

RIKEN BNL Research Center

Workshop on Generalized Parton Distributions for Nucleon Tomography in the EIC Era

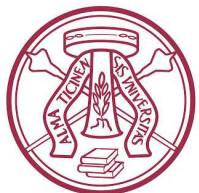
Hosted by Brookhaven National Laboratory
January 17–19, 2024



THE D-TERM FORM FACTOR FROM DISPERSION RELATIONS IN DEEPLY VIRTUAL COMPTON SCATTERING

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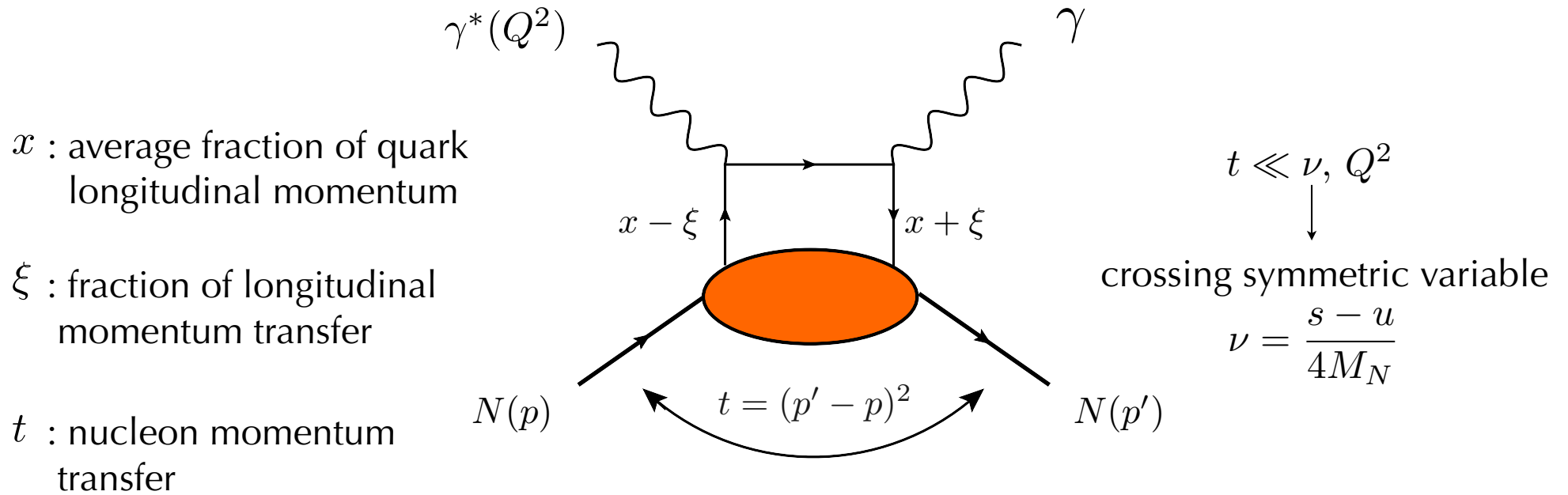
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Outline

- s-channel Dispersion Relations (DRs) for twist-2 amplitudes in terms of GPDs
→ link between subtraction function in s-channel DRs and D-term form factor
- D-term form factor t-channel dispersion relations
- Results and comparison with phenomenological extractions

DVCS at leading twist



DVCS tensor at twist 2: $T^{\mu\nu} = \sum_{i=1}^4 A_i(\nu, t, Q^2) O_i^{\mu\nu}$

unpolarized quark

$$A_1 = \mathcal{H} + \mathcal{E}$$

$$A_2 = -\mathcal{E}$$

long. polarized quark

$$A_3 = \tilde{\mathcal{H}}$$

$$A_4 = \tilde{\mathcal{E}}$$

Twist-2 DVCS amplitudes

$$A_i(\xi, t) = \int_0^1 dx F_i^+(x, \xi, t) \left[\frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right] \quad (i = 1, \dots, 4)$$

- Involve singlet GPDs: $F_i^+ = \{H^+ + E^+, -E^+, \tilde{H}^+, \tilde{E}^+\}$

$$F_i^+(x, \xi, t) = F_i(x, \xi, t) - F_i(-x, \xi, t)$$

- Imaginary part: GPD at $x = \xi$

$$\text{Im } A_i(\xi, t) = -\pi F_i^+(\xi, \xi, t)$$

- Real part involves convolution integral:

$$\text{Re } A_i(\xi, t) = \mathcal{P} \int_0^1 dx F_i^+(x, \xi, t) \left[\frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

*the dependence on Q^2 is implicit

Dispersion relations at fixed t and Q^2

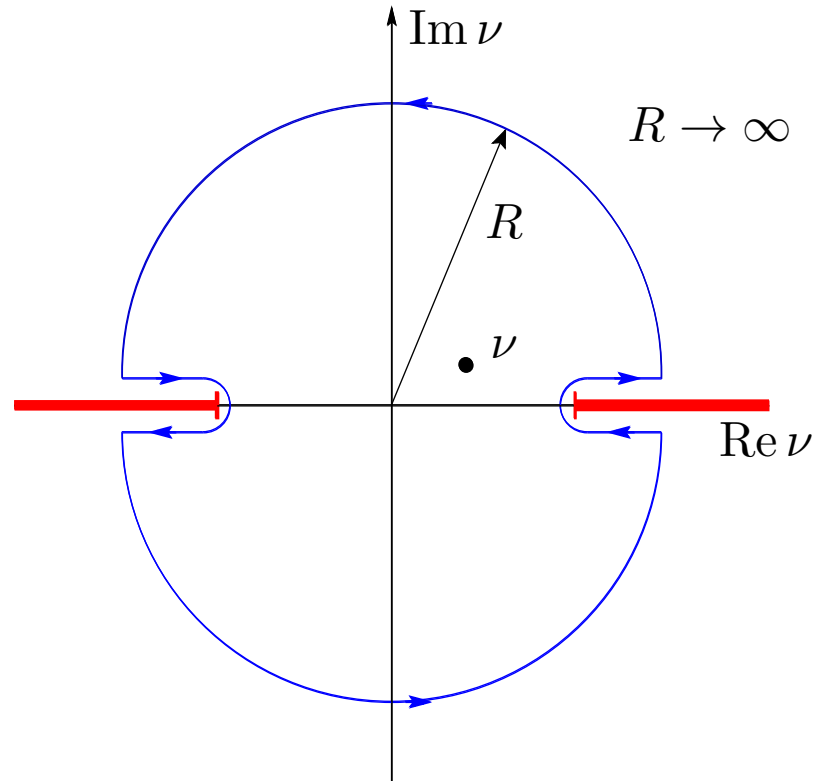
energy variables: $\nu = \frac{s - u}{4M_N} = \frac{Q^2}{4M_N\xi}$ and $\nu' = \frac{Q^2}{4M_Nx}$

$A_i(\nu, t)$: analytical functions in the complex ν plane, with cuts on the real axis

Dispersion relations at fixed t and Q^2

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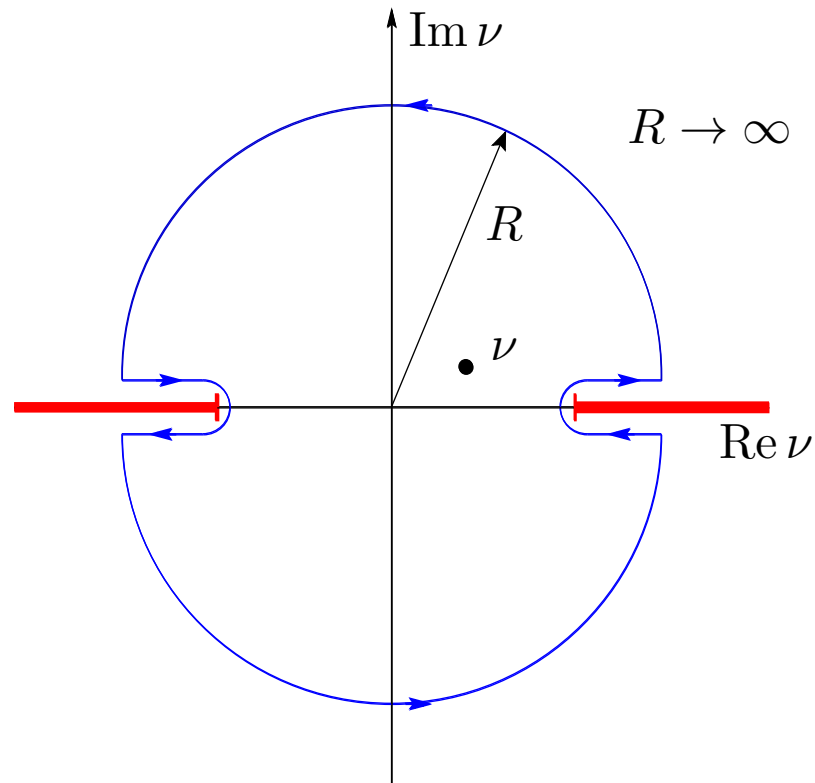
- Cauchy integral formula

$$A_i(\nu, t) = \frac{1}{2\pi i} \oint_C d\nu' \frac{A_i(\nu', t)}{\nu' - \nu}$$

Dispersion relations at fixed t and Q^2

energy variables: $\nu = \frac{s - u}{4M_N} = \frac{Q^2}{4M_N\xi}$ and $\nu' = \frac{Q^2}{4M_Nx}$

$A_i(\nu, t)$: analytical functions in the complex ν plane, with cuts on the real axis



- Cauchy integral formula

$$A_i(\nu, t) = \frac{1}{2\pi i} \oint_C d\nu' \frac{A_i(\nu', t)}{\nu' - \nu}$$

- Crossing symmetry and analyticity

$$A_i(\nu, t) = A_i(-\nu, t)$$

$$A_i(\nu^*, t) = A_i^*(\nu, t)$$

Unsubtracted Dispersion Relations

$$\operatorname{Re} A_i(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \operatorname{Im} A_i(\nu', t) \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \quad (i = 1, \dots, 4)$$

non-convergent integrals for A_2

Unsubtracted Dispersion Relations

$$\operatorname{Re} A_i(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \operatorname{Im} A_i(\nu', t) \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \quad (i = 1, \dots, 4)$$

non-convergent integrals for A_2



Subtracted Dispersion Relations

$$\operatorname{Re} A_2(\nu, t) = A_2(0, t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \operatorname{Im} A_2(\nu', t) \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)}$$



subtraction at $\nu = 0$

Dispersion relations in terms of GPDs

once subtracted fixed-t DR in the variable x

$$\text{Re } A_2(\xi, t) = \Delta(t) + \frac{2}{\pi} \mathcal{P} \int_0^1 \frac{dx}{x} \frac{\text{Im } A_2(x, t)}{(\xi^2/x^2 - 1)}$$

- imaginary part in terms of GPDs: $\text{Im } A_2(\xi, t) = \pi E^+(x, \xi = x, t)$

$$\text{Re } A_2(\xi, t) = \Delta(t) - \mathcal{P} \int_0^1 dx E^+(x, x, t) \left[\frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

