Gravitational form factors on the lattice

RBRC Workshop on GPDs for Nucleon Tomography in the EIC Era

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Outline

Gravitational structure of the nucleon

- Gravitational form factors (GFFs)?
- Why are GFFs interesting?

GFFs on the lattice

- Overview of calculation

Results

- GFFs of proton (w/ flavor decomp)
- Experimental comparison

Tomography

- Mechanical densities & radii

2307.11707

Gravitational form factors of the pion from lattice QCD

Daniel C. Hackett, Patrick R. Oare, Dimitra A. Pefkou, and Phiala E. Shanahan
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

The two gravitational form factors of the pion, $A^\pi(t)$ and $D^\pi(t)$, are computed as functions of the momentum transfer squared t in the kinematic region $0 \leq -t < 2$ GeV^2 on a lattice QCD ensemble with quark masses corresponding to a close-to-physical pion mass $m_\pi \approx 170$ MeV and $N_f = 2+1$ quark flavors. The flavor decomposition of these form factors into gluon, up/down light-quark, and strange quark contributions is presented in the $\overline{\text{MS}}$ scheme at energy scale $\mu = 2$ GeV, with renormalization factors computed non-perturbatively via the RI-MOM scheme. Using monopole and z-expansion fits to the gravitational form factors, we obtain estimates for the pion momentum fraction and D-term that are consistent with the momentum fraction sum rule and the next-to-leading order chiral perturbation theory prediction for $D^\pi(0)$.

2310.08484

Gravitational form factors of the proton from lattice QCD

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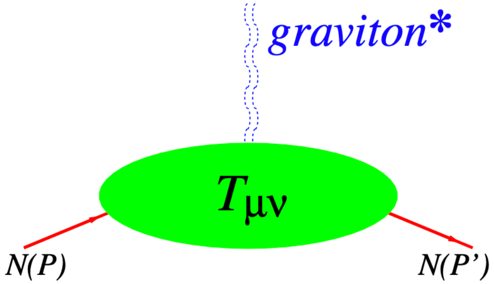
The gravitational form factors (GFFs) of a hadron encode fundamental aspects of its structure, including its shape and size as defined from e.g., its energy density. This work presents a determination of the flavor decomposition of the GFFs of the proton from lattice QCD, in the kinematic region $0 \leq -t \leq 2$ GeV^2. The decomposition into up-, down-, strange-quark, and gluon contributions provides first-principles constraints on the role of each constituent in generating key proton structure observables, such as its mechanical radius, mass radius, and D-term.

Gravitational structure of hadrons

Gravitational form factors (GFFs)



GFFs are EMT form factors

$$T^{\{\mu\nu\}} = 2\,\mathrm{Tr}\left[-G^{\alpha\mu}G_{\alpha}^{\nu} + \frac{1}{4}g^{\mu\nu}G^{\alpha\beta}G_{\alpha\beta}\right] + \bar{q}\,\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\,q$$

Nucleon:

$$\langle N(p')|T^{\{\mu\nu\}}|N(p)\rangle = \bar{U}(p')\left[A(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M}\right]U(p)$$

Why are these interesting?

$a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$
$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$
$U, \bar{U}$ = Dirac spinors
$P = (p' + p)/2$
$\Delta = p' - p$
$t = \Delta^2$

Global properties

$$\langle N(p')|T^{\{\mu\nu\}}|N(p)\rangle = \bar{U}(p')\left[A(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M}\right]U(p)$$

$\partial_\mu T^{\mu\nu} = 0 \rightarrow$ GFFs are scale- and scheme-independent

Forward GFFs are fundamental, global properties:

$A(0) = 1 \Leftrightarrow \langle p|T^{tt}|p\rangle = M$

$J(0) = \frac{1}{2} =$ Total spin

$B(0) = 2J(0) - A(0) = 0$ "vanishing of the anomalous gravitomagnetic moment"

$D(0) =$???* (internal forces)

Flavor decomposition

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{q}\,\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\,q$

Momentum fraction

$A_{q,g}(0) = \langle x \rangle_{q,g}$

$A_g(0) + \sum_q A_q(0) = 1$

Spin fraction

$J = (A+B)/2$

$J_g(0) + \sum_q J_q(0) = \frac{1}{2}$

$$\left\langle N(p') \middle| T_{g,q}^{\{\mu\nu\}} \middle| N(p) \right\rangle = \bar{u}(p') \left[A_{g,q}(t) \gamma^{\{\mu}P^{\nu\}} + B_{g,q}(t) \frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t) \frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t) M g^{\mu\nu} \right] u(p)$$

Internal forces

$D(0) = D_g(0) + \sum_q D_q(0)$

Not conserved $\sum_q \bar{c}_q + \bar{c}_g = 0$

Power-divergent mixing

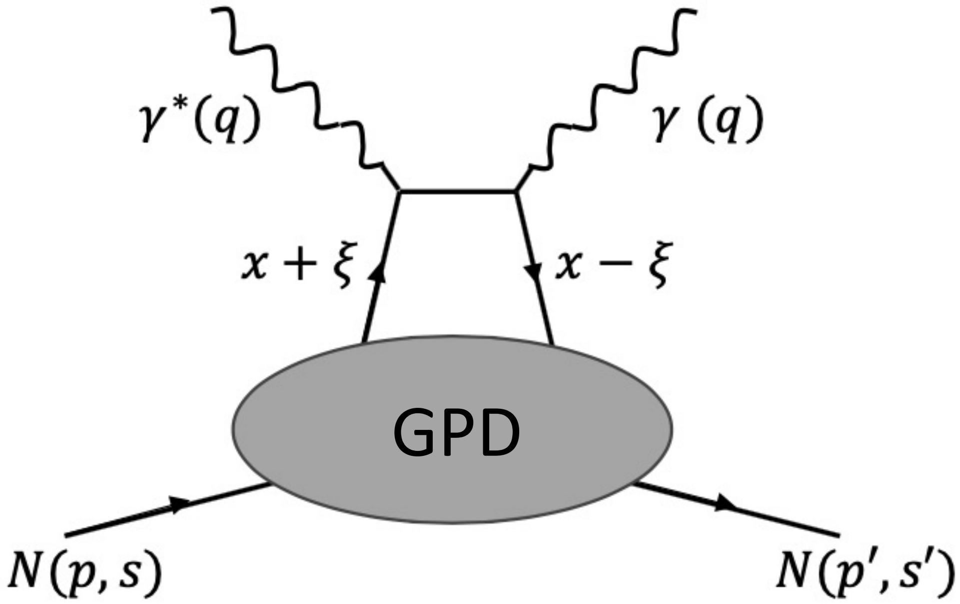
Relation to GPDs

GFFs ⊂ GFFs

Gravitational Form Factors → GFFs (left); Generalized Form Factors → GFFs (right)

Gravitational
Form Factors

Generalized
Form Factors



GFFs are Mellin moments of GPDs, e.g.

$$\int dx\, x\, H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t) \qquad \int dx\, H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)$$

$$\int dx\, x\, E_q(x,\xi,t) = B_q(t) - \xi^2 D_q(t) \qquad \int dx\, E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

→ relate to experiment via factorization

GFFs on the lattice

General idea: bare matrix elements

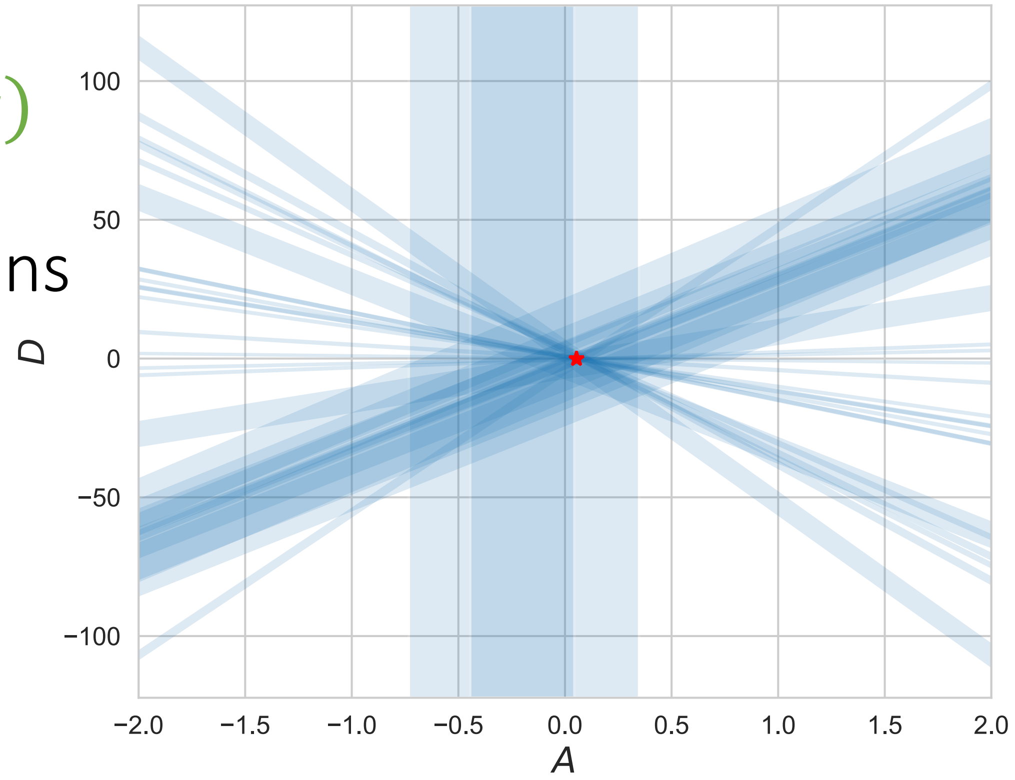
A three-point function for $\Delta = p' - p$

$$\langle \chi(p', t_f)\, T^{\mathrm{b}}(\Delta, \tau)\, \bar{\chi}(p, 0) \rangle \sim Z_{p'}\, Z_p\, \langle p' | T^{\mathrm{b}}(\Delta) | p \rangle\, e^{-E'(t_f - \tau) - E\tau} + (\text{excited states})$$

constrains the bare GFFs at $t = \Delta^2$

$$\langle p' | T^{\mathrm{b}}(\Delta) | p \rangle = c_A\, A^{\mathrm{b}}(t) + c_J\, J^{\mathrm{b}}(t) + c_D\, D^{\mathrm{b}}(t)$$

$\Rightarrow$ measure and analyze many three-point functions



General idea: renormalization

(Flavor singlet) EMTs mix & renormalize multiplicatively

$q = u + d + s$

$$\begin{bmatrix} T_q^{\overline{MS}} \\ T_g^{\overline{MS}} \end{bmatrix} = \begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix} \begin{bmatrix} T_q^{\text{bare}} \\ T_g^{\text{bare}} \end{bmatrix}$$

Assert RI-MOM conditions at scale $\mu^2 = p^2$

$$\langle q(p)\, T_f(0)\, \bar{q}(p) \rangle_{\text{lattice}} = Z_q\, R_{fq}^{\text{RI}}\ \langle q(p)\, T_{q,g}(0)\, \bar{q}(p) \rangle_{\text{tree}}$$

$$\langle A(p)\, T_f(0)\, A(p) \rangle_{\text{lattice}} = Z_g\, R_{fg}^{\text{RI}}\ \langle A(p)\, T_{q,g}(0)\, A(p) \rangle_{\text{tree}}$$

…then apply perturbative matching to $\overline{\text{MS}}$ and run to $\mu = 2\ \text{GeV}$

Ensembles

Gauge action: tadpole-improved Luscher-Weisz

Fermion action: $2+1$ flavors, stout-smeared clover

	L/a	T/a	β	am_l	am_s	a [fm]	m_π [MeV]
A	48	96	6.3	-0.2416	-0.2050	0.091(1)	169(1)
B	12	24	6.1	-0.2800	-0.2450	0.1167(16)	450(5)

Bare matrix elements

Glue: 2511 configs

Quarks: 1381 configs (subset)

["a091m170" (JLab/W&M/MIT/LANL)]

Renormalization

Conn. quark: 240 configs

Disco./glue: 20000 configs

Lattice EMT operators

Quark: $T_q^{\{\mu\nu\}} = \bar{q}\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}} q$

Discretized covariant derivative

$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

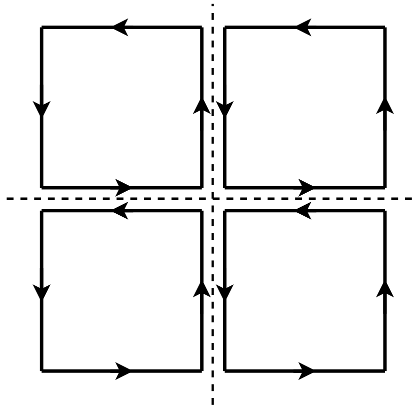
$(\overrightarrow{D}_\mu \psi)(x) = \frac{1}{2}\left[U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu)\right]$

$(\bar{\psi}\overleftarrow{D}_\mu)(x) = \frac{1}{2}\left[\bar{\psi}(x+\mu)U_\mu^\dagger(x) - \bar{\psi}(x-\mu)U_\mu(x-\mu)\right]$

Glue: $T_g^{\{\mu\nu\}} = \frac{2}{g^2}\,\mathrm{Tr}[G^{\alpha\{\mu} G^{\nu\}\alpha}]$

Clovers flowed to $t/a^2 = 2$

$G_{\mu\nu} \sim \left(Q_{\mu\nu} - Q_{\mu\nu}^\dagger\right)$

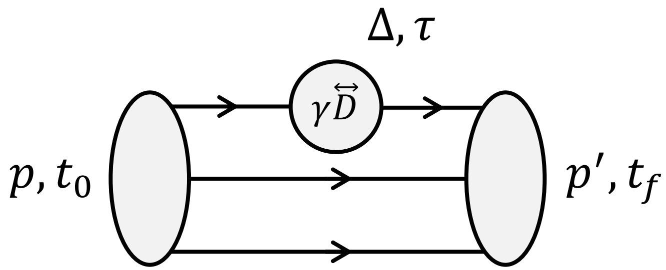


Operator Bases

$\tau_1^{(3)}$: $\frac{1}{2}(T^{xx} + T^{yy} - T^{zz} + T^{tt})$, $\frac{1}{\sqrt{2}}(T^{zz} + T^{tt})$, $\frac{1}{\sqrt{2}}(T^{xx} - T^{yy})$

$\tau_3^{(6)}$: $\left\{\frac{i^{\delta_{\mu 0}}}{\sqrt{2}}(T^{\mu\nu} + T^{\nu\mu}),\ 0 \le \mu \le \nu \le 3\right\}$

Three-point functions

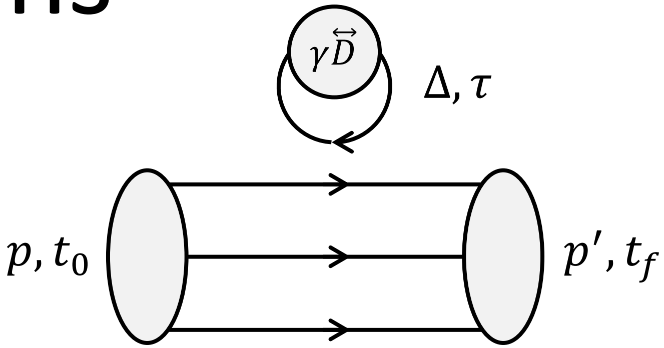


Connected Quark (u, d)

Sequential source (thru sink)

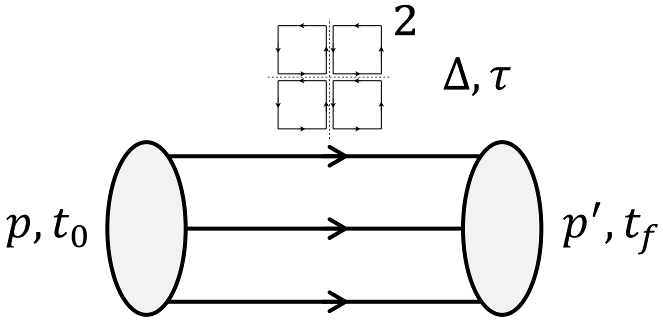
- 3 sink momenta
- Nucleon: 1 spin channel
- Sources / cfg varies w/ t_f

t_s	6	7	8	9	10	11	12	13	14	16	18
N_s	9	16	16	16	16	16	16	16	32	32	32



Disconnected Quark $(u = d, s)$

- 1024 sources / cfg
- 4 spin channels
- Hierarchical probing w/ 512 Hadamard vectors
- 2 Z_4 noise shots / cfg



Glue (disconnected)

- 1024 sources / cfg
- 4 spin channels

Strategy: use all available data!

- all $p^2 \leq 10\,(2\pi/L)^2$
- all $\Delta^2 \leq 25(2\pi/L)^2$
- all operators

Extract bare matrix elements

1. Construct ratios

$$R(p, p'; \tau, t_f) = \frac{C^{3\text{pt}}(p, p'; t_f, \tau)}{C^{2\text{pt}}(p'; t_f)} \sqrt{\frac{C^{2\text{pt}}(p; t_f - \tau)}{C^{2\text{pt}}(p'; t_f - \tau)} \frac{C^{2\text{pt}}(p'; \, t_f)}{C^{2\text{pt}}(p; t_f)} \frac{C^{2\text{pt}}(p'; \, \tau)}{C^{2\text{pt}}(p; \tau)}}$$

$$= \# \, \langle p' | T | p \rangle + O\left(e^{-\Delta E \, \tau - \Delta E' (t_f - \tau)}\right)$$

2. Bin ratios together w/ same kinematic coeffs →

Number of distinct ratios:

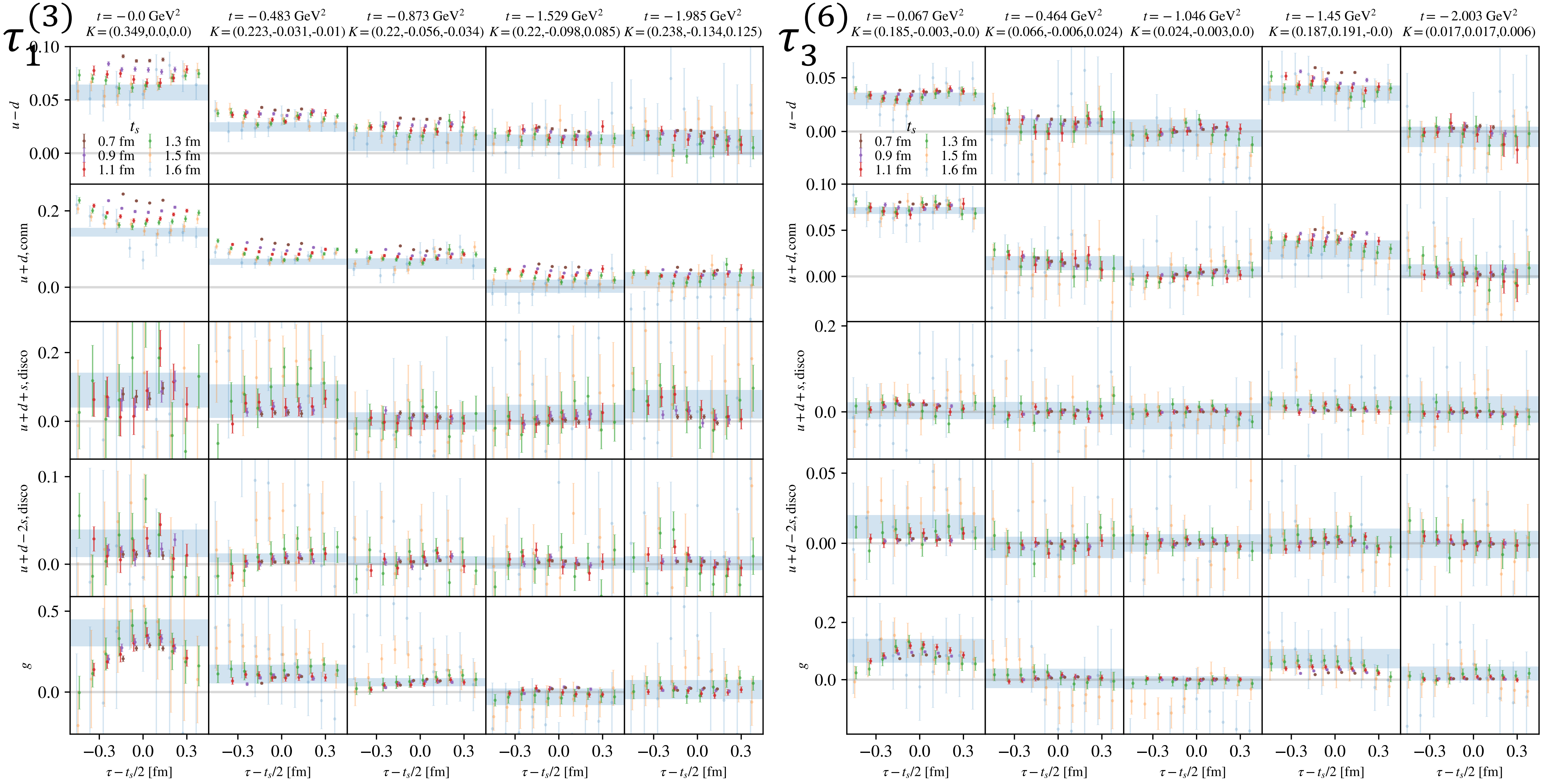
conn q:	6982 →	3081
disc q/g:	1200296 →	11452

