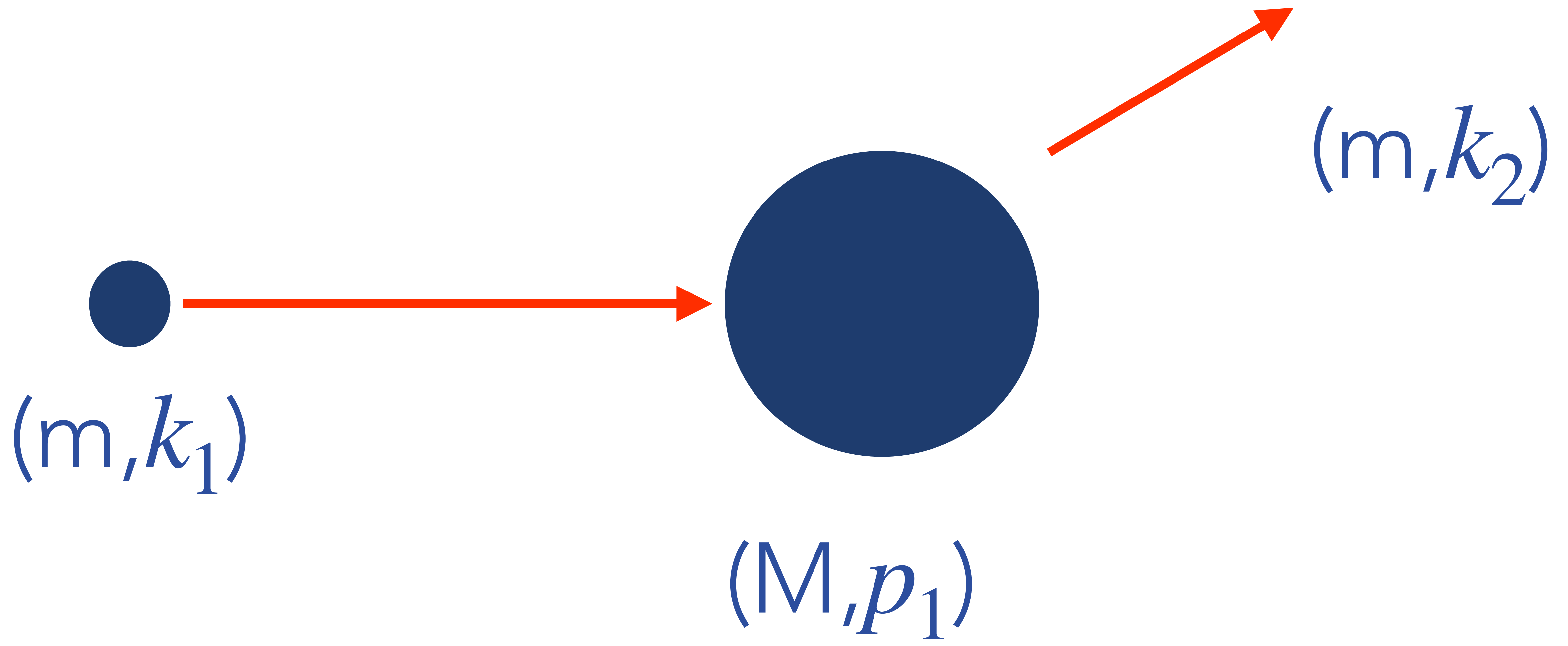

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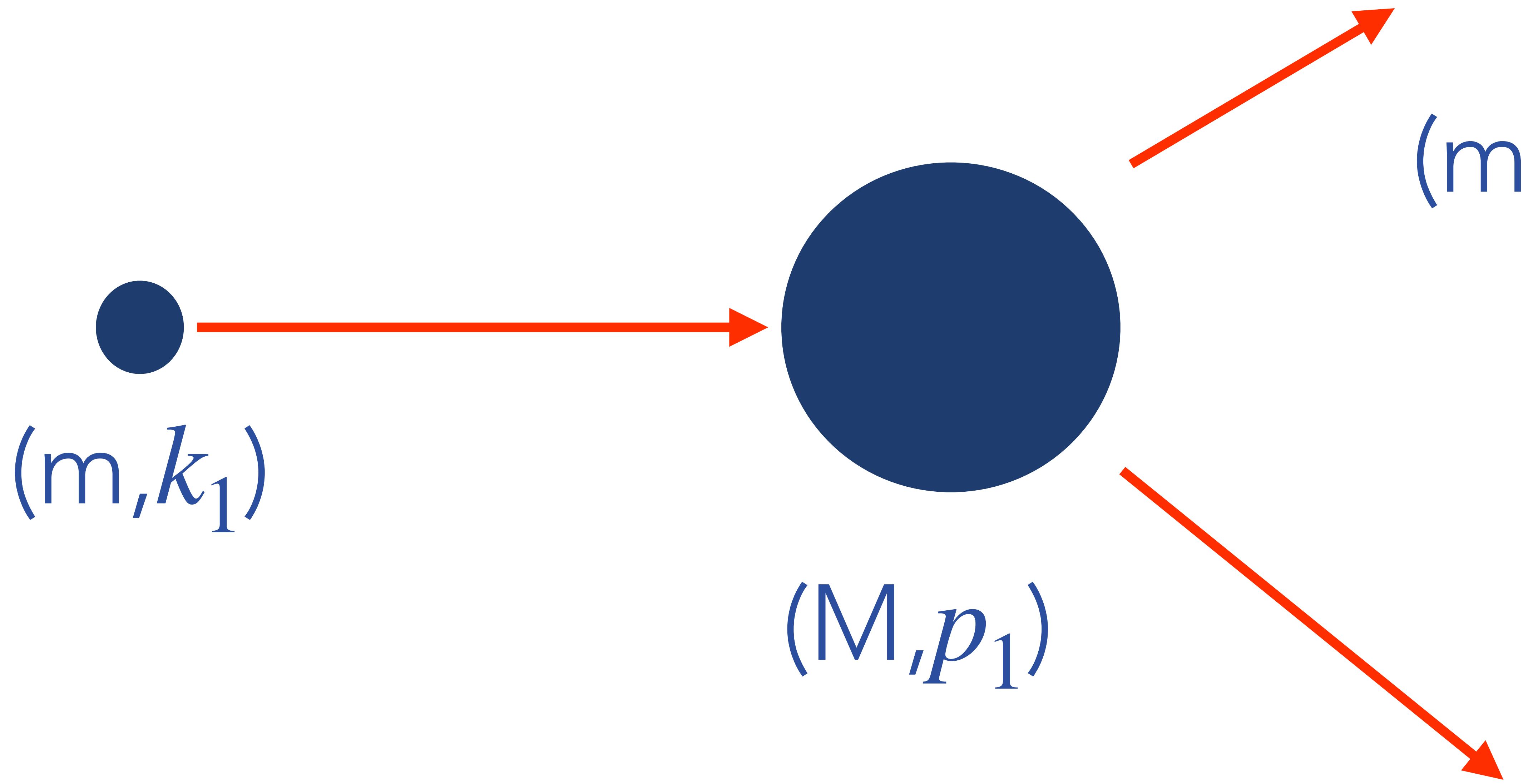

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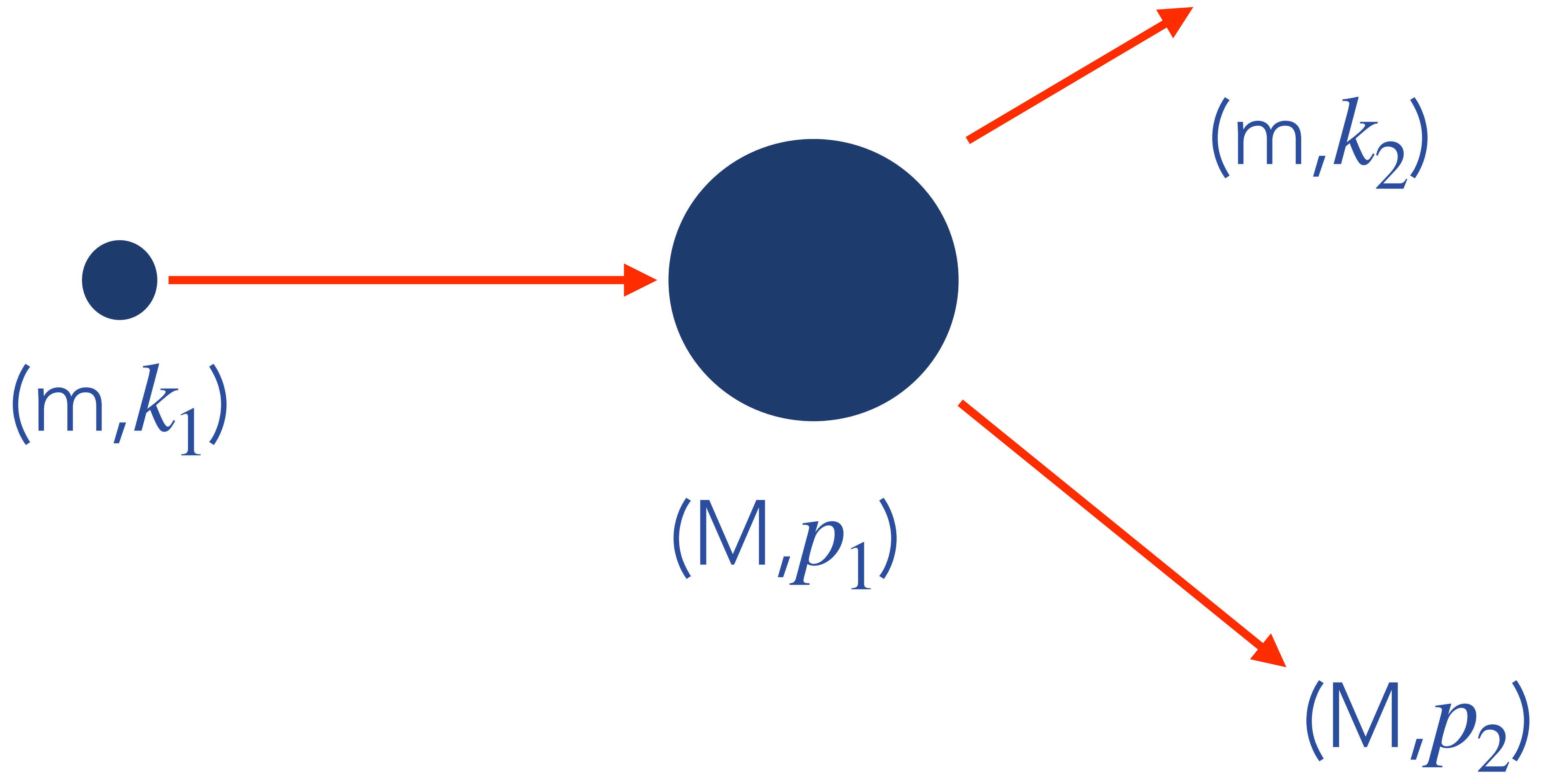

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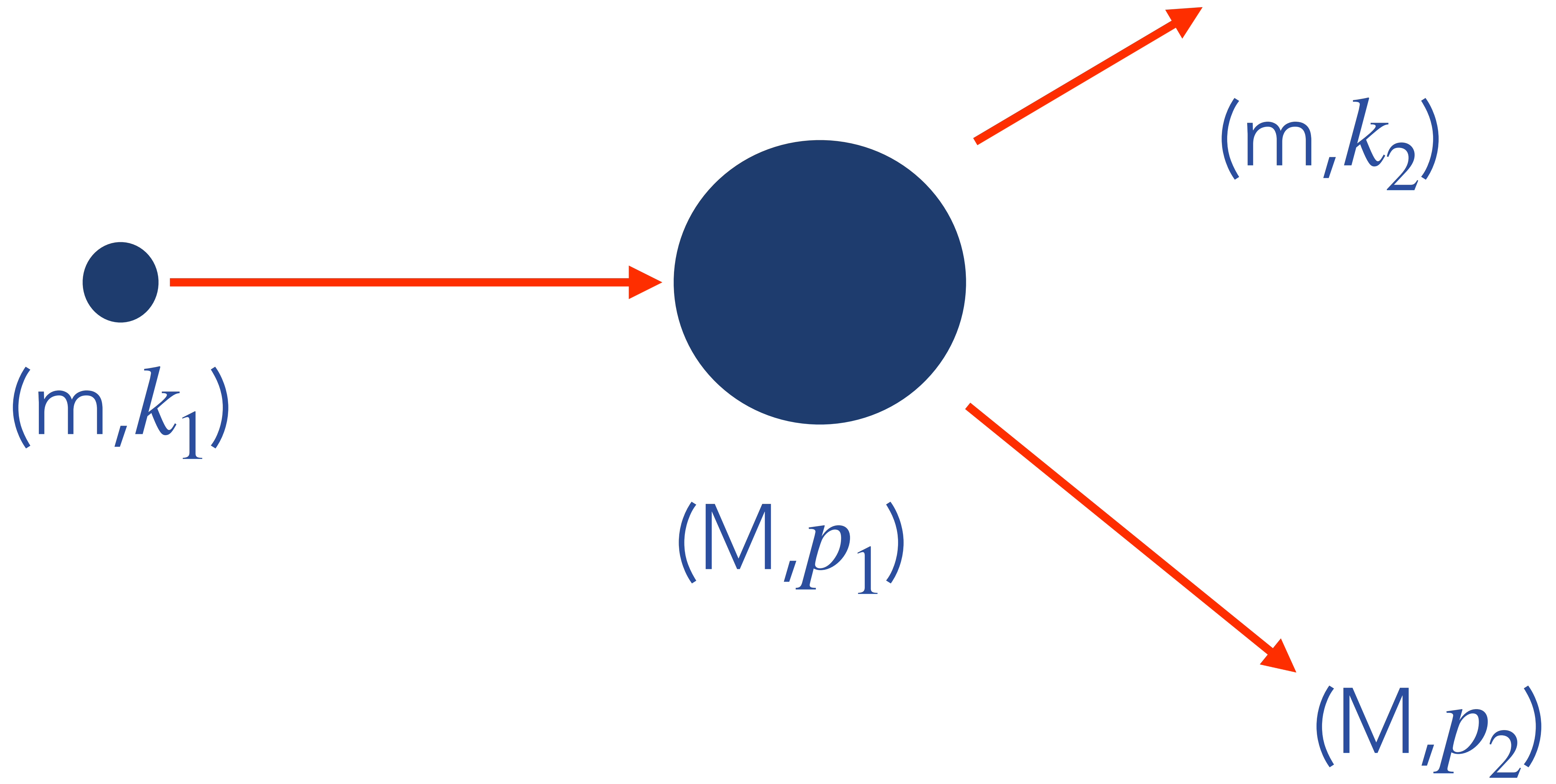

→ **Covariant Approach**

- Using covariant approach → calculated tree level, one-loop level and quadratic level scattering amplitudes squared along with the parity violating asymmetry.

QED

WEAK

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QED

$$i\mathcal{M} = \bar{u}(k_2)(-ie\gamma_\mu)u(k_1)\frac{-i}{t}\bar{u}(p_2)(-ie\Gamma^\mu_{\gamma-p})u(p_1)$$

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$f_1^p(t)$, $f_2^p(t)$ **and** $g_1^p(t)$ → weak electric, magnetic and axial vector form factors.

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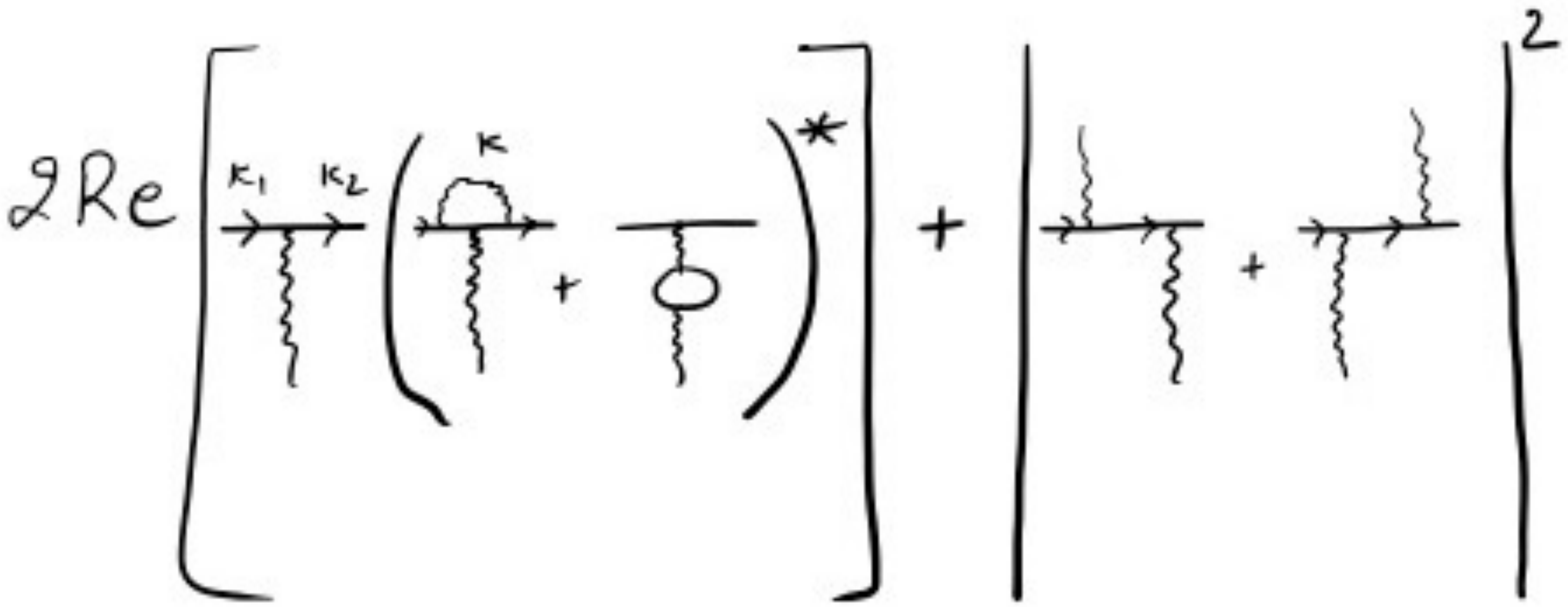
$a_v = \frac{I^3 - 2\sin^2\theta_W Q_f}{2\sin\theta_W\cos\theta_W}$, $a_p = \frac{I^3}{2\sin\theta_W\cos\theta_W}$, where $Q_f = -1(e^-)$ and $I_3 = -\frac{1}{2}$.