- real part from convolution integral:

$$\text{Re } A_2(\xi, t) = -\mathcal{P} \int_0^1 dx E_i^+(x, \xi, t) \left[\frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

- difference between convolution and dispersion integrals:

$$\Delta(t) = \mathcal{P} \int_0^1 dx [E^+(x, x, t) - E^+(x, \xi, t)] \left[\frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

Subtraction Function

$$\Delta(t) = \mathcal{P} \int_0^1 dx [E^+(x, x, t) - E^+(x, \xi, t)] \left[\frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

⇒ Subtraction function is independent of ξ → formally put $\xi = 0$

$$\Delta(t) = 2 \mathcal{P} \int_0^1 dx \frac{1}{x} [E^+(x, x, t) - E^+(x, 0, t)]$$

⇒ Time-Reversal invariance: GPD even in ξ : $E(x, x, t) = E(x, -x, t)$

$$\Delta(t) = 2 \mathcal{P} \int_{-1}^1 dx \frac{1}{x} [E(x, x, t) - E(x, 0, t)]$$

Subtraction Function: relation with D-term

$$\begin{aligned} \int_{-1}^1 \frac{dx}{x} [E(x, x + \xi, t) - E(x, \xi, t)] &= \int_{-1}^1 \frac{dx}{x} \sum_{n=1}^{\infty} \frac{x^n}{n!} \frac{\partial^n}{\partial \xi^n} E(x, \xi, t) \\ &= \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \frac{\partial^{n+1}}{\partial \xi^{n+1}} \left[\int_{-1}^1 dx x^n E(x, \xi, t) \right] \end{aligned}$$

⇒ Polinomiality of Mellin moments of GPDs

$$\int_{-1}^1 dx x^n E(x, \xi, t) = e_0^{(n)}(t) + e_2^{(n)}(t) \xi^2 + \dots + e_{n+1}^{(n)}(t) \xi^{n+1}$$

$$\int_{-1}^1 \frac{dx}{x} [E(x, x + \xi, t) - E(x, \xi, t)] = \sum_{n=0}^{\infty} e_{n+1}^{(n)}(t)$$

⇒ Highest moment generated by Polyakov-Weiss D-Term $e_{n+1}^{(n)} = - \int_{-1}^1 dz z^n D(z, t)$

$$\Delta(t) = 2\mathcal{P} \int_{-1}^1 dx \frac{1}{x} [E(x, x, t) - E(x, 0, t)] = -2 \int_{-1}^1 dz \frac{D(z, t)}{1-z}$$

Subtraction Function: relation with D-term

$$\Delta(t, Q^2) = -2 \int_{-1}^1 dz \frac{D(z, t)}{1-z}$$

⇒ Gegenbauer expansion of D-term $D(z, t) = (1-z^2) \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} d_n(t) C_n^{(3/2)}(z)$

$$\Delta(t) = -4 \sum_{\{n \text{ odd}\}}^{\infty} d_n(t)$$

⇒ Relation to EMT for factors $d_1(t) = \frac{5}{4} D(t) = 5 C(t)$

Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

$$\text{Re } A_2(\nu, t) = \Delta(t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \text{Im} A_2(\nu', t) \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)}$$

Dispersion Relations for DVCS amplitudes

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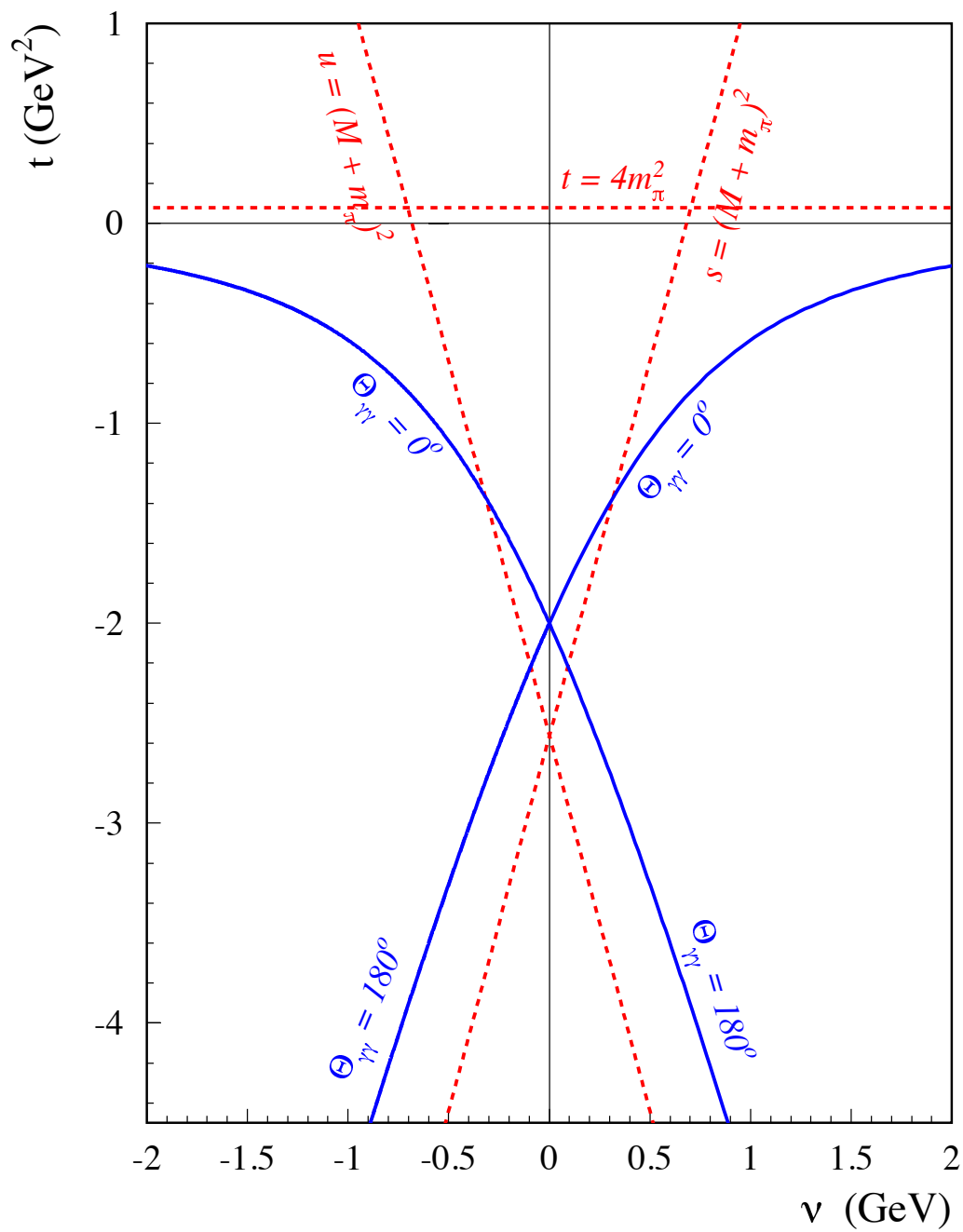
$$\text{Re } A_2(\nu, t) = \Delta(t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \text{Im} A_2(\nu', t) \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)}$$

- t-channel DRs for subtraction function

$$\Delta(t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dt' \frac{\text{Im}_t A_2(0, t')}{t' - t} + \frac{1}{\pi} \int_{-\infty}^{-a} dt' \frac{\text{Im}_t A_2(0, t')}{t' - t}$$