3. Fit using summation method

$$\Sigma(t_f) = \sum_{\tau = \tau_{\text{cut}}}^{t_f - \tau_{\text{cut}}} R(\tau, t_f) = (\text{const}) + \# \, \langle p' | T | p \rangle \, t_f + O\left(e^{-\delta E \, t_f}\right)$$

... w/ Bayesian model averaging over fit ranges, τ_{cut}

Example nucleon ratios



R

Renormalization

$$R_{qq}^{\mathrm{RI}}(\mu_R^2) = \frac{C_{q,\mu\nu}^{q,\mathrm{amp}}}{Z_q \Lambda_{\mu\nu}^q}\bigg|_{\tilde{p}^2=\mu_R^2}$$

$$R_{gg}^{\mathrm{RI}}(\mu_R^2) = \frac{C_{g,\mu\nu\alpha\beta}^{g,\mathrm{amp}}}{Z_g \Lambda_{\mu\nu\alpha\beta}^g}\bigg|_{\tilde{p}_\alpha=0,\tilde{p}^2=\mu_R^2}^{\alpha=\beta,\alpha\neq\mu,\alpha\neq\nu}$$

$$R_{qg}^{\mathrm{RI}}(\mu_R^2) = \frac{C_{q,\mu\nu\alpha\beta}^{g,\mathrm{amp}}}{Z_g \Lambda_{\mu\nu\alpha\beta}^g}\bigg|_{\tilde{p}_\alpha=0,\tilde{p}^2=\mu_R^2}^{\alpha=\beta,\alpha\neq\mu,\alpha\neq\nu}$$

$$R_{gq}^{\mathrm{RI}}(\mu_R^2) = \frac{C_{g,\mu\nu}^{q,\mathrm{amp}}}{Z_q \Lambda_{\mu\nu}^q}\bigg|_{\tilde{p}^2=\mu_R^2}$$

$$R_{v}^{\mathrm{RI}}(\mu_R^2) = \frac{C_{v,\mu\nu}^{q,\mathrm{amp}}}{Z_q \Lambda_{\mu\nu}^q}\bigg|_{\tilde{p}^2=\mu_R^2}$$

Compute amputated 3pt functions $C^{\cdots,amp}$ → RI factors:

- Landau gauge
- Flow to $t/a^2 = 1.2$ to match physical extent

Matching to $\overline{MS}$ gives:

$$\left(Z_v^{\overline{MS}}\right)^{-1}(\mu^2) = C_v^{\mathrm{RI}/\overline{MS}}(\mu^2, \mu_R^2)\, R_v^{\mathrm{RI}}(\mu_R^2)$$

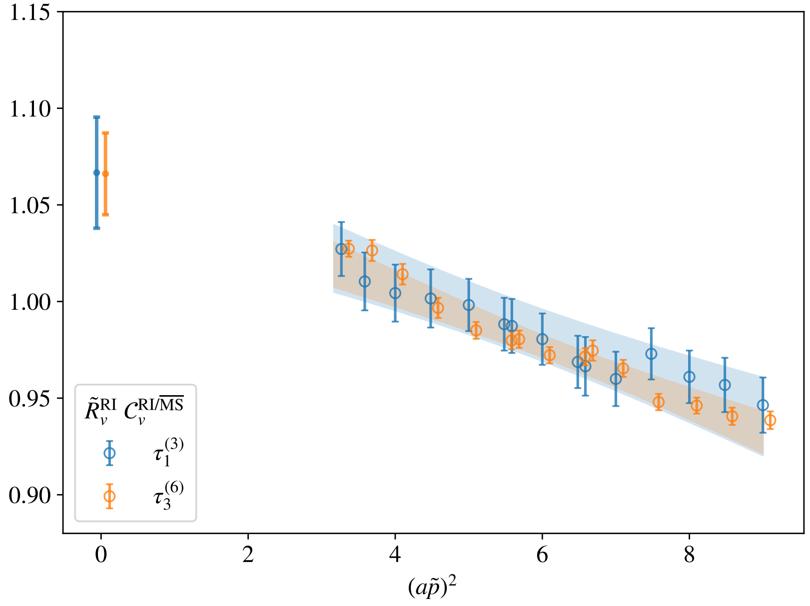
$$\begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix}^{-1}(\mu^2) = \begin{bmatrix} R_{qq}^{\mathrm{RI}} & R_{qg}^{\mathrm{RI}} \\ R_{gq}^{\mathrm{RI}} & R_{gg}^{\mathrm{RI}} \end{bmatrix}(\mu_R^2) \begin{bmatrix} C_{qq}^{\mathrm{RI}/\overline{MS}} & C_{qg}^{\mathrm{RI}/\overline{MS}} \\ C_{gq}^{\mathrm{RI}/\overline{MS}} & C_{gg}^{\mathrm{RI}/\overline{MS}} \end{bmatrix}(\mu^2, \mu_R^2)$$

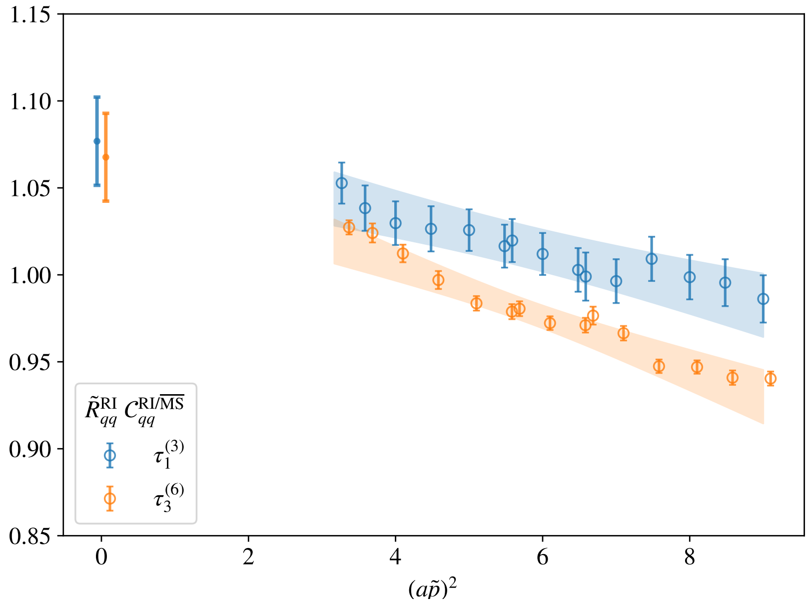
$\Lambda^{q,g}$ ~ tree-level 3pts
$q = u + d + s$
$v = u + d - 2s$

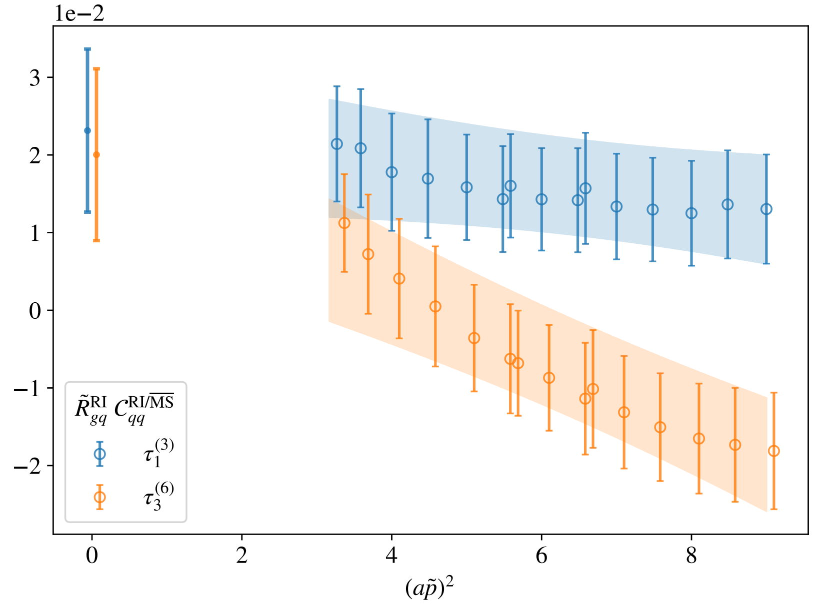
Model and fit residual $(ap)^2$ dependence in each of product $R^{\mathrm{RI}}\, C^{\mathrm{RI}/\overline{MS}}$

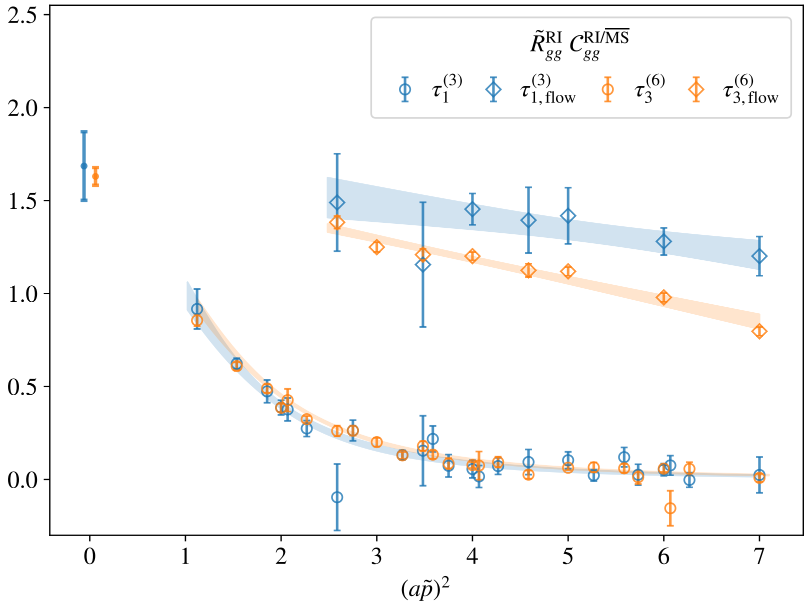
Fitting for renormalization coeffs

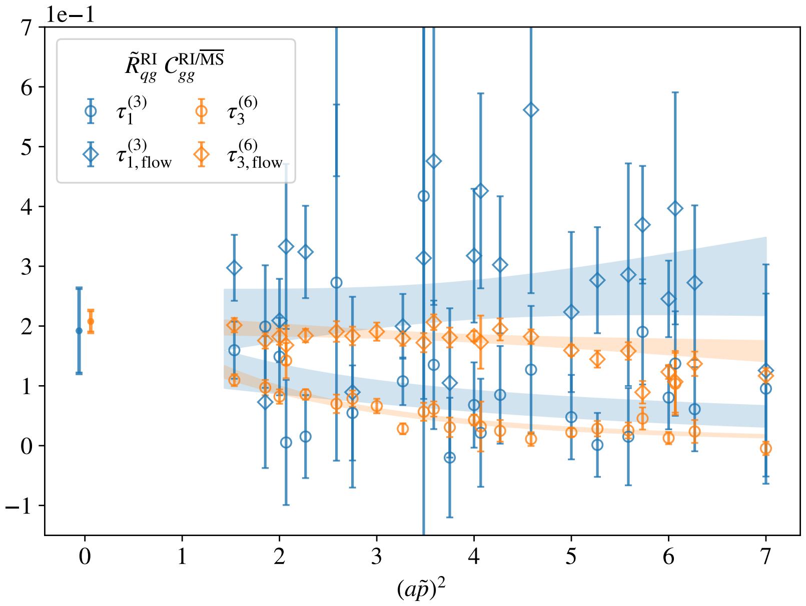
Model discretization artifacts as polynomials, inverse polynomials







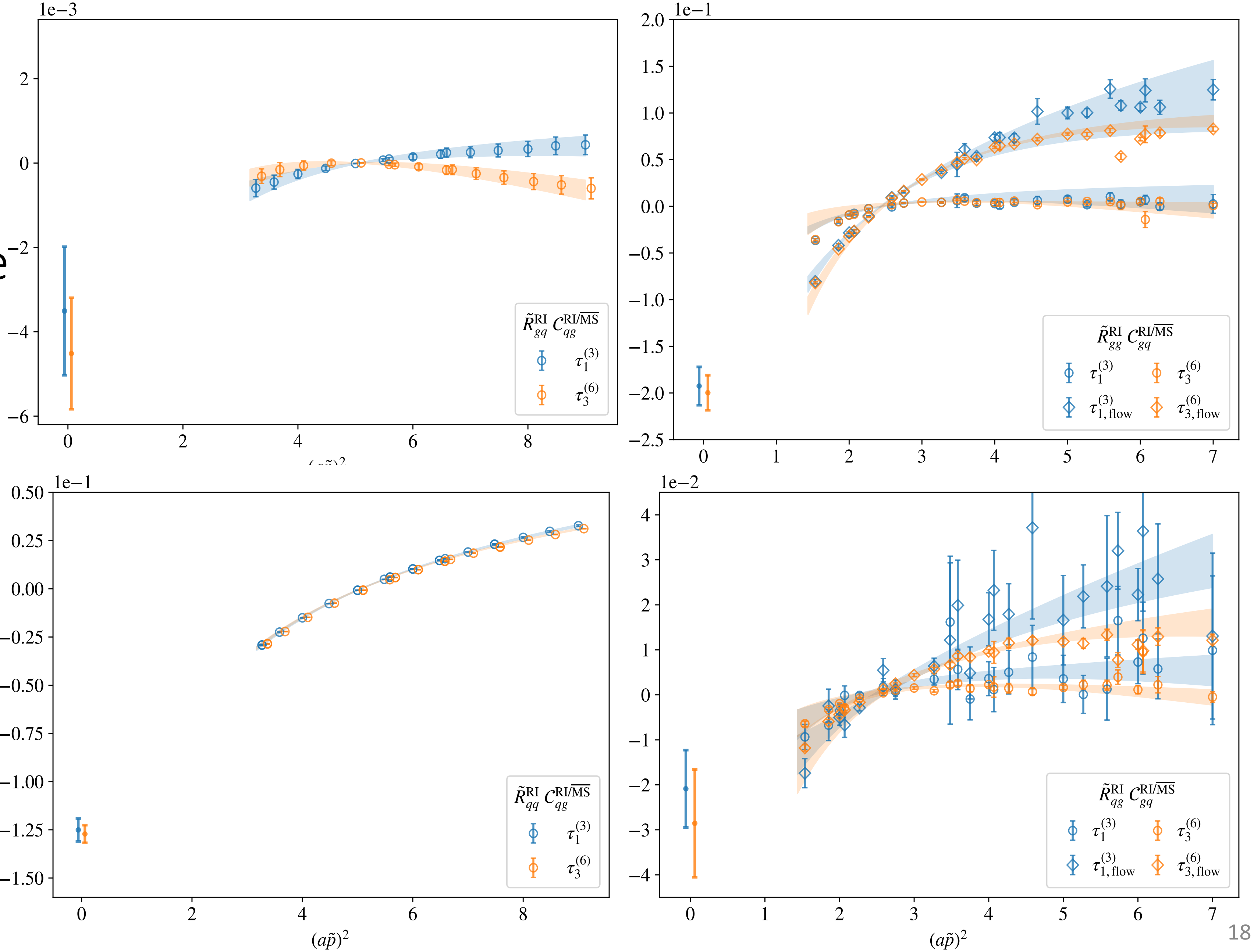




Fitting for renormalization coeffs

Model discretization artifacts as polynomials, inverse polynomials

+ logs for nonperturbative effects



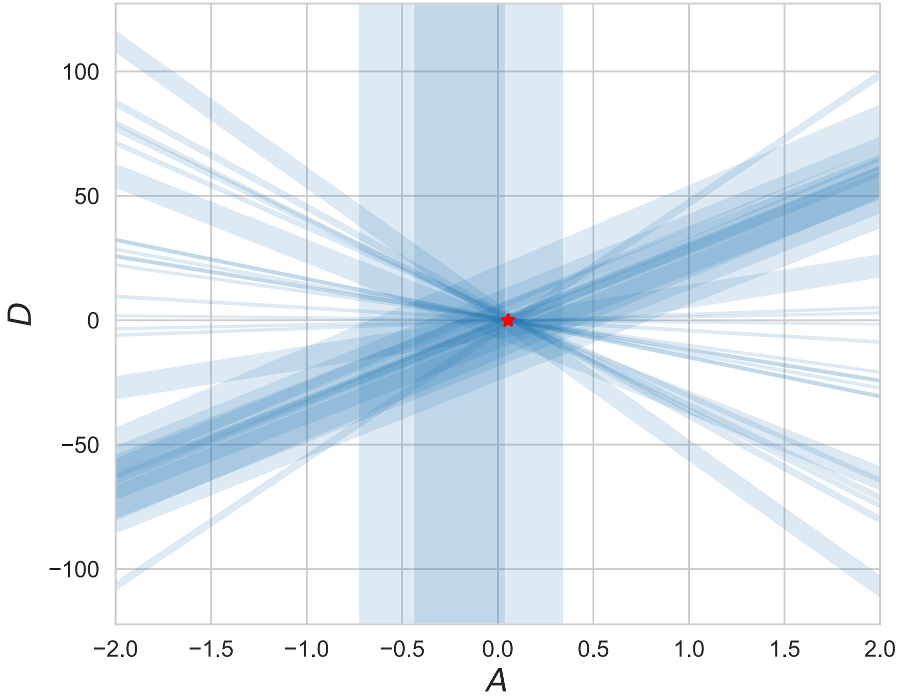
Computing the GFFs

Have:

- Bare matrix elements for g, u, d, s, binned by t
- Singlet mixing matrix + non-singlet Z factor

