Covariant framework to parametrize realistic deuteron wave functions

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Introduction

- **Original idea**: a covariant and separable but non-local model of nucleon-nucleon interactions.
  - Solve for deuteron from Bethe-Salpeter equation.
  - Calculate deuteron observables in manifestly covariant way.
  - Get generalized parton distributions that obey polynomiality.

- **Modified idea**: the formalism of the original idea can encode approximate parametrization of **realistic wave functions**.
  - Get **manifest covariance** (and GPD polynomiality) with existing, precision wave functions!
  - I use Argonne V18 as an example.

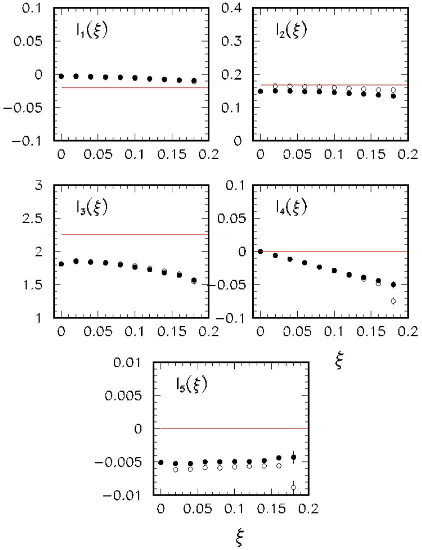
- I'll explain the original idea first, and then how I adapted the framework to get covariant results from AV18.

Why covariance matters

- Generalized parton distributions exhibit **polynomiality**.

$$\int \mathrm{d}x\, x H_1(x,\xi,t) = \mathcal{G}_1(t) + \xi^2 \mathcal{G}_3(t) \qquad \text{etc.}$$

  - Required for unambiguous extraction of energy-momentum tensor from GPDs.
- Polynomiality requires covariance.
  - X. Ji, J. Phys. G24 (1998) 1181
- Finite Fock expansion (standard method) violates covariance.
  - Example: landmark calculation of Cano and Pire EPJA 19 (2004) 423



Non-local Lagrangian

- Adapted from **non-local NJL model**.
  - Bowler & Birse, Nucl. Phys. A582 (1995) 655
  - Modified to be a nucleon-nucleon interaction.
- V and T currents in *isosinglet* channel:

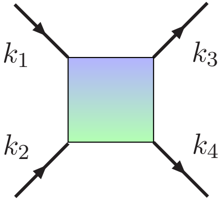
$$B_V^\mu(x) = \frac{1}{2}\int \mathrm{d}^4z\, f(z)\psi^\intercal\left(z+\frac{z}{2}\right)C^{-1}\tau_2\gamma^\mu\psi\left(z-\frac{z}{2}\right)$$
$$B_T^{\mu\nu}(x) = \frac{1}{2}\int \mathrm{d}^4z\, f(z)\psi^\intercal\left(z+\frac{z}{2}\right)C^{-1}\tau_2\,\mathrm{i}\sigma^{\mu\nu}\psi\left(z-\frac{z}{2}\right)$$

  - $f(z)$ a spacetime form-factor; regulates UV divergences.
  - C is charge conjugation matrix.
  - τ_2 isospin matrix.
- Interaction Lagrangian:

$$\mathscr{L}_I = g_V B_V^\mu (B_{V\mu})^* + \frac{1}{2} g_T B_T^{\mu\nu}(B_{T\mu\nu})^*$$

Kernel

- Momentum-space Feynman rule for interactions:



$$= \left\{ g_V \gamma^\mu C \otimes C^{-1} \gamma_\mu + \frac{g_T}{2} \sigma^{\mu\nu} C \otimes C^{-1} \sigma_{\mu\nu} \right\} \widetilde{f}(k_1 - k_2) \widetilde{f}(k_3 - k_4)$$

  - **Separable interaction**: initial & final momentum dependence factorize.
  - (isospin dependence suppressed to compactify formula)
- $\widetilde{f}(k)$ is Fourier transform of $f(z)$, chosen:

$$\widetilde{f}(k) \equiv \frac{\Lambda}{k^2 - \Lambda^2 + \mathrm{i}0}$$

  - Λ is the regulator scale (non-locality scale).

