Lattice QCD calculation of the pion generalized parton distributions (GPDs)

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Outline

- Introduction: GPDs, Lattice QCD
- Frame-independent approach
- Lattice simulation in zero-skewness
- Quasi-GPDs
- LaMET -> Valence GPDs

Introduction: GPDs

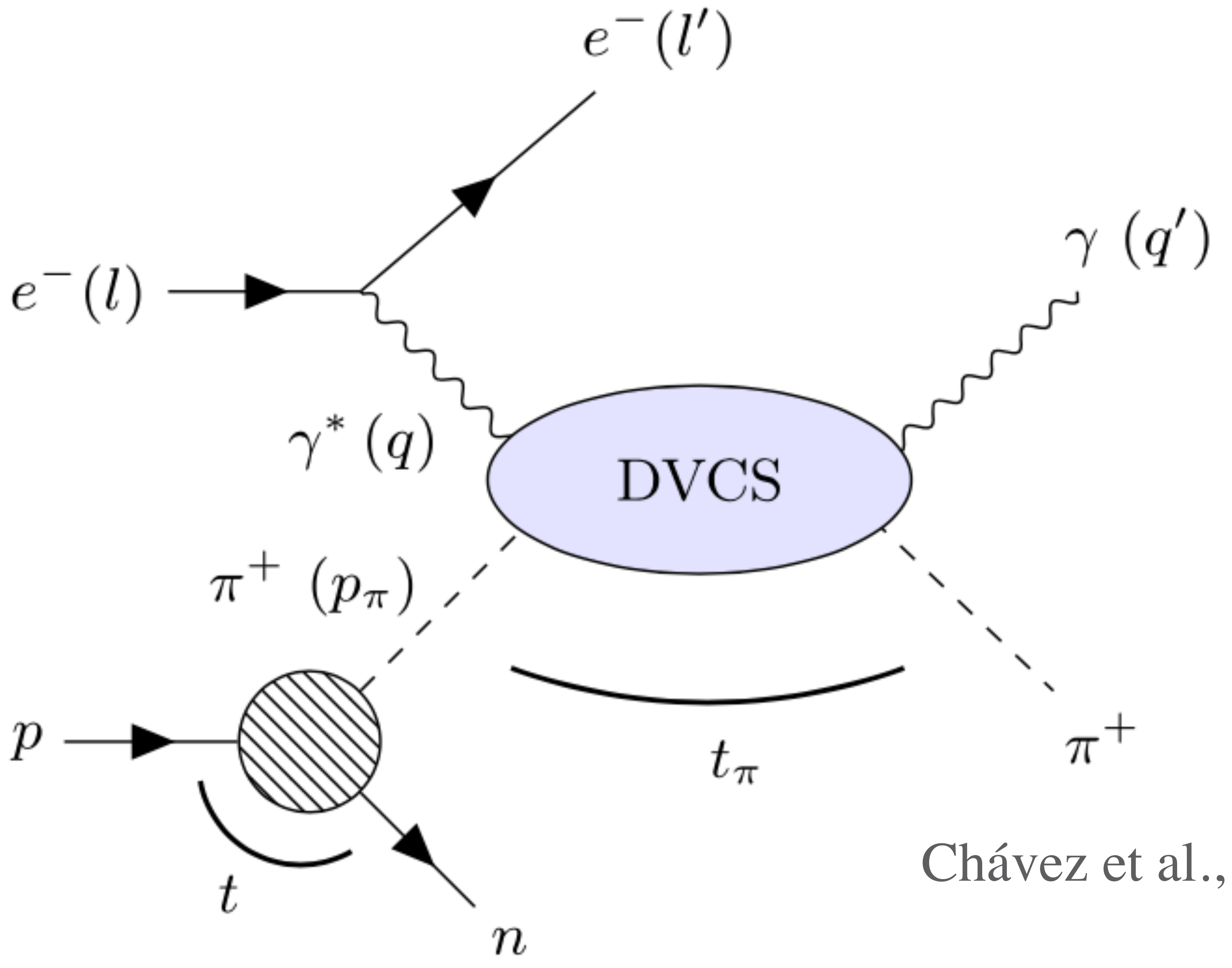
1D

Form Factor (pion-electron scattering)

Parton Distribution Functions (Drell-Yan process)

