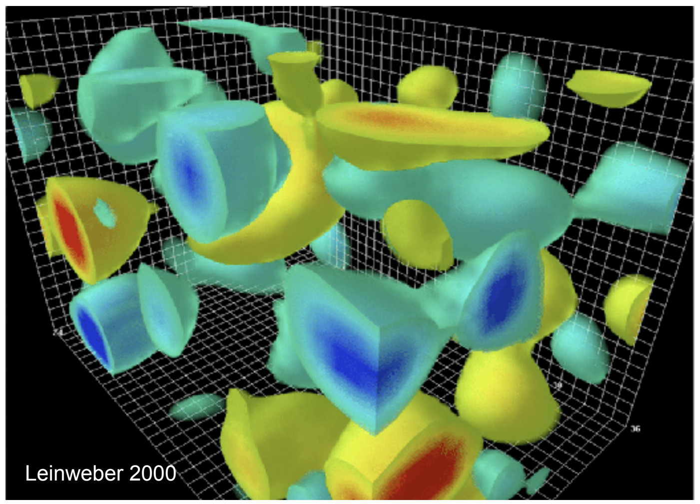
GPDs and gluonic structure from the instanton vacuum

C. Weiss (JLab), GPDs for Nucleon Tomography in the EIC Era, BNL,19-Jan-2024

Jefferson Lab



→ Chiral symmetry breaking
Mass generation
Long-range hadron structure

→ Hadronic matrix elements of QCD operators $\langle N' | \hat{\mathcal{O}}_{\mathrm{QCD}} | N \rangle$

GPDs Twist-2, 3

Gluonic structure

Diakonov, Polyakov, Weiss, NPB 461, 539 (1996) [INSPIRE]
J-Y Kim, Weiss, PLB 848 (2024) 138387 [INSPIRE]

Introduction

Hadron structure ↔ ChSB ↔ topological fields

Program

Instanton vacuum

Effective dynamics from chiral symmetry breaking

Hadronic correlation functions

Effective operators from QCD operators

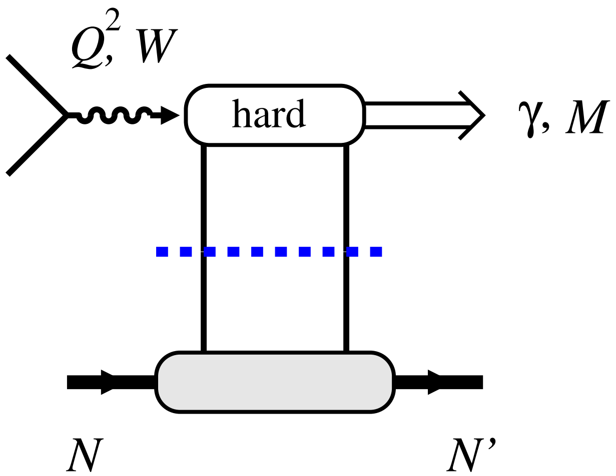
Hadronic matrix elements of QCD operators

Twist-2 operators → parton picture

Twist-3 spin-orbit → chiral interactions

$F^2, F\tilde{F}$ → trace anomaly, U(1) anomaly

Introduction: Matrix elements of QCD operators



$\bar{\psi}\gamma^{\alpha}\nabla^{\beta_1}\ldots\nabla^{\beta_n}\psi$

$F^{\alpha_1\beta}\ldots F^{\beta\alpha_n}$

F^2, $F\tilde{F}$, higher-dim, …

QCD operators from factorization

Processes: DIS, hard exclusive, heavy quarkonium production, …

Local spin-n or light-ray operators [Here: Local twist-2, 3, 4]

Involve QCD gauge potential or fields

Scale-dependent

Need hadronic matrix elements! [Typical scale $\mu \sim 1$ GeV]

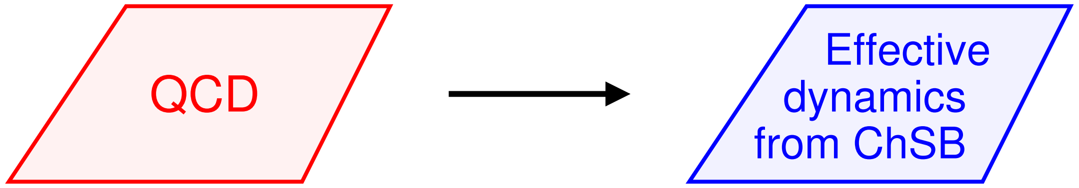
Hadron structure from chiral symmetry breaking

Chiral symmetry breaking governs long-range hadron structure:
Pion as massless excitation, chiral interactions. Systematic construction in Chiral EFT

Chiral symmetry breaking associated with nucleon mass generation:
Constituent quark picture, supported by extensive phenomenology, connected with parton picture

Can we use this knowledge to compute/estimate matrix elements of QCD operators?

Introduction: Effective operators



$\hat{\mathcal{O}}_{\text{QCD}} \longrightarrow \hat{\mathcal{O}}_{\text{eff}}$

Need "effective operators" representing QCD operators in effective dynamics!

$V^{\alpha}, T^{\alpha\beta} \longrightarrow V^{\alpha}_{\text{eff}}, T^{\alpha\beta}_{\text{eff}}$

Conserved currents (vector current, EM tensor): Effective operators from global symmetries ✓

$\bar{\psi}\gamma^{\{\alpha_1}\nabla^{\alpha_2}\dots\nabla^{\alpha_n\}}\psi$
$F^{\{\alpha_1\beta}\dots F^{\beta\alpha_n\}}$ $\longrightarrow$ **?**

Twist-2 QCD operator: Effective operators determined by dynamics!