- Kernel encodes channels with multiple quantum numbers:

$$\gamma^\mu C \otimes C^{-1}\gamma_\mu = \left(\gamma^\mu - \frac{\not{p}p^\mu}{p^2}\right) C \otimes C^{-1} \left(\gamma_\mu - \frac{\not{p}p_\mu}{p^2}\right) + \frac{1}{p^2}\not{p}C \otimes C^{-1}\not{p}$$

spin-one spin-zero

$$\sigma^{\mu\nu} C \otimes C^{-1}\sigma_{\mu\nu} = \frac{1}{p^2}\sigma^{\mu p} C \otimes C^{-1}\sigma_{\mu p} + \left(\sigma^{\mu\nu} - \frac{\sigma^{\mu p}p^\nu - \sigma^{\nu p}p^\mu}{p^2}\right) C \otimes C^{-1} \left(\sigma^{\mu\nu} - \frac{\sigma^{\mu p}p^\nu - \sigma^{\nu p}p^\mu}{p^2}\right)$$

even parity odd parity

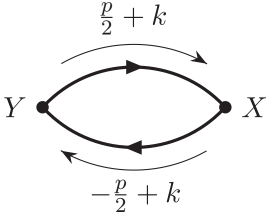
  - p is center-of-mass momentum (deuteron momentum)
- Need only structures with deuteron quantum numbers:

$$\gamma_V^\mu \equiv \gamma^\mu - \frac{\not{p}p^\mu}{p^2} \qquad \gamma_T^\mu \equiv \frac{\mathrm{i}\sigma_{\mu p}}{\sqrt{p^2}}$$

  - Other structures fully decouple in the T-matrix equation!

- Bubble diagrams defined via:

$$-\,\mathrm{i}\left(g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^{2}}\right)\Pi_{XY}(p^{2}) =$$

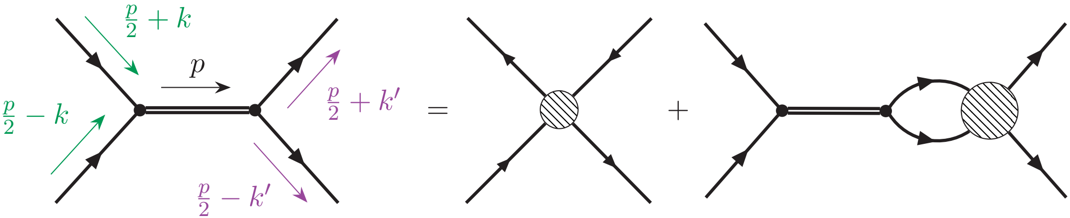


  - Either γ_V^μ or γ_T^μ can be on either end.
  - The regulator $\widetilde{f}(k)$ appears twice inside loop integral—makes it UV finite.
- Can define bubble matrix:

$$\Pi = \begin{bmatrix} \Pi_{VV}(p^2) & \Pi_{VT}(p^2) \\ \Pi_{TV}(p^2) & \Pi_{TT}(p^2) \end{bmatrix}$$

  - Essential ingredient in calculations to follow.

- Bethe-Salpeter equation (BSE) for T-matrix given by:



- **Separability** of interaction permits a simple matrix form:

$$T = G - G\Pi T$$

  - Actual T-matrix related to simplified matrix via:

$$\mathcal{T}(p,k,k') = \frac{\Lambda}{k^2-\Lambda^2}\frac{\Lambda}{k'^2-\Lambda^2}\left\{T_{11}\gamma_V\otimes\gamma_V + T_{12}\gamma_V\otimes\gamma_T + T_{21}\gamma_T\otimes\gamma_V + T_{22}\gamma_T\otimes\gamma_T\right\}$$

  - Simplified kernel matrix:

$$G = \begin{bmatrix} g_V & 0 \\ 0 & g_T \end{bmatrix}$$

Deuteron bound state pole

- T-matrix solution given by:

$$T = (1 + G\Pi)^{-1} G$$

- Deuteron bound state pole exists where:

$$\det(1 + G\Pi) = 0$$

  - Use physical deuteron mass to fix g_V in terms of Λ and g_T.
- Residues at this pole give reduced form of deuteron vertex:

$$T(p^2 \approx M_{\rm D}^2) \approx -\frac{1}{p^2 - M_{\rm D}^2} \begin{bmatrix} \alpha^2 & \alpha\beta \\ \alpha\beta & \beta^2 \end{bmatrix}$$

  - α and β are coefficients in deuteron Bethe-Salpeter vertex.
  - The k and k' dependence is fixed and **separable**.

- The result of all this is a deuteron Bethe-Salpeter vertex:

$$\Gamma_{\mathrm{D}}^{\mu}(p,k) = \frac{\Lambda}{k^2 - \Lambda^2 + \mathrm{i}0}\Big\{\alpha\gamma_V^{\mu} + \beta\gamma_T^{\mu}\Big\}C\tau_2$$

  - Simple k dependence fixed by separable interaction.
  - Can be used to **covariantly** calculate all sorts of observables.
- Relationship to fundamental model parameters:

$$(\Lambda, g_V, g_T) \to (M_{\mathrm{D}}, \alpha, \beta)$$

  - Eliminate one model parameter by fixing M_{D} to empirical value.
  - Could fix other parameters via observables, e.g., charge radius & quadrupole moment.
- A curious thing happens if we look at the non-relativistic limit …

Non-relativistic reduction

- Non-relativistic, momentum-space wave function:

$$\psi_{\text{NR}}(\boldsymbol{k},\lambda) \sim \frac{-1}{\sqrt{8M_{\text{D}}}} \frac{\bar{u}(\boldsymbol{k},s_1)(\Gamma_{\text{D}}\cdot\varepsilon_\lambda)\bar{u}^{\mathsf{T}}(-\boldsymbol{k},s_2)}{\boldsymbol{k}^2+m\epsilon_{\text{D}}}$$