} Generalized Parton Distributions

3D



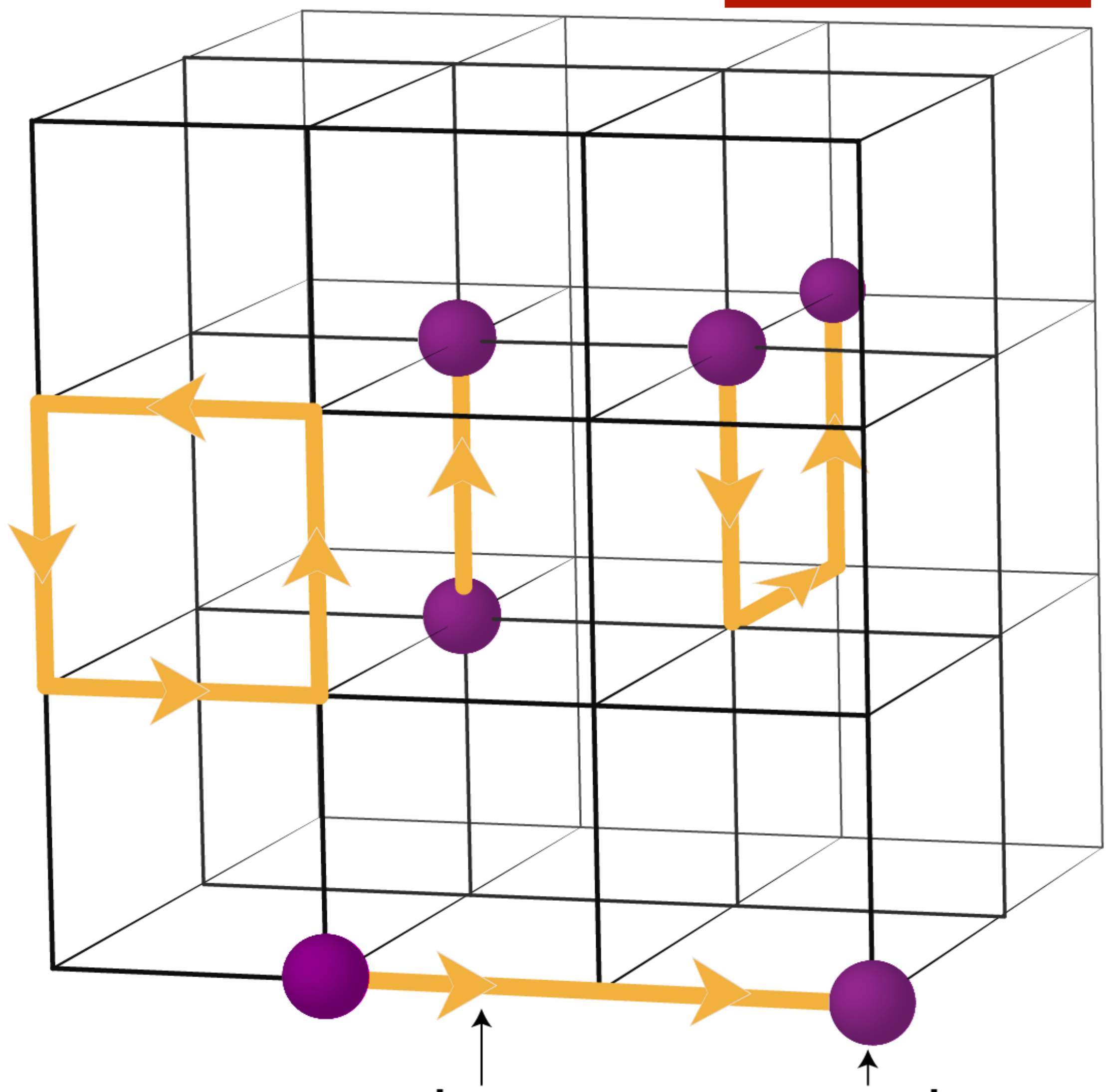
Chávez et al., PRL 128 (2022) 202501

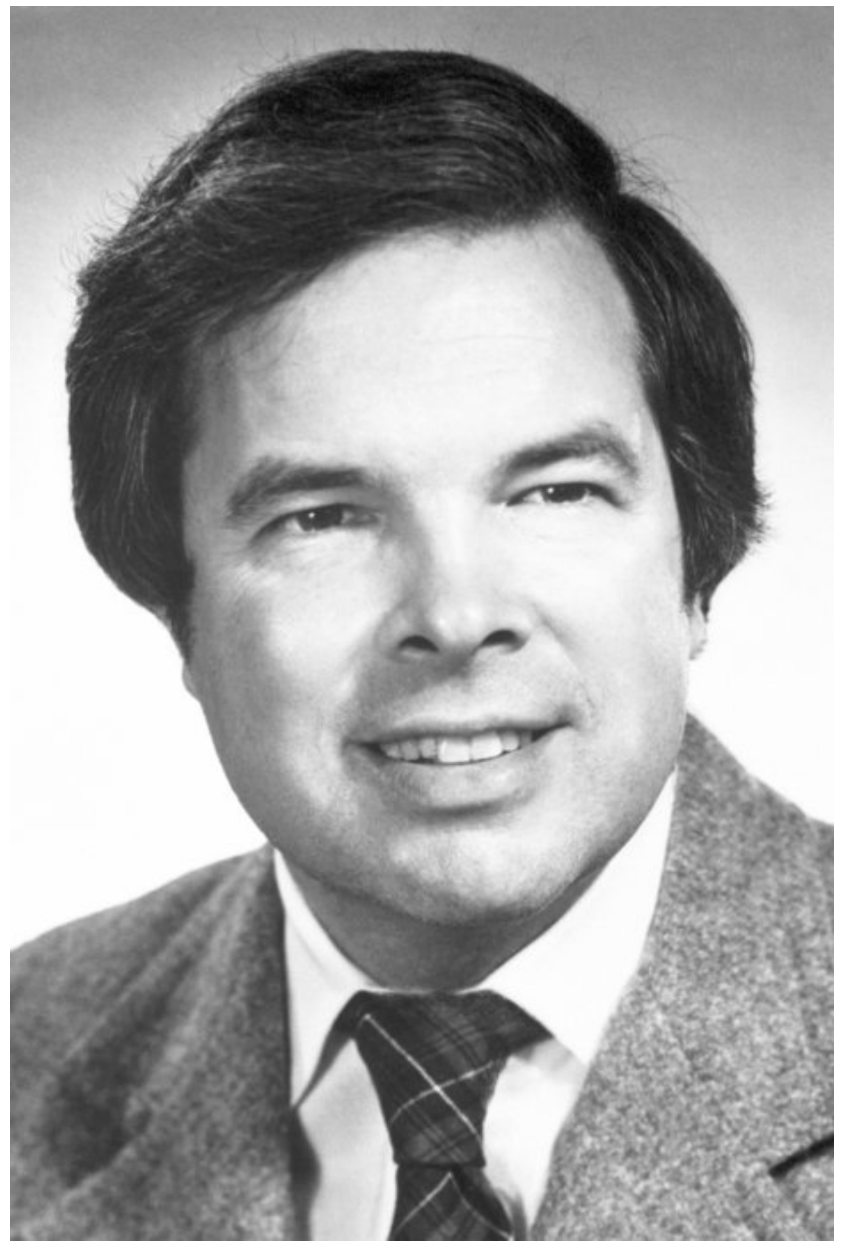
+ **Models**

!!! Lattice QCD: from first principle

Introduction: Lattice QCD

1974





Kenneth G. Wilson

From first principle

- Discretizing Spacetime -> $N_s^3 \times N_t$
- QCD Lagrangian $\mathscr{L}$ -> action $\mathcal{S}[\phi, \bar{\phi}, U]$
- Path Integral and Partition Function

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathscr{D}[U]\, \mathscr{D}[\phi]\, \mathscr{D}[\bar{\phi}]\, \mathcal{O}\, e^{-\mathcal{S}}$$

Computational cost

$$N_{\text{color}} \otimes N_{\text{flavor}} \otimes N_{\text{spin}} \otimes N_{\text{space}}^3 \otimes N_{\text{time}} \gtrsim 10^9 \quad \otimes \quad N_{\text{polarization}}, N_{\text{momentum}}, N_{\text{temperature}}, \cdots$$

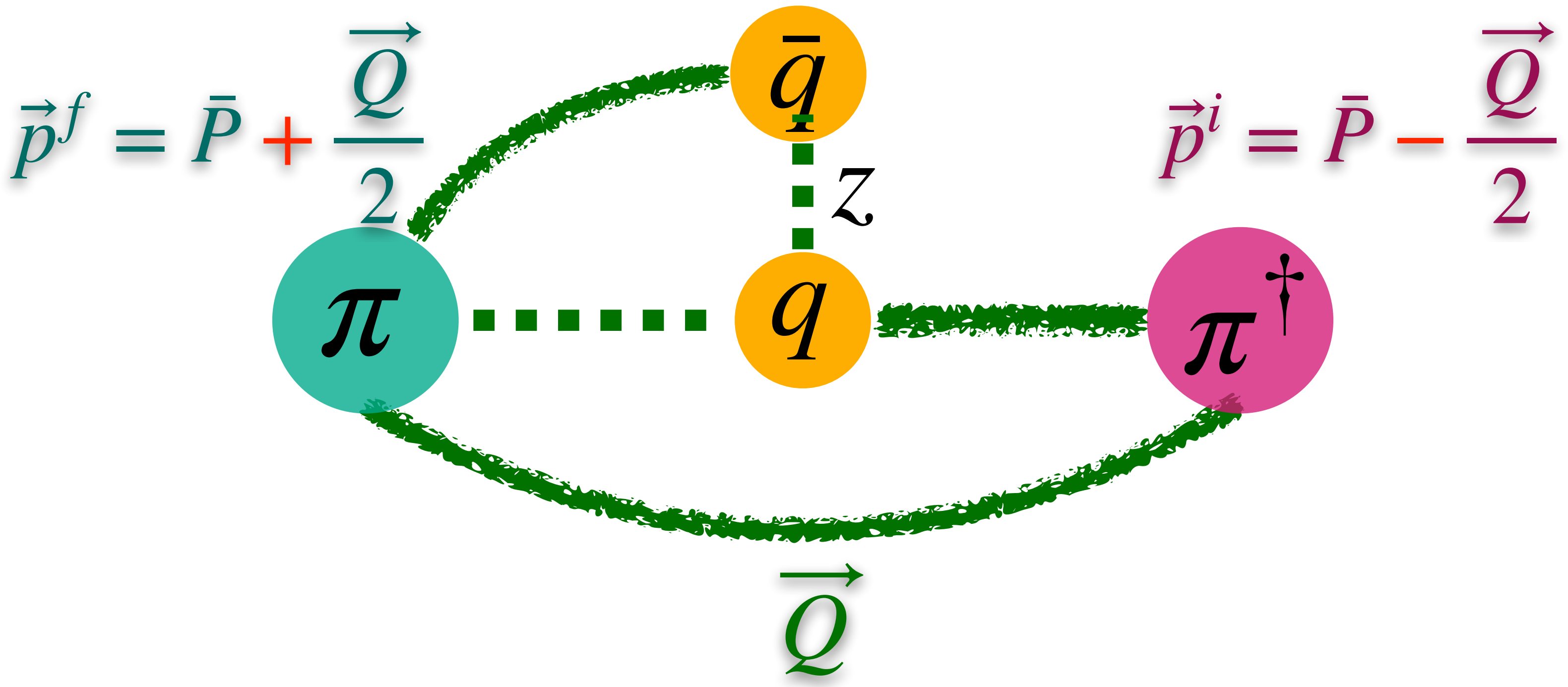
Several months ~ years

Frame-independent approach

$N_{\text{color}} \otimes N_{\text{flavor}} \otimes N_{\text{spin}} \otimes N^3_{\text{space}} \otimes N_{\text{time}} \gtrsim 10^9$

Lattice: each $\vec{p}^f$ requires a seperate calculation -> Goal: reduce the number of $\vec{p}^f$