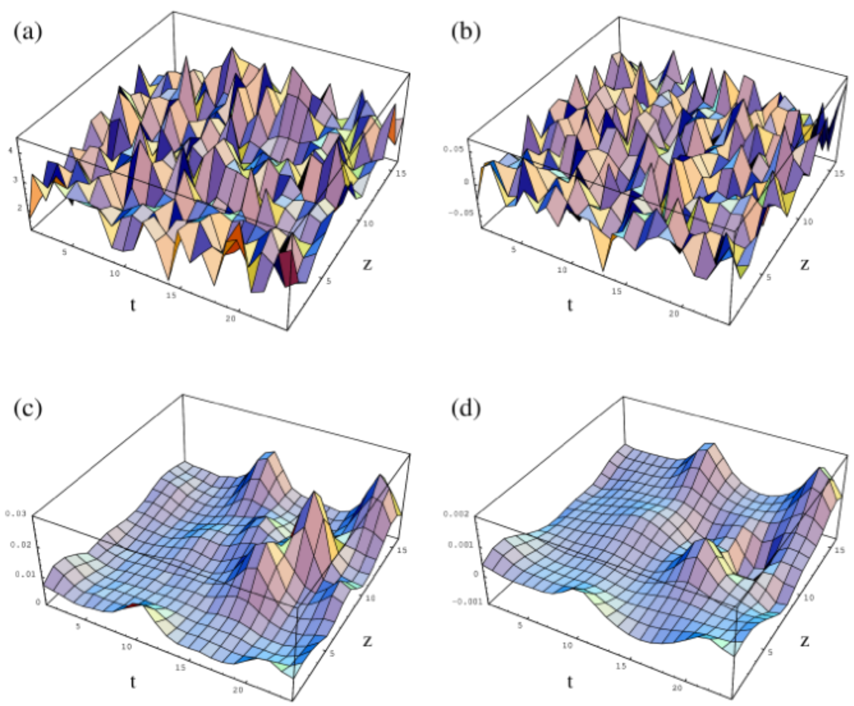
$\psi\gamma^{[\alpha}\gamma_5\nabla^{\beta]}\psi \longrightarrow$ **?**

$= -\frac{1}{4}\epsilon^{\alpha\beta\gamma\delta}\partial_{\gamma}[\bar{\psi}\gamma_{\delta}\tau\,\psi]$

Twist-3 QCD operator (spin-orbit correlations): Relations from QCD equations of motion, should be satisfied by effective operators

Effective operators need to be derived from QCD, consistently with effective dynamics!

Introduction: Topological gauge fields



Chu et al, PRL 70 (1993) 225; PRD 49 (1994) 6039

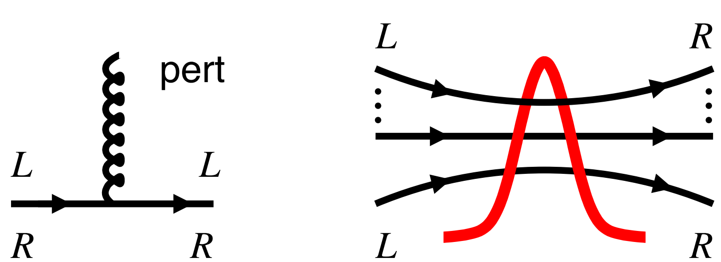
Topological fluctuations of QCD gauge fields

Topological charge $\frac{1}{32\pi^2}\int_{\bar{R}} d^4x\, \tilde{F}F(x) \approx \pm 1$

Instantons: Classical solutions of YM equations, self-dual fields $\tilde{F} = \pm F$, localized

Typical size $\bar{\rho} \approx$ 0.3 fm, separation $\bar{R} \approx$ 1 fm

Strong fields: $(F^2)^{1/2} \approx (32\pi^2/\pi^2\bar{\rho}^4)^{1/2} \sim 2\,\mathrm{GeV}^2$, semiclassical



Induce zero mode of fermion field $i\gamma\nabla_{\rm top}\Phi_{\pm} = 0$
Definite chirality $\gamma_5\Phi_{\pm} = \pm\,\Phi_{\pm}$

→ Chiral symmetry breaking in QCD

Evidence direct and indirect:

LQCD cooling: Polikarpov, Veselov 1988; Campostrini et al. 1990; Chu, Negele et al 1993; DeGrand et al 1997; de Forcrand et al 1997, …, Athenodorou et al 2018

Correlation functions: Shuryak 1982; Diakonov, Petrov 1984; Shuryak, Schafer 1993, …

Introduction: Program

- Construct description of QCD vacuum based on topological gauge fields: Instanton vacuum

Shuryak 82+,
Diakonov, Petrov 1984+

- Derive effective dynamics from chiral symmetry breaking → hadron structure

Diakonov, Petrov 1986;
Shuryak, Schafer 1993+,
Kacir, Prakash Zahed 1996

- Derive effective operators resulting from QCD operators in same scheme → Twist-2,3 GPDs, gluonic structure