LEPTONIC TENSOR AND INTRODUCTION TO COVARIANT APPROACH



LEPTONIC TENSOR AND INTRODUCTION TO COVARIANT APPROACH

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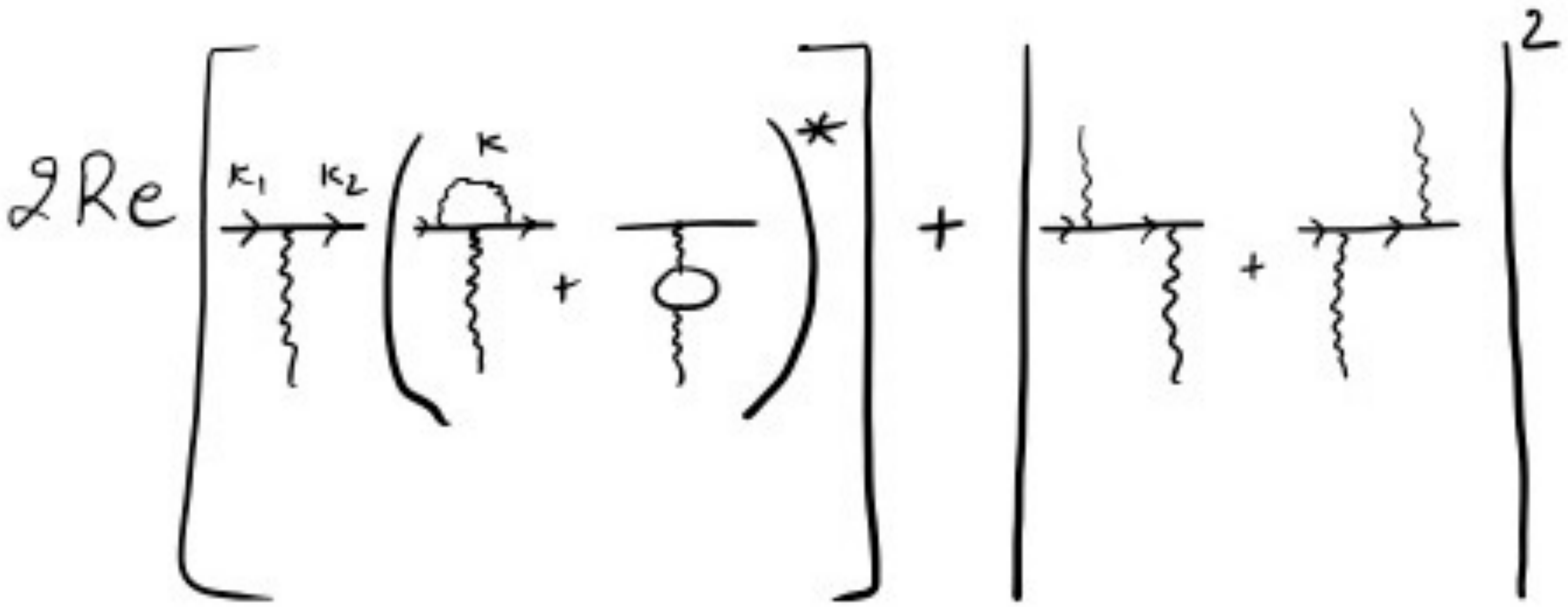

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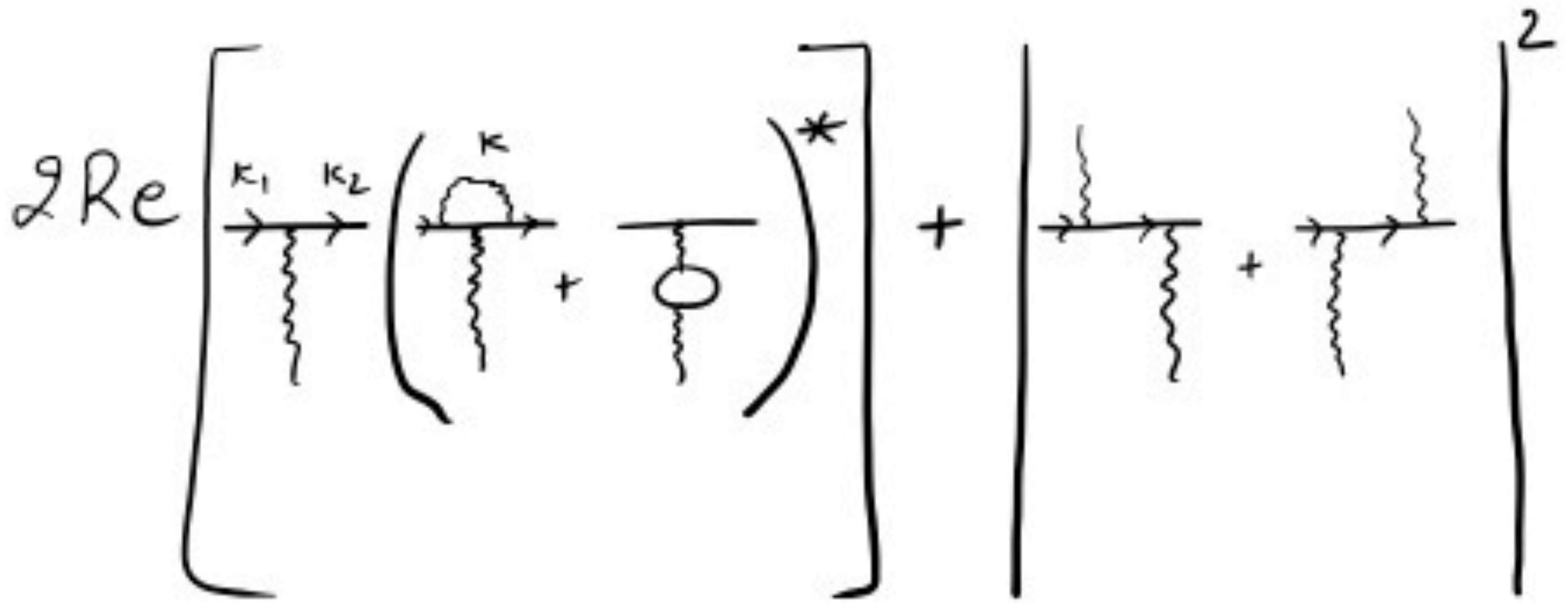
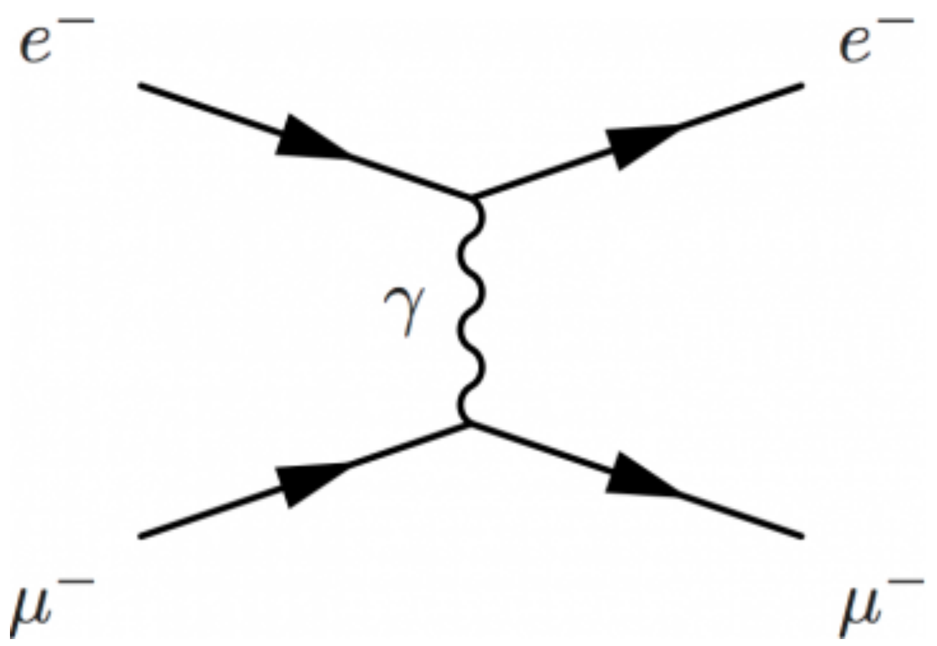
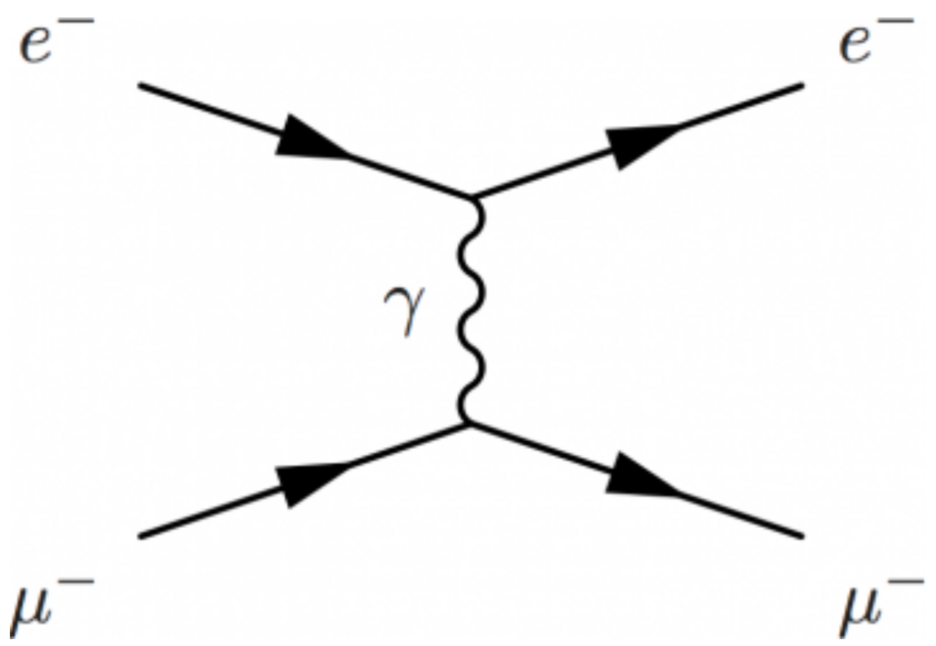
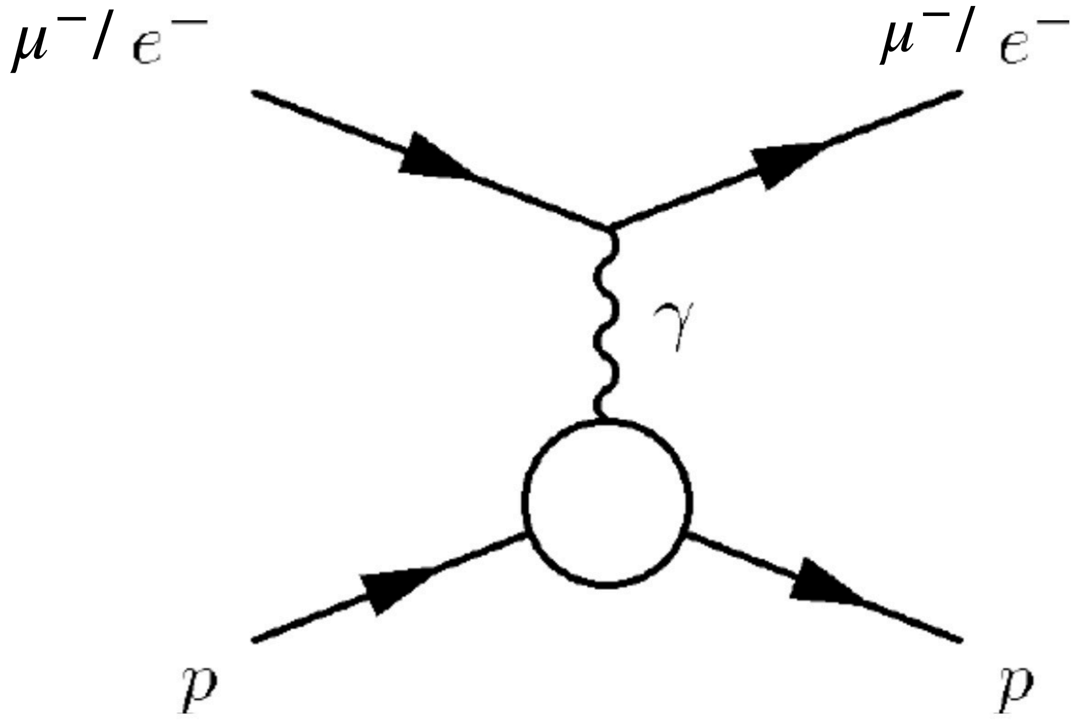


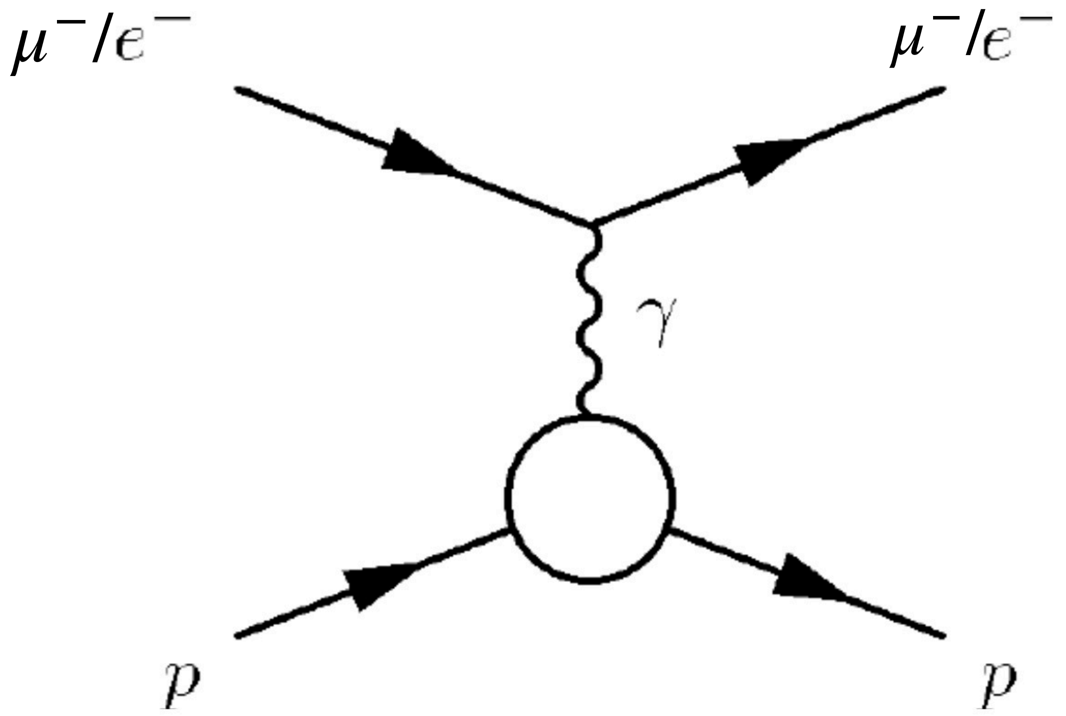
- Recently used by Afanasev et al. (Phys. Rev. D **66**) to calculate QED radiative corrections in processes of exclusive Pion electroproduction.

WHAT IS A COVARIANT APPROACH?









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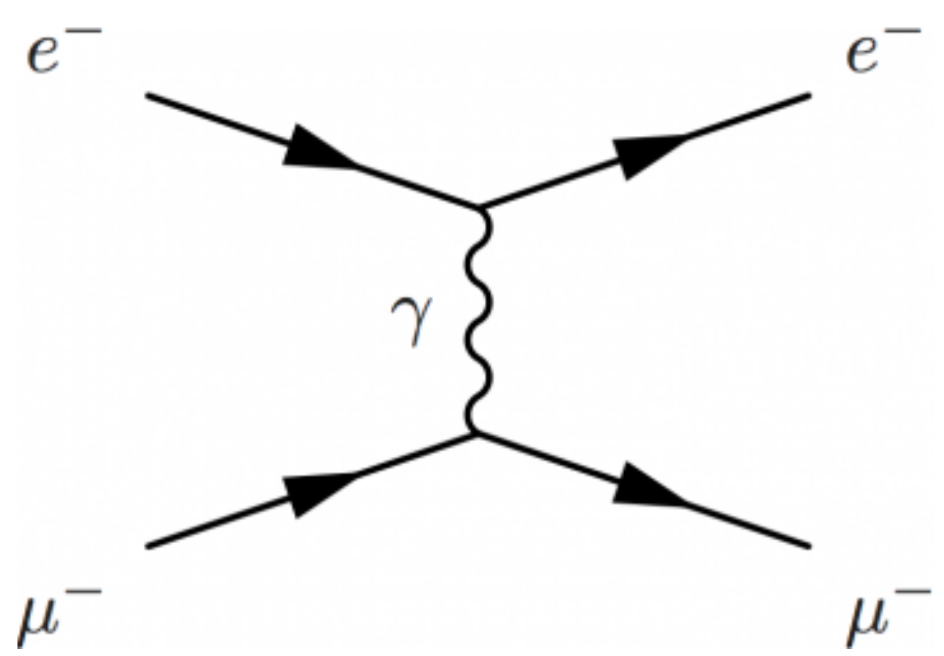

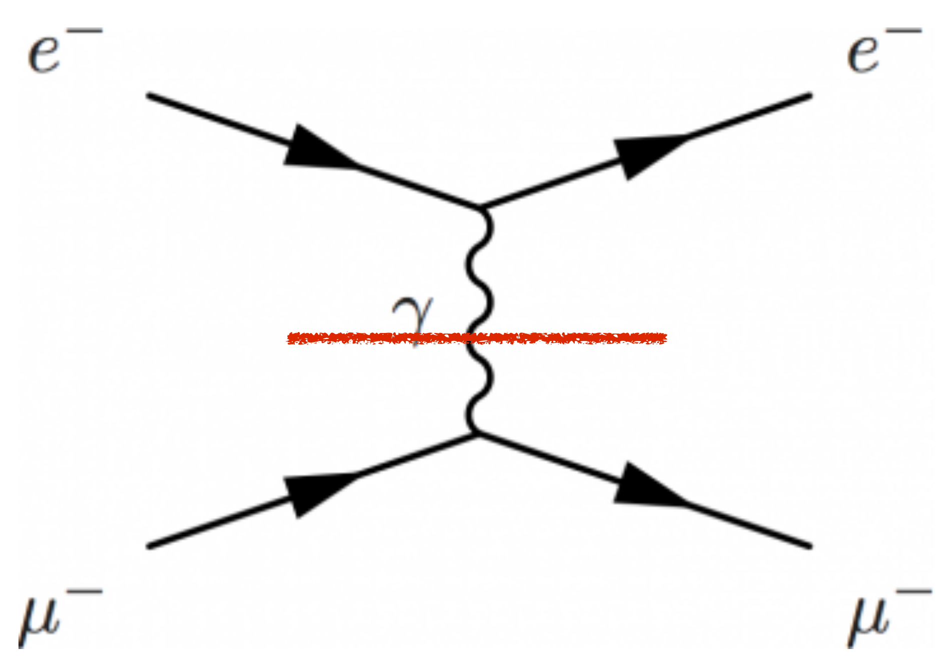

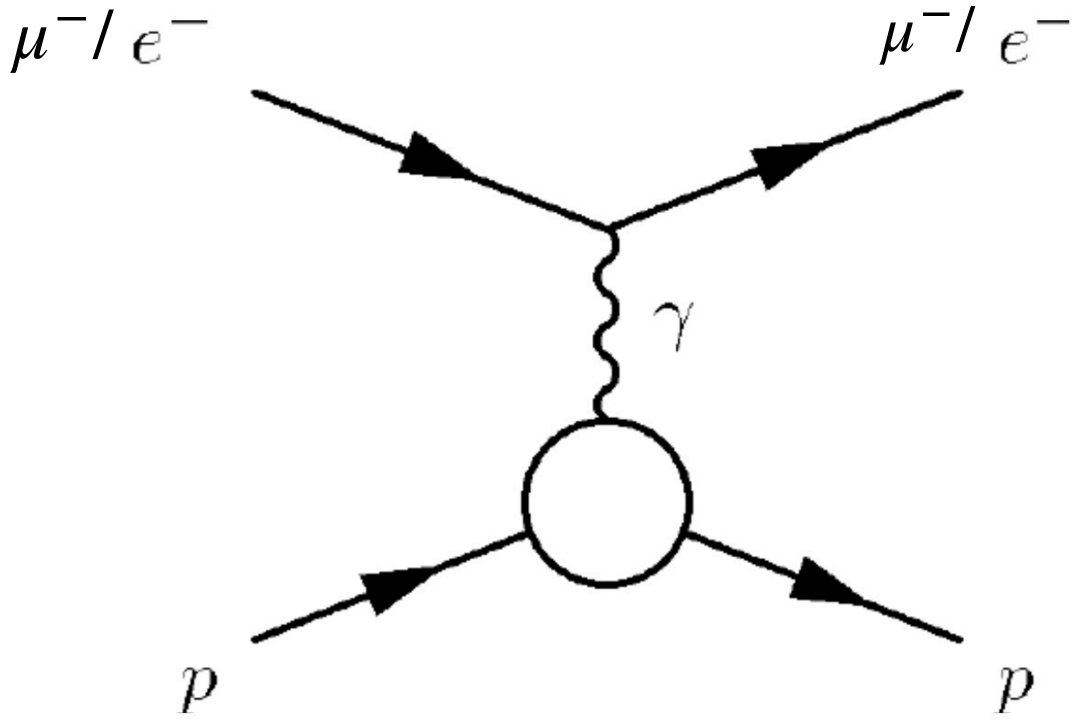

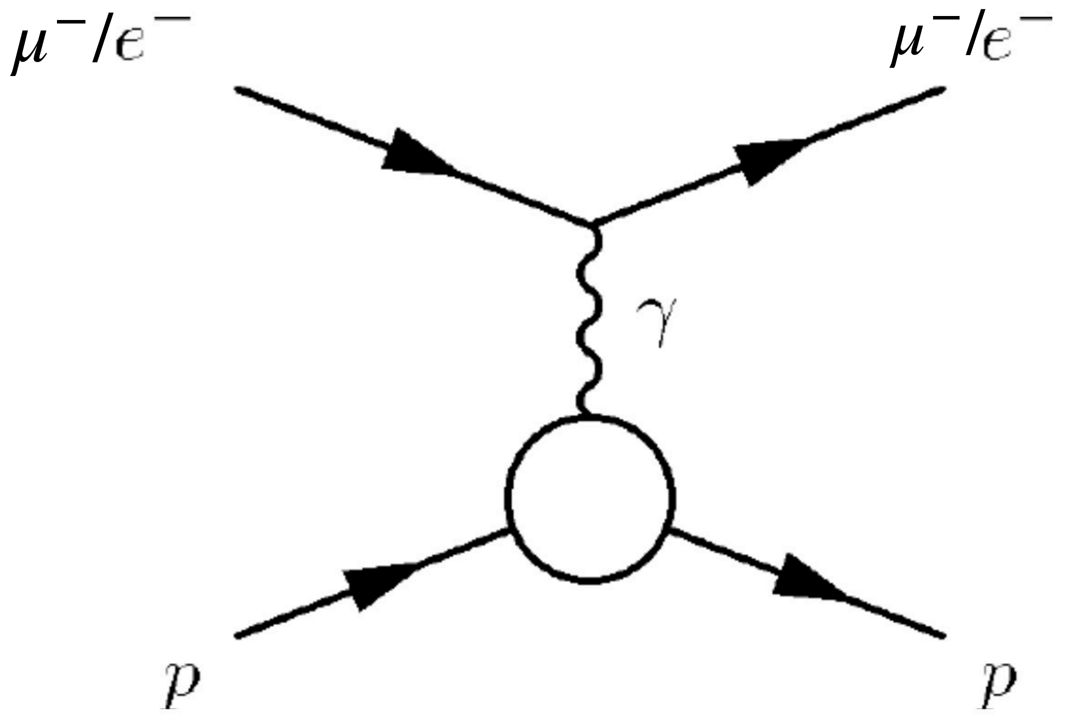

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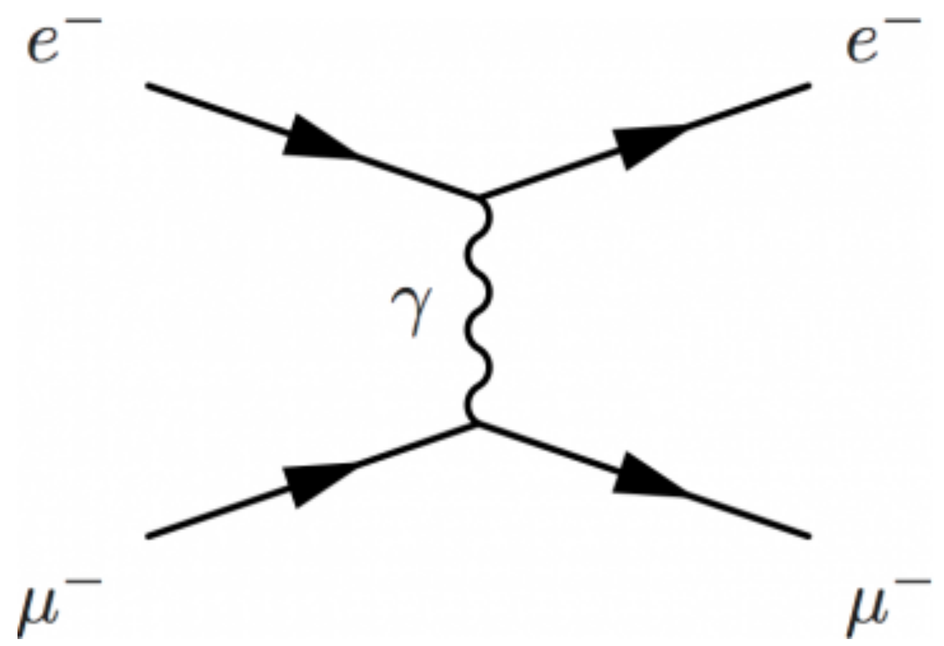