→ Renormalized set of linear constraints on GFFs in each t-bin

Fit to extract GFFs at discrete values of t



Fit GFFs to model functions

n-pole $G(t) \sim \frac{\alpha}{(1-t/\Lambda^2)^n}$

z-expansion $G(t) \sim \sum_{k=0}^{k_{\max}=2} \alpha_k [z(t)]^k$

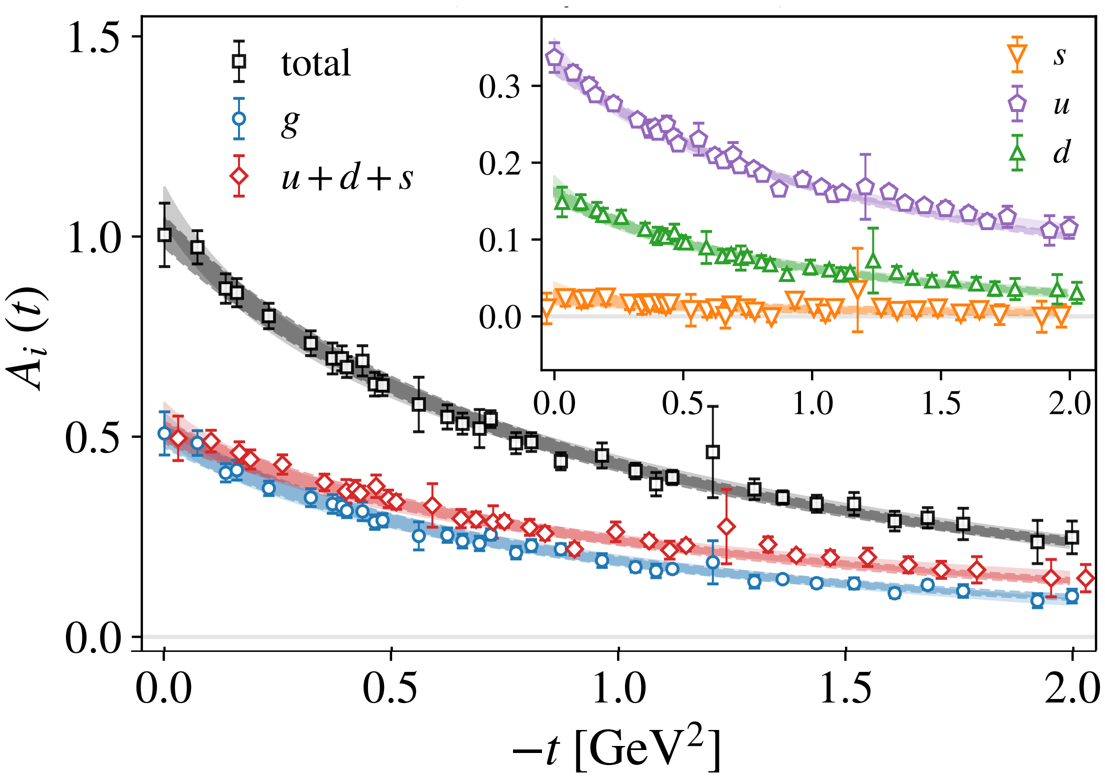
$z(t) = \frac{\sqrt{t_{\text{cut}}-t} - \sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}-t} + \sqrt{t_{\text{cut}}-t_0}}$ $\quad t_{\text{cut}} = 4M_\pi^2$ $\quad t_0 = t_{\text{cut}}\left(1 - \sqrt{1 + (2\ \text{GeV}^2)}\right)$

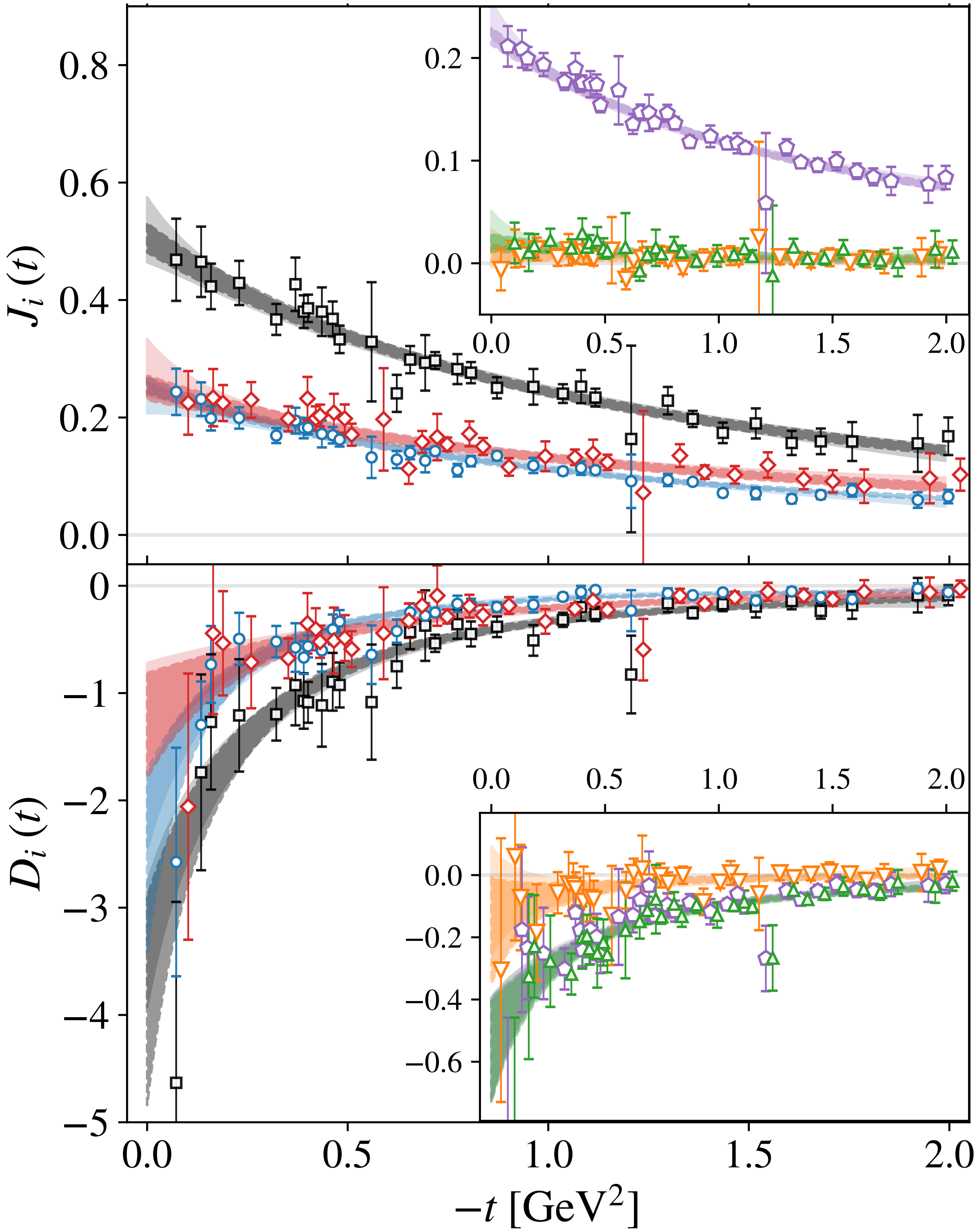
Results

Nucleon GFFs

Dark bands: dipole

Light bands: z-expansion





Forward limits

	Dipole			z-expansion		
	A_i	J_i	D_i	A_i	J_i	D_i
u	0.3255(92)	0.2213(85)	−0.56(17)	0.349(11)	0.238(18)	−0.56(17)
d	0.1590(92)	0.0197(85)	−0.57(17)	0.171(11)	0.033(18)	−0.56(17)
s	0.0257(95)	0.0097(82)	−0.18(17)	0.032(12)	0.014(19)	−0.08(17)
$u+d+s$	0.510(25)	0.251(21)	−1.30(49)	0.552(31)	0.286(48)	−1.20(48)
g	0.501(27)	0.255(13)	−2.57(84)	0.526(31)	0.234(27)	−2.15(32)
Total	1.011(37)	0.506(25)	−3.87(97)	1.079(44)	0.520(55)	−3.35(58)

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Sum rules (consistency check)

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Sum rules (consistency check)

cf. global fit result

$A_g(0) = 0.414(8)$

[Hou et al. 1912.10053]

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Sum rules (consistency check)

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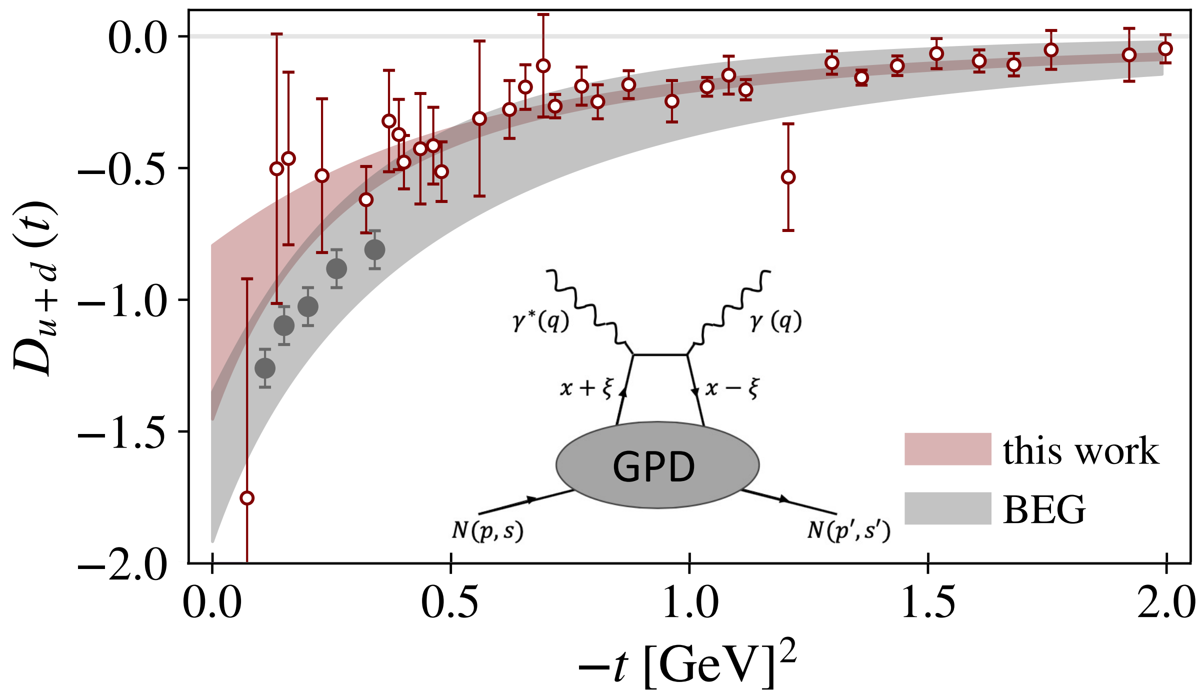
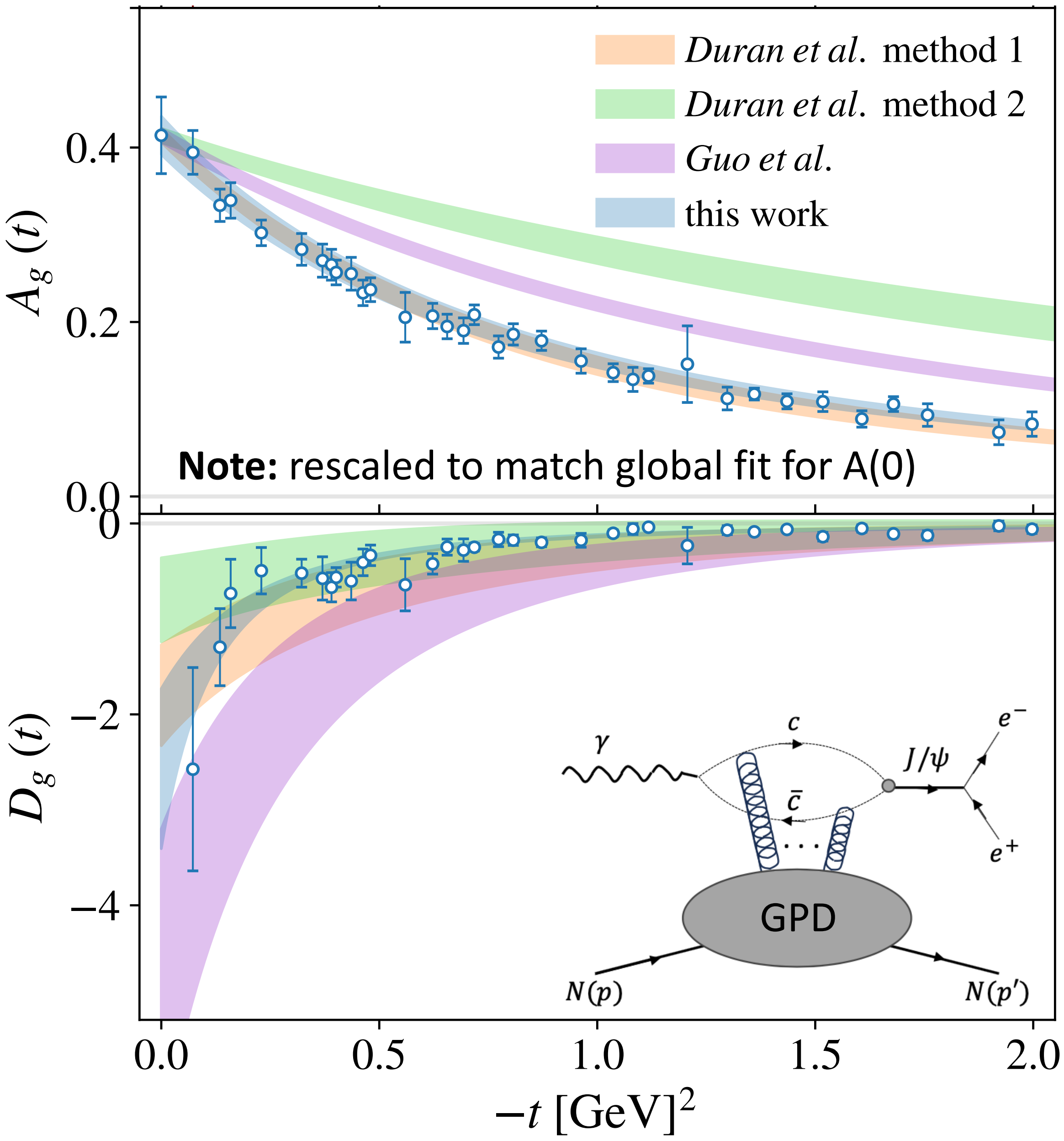
[Hou et al. 1912.10053]

First determination!
Satisfies χPT bound
$D(0)/M \leq -1.1(1)\ \mathrm{GeV}^{-1}$

Nucleon vs. experiment





BEG = [Burkert Elouadrhiri Girod 2018] (DVCS)

[Duran et al. 2207.05212] (J/ψ)

- Method 1: holographic QCD (Mamo Jahed, PRD 21,22)
- Method 2: GPD (Guo Ji Liu, PRD 2021)

[Guo et al. 2305.06992]

- Updated GPD analysis + GlueX data

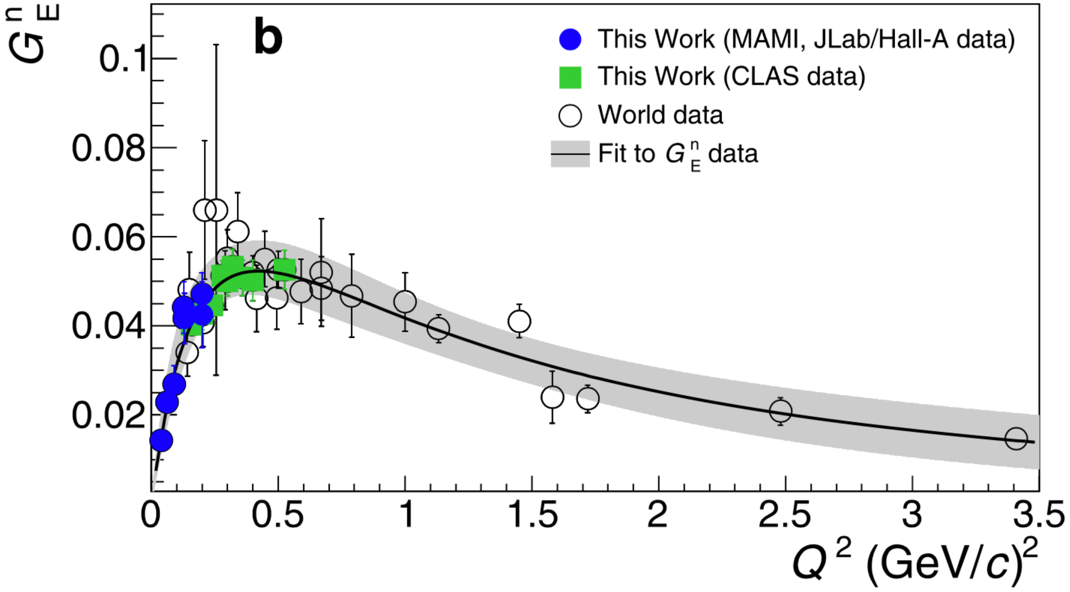
Tomography

(G)FFs and Tomography

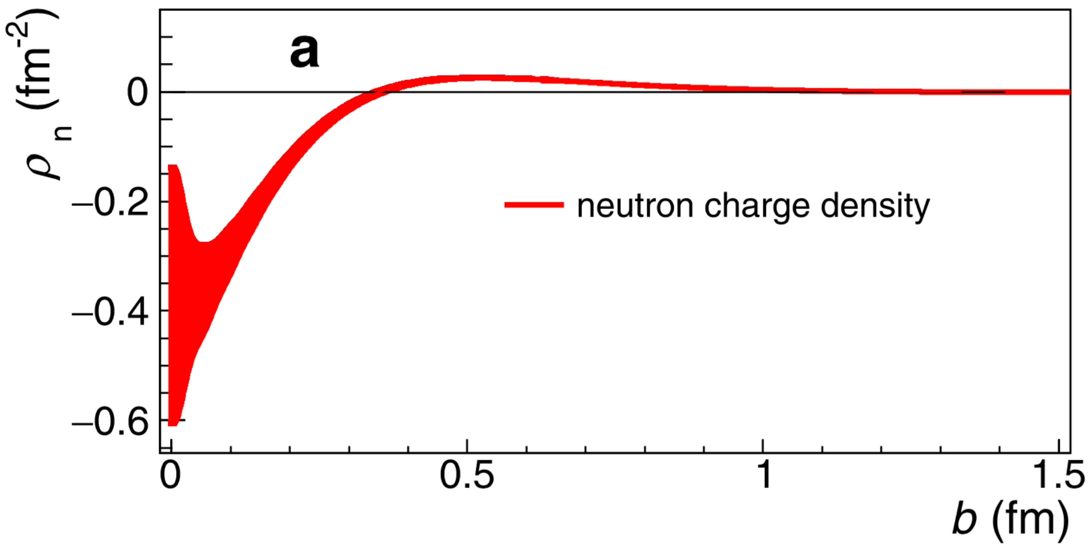
Fourier-transformed form factors provide information about spatial densities

Example: electric charge density in the neutron from G_E^n

[Atac, Constantinou, Meziani, Paolone, Sparveris 2103.10840]



Fourier transform →



Applies also for GFFs → mechanical densities

Mechanical densities from GFFs

$$[f(t)]_{\mathrm{FT}} = \int \frac{d^3\mathbf{\Delta}}{(2\pi)^3} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} f(t)$$

1. Parametrize $T_{\mu\nu}(t)$ with GFFs
2. Fourier transform $T_{\mu\nu}(t) \to T_{\mu\nu}(r)$
3. Identify

$$T_{\mu\nu}(r) = \begin{bmatrix} T_{tt}(r) & \\ & T_{ij}(r) \end{bmatrix} = \begin{bmatrix} \epsilon(r) & \\ & \left(\frac{r_i r_j}{r^2} - \frac{1}{d}\delta_{ij}\right) s(r) + \delta_{ij} p(r) \end{bmatrix}$$

→ Spatial densities

energy $\epsilon(r) = M\left[A(t) - \frac{t}{4M^2}\left(D(t) + A(t) - 2J(t)\right)\right]_{FT}$

pressure $p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr} [D(t)]_{FT}$

shear forces $s(r) = -\frac{1}{4M} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr} [D(t)]_{FT}$