- Working out the Dirac matrix algebra and using the limit $\boldsymbol{k}^2 \ll m^2$ will give:

$$\psi_{\text{NR}}(\boldsymbol{k},\lambda) = 4\pi\Big\{u(k)Y^{\lambda}_{101}(\hat{k}) + w(k)Y^{\lambda}_{121}(\hat{k})\Big\}$$

$$u(k) = \sum_{j=0}^{1} \frac{C_j}{\boldsymbol{k}^2+B_j^2}$$

$$w(k) = \sum_{j=0}^{1} \frac{D_j}{\boldsymbol{k}^2+B_j^2}$$

$$B_0 = \sqrt{m\epsilon_{\text{D}}}$$

$$B_1 = \Lambda$$

$$C_0 = \frac{m}{\sqrt{4\pi M_{\text{D}}}} \frac{\Lambda}{\Lambda^2-\epsilon_{\text{D}}m}\left(\alpha+\beta-\frac{(\alpha-\beta)\epsilon_{\text{D}}m}{12m^2}\right)$$

$$C_1 = -\frac{m}{\sqrt{4\pi M_{\text{D}}}} \frac{\Lambda}{\Lambda^2-\epsilon_{\text{D}}m}\left(\alpha+\beta-\frac{(\alpha-\beta)\Lambda^2}{12m^2}\right)$$

$$D_0 = -\frac{m}{\sqrt{4\pi M_{\text{D}}}} \frac{\Lambda}{\Lambda^2-\epsilon_{\text{D}}m} \frac{\sqrt{2}(\alpha-\beta)\epsilon_{\text{D}}m}{6m^2}$$

$$D_1 = \frac{m}{\sqrt{4\pi M_{\text{D}}}} \frac{\Lambda}{\Lambda^2-\epsilon_{\text{D}}m} \frac{\sqrt{2}(\alpha-\beta)\Lambda^2}{6m^2}$$

  - $u(k)$ is S-wave, $w(k)$ is D-wave.

Approximating non-relativistic wave functions

- The curious thing is that this is a standard parametrization for deuteron wave functions!

$$u(k) = \sum_{j=0}^{N} \frac{C_j}{\boldsymbol{k}^2 + B_j^2} \qquad w(k) = \sum_{j=0}^{N} \frac{D_j}{\boldsymbol{k}^2 + B_j^2}$$

  - First used by Paris group, Lacombe *et al.*, PLB 101 (1981) 139
  - Typically $N > 1$ of course.
- One requires $B_0 = \sqrt{\epsilon_{\mathrm{D}} m}$ to get the right asymptotic behavior. (**Check!**)
- One also requires the following sum rules for correct behavior at the origin:

$$\sum_{j=0}^{N} C_j = \sum_{j=0}^{N} D_j = \sum_{j=0}^{N} D_j B_j^{-2} = \sum_{j=0}^{N} D_j B_j^2 = 0$$

  - Model as given **fails** unless $\alpha = \beta$, meaning no D wave.
- But we can fix this by having N copies of the separable kernel!

Making N copies of the separable kernel

- Just have N copies of the original separable interaction with different Λ_n:

$$\mathcal{K}(k,k') = \sum_{n=1}^{N} \frac{\Lambda_n}{k^2 - {\Lambda_n}^2 + \mathrm{i}0} \frac{\Lambda_n}{k'^2 - {\Lambda_n}^2 + \mathrm{i}0} \left\{ g_{Vn} \gamma^\mu C \otimes C^{-1} \gamma_\mu + \frac{g_{Tn}}{2} \sigma^{\mu\nu} C \otimes C^{-1} \sigma_{\mu\nu} \right\}$$

  - Kernel now has $3N$ parameters: $\{\Lambda_n, g_{Vn}, g_{Tn} | n \in \{1,\ldots,N\}\}$.
- Simplified forms of kernel, T-matrix, and bubble are now all $2N \times 2N$ matrices.
  - The different Λ_n mix, but the T-matrix equation is still separable and can be solved algebraically.
- Deuteron vertex is now:

$$\Gamma^\mu_{\mathrm{D}}(p,k) = \sum_{n=1}^{N} \frac{\Lambda_n}{k^2 - {\Lambda_n}^2 + \mathrm{i}0} \left\{ \alpha_n \gamma^\mu_V + \beta_n \gamma^\mu_T \right\} C \tau_2$$

  - Non-relativistic reduction now has $N+1$ terms in S and D waves!