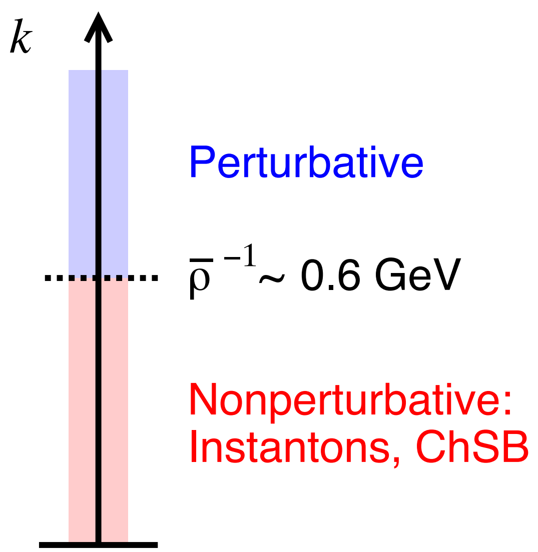
← here

Use systematic parametric approximation: Packing fraction, $1/N_c$

Obtain effective operators consistent with effective dynamics

Preserve operator relations from QCD equation of motion, trace and U(1) anomaly

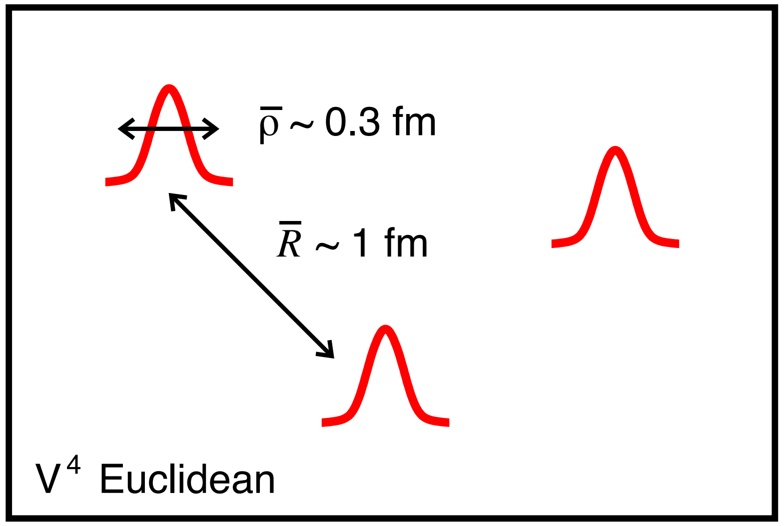
Instanton vacuum: Gauge fields



Separation of modes

$k > \bar\rho^{-1}$: Integrate perturbatively:
Renormalization, $\bar\rho^{-2} \gg \Lambda^2_{\rm QCD}$

$k < \bar\rho^{-1}$: Integrate nonperturbatively:
Instantons + massive fermions



Instanton ensemble

$$A(x) = \sum_I A_I(x\,|\,z_I, \rho_I, O_I) + \sum_{\bar I} A_{\bar I}(\,..\,)$$

gauge potential → classical top. fields

$$\int [DA] \rightarrow \int \prod_{I,\bar I} dz_I\, d\rho_I\, dO_I\, d_0(\rho_I)$$

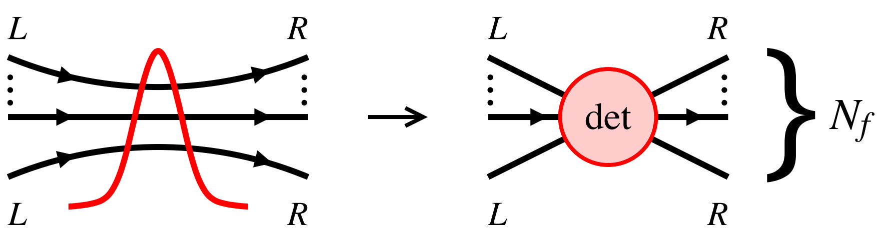
functional integral → collective coordinates

Stable system emerges due to instanton interactions
Implementations: Variational principle Diakonov, Petrov 1984; numerical simulations Shuryak 1988+

Small parameter: Packing fraction $\pi^2 \bar\rho^4 / \bar R^4 \approx 0.1$

All dynamical scales "emerge" from $\Lambda_{\rm QCD}$ via instanton density
Preserves renormalization properties of QCD

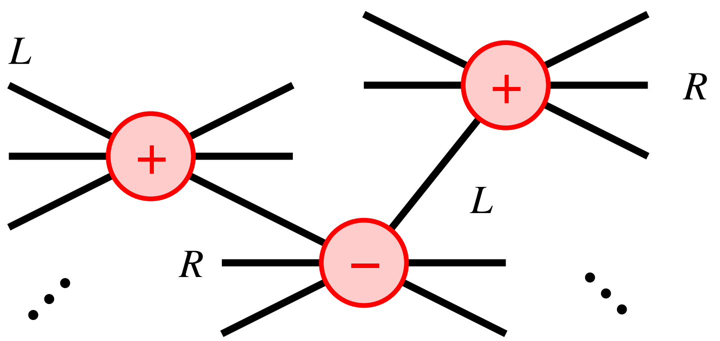
Instanton vacuum: Fermions and ChSB



Fermions in 1-instanton background

Zero mode induces multifermion vertex ('tHooft)

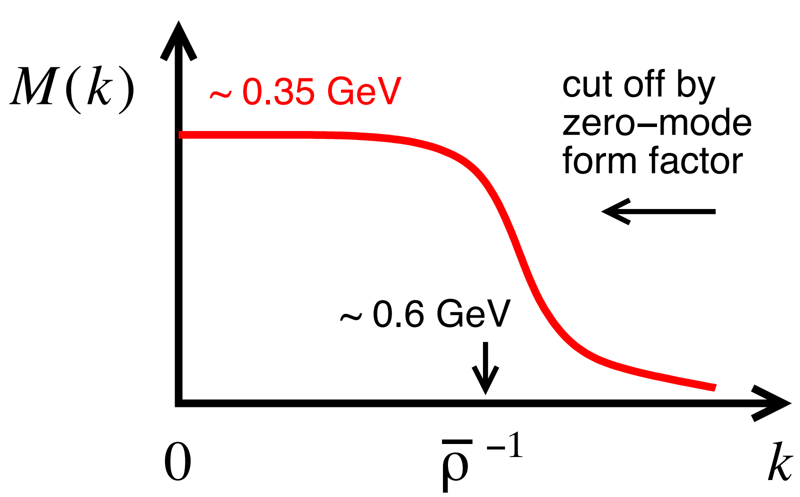
$$\int dO_I \, \bar\psi \,|\, \Phi_I)(\Phi_I \,|\, \psi \;\propto\; \det_{ab} \bar\psi^a_L \psi^b_R \quad \times \text{ form factor}$$



Chiral symmetry breaking in instanton medium

Fermions hop between zero modes — chirality flip
Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; ...