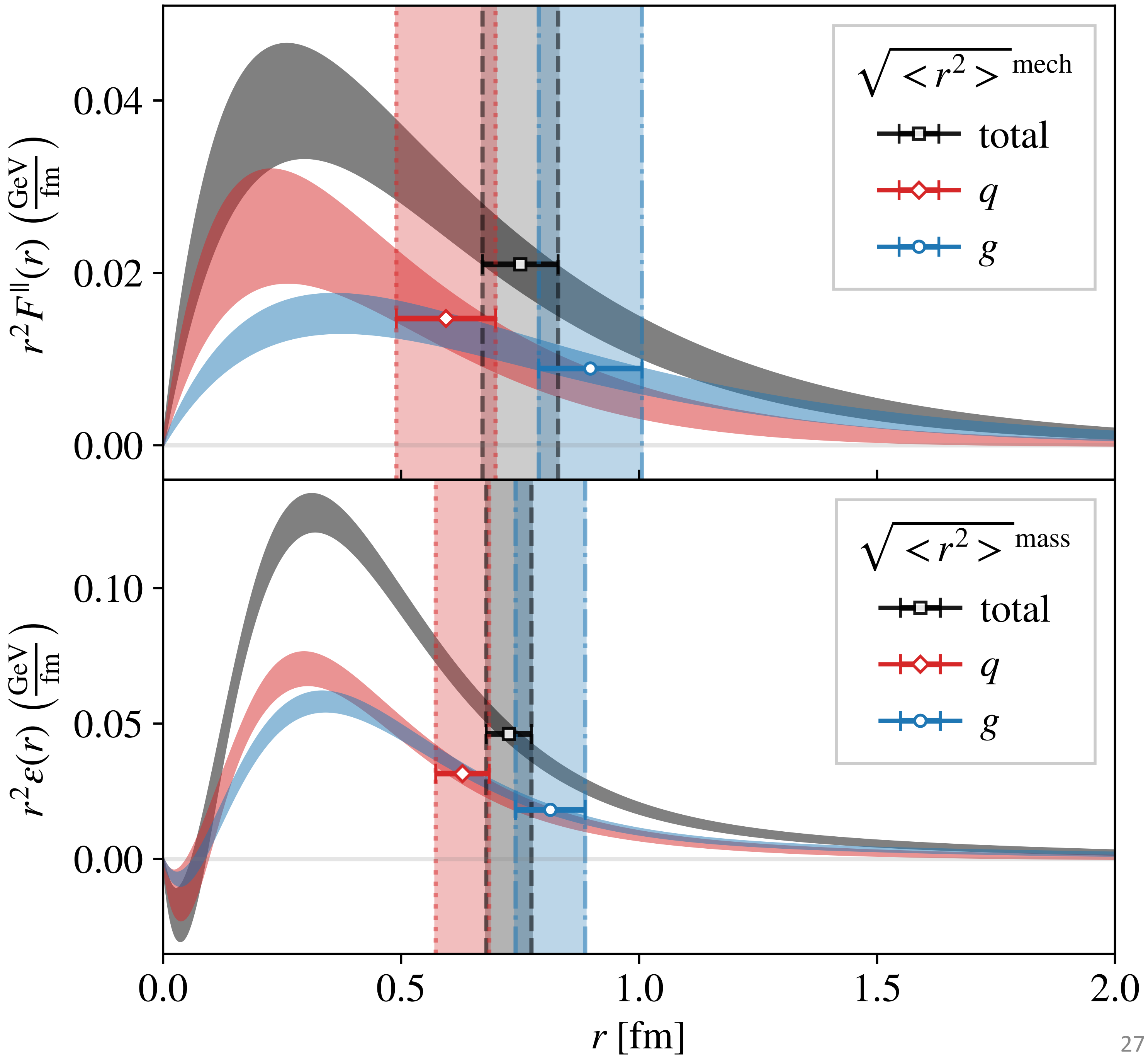
longitudinal force $F^{\parallel}(r) = p(r) + 2s(r)/3$

Caveat: physical significance of these analogies is under debate

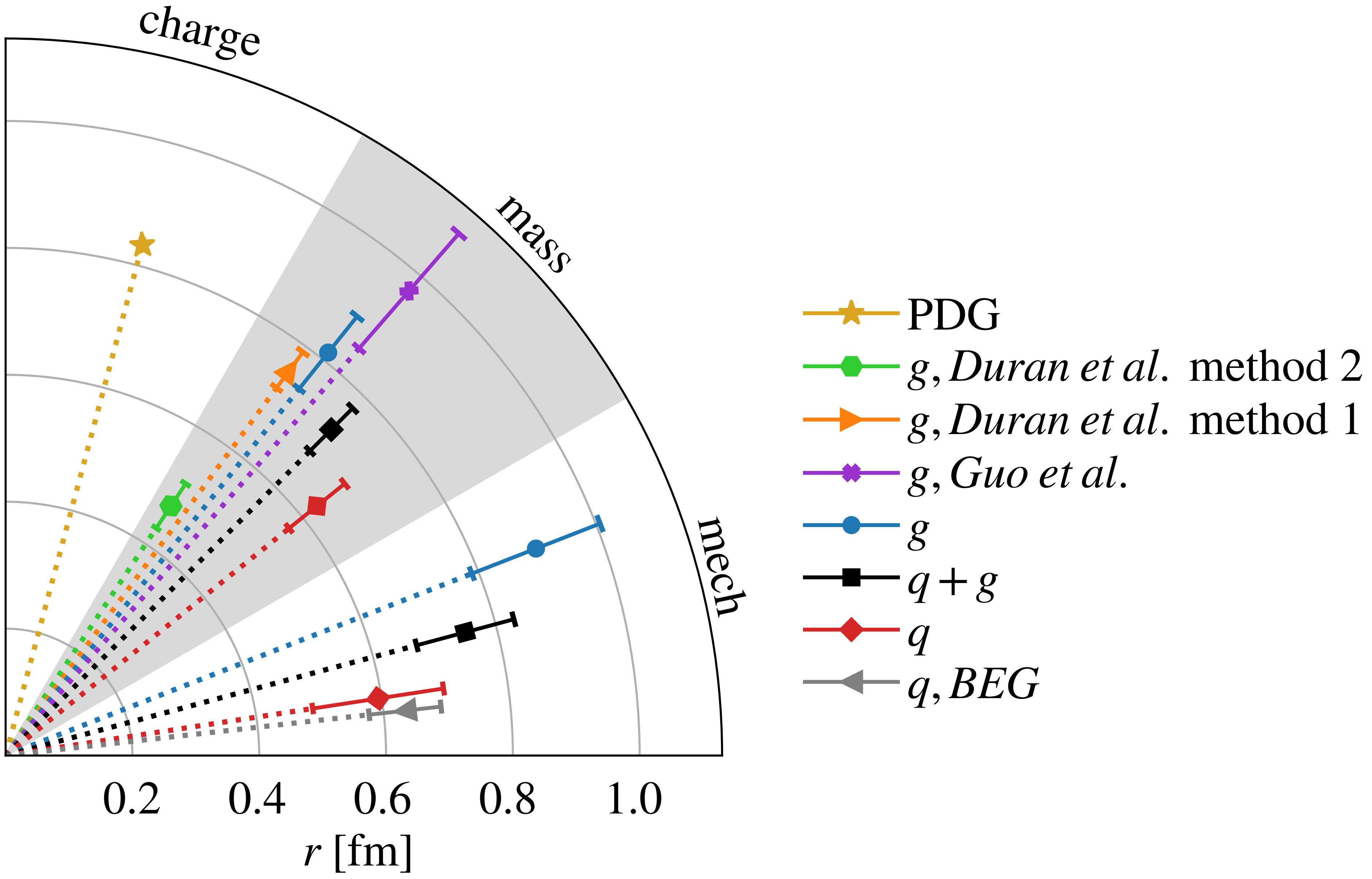
Densities & radii

$$\langle r_i^2 \rangle^{\text{mass}} = \frac{\int d^3\mathbf{r}\ r^2\, \epsilon_i(r)}{\int d^3\mathbf{r}\ \epsilon_i(r)}$$

$$\langle r_i^2 \rangle^{\text{mech}} = \frac{\int d^3\mathbf{r}\ r^2 F_i^{\parallel}(r)}{\int d^3\mathbf{r}\ F_i^{\parallel}(r)}$$



How big is a proton?



Conclusion

First lattice calculation of:

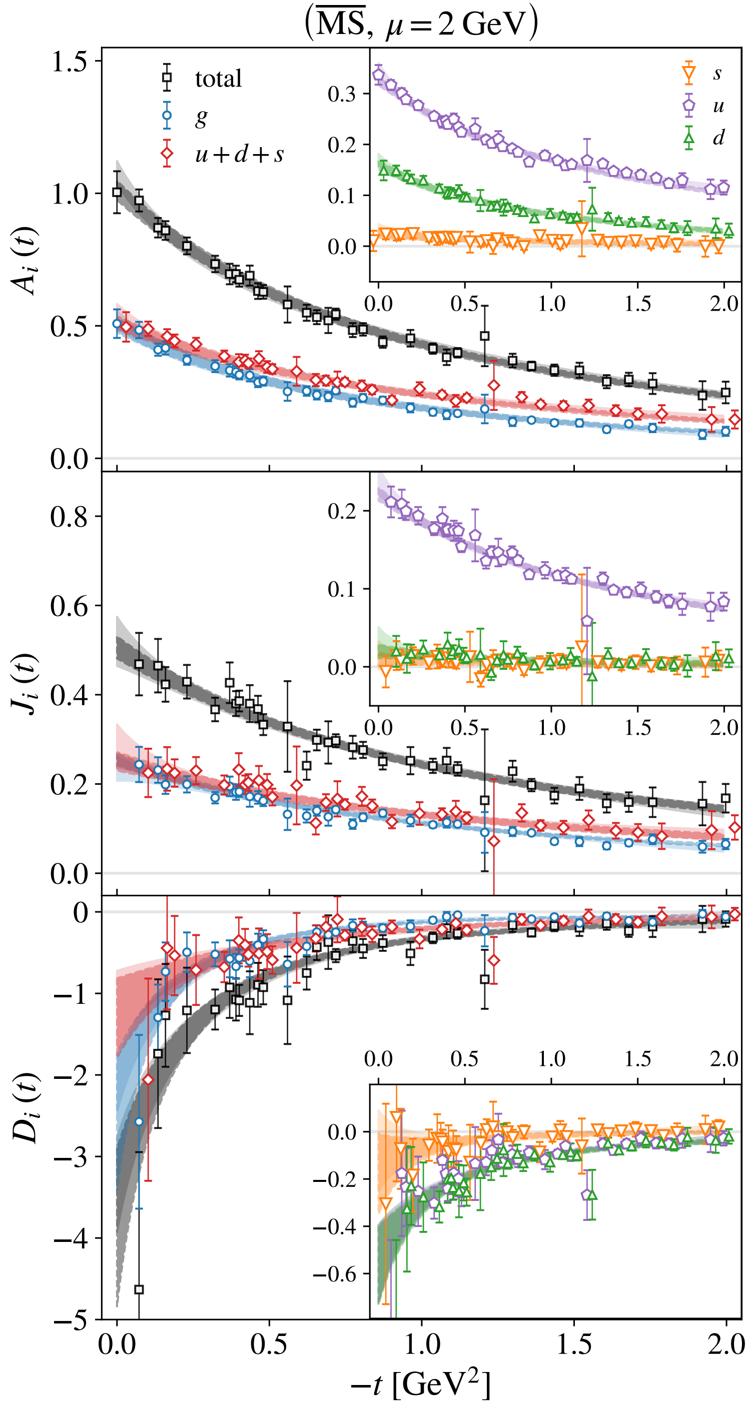
- complete flavor decomposition of nucleon GFFs
- *total* GFFs → *physical* (i.e. RGI) densities, radii
- $D(0)$

New first-principles descriptions of size and shape of nucleon

Results can help discriminate between different experimental extractions

Towards a precision calculation, need:

- Multiple ensembles to take continuum, physical-mass limits
- Improved renormalization (GIRS? Flow? Sum rules?)
- Variational method to fully control excited state effects



Backup

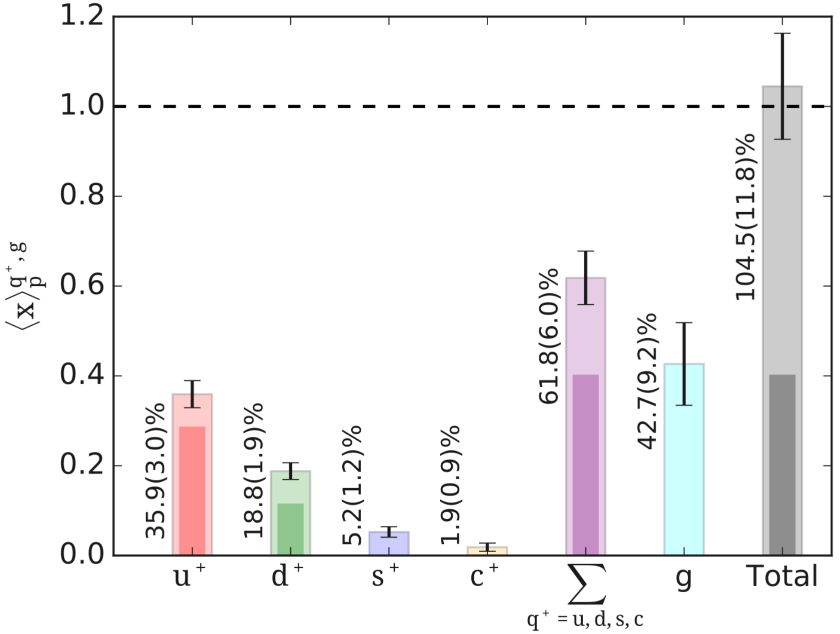
Forward GFFs & decompositions

$$\left\langle N(p')\middle|T_{g,q}^{\{\mu\nu\}}\middle|N(p)\right\rangle = \bar{U}(p')\left[A_{g,q}(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)M g^{\mu\nu}\right]U(p)$$

Momentum fraction

$A_{q,g}(0) = \langle x\rangle_{q,g}$

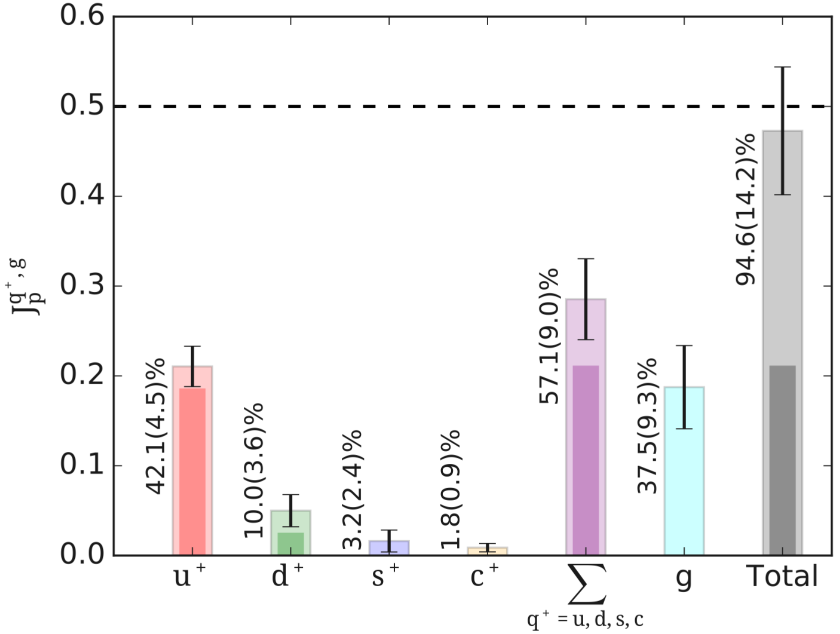
$\sum_q A_q(0) + A_g(0) = 1$



[ETMC 2003.08486]

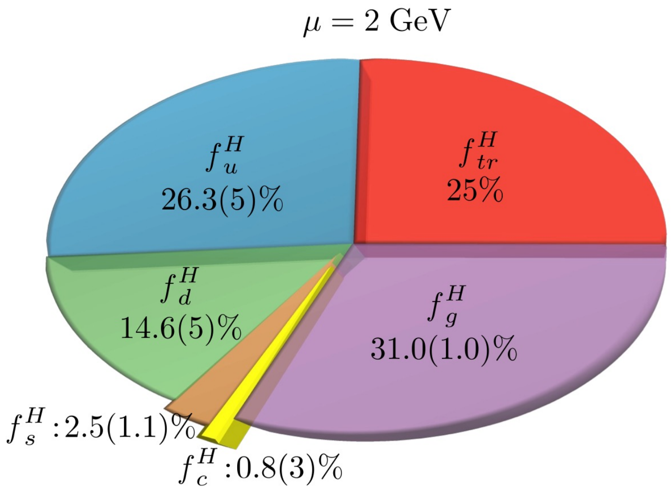
Spin fraction

$\sum_q J_q(0) + J_g(0) = 1/2$



[ETMC 2003.08486]

…and involved in others, e.g. Ji's rest energy decomp



[Liu 2103.15768]

$D(0)$: "the last global unknown"

[Polyakov Schweitzer 1805.06596]

em: $\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\vert J^\mu_{\text{em}}\vert N\rangle$	$\longrightarrow$	Q	$=$	$1.602176487(40) \times 10^{-19}\text{C}$
			μ	$=$	$2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\vert J^\mu_{\textbf{weak}}\vert N\rangle$	$\longrightarrow$	g_A	$=$	$1.2694(28)$
			g_p	$=$	$8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\textbf{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\textbf{grav}}\vert N\rangle$	$\longrightarrow$	m	$=$	$938.272013(23)\,\text{MeV}/c^2$
			J	$=$	$\frac{1}{2}$
			$\boldsymbol{D}$	$=$	**?**

Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.