Using the separable kernel as a parametrization

- The popular non-relativistic parametrization is:

$$u(k) = \sum_{j=0}^{N} \frac{C_j}{\boldsymbol{k}^2 + B_j^2} \qquad w(k) = \sum_{j=0}^{N} \frac{D_j}{\boldsymbol{k}^2 + B_j^2}$$

  - Here $B_0 = \sqrt{\epsilon_{\mathrm{D}} m}$ and $B_n = \Lambda_n$ (for $n > 0$).
- From the kernel coupling strengths:

$$\{g_{Vn}\}, \{g_{Tn}\} \rightarrow \{\alpha_n\}, \{\beta_n\} \rightarrow \{C_j\}, \{D_j\}$$

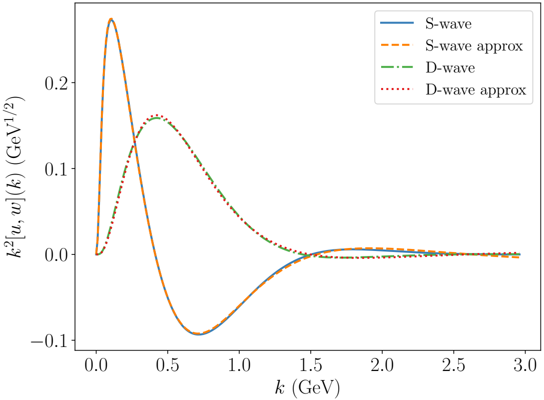
  - Won't fill up a slide with all the formulas (see preprint when it comes out).
- The formulas are linear & invertible!

$$\{C_j\}, \{D_j\} \rightarrow \{\alpha_n\}, \{\beta_n\} \rightarrow \{g_{Vn}\}, \{g_{Tn}\}$$

- So why not **start with** the C_j, D_j and B_j from a **well-established wave function**?
  - Automatically get precision of established wave function.
  - Get correct constraints by starting with $\{B_j, C_j, D_j\}$ that obey them.
  - Guaranteed **Lorentz covariance** from using separable framework.

Approximating Argonne V18

- Example: Argonne V18 fit using $N = 7$.
  - Get $u(k)$ and $w(k)$ from ANL website*.
  - B_n $(n > 0)$ were allowed to float in fit.
  - $B_0 = \sqrt{\epsilon_D m}$.
  - $r \to 0$ constraints were enforced.



- From $\{B_j, C_j, D_j\}$ get $\{\Lambda_n, g_{Vn}, g_{T,n}\}$.
  - See future preprint for numerical values!

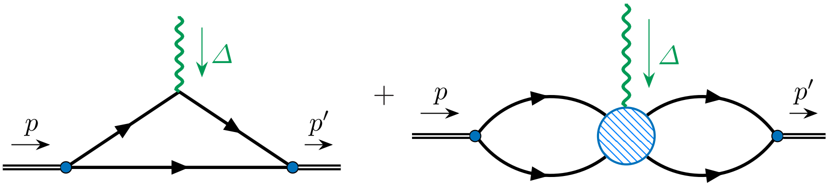
- Now have **covariant Lagrangian** that reproduces AV18 wave function in NR limit!

*: https://www.phy.anl.gov/theory/research/av18/

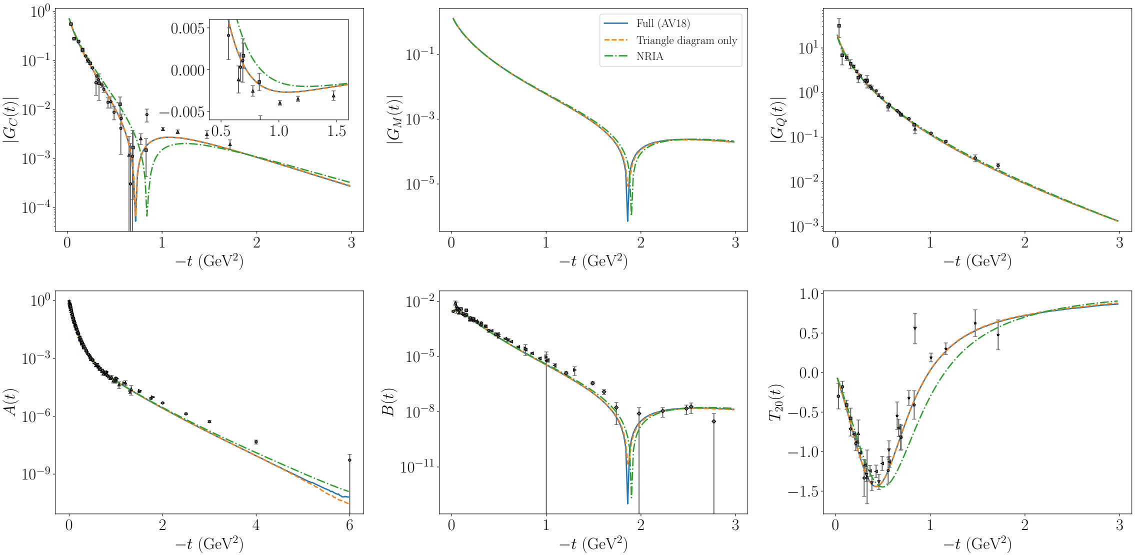
Things to do with this framework

- Electromagnetic form factors (**obtained!**)
- Gravitational form factors (in progress)
  - Manifest covariance helpful here.
  - Previous non-covariant work (AF & Cosyn, PRD) found inconsistencies in EMT components.
- Collinear parton distributions (**obtained!**)
- b_1 structure function (**obtained!**)
- Generalized parton distributions (in progress)
  - GPDs are the **main goal** of this project.
  - Existing deuteron GPDs violate polynomiality.
  - Manifest covariance of this framework *guarantees* polynomiality.