\uparrow
 $-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$

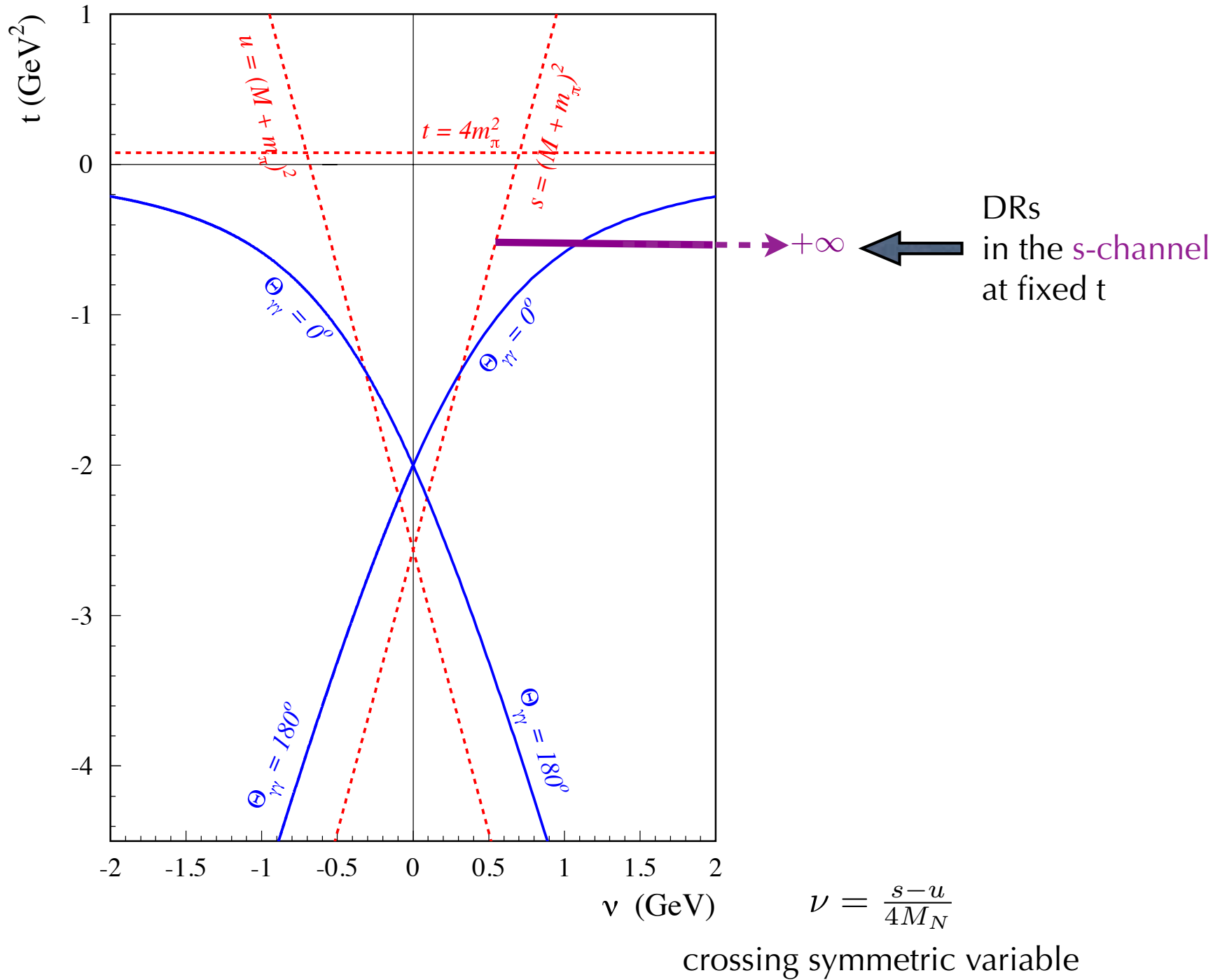
Fixed $Q^2 = 2 \text{ GeV}^2$



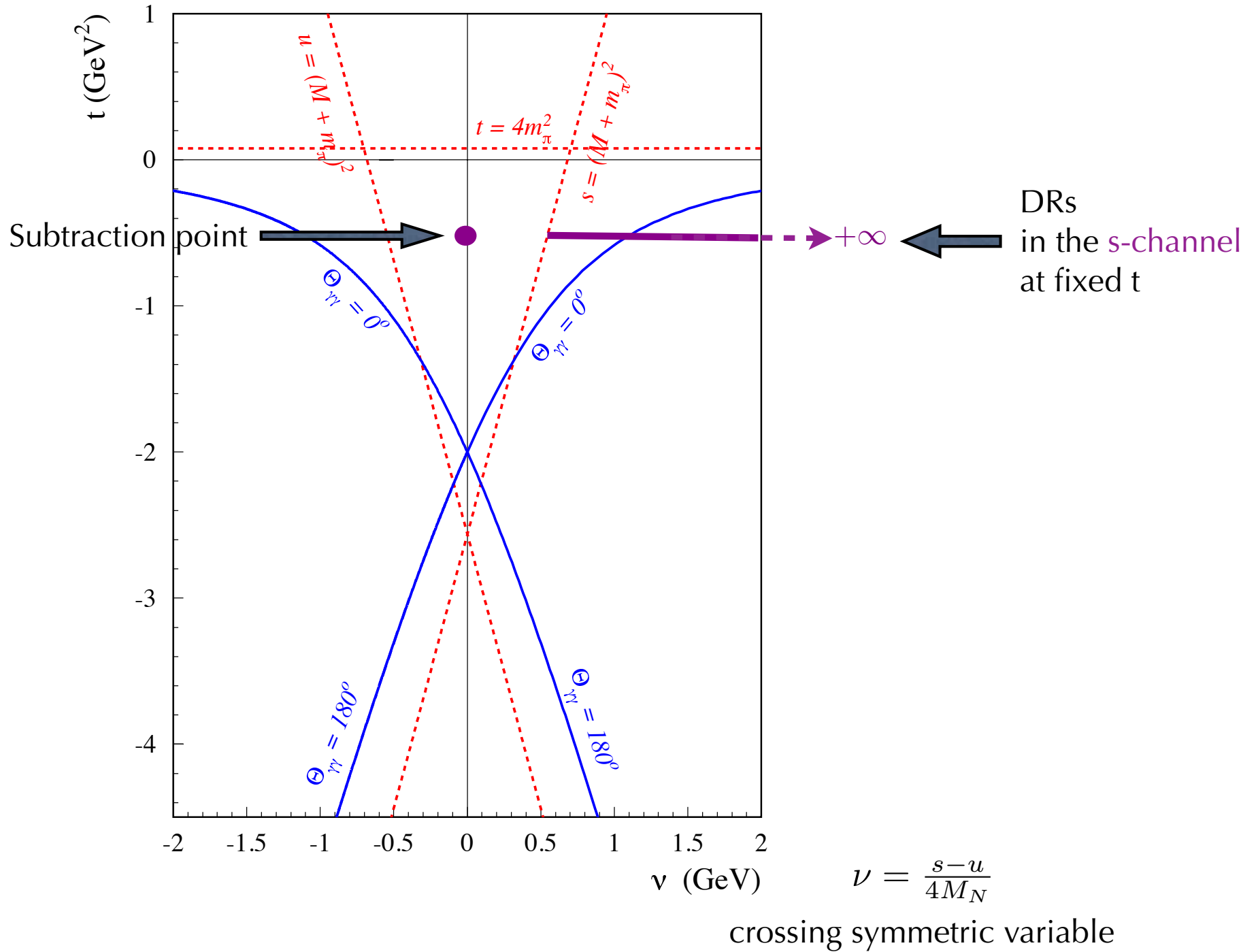
$$\nu = \frac{s-u}{4M_N}$$

crossing symmetric variable

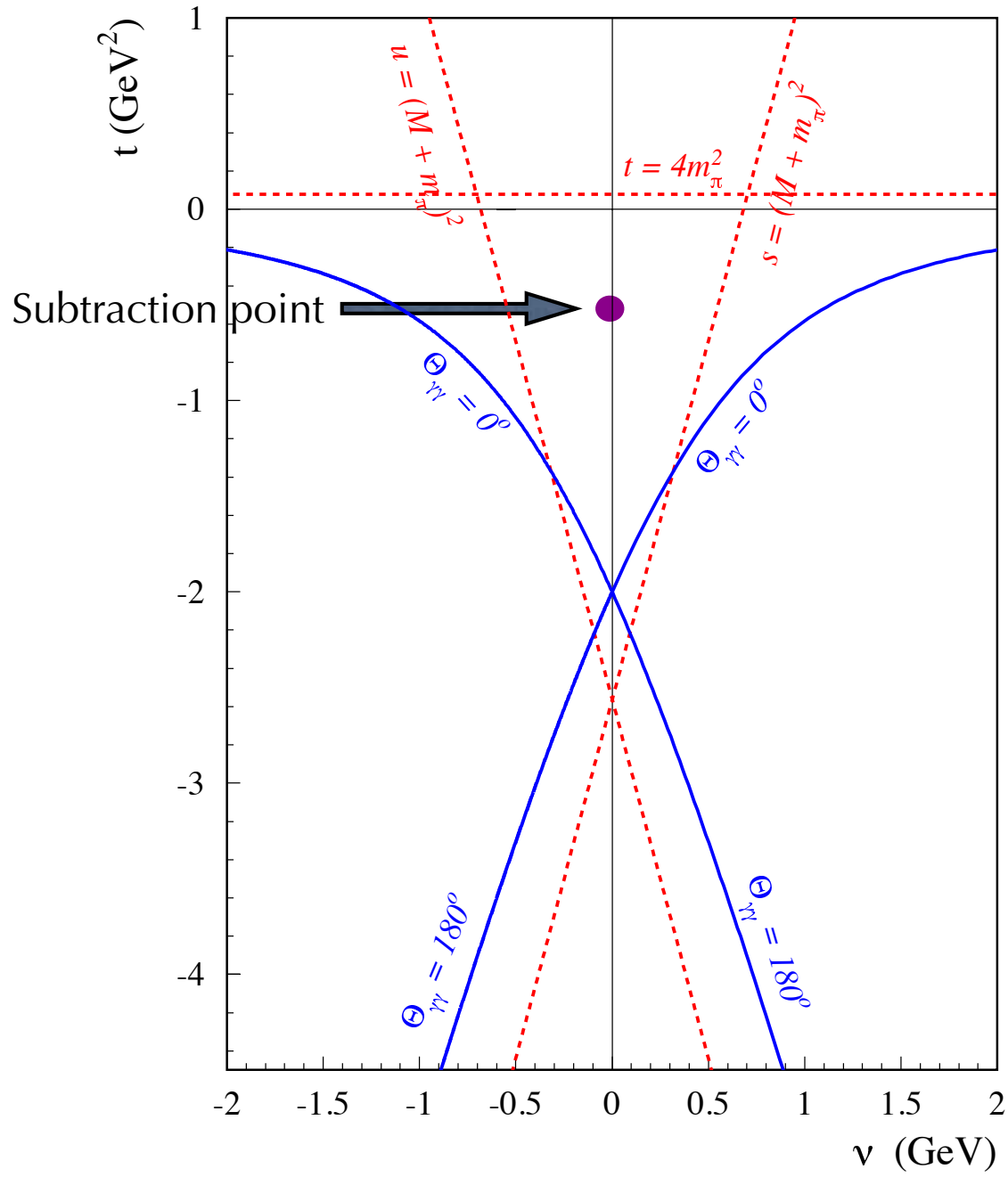
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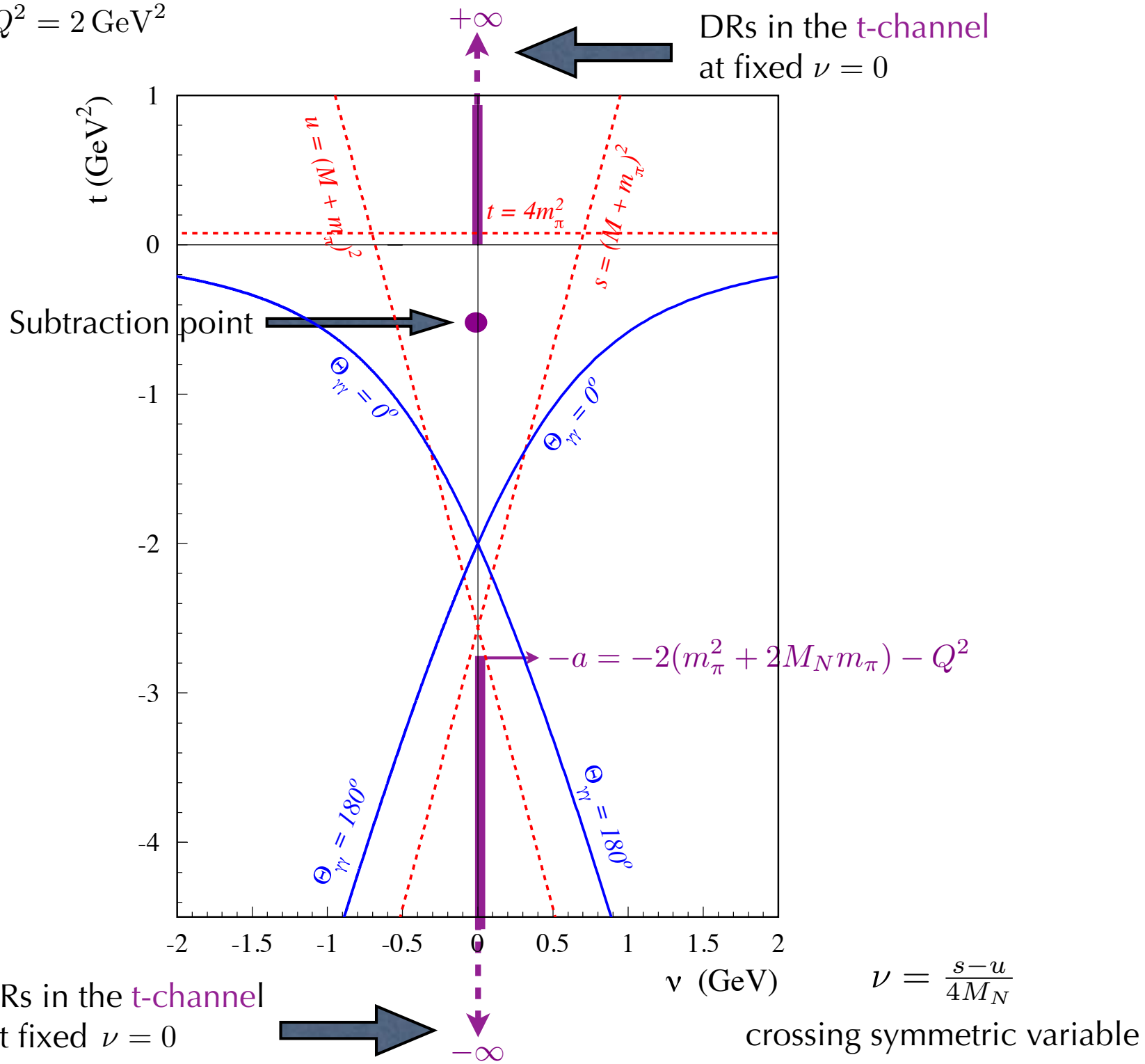
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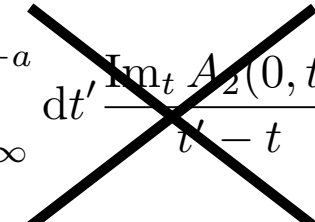
Dispersion Relations for DVCS amplitudes