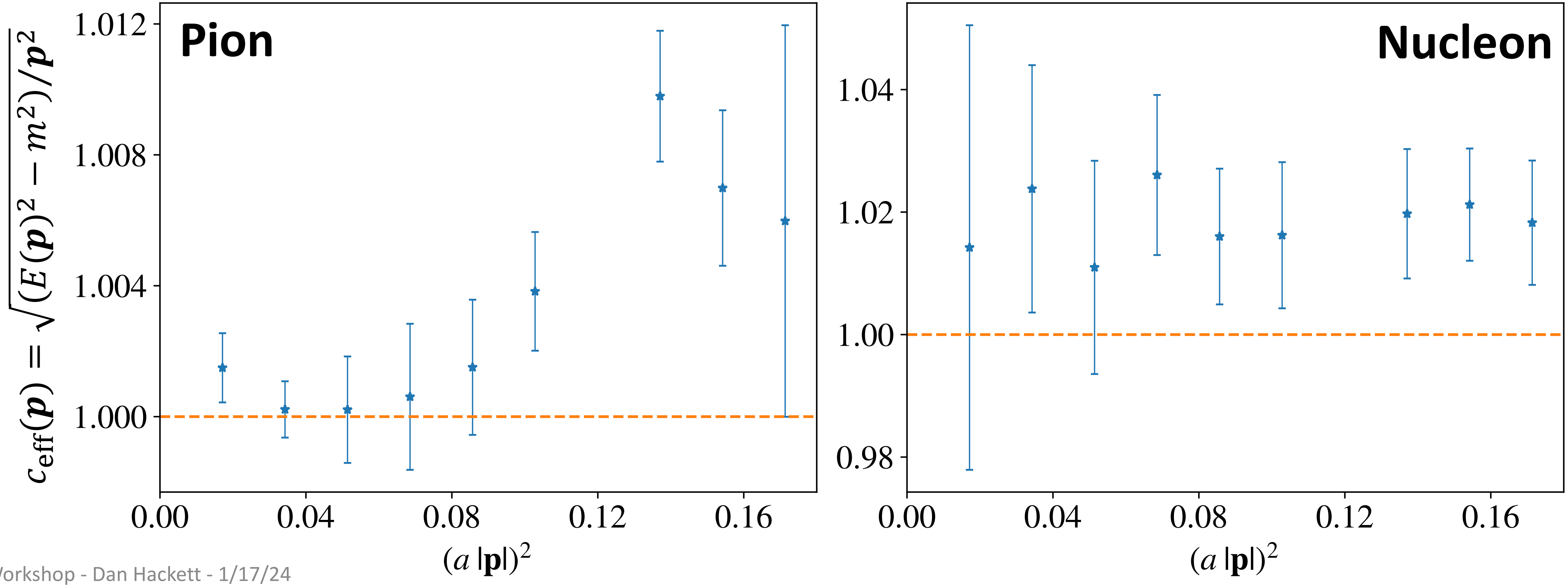
Two-point functions

Compute on 2511 configs, 1024 srcs/cfg (2x offset $4^3 \times 8$ grids)

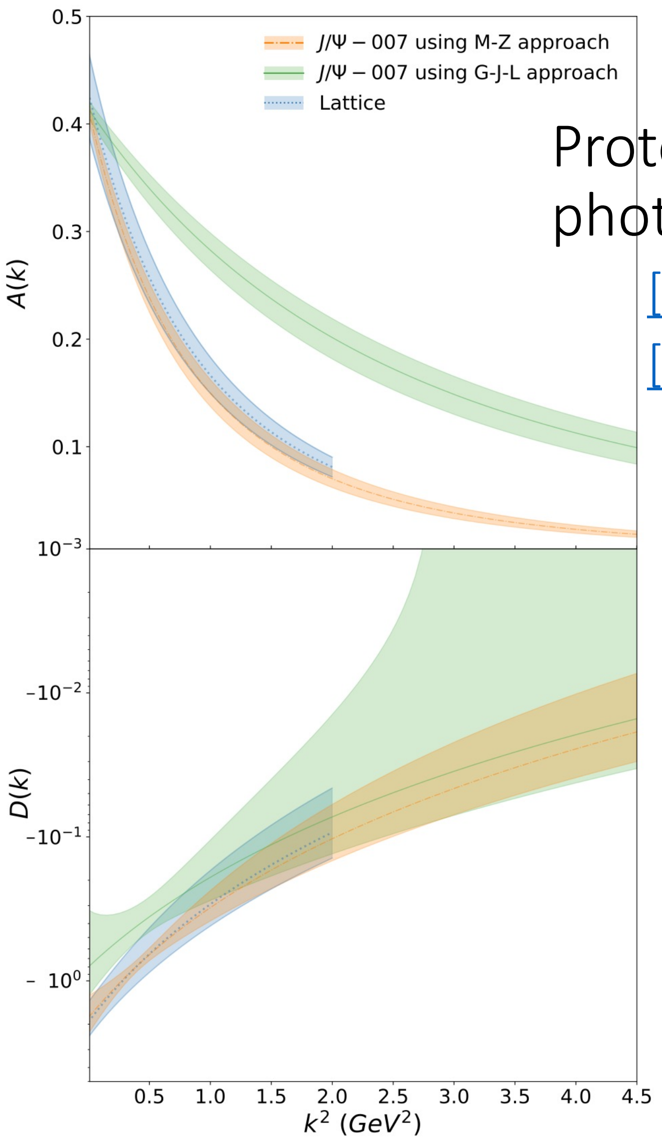
Note: only one interpolating operator; both diagonal spin channels

Relativistic dispersion obeyed at ~ % level

→ Neglect errors in $aM_\pi = 0.0779$ and $aM_N = 0.4169$



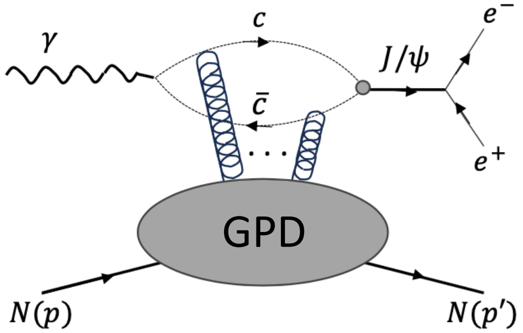
Experimental results



Proton: glue from J/Ψ photoproduction

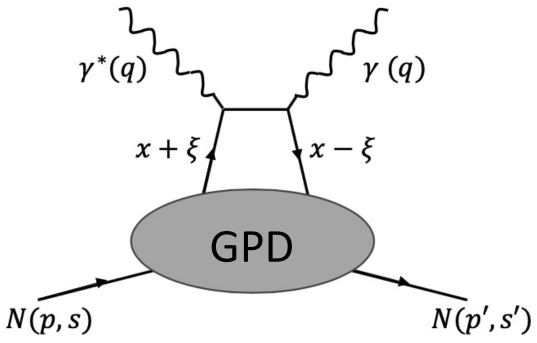
[Duran et al. 2207.05212]

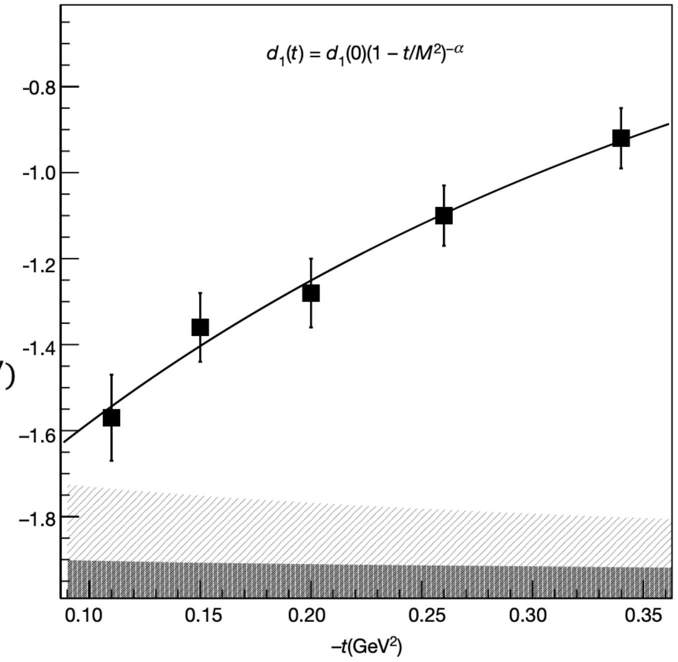
[Guo et al. 2305.06992]

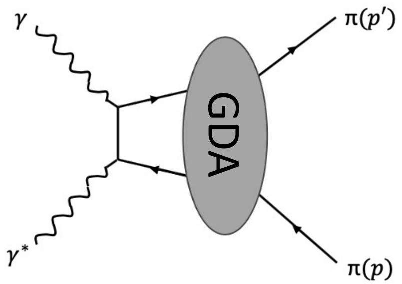


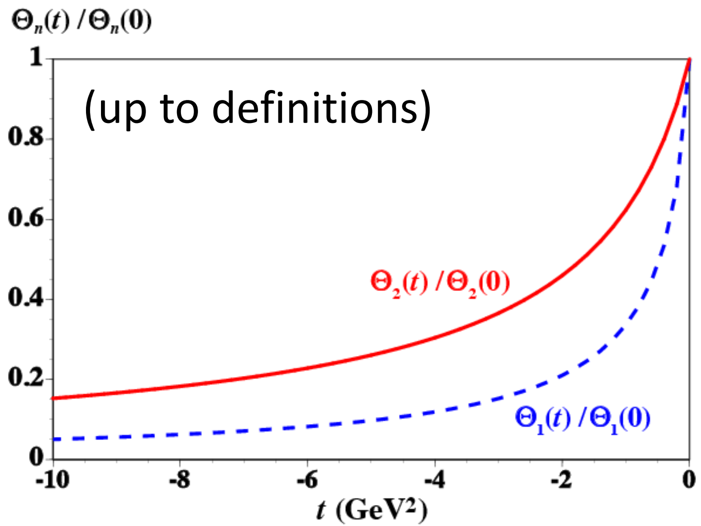
Proton: quark D from DVCS

[Burkert Elouadrhiri Girod 2018]





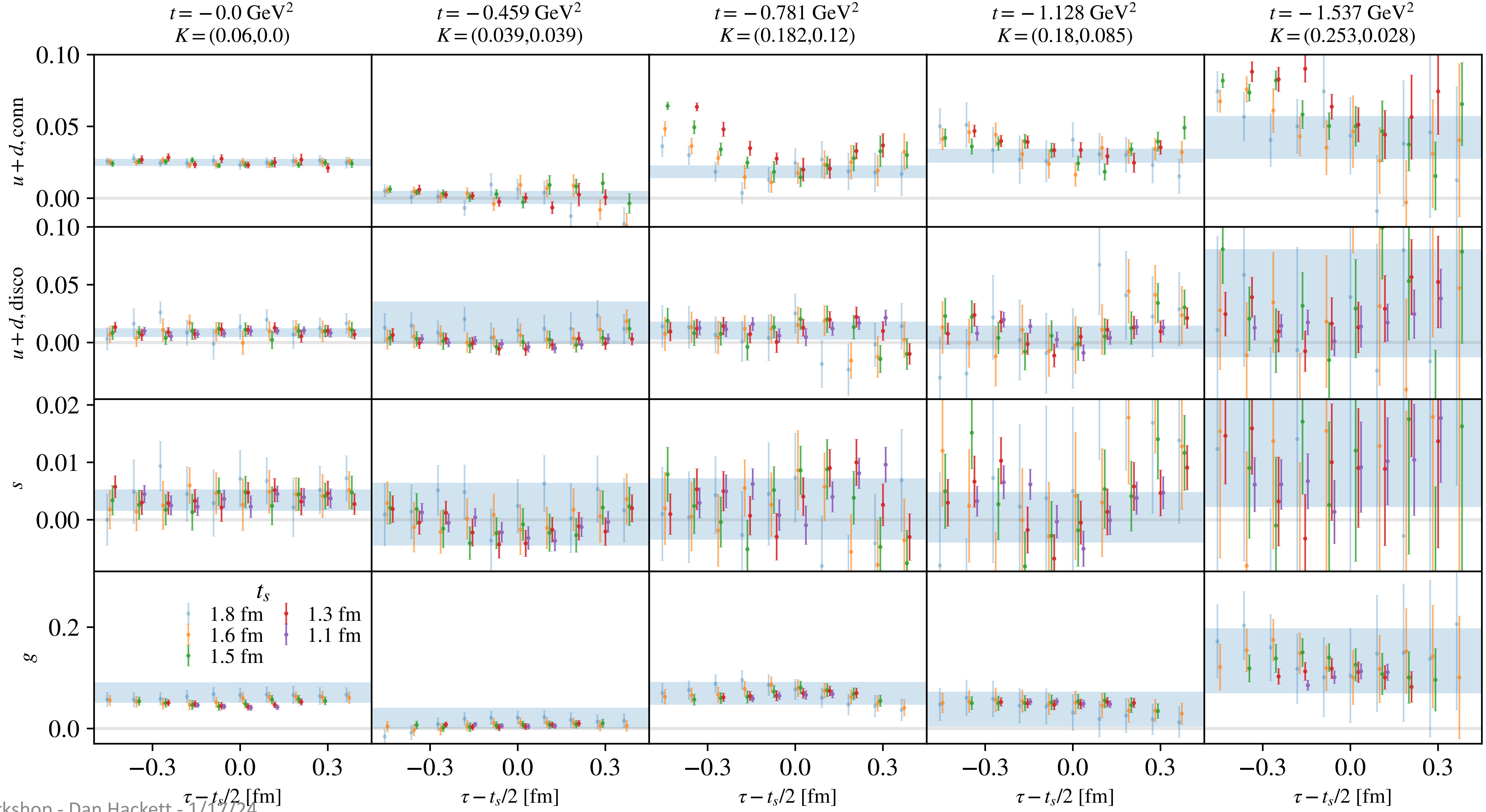




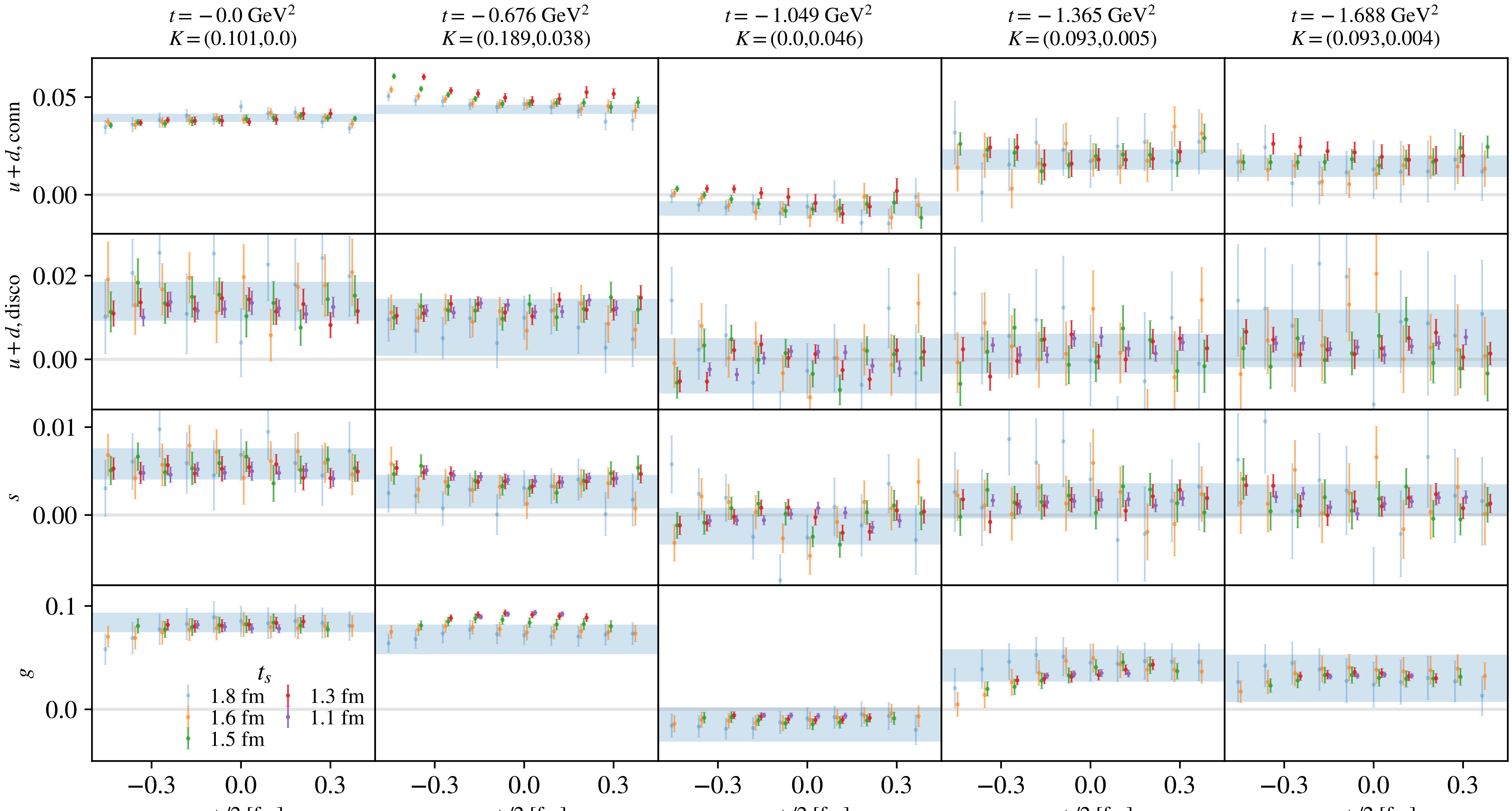
Pion: quark from $\gamma\gamma \to \pi^0\pi^0$

[Kumano Song Teryaev 1711.08088]

Example pion ratios: $\tau_1^{(3)}$

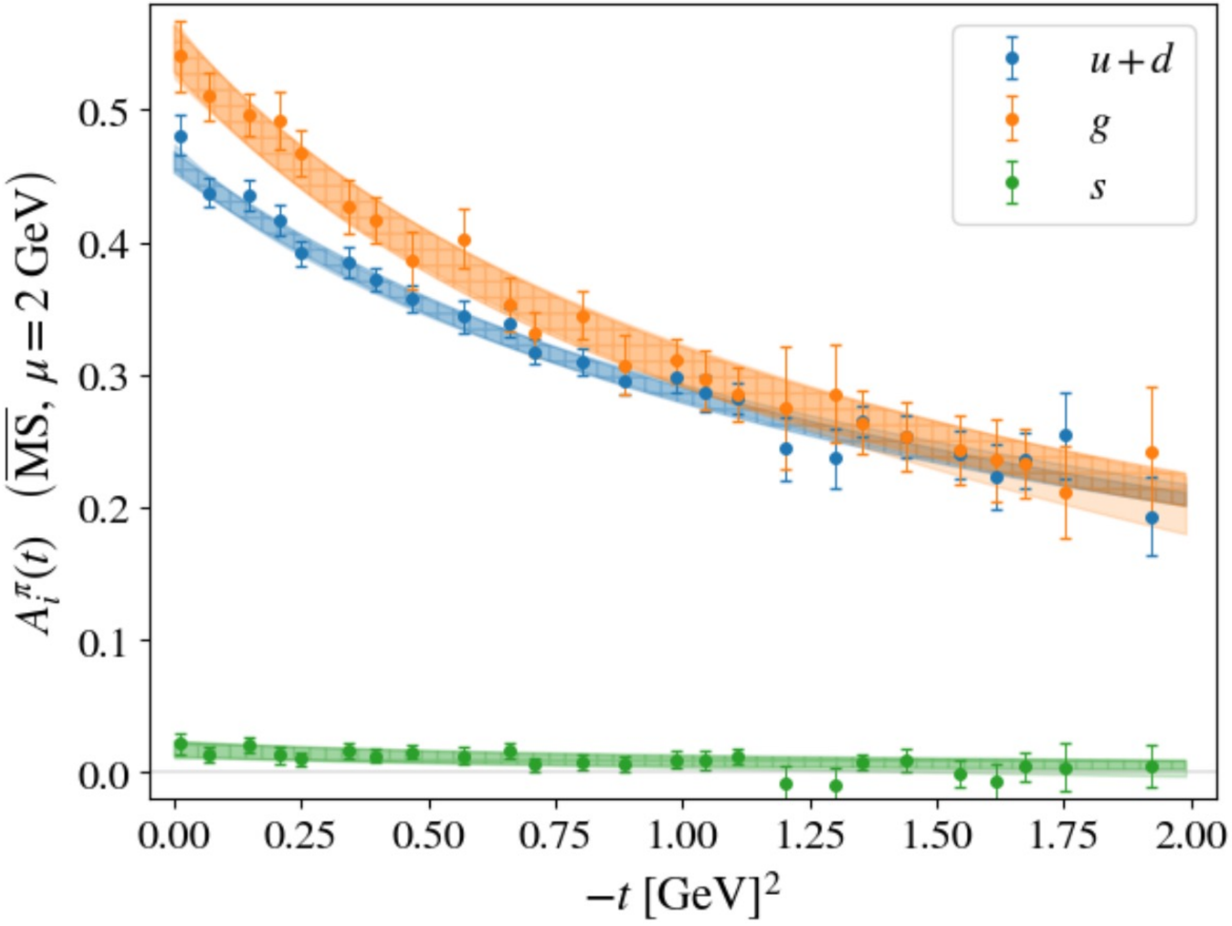


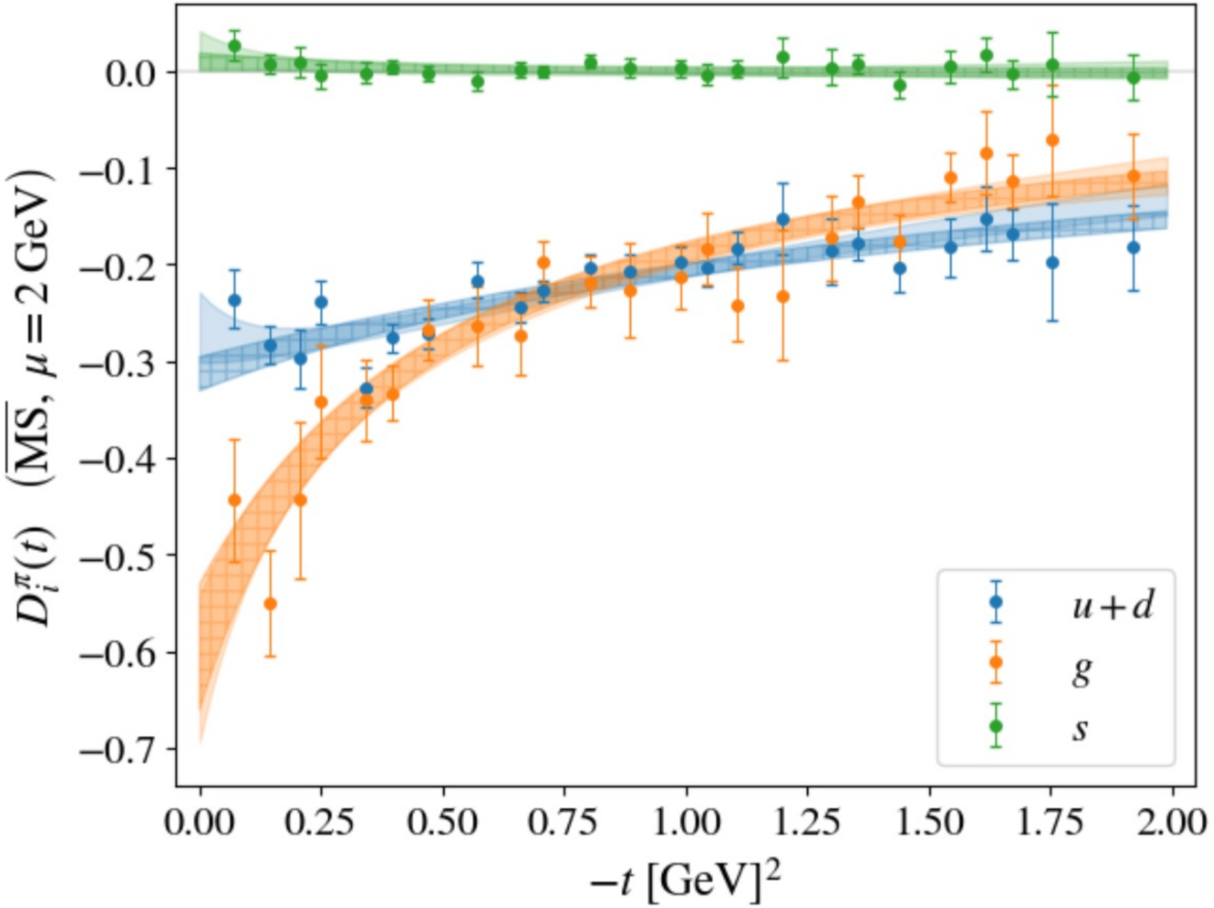
Example pion ratios: $\tau_3^{(6)}$



Pion GFFs (flavor decomp)