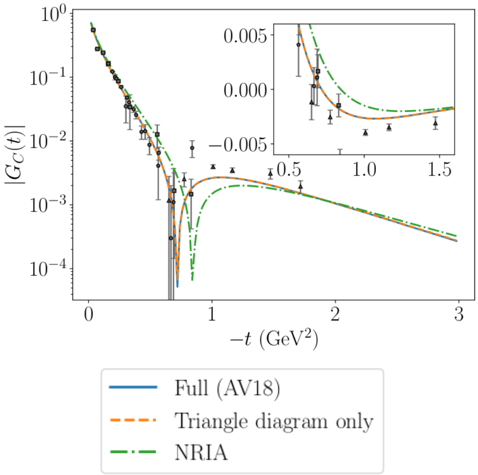
- Sum of triangle and bicycle diagrams



  - Bicycle diagram comes from gauge invariance.
  - Non-local interaction requires Wilson lines.
- Diagrams can be evaluated *exactly* within the present framework!
  - Symbolic algebra program needed though—results are *long* (hundreds of lines of generated Fortran code)
  - Results are covariant too.

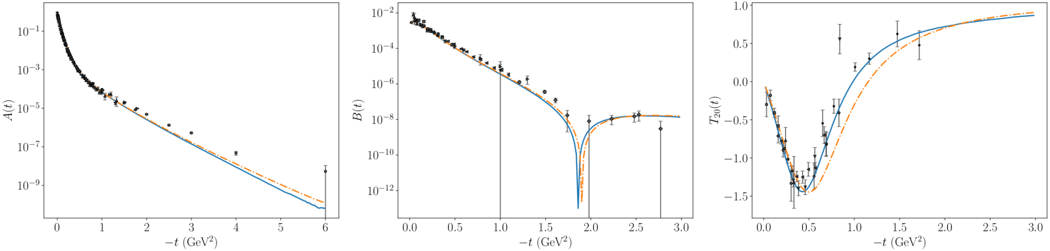


$|G_C(t)|$
$-t$ (GeV^2)
$|G_M(t)|$
Full (AV18)
Triangle diagram only
NRIA
$|G_Q(t)|$
$A(t)$
$B(t)$
$T_{20}(t)$



- *Vast* improvement over non-relativistic impulse approximation (NRIA)!
  - Despite same NR limit.
  - Likely just form relativistic kinematics.
- Bicycle diagram makes negligible contribution.
- Reasonable description of data—sanity check passed for framework!

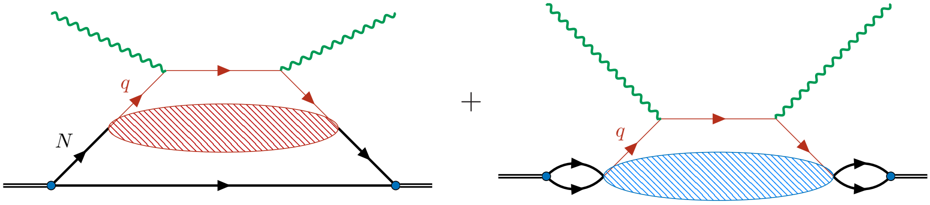
- Good agreement with elastic structure functions



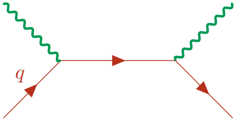
- Static moments a bit large …

	Separable framework	Empirical value
Charge radius	2.156 fm	2.12799 fm
Magnetic moment	0.876 μ_N	0.8574382284 μ_N
Quadrupole moment	0.301 fm^2	0.2859 fm^2

- Sum of **convolution** and **interaction** diagrams:



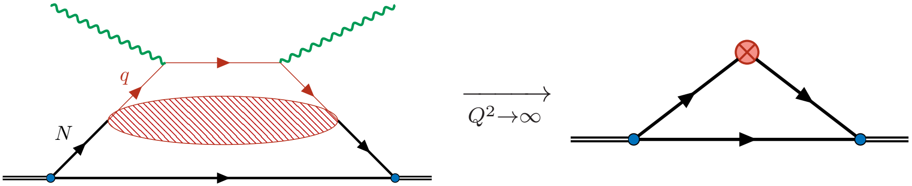
- Diagrams evaluated in the Bjorken limit:



$$\xrightarrow[Q^2\to\infty]{} \sum_q e_q^2 \frac{1}{2} \int \frac{\mathrm{d}z}{2\pi}\, \mathrm{e}^{\mathrm{i}x_d p^+ z^-} \bar{q}\left(-\frac{z^-}{2}\right)\gamma^+ q\left(\frac{z^-}{2}\right)$$

  - Here $0 < x_d < 1$, in contrast to usual normalization.
  - $0 < x_{\mathrm{Bj}} = \frac{M_d}{m_N} x_d < \frac{M_d}{m_N} \approx 2$ is the usual variable.
  - x_d is easier to use in calculations.
  - Compare empirical data in terms of x_{Bj}.