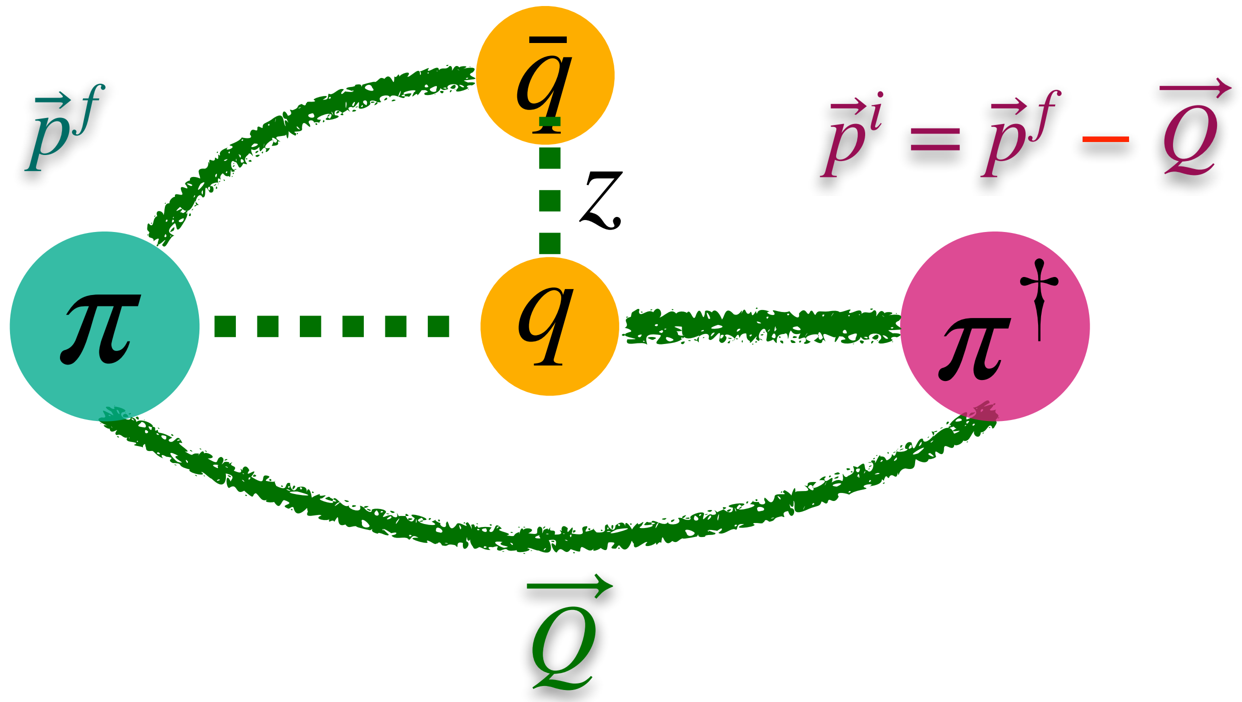
Traditional: Symmetric



- One $\vec{p}^f$ — only one $Q^2 = -t$ is useful
- Each Q^2 requires a seperate calculation

Newly proposed: Asymmetric

Bhattacharya, Constantinou et al., PRD 106 (2022)



- One $\vec{p}^f$ — several Q^2 are useful

Fix $\vec{p}^f$, vary Q^2 in one calc

Vary $\vec{p}^f$ in several calcs

Several Q^2

Computational cost

Frame-independent approach

- Lorentz invariant amplitudes A_i's

$$F^\mu(P,z,Q) = \frac{1}{\sqrt{E^i E^f}}(P^\mu A_1 + m^2 z^\mu A_2 + Q^\mu A_3), \qquad P^\mu = (p_f^\mu + p_i^\mu)/2, \quad Q^\mu = p_f^\mu - p_i^\mu .$$

$\longrightarrow$ $A_i(\mathrm{Breit}) \sim A_i(\mathrm{Non-breit})$

- Frame-independent GPD H

$$H(P,z,\Delta) = A_1 + \frac{z\cdot Q}{z\cdot P}A_3$$

$$A_3(-z\cdot Q) = -A_3(z\cdot Q) \xrightarrow{\xi=0} A_3(z\cdot Q=0)=0$$

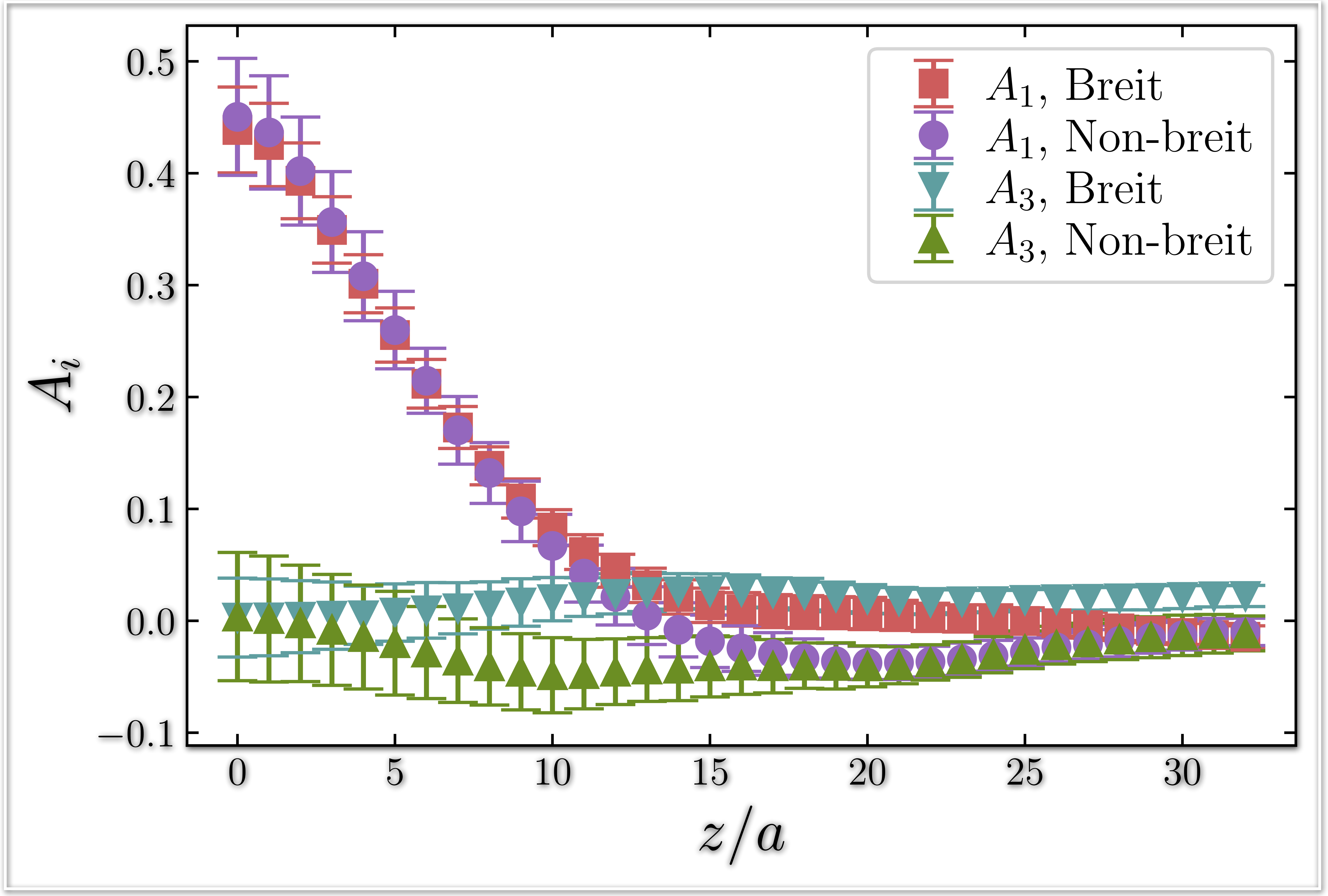
$$\boxed{H(P,z,\Delta) = A_1}$$

Comparison of A_i obtained from both frames

$$P_z = 0.968\ \text{GeV}^2,\ Q^2 = \begin{cases} 0.938\ \text{GeV}^2, & \text{Breit} \\ 0.952\ \text{GeV}^2, & \text{Non-breit} \end{cases}$$

$A_1(\text{Breit}) \sim A_1(\text{Non}-\text{breit})$

$A_3(z \cdot Q = 0) = 0$



- Work on the Non-breit frame

$$F^\mu(P, z, \Delta) = \frac{1}{\sqrt{E^i E^f}}(P^\mu A_1 + m^2 z^\mu A_2 + Q^\mu A_3).$$