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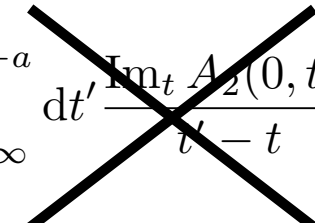
Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

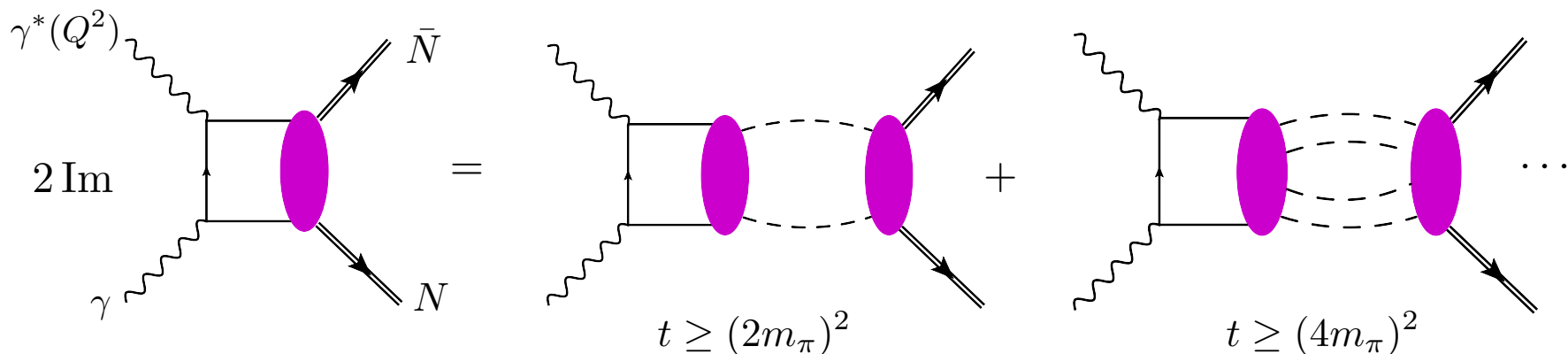
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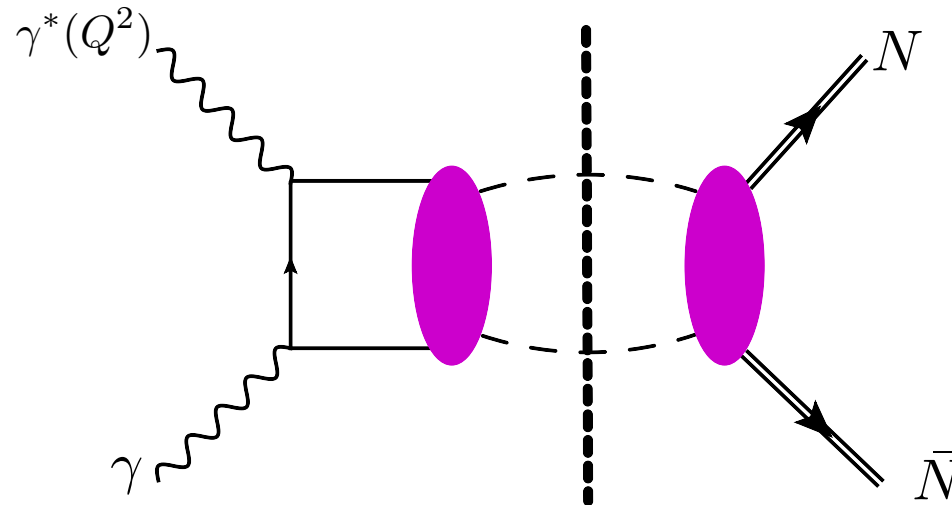
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Unitarity relation in t-channel

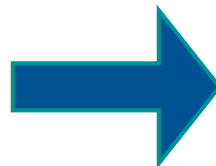


Unitarity relation in the t-channel: two-pion intermediate state



- Charge conjugation and Parity

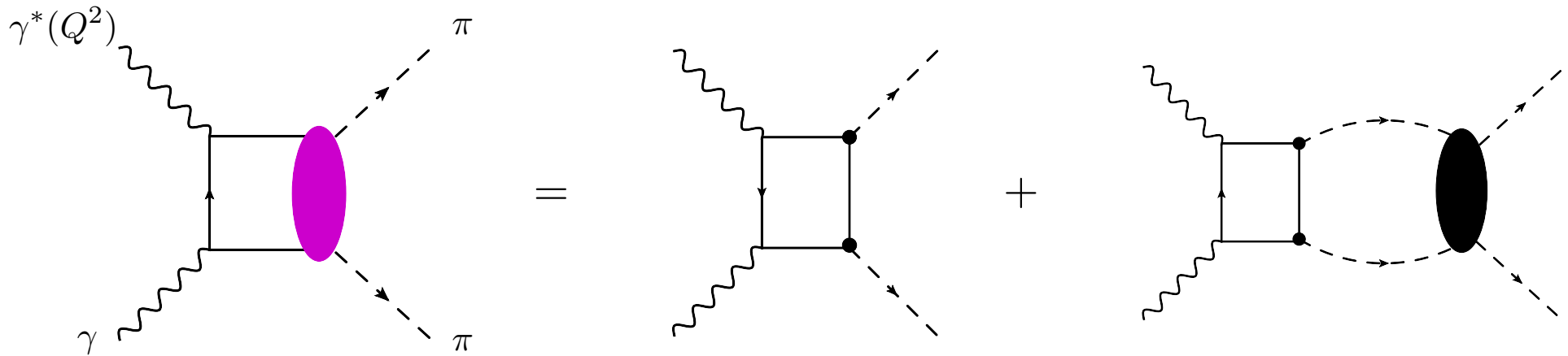
- Partial wave expansion
with $\nu = 0 \rightarrow \theta_t = 90^\circ$



two-pion intermediate state with

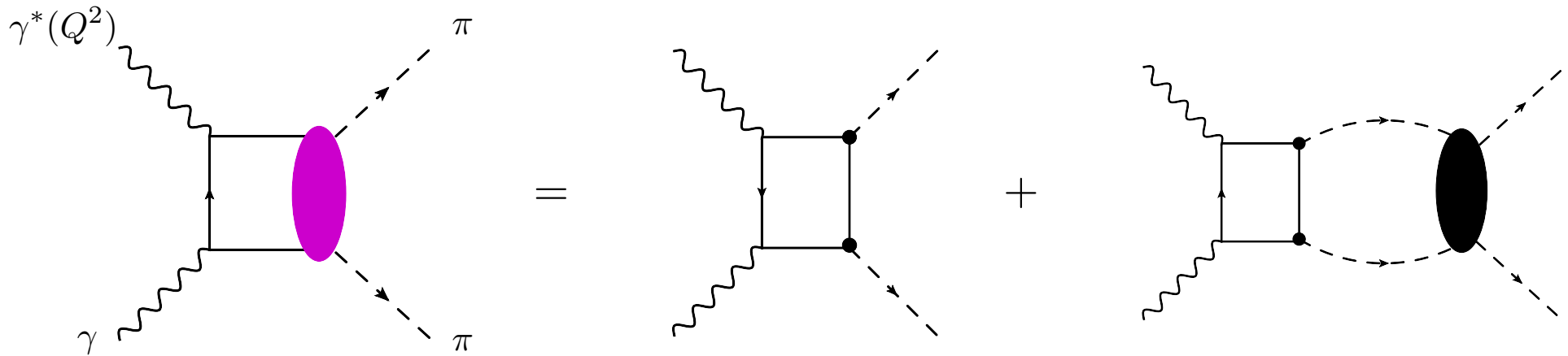
$$I = 0 \quad J = 0, 2, \dots$$

$\gamma\gamma^* \rightarrow \pi\pi$: two-pion GDAs



$$\Phi_q^{\pi\pi} = 6 z(1-z) \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} \tilde{B}_{nl}^q(t) C_n^{(3/2)}(2z-1) P_l(\cos \theta_{\pi\pi})$$

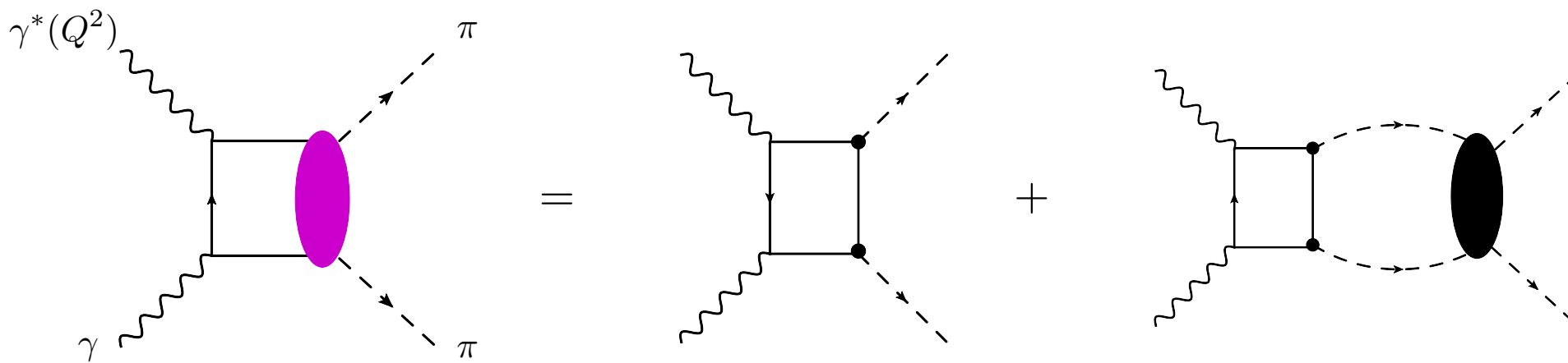
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take contribution only from S and D waves \longrightarrow DRs for $d_1(t)$