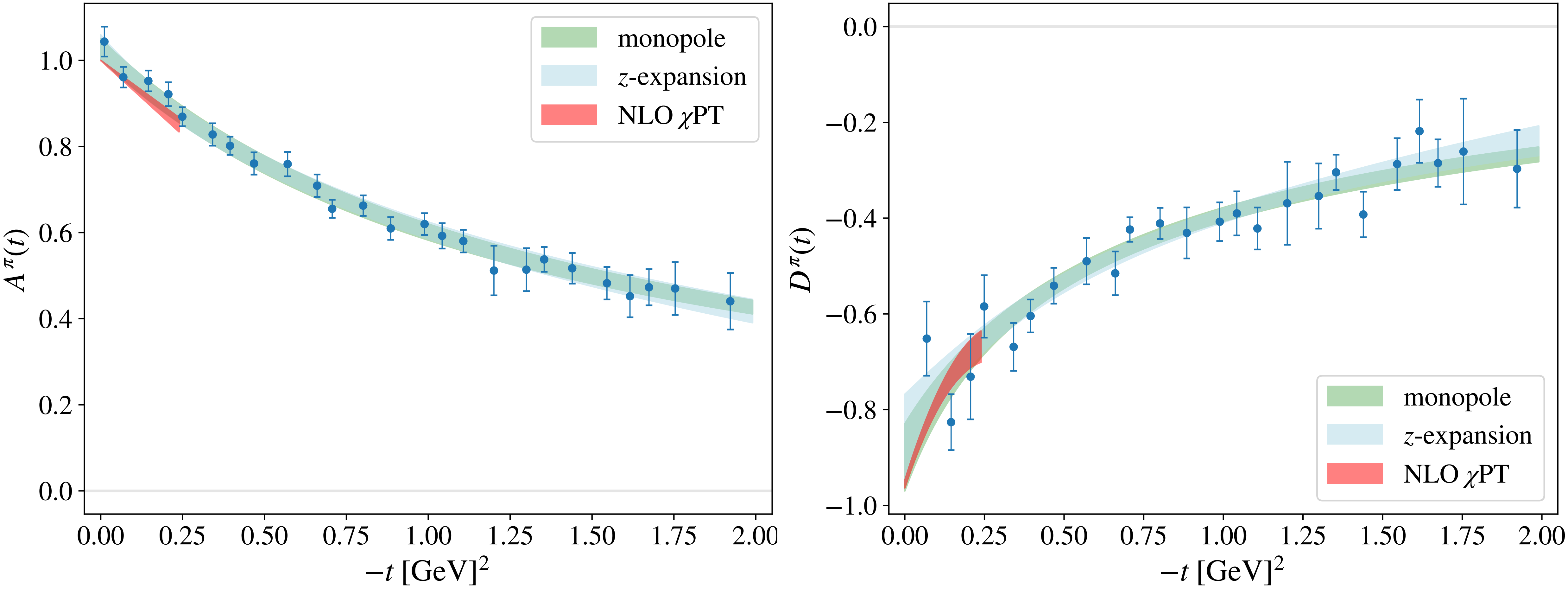
Hatched bands: monopole   Solid bands: z-expansion



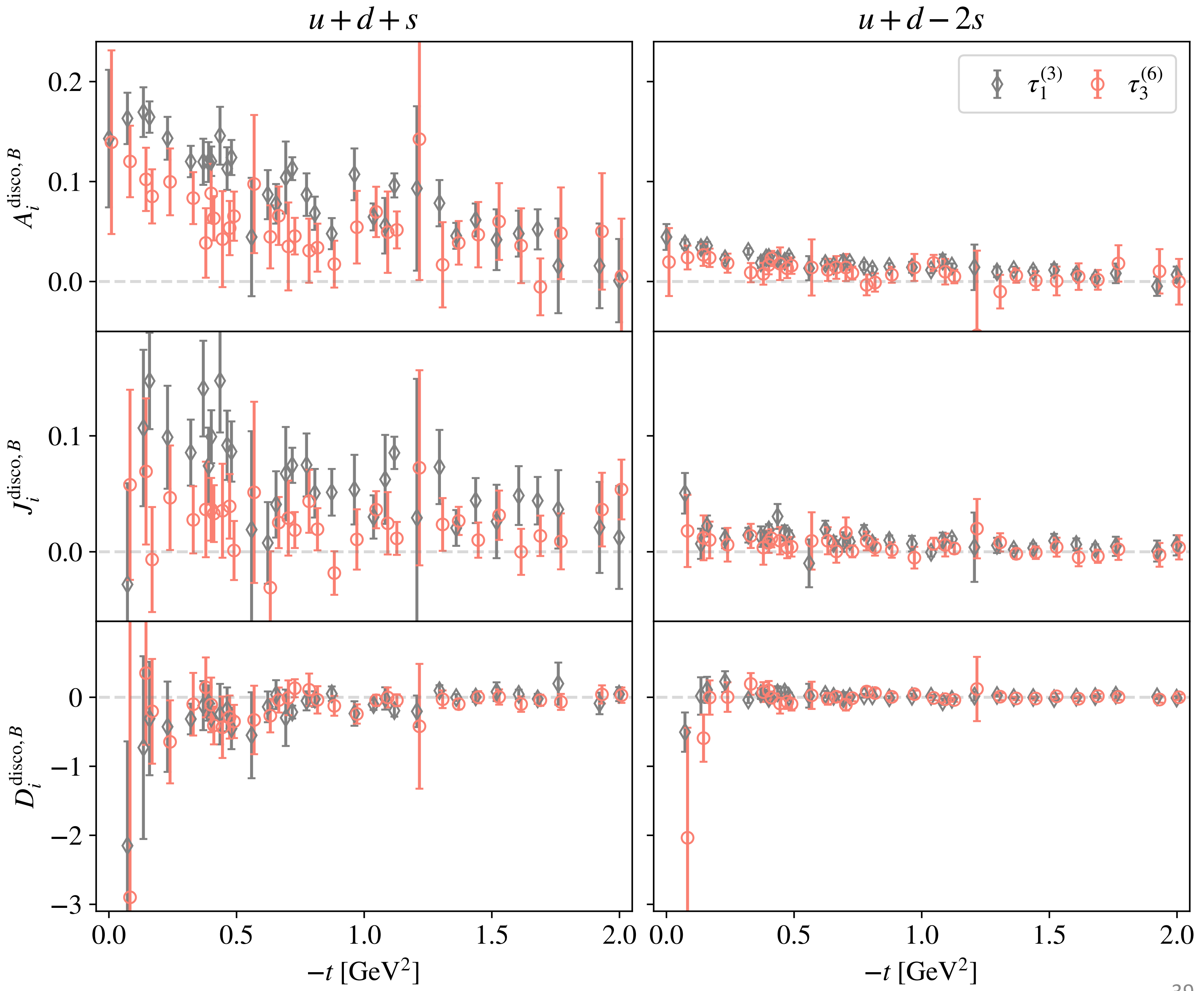


Pion GFFs (total)

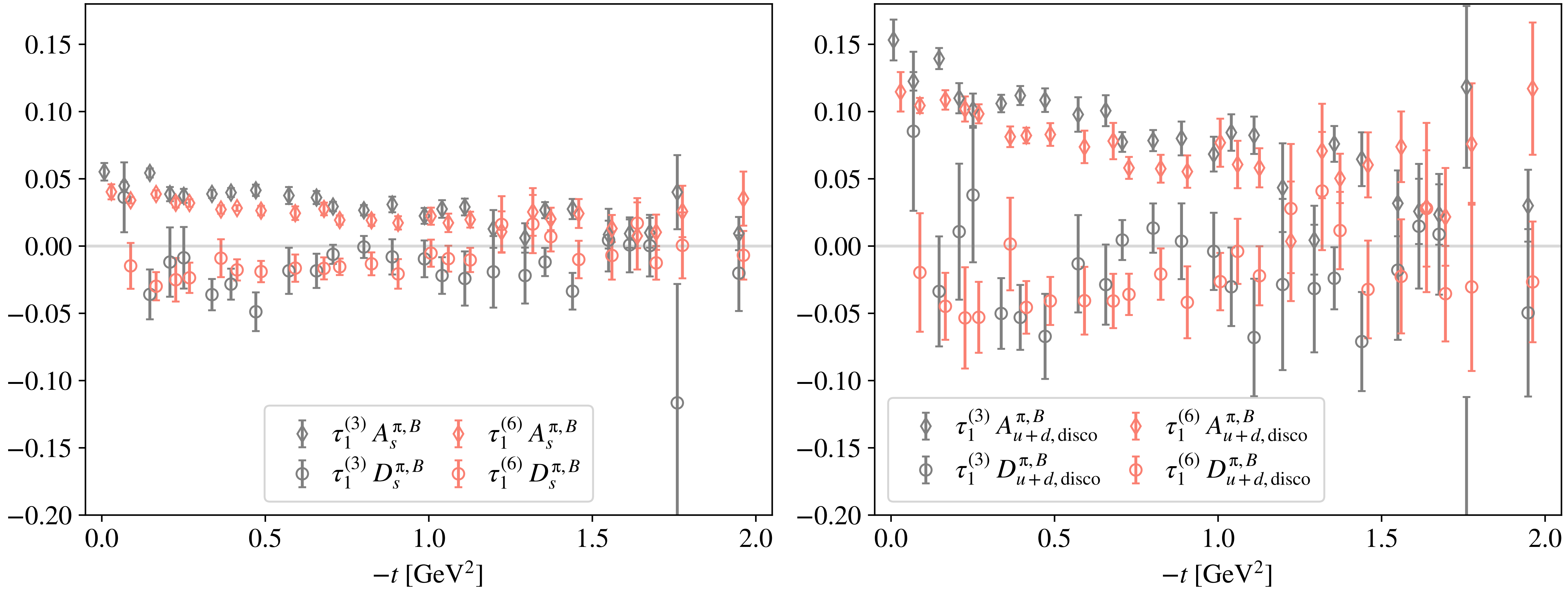
Error on χPT estimate due to different estimates for LECs [Donaghue Leutwyler 1991]