$$H(P, z, \Delta) = A_1 + \frac{z \cdot Q}{z \cdot P} A_3.$$

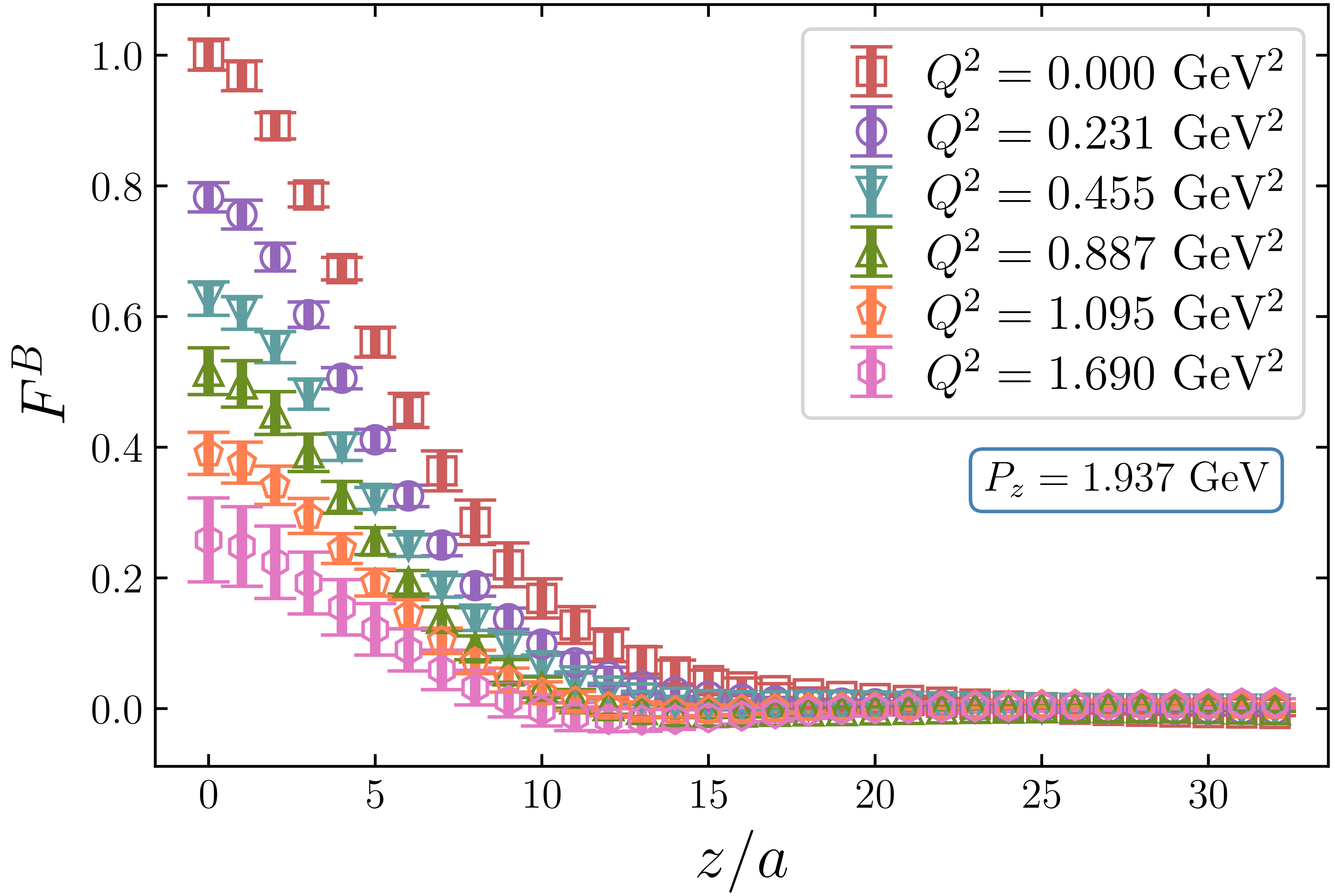
↓

$$F^t(P, z, \Delta) = \frac{E^i + E^f}{2\sqrt{E^i E^f}} A_1,$$

$$H(P, z, \Delta) = A_1 = \frac{2\sqrt{E^i E^f}}{E^i + E^f} F^t$$

Bare Matrix elements

- Large momenta: $P_z = 1.453, 1.937\ \mathrm{GeV}$,
- Varying different momentum transfer $Q^2 = -t$



→ **Quasi-GPDs**

Renormalization: Hybrid scheme

Ji et al., NPB 964 (2021) 115311

- RI/MOM, ratio schemes — short distance
- Hybrid scheme — long distance

Logarithmic   Handle the renormalon ambiguity

$$F^B(z,a) = Z(a)e^{-\delta m(a)|z|}e^{-\bar{m}_0|z|}F^R(z)\,.$$

Linear

Gao et al., PRL 128 (2022) 142003

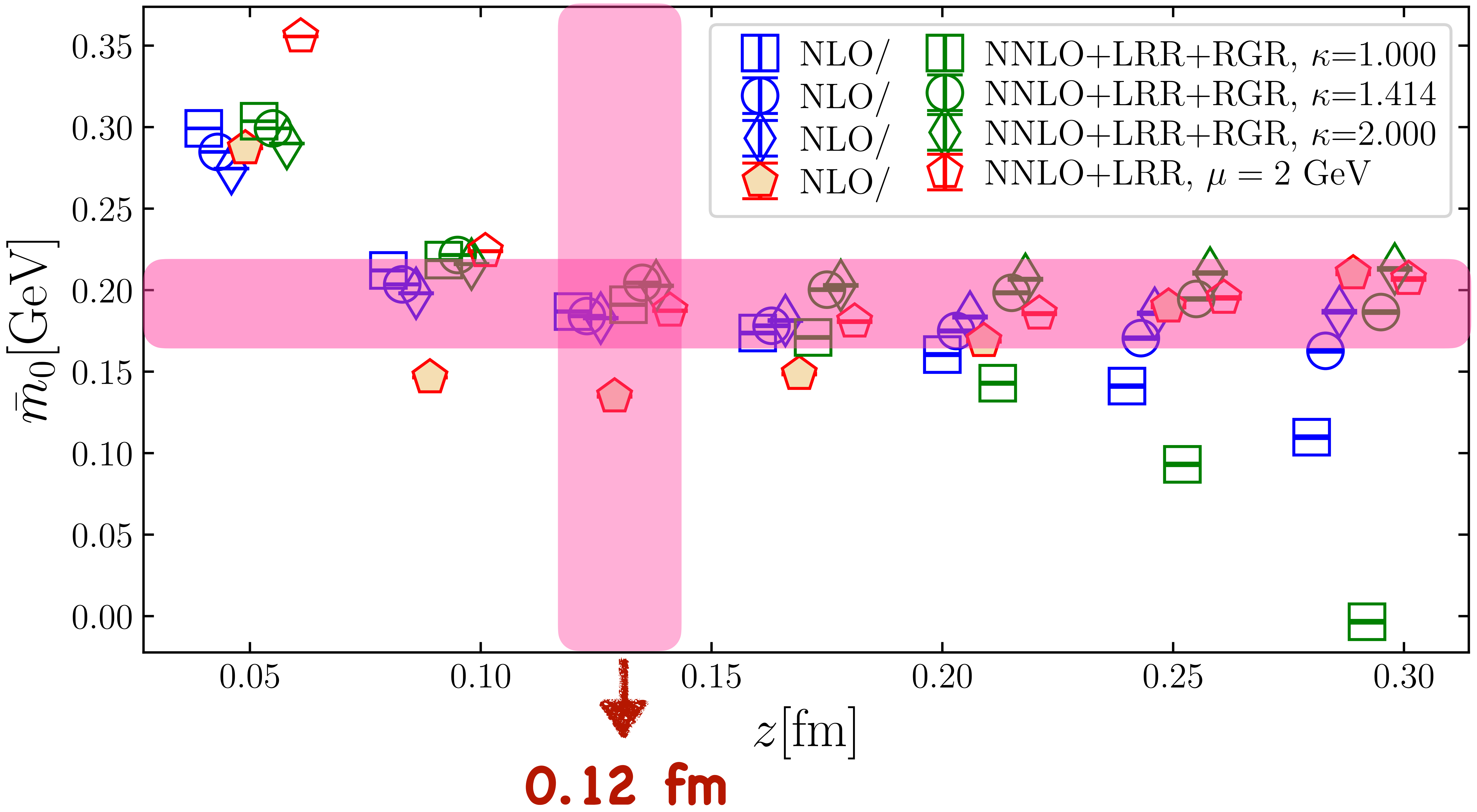
Hybrid scheme, F^R
$$\begin{cases} z \le z_S: & \dfrac{F^R(z,\vec{p},\vec{q})}{F^R(z,0,0)} = \dfrac{F^B(z,\vec{p},\vec{q})}{F^B(z,0,0)}, \quad \text{Ratio scheme} \\ z \ge z_S: & \dfrac{F^R(z,\vec{p},\vec{q})}{F^R(z_S,0,0)} = e^{(\delta m+\bar{m}_0)|z-z_S|}\dfrac{F^B(z,\vec{p},\vec{q})}{F^B(z_S,0,0)}\,. \end{cases}$$

Hybrid-scheme

- $a\delta m(a) = 0.1508(12)$ for $a = 0.04$ fm lattice.
- $\bar{m}_0$: F^B at $P_z = 0, \vec{q} = \vec{0}$; $e^{(\delta m + \bar{m}_0)\Delta z} \dfrac{F^B(z+\Delta z)}{F^B(z)} = \dfrac{C_0(\mu_0^2 (z+\Delta z)^2)}{C_0(\mu_0^2 z^2)}$.