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take contribution only from S and D waves \longrightarrow DRs for $d_1(t)$

unitarized S- and D- waves: dispersive (Omnès) representation

S wave

$$\tilde{B}_{10}(t) = -B_{12}(0) \frac{3C - \beta^2}{2} f_0(t)$$

D wave

$$\tilde{B}_{12}(t) = \beta^2 B_{12}(0) f_2(t)$$

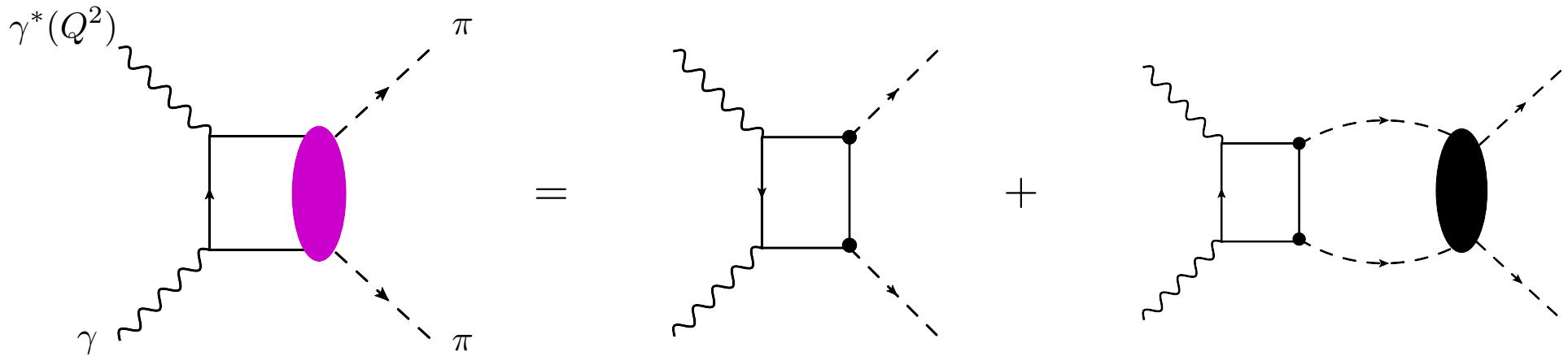
$$B_{12}(0) = \frac{10}{9} \int dx x \frac{1}{N_f} \sum_f [q_\pi^f(x) + \bar{q}_\pi^f(x)]$$

pion PDFs

$$f_l(t) = \exp \left[\frac{t}{\pi} \int_{4m_\pi^2}^{\infty} dt' \frac{\delta_l^0(t')}{t'(t' - t - i\epsilon)} \right]$$

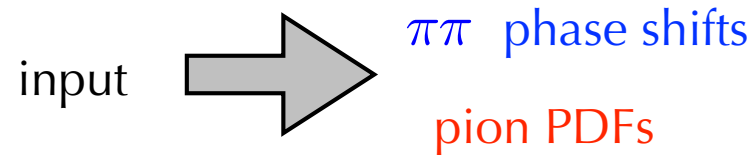
$\pi\pi$ phase shifts

$\gamma\gamma^* \rightarrow \pi\pi$: two-pion GDAs

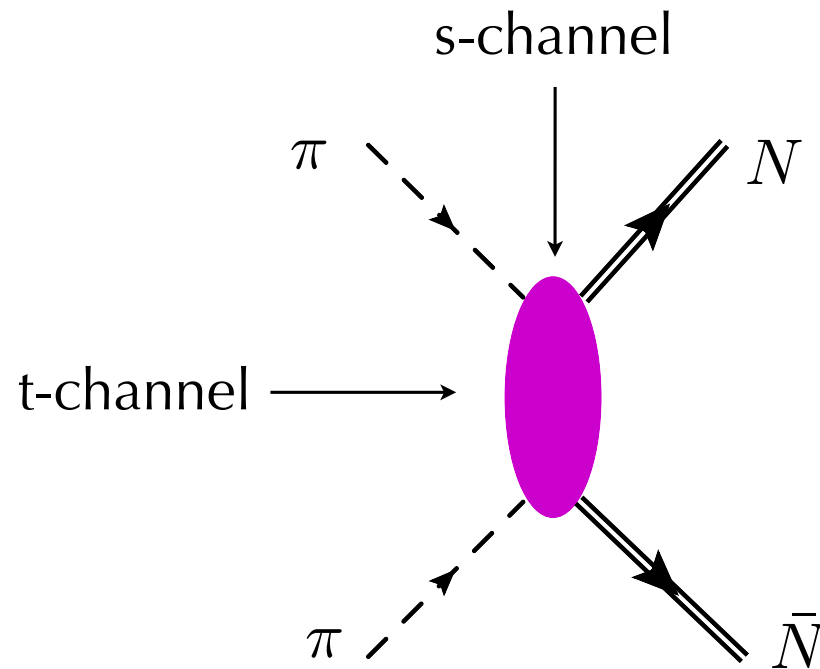


$$\Phi_q^{\pi\pi} = 6 z(1-z) \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} \tilde{B}_{nl}^q(t) C_n^{(3/2)}(2z-1) P_l(\cos \theta_{\pi\pi})$$

unitarized S- and D- waves: dispersive (Omnès) representation

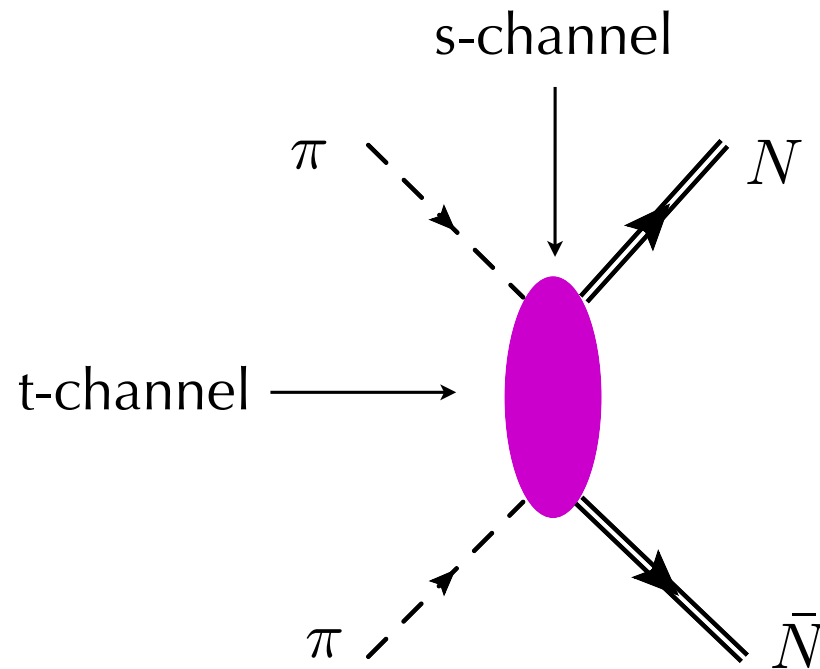


$\pi\pi \rightarrow N\bar{N}$ scattering amplitudes

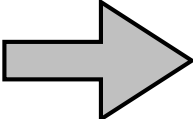


analytical continuation of s-channel partial-wave helicity amplitudes
calculated from DRs

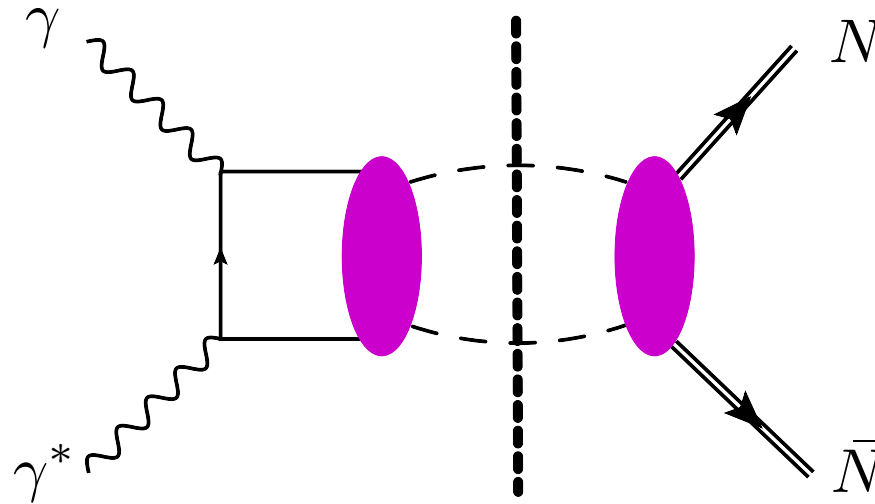
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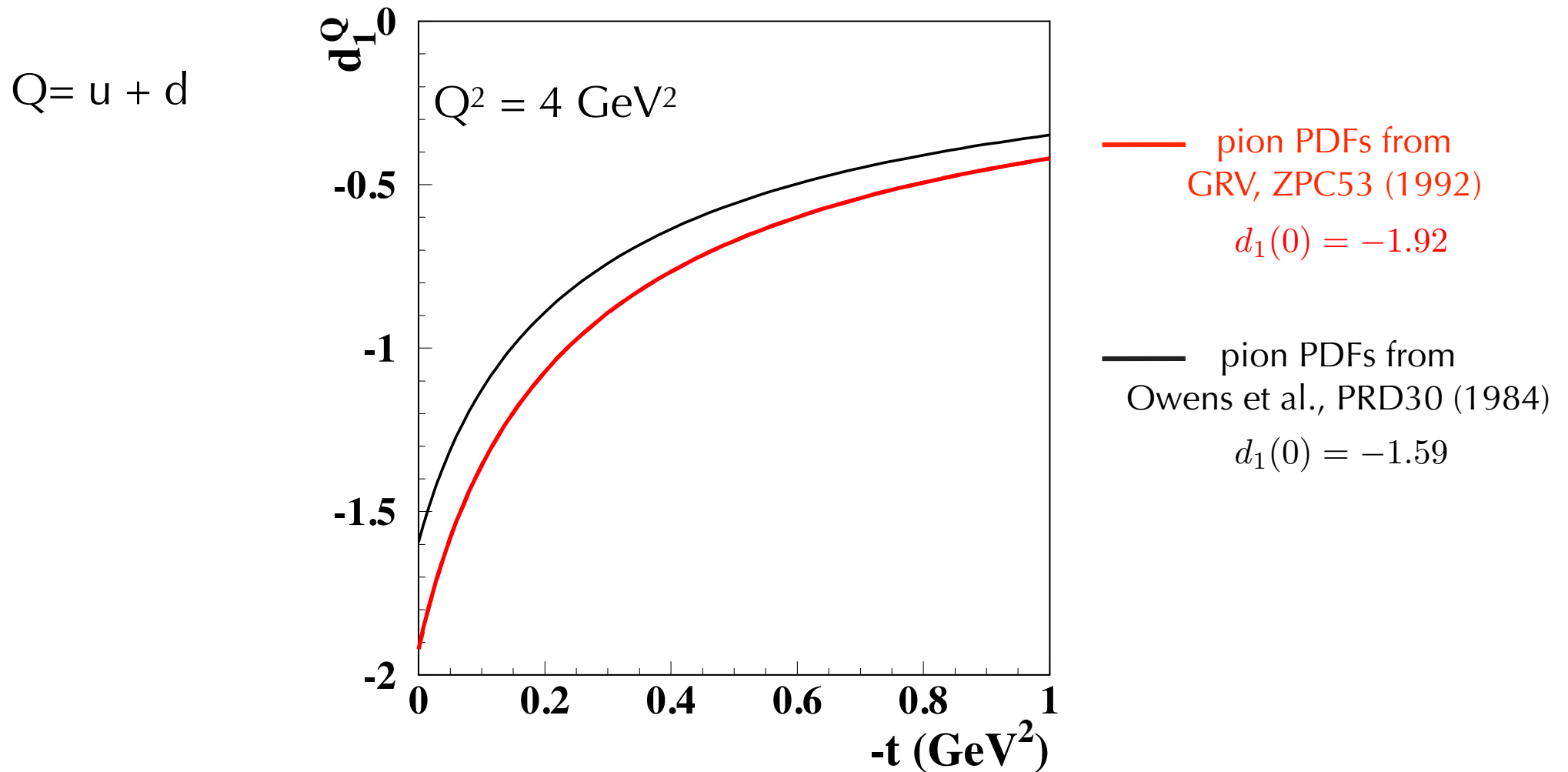
input  $\pi\pi$ phase shifts