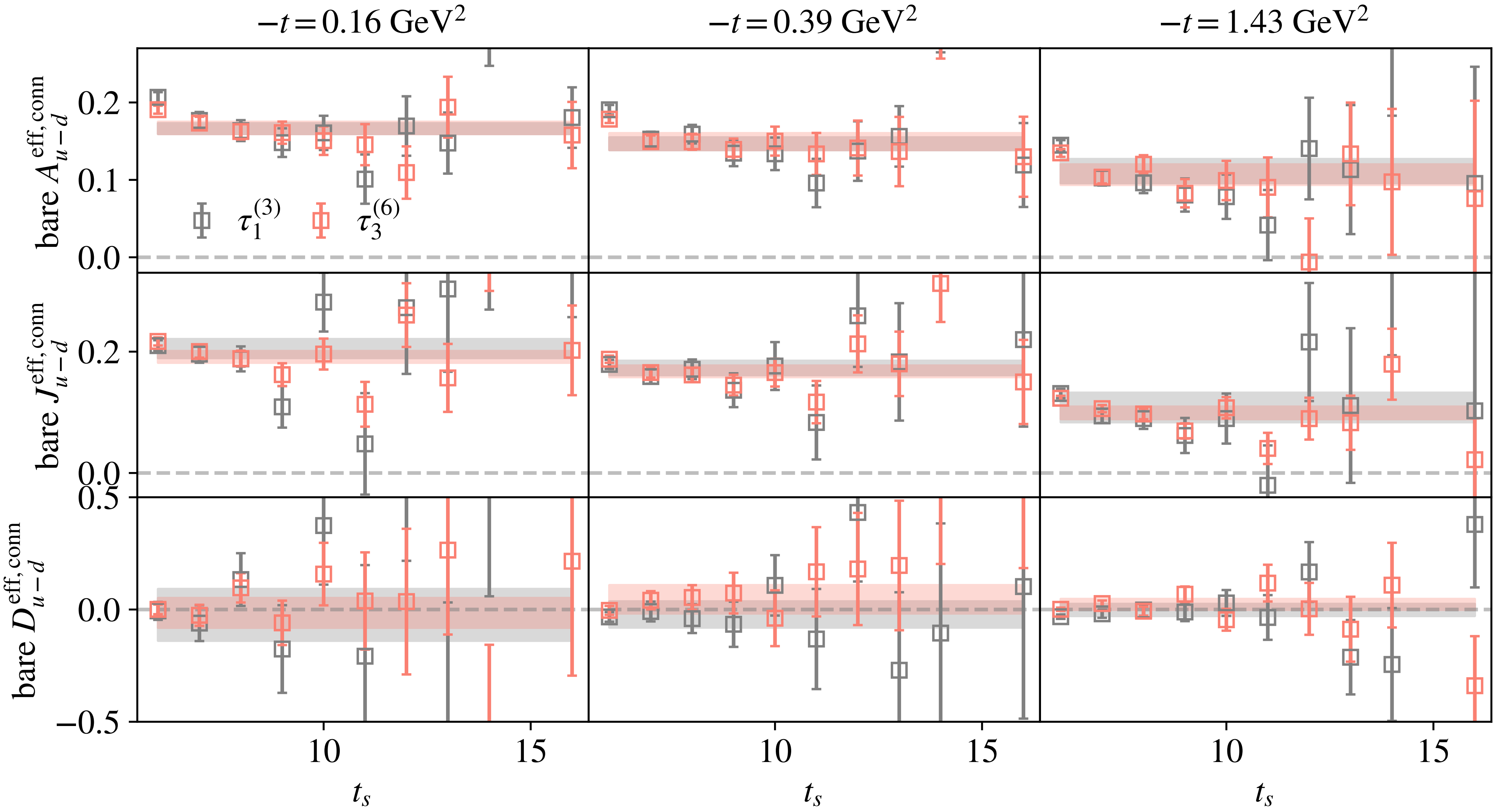
Nucleon: bare disconnected GFFs



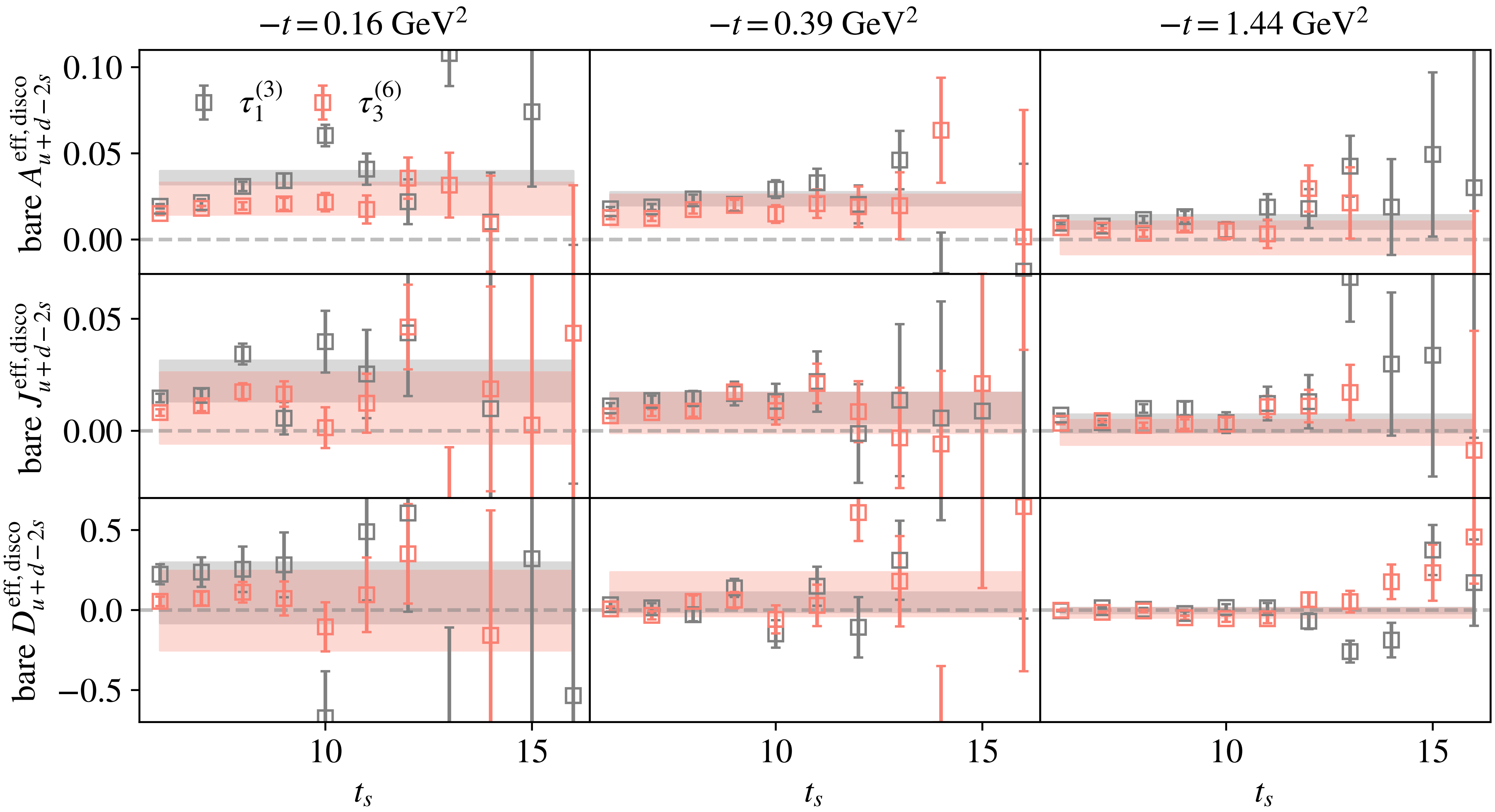
Pion: bare disconnected GFFs



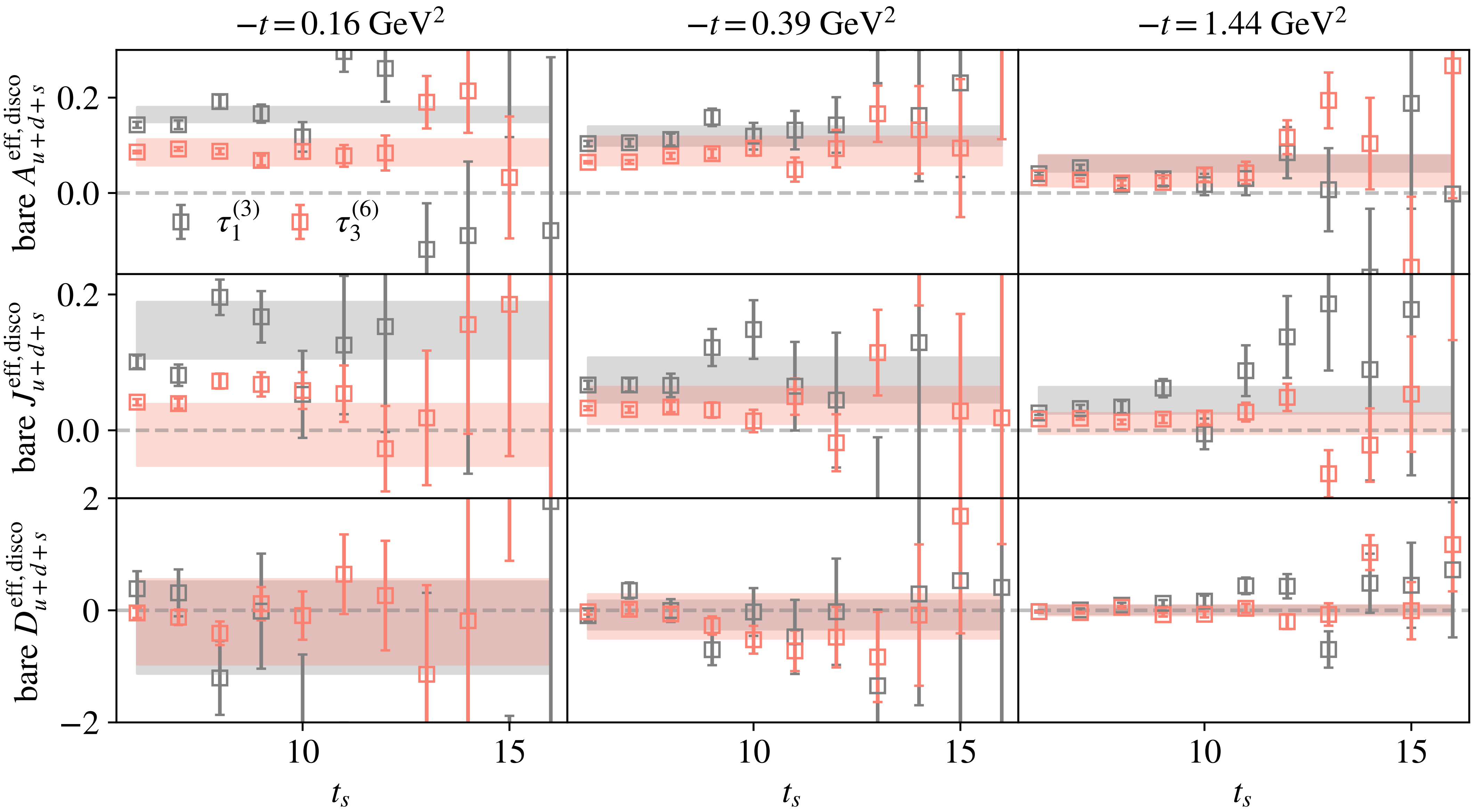
Nucleon: effective GFFs



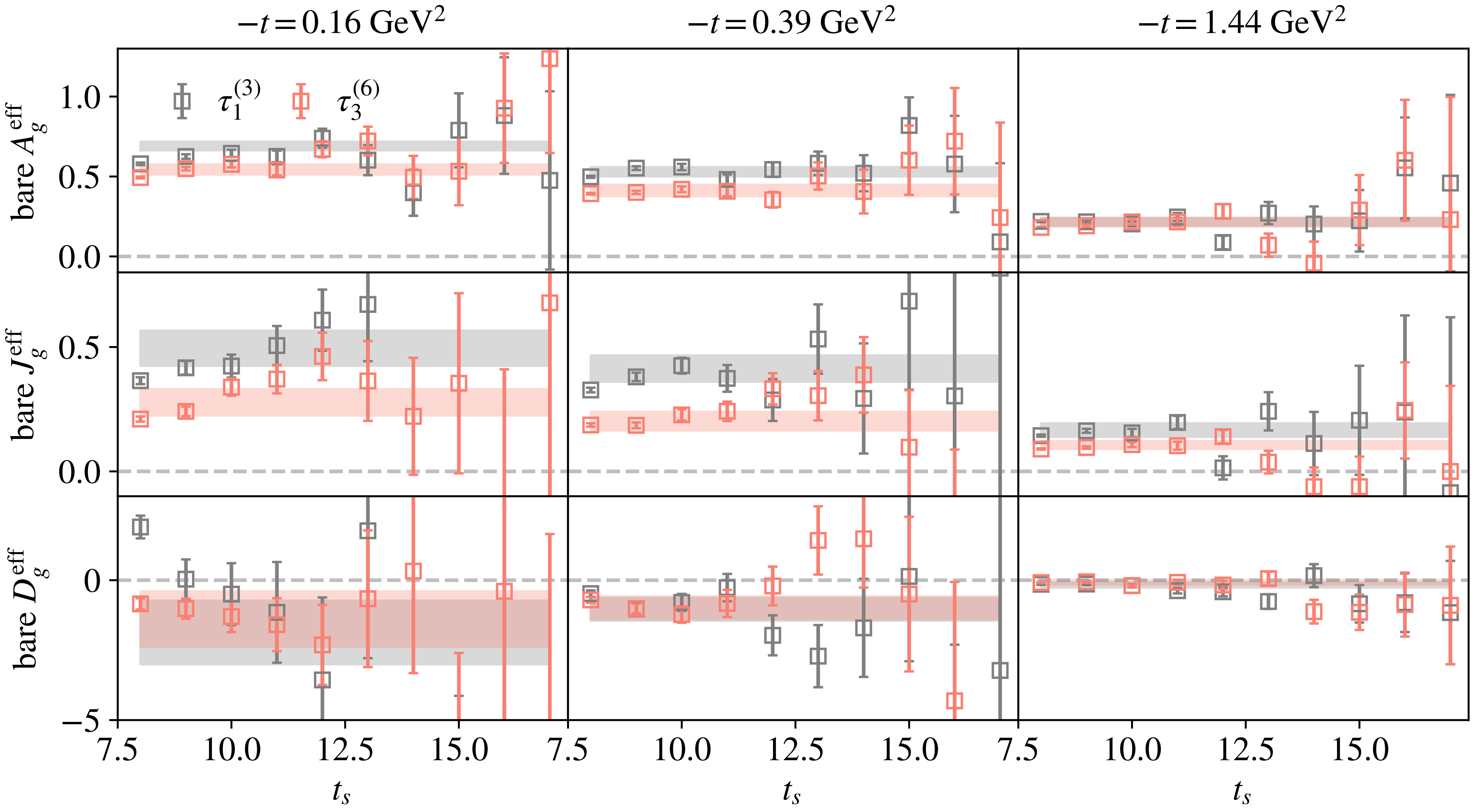
Nucleon: effective GFFs



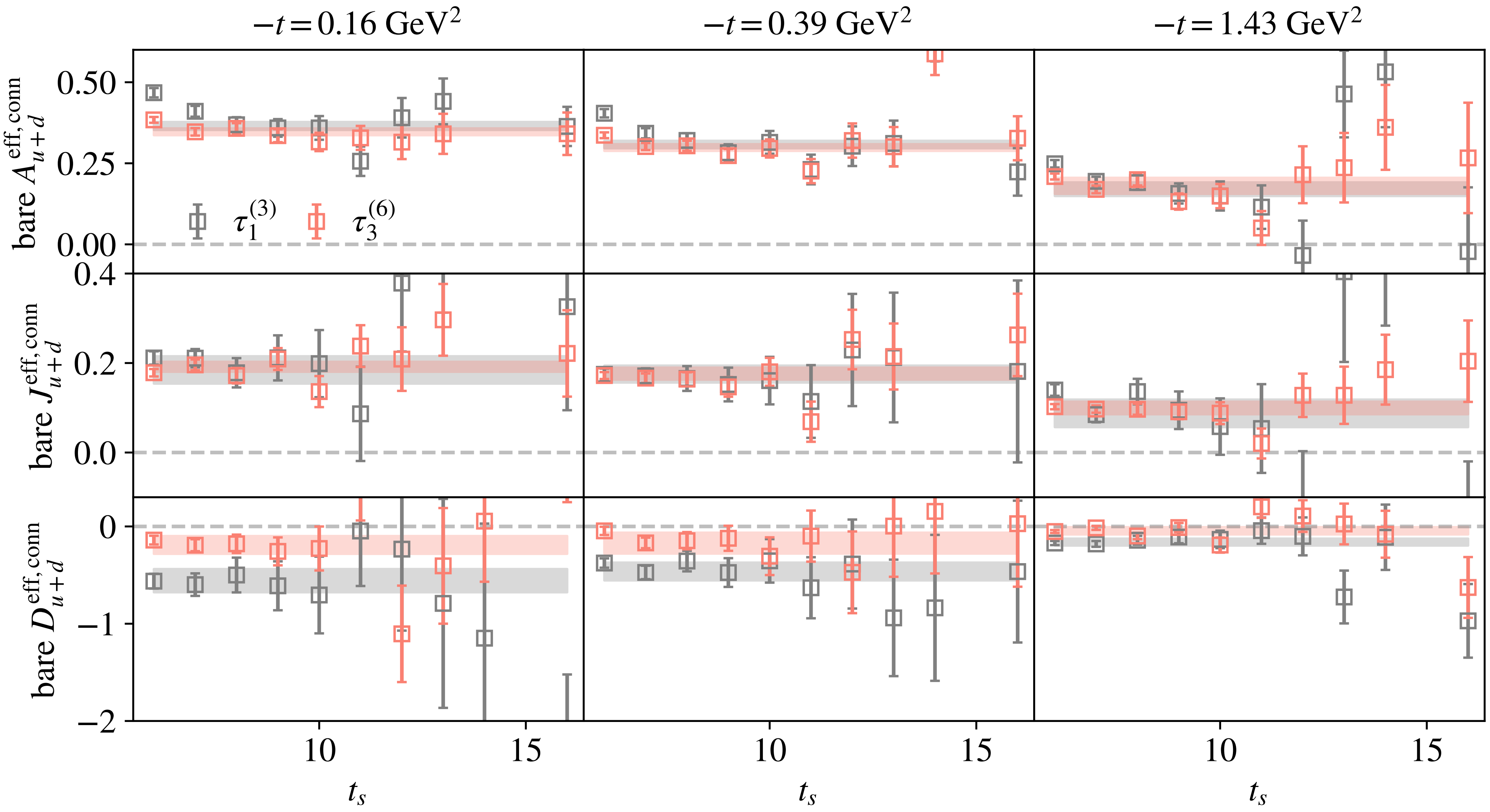
Nucleon: effective GFFs



Nucleon: effective GFFs



Nucleon: effective GFFs



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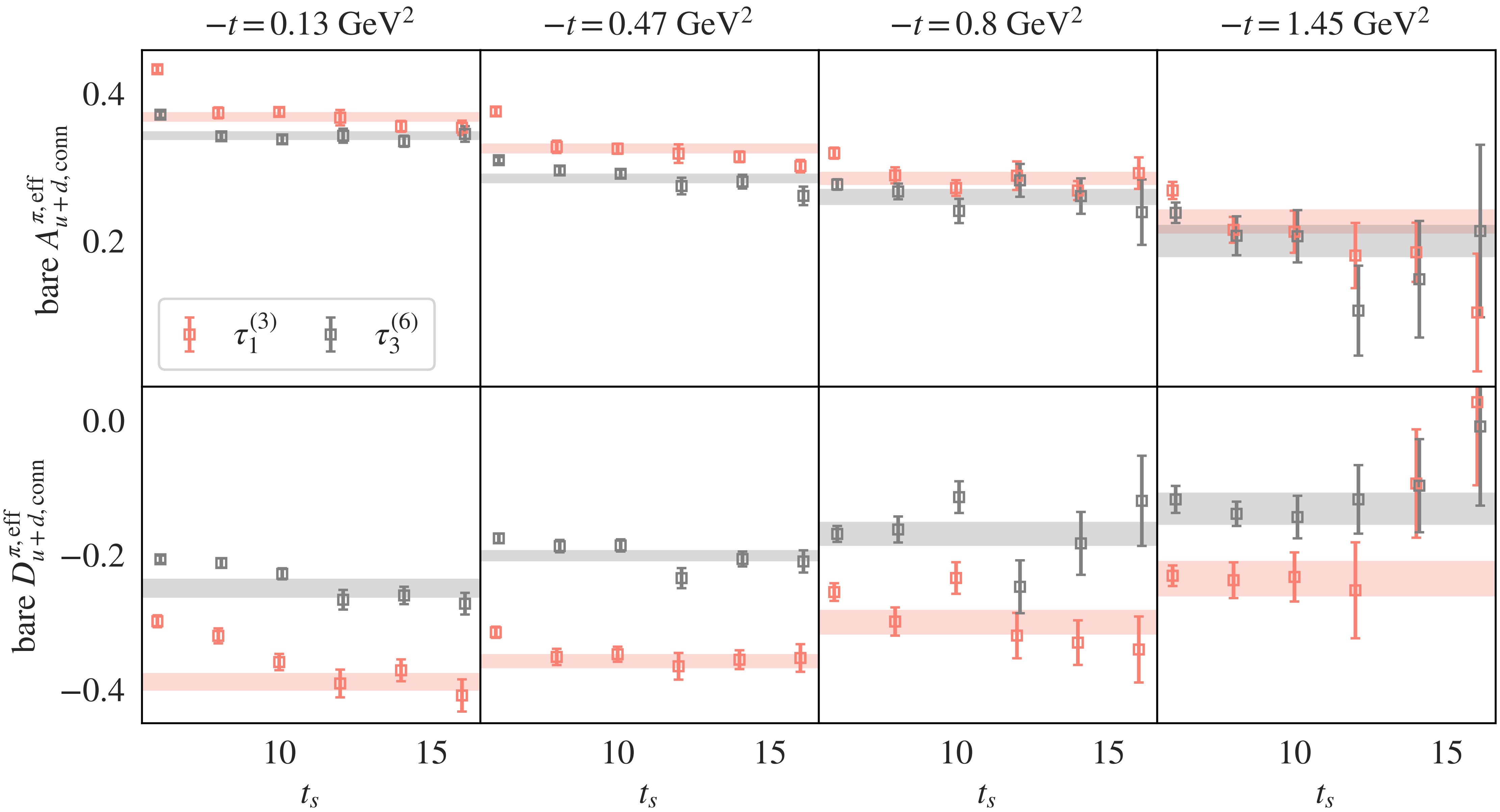

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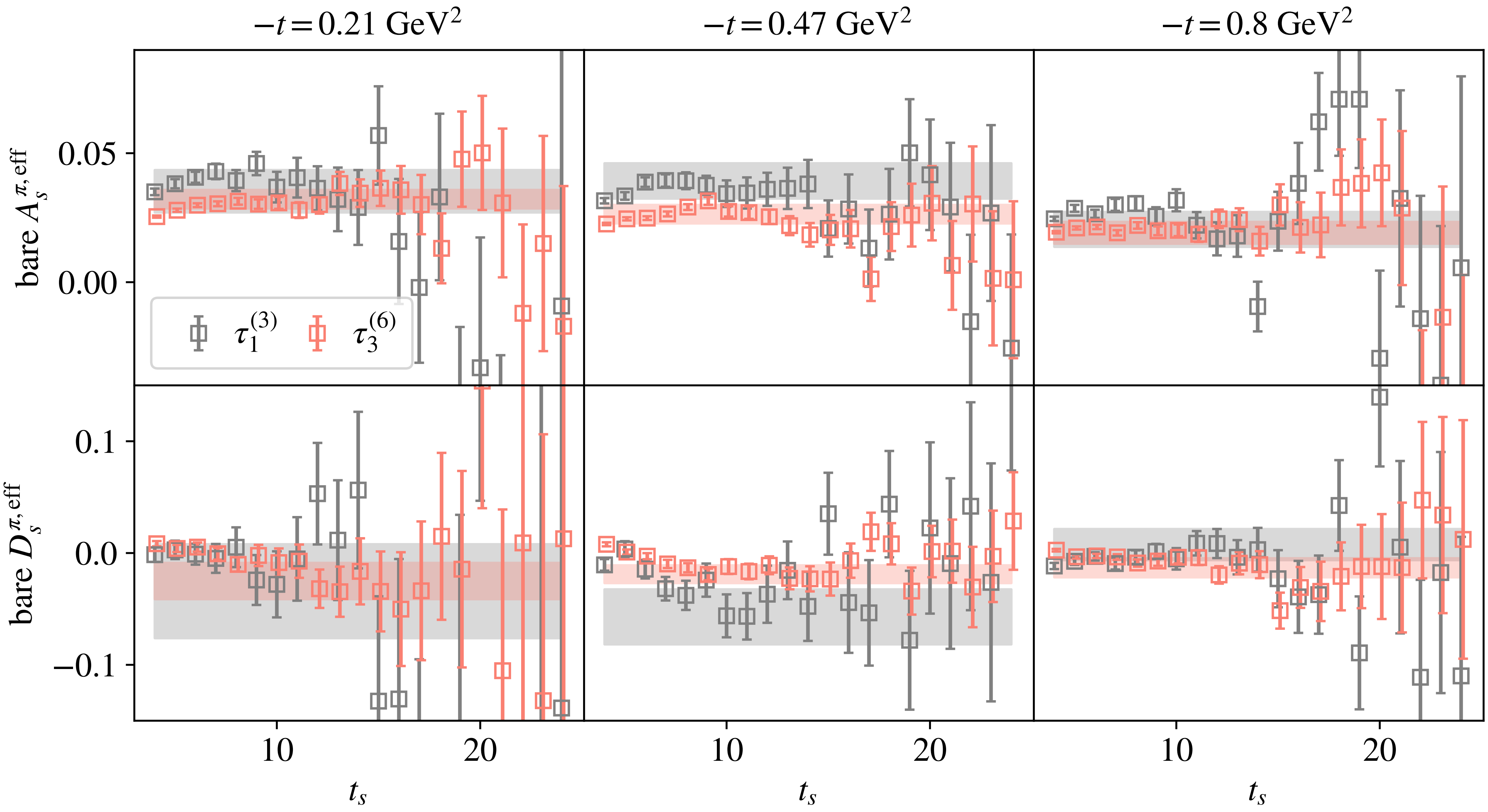