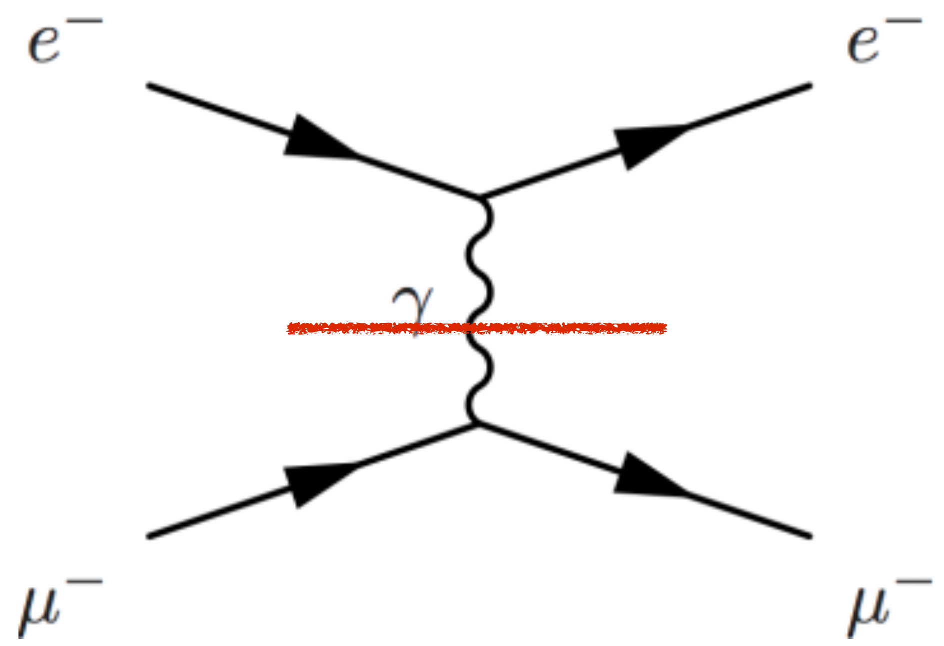

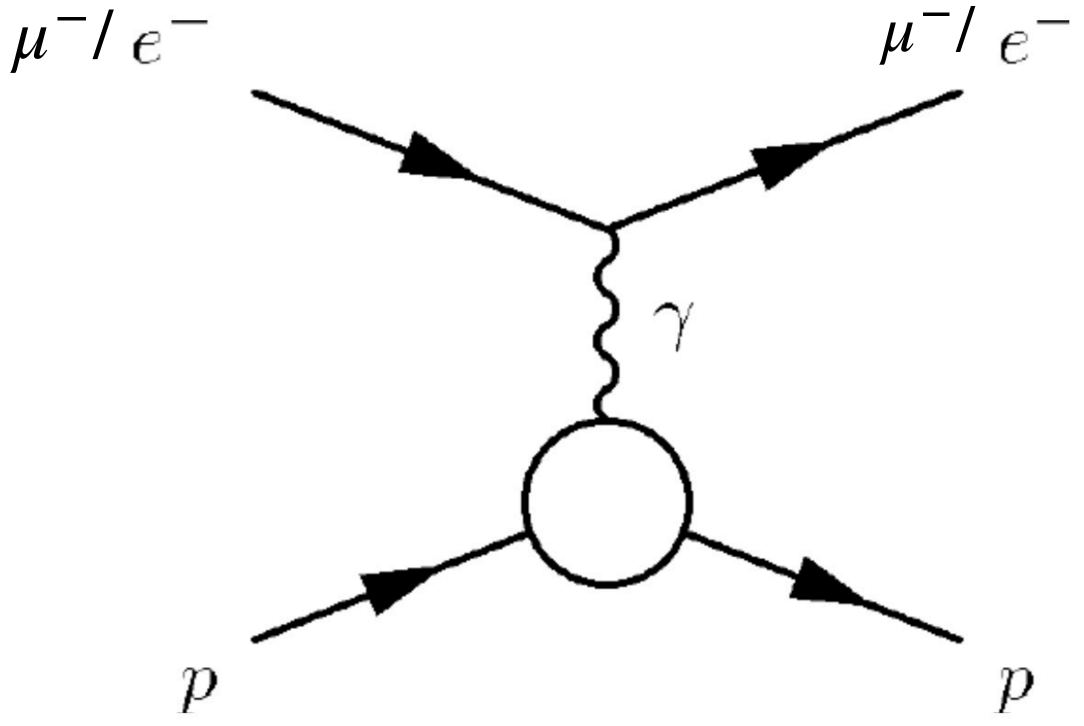

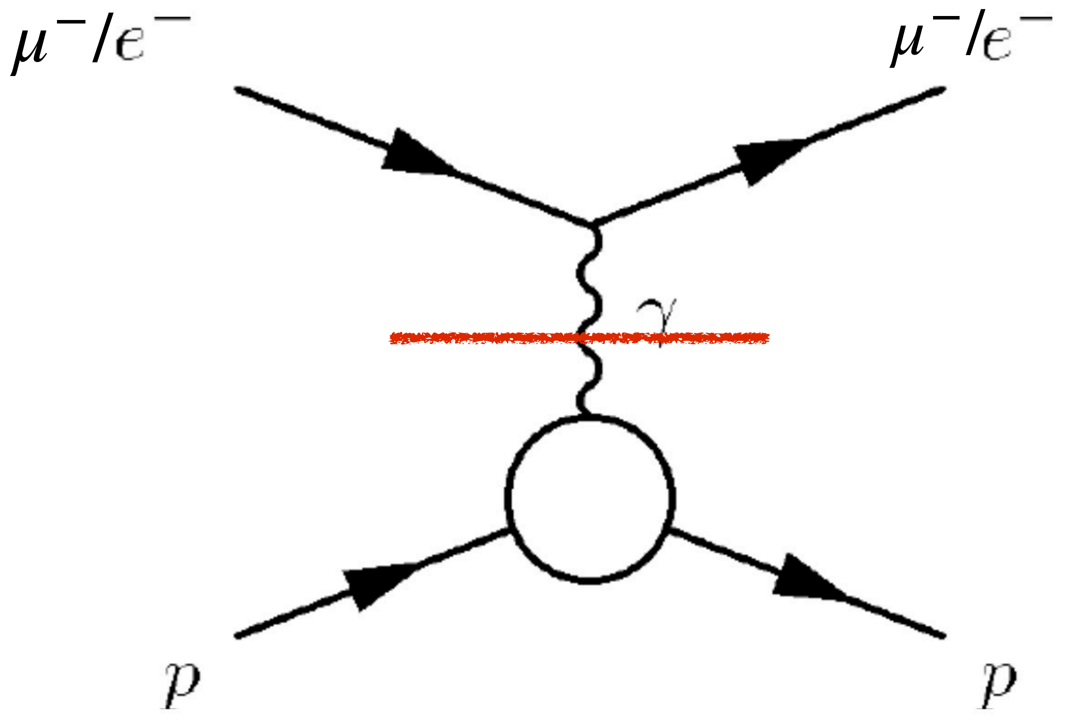

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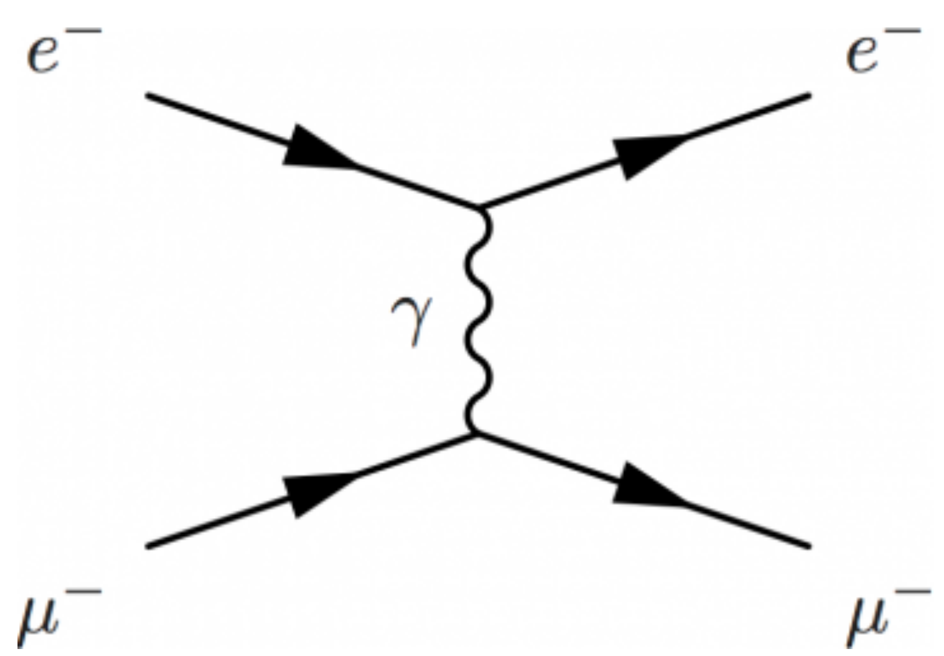

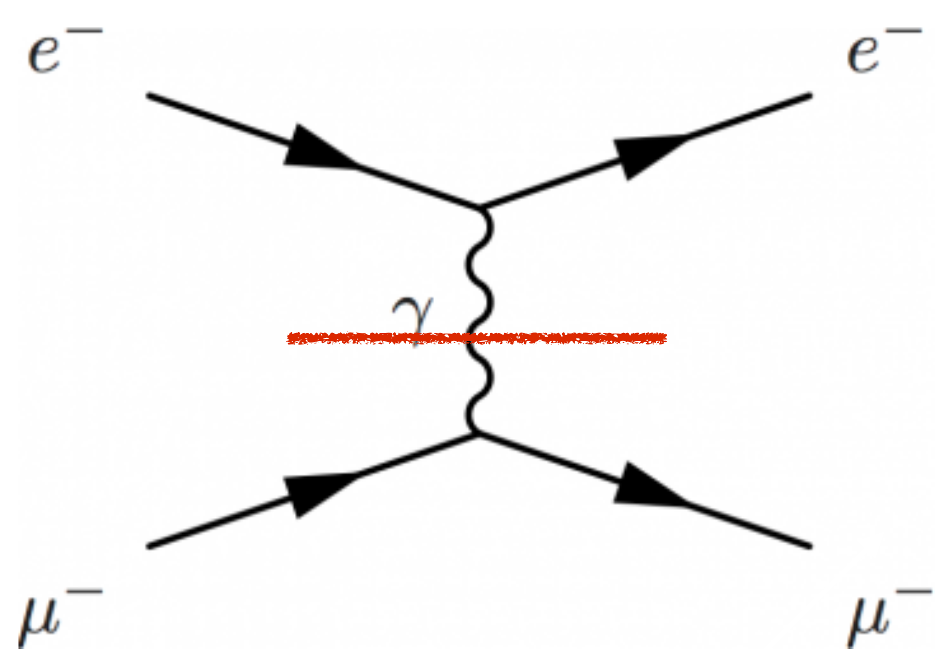

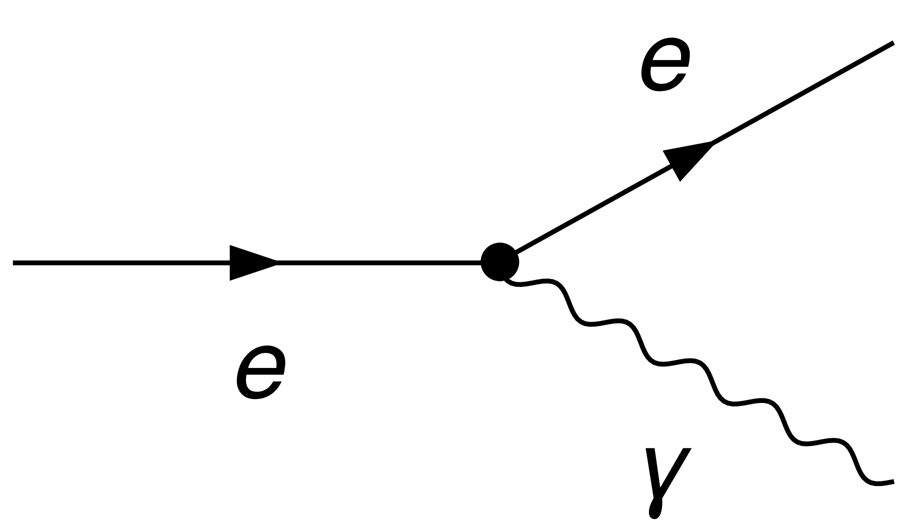

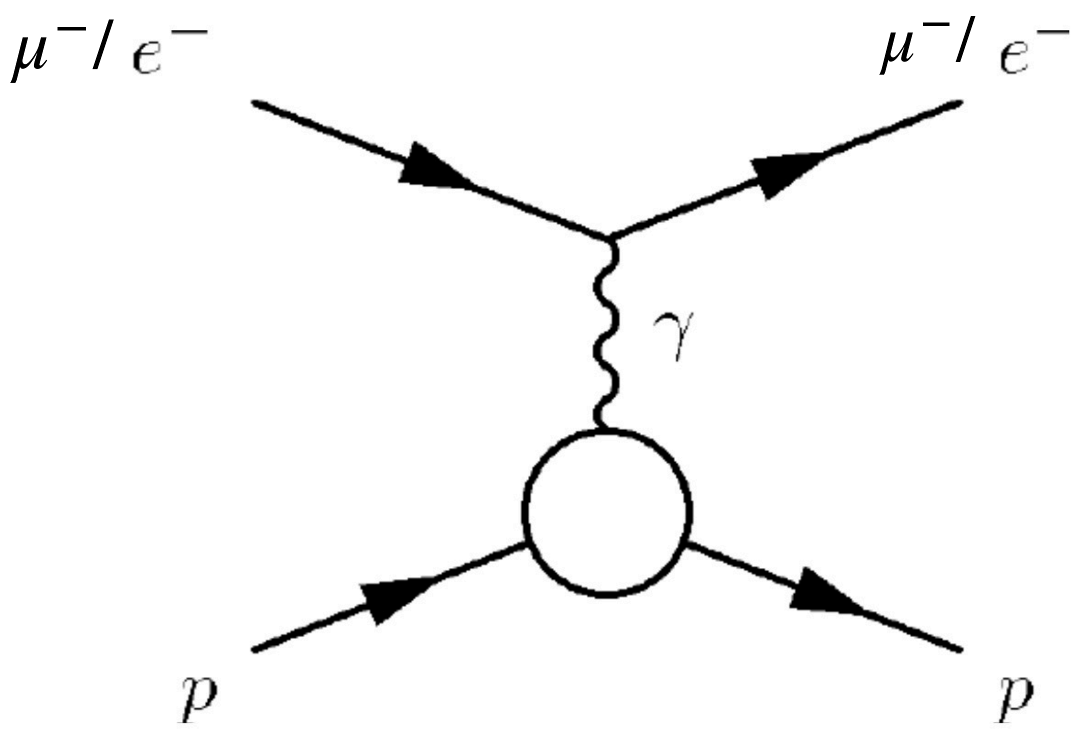

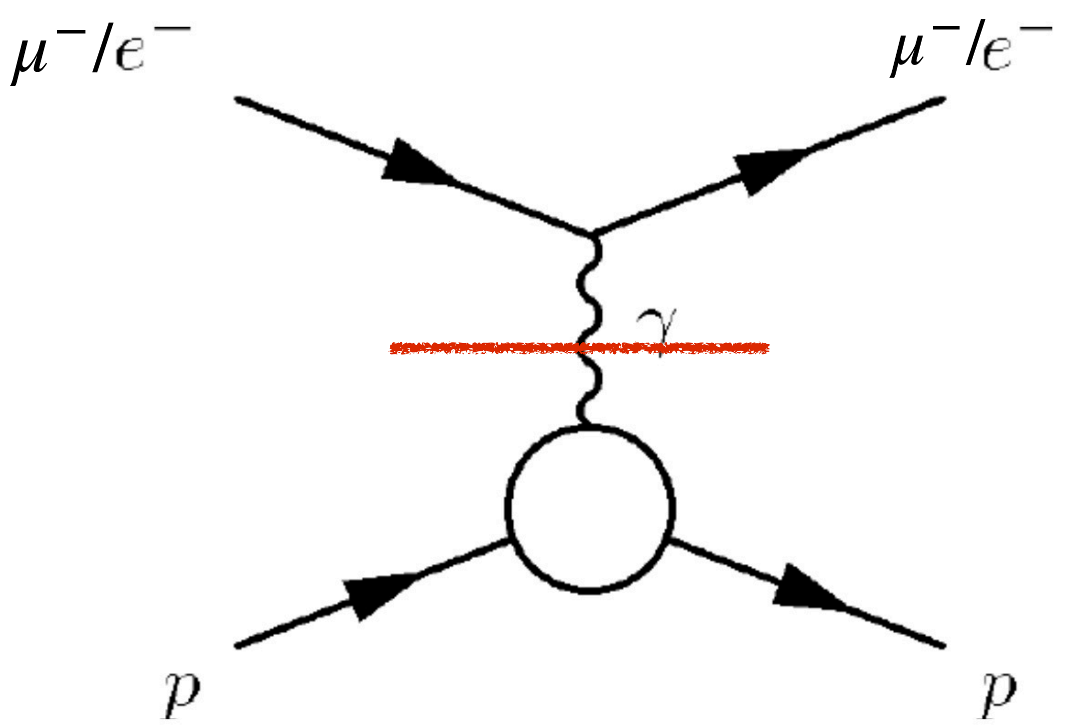

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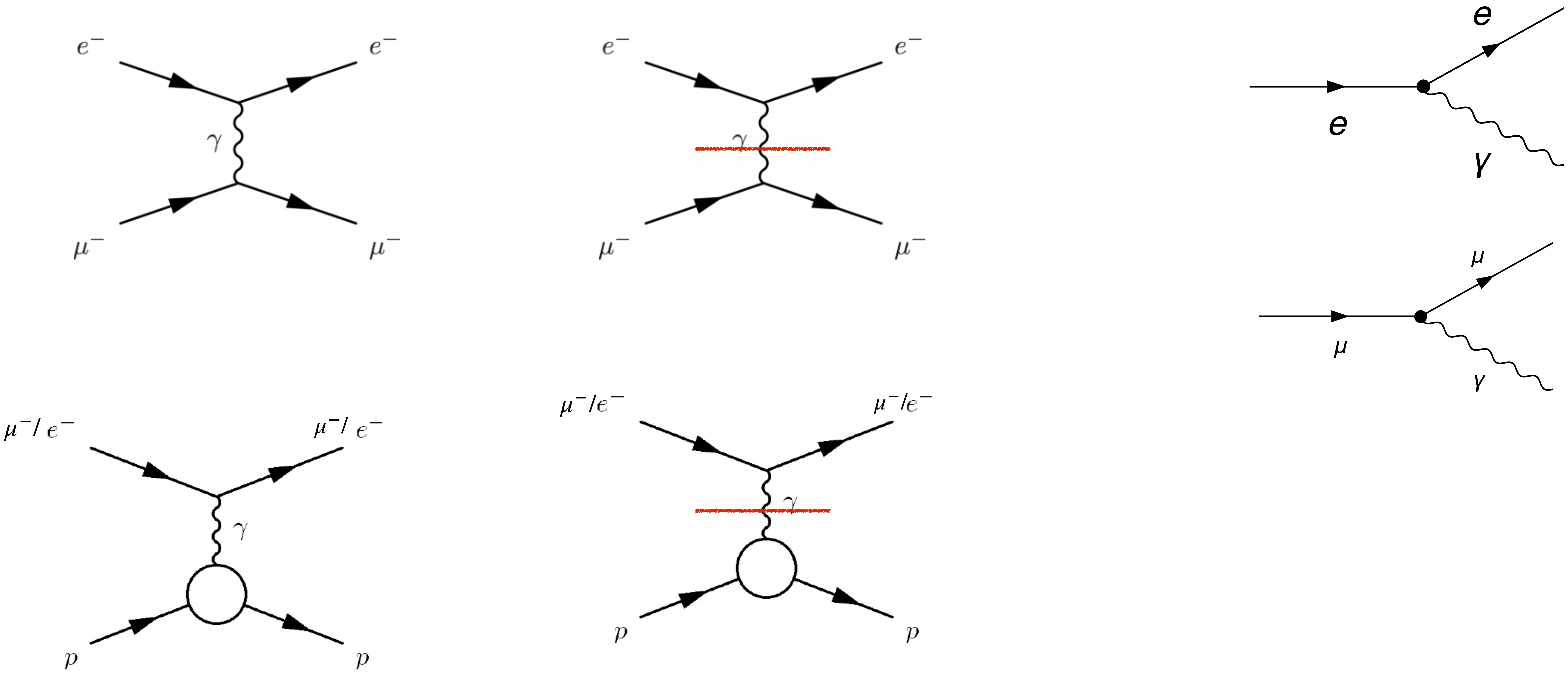