- Effective **triangle diagram** (Bjorken limit):



- Convolution formula results:

$$H_i(x_d, \xi, t, Q^2; \lambda) = \sum_{N=p,n} \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} \left[h_i(y, \xi, t; \lambda) H_N \left(\frac{x_d}{y}, \frac{\xi}{y}, t, Q^2 \right) \right.$$

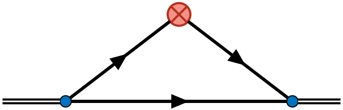
$$\left. + e_i(y, \xi, t; \lambda) E_N \left(\frac{x_d}{y}, \frac{\xi}{y}, t, Q^2 \right) \right]$$

  - Forward limit ($t \to 0$) gives standard PDF convolution:

$$q_d(x_d, Q^2; \lambda) = \sum_{N=p,n} \int_{x_d}^{1} \frac{\mathrm{d}y}{y} f(y; \lambda) q_N \left(\frac{x_d}{y}, Q^2 \right)$$

Light cone density (triangle diagram)

- **Light cone density**: a PDF assuming pointlike nucleons.



$$\xrightarrow[\text{pointlike nucleons}]{} f(y;\lambda)$$

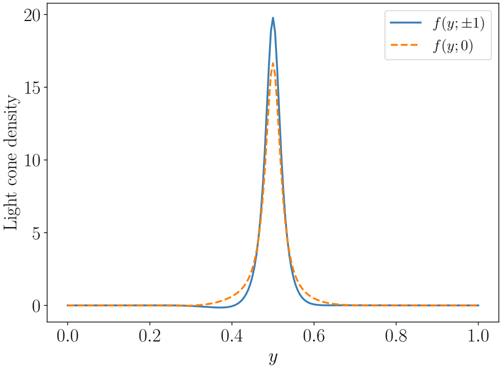
- Nucleon sum rule obeyed:

$$\sum_{N=p,n} \int_0^1 \mathrm{d}y\, f(y;\lambda) = 2$$

- Momentum sum rule violated!

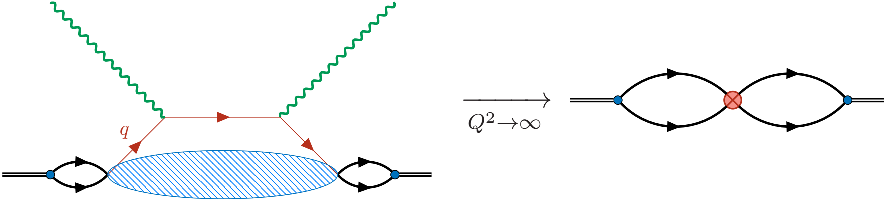
$$\sum_{N=p,n} \int_0^1 \mathrm{d}y\, y f(y;\lambda) = \begin{cases} 1.0036 & : \quad \lambda = \pm 1 \\ 0.9984 & : \quad \lambda = 0 \end{cases}$$

  - **Interaction carries momentum**

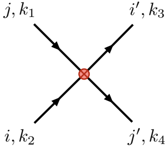


Bicycle diagram

- Effective **bicycle diagram** (Bjorken limit):



- New Feynman rule for operator insertion:



$$= -\frac{1}{2}\sum_X g_X \mathcal{S}\Big\{ \widetilde{h}^+_X(k,0)\big(\delta(xp^+ - k_1^+) + \delta(xp^+ - k_2^+)\big)\widetilde{f}_X(k') - \widetilde{f}_X(k)\widetilde{h}^+_X(k',0)\big(\delta(xp^+ - k_3^+) + \delta(xp^+ - k_4^+)\big)\Big\}\big(\gamma^\nu_X C\tau_2\big)_{i'j'}\big(C^{-1}\tau_2\overline{\gamma}_{X\nu}\big)_{ij}$$

  - …assuming pointlike nucleon.
  - Use convolution formula to fold in NN vertex structure???

- Mellin moment of non-local correlator:

$$\int \mathrm{d}x_d \, x_d^{s-1} \overline{q}\left(-\frac{z^-}{2}\right) \frac{\gamma^+}{2} q\left(\frac{z^-}{2}\right) = \frac{1}{(2p^+)^s} \overline{q}(0)\gamma^+ (\mathrm{i}\overleftrightarrow{\partial}^+)^{s-1} q(0)\,.$$

  - Quark interaction with (entirely hypothetical) spin-s gauge field.
  - Operator inserted on the NN kernel.