Intermediate summary: input for t-channel DRs



- two pion intermediate states \longrightarrow upper limit of integration is $t = 0.78 \text{ GeV}^2$
- partial wave expansion and take $I = 0, J = 0, 2 \longrightarrow$ DRs for $d_1(t)$
- $\gamma^* \gamma \rightarrow \pi\pi$: GDAs with input from first moment of flavor singlet pion PDFs and $\pi\pi$ phase shifts
- $\pi\pi \rightarrow N\bar{N}$: analytical continuations of pion-nucleon scattering amplitudes with input from $\pi\pi$ phase shifts

D-term form factor: dependence on pion PDFs



χ QSM

$$d_1^Q(0) = -2.35$$

Schweitzer et al., (2007)

Skyrme model

$$d_1^Q(0) = -4.48$$

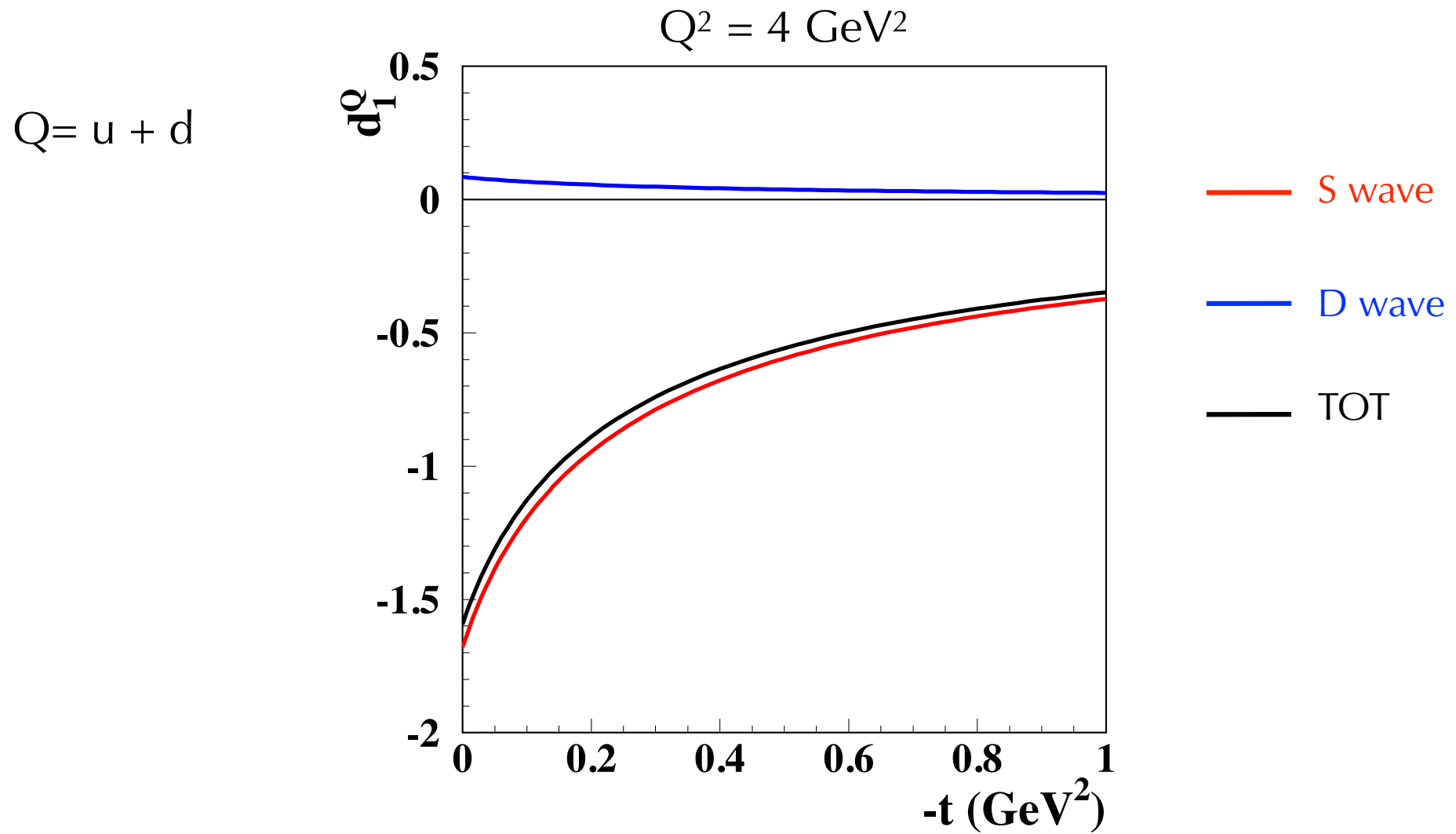
Schweitzer et al., (2007)

Effective LFWFs

$$d_1^Q(0) = -2.01$$

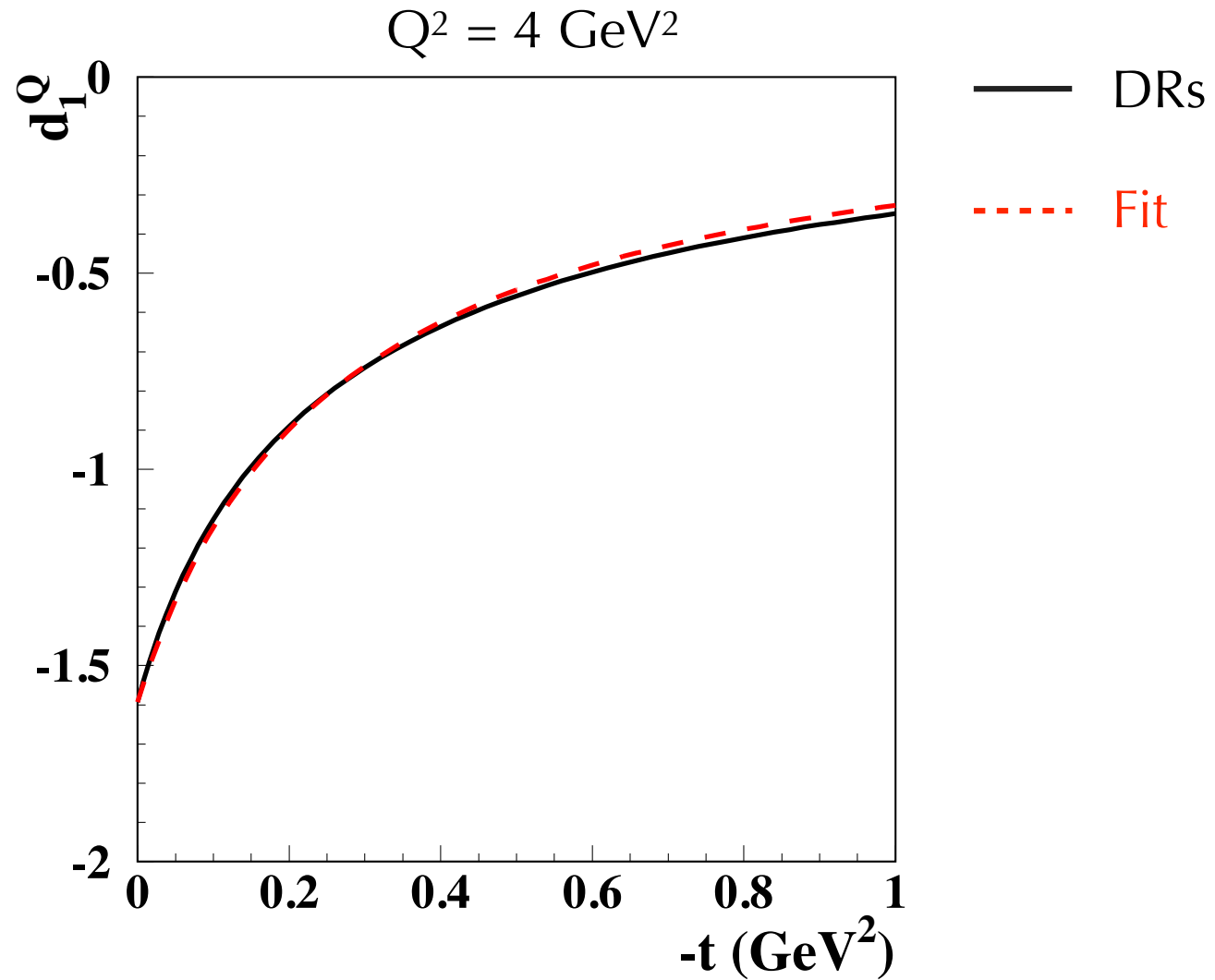
Mueller and Hwang, (2014)

D-term form factor: partial-wave decomposition



D-term form factor: t-dependence

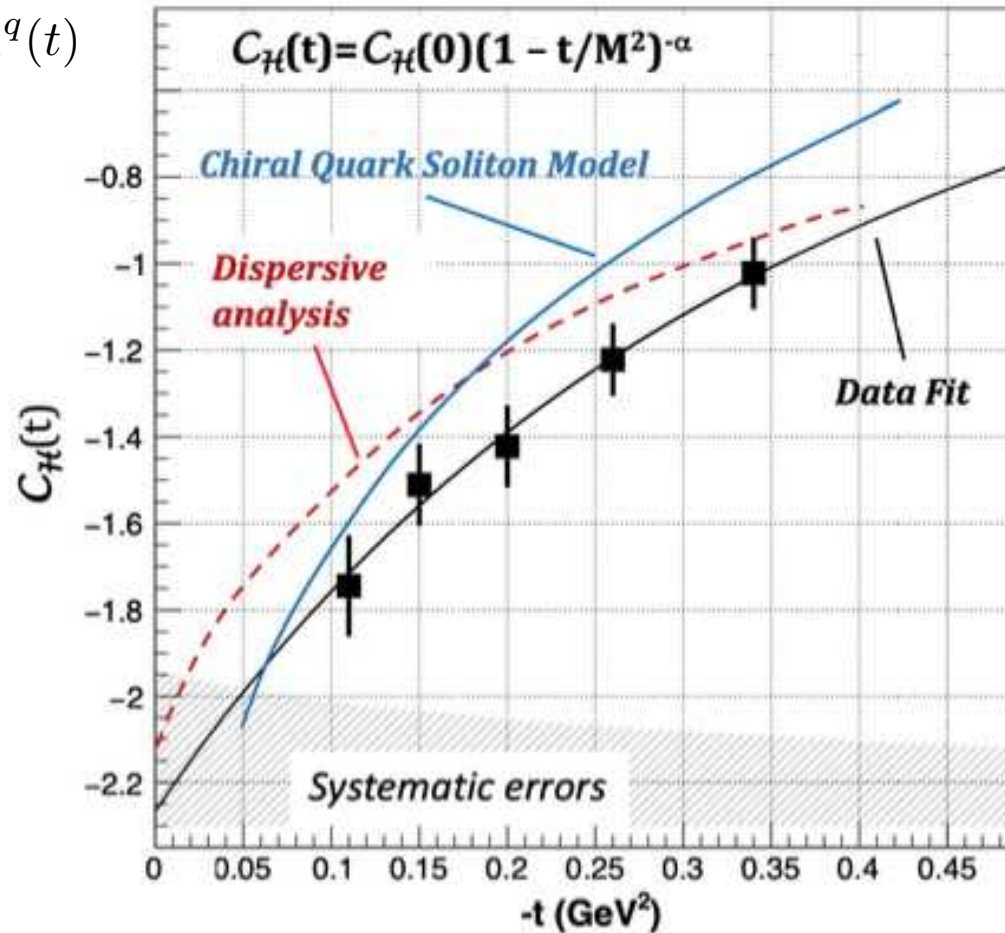
$$Q = u + d$$



Fit: $d_1(t) = \frac{d_1(0)}{[1 - t/(\alpha M_D^2)]^\alpha}$ with $M_D = 0.487 \text{ GeV}$
 $\alpha = 0.841$