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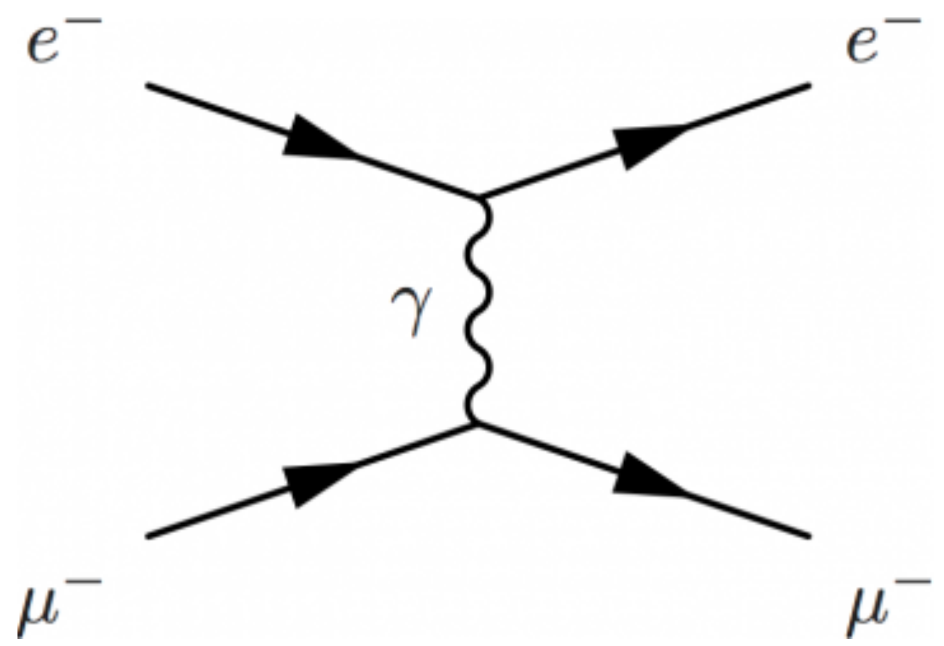
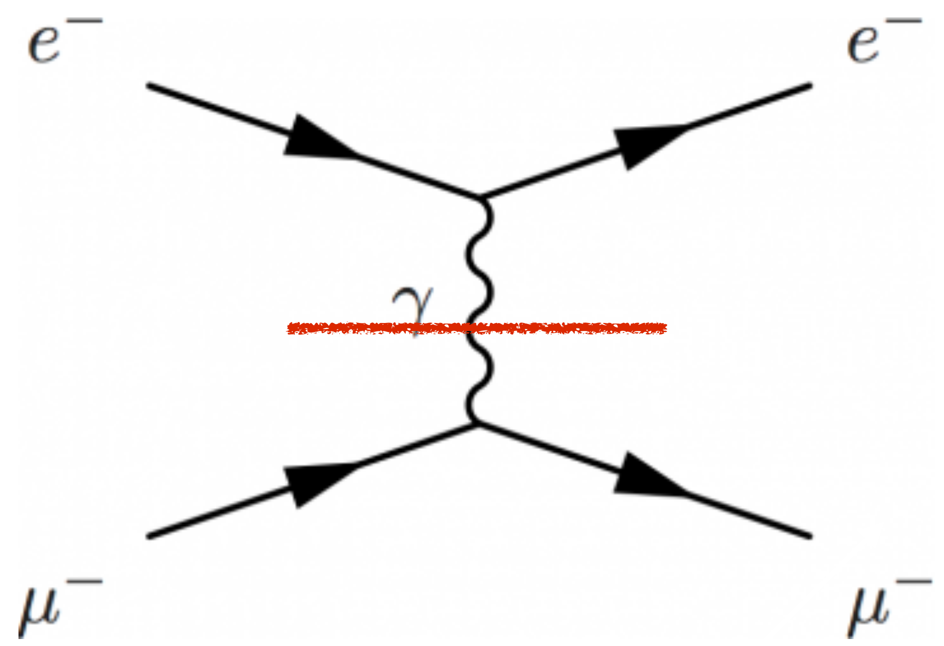
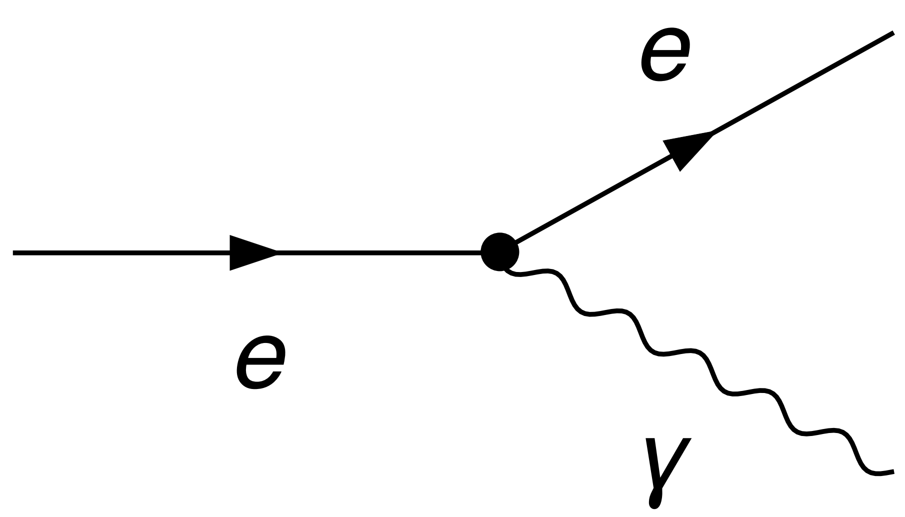
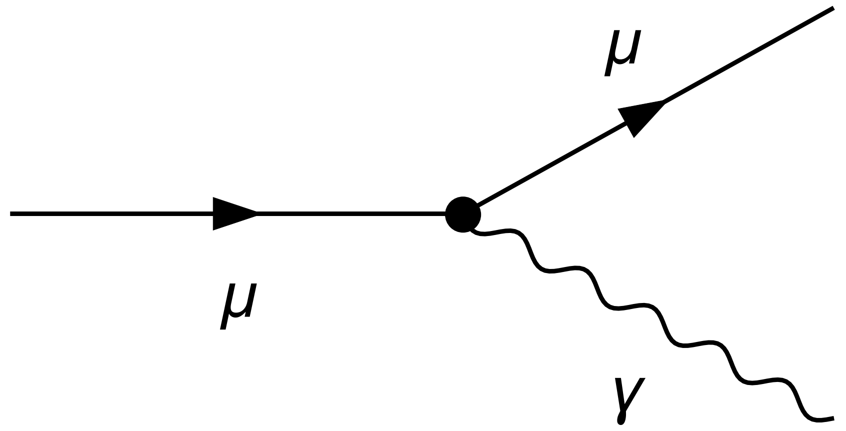
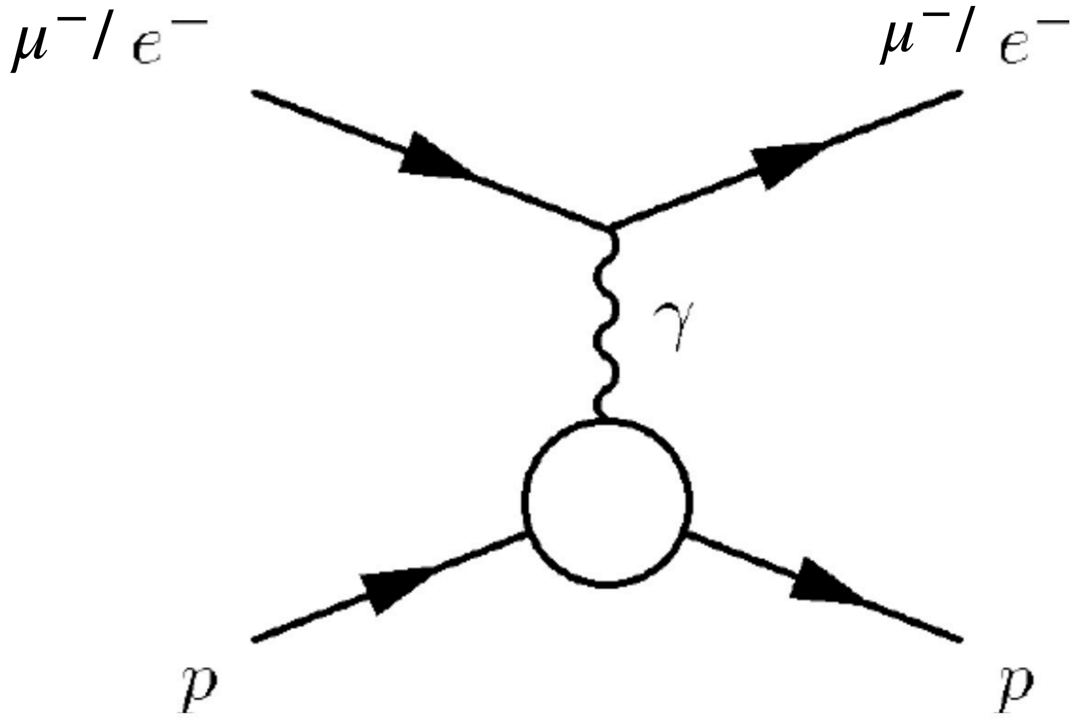
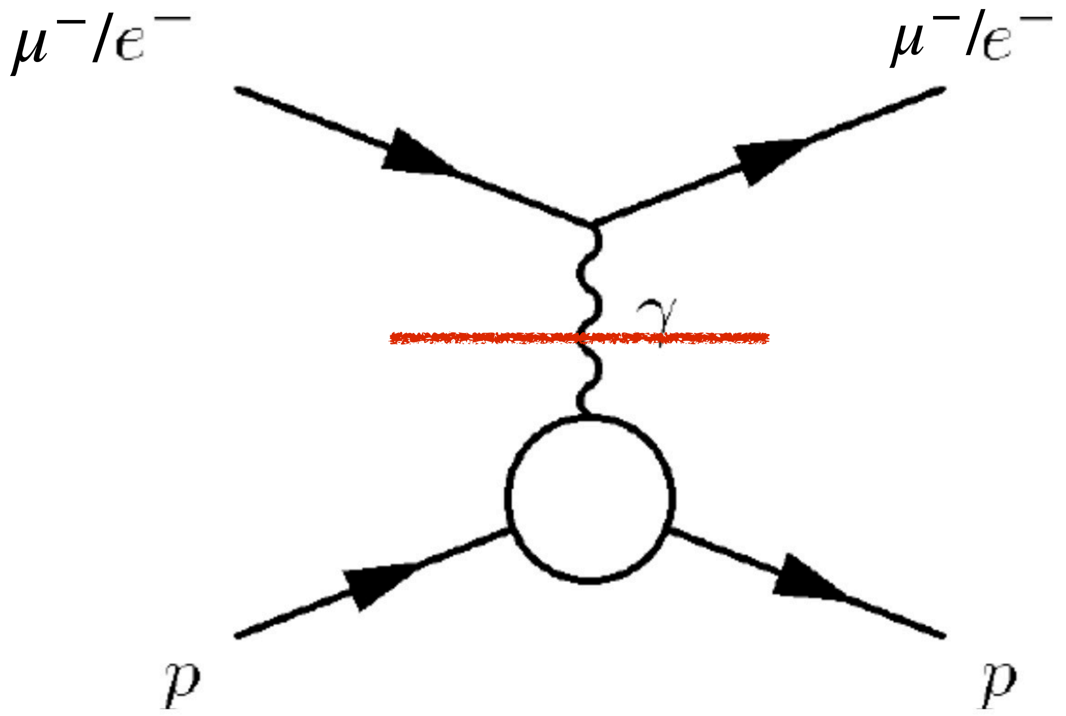


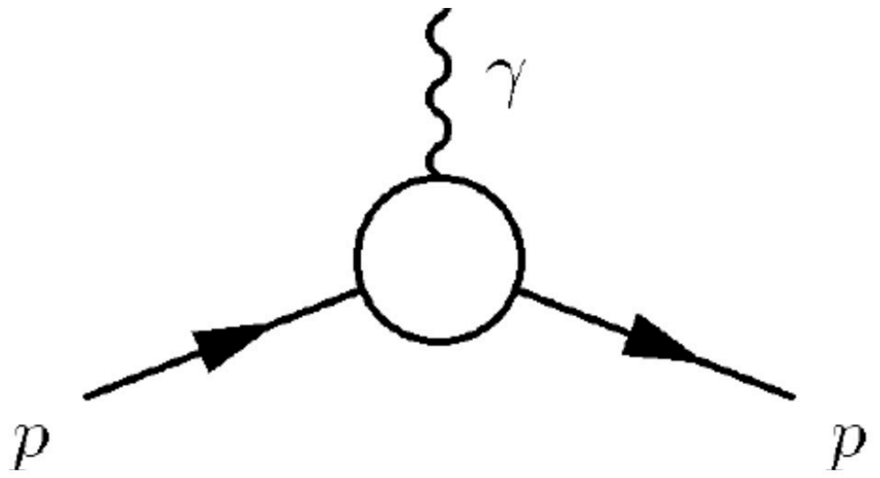












COVARIANT APPROACH WITH LEPTONIC-HADRONIC TENSORS

- The differential cross section of general lepton-lepton/hadron scattering can be obtained by:

$$d\sigma \sim L^{\mu\nu} L_{\mu\nu} \text{ or } d\sigma \sim L^{\mu\nu} W_{\mu\nu}$$

- where $W_{\mu\nu}$ is the hadronic tensor which in case of elastic $e^{-}p$ scattering:

$$W_{\mu\nu} = H_1 g_{\mu\nu} + H_2 p_{1\mu} p_{1\nu} + H_3 p_{2\mu} p_{2\nu} + H_4 p_{1\mu} p_{2\nu} + H_5 p_{2\mu} p_{1\nu} + H_6 \epsilon_{\mu,\nu,p_1,p_2}$$

where p_1 and p_2 are incoming and outgoing protons momenta. H_1, H_2, H_3, H_4, H_5 and H_6 are the hadronic structure functions which can be extracted from experimental data.

QED AND ELECTROWEAK HADRONIC COUPLINGS WITH FORM FACTORS

$$\Gamma^{\mu}_{\gamma-p}(q^2) = ieCnp2\left(f2p\gamma^{\mu} + gp\gamma_L\gamma^{\mu}\omega_- + gp\gamma_R\gamma^{\mu}\omega_+ - \frac{f2p(p_1^{\mu} + p_2^{\mu})}{2m_p}\right)$$

$$\Gamma^{\mu}_{Z-p}(q^2) = -ieCnp2\left(F2W\gamma^{\mu} + gpz_L\gamma^{\mu}\omega_- + gpz_R\gamma^{\mu}\omega_+ - \frac{F2W(p_1^{\mu} + p_2^{\mu})}{2m_p}\right)$$

$F2W = \frac{F2Vp - 4\sin^2\theta_W f2p}{4\cos\theta_W \sin\theta_W}$ → **EW form factor**

$gpz_{(L,R)} = \frac{F1Vp - 4\sin^2\theta_W f1p \pm G1p}{4\sin\theta_W \cos\theta_W}$

$gp\gamma_L = gp\gamma_R = f1p(0)$ → **Electric form factor**

$G1p = 1.267$ → **Axial form factor**

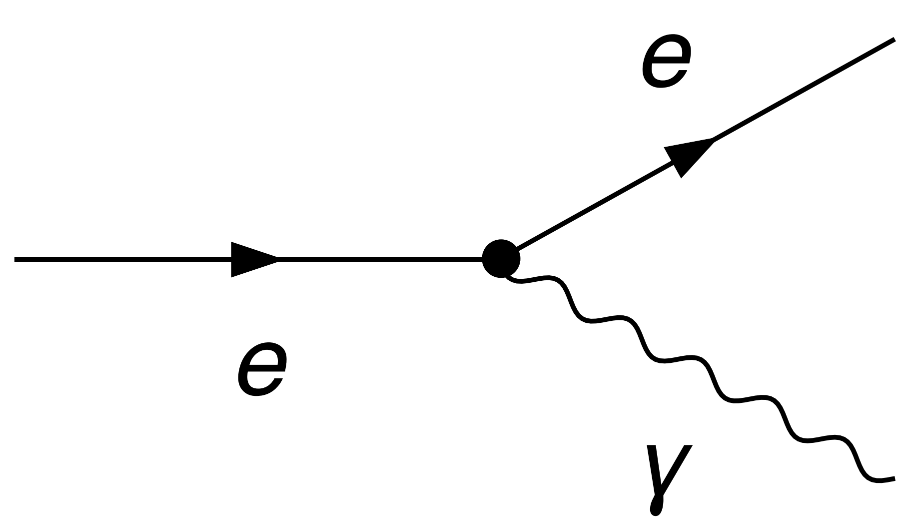
$Cnp2 = \left(\frac{\Lambda^2}{\Lambda^2 - t}\right)^2, \quad \Lambda = \sqrt{0.83\ m_p^2}$