C_0: **NLO/NNLO, LRR, RGR**

$\mu_0 = 2\kappa e^{-\gamma_E}/z \to \mu = 2$ GeV



Renormalization

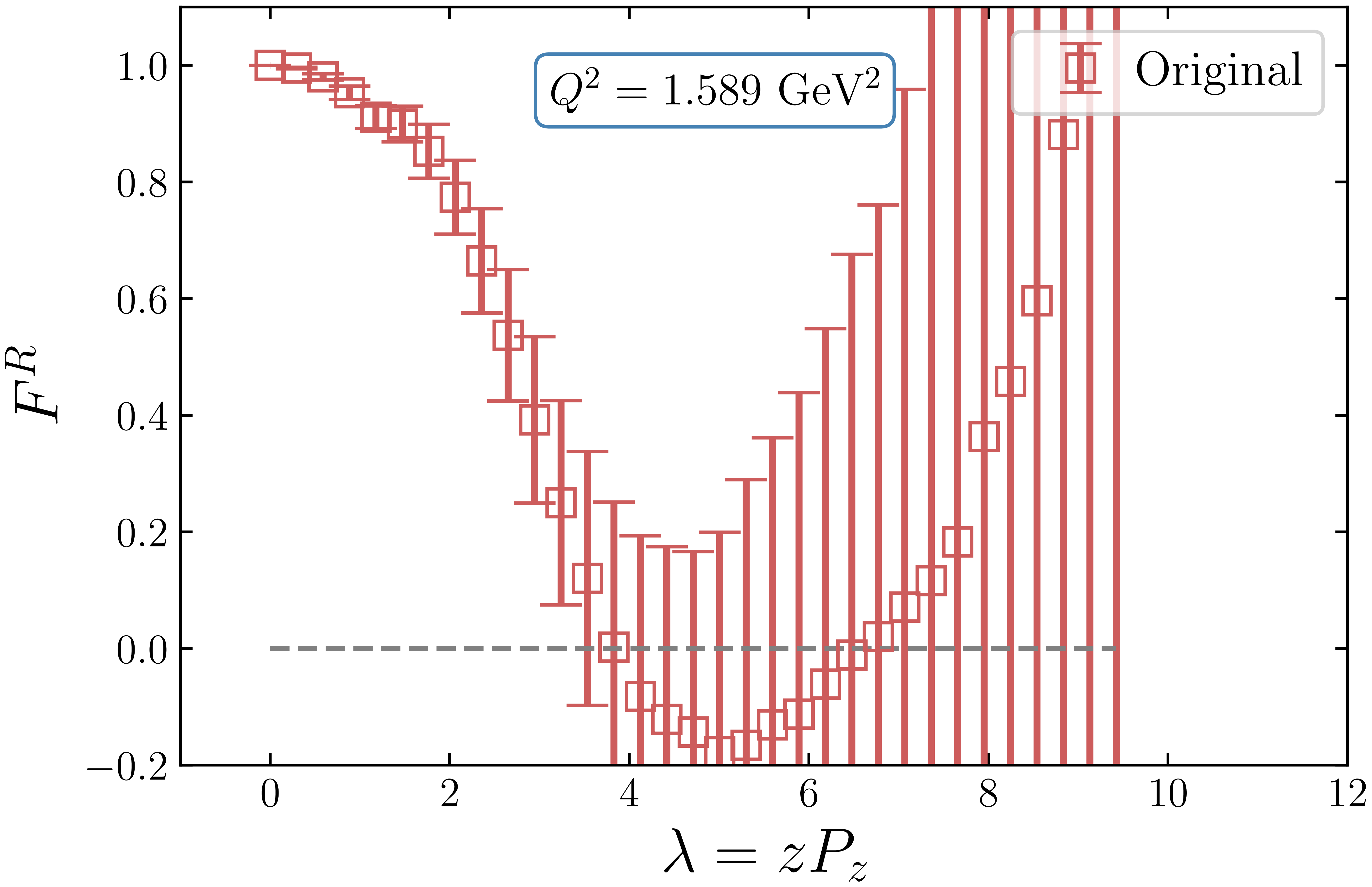
Lattice artifacts

↓

Unphysical oscillations
in quasi-GPDs

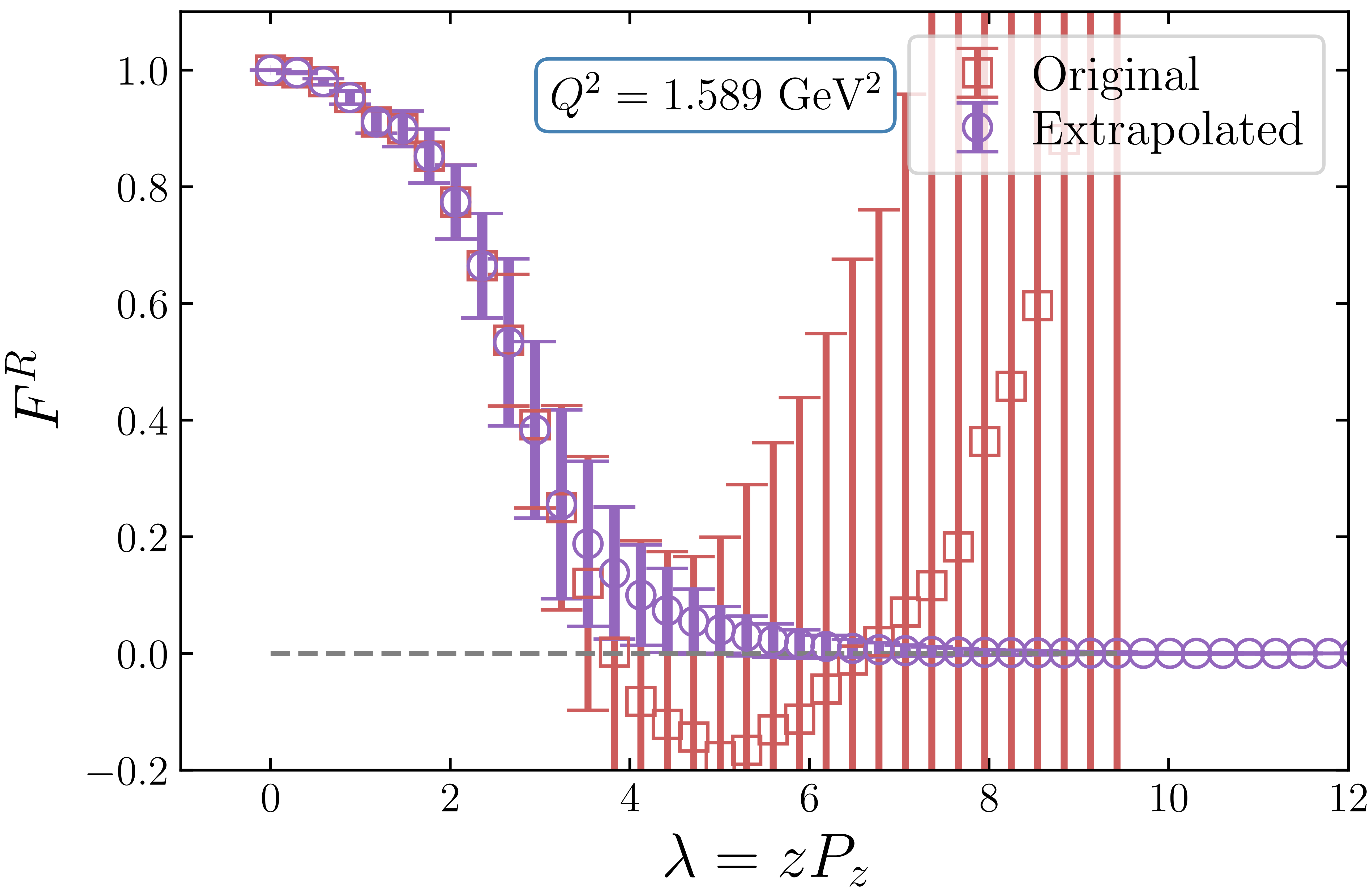
↓

Extrapolation



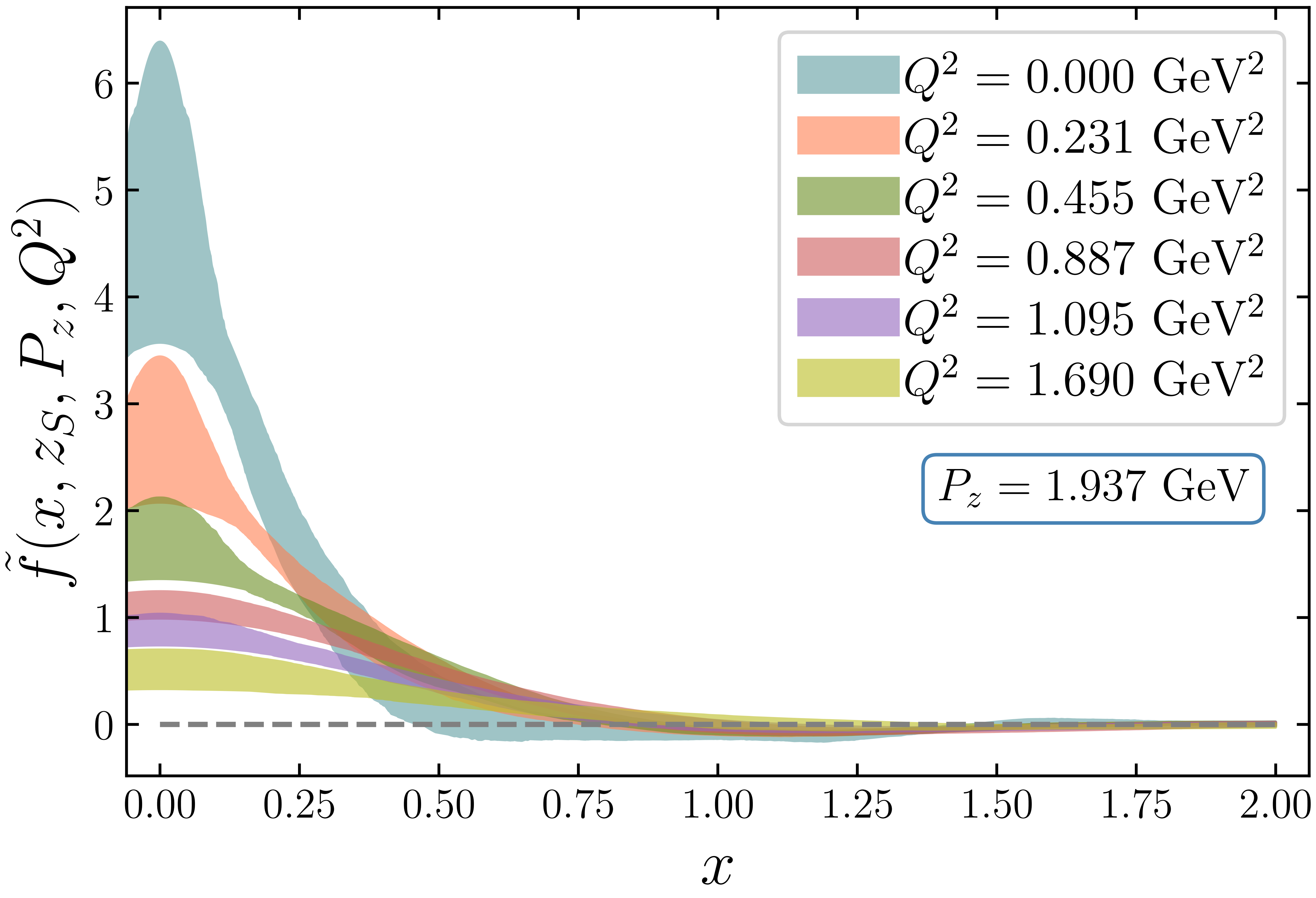
Extrapolation

$$F^R = A\frac{e^{-mz}}{\lambda^d}$$



Fourier transform -> Quasi-GPDs

$$\tilde{f}(x, z_S, P_z, Q^2 = -t) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} F^R(\lambda, z_S, P_z, Q^2 = -t), \quad \lambda = zP_z$$



$Q^2 = -t$ increases

↓

Smaller, decay slower

Perturbative matching: LaMET