Extraction of D-term form factor

$$C_H(t) = -2 \sum_q e_q^2 \Delta^q(t)$$



Extraction from data:

- neglecting gluon contribution
- assuming:

$$C_H(t) = 8 \sum_q e_q^2 \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} d_n(t) \approx \frac{10}{9} d_1^Q(t)$$

$$\text{Fit to data: } C_H(t) = \frac{C_H(0)}{(1 - t/M^2)^\alpha}$$

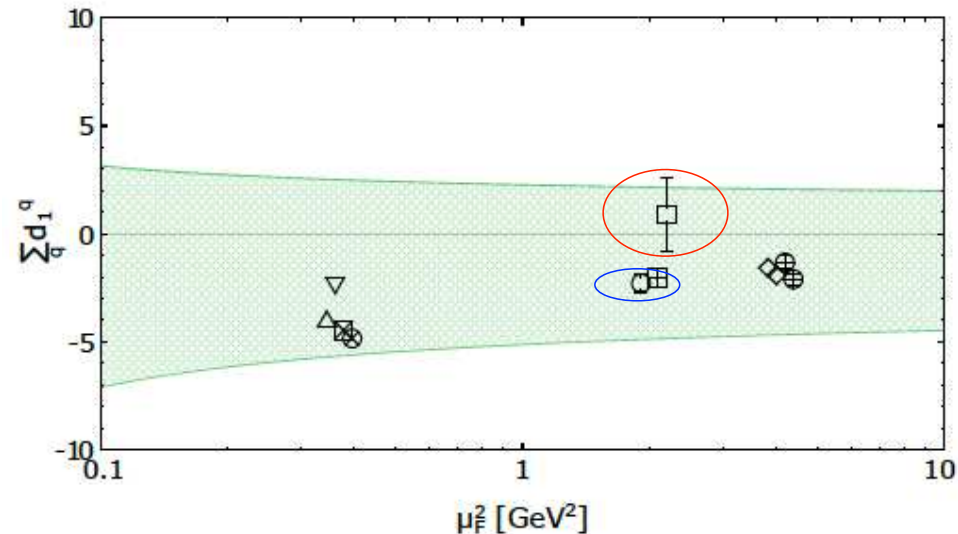
$$C_H(0) = -2.27 \pm 0.16 \pm 0.36 \quad \lambda = 2.76 \pm 0.23 \pm 0.48$$

$$M^2 = 1.02 \pm 0.13 \pm 0.21 \text{ GeV}^2$$

Necessary to verify model assumptions in the exp extraction
with more data coming from JLab, COMPASS and the future EIC, ElcC

Kumericki, Nature 570 (2019) 7759; Dutriex et al, Eur. Phys. J. C81 (2021) 4

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param.,
Girod et al.

CLAS data, with neural networks
Kumericki

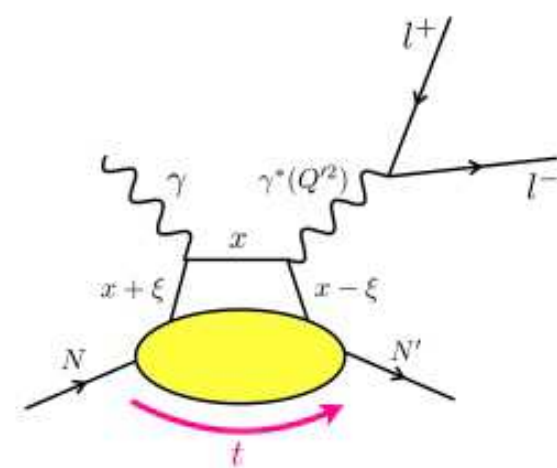
$$\sum_q d_1^q < 0$$

in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV ²	# of flavours	Type
	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
	0.88 ± 1.69	2.2	2	from experimental data
	-1.59	4	2	t -channel saturated model
	-1.92	4	2	t -channel saturated model
	-4	0.36	3	χ QSM
	-2.35	0.36	2	χ QSM
	-4.48	0.36	2	Skyrme model
	-2.02	2	3	LFWF model
	-4.85	0.36	2	χ QSM
	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)

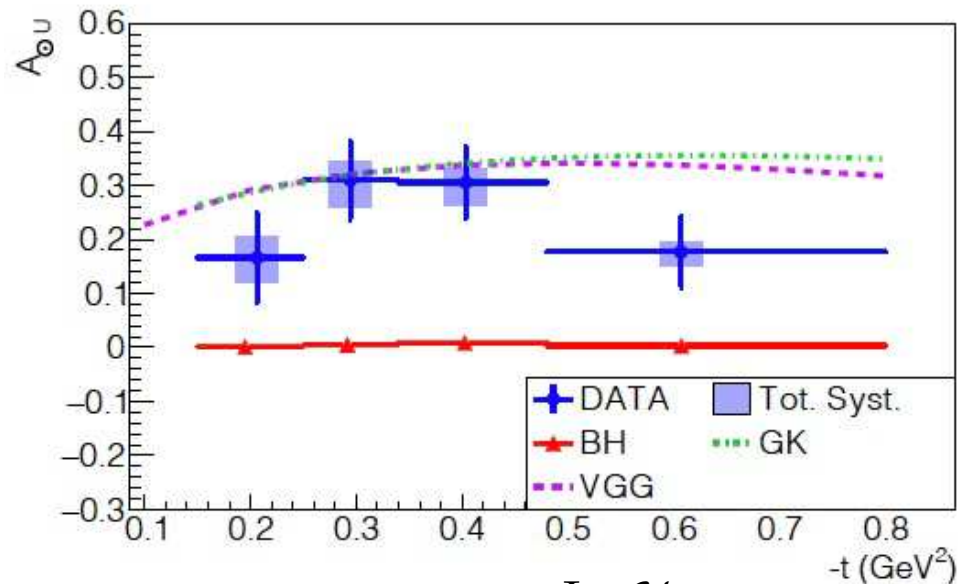
D-term from observables: Timeline Compton Scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)



photon polarization asymmetry

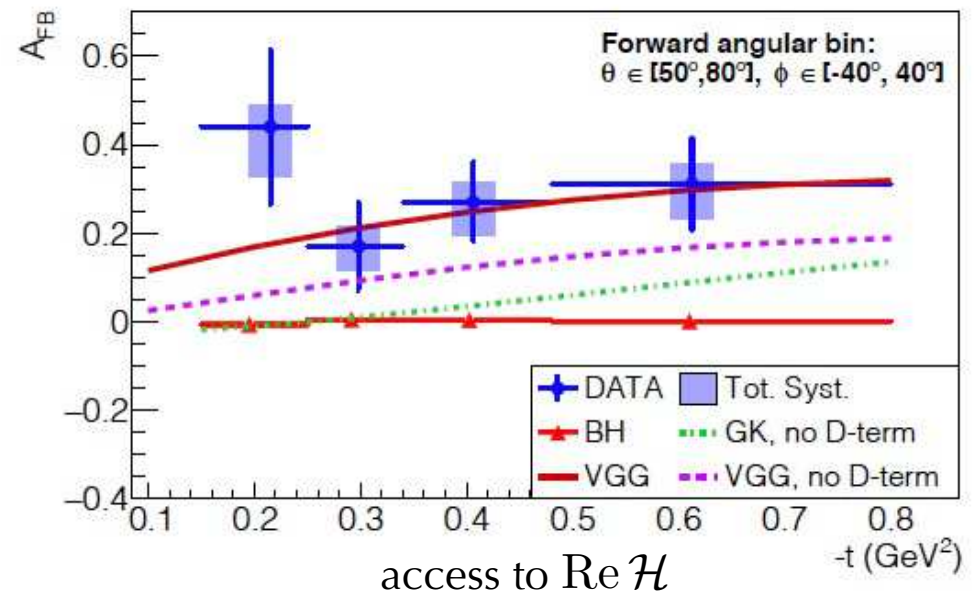
$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$