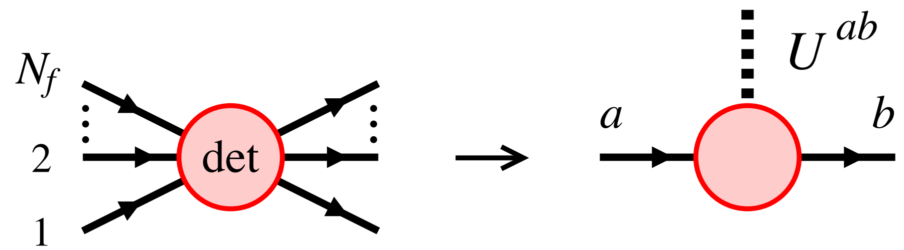
Dynamical quark mass $M \approx$ 0.3-0.4 GeV, active for momenta $k \lesssim \bar\rho^{-1} \approx$ 0.6 GeV



Ground state determined in $1/N_c$ expansion, saddle point approximation

Effective dynamics of massive quarks with multi-fermion interactions

Instanton vacuum: Bosonization



Auxiliary field $U^{ij} \sim \bar{\psi}^i_L \psi^j_R$

[Diakonov, Petrov 1986; Kacir, Prakash, Zahed 1995; …]

$$\int DU \int D\bar{\psi} D\psi \ \exp \int d^4x \ \bar{\psi}(x) \left[i\hat{\partial}\psi - M \, U^{\gamma_5}(x) \right] \psi(x)$$

Yukawa-type interaction of massive quarks with chiral field

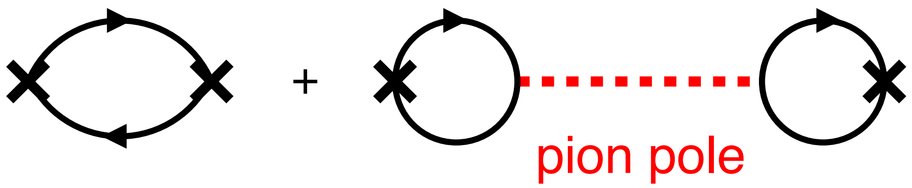
$$U^{\gamma_5}(x) \equiv \frac{1+\gamma_5}{2} U(x) + \frac{1-\gamma_5}{2} U^{\dagger}(x)$$

Form of interaction constrained by chiral invariance

Effective dynamics of massive quarks and Goldstone bosons with chiral interactions

Quark-pion coupling $g_{\pi qq} = M/F_\pi \approx 4$, strongly coupled system!
Solved nonperturbatively using $1/N_c$ expansion — saddle point approximation

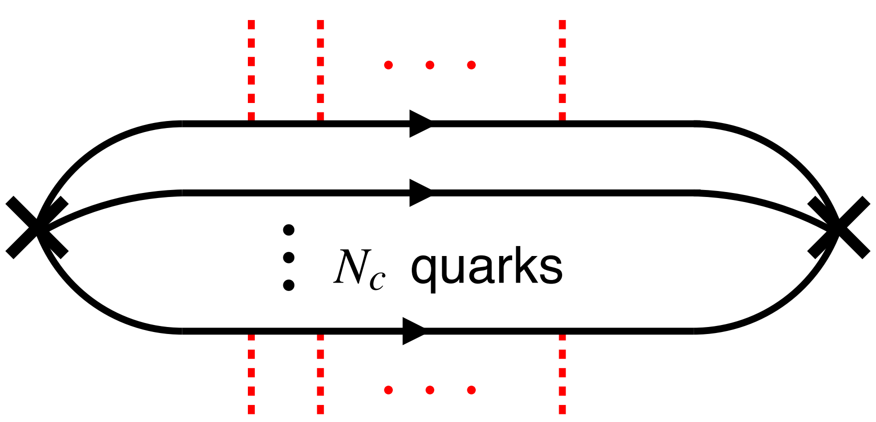
$$U(x) = e^{i\tau^a \pi^a(x)/F_\pi}$$



Meson correlators

Saddle point $U = 1$: Vacuum fields

Pseudoscalar: Pion pole (isovector), η' mass (isoscalar)



Baryon correlators

Saddle point $U \neq 1$: Classical pion field (“soliton”)
[Diakonov, Petrov, Pobylitsa 1988]

Fixed time: Quarks moving in self-consistent pion field with single-particle Hamiltonian $H = \alpha\mathbf{p} + \beta M U_{\text{class}}$
[Kahana, Ripka1984]

Realization of general relativistic mean-field picture at large N_c. Unifies “quark model” and “chiral soliton”

[Witten 1983]

Instanton vacuum: Nucleon structure

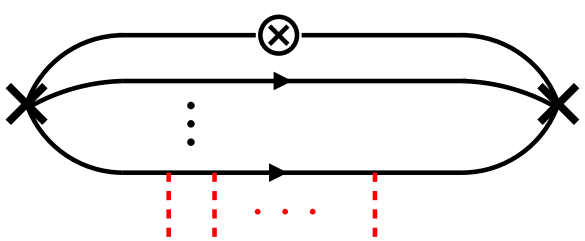
3-point functions include disconnected quark diagrams… connected by chiral field!

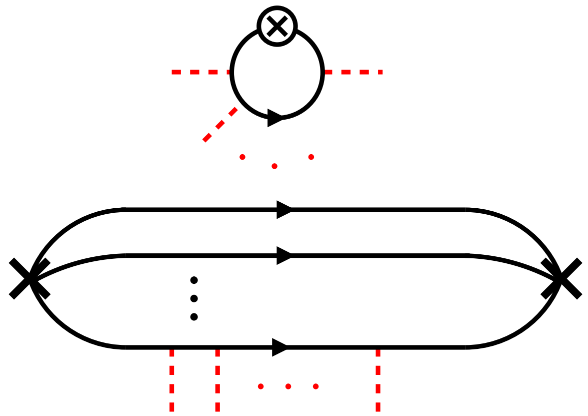
Example: Axial and vector and current matrix elements for SU(2) flavor