- How does the non-local kernel interact with a spin-s gauge field?
  - Got the photon insertion rule from a gauge link.
  - Assert invariance under spin-s gauge transforms.
  - Spin-s gauge link???

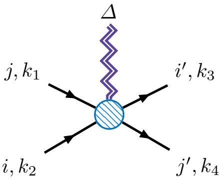
- Higher-spin gauge fields transform like

$$\delta h_{\mu_1\mu_2\ldots\mu_s}(x) = \partial_{\mu_1}\xi_{\mu_2\ldots\mu_s}(x) + \partial_{\mu_2}\xi_{\mu_1\ldots\mu_s}(x) + \ldots + \partial_{\mu_s}\xi_{\mu_1\ldots\mu_{s-1}}(x)$$

  with $\xi_{\mu_1\ldots}(x)$ totally symmetric.
  - Fronsdal, PRD (1978); de Wit & Freedman, PRD (1980)
  - Not compatible with Poincaré symmetry (Coleman & Mandula, Phys. Rev. (1967))
- My *guess* for gauge link (would love a proper derivation!):

$$\psi(x+y) \to \exp\left\{\mathrm{i}\int_x^{x+y} \mathrm{d}z^{\mu_1}\, h_{\mu_1\ldots\mu_s}(z)\, \mathrm{i}\partial^{\mu_2}\ldots\mathrm{i}\partial^{\mu_s}\right\}\psi(x+y)$$

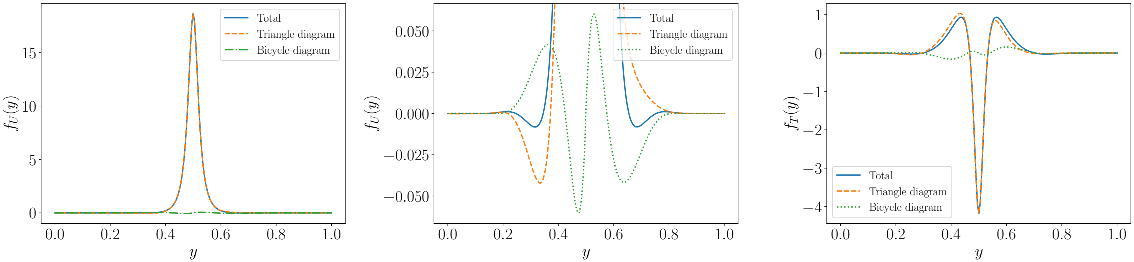
  - Gives EM case for $s = 1$.
  - Gives linearized gravity case for $s = 2$ (Green, PRD (2008); S. Wikeley's PhD thesis).
- Resulting Feynman rule:



$$= \begin{aligned} &-\mathrm{i}\textstyle\sum_X g_X \mathcal{S}\Big\{\widetilde{h}_X^{\mu_1}(k,\Delta)\big(k_1^{\mu_2}k_1^{\mu_3}\ldots k_1^{\mu_s} + k_2^{\mu_2}k_2^{\mu_3}\ldots k_2^{\mu_s}\big)\widetilde{f}_X(k') \\ &-\widetilde{f}_X(k)\widetilde{h}_X^{\mu_1}(k',\Delta)\big(k_3^{\mu_2}k_3^{\mu_3}\ldots k_3^{\mu_s} + k_4^{\mu_2}k_4^{\mu_3}\ldots k_4^{\mu_s}\big)\Big\}\Big(\gamma_X^{\nu}C\tau_2\Big)_{i'j'}\Big(C^{-1}\tau_2\overline{\gamma}_{X\nu}\Big)_{ij} \end{aligned}$$

  - Inverse Mellin transform of $++\ldots$ component gives rule from two slides ago.

Light cone density (triangle+bicycle)



- Momentum sum rule obeyed.
  - Saved by the bicycle diagram!
  - Also makes LCD symmetric.
- Slight negative support.
  - Either a flaw with the framework …
  - …or a feature of renormalization?
    cf. Collins, Rogers & Sato, PRD (2022)

$$f_U(y) = \frac{f(y,0) + f(y,+1) + f(y,-1)}{3}$$

$$f_T(y) = f(y,0) - \frac{f(y,+1) + f(y,-1)}{2}$$

Collinear parton distributions

- Deuteron PDFs via convolution:

$$q_d(x_d, Q^2; \lambda) = \sum_{N=p,n} \int_{x_d}^{1} \frac{\mathrm{d}y}{y}\, q_N\left(\frac{x_d}{y}, Q^2\right) f(y;\lambda)$$