$F(1,2)Vp = f(1,2)p - f(1,2)n$

Tree Level Graphs

Full Electroweak Tree level Graphs

Tree Level Graphs



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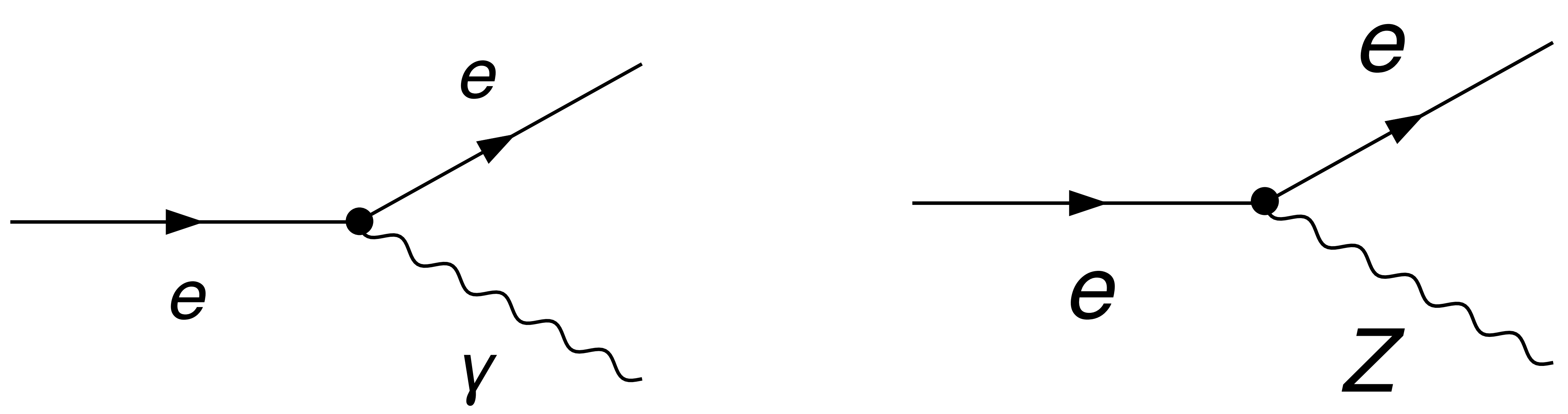
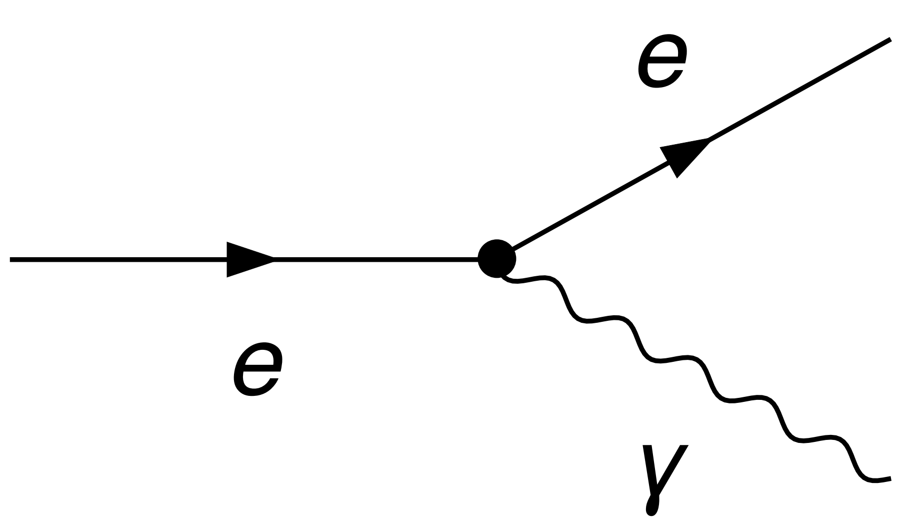
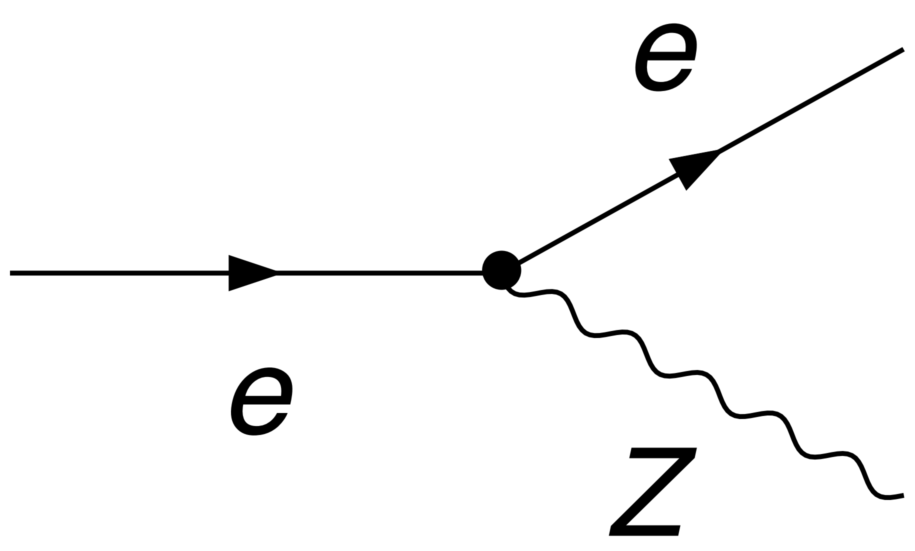
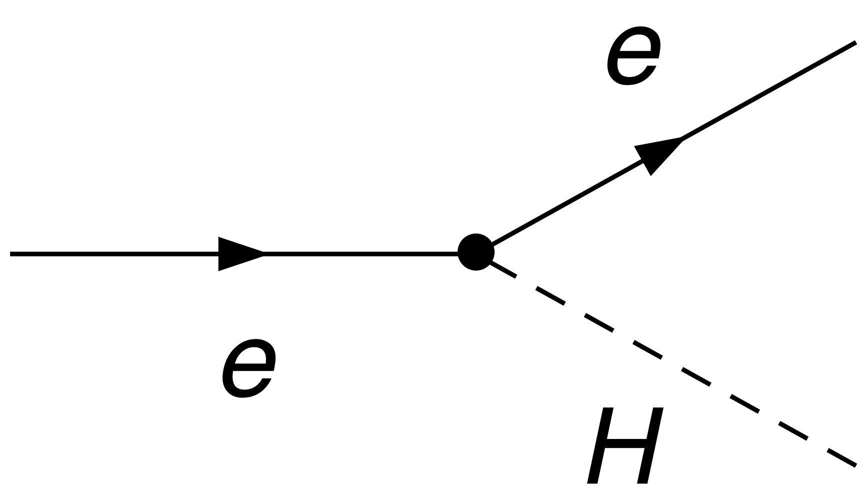


Tree Level Graphs

Full Electroweak Tree level Graphs







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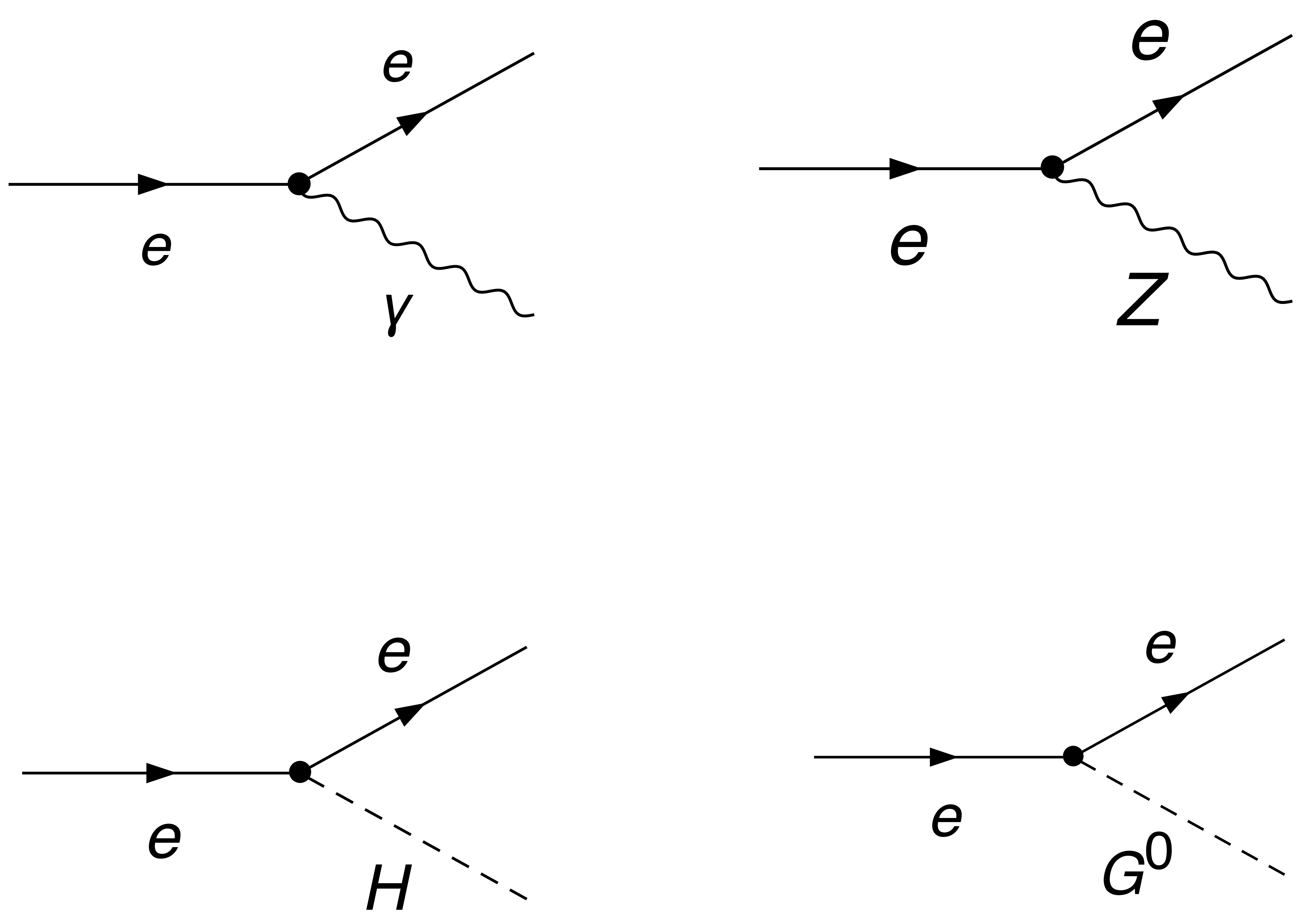

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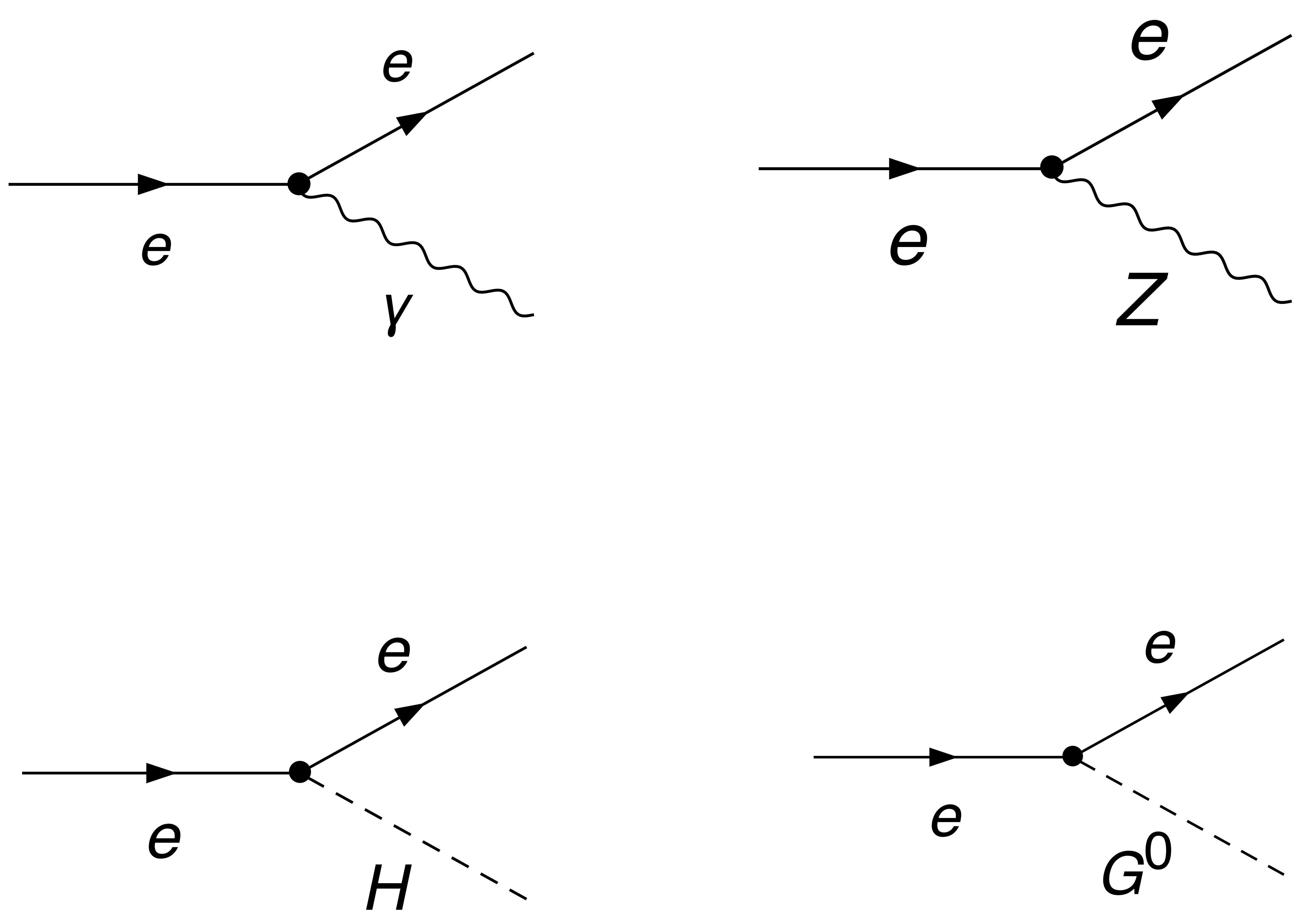
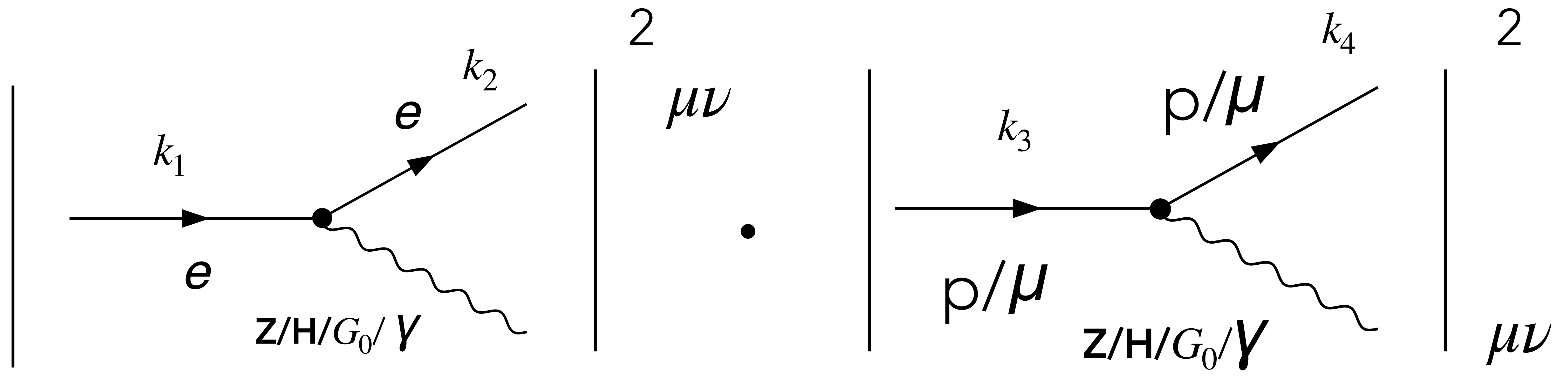


TREE-LEVEL LEPTONIC TENSOR (α-ORDER)

$|M|^2 \propto$ 

- For **tree-level** upper part of the diagram (say $e^- p$ scattering), one can calculate leptonic tensor which is:

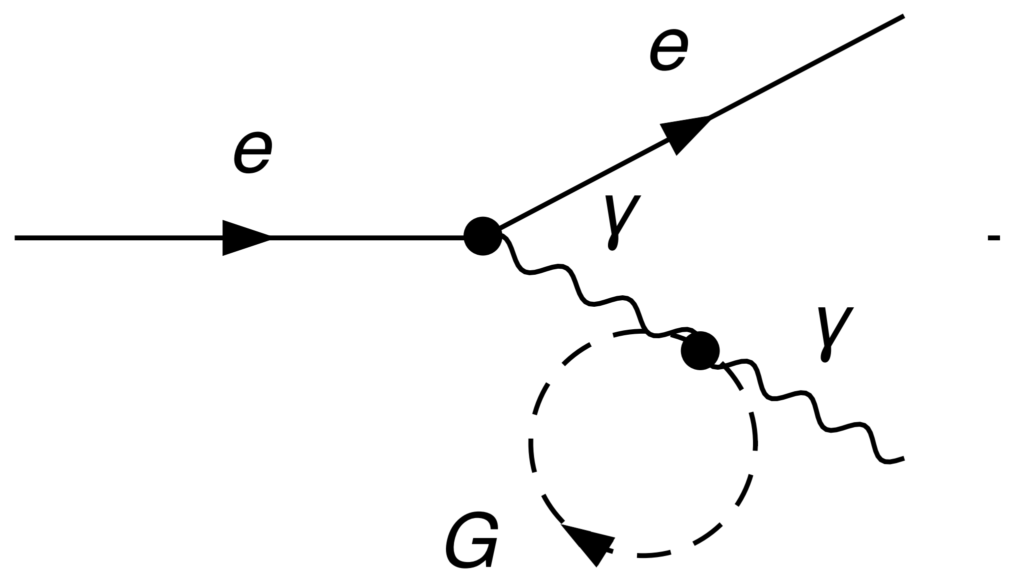
$$L^0_{\mu\nu} \propto 4\pi\alpha((l_1)g_{\mu\nu} + (l_2)k_{2\mu}k_{1\nu} + (l_3)k_{1\mu}k_{2\nu} + \ldots)$$

where k_1, k_2 are incoming and outgoing e^- momenta and $l_{1,2..}$ are tree level leptonic tensor structure functions.