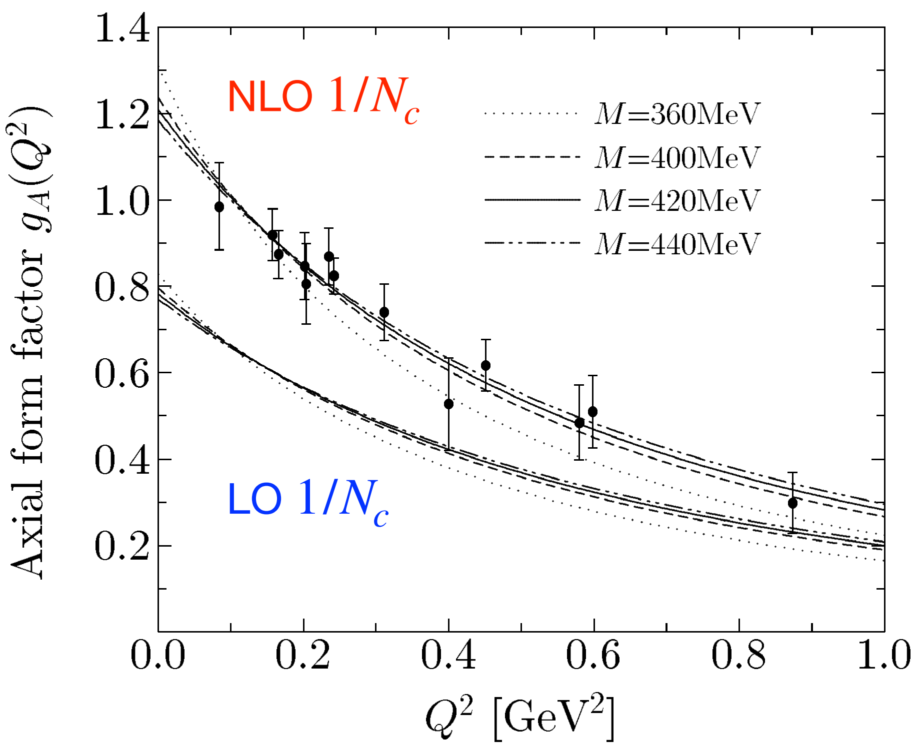
[Review: Christov et al 1995. Many more results: Sigma term, $N \to \Delta$, SU(3), …]

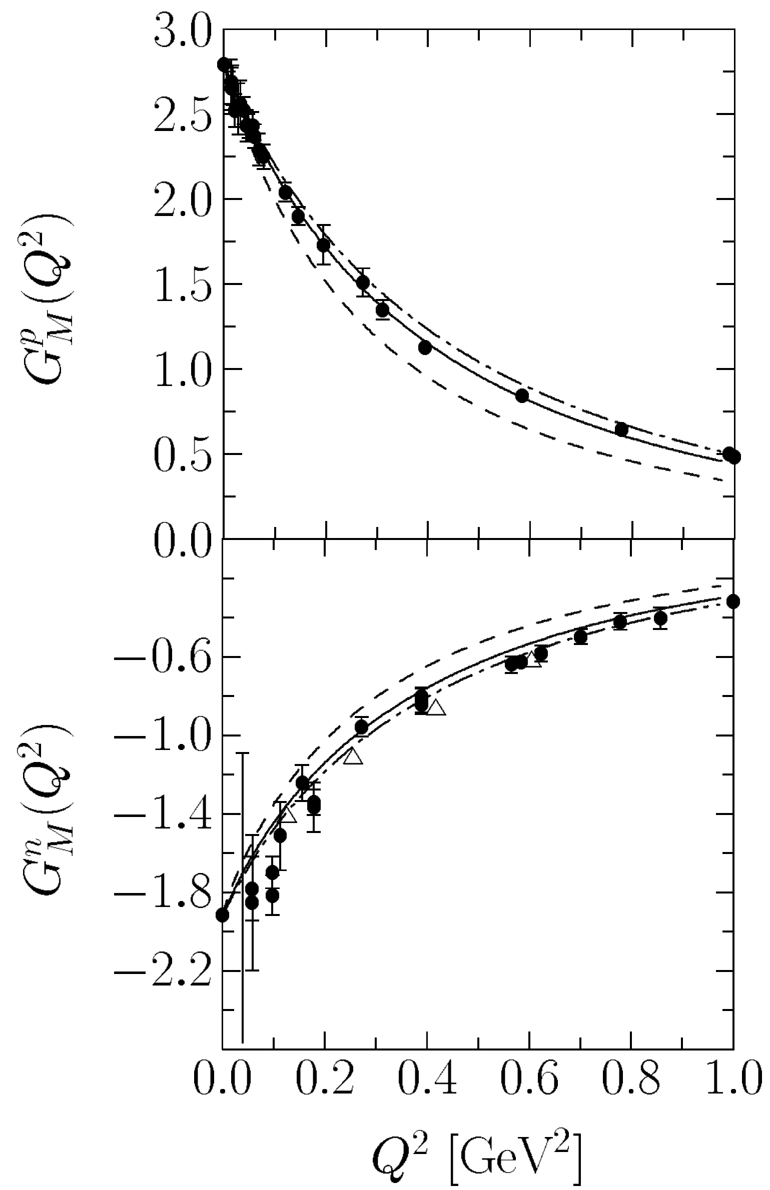
Reasonable description of nucleon static properties, current matrix elements

→ **Talk H-C Kim**

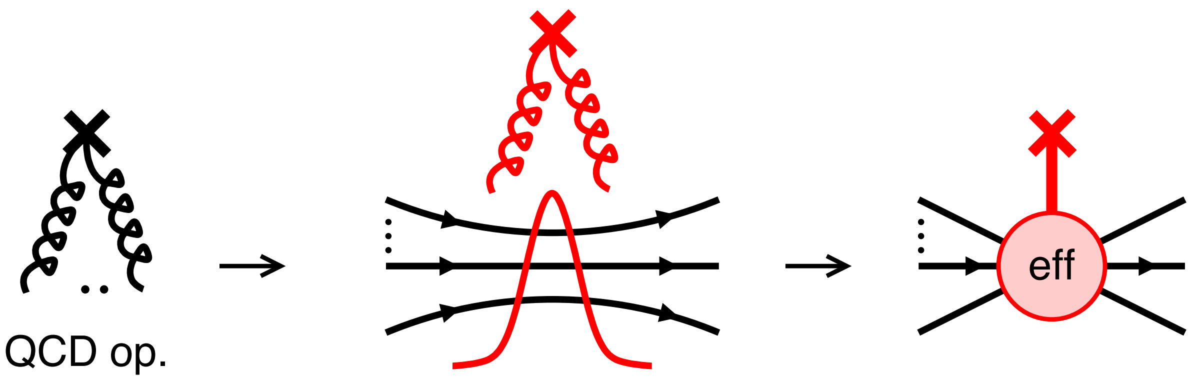








QCD operators: Effective operators



Gluon operator

$\mathscr{F}[A]$ local QCD gluon operator

Normalized at scale $\mu = \bar{\rho}^{-1}$

Gluon operator in instanton vacuum correlation functions

$$\mathscr{F}[A] \to \sum_{I+\bar{I}} \mathscr{F}[A_I] \; + \; \mathscr{O}(\rho^4/R^4)$$