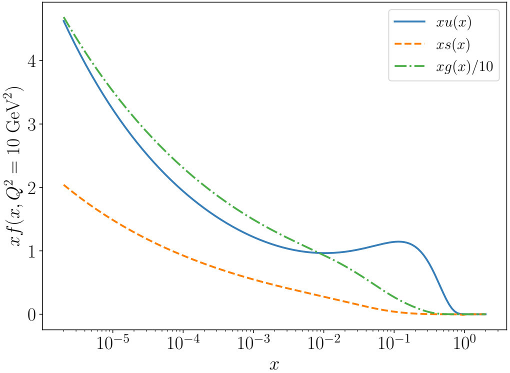
- Use JAM PDFs for nucleon.
  - C. Cocuzza *et al.*, PRD106 (2022) L031502

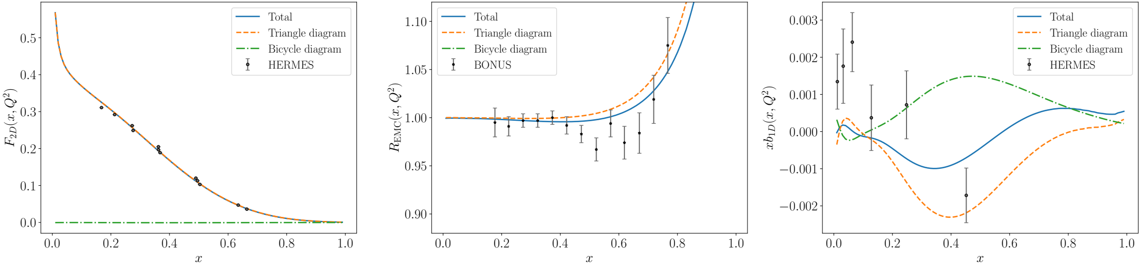
- Same PDF for triangle & bicycle diagrams.

  - Plot in terms of

$$x = x_{\mathrm{Bj}} \equiv \frac{Q^2}{2m_N\nu} = \frac{M_d}{m_N} x_d \approx 2x_d$$

    - This is the *standard* x variable.

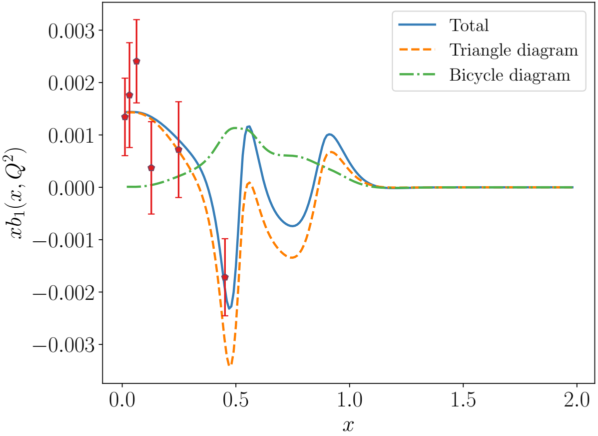




- Use free nucleon PDFs inside both triangle & bicycle diagrams.
  - Use JAM22 PDFs.
- **Unpolarized** $F_{2D}(x_{\mathrm{Bj}}, Q^2)$ structure function.
  - Looks reasonable.
  - But no EMC effect when using free PDFs.
- **Tensor-polarized** $b_{1d}(x_{\mathrm{Bj}}, Q^2)$ structure function looks standard.
  - Total (blue curve) looks similar to Cosyn &al., PRD (2017).
  - Can't explain HERMES b_1 data.

- 2005 HERMES data at low Q^2.
  - From 0.51-4.69 GeV^2.
  - Lower x points at lower Q^2.
  - PRL 95 (2005) 242001
- Partonic picture may not be valid for low-x data here.
- So, maybe fold in Bodek parametrization of nucleon structure functions?
  - Includes resonances at low Q^2.
  - Bodek *et al.*, PRD (1979)
- Something surprising happens when I do this…

Separable model with Bodek structure functions



- Nucleon F_1 at $Q^2 = 2.5$ GeV^2.
  - Bodek *et al.*, PRD (1979)
- Model **was not fit to these data**.
  - Parameters fixed on slide 15.
  - I'm actually surprised by this result!
- Seems too good to be true?
  - Cosyn (PRD (2017)) don't see this.
  - Model $f_T(y)$ responsible?
  - Maybe due to negative support?
  - Specifically needs Bodek F_1.
  - Using PDFs doesn't work.
  - Found no code errors yet.

- **Result is preliminary and subject to double-checking!**

Outlook

- Presented a framework for **covariant calculations** using **realistic deuteron wave functions**.
  - Used Argonne's AV18 wave function as an example.
  - Covariance means GPDs *will obey polynomiality*.
- Reproduced known deuteron properties in this framework
  - Necessary sanity check.
  - Learned an important lesson: **bicycle diagrams** must be accounted for!
- **Maybe** able to explain HERMES $b_1(x, Q^2)$ data.
  - I'm double-checking the result though.
- Much more to be done:
  - Energy momentum tensor and gravitational form factors.
  - **Generalized parton distributions** (the main purpose of this project!)

Thank you for your time!