NLO level Graphs

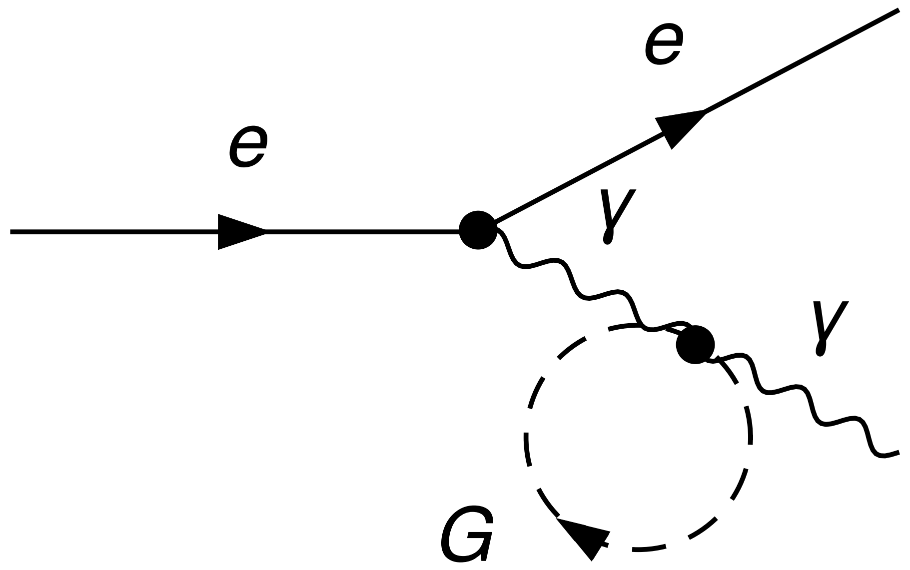
One loop level
Examples

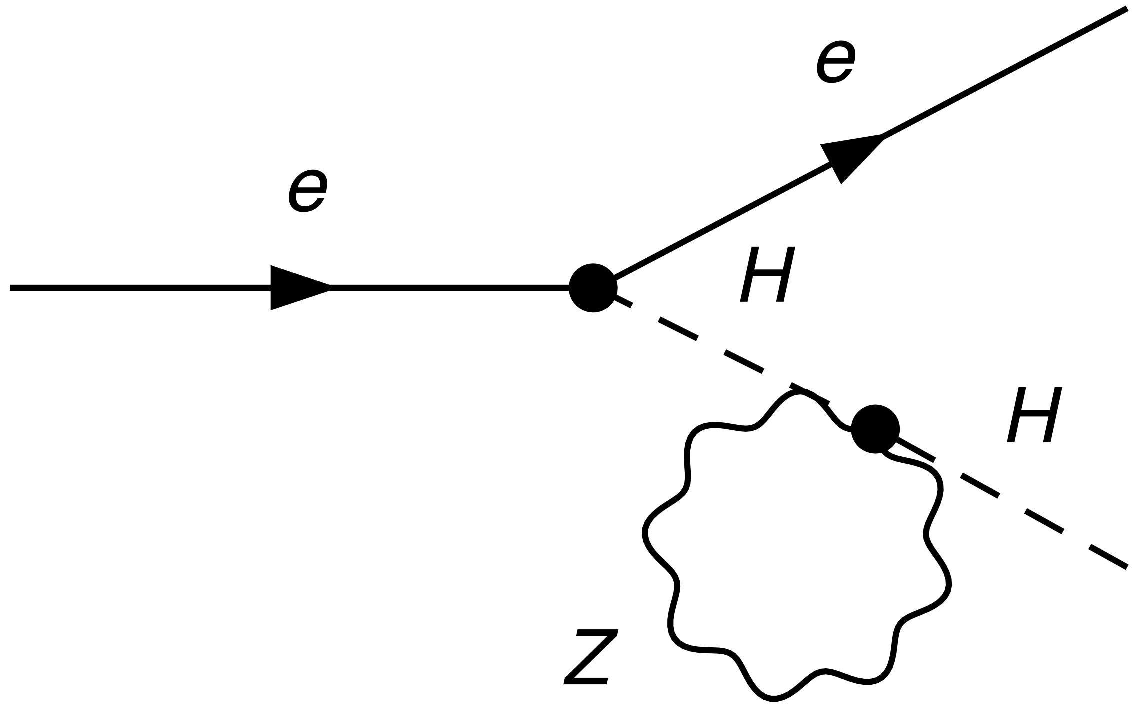
NLO level Graphs



One loop level
Examples

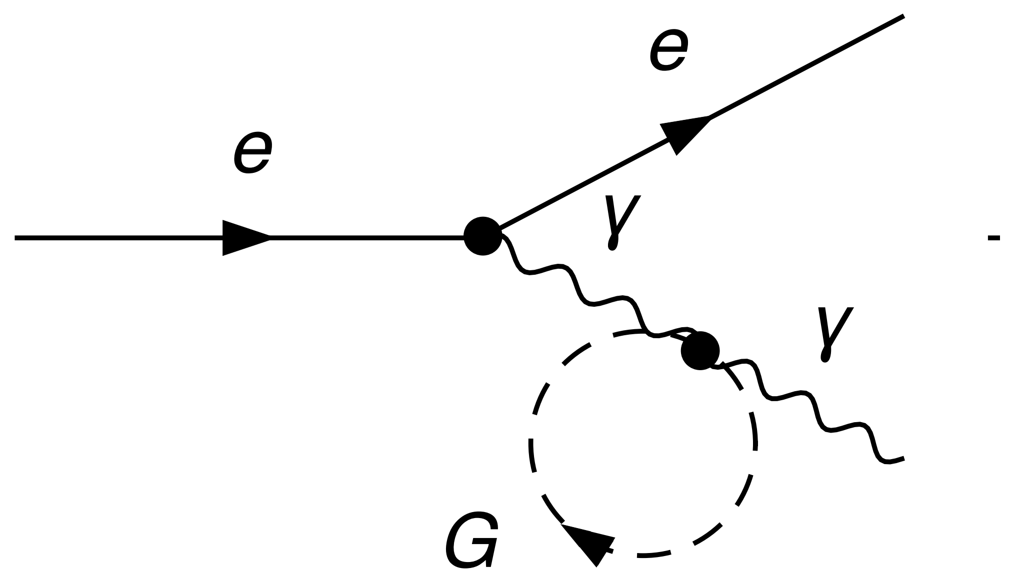
NLO level Graphs

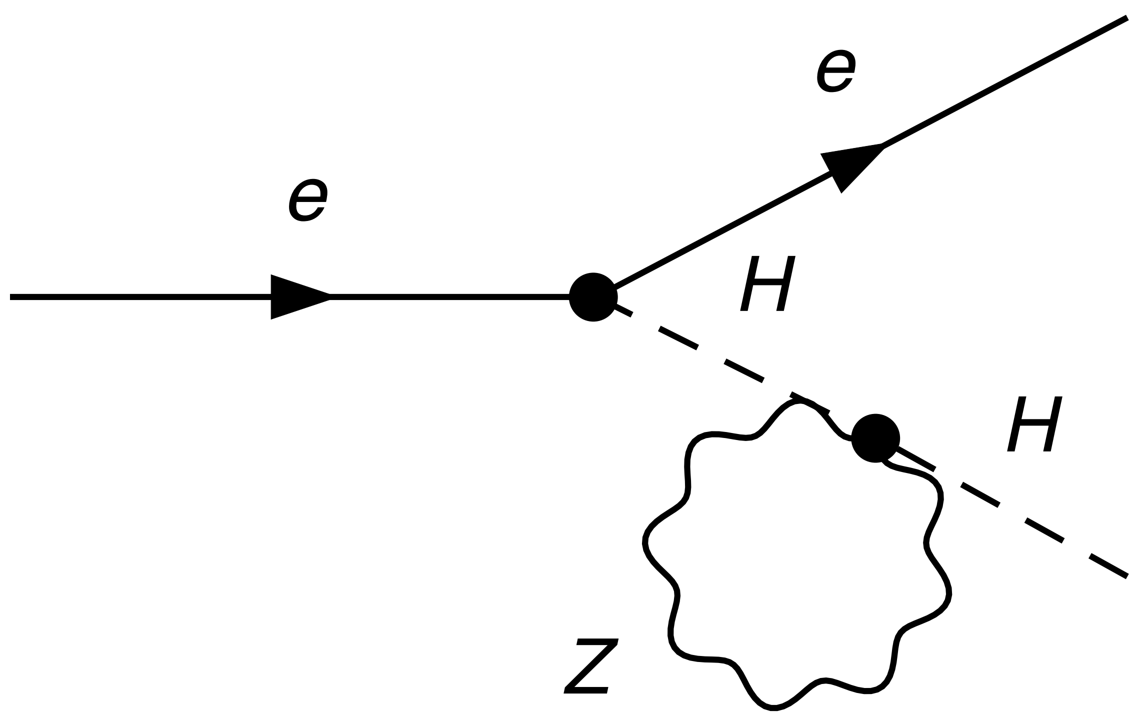




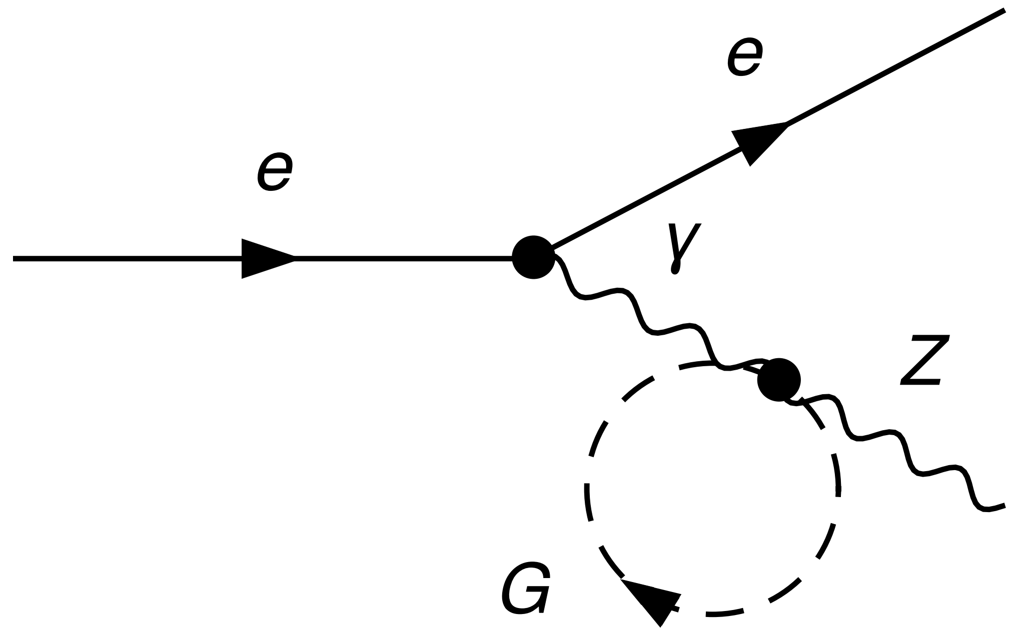
One loop level
Examples

NLO level Graphs



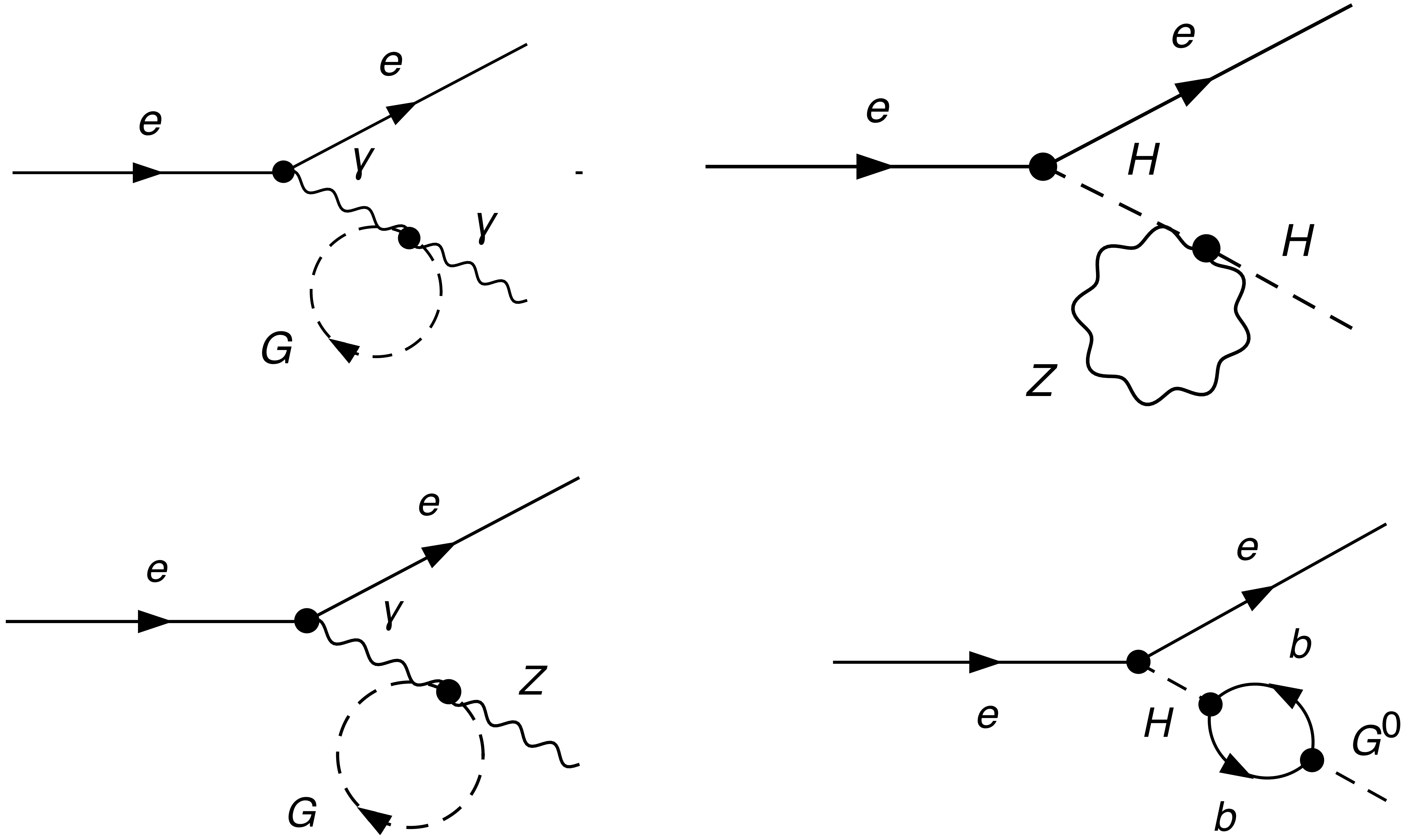


One loop level Examples



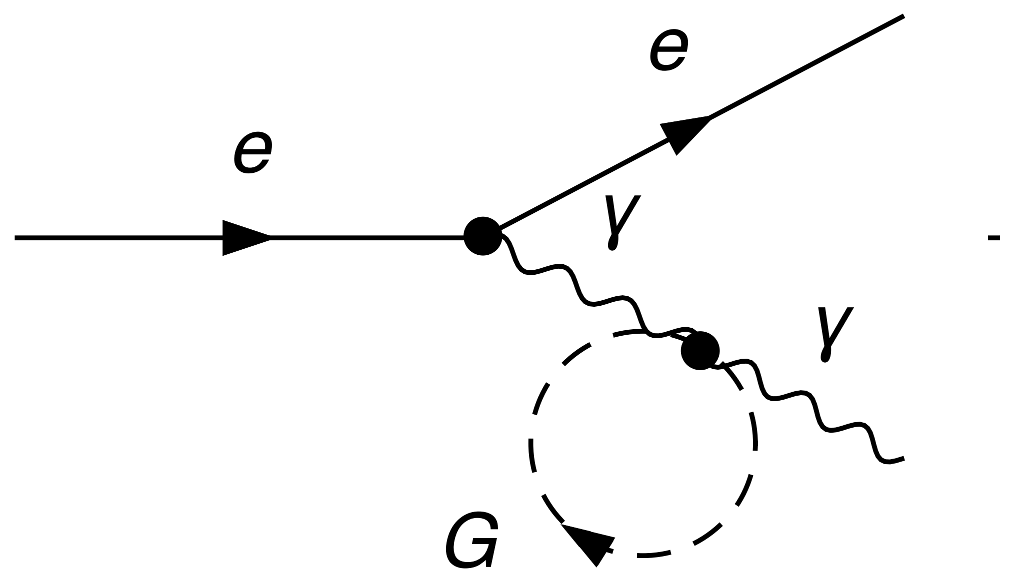
NLO level Graphs

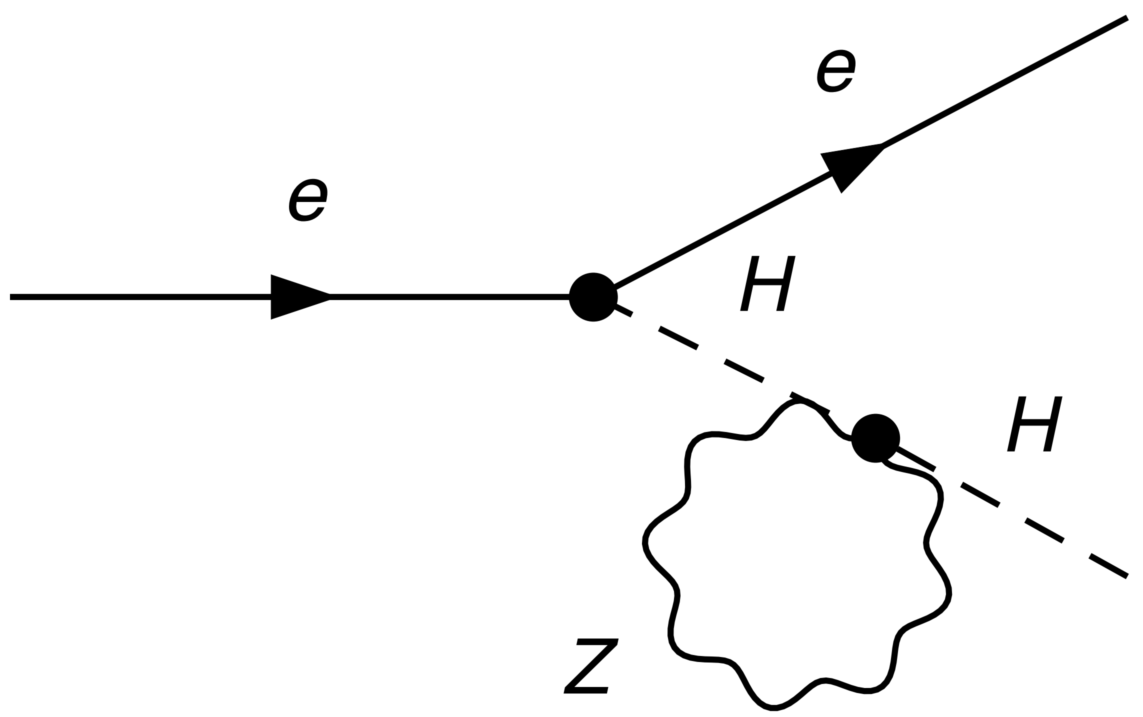
One loop level
Examples

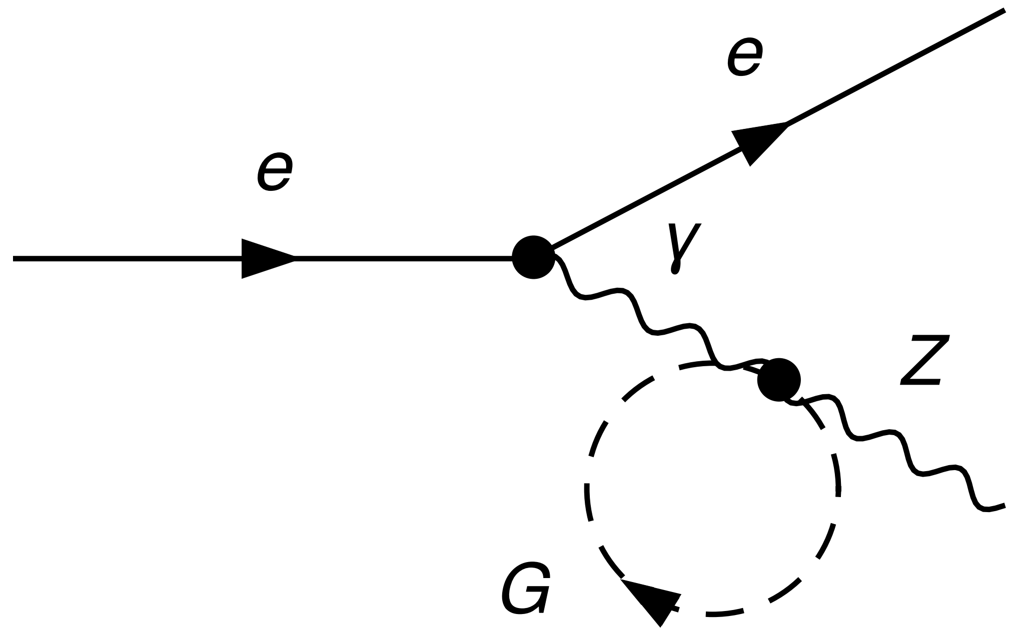


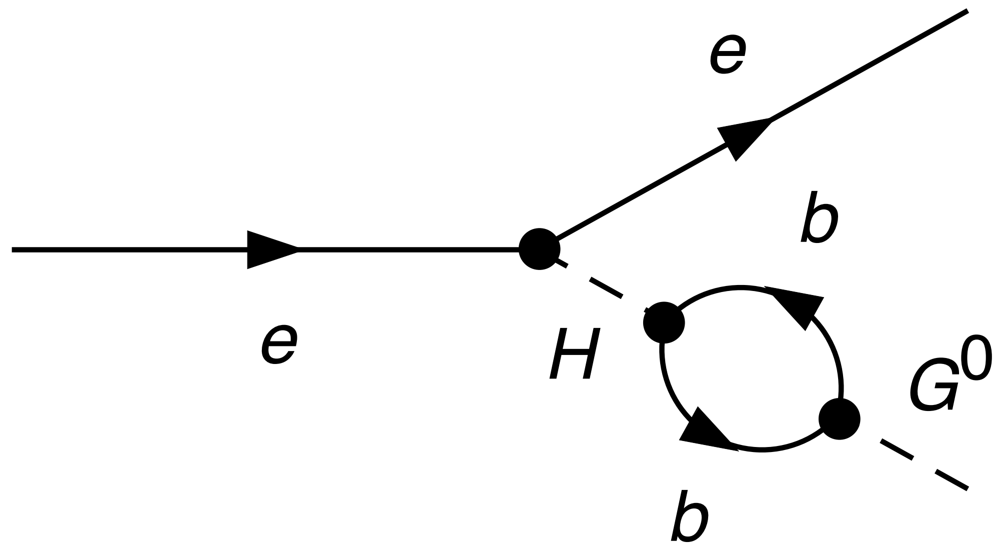
NLO level Graphs

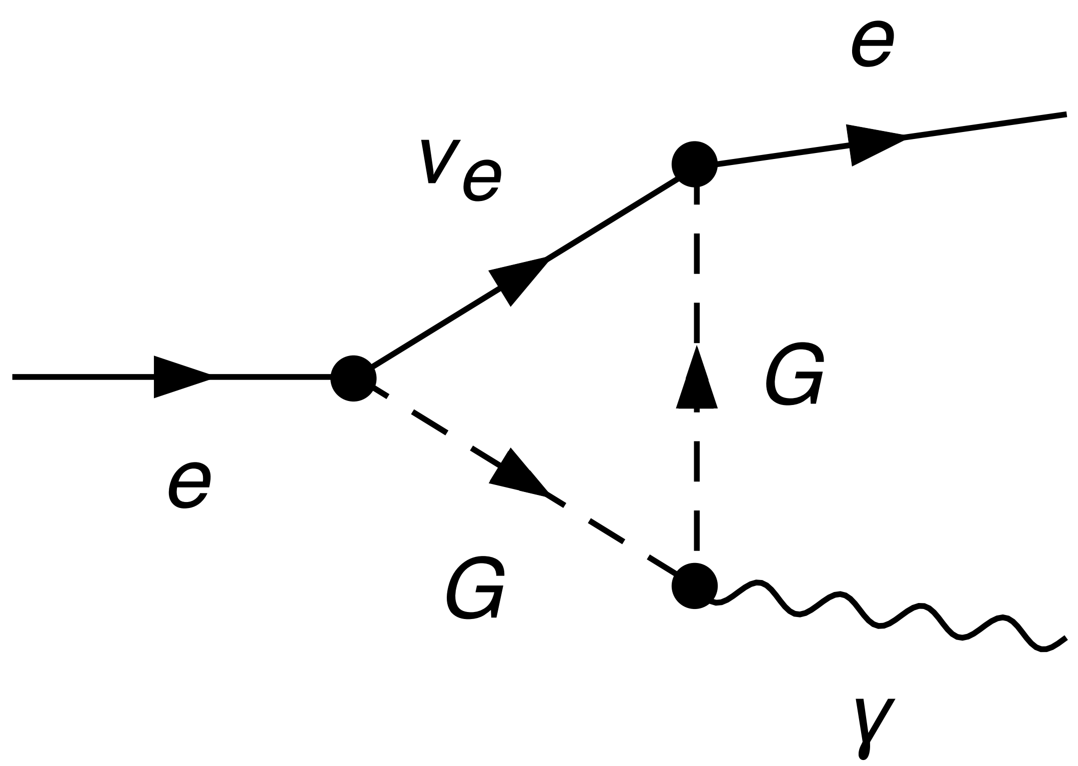
One loop level
Examples





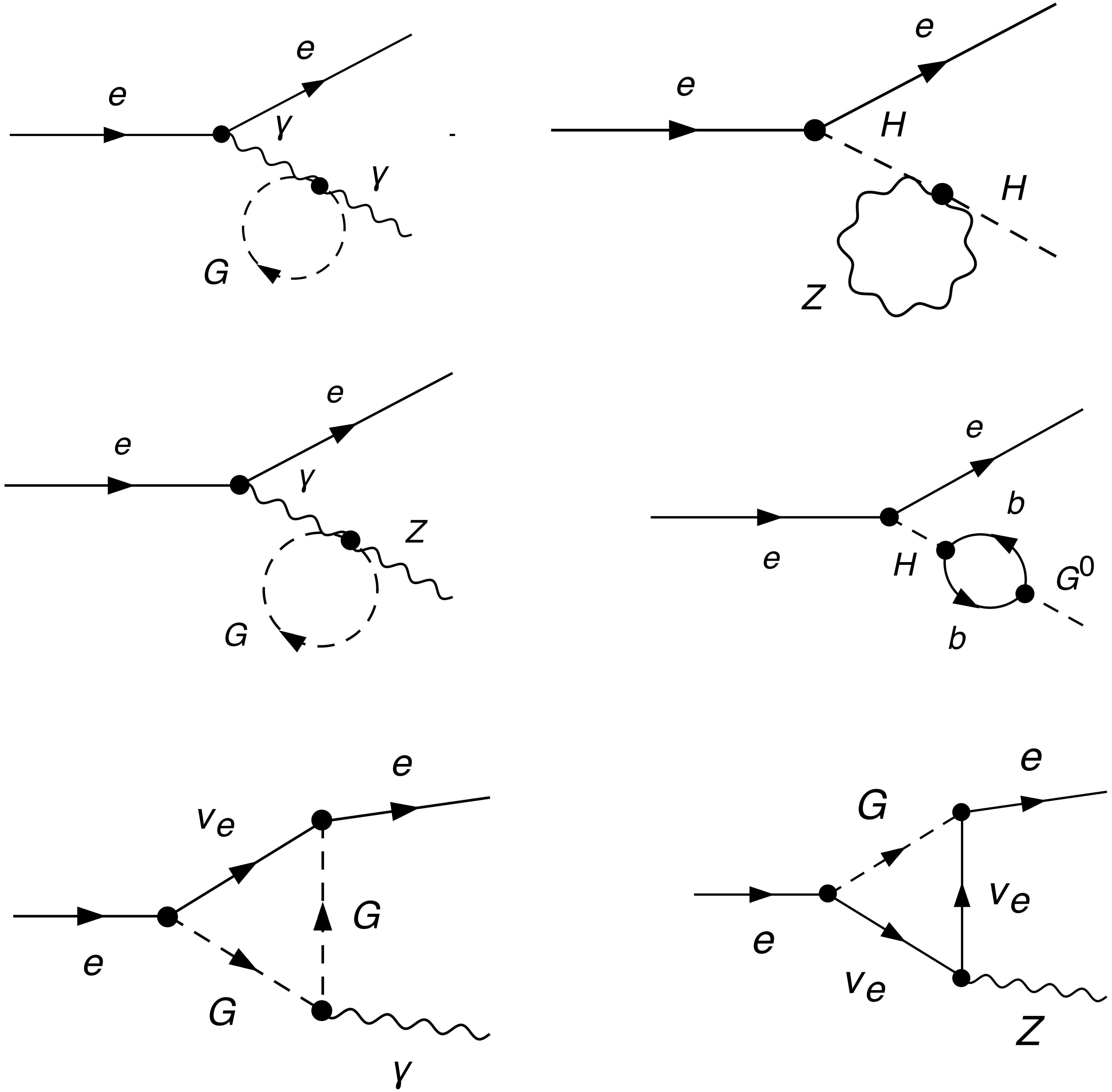






NLO level Graphs

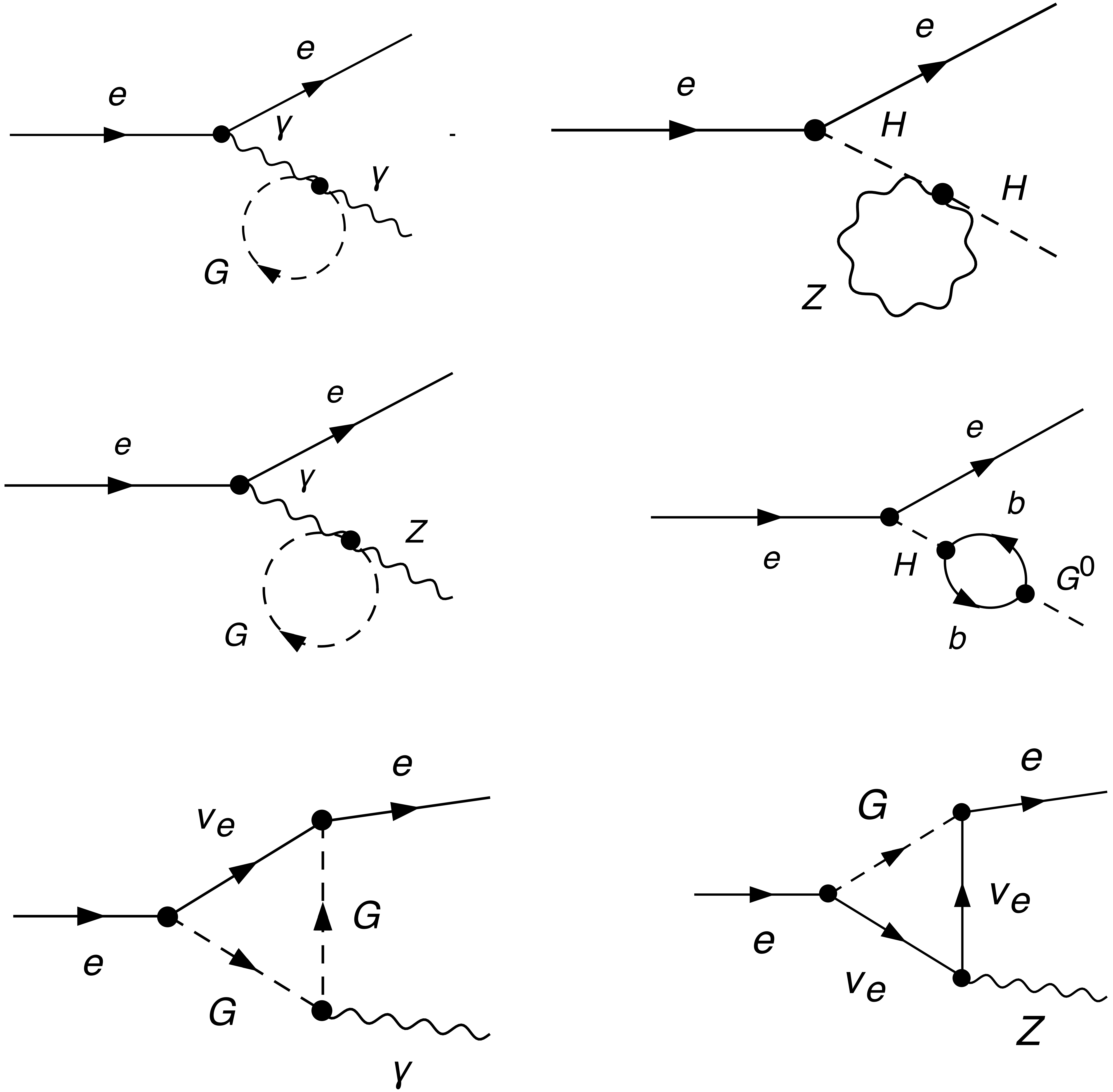
One loop level
Examples



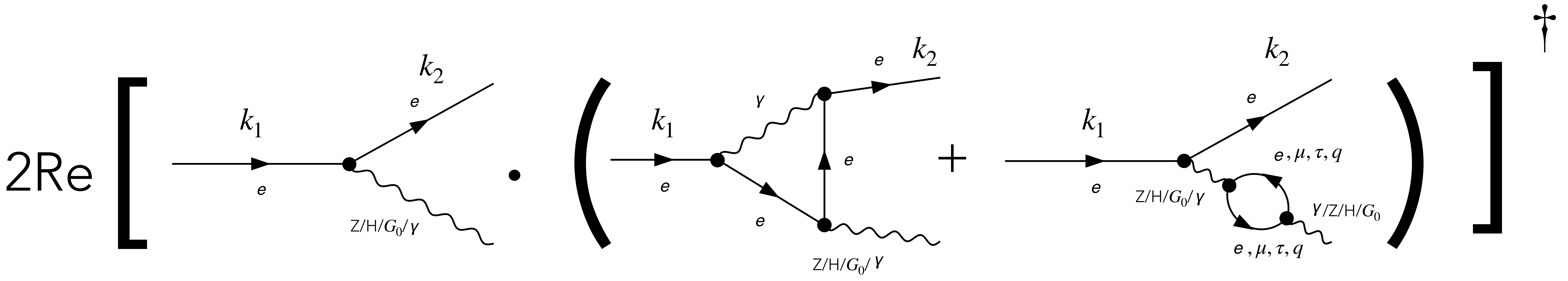
NLO level Graphs

One loop level
Examples

307 graphs



NEXT TO THE LEADING ORDER (NLO) LEPTONIC TENSOR (α^2-ORDER)