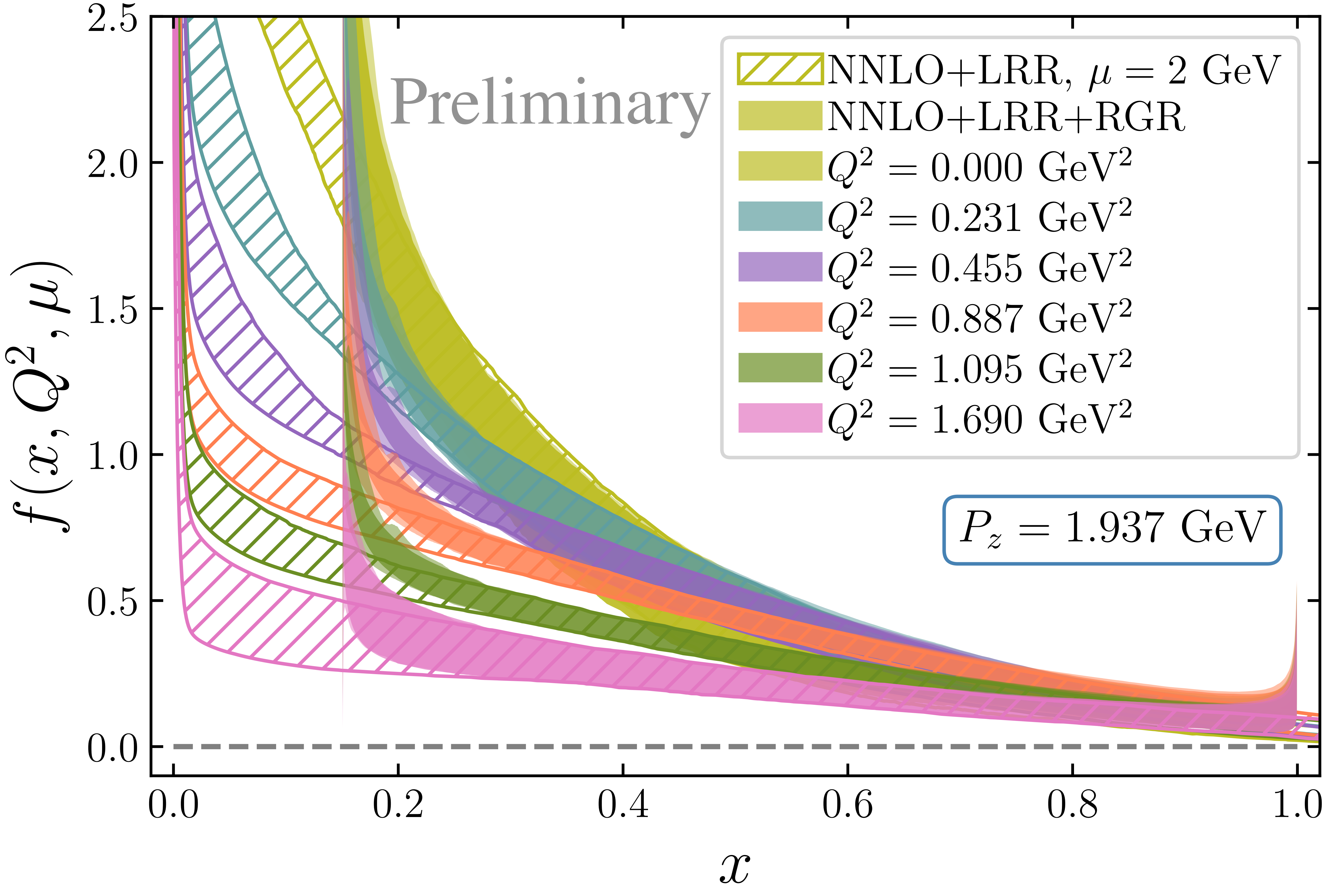
Ji, PRL 110 (2013) 262002

$$f(x,\xi=0,Q^2,\mu)=\int_{-\infty}^{\infty}\frac{\mathrm{d}y}{|y|}C^{-1}\left(\frac{x}{y},\frac{\mu}{yp_z},|y|\lambda_S\right)\tilde{f}(y,\xi=0,Q^2,\mu)+\mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xp_z)^2},\frac{\Lambda^2_{\mathrm{QCD}}}{[(1-x)p_z]^2}\right)$$

$$\approx\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{|k|}\int_{-\infty}^{\infty}\frac{\mathrm{d}y}{|y|}C^{-1}_{\mathrm{evo}}\left(\frac{x}{k},\frac{\mu}{\mu_0}\right)C^{-1}_{\mathrm{mat}}\left(\frac{k}{y},\frac{\mu_0}{yp_z},|y|\lambda_S\right)\tilde{f}(y,\xi=0,Q^2,\mu_0)\,.$$