Evaluated in gluon field of single $I(\bar{I})$

$$\big\langle \ldots \mathscr{F}[A] \; \ldots \big\rangle_{\text{inst}} \; \to \; \big\langle \ldots \mathscr{F}_{\text{eff}}[\bar{\psi}, \psi] \ldots \big\rangle_{\text{eff}}$$

Converted to effective fermionic operator
in effective theory of massive fermions

$$\mathscr{F}_{\text{eff}}[\bar{\psi}, \psi] \; = \; N \int dz_I \, dO_I \, d\rho_I \; d_{\text{eff}}(\rho_I) \;\; \mathscr{F}[A_I] \;\; \bar{\psi} \,|\, \Phi_I)(\Phi_I \,|\, \psi$$

Coupling through zero modes
↔ ‘tHooft vertex

Construction possible in saddle-point approximation $1/N_c$

[Diakonov, Polyakov, Weiss, 1995]

Advantages of effective operator representation:
Universal, relations between hadronic matrix elements, insight into origin of gluonic structure

QCD operators: Twist-2

$$O^q_{\alpha_1 \ldots \alpha_n} = \bar\psi \gamma_{\{\alpha_1} \nabla_{\alpha_2} \ldots \nabla_{\alpha_n\}} \psi - \text{traces}$$

Twist-2 spin-n QCD operators, scale $\mu = \bar\rho^{-1}$

$$O^g_{\alpha_1 \ldots \alpha_n} = F_{\beta\{\alpha_1} D_{\alpha_2} \ldots D_{\alpha_{n-1}} F_{\alpha_n\}\beta} - \text{traces}$$

Matrix elements = moments of PDFs/GPDs

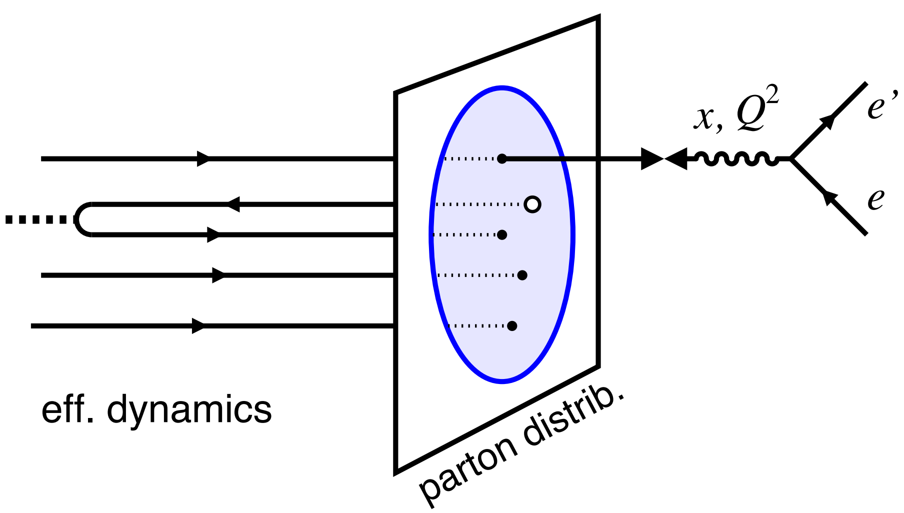
↓ effective theory

$$(O^q_{\text{eff}})_{\alpha_1 \ldots \alpha_n} = \bar\psi \gamma_{\{\alpha_1} \partial_{\alpha_2} \ldots \partial_{\alpha_n\}} \psi + \mathcal{O}(\rho^4/R^4)$$

Quark/antiquark operators $\mathcal{O}(1)$

$$(O^g_{\text{eff}})_{\alpha_1 \ldots \alpha_n} = 0 + \mathcal{O}(\rho^4/R^4)$$

Gluon operators suppressed in packing fraction



$\int dx\, x\, [q + \bar q](x)$ = 1: Momentum sum rule saturated by quarks + antiquarks…. consistent

Nucleon's antiquark content $\mathcal{O}(1)$, rich spin-flavor dependence [Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]

Twist-2: Instanton field subsumed in massive quarks/antiquarks, no direct effect

QCD operators: Twist-3 non-forward

$$O^{\alpha\beta}(x) = \bar{\psi}(x)\,\gamma^{[\alpha}\,\nabla^{\beta]}\tau\,\psi(x)$$

Twist-3 QCD operator, quark spin density in EMT
$\nabla \equiv \partial - iA$ contains gauge potential

$$= -\frac{1}{4}\,\epsilon^{\alpha\beta\gamma\delta}\,\partial_\gamma[\bar{\psi}(x)\,\gamma_\delta\gamma_5\tau\,\psi(x)]$$

Relation from QCD equations of motion $\hat{\nabla}\psi = 0$

↓ effective theory

$$O^{\alpha\beta}_{\mathrm{eff}}(x) = \bar{\psi}(x)\left(\gamma^{[\alpha}\partial^{\beta]}\tau + \frac{i}{4}M\sigma^{\alpha\beta}[U^{\gamma_5}(x),\tau]\right)\psi(x)$$

Effective operator in chiral theory (bosonized).
Instantons induce "potential" term containing interaction of massive quarks with chiral field

$$= -\frac{1}{4}\,\epsilon^{\alpha\beta\gamma\delta}\,\partial_\gamma[\bar{\psi}(x)\,\gamma_\delta\gamma_5\tau\,\psi(x)]_{\mathrm{eff}}$$

Same operator relation as in QCD, now from effective equations of motion $[\hat{\partial} - MU^{\gamma_5}]\,\psi = 0$

J-Y Kim, Weiss 2023

Similar effect in QCD operator with $\gamma^\alpha\gamma_5$ describing quark spin-orbit correlations   Lorce 2014

Twist-3 non-forward: Instanton field induces spin-flavor dependent chiral interactions, $\mathcal{O}(1)$ dynamical effect on quark spin density and spin-orbit correlations