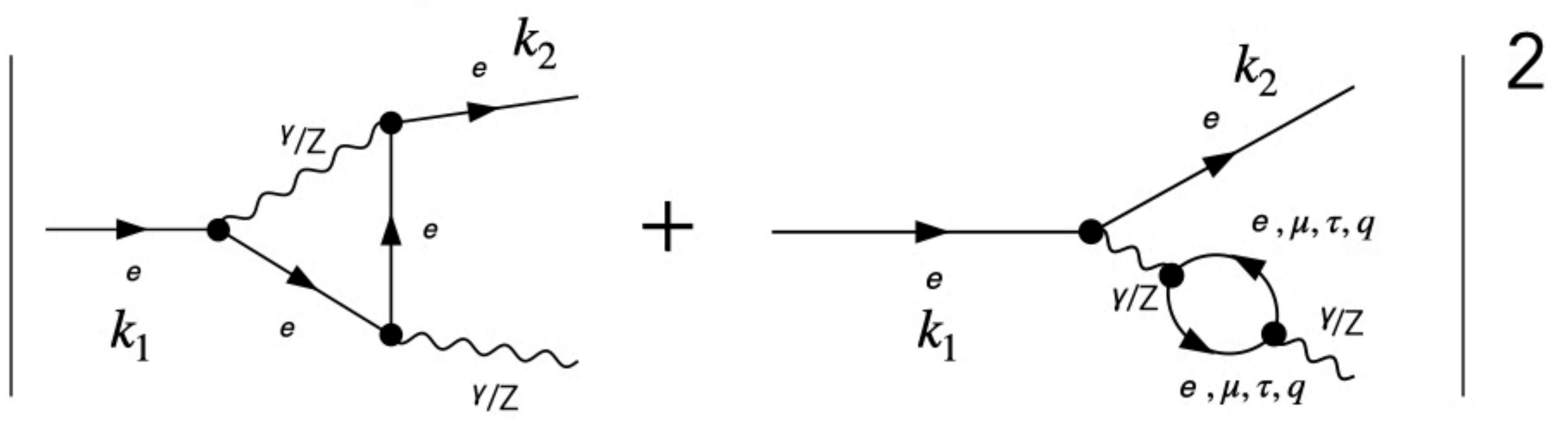
- The **NLO** leptonic tensor can be obtained by multiplying tree-level upper diagram with the sum of one-loop level self energy (SE) and triangular diagrams.

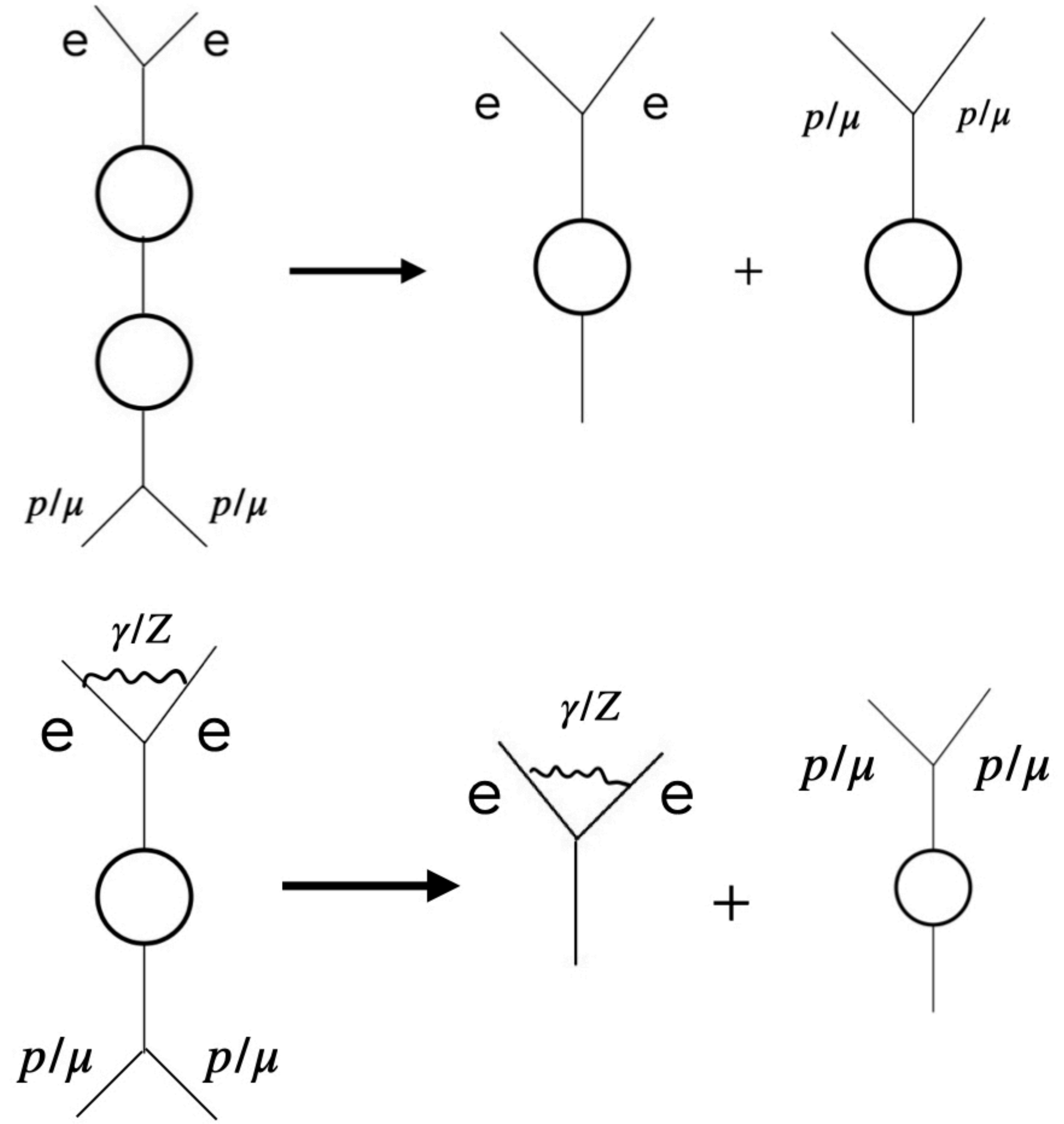
$$L_{\mu\nu}^{NLO} = (m_1)g_{\mu\nu} + (m_2)k_{1\nu}k_{2\mu} + (m_3)k_{1\mu}k_{2\nu} + (m_4)k_{1\mu}k_{1\nu} + (m_5)k_{2\mu}k_{2\nu} + \ldots..$$

Where $m_{1,2,3...}$ are leptonic structure functions which depend on the momentum transfer "t" and written in terms of Passarino-Veltman integral functions. We used LoopTools Mathematica package to calculate them.

- In total 307 graphs SE and triangular graphs.

NEW RESULTS: **QED AND ELECTROWEAK NNLO LEVEL** LEPTONIC TENSOR (α^3-ORDER)

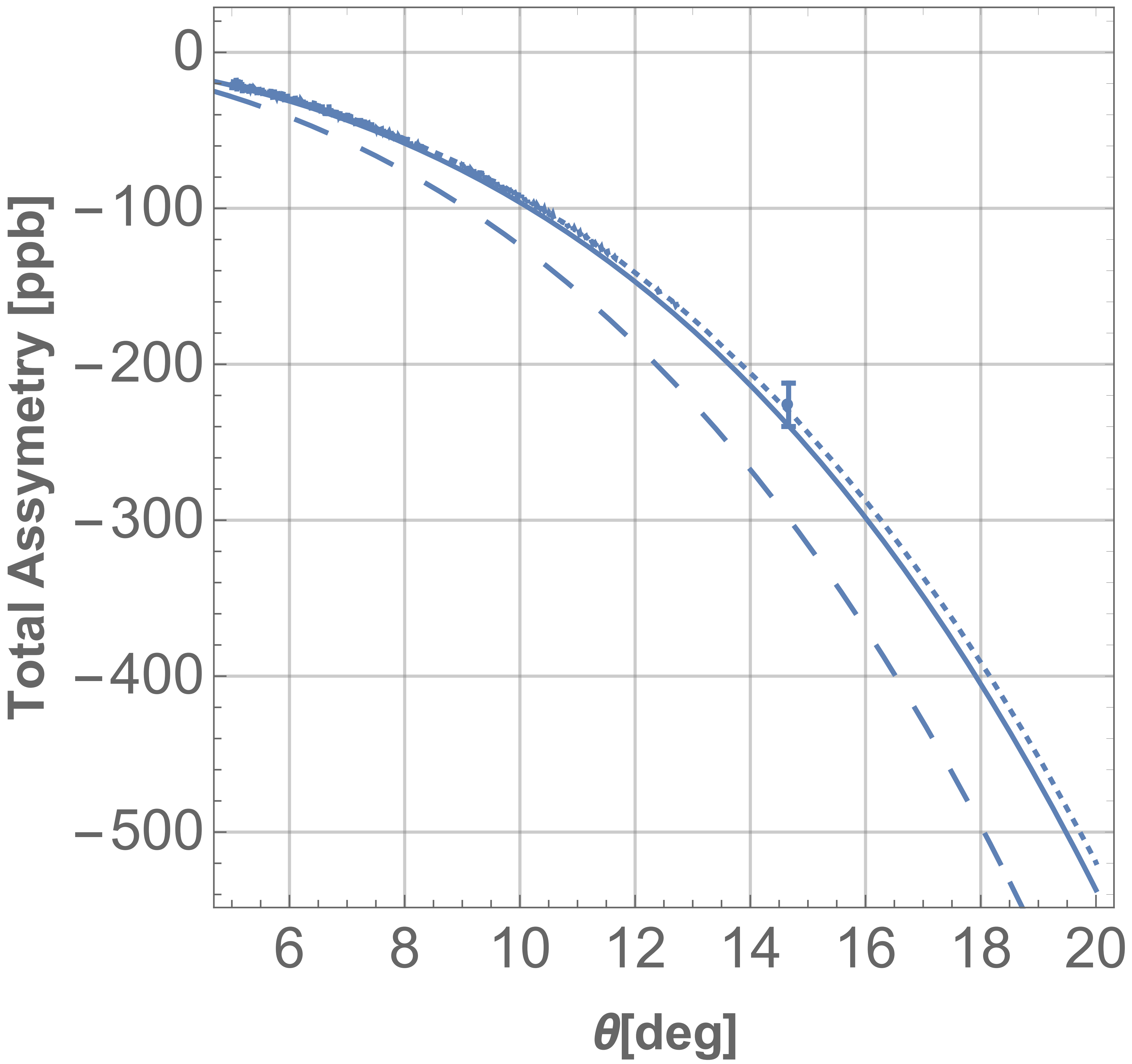




$$L_{\mu\nu}^{NNLO} = (n_1)g_{\mu\nu} + (n_2)k_{1\nu}k_{2\mu} + (n_3)k_{1\mu}k_{2\nu} + (n_4)k_{1\mu}k_{1\nu} + (n_5)k_{2\mu}k_{2\nu} + \ldots$$

- FEYNARTS and FORMCALC as base languages to calculate leptonic tensor structure functions.
- Kept the mass of electron.