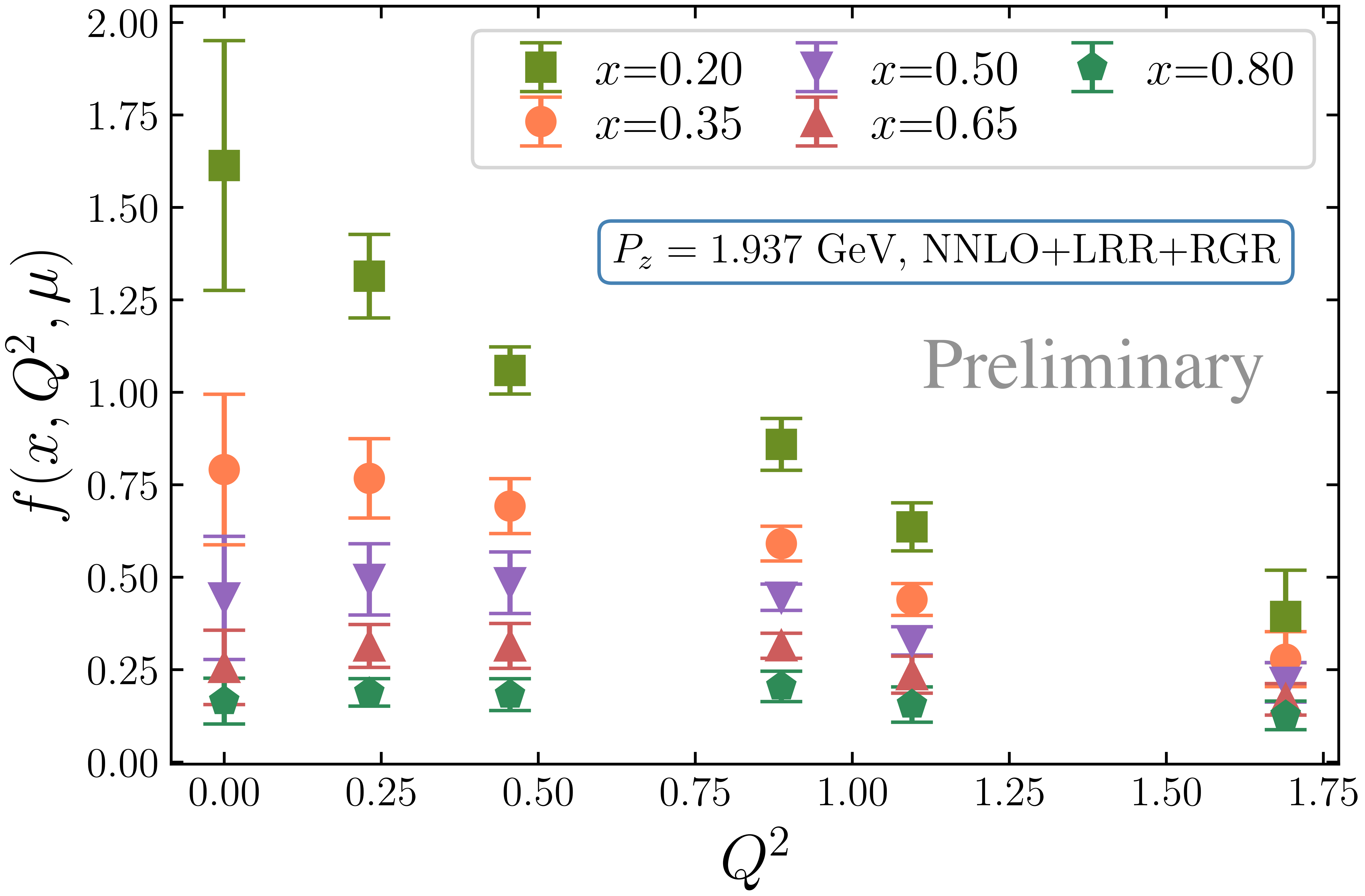
$$\left[f(x,\xi=0,Q^2,\mu)\right]_x\quad\approx\quad\sum_{k,y}\left[C^{-1}_{\mathrm{evo}}\right]_{xk}\left[C^{-1}_{\mathrm{mat}}\right]_{ky}\left[\tilde{f}(y,\xi=0,Q^2,\mu)\right]_y\,.$$

Valence GPDs: $Q^2(-t)$-dependence



- Fixed scale & RGR:
  - $x > 0.3$: identity
  - $? < x < 0.3$: correction
- Scale variation (lighter filled bands):
  - $? < x < 0.3$

Valence GPDs: x-dependence

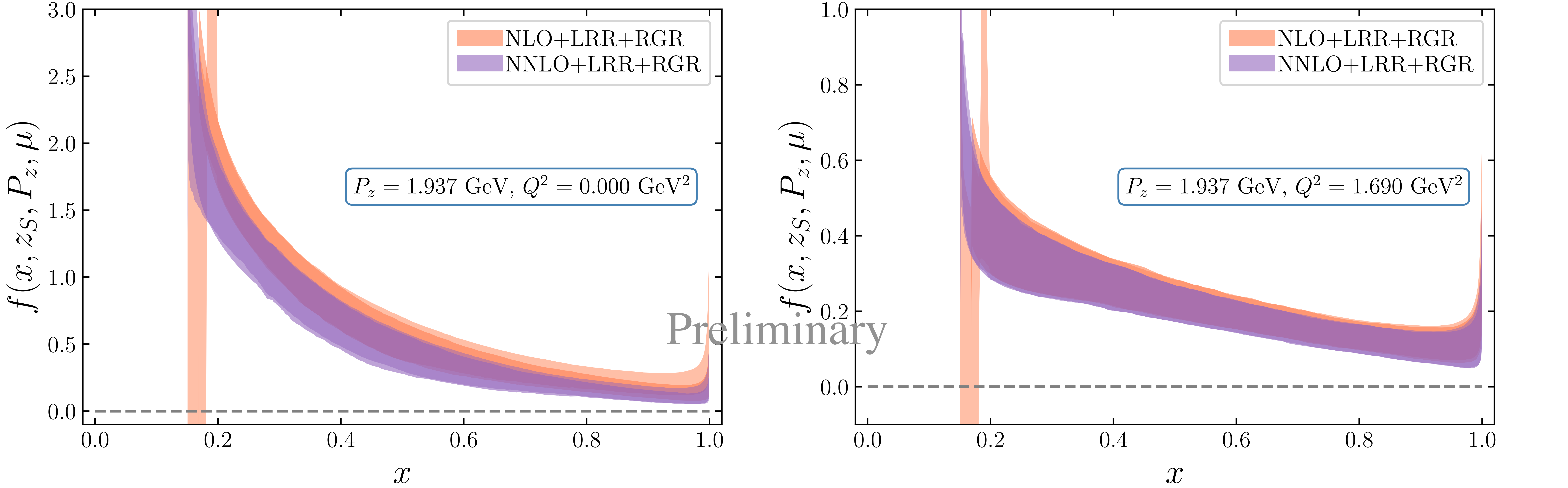


x increases

↓

Declining trend -> stable plateau

Valence GPDs: Order-dependence



$Q^2 = -t$ increases → Order-dependence reduces

NNLO: better perturbative convergence

Summary

- Confirm the frame-independence of the amplitudes A_i's
- Get the quasi-GPDs
  - hybrid-scheme renormalization
  - NNLO + LRR + RGR
- Get the pion valance GPDs with LaMET
  - fixed scale $\mu = 2$ GeV & DGLAP evolution
  - Q^2-, x-, and Order-dependence

Thanks for your attention!

Backup

Amplitude

Non-breit

$$F^t_{\mathrm{M}}(z,p,\Delta) = \frac{1}{\sqrt{E^iE^f}}\left(p^t_{\mathrm{M}}A_1 + \Delta^t_{\mathrm{M}}A_3\right),$$

$$F^x_{\mathrm{M}}(z,p,\Delta) = \frac{1}{\sqrt{E^iE^f}}\left(p^x_{\mathrm{M}}A_1 + \Delta^x_{\mathrm{M}}A_3\right),$$

$$F^y_{\mathrm{M}}(z,p,\Delta) = \frac{1}{\sqrt{E^iE^f}}\left(p^y_{\mathrm{M}}A_1 + \Delta^y_{\mathrm{M}}A_3\right),$$

$$F^z_{\mathrm{M}}(z,p,\Delta) = \frac{1}{\sqrt{E^iE^f}}\left(p^z_{\mathrm{M}}A_1 + m^2 z^z_{\mathrm{M}}A_2 + \Delta^z_{\mathrm{M}}A_3\right).$$

Breit

$$F^t_{\mathrm{M}}(z,p,\Delta) = \frac{E^i + E^f}{2\sqrt{E^iE^f}}A_1 = A_1,$$

$$F^x_{\mathrm{M}}(z,p,\Delta) = \frac{p^f_x - p^i_x}{\sqrt{E^iE^f}}A_3,$$

$$F^y_{\mathrm{M}}(z,p,\Delta) = \frac{p^f_y - p^i_y}{\sqrt{E^iE^f}}A_3,$$

$$F^z_{\mathrm{M}}(z,p,\Delta) = \frac{p^i_z + p^f_z}{2\sqrt{E^iE^f}}A_1 + \frac{m^2 z_z}{\sqrt{E^iE^f}}A_2,$$

Lattice Setup

- $N_s^3 \times N_t = 64^3 \times 64, \qquad a = 0.04$ fm
- HISQ action + Wilson-Clover action $\Rightarrow m_\pi^{\text{val}} = 0.3$ GeV
- Using boost smearing to enhance the signal
- Momentum transfer Q^2: 0 ~ 1.7 GeV^2

<table>
<tr><th>Frame</th><th>n_p^z, p^z(GeV)</th><th>Γ</th><th>n_q</th><th>$N_{\text{cfg}} * N_{\text{src}}$</th></tr>
<tr><td>Breit</td><td>(0, 1, 2), 1.083</td><td>$\gamma_x, \gamma_y, \gamma_t$</td><td>(0, -2, 0)</td><td>115*32=3680</td></tr>
<tr><td rowspan="5">Non-breit</td><td>(0, 0, 0), 0</td><td rowspan="5">$\gamma_x, \gamma_y, \gamma_t$</td><td rowspan="5">(0, 0, 0), (0, -1, 0)
(-1, -1, 0), (0, -2, 0)
(-1, -2, 0), (-2, -2, 0)</td><td rowspan="4">314*96=30144</td></tr>
<tr><td>(0, 0, 1), 0.484</td></tr>
<tr><td>(0, 0, 2), 0.968</td></tr>
<tr><td>(0, 0, 3), 1.453</td></tr>
<tr><td>(0, 0, 4), 1.937</td><td>564*96=54144</td></tr>
</table>