QCD operators: F^2 and $F\tilde{F}$

$$\langle N' |\, F^2(0)\, | N \rangle \;=\; C(t)\, m_N \bar{u}' u$$

Nucleon form factor of F^2

$$C(0) \;=\; -\frac{32\pi^2}{b}$$

Relation from QCD trace anomaly $T_{\alpha\alpha}(x) = -\dfrac{b}{32\pi^2} F^2(x)$

$b = \frac{11}{3} N_c - \frac{2}{3} N_f$ beta function coefficient

[Shifman, Vainshtein, Zakharov 1978; Novikov SVZ 1981]

Realized in instanton vacuum! Effective operator $F^2 \propto N_+ + N_- \equiv N$ instanton number.
Fluctuations in ensemble controlled by trace anomaly $\langle N^2 \rangle - \langle N \rangle^2 = 4/b\, \langle N \rangle$ [Diakonov, Polyakov, Weiss, 1995]

$$\langle N' |\, F\tilde{F}(0)\, | N \rangle \;=\; \tilde{C}(t)\, m_N\, \bar{u}' i \gamma_5 u$$

Nucleon form factor of $F\tilde{F}$

$$\tilde{C}(0) \;=\; \frac{32\pi^2\, g_A^{(0)}}{N_f}$$

Relation from axial anomaly $\partial_\alpha J_{5\alpha}(x) = \dfrac{N_f}{16\pi^2} \tilde{F}F(x) + \mathcal{O}(m_f)$

Realized in instanton vacuum! Effective operator $F\tilde{F} \propto N_+ - N_-$ topological charge fluctuations

Instanton vacuum realizes low-energy theorems from trace and axial anomaly.
Framework suitable for analyzing hadronic form factors of $F^2, F\tilde{F}$

[See also Zahed 2021+]

Summary

- Topological fluctuations of gauge fields play essential role in low-energy QCD:
  Fermionic zero modes → chiral symmetry breaking → hadron structure/interactions

- Instanton vacuum provides effective description: Packing fraction as small parameter, all scales generated dynamically from Λ_{QCD}

- Instanton vacuum enables systematic derivation of effective dynamics from chiral symmetry breaking and effective operators representing QCD operators, including gluon operators: Equation-of-motion relations, scale and axial anomaly

- Twist-2: Instanton field subsumed in massive quarks/antiquarks, no direct effect

- Twist-3 non-forward: Instanton field produces spin-flavor dependent chiral interactions, $\mathcal{O}(1)$ effect

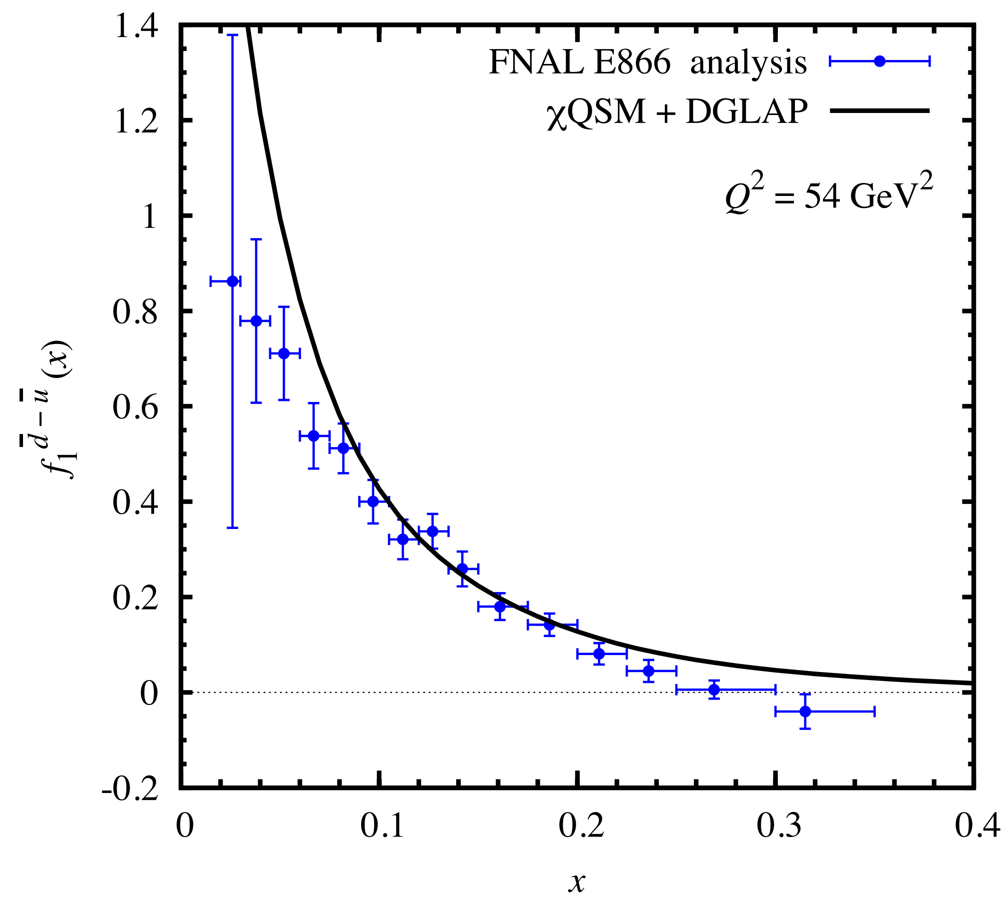
- Essential tool for exploring GPDs and gluonic structure of nucleon.
  In progress: Spin-orbit correlations, chiral-odd structures, nucleon form factors $F^2, F\tilde{F}$

- Other applications (not covered here): Twist-3,4 operators from DIS power corrections; higher-dim operators from BSM physics

- Beyond instantons: Include non-topological fluctuations of gauge fields (“$I\bar{I}$ molecules”), important for Wilson loops, coupling to heavy quarks [Shuryak, Zahed 2023]