Tree level, NLO and NNLO level A_{PV} for $e^{-}p$ scattering versus θ_{CM} using QWEAK kinematics



$(e^{-}p)$ Tree level, NLO and NNLO level A_{PV} versus θ_{CM}

$(E_{beam} = 1.16\ GeV,\ \theta_{CM} = 14.6^{0})$

- - - - Tree $A_{PV} \sim -298.9\ ppb$

—— NLO $A_{PV} \sim -239\ ppb$

...... NNLO $A_{PV} \sim -230\ ppb$

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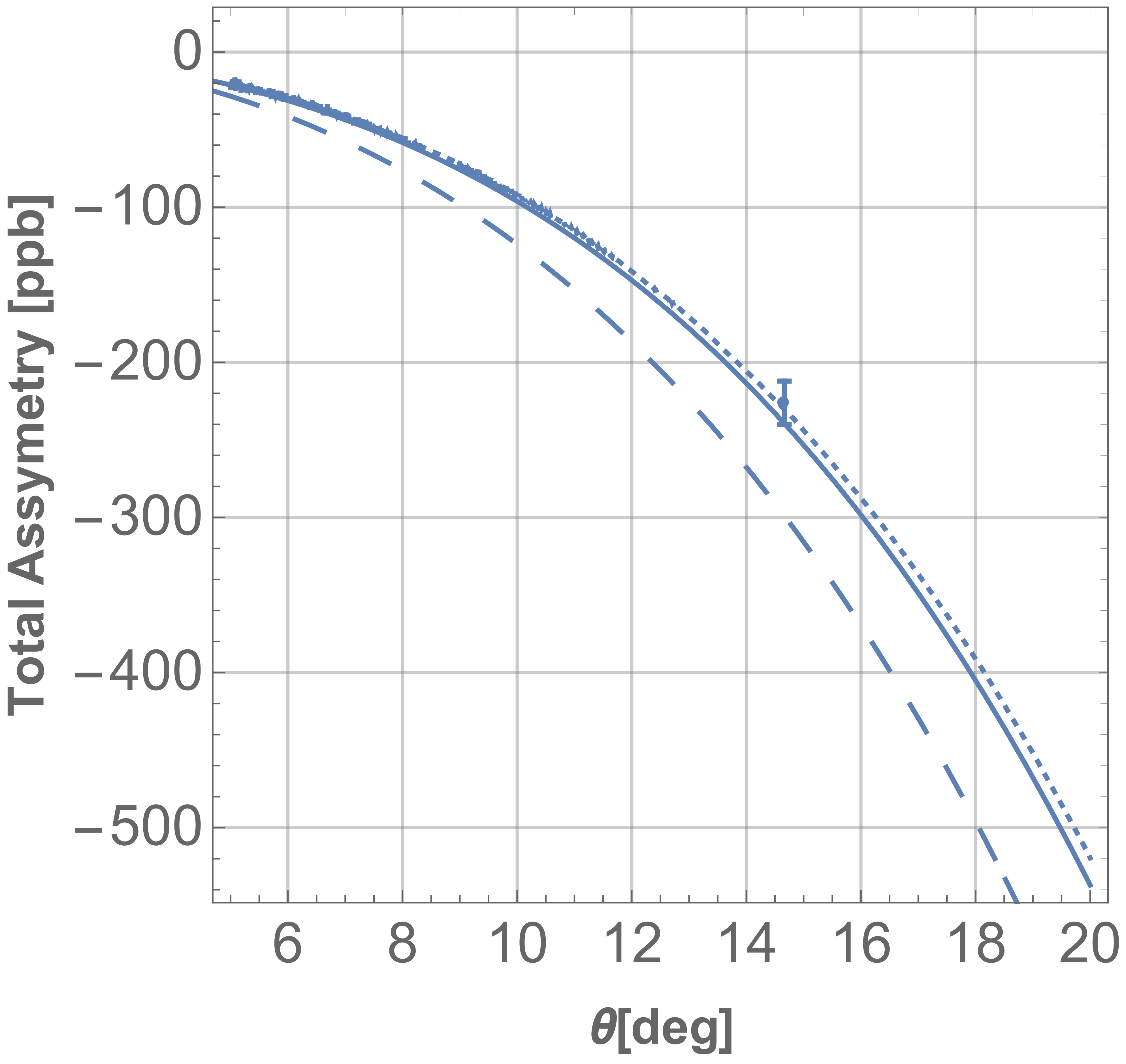
Tree level, NLO and NNLO level A_{PV} for $e^{-}p$ scattering versus θ_{CM} using QWEAK kinematics



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QWEAK Measured

$\sim -226.5 \pm 7.3(statistical) \pm 5.8(systematic)\ ppb$

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NLO AND NNLO LEVEL CORRECTION FACTORS

- The correction factors depend upon the scattering angle θ which appears in momentum transfer as

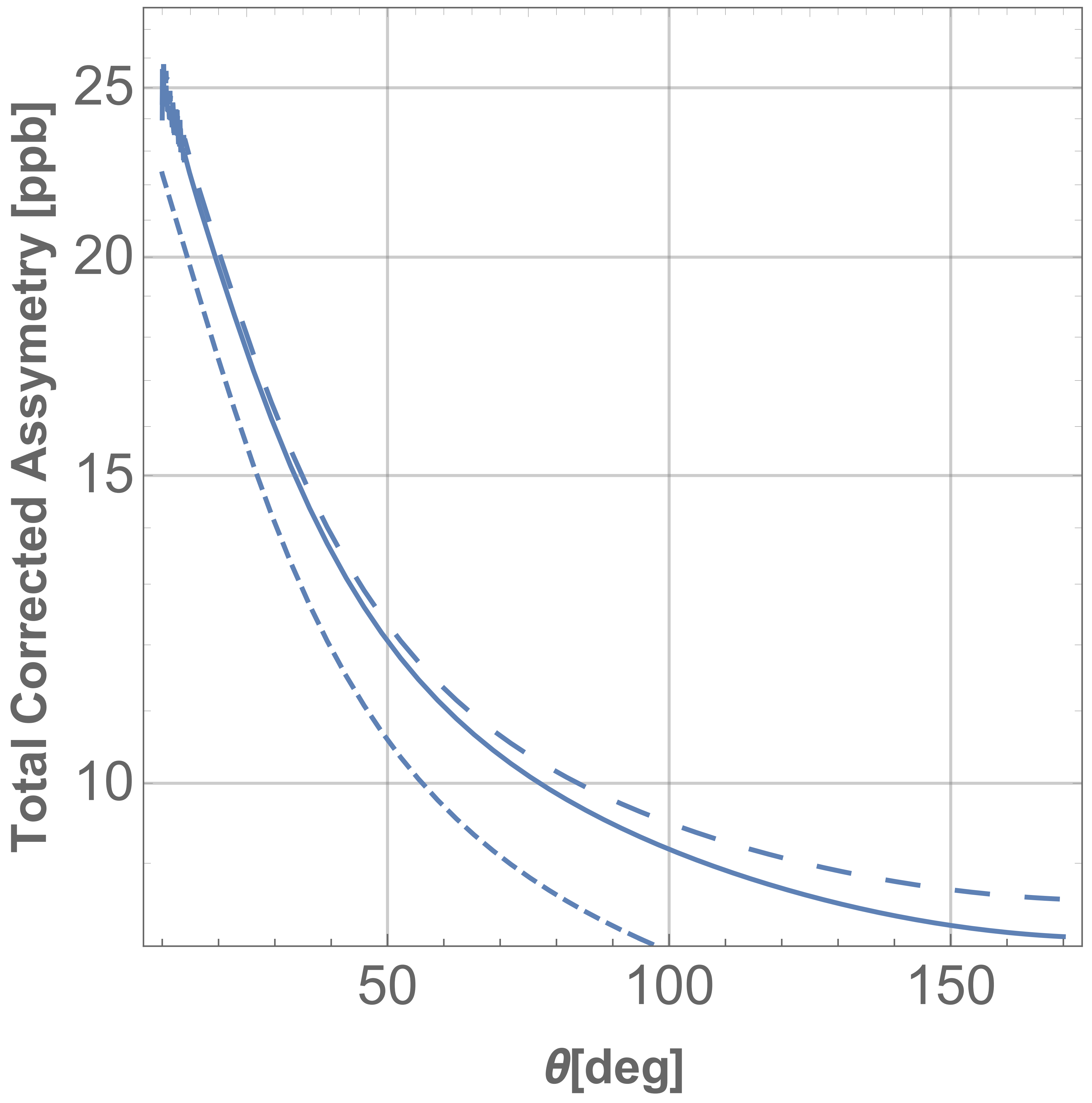
$$t = (k_2 - k_1)^2 = -2\ p_{in}^2\ [1 - Cos(\theta)]$$

where,

$$p_{in} = p_{out}$$

$$Corrected\ A_{PV}\% = \left(\frac{Tree\ level\ A_{PV} - (NLO, NNLO) A_{PV}}{Tree\ level\ A_{PV}} \right) \times 100$$

NLO and NNLO level Corrected A_{PV} Asymmetry with Correction factors in percentage for QWEAK kinematics



NLO and NNLO level Corrected A_{PV} versus θ_{CM}

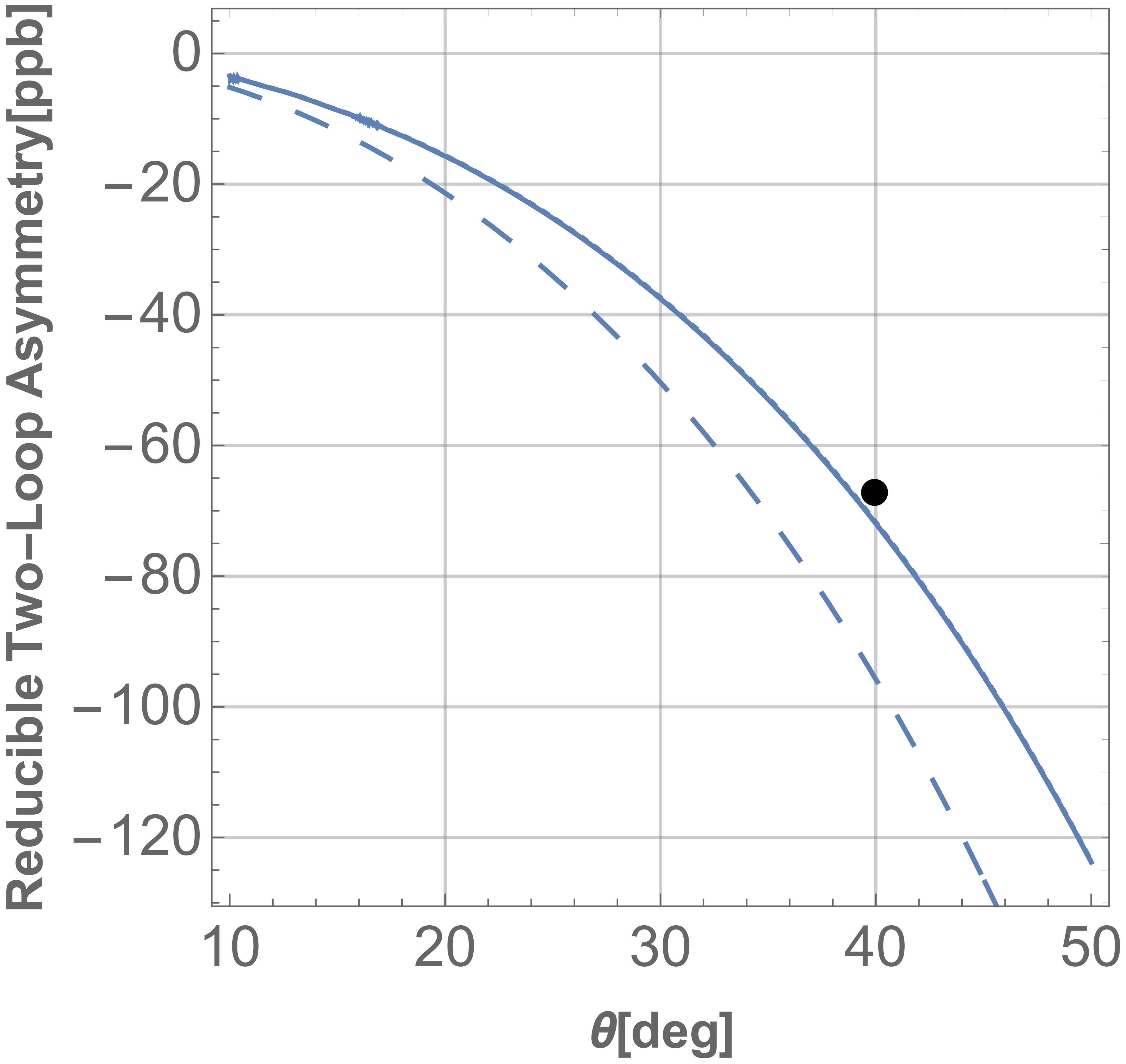
$(\theta = 14.6^0)$

.... NLO level Corrected A_{PV} $= 19.9\ \%$

—— Quadratic level Corrected $A_{PV} = 22.5\ \%$

- - - - Total Corrected A_{PV} $= 22.9\,\%$

Tree level, NLO and NNLO level A_{PV} for $e^{-}p$ scattering versus θ_{CM} using P2 kinematics



$(E_{beam} = 155\ MeV,\ \theta_{CM} = 39.97^0)$

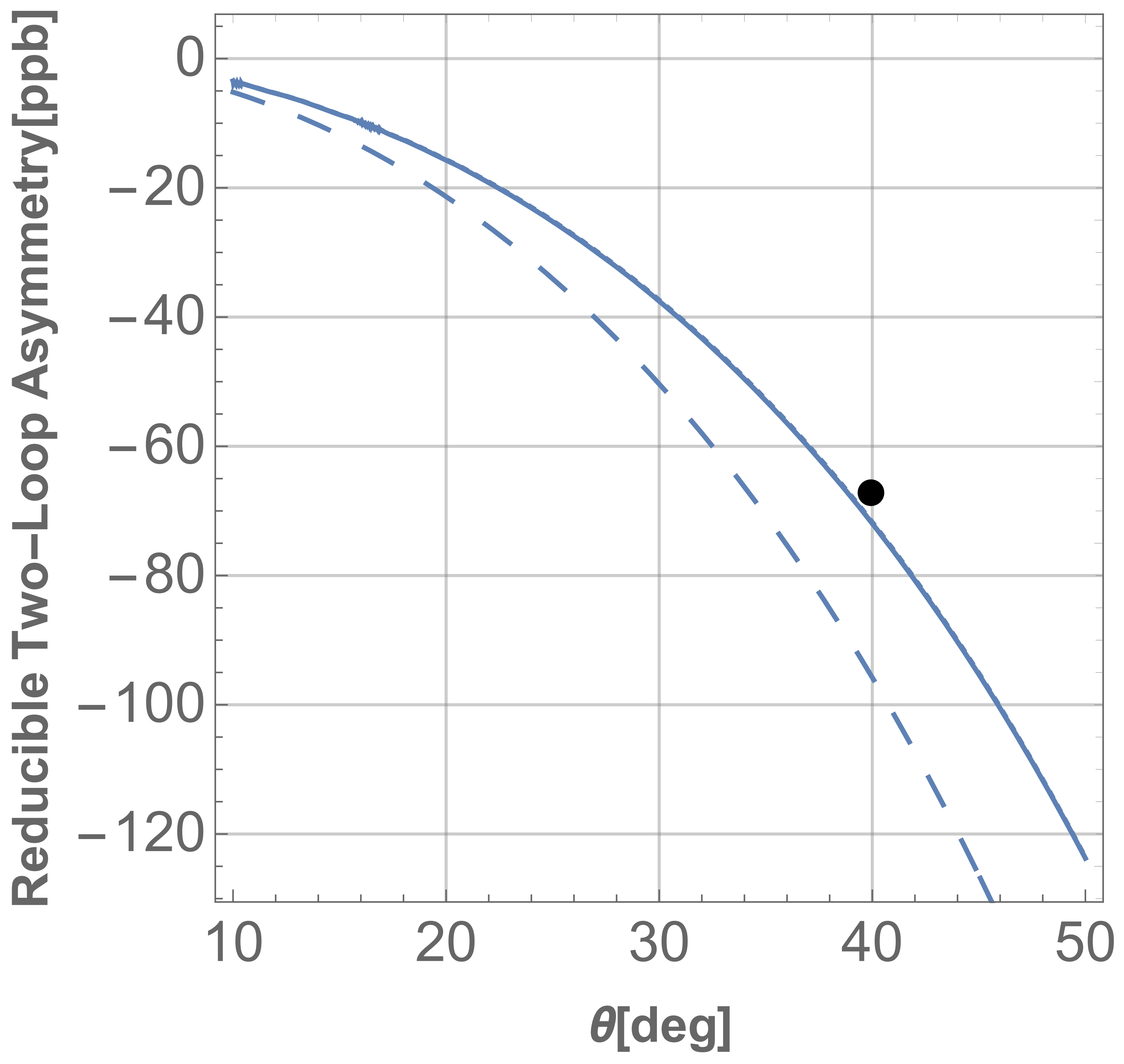
- - - - Tree $A_{PV} \sim -95.6\ ppb$

—— NLO $A_{PV} \sim -71.81\ ppb$

······· NNLO (Reducible Two loop) $A_{PV} \sim -71.6\ ppb$

$(e^{-}p)$ Tree level, NLO and NNLO level A_{PV} versus θ_{CM}

Tree level, NLO and NNLO level A_{PV} for $e^{-}p$ scattering versus θ_{CM} using P2 kinematics



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······· NNLO (Reducible Two loop) $A_{PV} \sim -71.6\ ppb$

P2 Proposed $A_{PV} \sim -67.34\ ppb$

RESULTS:

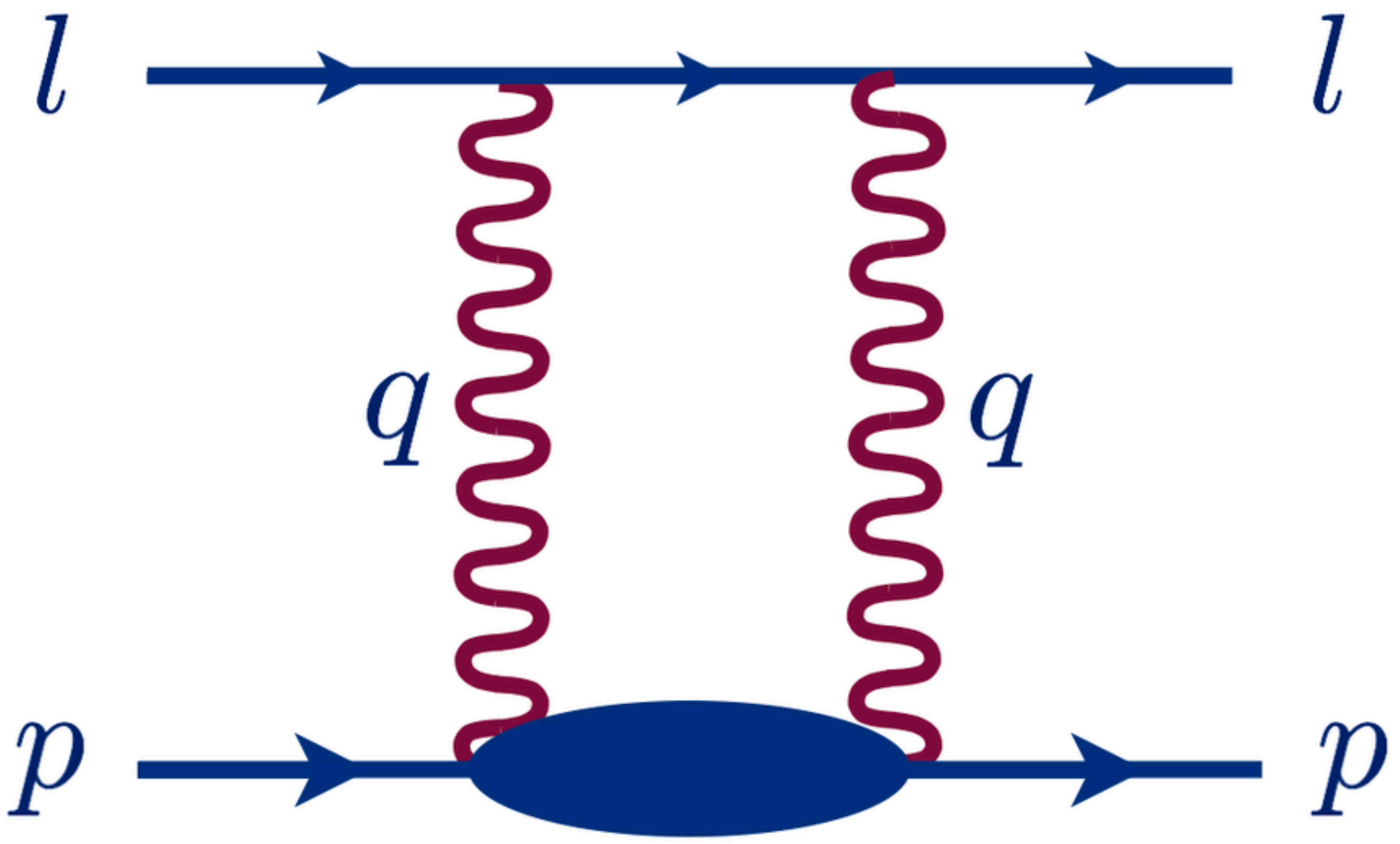
- For completeness, work in progress to include soft and hard photon bremsstrahlung cross sections in the results.
- Next goal is to consider the polarized proton target and study the A_{PV} effects in electron-proton scattering.
- We make predictions for the e^-p NNLO level radiative corrections. These theoretical predictions will be important for many experimental programs such as QWEAK, P2, EIC and MOLLER (background studies) searching for physics beyond the Standard Model at the precision frontier.

- We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC) for this project.

REFERENCES FOR BOX DIAGRAMS

[1] M. Gorchtein, Phys. Rev. C **73**, 055201 (2006)

[2] Peter G. Blunden et al., Physical Review Letters 91(14)



Backup Slides

ELECTROWEAK LEPTONIC TENSOR STRUCTURE FUNCTIONS

- In case of **tree level** polarized e^-p scattering:
  - With photon (γ) as a mediator → **Five** leptonic tensors

    $g^{\mu\nu},\ k_2^\mu k_1^\nu,\ k_1^\mu k_2^\nu,\ \epsilon^{s_1\mu\nu k_1},\ \epsilon^{s_1\mu\nu k_2}$

    where s_1 → helicity reference vector of the incoming electron.
  - With Z boson or γZ mixing → **Eight** leptonic tensors

    $g^{\mu\nu},\ k_2^\mu s_1^\nu,\ k_2^\nu s_1^\mu,\ {k_1}^\mu {k_2}^\nu,\ k_2^\mu k_1^\nu,\ \epsilon^{s_1\mu\nu k_1},\ \epsilon^{s_1\mu\nu k_2},\ \epsilon^{\mu\nu k_1 k_2}$

NLO LEVEL AND QUADRATIC LEVEL LEPTONIC TENSOR STRUCTURE FUNCTIONS

- In case of **one loop level** polarized $e^{-}p$ scattering:
  - With photon (γ), Z boson or γZ mixing → **19** leptonic tensors
- In case of **quadratic level** polarized $e^{-}p$ scattering:
  - With photon (γ), Z boson or γZ mixing → **21** leptonic tensors