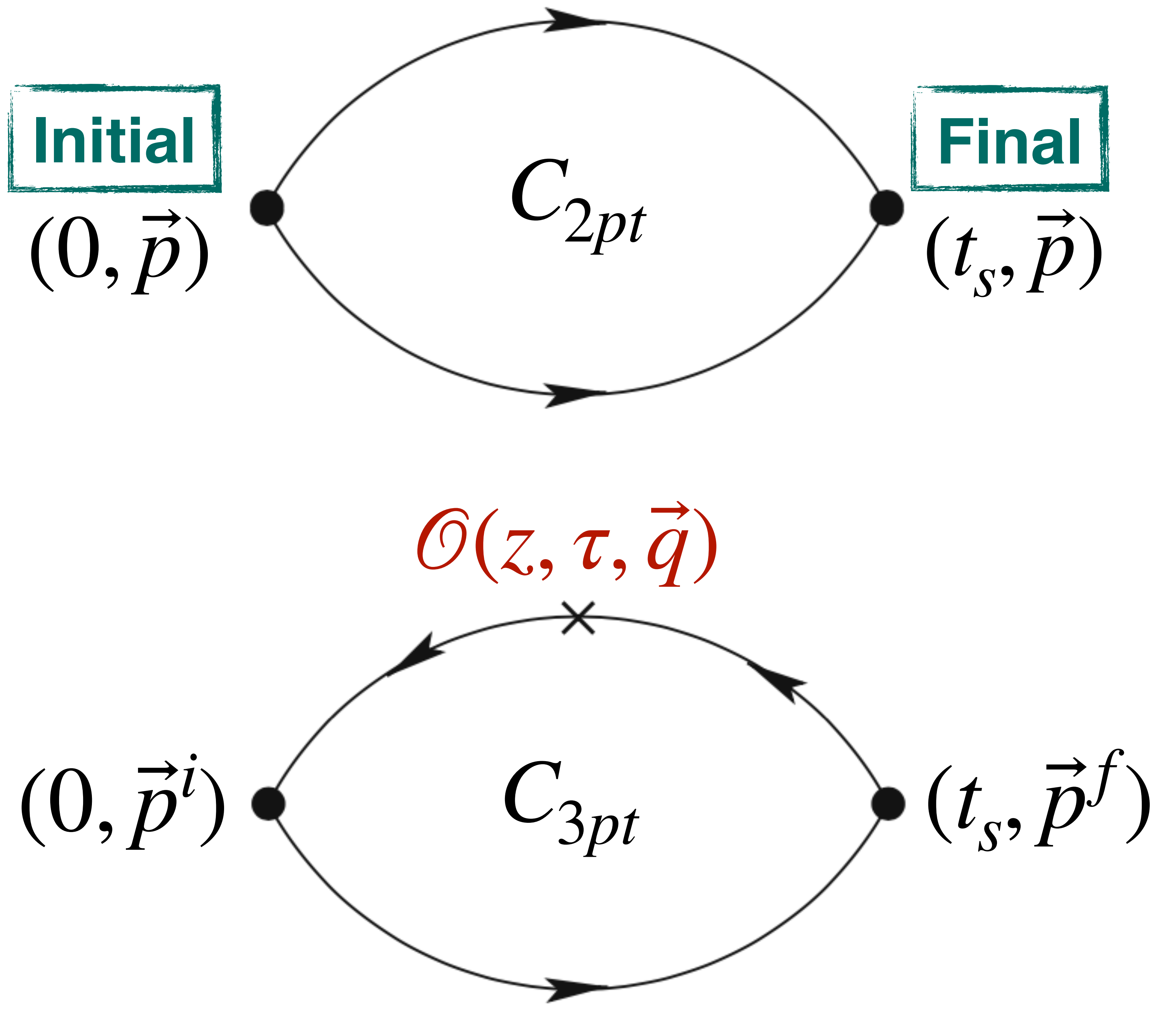
How to get the bare matrix elements from lattice

$$C_{2pt}(t,\vec{p}) = \left\langle H(t_s,\vec{p})H^{\dagger}(0,\vec{p})\right\rangle$$

Insert $\mathcal{O}(\tau,\vec{q})$ ↓ $\vec{p}^f = \vec{p}^i + \vec{q}$

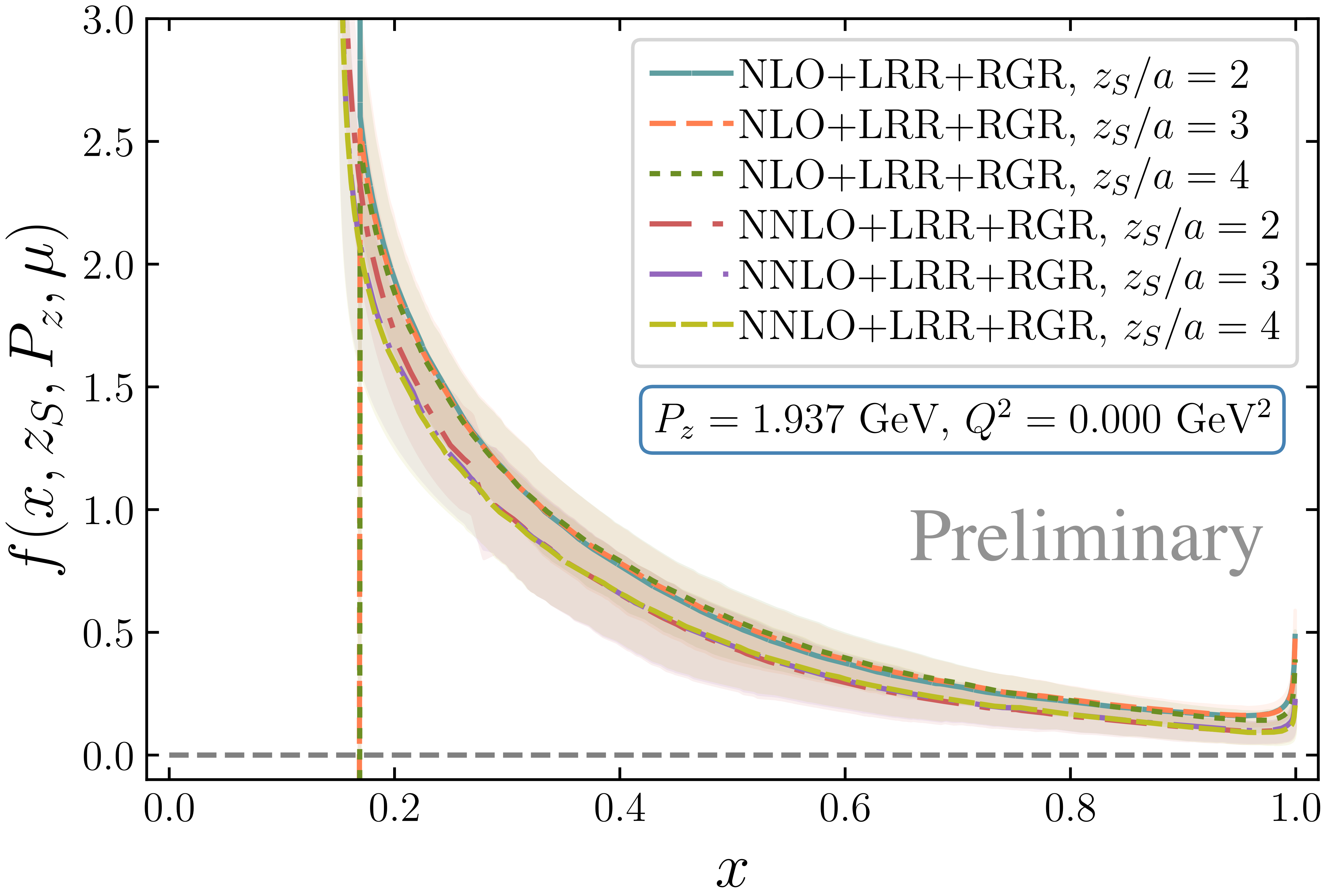
$$C_{3pt}(z;\tau,t_s;\vec{p}^i,\vec{p}^f) = \left\langle H(t_s,\vec{p}^f)\hat{\mathcal{O}}_{\Gamma}(z,\tau,\vec{q})H^{\dagger}(0,\vec{p}^i)\right\rangle$$

$\Gamma : \hat{1}, \gamma^{\mu}, \sigma^{\mu\nu}$



→ $F^B = \left\langle E_0,\vec{p}^f \,|\, \hat{\mathcal{O}}_{\gamma^{\mu}}(z,\tau,\vec{q}) \,|\, E_0,\vec{p}^i\right\rangle$ from ~ C_{3pt} / C_{2pt}

Valence GPD: z_S-dependence



Valence GPD: P_z-dependence