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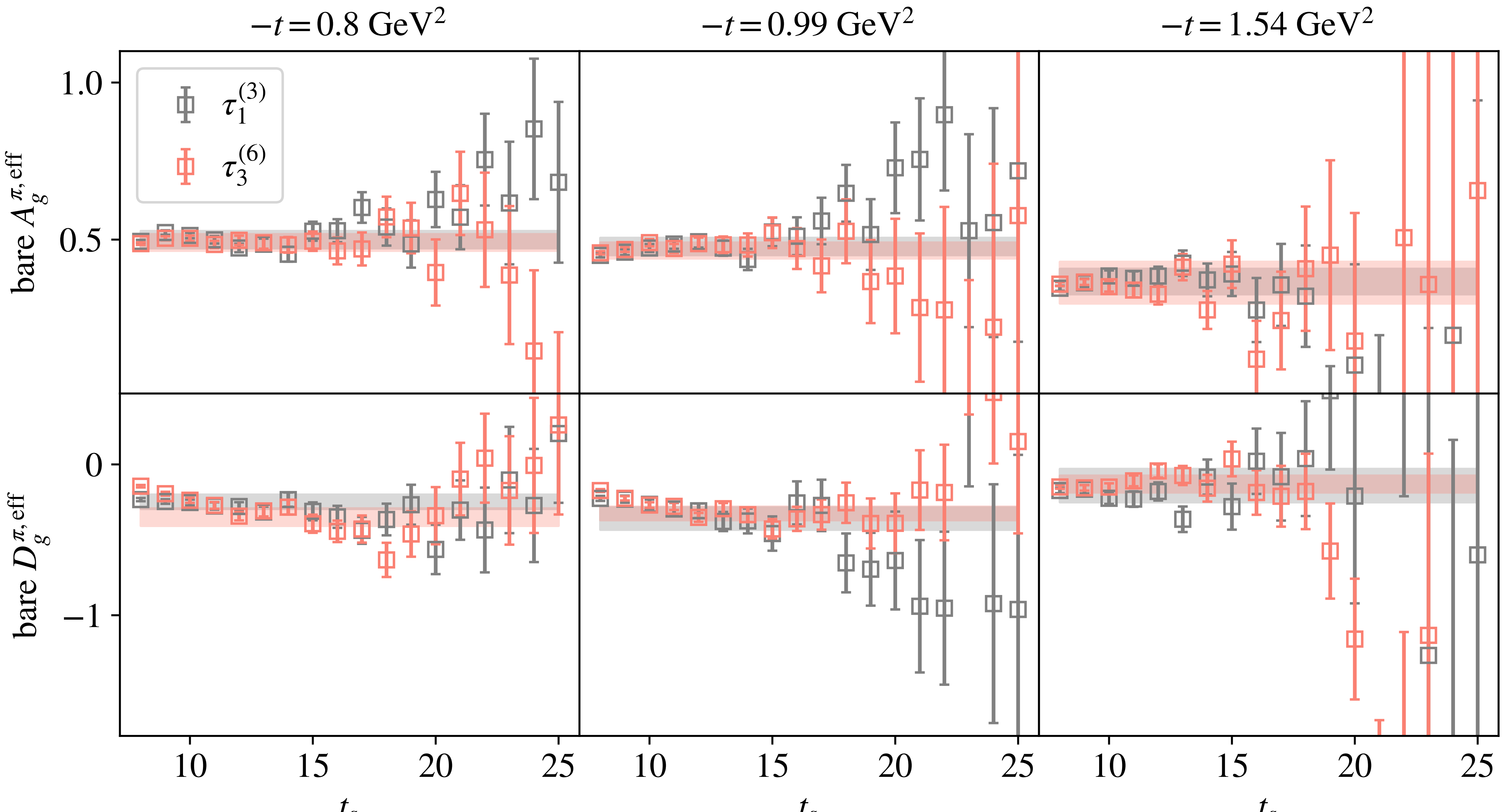

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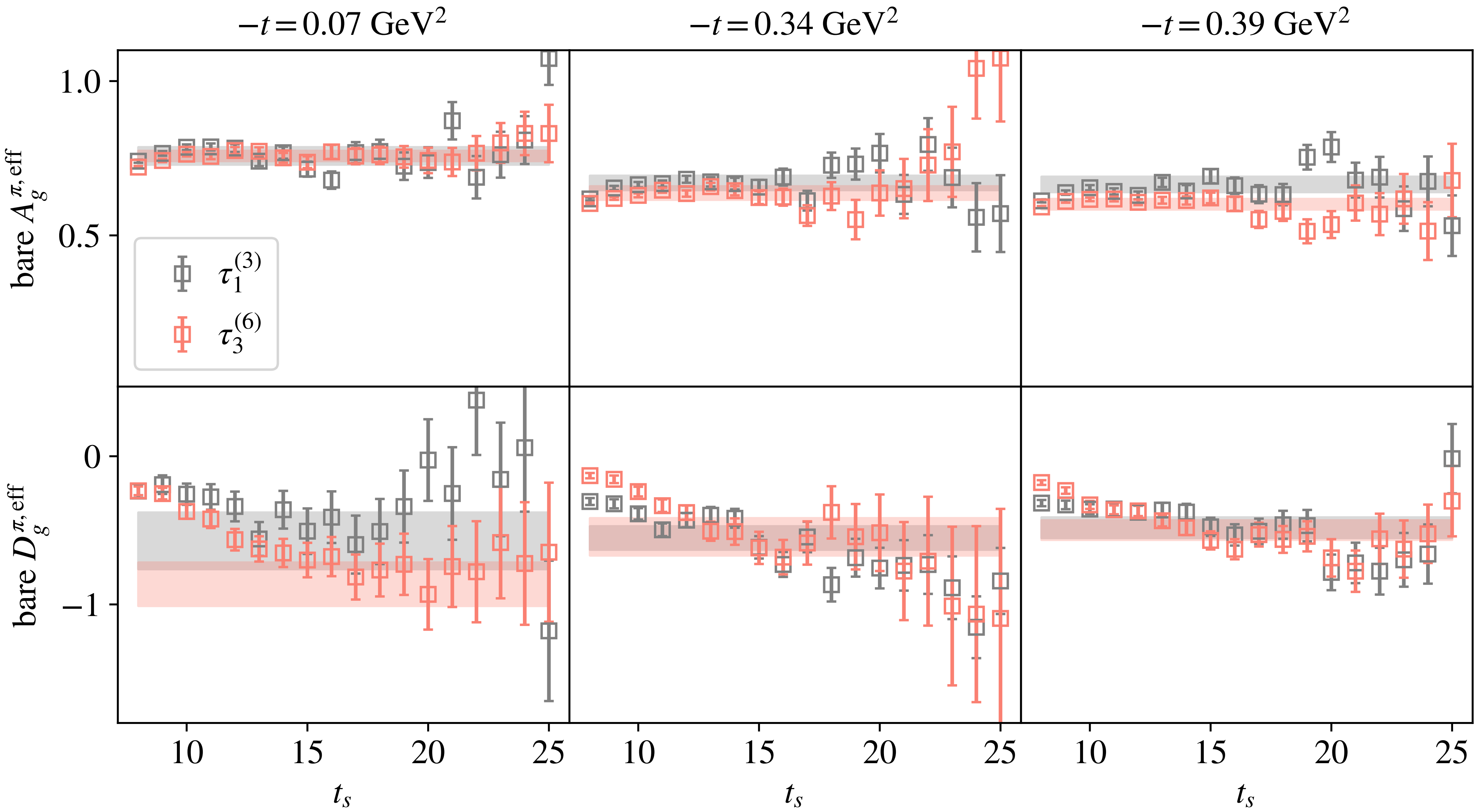
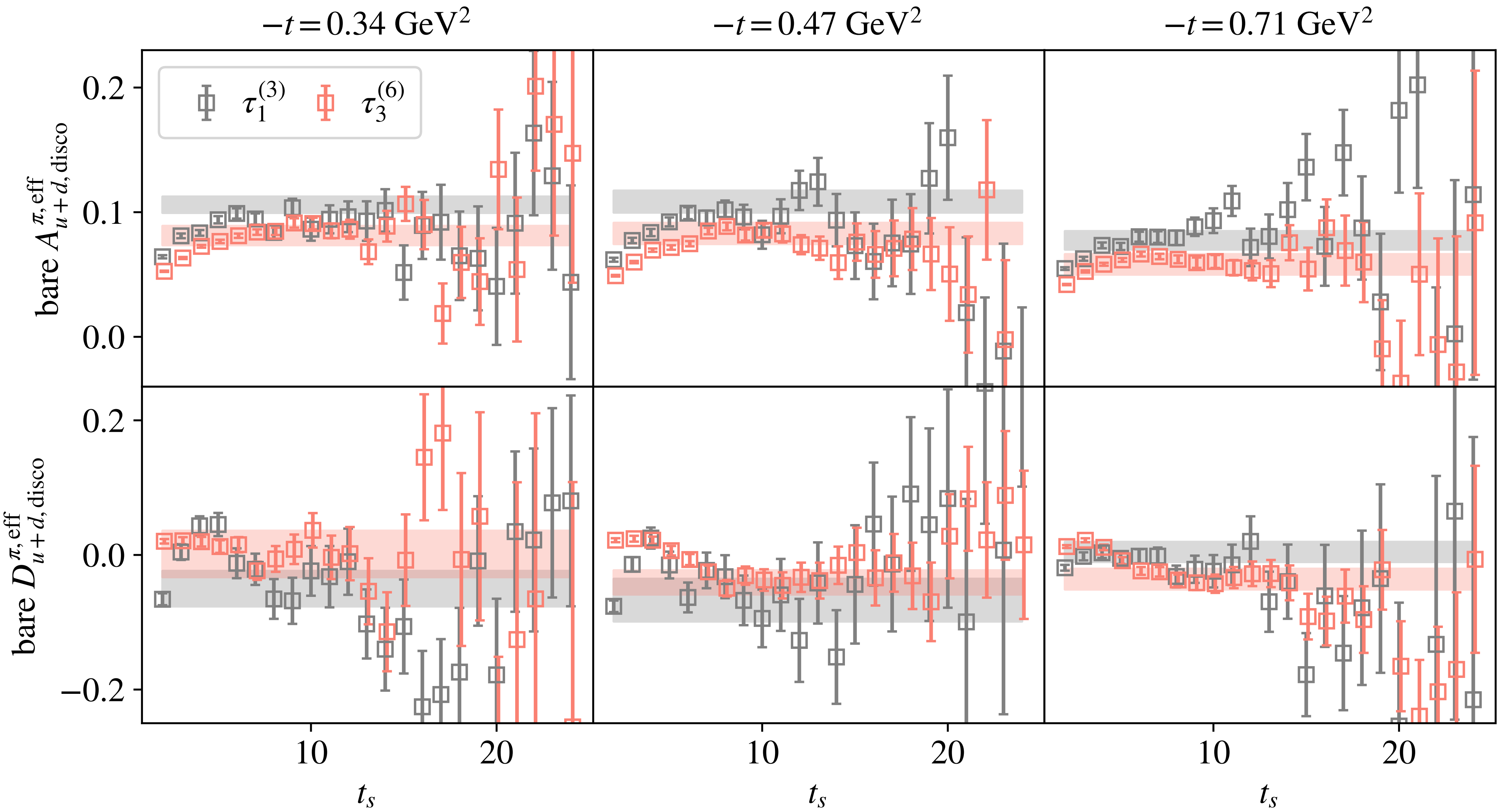
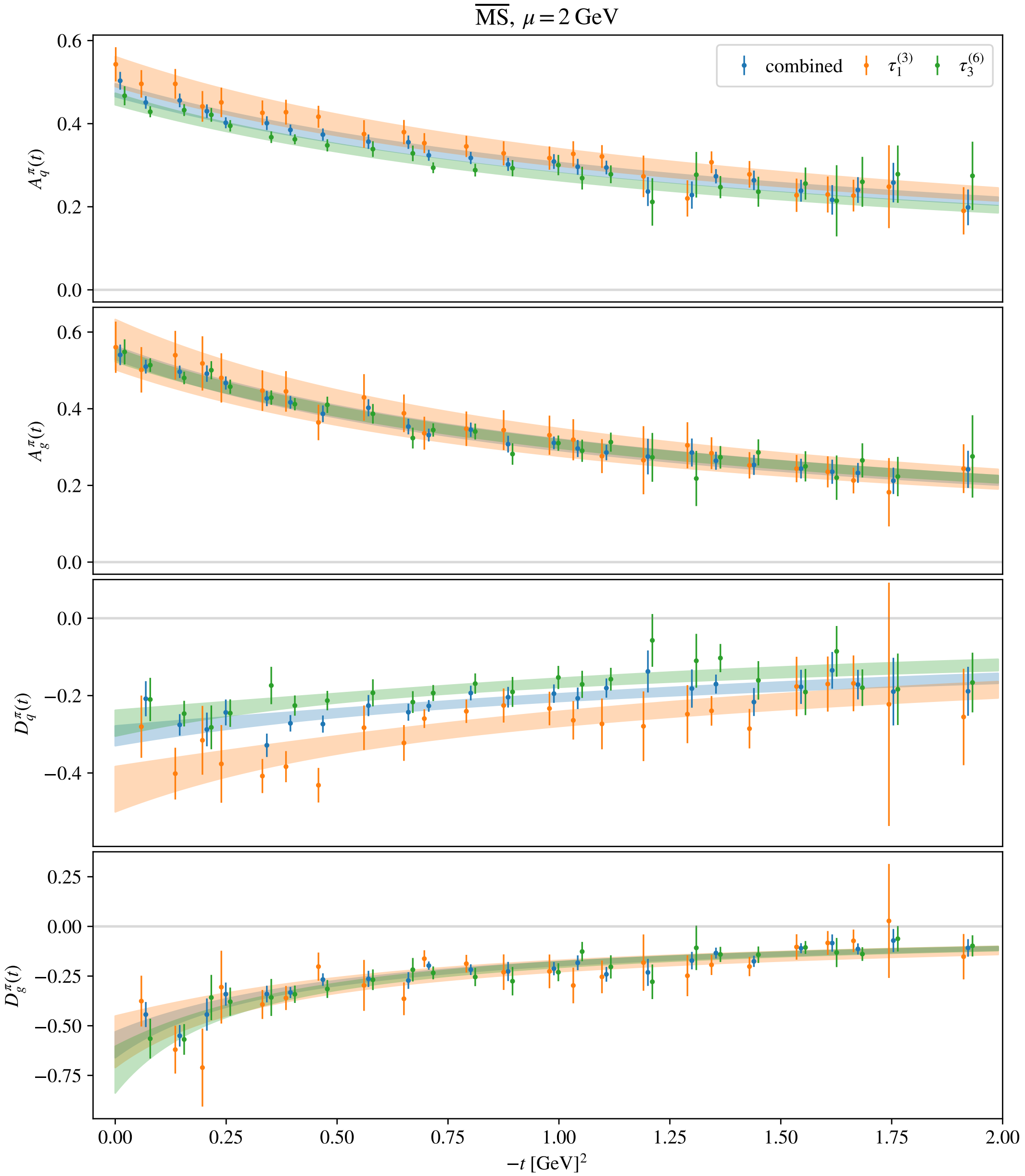
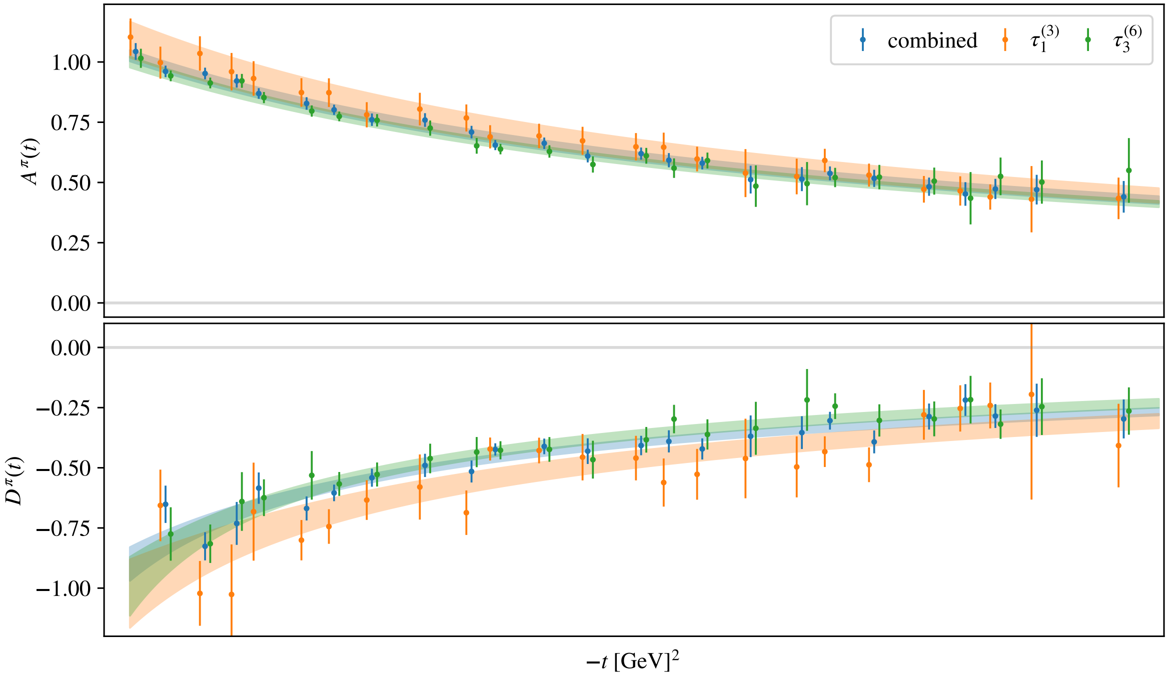


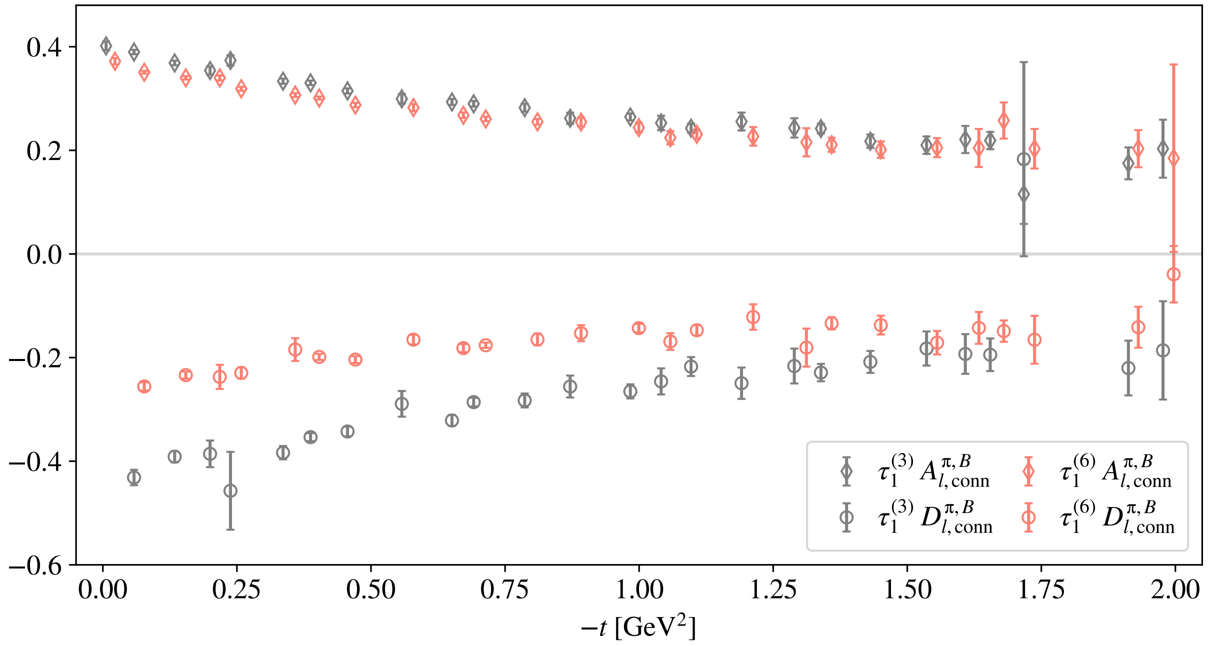
Pion: effective GFFs



Pion: split irreps







Nucleon: split irreps