access to $\text{Im } \mathcal{H}$

forward-backward asymmetry

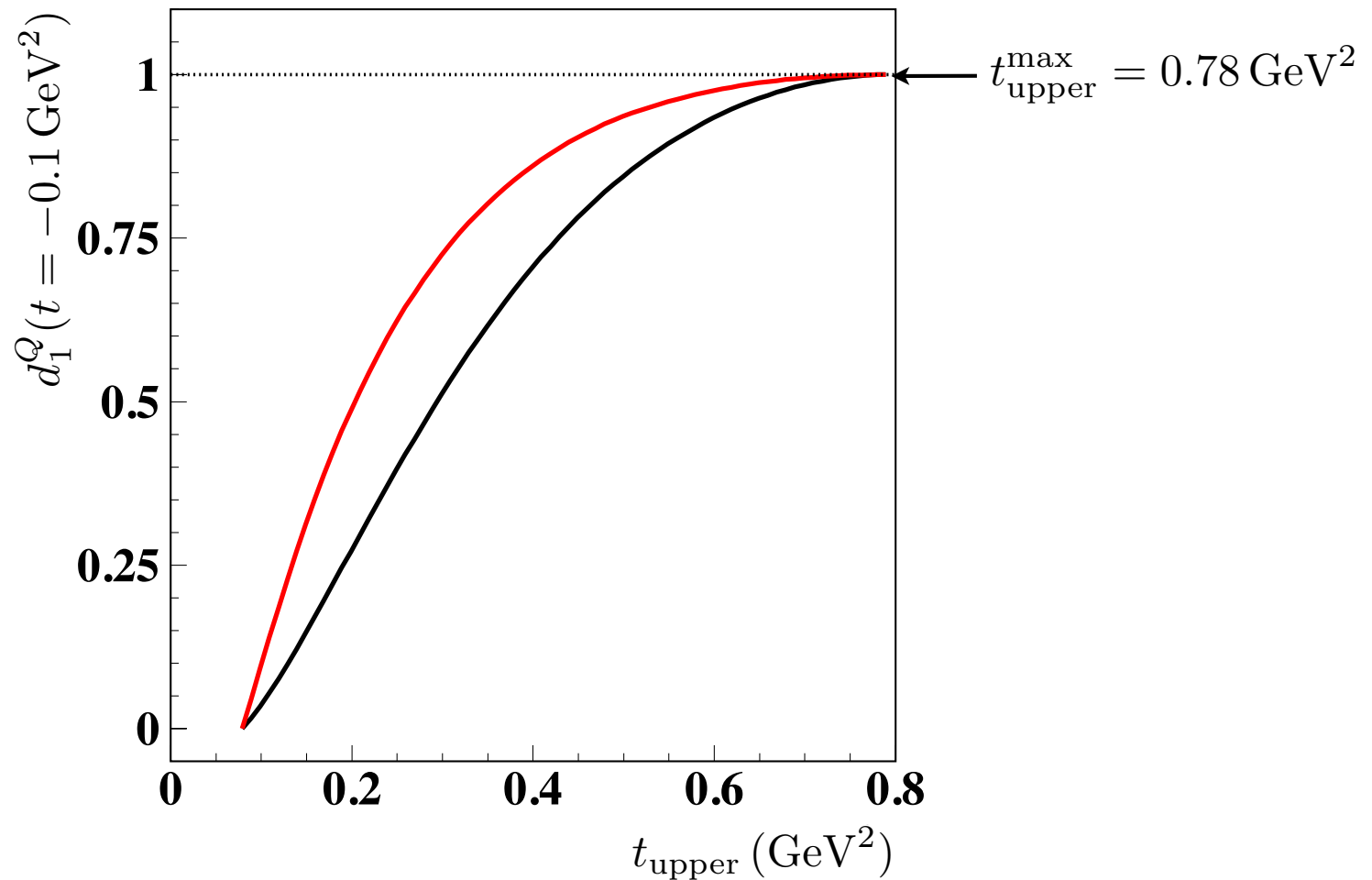
$$A_{FB} = \frac{d\sigma(\theta, \phi) - d\sigma(180^\circ - \theta, 180^\circ + \phi)}{d\sigma(\theta, \phi) + d\sigma(180^\circ - \theta, 180^\circ + \phi)}$$



access to $\text{Re } \mathcal{H}$

- ✓ Input from D-term from dispersion relations
- ✓ New promising path towards the extraction of $\text{Re } \mathcal{H}$ and then the D-term
- ✓ Further data from JLab12 and future EIC

Convergence of Dispersion Integrals



— unsubtracted DRs

$$d_1(t) = -\frac{1}{\pi} \int_{4m_\pi^2}^{t_{\text{upper}}} dt' \frac{\text{Im}_t A_2(0, t')}{t' - t}$$

— subtracted DRs

$$d_1(t) = d_1(0) - \frac{t}{\pi} \int_{4m_\pi^2}^{t_{\text{upper}}} dt' \frac{\text{Im} A_2(0, t')}{t'(t' - t)}$$

↓
subtraction constant as input parameter

Summary

- Dispersion Relations for DVCS amplitudes
constraints from analyticity, crossing, built in

- Subtraction functions for twist-2 DVCS amplitudes
relation to D-term form factor

- D-term from t-channel Dispersion Relations

D-term form factor \longrightarrow two-pion correlated state with $l=0, J=0, 2$

model independent representation

with input from two-pion GDAs and pion-nucleon scattering

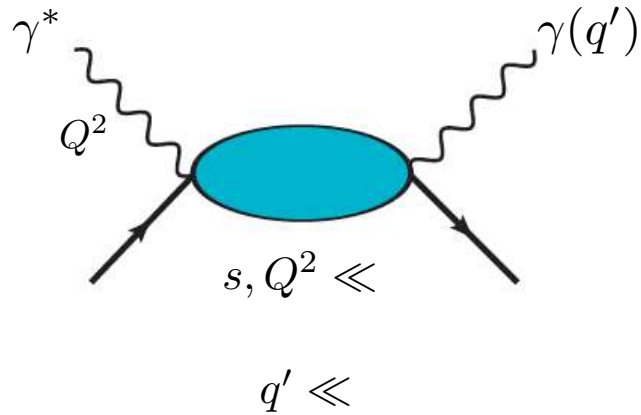
- DR predictions for $d_1(t) = \frac{5}{4}D(t) = 5C(t)$

slow convergence of unsubtracted dispersion integral

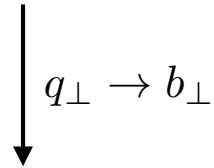
DR results in good agreement with phenomenological evidence of D-term

BACKUP SLIDES

VCS generalized pol.

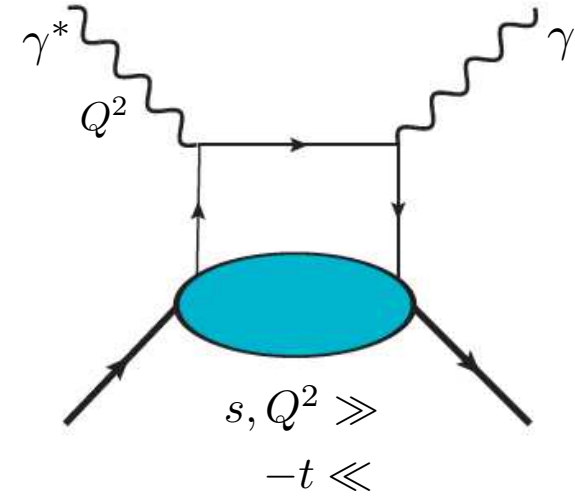


electron scattering by a target which is in constant electric and magnetic fields

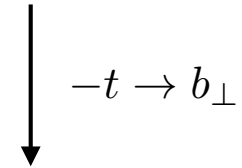


Spatial distribution of electric and magnetic polarization density

DVCS generalized parton distributions



EMT form factors



Spatial distribution of mechanical properties and gluons fields

VCS in low energy region

$$T^{\text{VCS}} = \varepsilon_\mu \varepsilon'_\nu{}^* \sum_{i=1}^{12} F_i(Q^2, \nu, t) \rho_i^{\mu\nu} \quad (i = 1, \dots, 12)$$

$F_i(Q^2, \nu, t) = F_i^{\text{pole}} + F_i^{\text{inel}}$ analytical functions with cuts and poles on the real axis

→ for 10 functions **UN**subtracted DRs

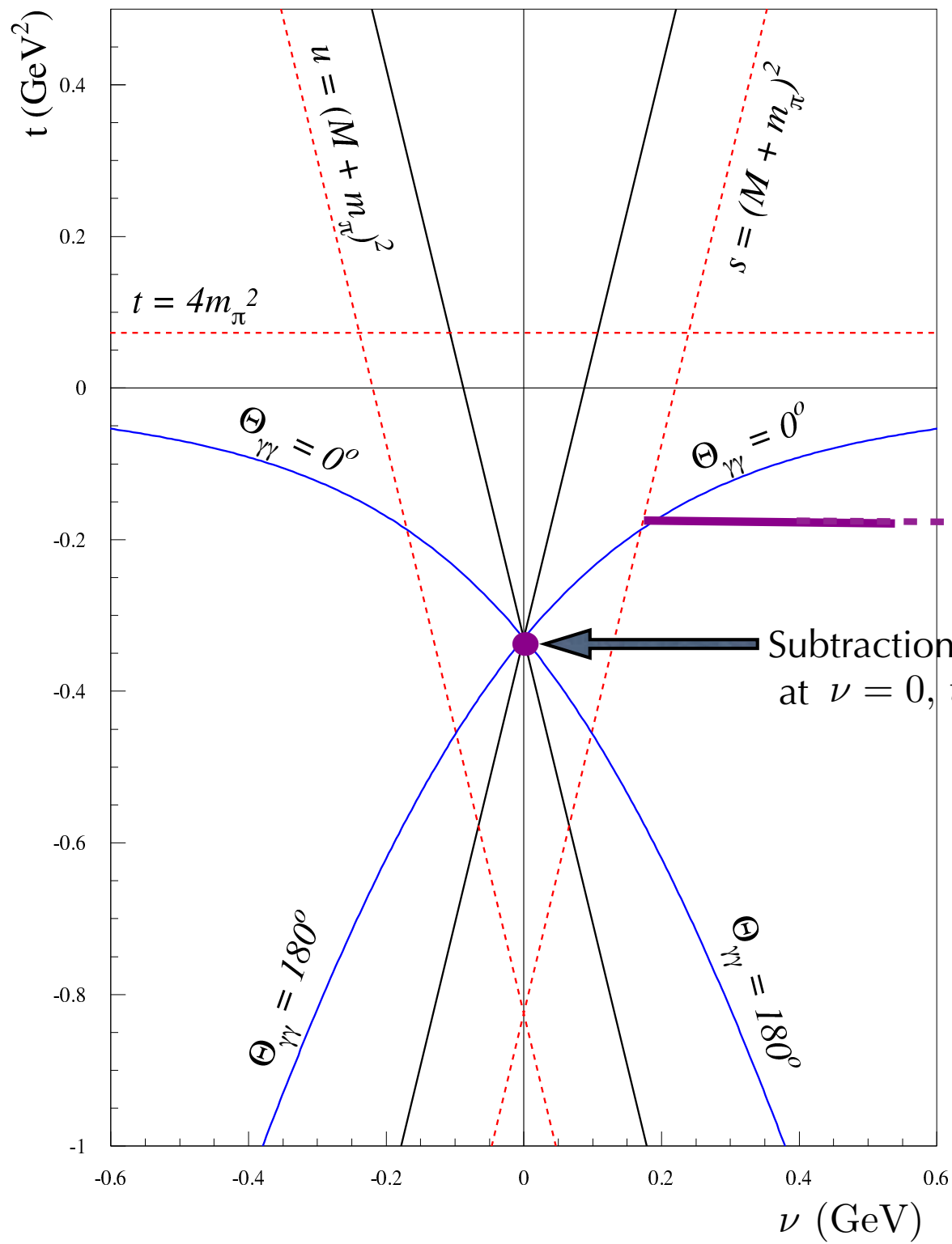
$$\text{Re}F_i^{\text{NB}}(Q^2, \nu, t) = \frac{2}{\pi} \mathcal{P} \int_{\nu_{thr}}^{\infty} \text{Im}_s F_i(Q^2, \nu', t) \frac{\nu' d\nu'}{\nu'^2 - \nu^2}$$

↑
unitarity input: $\gamma^* N \rightarrow X$

→ 2 functions **sub**tracted DRs

$$\text{Re}F_i^{\text{inel}}(Q^2, \nu, t) = F_i^{\text{inel}}(Q^2, 0, t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_{thr}}^{\infty} \text{Im}_s F_i(Q^2, \nu', t) \frac{\nu' d\nu'}{\nu'(\nu'^2 - \nu^2)}$$

↑
subtraction functions (scalar GPs)



Fixed
 $Q^2 = 0.33 \text{ GeV}^2$

DRs
 in the s -channel
 at fixed t

Subtraction constants (scalar GPs)
 at $\nu = 0, t = -Q^2$

$+\infty$