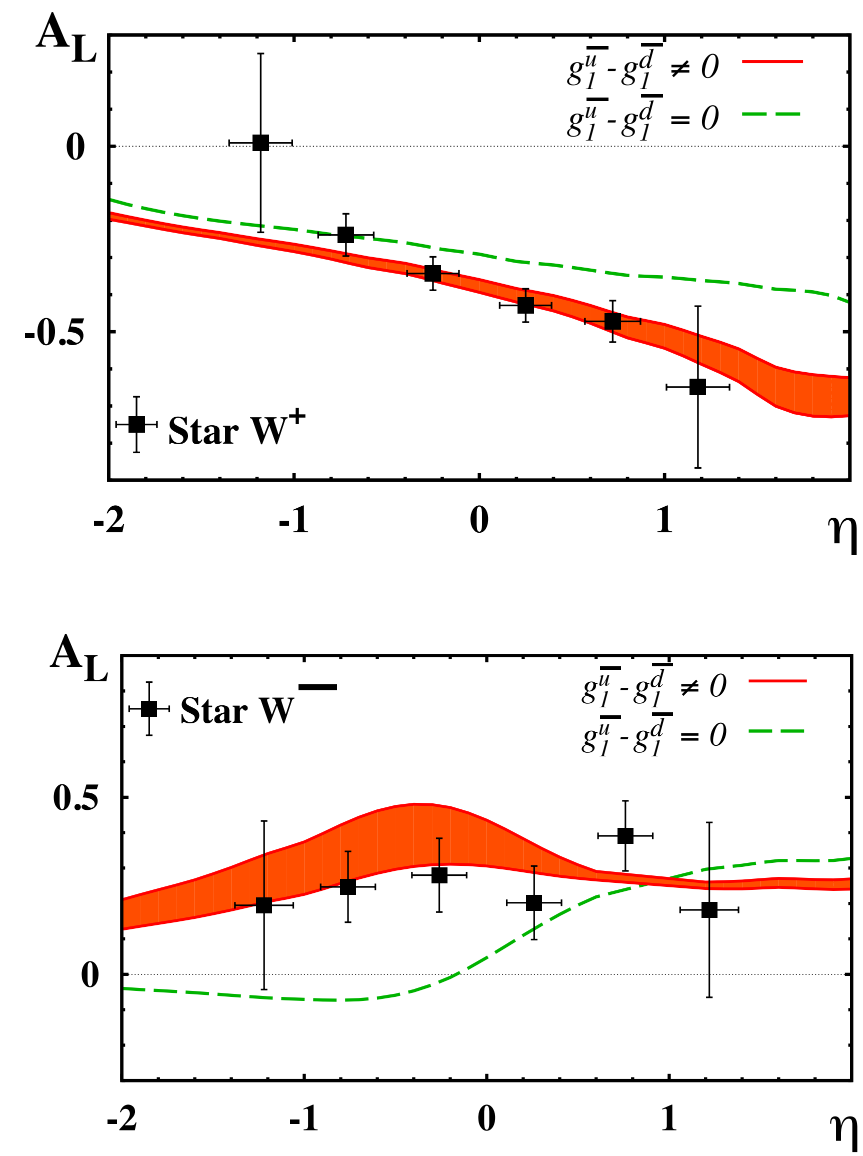
Supplemental material

QCD operators: Antiquark distributions in nucleon



Unpolarized flavor asymmetry $\bar{d} - \bar{u}$
from chiral soliton (← instanton vacuum)
[Pobylitsa, Polyakov, Goeke, Watabe, Weiss, 1998]

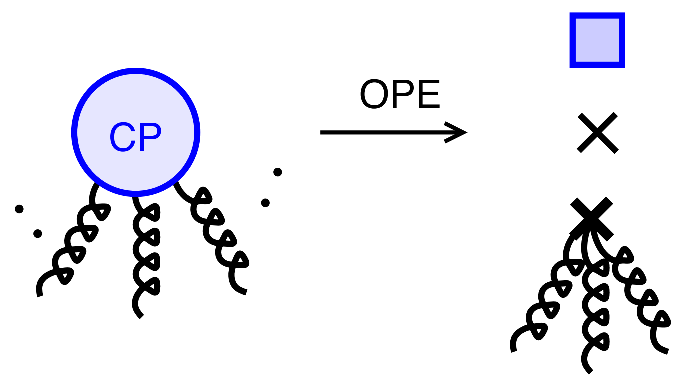
FNAL E866 Drell-Yan data + analysis



Polarized flavor asymmetry $\Delta\bar{u} - \Delta\bar{d}$ predicted
by chiral soliton (← instanton vacuum)
[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]

RHIC $W^{\pm}$ production data
[Adamczyk et al (STAR) 2014]

QCD operators: $\tilde{F}FF$



Dimension-6 gluon operator

$f^{abc}\tilde{F}^a_{\mu\nu}F^b_{\mu\rho}F^c_{\nu\rho}(x)$ Dimension-6 CP-odd gluon operator, essentially non-abelian structure

Appears in scenario of hadronic CP violation [Weinberg 89]

Need estimates of hadronic matrix elements!
[Bigi, Uraltsev 1991, Hatta 2020]

Instanton vacuum estimate

$$\frac{\int d^4x\,\tilde{F}FF(x)_{I(\bar{I})}}{\int d^4x\,\tilde{F}F(x)_{I(\bar{I})}} = -\frac{12}{5\bar{\rho}^2}$$

Operators $\tilde{F}FF$ and $\tilde{F}F$ proportional in field of single $I(\bar{I})$, effective operators simply related
[Weiss 2021]

$$\frac{A_{\tilde{F}FF}(0)}{32\pi^2} = \left(-\frac{12}{5\bar{\rho}^2}\right)\frac{g_A^{(0)}}{N_f}$$

Nucleon matrix element of $\tilde{F}FF$ inferred from $\tilde{F}F \leftrightarrow g_A^{(0)}$

$$\frac{12}{5\bar{\rho}^2} = 0.86\,\text{GeV}^2 = (0.22\,\text{fm})^{-2}$$

Large numerical value due to localization of instanton field

Further conclusion (paradox): Neutron EDM induced by $\tilde{F}FF$ is proportional to that induced by $\tilde{F}F$ and therefore chirally suppressed [Chiral behavior of neutron EDM: Crewther, DiVecchia, Veneziano, Witten 1979]